Static Analysis of Non-minimal Tensegrity Prisms

Introduction

A minimal *k*-bar tensegrity prism has *k* bars and *3k* strings. A non-minimal prism has an extra *k* strings connecting the diagonals. The top and bottom of the structure are *k*-sided polygons, which is formed by connecting strings around one end of the bars. To determine a tensegrity prism, one has to define its following properties:

1. *k*: the number of bars
2. *α*: the twist angle (angle of rotation of the top and bottom polygons)
3. *r*: the radii of the top/bottom polygons.

For simplicity purpose, Matlab code in Appendix B only handles prisms that has the same radii for top and bottom polygons.

To analyze the statics of a prism structure, some required data are needed, listed as the following:

1. *Q*: free node positions
2. *P*: fixed node positions
3. *C*: connectivity matrix (couple nodes with members)
4. *U*: external loads

In the case of tensegrity prisms, all *2k* nodes are considered free nodes. Thus, the connectivity matrix will have the size of *4k x 2k*. Each row represents a member and each column represents a node.

Results and Analysis

After several trials of the Matlab function f_non_min_prism, which analyzes the tensegrity prism using code written by Professor Thomas Bewley, the following observations were made:

1. The structure is potentially inconsistent and undetermined;
2. There are always 3 degrees of freedom no matter the number of bars;
3. The structure is pretensionable.

The static analysis code uses a Singular Value Decomposition to solve for the force density (force per unit length) in the members: $\mathbf{DXC_Q=U}$, written as $\mathbf{A_{se}X=U}$, where **D** is the normalized direction matrix of the member, **X** is the unknown member force density vector, $\mathbf{C_Q}$ is the connectivity matrix associated with free nodes and **U** is the matrix of the external force acting on free nodes. The left hand side of the original equation is then rewritten as $\mathbf{A_{se}X}$.

$\mathbf{A_{se}}$ has *dim x q* rows, where *dim* is the dimension of the problem (2D or 3D) and *q* is the number of free nodes. It has *b+s* columns, where *b* is the number of bars and *s* is the number of strings. When $\mathbf{A_{se}}$ has some rows that are linearly dependent of other rows, that is, the rank of $\mathbf{A_{se}}$ is less then the number of rows, $\mathbf{A_{se}}$ is called potentially inconsistent. The system has either 0 or 1 solution, depending if the external force vector **u** is spanned by the column space of $\mathbf{A_{se}}$. Therefore, when a tensegrity prism is potentially inconsistent, it means that the system has soft modes. A disturbance on the nodes, especially when **u** is in the null space of the column space of $\mathbf{A_{se}}$, could cause large deformation in the structure. Additional strings could be added to remove the soft modes.

When $\mathbf{A_{se}}$ has some columns that are linearly dependent of other columns, the system is called undetermined, implying that the number of unknowns is more than the number of equations. When a system is undetermined, it has some degrees of freedom (DOF) which means control authority of the force density in members. The number of degrees of freedom is the difference between the number of columns of $\mathbf{A_{se}}$ and rank of $\mathbf{A_{se}}$. In the case of tensegrity prisms, trials of the code showed that the DOF of a tensegrity prism is independent of the number of bars it has. The number of DOF of a tensegrity prism is always 3. The physical meaning of DOF here, is that the strings forming the top, bottom and side of the prism have same tension and can be adjusted. And this means that the system is pretensionable. In the Matlab code, the system is pretensioned with tau_min=0.1.

Reference

Skelton, R.E., & de Oliveira, M.C. (2009) Tensegrity Systems. Springer.

Bewley, T (2020) Stabilization of low-altitude balloon systems, Part 2: riggings with multiple taut ground tethers, analyzed as tensegrity systems