

Self-Adjoint Differential Equations

A

Let us consider the following system of linear partial differential equation to demonstrate the property of self-adjointness:

$$\mathbf{A}(\mathbf{u}) = \mathcal{L}\mathbf{u} + \mathbf{b} = \mathbf{0} \quad (\text{A.1})$$

where \mathcal{L} is a linear differential operator. For the above equation to be self-adjoint the operator \mathcal{L} requires

$$\int_{\Omega} \boldsymbol{\psi}^T (\mathcal{L}\boldsymbol{\gamma}) d\Omega = \int_{\Omega} \boldsymbol{\gamma}^T (\mathcal{L}\boldsymbol{\psi}) d\Omega + \text{b.t} \quad (\text{A.2})$$

for any two functions $\boldsymbol{\psi}$ and $\boldsymbol{\gamma}$. In the above equation b.t stands for boundary integral terms.