List of Figures

1.1	Coordinate direction and the infinitesimal control volume.	8
1.2	Basis function in linear polynomials for a patch of triangular elements.	18
1.3	(a) Three-node triangular element and (b) shape function for N_1 .	19
1.4	Linear triangular elements for Poisson equation example: (a) "connected" equations for node 1; (b) Type 1 and Type 2 element shapes in mesh.	21
1.5	Potential flow solution around an aerofoil. Mesh and streamline plots.	24
1.6	Free surface potential flow, illustrating an axisymmetric jet impinging on a hemispherical thrust reverser (from Sarpkaya and Hiriart [20]).	25
1.7	Finite volume weighting. Vertex centered method.	26
1.8	Finite volume domain and integrations for vertex centered method: "connected" equations for node 1; (b) Type 1 and Type 2 element boundary integrals.	27
2.1	A linear shape function for a one-dimensional problem.	35
2.2	Approximations to $U d\phi/dx - k d^2\phi/dx^2 = 0$ for $\phi(0) = 1$ and $\phi(L) = 0$ for various Peclet numbers. Solid line: exact solution; dotted line with triangular symbol: standard Galerkin solution.	36
2.3	Approximations to $U d\phi/dx - k d^2 \phi/dx^2 = 0$ for $\phi = 0$ at $x = 0$ and $\phi = 1$ at $x = L$ for various Peclet numbers.	38
2.4	Petrov-Galerkin weight function $W_a=N_a+\alphaW_a^{\star}$. Continuous and discontinuous definitions.	39
2.5	Critical (stable) and optimal values of the "upwind" parameter α for different values of $Pe = Uh/2k$.	40
2.6	Application of standard Galerkin and Petrov-Galerkin (optimal) approximation: (a) variable source term equation with constants k and h ; (b) variable source term with a variable U .	41
2.7	A one-dimensional pure convective problem $(k=0)$ with a variable source term Q and constant U . The Petrov-Galerkin procedure results in an exact solution but simple finite difference upwinding gives substantial error.	42
2.8	Assembly of one-dimensional quadratic elements.	48

xviii List of Figures

2.9	A two-dimensional, streamline assembly. Element size \boldsymbol{h} and streamline directions.	50
2.10	"Streamline" procedures in a two-dimensional problem of pure convection. Bilinear elements [35]: (a) boundary conditions for test problem; solutions for $\theta=45^\circ$ (top) and $\theta=65^\circ$.	52
2.11	The wave nature of a solution with no conduction. Constant wave velocity $\ensuremath{\mathcal{U}}$.	55
2.12	Mesh updating and interpolation: (a) forward and (b) backward.	58
2.13	Distortion of convected shape function.	60
2.14	A simple characteristic-Galerkin procedure.	60
2.15	Stability limit for lumped mass approximation and optimal upwind parameter.	65
2.16	Advection of a Gaussian cone in a rotating fluid by characteristic-Galerkin method: (a) original form; (b) form after one revolution using consistent M matrix; and (c) form after one revolution using lumped mass (Lax-Wendroff).	67
2.17	Characteristic-Galerkin method in the solution of a one-dimensional wave progression. Effect of using a lumped mass matrix and one of consistent iteration: (a) Courant number $= 0.5$; (b) Courant number $= 0.1$.	68
2.18	A Gaussian distribution advected in a constant velocity field. Boundary condition causes no reflection.	69
2.19	Progression of a wave with velocity $U = \phi$.	72
2.20	Development of a shock (Burger equation): (a) profile at time $t=0$; (b) characteristics; (c) profile at time $t=1$; (d) profile at time $t=2$.	73
2.21	Propagation of a steep wave by Taylor-Galerkin process: (a) explicit methods $C=0.5$, step wave at $Pe=12,500$ and (b) explicit methods $C=0.1$, step wave at $Pe=12,500$.	75
2.22	Propagation of a steep front in Burger's equation with solution obtained using different values of $C_{\rm Lap}$.	76
2.23	Solution of pure convection in element-by-element manner. Source term \mathcal{Q} and constant u : (a) one-dimensional solution sequence; (b) two-dimensional solution sequence.	77
2.24	Solution of one-dimensional convection-diffusion problem: (a) one-dimensional linear element solution; (b) one-dimensional quadratic element solution.	79
3.1	Element sizes at different nodes of a linear triangle.	102
3.2	Fictitious and real boundaries.	111

