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WARWICK BUSINESS SCHOOL
THE UNIVERSITY OF WARWICK

For the Open Minded

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Introduction to Linear Programming

1

Formulation of Linear Programming Problems, Solving LPs with Spreadsheets

You will have mastered the material discussed at the lecture WHEN, given a practical problem, you can

- identify decision variables
- formulate an objective function
- formulate constraints on the decision variables
- set up the problem in an attractive (spreadsheet) format
- test a solution given for feasibility

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Introduction to Linear Programming (LP)

- LP is a technique for dealing with constrained optimisation problems, e.g., the allocation of limited resources in an optimal manner.

- **Three parts of LP models**
  - Seek to optimise some objective (e.g., maximise returns, minimise costs)
  - By modifying a set of decision variables (e.g., product mix, delivery quantity)
  - Subject to a set of constraints (labour, funds, materials, time)
- All 'relationships' are of linear type.

Some Typical LP Applications

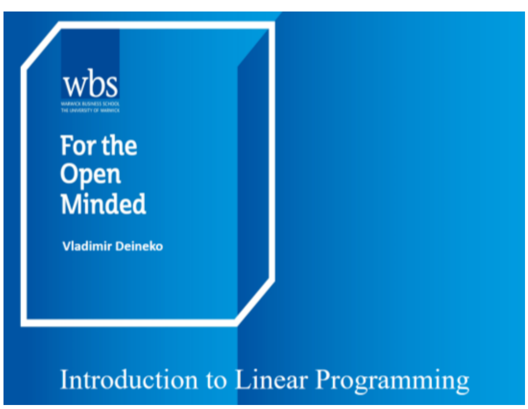
- **Manufacture**
  - Decide a product mix to maximise profits, subject to production capability.
- **Finance**
  - Select an investment portfolio from a variety of stock and bond investment alternatives to maximise the return on investment.
- **Marketing**
  - Determine how best to allocate a fixed advertising budget among alternative advertising media (such as radio, TV, newspapers and magazines) to maximise the advertising effectiveness or to minimise the total cost of advertising.

Some Typical LP Applications

- **Blending**
  - Determine the quantities of ingredients to achieve a desired mix of animal food at minimum cost.
- **Logistics**
  - A company has warehouses in a number of locations. Given a set of customer demands for its products, the company wants to determine which warehouse should ship how much product to which customers, so that the total transportation costs are minimised.

Some Unusual LP Applications

Please google and send to me the links to the most unexpected LP applications



Example 1 **(Paint Problem) A company makes two kinds of paint in bulk by mixing two raw materials in different proportions; the availability of materials, demand for the final product, and the selling prices, are:**

	Paint 1 needs	**Paint 2 needs**	**Resources Available** (unspecified units)
Raw material 1 (unspecified units)	1	2	6
Raw material 2 (unspecified units)	2	1	8
Price/tonne (£1000)	3	2	

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Question: **Which product mix maximises the company's gross income?**

Formulation of LP Model

- **Define appropriate decision variables**
  - A variable for every separately identifiable entity. Specify the units of measurement
  - Use descriptive letters (e.g., R and T) or mathematical symbols (e.g. x_1, x_i, x_{ij})

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Question: Which product mix maximises the company's gross income?

What do we want to find?

"Product mix" -> amount of Paint 1 and amount of Paint 2

x_1=amount of Paint 1

x_2=amount of Paint 2

Formulation of ~~LP~~ Models

- **Formulate objective function**
  - Maximise or minimise a function (should include at least one decision variable)

Formulation of ~~LP~~ Model

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What do we want to achieve?   What is our objective?

Maximal gross income (money obtained from selling the paint)

Income depends on the amount of paint produced

Income is a **function** of the amount of Paint 1 and amount of Paint 2

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Income depends on the amount of paint produced

Income depends on the amount of Paint 1 and amount of Paint 2

- **<u>Our Decision Variables</u>**

x_1 amount of paint 1 produced

x_2 amount of paint 2 produced

- **<u>Objective function:</u>**

maximise gross income

max z=function of x_1 and x_2

max z=$\mathbf{f}(x_1, x_2)$

function z=f(x1, x2) is given… means

given x_1 and x_2, you can calculate the value of f(x1, x2)

	Paint 1	Paint 2	Resources Available
Raw material 1	1	2	6
Raw material 2	2	1	8
Price/tonne (£1000)	3	2	

amount of paint 1 produced is 10 tonnes

x_1=10

amount of paint 2 produced is 20 tonnes

x_2=20

Calculate gross income z=

z=3*10+2*20 (thousand)

- **<u>Objective function:</u>**

$$\max z=3x_1+2x_2$$

Formulation of ~~LP~~ Model

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Function that can be written in the form
$f(x_1, x_2, x_3\ldots, x_n)= c_0+ c_1x_1+ c_2x_2 + c_3x_3 + \ldots + c_nx_n$
is called a linear function

The objective function in our model is a linear function

Formulation of ~~LP~~ Models

- **Formulate constraints**
  - It is a good practice to start with formulating constraints in plain English, e.g. “budget used should not accede money raised”
  - Represent (translate) then the constraints in an algebraic form
  - **All terms in the constraint must have the same units.**

Formulation of ~~LP~~ Model

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Question: Which product mix maximises the company’s gross income?

