APPENDIX

G

Boundary Layer–Inviscid Flow Coupling

A few references on the topic of boundary layer–inviscid flow coupling are given in Chapter 7. In this appendix we shall briefly explain a simple procedure of this flow coupling procedure. To understand the process of coupling the Euler and integral boundary solutions we shall consider a typical flow pattern around a wing as shown in Fig. G.1. Both turbulent and laminar regimes are shown in this figure.

We summarize the procedure as follows.

Step 1 Solve the Euler equations in the domain considered around the aerofoil. Here any mesh can be used independently of the mesh used for the boundary layer solution. The solution thus obtained will give a pressure distribution on the surface of the wing.

Step 2 Solve the boundary layer using an integral approach over an independently generated surface mesh. If the surface nodes do not coincide with the Euler mesh, the pressure needs to be interpolated to couple the two solutions. The laminar portion near the boundary (Fig. G.1) is calculated by the "Thwaites compressible" method and the turbulent region is predicted by the "lag-entrainment" integral boundary layer model.

Step 3 The Euler and integral solutions are coupled by transferring the outputs from one solution to the other. As indicated in Fig. G.1, direct and semi-inverse couplings can be used for different regions. The semi-inverse coupling is introduced here mainly to stabilize the solution in the turbulent region close to separation. Fig. G.2 shows the flow diagrams for the present boundary layer–inviscid coupling.

Further details on the Thwaites compressible method and semi-inverse coupling can be found in the references discussed in Section 7.13, Chapter 7 (Le Balleur and coworkers).

In Fig. G.2, C_p is the coefficient of pressure; s is the coordinate along the surface; δ is the boundary layer thickness; θ is the momentum thickness; C_f is the skin friction coefficient; H is the velocity profile shape parameter; ρ is the density; V_N is the transpiration velocity; K^* is a factor developed from stability analysis; the subscript v marks the viscous boundary layer region; δ^* is the displacement thickness; the superscript i indicates the inviscid region; and the superscript m indicates the current iteration.

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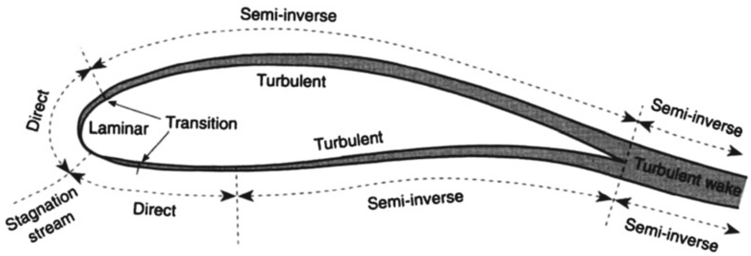

FIGURE G.1

Flow past an aerofoil. Typical problem for boundary layer–inviscid flow coupling.

Following are useful relations for some of the above quantities:

$$H = \frac{\delta^*}{\theta}, \qquad \delta^* = \int_0^\infty \left(1 - \frac{\rho u}{\rho_v u_v}\right) \mathrm{d}n, \qquad K^* = \frac{\beta\lambda}{2\pi\theta}, \qquad \beta = \sqrt{1 - M^2} \tag{G.1}$$

where n is the normal direction from the wing surface.

We have the following equations to be solved in the integral boundary layer lag-entrainment model.

Continuity

$$\theta \frac{\mathrm{d}\bar{H}}{\mathrm{d}s} = \frac{\mathrm{d}\bar{H}}{\mathrm{d}H}\left[C_e - H_1\left(\frac{C_f}{2} - (H+1)\frac{\theta}{u_v}\frac{\mathrm{d}u_v}{\mathrm{d}s}\right)\right] \tag{G.2}$$

Momentum

$$\frac{\mathrm{d}\theta}{\mathrm{d}s} = \frac{C_f}{2} - (H + 2 - M^2)\frac{\theta}{u_v}\frac{\mathrm{d}u_v}{\mathrm{d}s} \tag{G.3}$$

Lag-entrainment

$$\begin{aligned} \theta \frac{\mathrm{d}C_e}{\mathrm{d}s} &= F\left[\frac{2.8}{H + H_1}\left((C_\tau)^{0.5}_{\mathrm{EQ_o}} - \lambda C_f^{0.5}\right) + \left(\frac{\theta}{u_v}\frac{\mathrm{d}u_v}{\mathrm{d}s}\right)_{\mathrm{EQ}}\right. \\ &\quad \left. - \frac{\theta}{u_v}\frac{\mathrm{d}u_v}{\mathrm{d}s}(1 + 0.075M^2)\frac{(1 + 0.2M^2)}{(1 + 0.1M^2)}\right] \end{aligned} \tag{G.4}$$

where F is a function of C_e and C_f and given as

$$F = \frac{\left(0.02C_e + C_e^2 + \dfrac{0.8C_f}{3}\right)}{(0.01 + C_e)} \tag{G.5}$$

