
Abstract Data Type

**IT5003: Data Structures and Algorithms
(AY2019/20 Semester 1)**

Lecture Overview

- Abstraction in Programs
- Abstraction Data Type
 - Definition
 - Benefits
- Abstraction Data Type Examples
 - Floating Point Number
 - Complex Number

Abstraction

- The process of isolating implementation details and extracting only **essential property** from an entity
- Program = data + algorithms
- Hence, abstractions in a program:
 - **Data abstraction**
 - What operations are needed by the data
 - **Functional abstraction**
 - What is the purpose of a function (algorithm)

Abstract Data Type (ADT)

- ❑ End result of data abstraction
- ❑ A collection of **data** together with a set of **operations** on that data
- ❑ **ADT = Data + Operations**
- ADT is a language independent concept
 - ❑ Different language supports ADT in different ways
 - ❑ In Python (OOP Language) the class construct is the best match
- Important Properties of ADT:
 - ❑ **Specification:**
 - The supported operations of the ADT
 - ❑ **Implementation:**
 - Data structures and actual coding to meet the specification

ADT: Specification and Implementation

- Specification and implementation are disjoint:
 - ❑ **One** specification
 - ❑ **One or more** implementations
 - Using different data structure
 - Using different algorithm
- Users of ADT:
 - ❑ Aware of the specification **only**
 - Usage only base on the specified operations
 - ❑ Do not care / Need not know about the actual implementation
 - i.e. Different implementation do **not** affect the user

Abstraction as Wall : Illustration

```
result = factorial(5)
print(result)
```

User of `factorial()`

- Users only need to know
 - ❑ `factorial()`'s purpose
 - ❑ Its parameters and return value
- Users **do not** need to know
 - ❑ `factorial()` internal coding
- Different `factorial()` coding
 - ❑ Does not affect its users!
- We can build a wall to shield `factorial()` from `main()`! →

```
def factorial( n ):
    if n == 0:
        return 1

    return n * factorial(n-1)
```

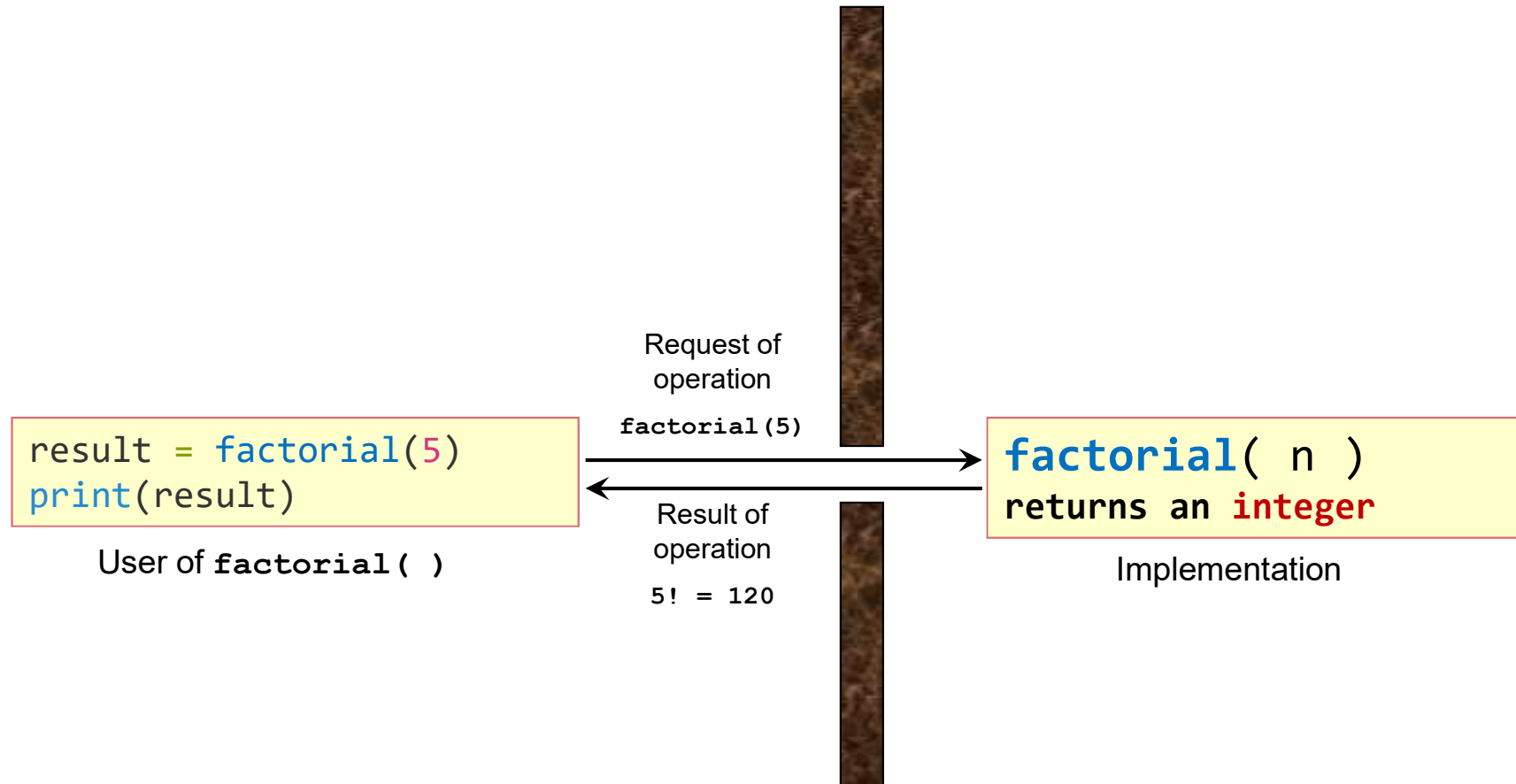
Implementation 1

```
def factorial( n ):
    result = 1

    for i in range(1, n+1):
        result *= i
    return result
```

