Mass-Weighted Averaged Turbulence Transport Equations

I

I.1 Turbulence models

In this appendix we provide two turbulence models commonly employed in the compressible flow calculations. Before discussing these models we write the Reynolds stress term and turbulent heat flux term in terms of turbulent eddy viscosity as

$$\tau_{ij}^R = \mu_T \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_j} - \frac{2}{3} \frac{\partial \tilde{u}_k}{\partial x_k} \delta_{ij} \right) - \frac{2}{3} \rho \kappa \delta_{ij} \tag{I.1}$$

and

$$q_j^R = -\mu_T \frac{c_p}{Pr_T} \frac{\partial \overline{T}}{\partial x_j} \tag{I.2}$$

One of the following turbulence models may be employed to calculate the turbulent viscosity

I.1.1 Spalart-Allmaras model

In this model the turbulent eddy viscosity is calculated as

$$\nu_T = \frac{\mu_T}{\rho} = \tilde{\nu} f_{v1} \tag{I.3}$$

where

$$f_{v1} = \frac{X^3}{X^3 + C_{v1}^3} \tag{I.4}$$

with

$$X = \frac{\tilde{\nu}}{\nu} \tag{I.5}$$

The viscosity variable $\tilde{\nu}$ is calculated from

$$\frac{\partial \tilde{\nu}}{\partial t} + \tilde{u_j} \frac{\partial \tilde{\nu}}{\partial x_j} = c_{b1}[1 - f_{t2}]\tilde{S}\tilde{\nu} + \frac{1}{\sigma} \left[\frac{\partial}{\partial x_j} \left\{ (\nu + \tilde{\nu}) \frac{\partial \tilde{\nu}}{\partial x_j} \right\} \right.$$
$$\left. + c_{b2} \frac{\partial \tilde{\nu}}{\partial x_j} \frac{\partial \tilde{\nu}}{\partial x_j} \right] - \left[c_{w1} f_w - \frac{c_{b1}}{K^2} f_{t2} \right] \left[\frac{\tilde{\nu}}{y} \right]^2 + f_{t1} \Delta \tilde{u}^2 \tag{I.6}$$

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The parameters used in the above equation are written as

$$
\begin{aligned}
\tilde{S} &= \tilde{\omega} + \frac{\tilde{\nu}}{K^2 y^2} f_{v2} \\
f_{v2} &= 1 - \frac{X}{1 + X f_{v1}} \\
f_w &= g \left[\frac{1 + c_{w3}^6}{g^6 + c_{w3}^6} \right]^{1/6} \\
g &= r + c_{w2}(r^6 - r) \\
r &= \min\left(\frac{\tilde{\nu}}{\tilde{S} K^2 d^2}, 10 \right) \\
f_{t2} &= c_{t3} \exp(-c_{t4} X^2) \\
f_{t1} &= c_{t1} g_t \exp\left(-c_{t2} \frac{\tilde{\omega}_t{}^2}{\Delta \tilde{u}^2} [y^2 + g_t^2 y_t^2] \right) \\
g_t &= \min\left(0.1, \frac{\Delta \tilde{u}}{\tilde{\omega}_t \Delta x} \right)
\end{aligned}
\tag{I.7}
$$

where y is the distance from a given node to the nearest wall, $\tilde{\omega}$ is the vorticity given as

$$
\tilde{\omega} = \left[\left(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3} \right)^2 + \left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1} \right)^2 + \left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2} \right)^2 \right]^{1/2} \tag{I.8}
$$

$\Delta \tilde{u}$ is the difference in velocity between the point and trip, y_t is distance from a node to a trip point or curve, $\tilde{\omega}_t$ is the vorticity magnitude at the trip point or curve, and Δx is the surface grid spacing at the trip. Other constants used in the model are $c_{b1} = 0.1355$, $c_{b2} = 0.622$, $\sigma = 2/3$, $K = 0.41$, $c_{w1} = c_{b1}/K^2 + (1 + c_{b2})/\sigma$, $c_{w2} = 0.3$, $c_{w3} = 2$, $c_{v1} = 7.1$, $c_{t1} = 1$, $c_{t2} = 2$, $c_{t3} = 1.1$, and $c_{t4} = 2$.

The major difference between the model given here and the one used in Section 8.2 is that here we have a trip curve (3D) or trip point (2D) to trigger turbulence. The trip curve is often defined at a 3% distance from the leading edge of a solid surface.

I.1.2 κ-ω model

The basic idea of the κ-ω model arises from the fact that vorticity is directly proportional to κ^2/l, i.e.,

$$
\omega = c \frac{\kappa^2}{l} \tag{I.9}
$$

where c is a constant. The eddy viscosity may therefore be written as

$$
\mu_T = \rho \frac{\kappa}{\omega} \tag{I.10}
$$

The transport equations for κ and ω may be written as

$$\frac{\partial}{\partial t}(\overline{\rho}\kappa) + \frac{\partial}{\partial x_i}(\overline{\rho}\kappa\tilde{u_i}) = \frac{\partial}{\partial x_i}\left(\mu_\kappa \frac{\partial \kappa}{\partial x_i}\right) + \frac{\partial}{\partial x_i}(\overline{\tau_{ij}}\tilde{u_j}) - \beta^*\overline{\rho}\kappa\omega \tag{I.11}$$

and

$$\frac{\partial}{\partial t}(\overline{\rho}\omega) + \frac{\partial}{\partial x_i}(\overline{\rho}\omega\tilde{u_i}) = \frac{\partial}{\partial x_i}\left(\mu_\omega \frac{\partial \omega}{\partial x_i}\right) + \alpha\frac{\omega}{\kappa}\frac{\partial}{\partial x_i}(\overline{\tau_{ij}}\tilde{u_j}) - \beta\overline{\rho}\omega^2 \tag{I.12}$$

where $\mu_\kappa = \mu + \mu_T/\sigma_\kappa$ and $\mu_\omega = \mu + \mu_T/\sigma_\omega$. The constants are $\alpha = 5/9$, $\beta = 3/40$, $\beta^* = 9/100$, $\sigma_\kappa = \sigma_\omega = 2$.

If necessary the turbulence models discussed in this section can be non-dimensionalized as discussed in Section 8.2.