wbs
WARWICK BUSINESS SCHOOL
THE UNIVERSITY OF WARWICK

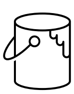
For the Open Minded

Vladimir Deineko

Mathematical Programming I

Solutions/Comments/Problems revisited

1



	Paint 1	Paint 2	Resources Available
Raw material 1	1	2	6
Raw material 2	2	1	8
Price/tonne (£1000)	2	3	

Constraints:

- Can't use more raw material than available:

Row material 1

$$x_1 \quad x_2$$

$$x_1+2x_2 \leq 6$$

Row material 2

$$2x_1+ x_2 \leq 8$$

Decision Variables

x_1 amount of paint 1 produced

x_2 amount of paint 2 produced

	Paint 1	Paint 2	Resources Available
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paint

raw material

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paint

2 r.m./ paint x x_2 paint = 2 x_2

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⦿ **Decision Variables**

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raw material

2 r.m./ paint x x_2 paint = 2 x_2

1.4. The National Coal Board has unlimited supplies of three grades of coal: A,B,C, which contain ash and phosphorus as impurities. A firm requires supplies containing not more than 3% ash and 0.03% phosphorus. The available coal satisfies the following specifications:

	% Phosphorus	%Ash	Profit(£/ton)
A	0.02	3	60
B	0.04	2	75
C	0.03	5	70

Formulate an LP model to determine how the Board should meet the firm's requirements and maximise its own profit.

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Do you have this solution?
1.Yes
2.No

Model 1

max $60x_1+75x_2+70x_3$
s.t.
$0.02x_1+0.04x_2+0.03x_3 \leq 0.03$
$3x_1+ \quad 2x_2 \quad +5x_3 \leq 3$
$x_i \geq 0$

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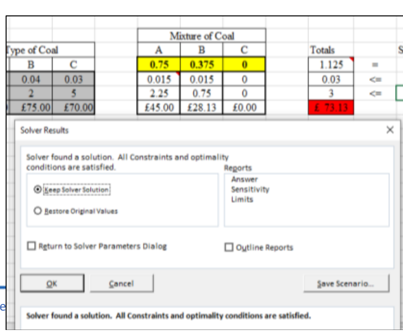

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Modelling…

Variables x_1, x_2, x_3

Amount of coal A, B, and C used to get the mixture x_1, x_2, x_3

$$\max \underbrace{60x_1+75x_2+70x_3}_{\text{£ Total Profit}}$$

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Constraints

How much phosphorus is in x_1 tonnes of coal A?

$x_1 \cdot 0.02/100$

How much phosphorus is in $x_1+x_2+x_3$ tonnes of the mixture coal A, B, and C?

$x_1 \cdot 0.02/100 + x_2 \cdot 0.04/100 + x_3 \cdot 0.03/100$

What is the percentage of phosphorus in $x_1+x_2+x_3$ tonnes of the mixture coal A, B, and C?

$(x_1 \cdot 0.02/100 + x_2 \cdot 0.04/100 + x_3 \cdot 0.03/100)/(x_1+x_2+x_3)$

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Formulate an LP model to determine how the Board should meet the firm's requirements and maximise its own profit.

Variables $x_1,\ x_2,\ x_3$

Amount of coal A, B, and C used to get the mixture $x_1,\ x_2,\ x_3$

$$\max \underbrace{60x_1+75x_2+70x_3}_{\text{£ Total Profit}}$$

Constraints

How much phosphorus is in x_1 tonnes of coal A?

$x_1 \cdot 0.02/100$

How much phosphorus is in $x_1+x_2+x_3$ tonnes of the mixture coal A, B, and C?

$x_1 \cdot 0.02/100 + x_2 \cdot 0.04/100 + x_3 \cdot 0.03/100$

What is the percentage of phosphorus in $x_1+x_2+x_3$ tonnes of the mixture coal A, B, and C?

