

HW2: Descriptive Statistics—Dispersion

1. You are given the following test scores for a Math quiz.

Score (X)	Frequency (f)	Xf	X ²	X ² f
20	2	40	400	800
19	1	19	361	361
18	4	72	324	1296
17	6	102	289	1734
16	7	112	256	1792
15	10	150	225	2250
14	15	210	196	2940
13	9	117	169	1521
12	6	72	144	864
11	2	22	121	242
10	1	10	100	100
9	2	18	81	162
8	1	8	64	64
7	0	0	49	0
	$\sum f = 66$	$\sum Xf = 952$		$\sum X^2f = 14126$

- (1) What is the range for the above data? (1 pt)
 (2) Calculate the variance and the standard deviation for the above distribution or indicate why they cannot be calculated. (2 pts)

Ans: (1) Range = Highest X – Lowest X = 20 – 8 = 12

(The frequency of score 7 = 0, so we do not take 7 as the lowest X to compute the range)

$$(2) \text{ Variance} = S^2 = \frac{\sum x^2}{N} = \frac{\sum X^2 - \frac{(\sum X)^2}{N}}{N} = \frac{\sum X^2 f - \frac{(\sum X f)^2}{\sum f}}{\sum f} = \frac{14126 - \frac{952^2}{66}}{66} = 5.97$$

$$\text{Standard deviation} = S = \sqrt{5.97} \approx 2.44$$

2. The following distribution of “happiness scores” was obtained from a sample of married women.

Happiness Score	Number of Wives (f)	Midpoint (X)	Xf	X ²	X ² f
0-2	4	1	4	1	4
3-5	8	4	32	16	128
6-8	10	7	70	49	490
9-11	12	10	120	100	1200
12-14	25	13	325	169	4225
15-17	32	16	512	256	8192
18-20	6	19	114	361	2166
21-23	2	22	44	484	968
24 and over	0				
	$\sum f = 99$		$\sum Xf = 1221$		$\sum X^2f = 17373$

- (1) What is the range for the above distribution? (1 pt)
 (2) Calculate the standard deviation and the variance for the above distribution or indicate why they cannot be calculated. (2 pts)

Ans:

(1) For grouped data, because we do not know the individual measurements, the range is taken to be the difference between the upper limit of the last interval and the lower limit of the first interval. In this question, we do not take “24 and over” into account, because there is no case in this category, so the highest measurement here is 23. → Range = 23 - 0 = 23

But some researches argue that this may overestimate the range, so they suggest using the midpoints of the upper- and lower-limit category to calculate range. → Range = 22 - 1 = 21

$$(2) \text{ Variance} = S^2 = \frac{\sum X^2f - \frac{(\sum Xf)^2}{\sum f}}{\sum f} = \frac{17373 - \frac{1221^2}{99}}{99} = 23.37$$

$$\text{Standard deviation} = S = \sqrt{23.37} \approx 4.83$$