Implementation 2

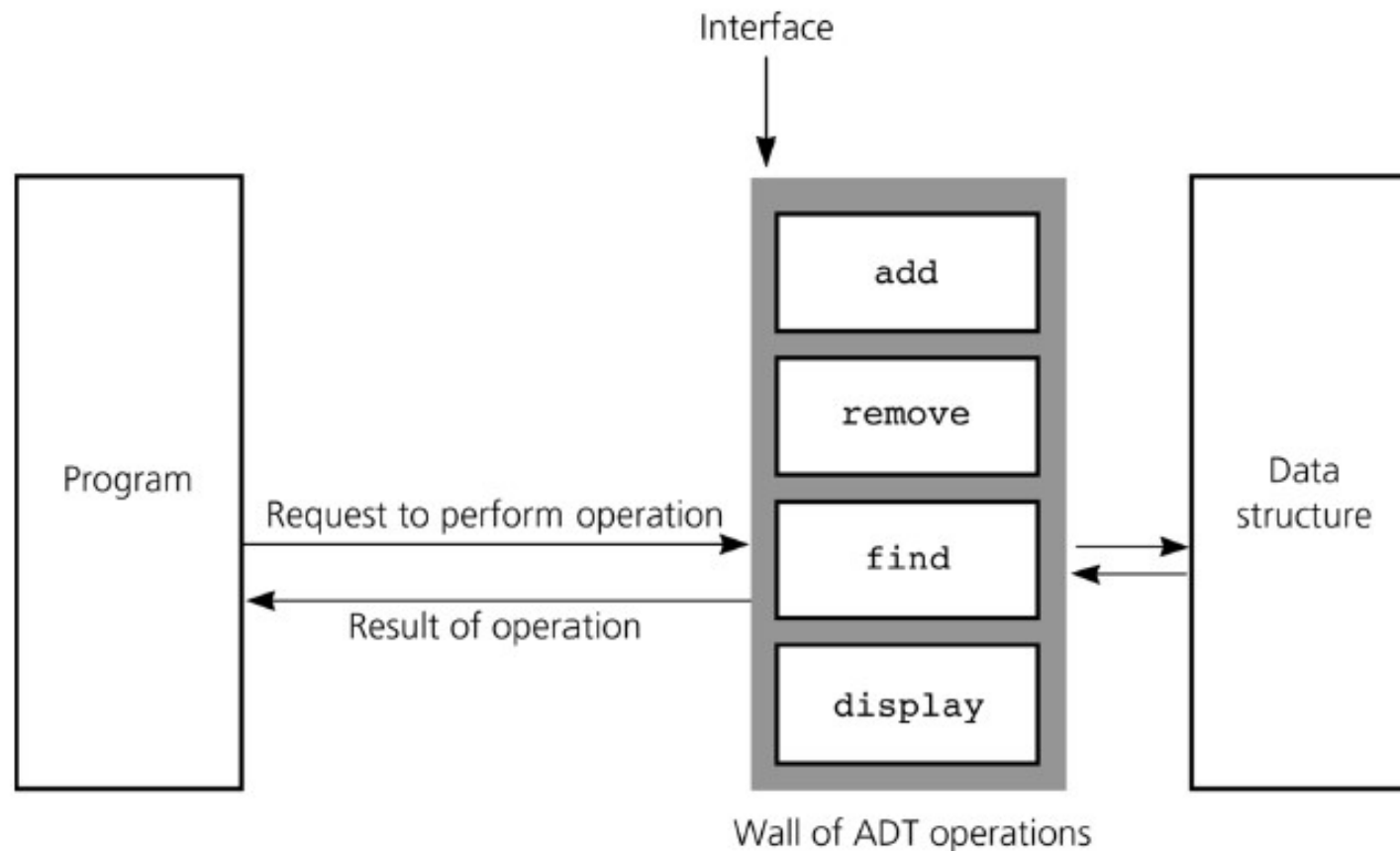
Specification: Slit in the Wall



- User only depends on specification
 - Function name, parameters and return type

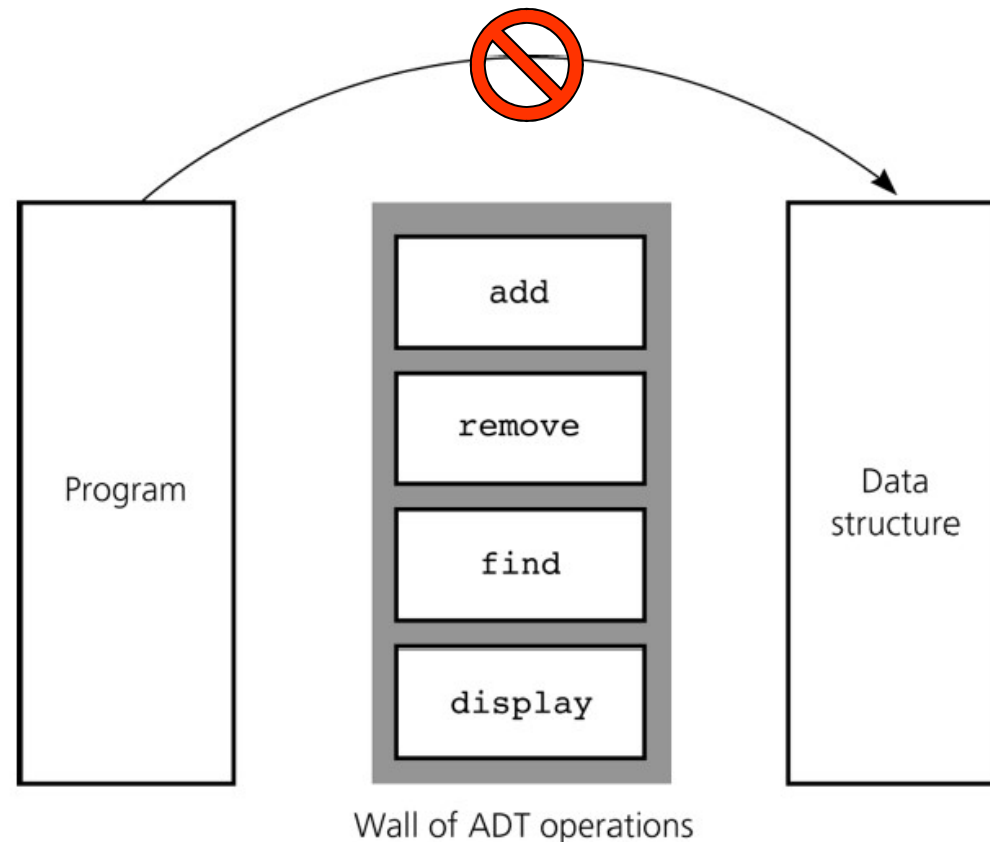
A Wall of ADT operations

- ADT operations provides:
 - ❑ Interface to data structure
 - ❑ Secure access



Abstraction Violation

- User programs **should not**:
 - ❑ Use the underlying data structure directly
 - ❑ Depend on implementation details



