2023/2024 | **Electrical Engineering, Mathematics and Computer Science** | **Master Applied Mathematics**

WI4011-17 | **Computational Fluid Dynamics** | **ECTS: 6**

Responsible Instructor

Name	E-mail
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Contact Hours / Week x/x/x/x
0/0/2/2

Education Period
3
4

Start Education
3

Exam Period
Exam by appointment

Course Language
English

Expected prior knowledge
The students should have had at least introductory courses on the following topics:
+ continuum mechanics
+ partial differential equations
+ tensor calculus linear algebra
+ basic concepts of the finite element method
+ basic concepts of iterative solvers and preconditioning
The students are also expected to be familiar with programming (e.g., in MATLAB or Python).

Course Contents
Part I:
Numerical methods for the convection-diffusion equation;
Stability, consistency and convergence of the numerical methods;
Extensions to incompressible Navier-Stokes equations;
Part II:
Basic iterative solvers and preconditioners for the Poisson problem;
Schwarz domain decomposition preconditioners for the Poisson problem:
Extension of Schwarz preconditioners to saddle point problems (Stokes and Navier-Stokes equations)

Study Goals
Learning objectives of the course: Upon completion, the student should be able to:
Part I:
1. Derive the equations of fluid dynamics using multivariable calculus and physical conservation laws
2. Explain the singularly-perturbed behavior of solutions to the equations of fluid dynamics
3. Discretize the convection-diffusion equation and derive discretization errors
4. Choose discretizations suitable for convection-dominated flows
5. Discretize the incompressible Navier-Stokes equations
Part II:
6. Construct and implement Schwarz domain decomposition methods
7. Explain the concept of the abstract theory of Schwarz methods
8. Apply the abstract Schwarz theory to one- and two-level Schwarz preconditioners to derive condition number bounds for the Poisson problem
9. Apply nonlinear and linear solution techniques to (Navier-)Stokes equations
10. Extend Schwarz methods to (Navier-)Stokes equations

Education Method
The course comprises 14 lectures (7 lectures for Part I taught by S. Jain, 7 lectures for Part II taught by A. Heinlein).

Course Relations
Some concepts discussed in the course WI4201 'Scientific Computing' are assumed as prior knowledge for the course WI4011-17. Even though parts are reviewed very briefly to be adapted to computational fluid dynamics problems, students are strongly recommended to take the course WI4201 before WI4011-17.

Literature and Study Materials
The following texts will serve as supplementary references to the class notes:
+ "The Finite Element Method" by T. J. R. Hughes
+ "Domain Decomposition Methods - Algorithms and Theory" by A. Toselli and O. B. Widlund
+ "Finite Element Methods for Incompressible Flow Problems" by V. John
+ "Principles of Computational Fluid Dynamics" by P. Wesseling
Additional current topics, which are not covered in standard text books yet, will also be discussed. Further references will be given during the course.

Assessment
The final grade of the course consists of the following components:
- Individual assignments (50%)
- Project presentation
- Project report (together 50%)

Final grade calculation: (0.5 * assignments) + (0.5 * presentation and report)

Resit/ Repair opportunities:
In case of an insufficient result, repair opportunities may be offered in accordance with TER Implementation Regulations Art 5, sub 5., for:
- Individual assignments: resubmit deliverable
- Presentation: resit presentation
- Report: resubmit improved report

To pass the course, the following is mandatory:
+ The assignments (written and programming exercises) must be completed with an average grade greater than 5.5.
+ The projects must be completed with grades greater than 5.5.

Disclaimer: information may change depending on unforeseen circumstances or measures (see: TER Art 29, sub 4).

Remarks
This course focuses on the mathematical background, analysis, and motivation of numerical methods for the challenging applications of the incompressible Navier-Stokes equations and convection-diffusion equations. It follows a bottom-up approach, developing the numerical techniques from scratch rather than building a complex numerical framework for simulating realistic fluid problems, as is often done in more engineering focused courses.

Co-Instructor

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