3.3	Inviscid flow past a NACA0012 aerofoil $\alpha=0$: (a) unstructured mesh with 1824 elements and 969 nodes; (b) details of mesh near stagnation point; (c) steady-state convergence for $M=0.5$ with two- and single-step schemes, fully explicit form; (d) steady-state convergence for $M=1.2$ for two-step scheme; (e) steady-state convergence for $M=1.2$ for single-step scheme.	114
3.4	Subsonic inviscid flow past a NACA0012 aerofoil with $\alpha=0$ and $M=0.5$: (a) density contours with TG scheme with no additional viscosity; (b) density contours with TG scheme with additional viscosity; (c) density contours with CBS scheme with no additional viscosity; (d) comparison of density along the stagnation line.	115
3.5	Vortex decay. Temporal convergence of the dual time-stepping quasi-implicit scheme with first-, second-, and third-order backward difference formula for $\Delta u^n/\Delta \tau$: (a) velocity; (b) pressure.	117
4.1	Incompressible flow in a lid-driven cavity. Geometry and meshes. (a) Geometry and boundary conditions; (b) nonuniform structured mesh (elements: 2888, nodes: 1521); (c) uniform structured mesh (elements: 20,000, nodes: 10,201); (d) nonuniform unstructured mesh (elements: 10,596, nodes: 5515).	130
4.2	Incompressible flow in a lid-driven cavity: (a) $Re = 100$, stream traces; (b) $Re = 100$, pressure contours; (c) $Re = 400$, stream traces, (d) $Re = 400$, pressure contours.	131
4.3	Incompressible flow in a lid-driven cavity: (a) $Re = 5000$, stream traces on the unstructured mesh; (b) $Re = 5000$, stream traces on the uniform structured mesh; © $Re = 5000$, pressure contours on the unstructured mesh; (d) $Re = 5000$, pressure contours on the uniform structured mesh.	132
4.4	Incompressible flow in a lid-driven cavity. Horizontal velocity distribution at different Reynolds numbers: (a) Re = 0; (b) Re = 400 ; (c) Re = 1000 ; (d) Re = 5000 .	133
4.5	Incompressible flow in a lid-driven cavity. Vertical velocity distribution at different Reynolds numbers: (a) $Re = 0$; (b) $Re = 400$; (c) $Re = 1000$; (d) $Re = 5000$.	134
4.6	Incompressible flow in a 3D lid-driven cavity. Mesh and contours at Re = 400: (a) unstructured mesh; (b) u_1 contours; (c) u_3 contours; (d) pressure contours.	135
4.7	Lid-driven cavity. Steady-state convergence histories for (a) Re = 400 and (b) 5000. Comparison between fully explicit and semi-implicit schemes.	136
4.8	Incompressible flow past a backward facing step. Geometry and boundary conditions.	136
4.9	Incompressible flow past a backward facing step. step: (a) unstructured mesh; (b) u_1 velocity contours; (c) pressure contours (Re = 229).	136

