Universidad Autónoma de Chihuahua

Facultad de Ingeniería

NOTEBOOK: Development of Topics Related to the Project

Notes Before Start Working on the Thesis

Student: Leonardo Rafael León Mora.

Thesis director: Dr. Daniel Espinobarro Velázquez.

Advisors:

- M.C. Carlos Hugo Larrinúa Pacheco.
- M.I. Joseph Isaac Ramírez Hernández.

Chihuahua, Chih., April 14, 2024.

BASIC RECAP

DEFINITIONS

1. DIFFERENTIAL EQUATIONS

DEFINITION 1.1.1 - Differential Equation

An equation containing the derivatives of one or more unknown functions (or dependent variables), with respect to one or more independent variables, is said to be a **differential equation (DE)**.

1.1 Notation

- **Leibniz notation**:

$$\frac{dy}{dx},\ \frac{d^2y}{dx^2},\ \frac{d^3y}{dx^3},\ \ldots$$

- **Prime notation**:

$$y',\ y'',\ y''',\ y^{(4)},\ \ldots$$

In general, the nth derivative of y is written as $\frac{d^ny}{dx^n}$ or $y^{(n)}$.

- **Newton's dot notation**: used in physical science and engineering, to denote derivatives with respect to time t,

$$\frac{d^2s}{dt^2} = -32\ \rightarrow\ \text{becomes}\ \rightarrow\ \ddot{s} = -32.$$

- **Subscript notation**: often used in partial derivatives indicating the independent variables.

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} - 2\frac{\partial u}{\partial t}\ \rightarrow\ \text{becomes}\ \rightarrow\ u_{xx} = u_{tt} - 2u_t.$$

1.2 Classification By Type

If a differential equation contains only ordinary derivatives of one or more unknown functions with respect to a single independent variable, it is said to be an **ordinary differential equation (ODE)**. An equation involving partial derivatives of one or more unknown functions of two or more independent variables is called a **partial differential equation (PDE)**.

1.3 Classification By Order

The order of a differential equation (either ODE or PDE) is the order of the highest derivative in the equation. For example,

$$\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^3 - 4y = e^x \ ,$$

is a second-order differential equation.

Part I

MATHEMATICS OF PHYSICS

2. INFINITE SERIES, POWER SERIES

2.1 The Geometric Series[1]

Geometric Progression, is a sequence of numbers where each term after the first is found by multiplying the previous term by a fixed, non-zero number called the common ratio. The **general form** of a geometric progression is:

$$a,\ ar,\ ar^2,\ ar^3,\ \ldots \tag{2.1}$$

where a is the *first term* and r is the *common ratio*. Geometric Progressions have some interesting properties:

- If $r > 1$, the terms will increase.
- If $0 < r < 1$, the terms will decrease.
- If $r = 1$ the sequence becomes constant (where each term is the same).
- If $r < 0$, the terms will alternate between positive and negative values.

i.e., Suppose a bouncing ball rises each time to $\frac{2}{3}$ of the height of the previous bounce. Then

$$1,\ \frac{2}{3},\ \frac{4}{9},\ \frac{8}{27},\ \frac{16}{81},\ \ldots, \tag{2.2}$$

Part II

NUMERICAL METHODS

3. INTRODUCTION

The term **Numerical Methods** refers to those techniques used to approximate the solution to a mathematical problem, these methods are used for mathematical processes such as integrals, differential equations, nonlinear equations in which the solution is close to the exact one and the error of that result can also be quantified by an approximation.

3.1 Steps for solving an Engineering Problem

- **Description**: present the problem exposing every single need related to it, write the background of it and the need for its solution.
- **Mathematical Model**: present an equation that best describes the situation when fitting the given data.
- **Solution of the Mathematical Model**: find the solution to the proposed equation.
- **Using the Solution**: propose a solution to the real problem based on the obtained result from the mathematical model.

4. MATHEMATICAL PROCESSES

4.1 Roots of Nonlinear Equations

4.2 Simultaneous Linear Equations

4.3 Curve Fitting by Interpolation

4.4 Differentiation

4.5 Curve Fitting by Regression

4.6 Numerical Integration

4.7 Ordinary Differential Equations

4.8 Partial Differential Equations

4.9 Optimization

4.10 Fast Fourier Transform

Bibliography

[1] Mary L Boas. *Mathematical methods in the physical sciences.* John Wiley & Sons, 2006.