Abstract Data Types: **When to use?**

- When you need to operate on data that are not directly supported by the language
 - E.g. Complex Number, Module Information, Bank Account etc (language dependent!)
- **Simple Steps:**
 1. **Design** an abstract data type
 2. Carefully **specify all operations** needed
 - Ignore/delay any implementation related issues
 3. **Implement** them

Abstract Data Types: **A**dvantages

- Hide the unnecessary details by **building walls around the data and operations**
 - So that changes in either will not affect other program components that use them
- Functionalities are less likely to change
- Localise rather than globalise changes
- Help manage software complexity
- Easier software maintenance

Abstract Data Types: **E**xamples

1. **Primitive Types** as ADTs

- A simple example

2. **Complex Number** ADT

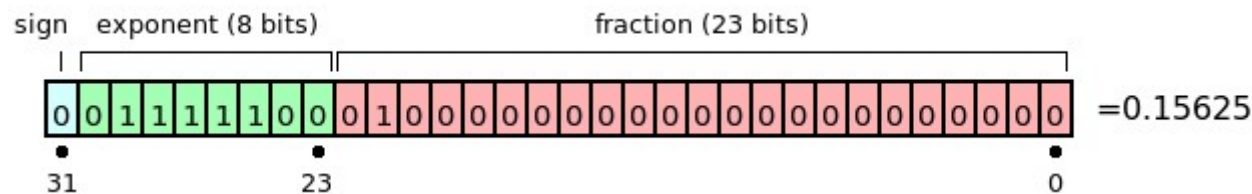
- A detailed example to highlight the advantages of ADT

■ All data structures covered later in the course are presented as ADTs

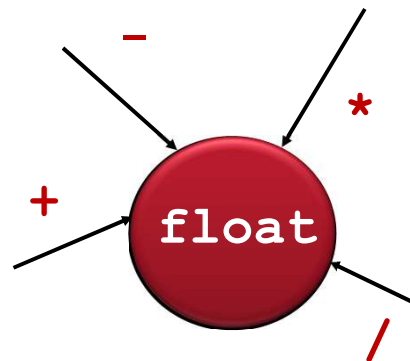
- **Specification**: Essential operations
- **Implementation**: Actual data structure and coding

ADT 1 : Primitive Data Types

- Predefined data types are examples of ADT
 - E.g. integer, floating point, string, etc
- Representation details are hidden to aid *portability*
 - E.g. float is usually implemented as