4.10	Incompressible flow past a backward facing step. Comparison between experimental [10] and numerical data, Re = 229.	137
4.11	Incompressible flow past a sphere: (a) unstructured mesh; (b) unstructured mesh, cross section.	138
4.12	Incompressible flow past a sphere: (a) u_1 contours, Re = 100; (b) pressure contours, Re = 100; (c) u_1 contours, Re = 200; pressure contours, Re = 200.	139
4.13	Incompressible flow past a sphere. Coefficient of pressure distribution on the surface along the flow direction $[15-17]$: (a) Re = 100; (b) Re = 200.	140
4.14	Transient flow past a circular cylinder, Re = 100: (a) unstructured mesh; (b) vertical velocity fluctuation at the exit mid-point; (c) drag history.	141
4.15	Lid-driven cavity. Horizontal and vertical velocity distributions along the centerline cross-sections, $Re = 1000$.	142
4.16	Interpolation error in a one-dimensional problem with linear shape functions.	144
4.17	Element elongation $\boldsymbol{\delta}$ and minimum and maximum element sizes.	147
4.18	Lid-driven cavity, Re = 5000. Adapted meshes using curvature- and gradient-based refinements and solutions: (a) curvature- based procedure (nodes: 2389, elements: 4599); (b) gradient- based procedure (nodes: 1034, elements: 1962); (c) compari- son of velocity at mid-vertical plane.	150
4.19	Transient incompressible flow around a cylinder at Re = 250. Adaptively refined mesh. Pressure contours and streamlines at various times after initiation of "vortex shedding": (a) $t = 6$ s; (b) $t = 11.5$ s; (c) $t = 16.5$ s.	152
4.20	Some useful velocity-pressure interpolations and their asymptotic, energy norm convergence rates: (a) continuous p interpolation; (b) discontinuous p interpolation.	154
5.1	Stress $\bar{\sigma}$, viscosity μ , and strain rate $\dot{\bar{\epsilon}}$ relationships for various materials: (a) linear, Newtonian, fluid; (b) non-Newtonian polymers; (c) viscoplastic-plastic metals.	164
5.2	Forming processes typically used in manufacture: (a) steady rate; (b) transient	166
5.3	Plane strain extrusion (extrusion ratio 2:1) with ideal plasticity assumed.	168
5.4	Steady-state rolling process with thermal coupling [40]: (a) geometry; (b) velocity profiles; (c) temperature distribution for different entry temperatures.	170
5.5	Punch indentation problem (penalty function approach) [4]. Updated mesh and surface profile with 24 isoparametric elements. Ideally plastic material; (a), (b), (c), and (d) show various	
	depths of indentation (reduced integration is used here)	171

5.6	(a) A material grid and updated and adapted meshes with material deformation (η percentage in energy norm). A transient extrusion problem with temperature and strain-dependent yield [42]. Adaptive mesh refinement uses T6/1D elements of Fig. 4.20 (b) Contours of state parameters at $t=2.9~\rm s$; (c) load versus time.	172
5.7	Deep drawing by a flat-nosed punch [50].	175
5.8	Finite element simulation of the superplastic forming of a thin sheet component by air pressure application. This example considers the superplastic forming of a truncated ellipsoid with a spherical indent. The original flat blank was 150×100 mm. The truncated ellipsoid is 20 mm deep. The original thickness was 1 mm. The minimum final thickness was 0.53 mm; 69 time steps were used with a total of 285 Newton-Raphson iterations (complete equation solutions) [53]: (a) mesh of 856 elements for sheet idealization; (b) mesh for establishing die geometry; (c) deformed sheets at various times.	176
5.9	Physical representation of Maxwell model using spring and dashpot.	178
5.10	Physical representation of Oldroyd-B model using spring and dashpots.	178
5.11	Viscoelastic flow past a circular cylinder. Geometry and boundary conditions.	181
5.12	Viscoelastic flow past a circular cylinder. Unstructured mesh (nodes: 10,619; elements: 20,384) (a) mesh; (b) mesh in the vicinity of the cylinder.	183
5.13	Stokes flow past a circular cylinder. $Re=0$, $De=0.0$. Contours of velocity components and pressure: (a) u_1 velocity contours, $u_{1_{\rm min}}=0$, $u_{1_{\rm max}}=2.94$; (b) u_2 velocity contours, $u_{2_{\rm min}}=-0.895$, $u_{2_{\rm max}}=0.893$; (c) pressure contours, $p_{\rm min}=-29.28$, $p_{\rm max}=36.04$.	183
5.14	Viscoelastic flow past a circular cylinder. $Re=0$, $De=0.5$. Contours of velocity, pressure, and elastic stresses: (a) u_1 velocity contours, $u_{1_{\min}}=0$, $u_{1_{\max}}=2.99$; (b) u_2 velocity contours, $u_{2_{\min}}=-0.929$, $u_{2_{\max}}=0.884$; (c) pressure contours, $p_{\min}=-28.35$, $p_{\max}=34.97$; (d) τ_{11}^{V} contours, $\tau_{11_{\min}}^{V}=-0.979$, $\tau_{11_{\max}}^{V}=76.45$; (e) τ_{12}^{V} contours, $\tau_{12_{\min}}^{V}=-18.59$, $\tau_{12_{\max}}^{V}=22.92$; (f) τ_{22}^{V} contours, $\tau_{22_{\min}}^{V}=-0.463$, $\tau_{22_{\max}}^{V}=16.82$.	184
5.15	Viscoelastic flow past a circular cylinder. Comparison of drag force distribution with other available numerical data. Dou and Phan-Thian1 = Plain Oldroyd-B formulation without stress splitting; Dou and Phan-Thian2 = EVSS; Dou and Phan-Thian3 = DEVSS-ω; Dou and Phan-Thian4 = DAVSS-ω; Dou and Phan-Thian4	104
	Thian5 = Extrapolated results for zero mesh size.	185