$((x_1 \cdot 0.02/100 + x_2 \cdot 0.04/100 + x_3 \cdot 0.03/100)/(x_1+x_2+x_3)) \cdot 100$

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	% Phosphorus	%Ash	Profit(£/tonne)
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Formulate an LP model to determine how the Board should meet the firm's requirements and maximise its own profit.

Variables $x_1,\ x_2,\ x_3$

Amount of coal A, B, ... **to get the mixture** $x_1,\ x_2,\ x_3$

$$\max \underbrace{60x_1+75x_2+70x_3}_{\text{£ Total Profit}}$$

Constraints

How m...

$x_1 \cdot 0.02/1$...

What is wrong with this constraint?

How much phosphorus is in x_1+x_2+... coal A, B, and C?

$x_1 \cdot 0.02/100 + x_2 \cdot 0.04/100 + x_3 \cdot 0$...$00$

What is the percentage of phosphorus in $x_1+x_2+x_3$ tonnes of the mixture coal A, B, and C?

$((x_1 \cdot 0.02/100 + x_2 \cdot 0.04/100 + x_3 \cdot 0.03/100)/(x_1+x_2+x_3)) \cdot 100 \le 0.03$

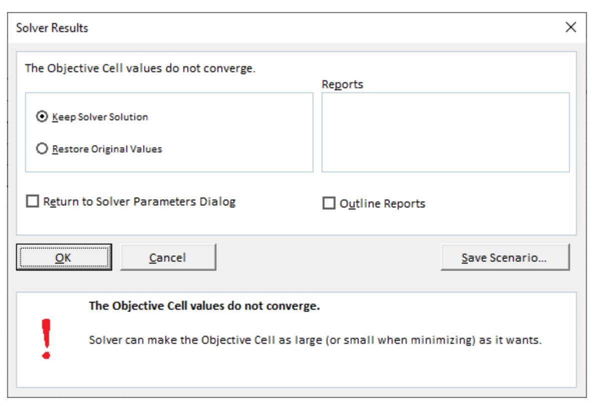
Are you happy with the model below? Vote

Have you tried to solve this model?
1. Yes
2. No

Model 2

$$\max\ 60x_1+75x_2+70x_3$$
s.t.
$$0.02x_1+0.04x_2+0.03x_3 \le 0.03(x_1+x_2+x_3)$$
$$3x_1+2x_2+5x_3 \le 3(x_1+x_2+x_3)$$
$$x_i \ge 0$$

Are you happy with the model below? Vote



Model 2

$$\max\ 60x_1+75x_2+70x_3$$
s.t.
$$0.02x_1+0.04x_2+0.03x_3 \le 0.03(x_1+x_2+x_3)$$
$$3x_1+2x_2+5x_3 \le 3(x_1+x_2+x_3)$$
$$x_i \ge 0$$

Final model

Correct Model

$$\max \underbrace{60x_1+75x_2+70x_3}_{\text{£ ?}}$$

s.t.

$$0.02x_1+0.04x_2+0.03x_3 \le 0.03(x_1+x_2+x_3)$$
$$3x_1+ \quad 2x_2 \quad +5x_3 \le 3(x_1+x_2+x_3)$$
$$(x_1+x_2+x_3)=1$$
$$x_i \ge 0$$

Final model

Correct Model

$$\max \underbrace{60x_1+75x_2+70x_3}_{\text{Profit from 1 tonne}}$$

s.t.

$$0.02x_1+0.04x_2+0.03x_3 \le 0.03(x_1+x_2+x_3)$$
$$3x_1+ \quad 2x_2 \quad +5x_3 \le 3(x_1+x_2+x_3)$$
$$(x_1+x_2+x_3)=1$$
$$x_i \ge 0$$

Final model

Correct Model

$$\max\ \underbrace{60x_1+75x_2+70x_3}_{}$$

s.t.