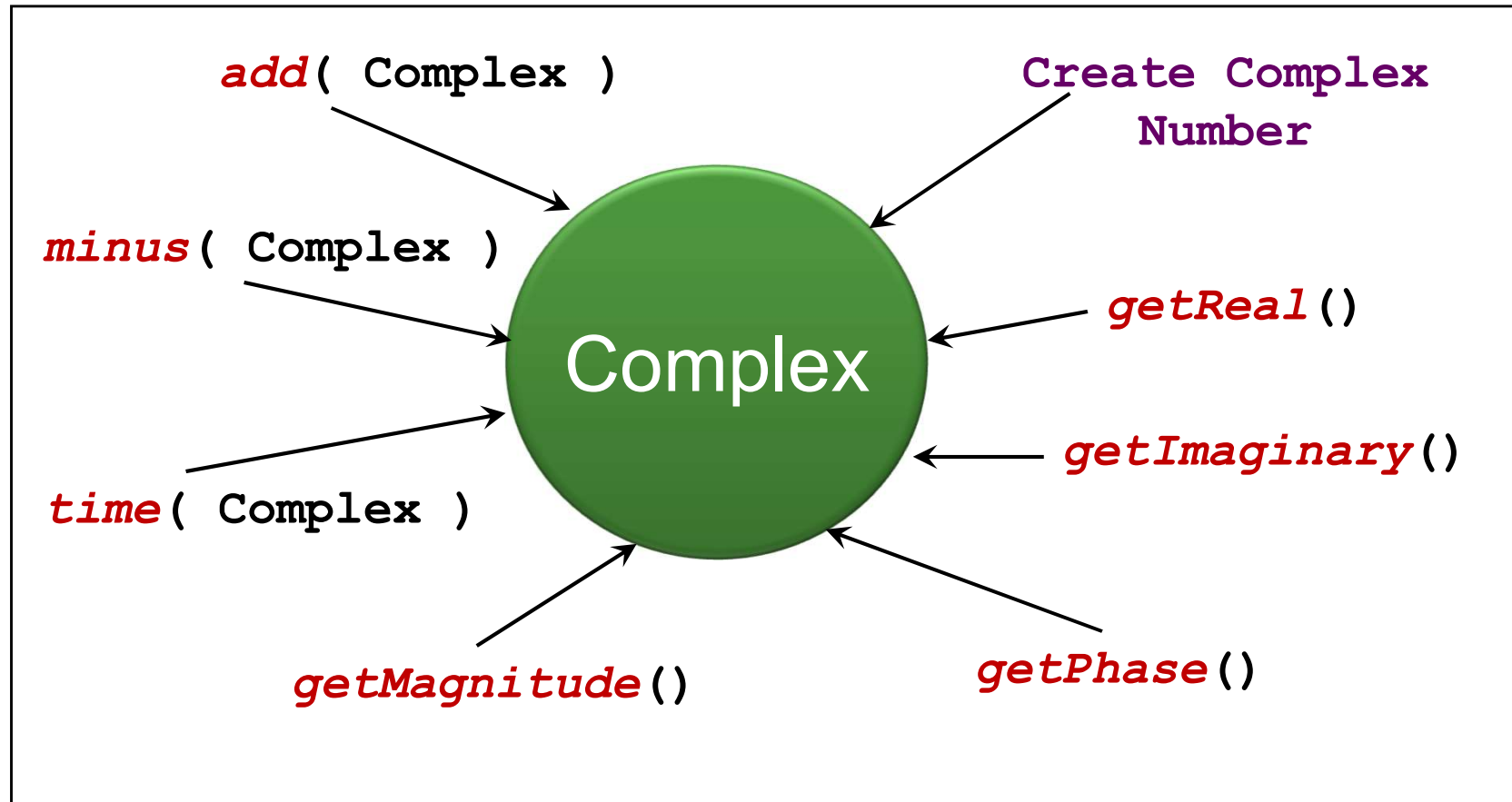


- However, as a user, you don't need to know the above to use float variable in your program