xxii List of Figures

5.16	Axisymmetric solutions to the bar impact problem: (a) initial shape; (b) linear triangles—displacement algorithm; (c) bilinear quadrilaterals—displacement algorithm; (d) linear triangles—CBS algorithm; (e) bilinear quadrilaterals—CBS algorithm.	186
5.17	Three-dimensional solution: (a) tetrahedral elements—standard displacement algorithm; (b) tetrahedral elements—CBS algorithm.	187
6.1	Typical problems with a free surface.	196
6.2	Broken dam problem. Problem definition and schematic of the free surface.	198
6.3	Broken dam problem. Mesh and contours after $t=2.0$: (a) mesh; (b) u_1 velocity contours; (c) u_2 velocity contours; (d) pressure contours.	199
6.4	Broken dam problem. Mesh and contours after $t=5.0$: (a) mesh; (b) u_1 velocity contours; (c) u_2 velocity contours; (d) pressure contours.	200
6.5	Broken dam problem. Comparison of numerical results with experimental data [45].	200
6.6	A typical problem of ship motion.	201
6.7	A submerged hydrofoil. Mesh updating procedure. Euler flow. Mesh after 1900 iterations.	204
6.8	A submerged hydrofoil. Mesh updating procedure. Euler flow: (a) pressure distribution; (b) comparison with experiment.	205
6.9	A submerged hydrofoil. Hydrostatic adjustment. Euler flow: (a) pressure contours and surface wave pattern; (b) comparison with experiment [68].	206
6.10	A submerged hydrofoil. Hydrostatic adjustment. Navier-Stokes flow: (a)–(d) magnitude of total velocity contours for different Reynolds numbers; (e) wave profiles for different Reynolds	
	numbers.	207
6.11	Submerged DARPA submarine model: (a) surface mesh; (b) wave pattern.	208
6.12	A sailing boat: (a) surface mesh of hull, keel, bulb, and rudder; (b) wave profile.	209
6.13	ALE description in Cartesian coordinates.	210
6.14	Solitary wave propagation. Problem definition.	212
6.15	Solitary wave propagation. Meshes at various time levels: (a) $t=0.0$; (b) $t=2.28$; (c) $t=4.58$, (d) $t=6.84$; (e) $t=9.12$.	213
6.16	Solitary wave propagation. Velocity vector distribution at various time levels: (a) $t=0.0$; (b) $t=2.28$; (c) $t=4.58$, (d) $t=6.84$; (e) $t=9.12$.	214
6.17	Wave heights with respect to time on the right and left side walls: (a) right wall; (b) left wall.	214