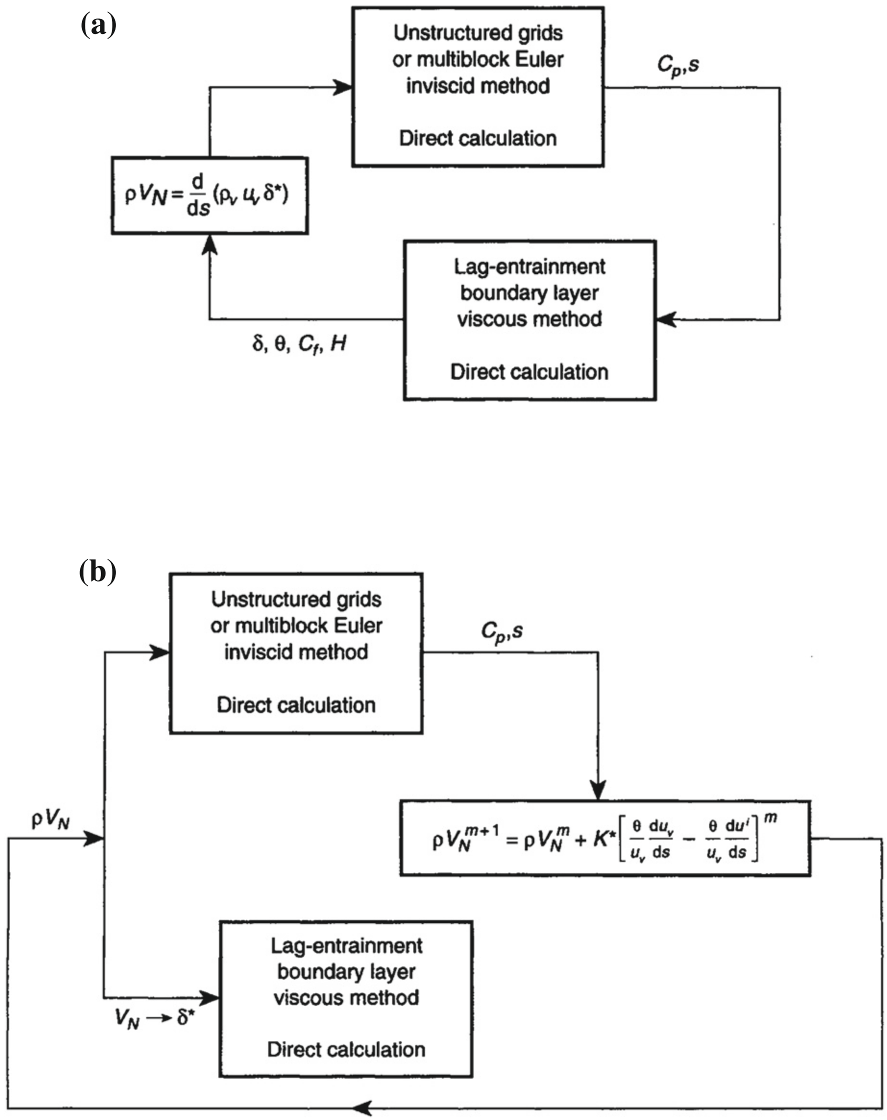

FIGURE G.2

Coupling techniques: (a) direct; (b) semi-inverse.

In the above equations, $\bar{H}$ and H_1 are the velocity profile shape parameters defined as

$$\bar{H} = \frac{1}{\theta}\int_0^\infty \left(1 - \frac{u}{u_v}\right) \mathrm{d}n, \qquad H_1 = \frac{\delta - \delta^*}{\theta} \tag{G.6}$$

C_e is the entrainment coefficient; u_v is the mean component of the streamwise velocity at the edge of the boundary layer; M is the Mach number; C_τ is the shear stress coefficient; λ is the scaling factor on the dissipation length; and the subscripts EQ and EQ_o denote, respectively, the equilibrium conditions and equilibrium conditions in the absence of secondary influences on the turbulence structure.

Once the above equations are solved, the transpiration velocity V_N is calculated as shown in Fig. G.2, added to the standard Euler boundary conditions on the wall, and plays the role of a surface source. The coupling continues until convergence. In practice, in one coupling cycle, several Euler iterations are carried out for each boundary layer solution.