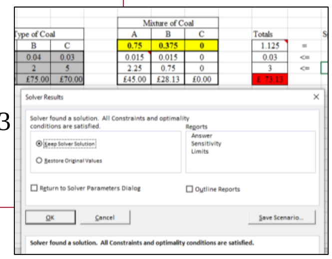
$$0.02x_1+0.04x_2+0.03x_3 \le 0.03(x_1+x_2+x_3)$$
$$3x_1+\quad 2x_2 \quad +5x_3 \le 3(x_1+x_2+x_3)$$
$$(x_1+x_2+x_3)=1$$
$$x_i \ge 0$$

Model 1 (Wrong model)

$$\max\ 60x_1+75x_2+70x_3$$

s.t.

$$0.02x_1+0.04x_2+0.03x_3 \le 0.03$$
$$3x_1+\quad 2x_2 \quad +5x_3 \le 3$$
$$x_i \ge 0$$

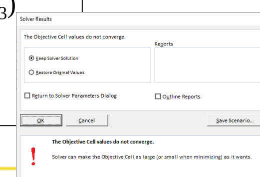


Model 2 (Incomplete model)

$$\max\ 60x_1+75x_2+70x_3$$

s.t.

$$0.02x_1+0.04x_2+0.03x_3 \le 0.03(x_1+x_2+x_3)$$
$$3x_1+\quad 2x_2 \quad +5x_3 \le 3(x_1+x_2+x_3)$$
$$x_i \ge 0$$



Your new task: solve the model graphically

$$\max\ 60x_1+75x_2+70x_3$$

s.t.

$$0.02x_1+0.04x_2+0.03x_3 \le 0.03(x_1+x_2+x_3)$$
$$3x_1+\quad 2x_2 \quad +5x_3 \le 3(x_1+x_2+x_3)$$
$$(x_1+x_2+x_3)=1$$
$$x_i \ge 0$$

Cargo problem

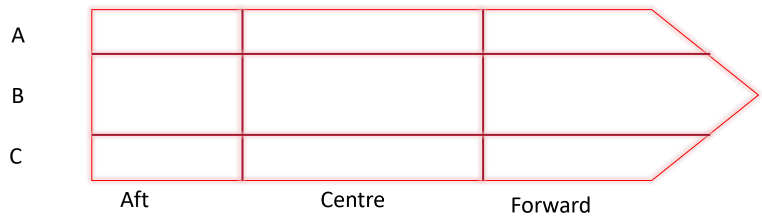
1.5. A ship has three cargo holds: forward, aft and centre. The capacity limits are:

Forward	2000 tons	100,000 cu ft
Centre	3000 tons	135,000 cu ft
Aft	1500 tons	30,000 cu ft

The following cargoes are offered and the shipowners may accept all or any part of each commodity

Commodity	Amount	Vol per ton	Profit(£/ton)
A	6,000	60	6
B	4,000	50	8
C	2,000	25	5

Formulate an LP model to determine how the cargo should be distributed to maximise profits.



Cargo problem

max $6(x_{1A}+x_{2A}+x_{3A})+8(x_{1B}+x_{2B}+x_{3B})+5(x_{1C}+x_{2C}+x_{3C})$

s.t.