6.18	Solitary wave propagation. Comparison of wave heights with experimental data [74].	215
6.19	Natural convection in a square enclosure. Streamlines and isotherms for different Rayleigh numbers. (a) $Ra = 10^4$; (b) $Ra = 10^5$; (c) $Ra = 10^7$.	217
6.20	Natural convection in a square enclosure. Adapted meshes for (a) $Ra=10^5$ and (b) $Ra=10^6$.	218
7.1	Boundaries of a computation domain. $\Gamma_{\it u},$ wall boundary; $\Gamma_{\it S},$ fictitious boundary.	228
7.2	Characteristic directions at inlet and exit for supersonic and subsonic flows.	230
7.3	Subsonic inviscid flow past a NACA0012 airfoil at Mach number of 0.25 and zero angle of attack. Smoothed density contours [68].	235
7.4	Subsonic inviscid flow past a NACA0012 airfoil at Mach number of 0.25 and zero angle of attack. Comparison between smoothed and unsmoothed pressure coefficients [68].	235
7.5	The Riemann shock tube problem [1,69]. The total length is divided into 100 elements. Profile illustrated corresponds to 70 time steps ($\Delta t = 0.25$). Lapidus constant $C_{\text{Lap}} = 1.0$.	236
7.6	Isothermal flow through a nozzle [1]. Forty elements of equal size used: (a) subsonic inflow and outflow; (b) supersonic inflow and outflow; (c) supersonic inflow-subsonic outflow with shock.	237
7.7	Transient supersonic flow over a step in a wind tunnel [5] (problem of Woodward and Colella [71]). Inflow Mach 3 uniform flow: (a) structured uniform mesh; (b) solution – contours of pressure at various times.	239
7.8	Inviscid flow past a NACA0012 aerofoil. Unstructured mesh. Number of nodes: 3753; number of elements: 7351. (a) Finite element mesh and domain; (b) mesh distribution in the vicinity of the aerofoil.	240
7.9	Inviscid flow past a NACA0012 aerofoil. Convergence histories to steady state.	240
7.10	Inviscid subsonic flow past a NACA0012 aerofoil. Pressure contours: (a) Mach number = 0.25; (b) Mach number = 0.5	241
7.11	Inviscid subsonic flow past a NACA0012 aerofoil. Pressure coefficient distribution: (a) Mach number = 0.25; (b) Mach number = 0.5	242
7.12	Inviscid transonic and supersonic flow past a NACA0012 aerofoil. Pressure contours: (a) Mach number = 0.85 ; (b) Mach number = 0.95 ; (c) Mach number = 1.2 .	242
7.13	Inviscid transonic and supersonic flow past a NACA0012 aerofoil. Pressure coefficient distribution: (a) Mach number = 0.85 ; (b) Mach number = 0.95 ; (c) Mach number = 1.2 .	243

xxiv List of Figures

7.14	Mesh enrichment. (a) Triangle subdivision. (b) Restoration of connectivity.	244
7.15	Supersonic, Mach 3, flow past a wedge. Exact solution forms a stationary shock. Successive mesh enrichment and density contours.	245
7.16	Reflection of a shock wave at a wall [11]: Euler equations. A sequence of meshes: (a) nodes: 279, elements: 478; (b) nodes: 265, elements: 479; (c) nodes: 285, elements: 528; and corresponding pressure contours, (d) to (f).	247
7.17	Hypersonic flow past a blunt body [11] at Mach 25, 22° angle of attack. (a) Sequence of meshes deployed; (b) the corresponding density; (c) the corresponding pressure contours. Initial mesh, nodes: 547, elements: 978; first mesh, nodes: 383, elements: 696; final mesh, nodes: 821, elements: 1574.	248
7.18	Supersonic flow past a full cylinder [61]. $M=3$: (a) geometry and boundary conditions; (b) adapted mesh, nodes: 12651, elements: 24,979; (c) Mach contours using second derivative shock capture; (d) Mach contours using anisotropic shock capture.	249
7.19	Supersonic flow past a full cylinder [61]. $M = 3$: comparison of (a) coefficient of pressure, (b) Mach number distribution along the mid-height and cylinder surface.	250
7.20	Interaction of an impinging and bow shock wave [77]. Adapted mesh and pressure contours.	251
7.21	Inviscid flow past an ONERA M6 wing. Density contours. Mach number = 0.78, angle of attack to horizontal = 2.8° .	253
7.22	Adaptive three-dimensional solution of compressible inviscid flow around a high-speed (Mach 2) aircraft [84]. Nodes: 70,000, elements: 125,000.	254
7.23	Three-dimensional analysis of an engine intake [84] at Mach 2 (14,000 elements): (a) mesh on analysis surface; (b) mesh on analysis surface; (c) pressure contours.	255
7.24	Supersonic car, THRUST SSC [35]. (a) car and (b) finite element surface mesh. (Image used in (a) courtesy of SSC Programme Ltd. Photographer Jeremy C.R. Davey.)	257
7.25	Supersonic car, THRUST SSC [35] pressure contours: (a) full configuration; (b) front portion.	258
7.26	Supersonic car, THRUST SSC [35] comparison of finite element and experimental results.	258
7.27	A transient problem with adaptive remeshing [88]. Simulation of a sudden failure of a pressure vessel. Progression of refinement and velocity patterns shown. Initial mesh 518 nodes.	259
7.28	A transient problem with adaptive remeshing [88]. Model of the separation of shuttle and rocket. Mach 2, angle of attack -4° , initial mesh 4130 nodes.	260