The float ADT

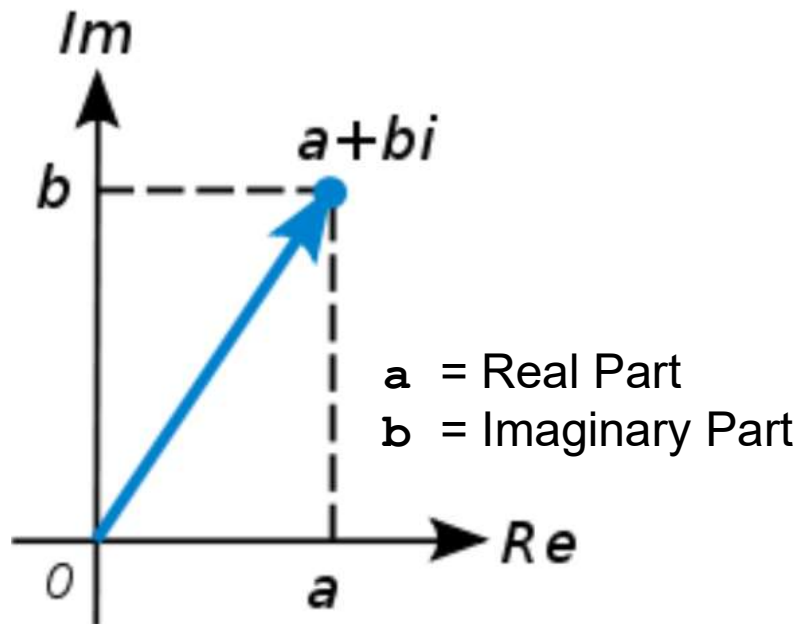
ADT 2 : **C**omplex **N**umber



The **C**omplex ADT

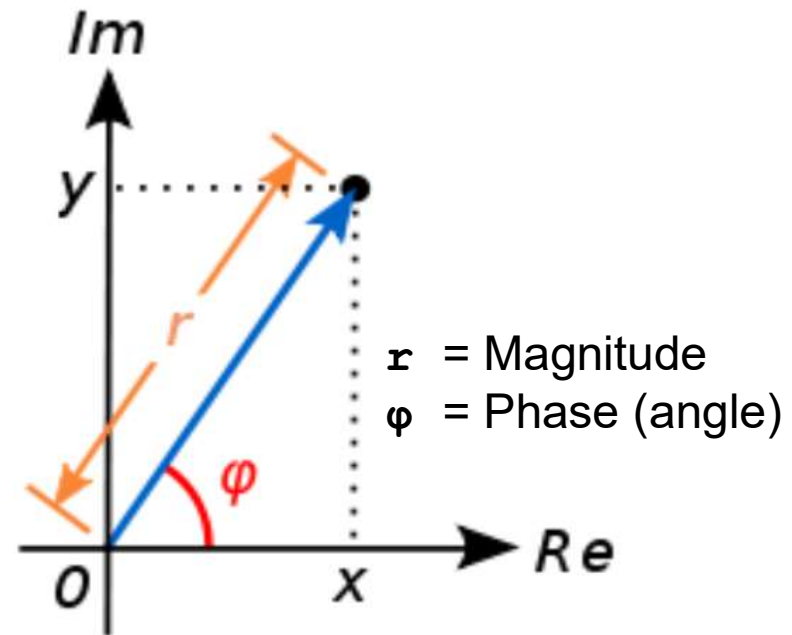
Complex Number: **R**epresentations

- Common representations of complex number:



Rectangular Form

$$(a + bi)$$



Polar Form

$$r (\cos \phi + i \sin \phi)$$

- Each form is easier to use in certain operations

Complex Number: Overview

■ Specification:

- ❑ Define the common expected operations for a complex number object

■ Implementation:

- ❑ Complex number can be implemented by at least two different internal representations
 - Keep the ***Rectangular form*** internally OR
 - Keep the ***Polar form*** internally
- Observes the ADT principle in action!

Complex Number: **D**esign

- Complex number can be implemented as two classes:
 - **Each utilize different internal representation**
- A better alternative:
 - Let us define a **abstract base class** which captures the essential operations of a complex number
 - The super class is independent from the actual representation
- We can then utilize:
 - **Inheritance** and **polymorphism** to provide different actual implementations without affecting the user

Abstract Base Class: **C**omplexBase

#imports not shown

class ComplexBase(ABC):

@abstractmethod

def getReal(self):
 pass

@abstractmethod

def getImaginary(self):
 pass

@abstractmethod

def getMagnitude(self):
 pass

@abstractmethod

def getPhase(self):
 pass

Abstract Base
Class

@abstractmethod

def add(self, other):
 pass

@abstractmethod

def minus(self, other):
 pass

@abstractmethod

def time(self, another):
 pass

@abstractmethod

def toRectangularString(self):
 pass

@abstractmethod

def toPolarFormString(self):
 pass

- **ComplexBase** is a "placeholder" class
 - Specifies all necessary operations but with no actual implementation

User Program Example: Preliminary

```
def main():
```

```
    c1 = To be replaced by actual implementations  
    c2 = of the ComplexBase class
```

```
    print("Complex number c1:")  
    print(c1.toRectangularString())  
    print(c1.toPolarFormString())
```

```
    print("Complex number c2:")  
    print(c2.toRectangularString())  
    print(c2.toPolarFormString())
```

```
    print("add c2 to c1")  
    c1.add(c2)
```

```
    print("Complex number c1:")  
    print(c1.toRectangularString())
```

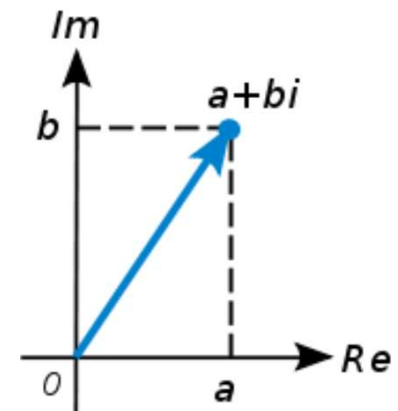
```
if __name__ == "__main__":  
    main()
```

As a user, we can use the methods without worrying about the actual implementation!