<u>Constraints:</u>

- Can’t use more raw material than available:

Row material 1

$$x_1 \quad x_2$$

$$x_1+2x_2 \leq 6$$

Row material 2

$$2x_1+x_2 \leq 8$$

Linear Programming Model

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Price/tonne (£1000)	3	2	

Question: Which product mix maximises the company's gross income?

Let

x_1 be the amount of paint 1 produced

x_2 be the amount of paint 2 produced,

then the problem can be formulated as the following linear programming model

$$\max\ z=3x_1+2x_2$$

subject to

$$\begin{aligned} x_1+\ 2x_2 &\le 6 \\ 2x_1+\ \ x_2 &\le 8 \end{aligned}$$

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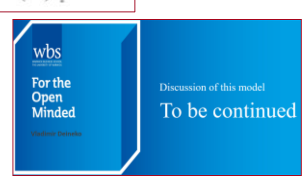
A **linear programming problem** (LP) is an optimisation problem for which we do the following:

1. We attempt to maximise (or minimise) a *linear* function of the *decision variables*. The function that is to be maximised or minimised is called the *objective function*.
$F(x_1, x_2, x_3, x_4 \ldots) = c_1x_1 + c_2x_2 + c_3x_3 + \ldots$

2. The value of the decision variables must satisfy a set of *constraints*. Each constraint must be a linear equation or linear inequality:
$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots \le (\ge =)\ b_1$
$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots \le (\ge =)\ b_2$



I will further discuss this model in my next lecture.

Your task: *Critically* to revise the model and be sure that we have not missed anything. Prepare your questions (if any).

Modelling with spreadsheets

Spreadsheets help to better understand the problem and see it from a different perspective

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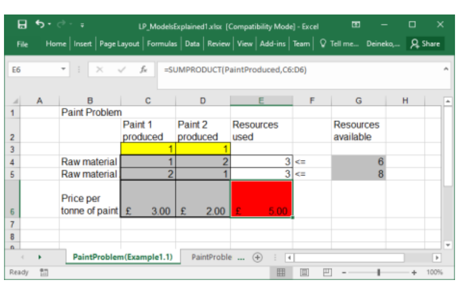
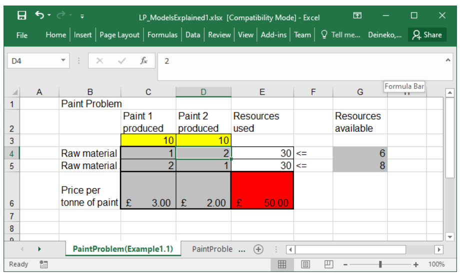


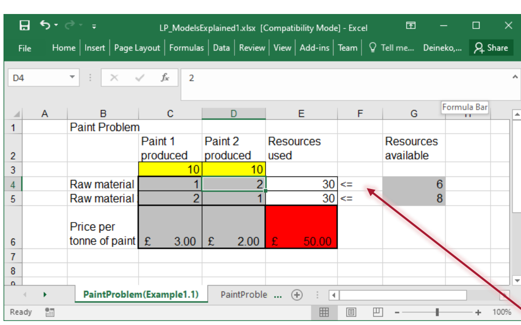
1. Summarise data in a table(s);
2. Define cells to contain the values of the decision variables;
3. Assume that you've decided on the values of the decision variables, e.g. 1 and 1.
4. Calculate the resources used and compare with the resources available;
5. Calculate the value of the objective function;

Spreadsheets help to understand the problem and see it from a different perspective



1. Is it possible to produce 10 tonnes of each paint?
2. What is missing?

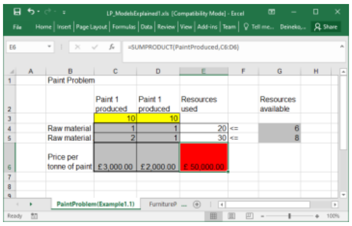
Spreadsheets help to understand the problem and see it from a different perspective



1. Solution that satisfies all constraints is called a **feasible** solution
2. Solution (10,10) is **unfeasible**
3. In the settings above we have not incorporated constraints checking;

This is just a text

Spreadsheets help to understand the problem and see it from a different perspective



1. Solution that satisfies all constraints is called a **feasible** solution

2. The set of feasible solutions (for the paint problem model we have formulated above) is the set of pairs (x_1,x_2) that satisfy the inequalities

$$\begin{aligned} x_1 + 2x_2 &\le 6 \\ 2x_1 + x_2 &\le 8 \end{aligned}$$

Investigate the model using a spreadsheet. Try to manually find the best feasible solution.

https://my.wbs.ac.uk/-/academic/212627/home/
Please work through Lesson 11 and think how you could represent the set of feasible solutions

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To be continued

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