Birth death and coalescent model cheat sheet

1 Birth-death

1.1 Assumptions

The birth-death can be thought of as a forwards-in-time process. It assumes that population size grows exponentially and that the trajectory is **stochastic**. As such, it is particularly well suited for high sampling proportions and at the early stages of an outbreak, where stochasticity plays a more important role. The expected trajectory of infected individuals, I(t), is I(t)=e^{rt}, where r is the epidemic growth rate and t is the time since the origin of the process.

1.2 Compound parameters

The birth-death usually has three parameters that are identifiable:

- r=λ - δ
- $\lambda\delta p$
- *origin*

  Where

- r: epidemic growth rate, r.
- λ: transmission (a.k.a birth) rate.
- δ: recovery (a.k.a death) rate. It is defined as the sum of the sampling rate ψ and, μ, the 'removal' rate. It is also the inverse of the duration of infection (i.e. $duration\ of\ infection = \frac{1}{\delta}$)
- p: sampling proportion.

  In BEAST2 the parameterisation usually involves the reproductive number, R_e, δ, and p. These three parameters relate to the compound parameters above by:

- $R_e = \frac{\lambda}{\delta}$
- $\delta = \psi + \mu$
- $p = \frac{\psi}{\psi+\mu}$

2 Coalescent exponential

2.1 Assumptions

In the coalescent, generally, the population trajectory is deterministic. As a result, this model assumes that the data represent a small proportion of the population (i.e. sampling intensity is low). It can be thought of as a backwards-in-time process that is conditioned on the samples. That is, it does not explicitly use sampling time information. As such, if sampling effort is not well understood, it may be a more sensible model than the birth-death above.

2.2 Compound parameters

This model has two compound parameters:

- $r = \lambda$ - δ.
- $\Phi = \frac{I(0)}{2\lambda}$

The new terms here are Φ, which is a 'scaled population size' and *I(0)*, which is the infected population size at present.

3 Identifiability

Both models have two compound parameters that we can identify. However, the number of individual parameters is three; λ, δ, p in the birth-death, and λ, δ, *I(0)*. For this reason, we **always** need some informative prior information on any individual parameter to work out the others. For example, R_e requires individual estimates of λ and δ.