APPENDIX

Computing the Drag Force and Stream Function

C

C.1 Drag calculation

The drag force is the resistance offered by a body which is equal to the force exerted by the flow on the body at equilibrium conditions. The drag force arises from two different sources. One is from the pressure p acting in the flow direction on the surface of the body (form drag) and the second is due to the force caused by viscosity effects in the flow direction. In general the drag force is characterized by a drag coefficient, defined as

$$C_d = \frac{D}{A_f \frac{1}{2}\rho_a u_a^2} \tag{C.1}$$

where D is the drag force, A_f is the frontal area in the flow direction, and the subscript a indicates the free stream value. The drag force D contains the contributions from both the influence of pressure and friction, i.e.,

$$D = D_p + D_f \tag{C.2}$$

where D_p is the pressure drag force and D_f is the friction drag force in the flow direction. The pressure drag, or form drag, is calculated from the nodal pressure values. For a two-dimensional problem, the solid wall may be a curve or a line and the boundary elements on the solid wall are one-dimensional with two nodes if linear elements are used. The pressure may be averaged over each one-dimensional element to calculate the average pressure over the boundary element. If this average pressure is multiplied by the length of the element, then the normal pressure acting on the boundary element is obtained. If the pressure force is multiplied by the direction cosine in the flow direction, we obtain the local pressure drag force in the flow direction. Integration of these forces over the solid boundary gives the drag force due to pressure D_p.

The viscous drag force D_f is calculated by integrating the viscous traction in the flow direction, over the surface area. The relation for the total drag force in the x_1 direction may be written for a two-dimensional case as

$$D_{x_1} = \int_{A_s} [(-p + \tau_{11})n_1 + \tau_{12} n_2]\mathrm{d}A_s \tag{C.3}$$

where n_1 and n_2 are components of the surface normal $\mathbf{n}$ as shown in Fig. C.1.

Finite Element Meth Fluid Dynam, 7E, Volume 3. http://dx.doi.org/10.1016/B978-1-85617-635-4.00027-3

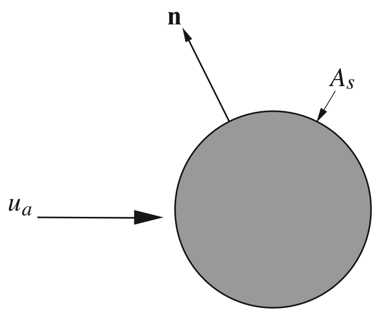

FIGURE C.1

Normal gradient of velocity close to the wall.

C.2 Stream function

In most two-dimensional fluid dynamics and convection heat transfer problems, it is often easier to understand the flow results if the streamlines are plotted. In order to plot these streamlines, or flow pattern, it is first necessary to calculate the stream function values at the nodes. Lines with constant stream function values are referred to as streamlines. The stream function is defined by the following relationships:

$$\begin{aligned} u_1 &= -\frac{\partial \psi}{\partial x_2} \\ u_2 &= \frac{\partial \psi}{\partial x_1} \end{aligned} \tag{C.4}$$

where ψ is the stream function. If we differentiate the first relation with respect to x_2 and the second with respect to x_1 and then sum, we get the differential equation for the stream function as

$$\frac{\partial^2 \psi}{\partial x_2^2} + \frac{\partial^2 \psi}{\partial x_1^2} = \frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2} \tag{C.5}$$

A solution to the above second order equation is straightforward for any numerical procedure. This equation is similar to Step 2 of the CBS scheme and an implicit procedure immediately gives the solution. Unlike the pressure equation of Step 2, the stream function of a solution needs to be calculated only once.