7.29	Separation of a generic shuttle vehicle and rocket booster [32]. (a) Initial surface mesh and surface pressure; (b) final surface mesh and surface pressure.	261
7.30	Refinement in the boundary layer: (a) a two-dimensional sub- layer of structured quadrilaterals; (b) a three-dimensional sub- layer of prismatic elements.	261
7.31	Viscous flow past a flat plate (Carter problem) [92]. Mach 3, $Re=1000$. (a) Mesh, nodes: 6750, elements: 13,172. Contours of (b) pressure and (c) Mach number.	263
7.32	Viscous flow past a flat plate (Carter problem) [92]. Mach 3, $Re=1000$. (a) Pressure distribution along the plate surface, (b) exit velocity profile.	264
7.33	Shock and boundary layer interaction [94]. Final mesh, nodes: 4198. (a) Initial and final (second) adapted mesh; (b) initial and final (second) pressure contours; (c) initial and final (second) Mach number contours; (d) surface pressure and skin tension.	265
7.34	Hybrid mesh for supersonic viscous flow past a NACA0012 aerofoil [95], Mach 2, and contours of Mach number: (a) initial mesh; (b) first adapted mesh; (c) final mesh; (d) mesh near stagnation point (shown opposite).	268
7.35	Structured grid in boundary layer for a two-component aerofoil [25]. Advancing boundary normals.	269
7.36	Transonic viscous flow past a NACA0012 aerofoil. Mach number 0.85, Reynolds number = 2000. (a) Finite element mesh; (b) structured layers close to the wall.	270
7.37	Transonic viscous flow past a NACA0012 aerofoil. Mach number 0.85, Reynolds number = 2000. Mach contours.	270
7.38	Transonic viscous flow past a NACA0012 aerofoil. Mach number 0.85, Reynolds number = 2000. (a) surface pressure and (b) friction coefficients distribution.	271
7.39	Hypersonic viscous flow past a double ellipsoid. Unstructured mesh with structured mesh layers close to the walls: (a) adapted mesh; (b) structured layers close to the wall; (c) close-up of structured layers.	272
7.40	Hypersonic viscous flow past a double ellipsoid. Density contours.	272
8.1	Random variation of velocity in a turbulent flow with respect to time.	284
8.2	(a) Structured mesh S1 (nodes: 15,625; elements: 69,120); (b) unstructured mesh U1 (nodes: 23,597; elements: 127,692).	292

xxvi List of Figures

8.3	Wall distance contours at a central section in the x_1 direction (uniformly structured mesh S1). Comparison between search procedure and implicit GMRES scheme. (a) Simple search procedure (structured mesh): $\phi_{min}=0.0, \phi_{max}=0.5000$; (b) Eikonal equation (structured mesh): $\phi_{min}=0.0, \phi_{max}=0.5000$; (c) simple search procedure (unstructured mesh): $\phi_{min}=0.0, \phi_{max}=0.4923$; (d) Eikonal equation (unstructured mesh): $\phi_{min}=0.0, \phi_{max}=0.4887$.	293
8.4	Turbulent incompressible flow through a rectangular channel using the Spalart-Allmaras model at $Re=12,300$. Logarithmic representation of time-averaged velocity profile. (Note: $u^+=u/u_\tau$ with $u_\tau=\sqrt{\tau_W/\rho}$ being the friction velocity; $y^+=yu_\tau/\nu$ with y being the shortest distance to the wall.)	294
8.5	Turbulent incompressible flow in a rectangular channel using the Spalart-Allmaras model at $Re = 12,300$. (a) Comparison of fully developed velocity profiles; (b) convergence to the steady	005
8.6	state. Turbulent flow past a two-dimensional backward facing step.	295
0.0	Problem definition.	296
8.7	Incompressible turbulent flow past a backward facing step. Velocity profiles at various downstream sections at $Re = 3025$: (a) one-equation model; (b) SA model; (c) two-equation model.	297
8.8	Incompressible turbulent flow past a backward facing step. (a) Structured mesh (elements: 8092, nodes: 4183), (b) velocity contours, (c) \hat{v} contours, and (d) pressure contours at $Re = 3015$ using the SA model.	298
8.9	Incompressible turbulent flow past a backward facing step. (a) Unstructured mesh (elements: 47,359, nodes: 24,336), (b) velocity contours, and (c) \hat{v} contours at $Re=3025$ using the SA model.	299
8.10	Incompressible turbulent flow past a circular cylinder. Finite element mesh: (a) overall mesh; (b) close-up of the cylinder.	300
8.11	Incompressible turbulent flow past a circular cylinder. Snapshots of variables at $Re = 10,000$ using the SA model: (a) u_1 contours; (b) p contours; (c) v_T contours.	301
8.12	Incompressible turbulent flow past a circular cylinder. (a) Drag and (b) lift coefficient distributions with respect to real time at $Re = 10,000$ using the SA model.	302
8.13	Incompressible turbulent flow past a circular cylinder. Time-averaged coefficient of pressure at $Re = 10,000$ using the SA model. Data for comparison from Ref. [20].	303
9.1	Typical examples of porous media.	310
9.2	Fluid saturated porous medium. Infinitesimal control volume.	311
9.3	Forced convection in a channel filled with a variable porosity medium. Geometry and boundary conditions.	317