Rectangular Form Representation

COMPLEX NUMBER

VERSION A



ComplexRectangular: Specification

```
class ComplexRectangular(ComplexBase):
```

```
    def __init__(self, real, imag):
```

```
        self._real = real
```

```
        self._imag = imag
```

The real and imaginary part are
kept as object attributes

```
    def getReal(self):
```

```
        return self._real
```

```
    def getImaginary(self):
```

```
        return self._imag
```

Methods in this class do not have the
abstract method decorator
→ we will give actual implementation

```
    def getMagnitude(self):
```

```
        return math.sqrt( self._real**2 + self._imag**2)
```

ComplexRectangular: Implementation

```
def getPhase(self):
    if self._real > 0 or self._imag != 0:
        den = math.sqrt(self._real**2 + self._imag**2)\
            + self._real
        radian = 2 * math.atan( self._imag / den)
    elif self._real < 0 and self._imag == 0:
        radian = math.pi / 2
    else:
        radian = None
    return radian

def add(self, other):
    self._real = self._real + other.getReal()
    self._imag = self._imag + other.getImaginary()

def minus(self, other ):
    self._real = self._real - other.getReal()
    self._imag = self._imag - other.getImaginary()
```

Algebra of complex numbers:

- (i) Addition:
 $(a + ib) + (c + id) = (a + c) + i(b + d)$
- (ii) Subtraction:
 $(a + ib) - (c + id) = (a - c) + i(b - d)$
- (iii) Multiplication:
 $(a + ib)(c + id) = (ac - bd) + i(ad + bc)$

ComplexRectangular: Implementation

```
def time( self, other ):
    realNew = self._real * other.getReal() \
              - self._imag * other.getImaginary()
    imagNew  = self._real * other.getImaginary() \
              + self._imag * other.getReal()
    self._real = realNew
    self._imag = imagNew

def toRectangularString(self):
    return "{:.3f}, {:.3f}i".format(self.getReal(),
    self.getImaginary())

def toPolarFormString(self):
    return "{0:.3f}(cos {1:.3f}, i sin {1:.3f})".format(self.getMagnitude(), self.getPhase())
```

■ Notes:

- ❑ We chose to avoid more advanced Python syntax (e.g. class property setter / getter, etc)
- ❑ Feel free to experiment after you understood the basic premise

User Program Example: **V**ersion **2.0**

```
def main():
```

```
    c1 = ComplexRectangular(30, 10)
    c2 = ComplexRectangular(20, 20)
```

```
    print("Complex number c1:")
    print(c1.toRectangularString())
    print(c1.toPolarFormString())
```

```
    print("Complex number c2:")
    print(c2.toRectangularString())
    print(c2.toPolarFormString())
```

```
    print("add c2 to c1")
    c1.add(c2)
```

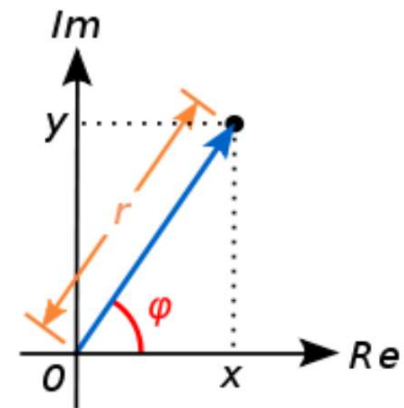
```
    print("Complex number c1:")
    print(c1.toRectangularString())
```

c1, c2 are initialized as
ComplexRectangular
objects

Observe that the
implementation details
doesn't affect the behavior
of an ADT, i.e. user
program is unchanged!