$x_{1A}+x_{2A}+x_{3A} \leq 6000$

$x_{1B}+x_{2B}+x_{3B} \leq 4000$

$x_{1C}+x_{2C}+x_{3C} \leq 2000$

$x_{1A}+x_{1B}+x_{1C} \leq 2000$

$x_{2A}+x_{2B}+x_{2C} \leq 3000$

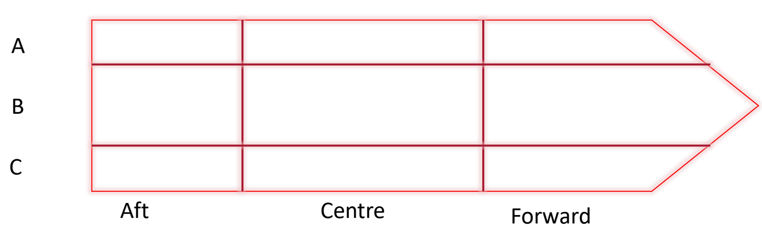
$x_{3A}+x_{3B}+x_{3C} \leq 1500$

$60x_{1A}+50x_{1B}+25x_{1C} \leq 100\,000$

$60x_{2A}+50x_{2B}+25x_{2C} \leq 135\,000$

$60x_{3A}+50x_{3B}+25x_{3C} \leq 30\,000$

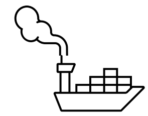
$x_{ij} \geq 0,\ i \in \{1,2,3\},\ j \in \{A,B,C\}$

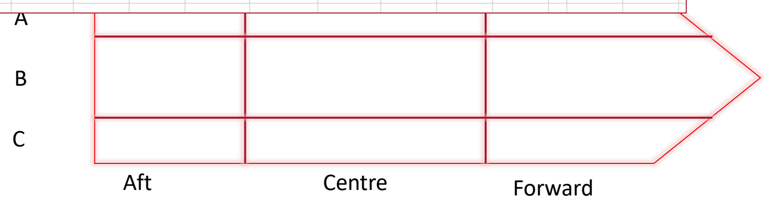


Cargo problem: Check data!!!

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1														
2				Weight (tons)						Volume (cu ft)			Volume	Profit
3	Commodity Available	Commodity	Forward	Center	Aft	TotalWeight	Profit	Commodity	Forward	Center	Aft		per ton	per ton
4	6000	A	305.55556	0	0	305.55556	£1,833.33	A	18333.33	0	0		60	£6.00
5	4000	B	1576.9231	2423.077	0	4000	£32,000.00	B	78846.15	121153.8	0		50	£8.00
6	2000	C	117.52137	576.9231	1250	1944.4444	£9,722.22	C	2820.513	13846.15	30000		24	£5.00
7				Weight per Section						Volume per section				
8			2000	3000	1250		£43,555.56		100000	135000	30000			
9				Weight Allowable						Volume allowable				
10			2000	3000	1500				100000	135000	30000			
11														
12														
13														
14		Named Ranges												
15	CommodityAvailable	='Cargo(TestPr2)'!$A$4:$A$6												
16	TotalWeight	='Cargo(TestPr2)'!$F$4:$F$6												
17	VolumeAllowable	='Cargo(TestPr2)'!$I$10:$K$10												
18	VolumePerSection	='Cargo(TestPr2)'!$I$8:$K$8												
19	WeightAllowable	='Cargo(TestPr2)'!$C$10:$E$10												
20	WeightPerSection	='Cargo(TestPr2)'!$C$8:$E$8												
21														
22														

List of used named ranges can help in understanding the model (Insert, Name, Paste List)





Comments on basic solutions

Solving Systems of Linear Equations

*Example*_3.8 Solve the system

$$\mathbf{x_1} + x_2 \qquad = 1$$

$$x_2 + \mathbf{x_3} = 3$$

$$x_1 + 2x_2 + x_3 = 4$$

For the system

$x_1 + x_2 \quad = 1$

$x_2 + x_3 = 3$

the solution $x_1=1$, $x_3=3$, $x_2=0$ is the ***basic solution*** with the set of ***basic variables*** $\{x_1,x_3\}$ and the set on ***nonbasic*** variables $\{x_2\}$.

Comments on basic solutions

$$\begin{aligned} \mathbf{x_1} + x_2 \quad\quad &= 1 \\ x_2 + \mathbf{x_3} &= 3 \\ x_1 + 2x_2 + x_3 &= 4 \end{aligned}$$

For the system

$x_1 = 1 - x_2$

$x_3 = 3 - x_2$

the solution $x_1=1$, $x_3=3$, $x_2=0$ is the ***basic solution*** with the set of ***basic variables*** $\{x_1, x_3\}$ and the set on ***nonbasic*** variables $\{x_2\}$.