9.4	Forced convection in a channel. Comparison of Nusselt number with experimental data for different particle Reynolds numbers. Points—experimental [24]; dashed line—numerical [24]; solid	
	line—CBS.	318
9.5	Forced convection in a channel. Comparison between the generalized model and the Forcheimmer and Brinkman extensions to Darcy's law.	319
9.6	Natural convection in a fluid-saturated variable-porosity medium. Problem boundary conditions.	320
9.7	Buoyancy driven flow in a fluid-saturated porous medium. Finite element mesh (nodes: 2601, elements: 5000).	321
9.8	Natural convection in a fluid-saturated porous, square enclosure. Vector plots and temperature contours for different Rayleigh and Darcy numbers, $Pr = 0.71$.	322
9.9	Natural convection in a fluid-saturated constant-porosity medium. Problem definition.	323
9.10	Natural convection in a fluid-saturated constant-porosity medium within an annular enclosure. Comparison of hot wall steady-state Nusselt number with the experimental and numerical data [32].	324
10.1	The shallow-water problem. Notation: (a) coordinates; (b) veloc-	
	ity distribution.	328
10.2	Shoaling of a wave: (a) problem statement; (b) solution, for 40, 80, and 160 elements at various times.	335
10.3	Propagation of waves due to dam break ($\mathcal{C}_{Lap}=0$). Forty ele-	
	ments in analysis domain. $C = \sqrt{gH} = 1$, $\Delta t = 0.25$.	336
10.4	A "bore" created in a stream due to water level rise downstream (A). Level at A, $\eta=1-\cos \pi t/30$ ($0 \le t \le 30$), 2 ($30 \le t$). Levels and velocities at intervals of 5 time units, $\Delta t=0.5$.	337
10.5	Steady-state oscillation in a rectangular channel due to periodic forcing of surface elevation at an inlet. Linear frictional dissipa-	30.
	tion [32].	338
10.6	Location map. Bristol Channel and Severn Estuary.	339
10.7	Finite element meshes. Bristol Channel and Severn Estuary.	340
10.8	Velocity vector plots (FL mesh).	341
10.9	Finite element mesh used in the Severn bore calculations (a) Full domain (b) Part of the domain between points A and B (c) Part of the domain beyond point B.	342
10.10	Severn tsunami. Generation during high tide. Water height contours (times after generation).	344
10.11	Wave-induced steady-state flow past a harbor [30].	345
10.12	Supercritical flow and formation of shock waves in symmetric channel of variable width contours of h. Inflow Froude number	
	= 2.5. Constriction: 15°.	345
10.13	Adjustment of boundary due to tidal variation.	346