Polar Form Representation

COMPLEX NUMBER VERSION B



ComplexPolar: Implementation

```
class ComplexPolar(ComplexBase):  
  
    def __init__(self, magnitude, phase):  
        self._mag = magnitude  
        self._phase = phase  
  
    def getReal(self):  
        return self._mag * math.cos(self._phase)  
  
    def getImaginary(self):  
        return self._mag * math.sin(self._phase)  
  
    def getMagnitude(self):  
        return self._mag  
  
    def getPhase(self):  
        return self._phase
```

The magnitude and phase from the complex plane origin are kept as object attributes

Note that the two parameters have different meaning compared to the `ComplexRectangular` version

Since we keep only magnitude and phase as attributes, the real and imaginary parts need to be calculated

ComplexPolar: Implementation

```
def _convertPhaseAngle(self, real, imag ):
    if real != 0:
        den = math.sqrt(real**2 + imag**2) + real
        radian = 2 * math.atan( imag / den)
    elif imag > 0:
        radian = math.pi / 2
    else:
        radian = -math.pi / 2
    return radian

def add(self, other):
    real = self.getReal() + other.getReal()
    imag = self.getImaginary() + other.getImaginary()

    self._mag = math.sqrt( real**2 + imag**2 )
    self._phase = self._convertPhaseAngle(real, imag)

def minus(self, other ):
    real = self.getReal() - other.getReal()
    imag = self.getImaginary() - other.getImaginary()

    self._mag = math.sqrt( real**2 + imag**2 )
    self._phase = self._convertPhaseAngle(real, imag)
```

An example of
"helper method",
used internally to
simplify coding

Convert to
rectangular form
for addition

Convert back to
polar form

Similar idea for
subtraction

ComplexPolar: Implementation

```
def time( self, another ):
    self._mag *= another.getMagnitude()
    self._phase += another.getPhase()
```

Multiplication in Polar form is real easy though!

```
def toRectangularString(self):
```

Code similar to `ComplexRectangular`. Not Shown.

```
def toPolarFormString(self):
```

Code similar to `ComplexRectangular`. Not Shown.

- At this point:
 - ❑ We have two **independent implementations** of complex number
 - ❑ They have different internal working, but support the same behavior

User Program Example: **V**ersion **3.0**

```
def main():
```

```
    c1 = ComplexPolar(31.62, 0.322)
    c2 = ComplexPolar(28.28, 0.785)
```

Note that **ComplexPolar**
constructs with magnitude
and phase

```
    print("Complex number c1:")
    print(c1.toRectangularString())
    print(c1.toPolarFormString())
```

```
    print("Complex number c2:")
    print(c2.toRectangularString())
    print(c2.toPolarFormString())
```

No change to
code otherwise

```
    print("add c2 to c1")
    c1.add(c2)
```

```
    print("Complex number c1:")
    print(c1.toRectangularString())
```

```
if __name__ == "__main__":
    main()
```

User Program Example: **Version 4.0**

```
def main():
```

```
    c1 = ComplexRectangular(30, 10)  
    c2 = ComplexPolar(28.28, 0.785)
```

```
    print("Complex number c1:")  
    print(c1.toRectangularString())  
    print(c1.toPolarFormString())
```

```
    print("Complex number c2:")  
    print(c2.toRectangularString())  
    print(c2.toPolarFormString())
```

```
    print("add c2 to c1")  
    c1.add(c2)
```

```
    print("Complex number c1:")  
    print(c1.toRectangularString())
```

```
if __name__ == "__main__":  
    main()
```

The **c1** and **c2** need not
be the same
implementation!

Can you figure out how **c1** and
c2 can interoperate?

Complex Number: **Summary**

- This example highlights:
 - Separation of **specification** and **implementation**
 - A specification can have multiple implementations

- Why is this useful?
 1. We can try out **different strategies** in implementation **without affecting the user**
 2. We can use the best implementation in a certain situation
 - E.g. If multiplication is going to be the most common operations in a complex number program, we can choose to use the **polar form** implementation

Summary

- Abstraction is a powerful technique
 - Data Abstraction
 - Function Abstraction
- Abstract Data Type
 - External Behavior
 - The specification
 - Internal Coding
 - The actual implementations

References

- [Carrano]
 - ❑ 4th / 5th Edition, Chapter 3

- [Koffman & Wolfgang]
 - ❑ Chapter 1.4

- Source:
 - ❑ The two diagrams of complex number representation are taken from <http://wikipedia.org>