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Basic Solutions

$$m \; \underbrace{\boxed{\;\; A \;\;}}_{} \!\!\!\overset{n}{} \qquad n \geq m$$

A *basic solution* to $A\mathbf{x}=\mathbf{b}$ is obtained by setting *n-m* variables (the *nonbasic variables*) equal to 0 and solving for the values of the remaining *m* variables (the *basic variables*). That assumes that setting *n-m* variables equal to 0 yields unique values for the remaining *m* variables.

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Basic Solutions

$$m \; \overset{n}{\boxed{\;A\;}} \qquad n \geq m$$

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$$\begin{aligned} \mathbf{x_1} + x_2 \phantom{{}+ x_3} &= 1 \\ x_2 + \mathbf{x_3} &= 3 \\ x_1 + 2x_2 + x_3 &= 4 \end{aligned}$$

For the system

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Basic Solutions

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$$\begin{aligned} x_1 + x_2 \phantom{{}+ x_3} &= 1 \\ x_2 + x_3 &= 3 \end{aligned}$$

For the system

$x_2 = 1 - x_1$

$x_3 = 2 + x_1$

the solution $x_2=1$, $x_3=2$, $x_1=0$ is the ***basic solution*** with the set of ***basic variables*** $\{x_2,x_3\}$ and the set on ***nonbasic*** variables $\{x_1\}$.

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Basic Solutions

$$m \; \underset{}{\overset{n}{\boxed{\quad A \quad}}} \qquad n \geq m$$

A *basic solution* to ***Ax=b*** is obtained by setting *n-m* variables (the *nonbasic variables*) equal to 0 and solving for the values of the remaining *m* variables (the *basic variables*). That assumes that setting *n-m* variables equal to 0 yields unique values for the remaining *m* variables.

$$\begin{aligned} x_1 + x_2 \phantom{{}+x_3} &= 1 \\ x_2 + x_3 &= 3 \end{aligned}$$

For the system

$x_1 = -2 + x_3$

$x_2 = 3 - x_3$

the solution $x_1=-2$, $x_2=3$, $x_3=0$ is the ***basic solution*** with the set of ***basic variables*** $\{x_1,x_2\}$ and the set on ***nonbasic*** variables $\{x_3\}$.

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Basic Solutions

$$m \; \underset{}{\overset{n}{\boxed{\quad A \quad}}} \qquad n \geq m$$

A *basic solution* to ***Ax=b*** is obtained by setting *n-m* variables (the *nonbasic variables*) equal to 0 and solving for the values of the remaining *m* variables (the *basic variables*). That assumes that setting *n-m* variables equal to 0 yields unique values for the remaining *m* variables.

$$\begin{aligned} x_1 + x_2 \phantom{{}+x_3} &= 1 \\ x_2 + x_3 &= 3 \end{aligned}$$

For the system

$x_1 = -2 + x_3$

$x_2 = 3 - x_3$

the solution $x_1=-2$, $x_2=3$, $x_3=0$ is the ***basic solution*** with the set of ***basic variables*** $\{x_1,x_2\}$ and the set on ***nonbasic*** variables $\{x_3\}$.

For the system

$x_2 = 1 - x_1$

$x_3 = 2 + x_1$

the solution $x_2=1$, $x_3=2$, $x_1=0$ is the ***basic solution*** with the set of ***basic variables*** $\{x_2,x_3\}$ and the set on ***nonbasic*** variables $\{x_1\}$.

For the system

$x_1 = 1 - x_2$

$x_3 = 3 - x_2$

the solution $x_1=1$, $x_3=3$, $x_2=0$ is the ***basic solution*** with the set of ***basic variables*** $\{x_1,x_3\}$ and the set on ***nonbasic*** variables $\{x_2\}$.

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