xxviii List of Figures

10.14	Heat convection and diffusion in tidal currents. Temperature contours at several times after discharge of hot fluid.	348
11.1	General wave domains.	361
11.2	Damper solutions for waves diffracted by circular cylinder. Comparison of relative errors for various outer radii ($ka = 1$). Relative error = $(abs(\eta_n) - abs(\eta_a))/abs(\eta_a)$.	364
11.3	Real part of elevations of plane wave diffracted by an ellipse, of aspect ratio 2, Bettess [22].	367
11.4	Waves scattered by an elastic sphere for $ka = 100$, Burnett and Halford [57].	369
11.5	Computed acoustical pressure contours for a hyperbolic duct $(\theta_0=70^\circ, ka=11, m_\phi=8)$. Conventional and wave envelope element solutions, Astley [59].	370
11.6	Refraction-diffraction solution: lines of equal wave height, lines every 0.25 unit [10].	373
11.7	Transient response of a dipole, Astley [66].	374
11.8	Finite element mesh and wave height magnification for Long Beach Harbor, Houston [85]: (a) finite element grid, grid 3; (b) contours of wave height amplification, grid 3, 232 s wave period.	378
11.9	Element mesh, contours of wave elevation, and wave transmission coefficients for floating breakwater, Hara [87].	380
11.10	Second-order wave elevations around cylinder—real and imaginary parts, Clark et al. [89].	382
12.1	Scattering of a plane wavelength 2 m by a perfectly conducting aircraft of length 18 m: (a) waves impacting aircraft, (b) computed distribution of radar cross-section (RCS), Morgan [10].	393
12.2	Short waves diffracted by a cylinder, modeled using special finite elements [26]: (a) cylinder mesh; (b) real potential; (c) imaginary potential. Reprinted from Ref. [26], with permission from Journal of Computational Acoustics.	396
12.3	Radar cross-section for <i>NACA 0012</i> on trailing edge at 1500 MHz [36]. Comparisons of UWVF and boundary integral results for frequency 1500 MHz, and TM and TE polarizations. <i>Figure reprinted from Ref. [36]</i> , with permission from SIAM.	400
12.4	Ultra weak variational element mesh, Huttunen et al. [20]. Reprinted from Ref. [20], with permission from Elsevier.	401
12.5	Cylindrical wave scattered by a circular region of different wave speeds [20]. Top figure (a) shows the analytical solution of the problem with $f=250$ kHz. Bottom figure (b) shows the UWVF solution of the problem using a uniform basis of 21 wave directions per node. Reprinted from Ref. [20], with permission from Elsevier.	402
	LIJUVIUI.	402

XXIX

12.16	Snapshot of surface elevation after 500 s [68]. Reproduced by permission from Ref. [68], copyright John Wiley and Sons Ltd.	417
13.1	Flow through a flexible pipe.	425
13.2	Nonreflecting boundary conditions in a one-dimensional domain. Solid arrow lines represent the outgoing characteristic and dashed arrow lines represent the incoming characteristic.	429
13.3	Propagation of a short (a) small and (b) large amplitude pressure pulse in a single vessel for inviscid (higher amplitude) and viscous flow (lower amplitude), with snapshots taken at $t=0.03$ s (solid lines), 0.045 s (dashed lines), and 0.06 s (dotted lines). For small amplitude pulses, propagation is approximately linear and the pulse does not distort. For large amplitude pulses, nonlinear effects distort the pulse and lead to shock formation. (c) Log-log plot of pressure peak decay Δp_{peak} over 20 cm due to viscosity with different vessel cross-sectional areas (A_0) and for initial pulse amplitudes of 100 , 1000 , and 2000 dynes/cm ² .	433
13.4	Reflected and transmitted waves in a single vessel due to (a) a step decrease and (b) a step increase in A_0 .	434
13.5	Reflected and transmitted waves in a single vessel due to (a) a step increase and (b) a step decrease in β with constant A_0 .	435
13.6	Schematic of a fluid–structure interaction problem. Superscripts f and s respectively indicate the fluid and solid and subscripts h , q , and $f-s$ respectively indicate a Dirichlet, Neumann, and	420
12.7	fluid-structure interface boundary.	436
13.7	Strong coupling of fluid and structure solvers using a staggered approach.	441
13.8	Artificially moved mesh surrounding a circular cylinder by solving the Laplace equation. (a) Cylinder moved to a position above and (b) below the original position.	445
13.9	Artificially moved mesh surrounding a circular cylinder by solving the Laplace equation and applying Laplace smoothing. (a) Cylinder moved to a position above and (b) below the original position.	446
14.1	(a) A typical human systemic arterial circulation network. (b) Representation of the heart within a one-dimensional frame-	440
	work.	453
14.2	An example of a realistic ventricular pressure constructed using two sigmoid functions and used (in combination with a valve model) as the input to the arterial model. Important physiolog-	450
140	ical features are labeled.	456
14.3	Representation of external pressure distribution on the coronary artery induced by the heart.	457
14.4	A simple ventricular-valve model describing the transformation of the characteristic variables at the aortic valve.	459
14.5	Various possible nodal branching/connection.	461

xxxi