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Source: The Economic Journal, Vol. 97, No. 387 (Sep., 1987), pp. 666-684

Published by: Wiley on behalf of the Royal Economic Society

Stable URL: http://www.jstor.org/stable/2232929

Accessed: 16-03-2016 10:40 UTC

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# ANTICIPATION AND THE VALUATION OF DELAYED CONSUMPTION\*

# George Loewenstein

'When calculating the rate at which a future benefit is discounted, we must be careful to make allowance for the pleasures of expectation.' (Marshall, 1891, p. 178)

Of the various assumptions underlying analyses of intertemporal choice, perhaps the assumption of positive discounting is the most widespread and noncontroversial. Empirical work which has sought to estimate individual discount rates (Hausman, 1979; Landsberger, 1971) has provided no grounds for questioning this assumption. In fact, a recent study which explicitly questioned the general applicability of positive discounting concluded that 'the case for positive time preference is absolutely compelling' (Olson and Bailey, 1981).

Yet it requires little effort to think of examples of behaviour in which negative discounting is apparent. The pleasurable deferral of a vacation, the speeding up of a dental appointment, the prolonged storage of a bottle of expensive champagne are all instances of this phenomenon. Indeed, if R. H. Strotz had begun his work thirty years ago with behaviour such as this in mind, he might have developed a critique of Discounted Utility theory (DU) equally as compelling as his work on myopia but pointing towards research very different from what has actually ensued from his work. Instead, in introducing the broad concept of 'time inconsistency', Strotz devoted his attention exclusively to a subdomain of instances in which the economic actor behaves more myopically in the present than he previously had planned. While the focus on impulsivity has offered important theoretical insights it may have impeded recognition of the existence and interest of other phenomena, such as low or negative discounting. A more inclusive theory of intertemporal choice should be able to account for both extremes of behaviour - myopic and far-sighted. Such a model is proposed here.

The model modifies DU by introducing an insight once recognised by economists: that anticipation of the future has an impact on immediate well-being. This observation can be traced to Bentham (1789), who included among the ingredients of utility, pleasures and pains that derive from anticipation. For Bentham, anticipation, like consumption itself, was an important source of pleasure and pain.

<sup>\*</sup> I thank Robert Abelson, Richard Levin, Sidney Winter, John Geanakoplos, Howard Kunreuther, Colin Camerer, Mark Machina and Robin Pope for their helpful suggestions. The support of the Institute for Advanced Study, in Princeton, is gratefully acknowledged.

<sup>&</sup>lt;sup>1</sup> In exploring the relationship between anticipation and time discounting, the current paper is akin to the recent work of Pope (1983), who examined the role of anticipation in risk aversion, and Wolf (1970), who discussed the implications for intertemporal choice of utility from memory.

Jevons who was one of the first to apply the Benthamite concept of utility to understanding intertemporal trade-offs, wrote: 'Three distinct ways are recognisable in which pleasurable or painful feelings are caused:

- (1) By the memory of past events;
- (2) By the sensation of present events;
- (3) By anticipation of future events.' (1905, p. 3)

The latter, which Jevons termed 'anticipal pleasure' and 'anticipal pain' were, if anything, the most important for understanding economic behaviour: 'The science of economics is very largely occupied in studying man's efforts to obtain anticipal pleasure by the provision of stocks of goods for future use: almost all the complicated practices of production and exchange resolve themselves ultimately into manifestations of these efforts' (1905, p. 65). In what follows, the term 'savouring' refers to positive utility derived from anticipation of future consumption; 'dread' refers to negative utility resulting from contemplation of the future.

## I. AN ILLUSTRATIVE STUDY

Fig. 1 summarises results from a survey in which 30 undergraduates were asked to specify the 'most you would pay now' to obtain (avoid) each of five outcomes, immediately, and following five different time delays. The outcomes were: (1) obtain four dollars; (2) avoid losing four dollars; (3) avoid losing one thousand

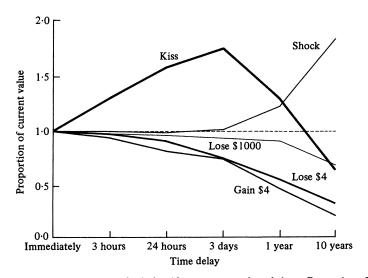


Fig. 1. Maximum payment to obtain/avoid outcomes at selected times. Proportion of current value (N=30).

dollars; (4) avoid receiving a (non-lethal) one hundred and ten volt shock, and (5) obtain a kiss from the movie star of your choice. Time delays were: (1) immediately (no delay); (2) in twenty-four hours; (3) in three days; (4) in one year; (5) in ten years. Subjects were asked to specify the most they would pay

for every combination of outcome and time delay. They were told to assume that all outcomes were certain to occur at the designated time. Summary statistics for the study are presented in Appendix 1.

It can be seen that the two non-monetary items, the kiss and the shock, both exhibit unusual patterns of devaluation. DU, with positive discounting, predicts that people will prefer to consume desired outcomes as soon as possible. This prediction is contradicted by the kiss item. Subjects on average were willing to pay more to experience a kiss delayed by 3 days than an immediate kiss or one delayed by three hours or one day. Data presented later in this paper show that the hump-shaped pattern of devaluation evident in Fig. 1 is common for desirable consumption that is fleeting.

Likewise, DU asserts that people prefer to delay undesirable outcomes whenever possible. The shock item contrasts sharply with this prediction. Subjects were, on average, willing to pay slightly more to avoid a shock that was delayed for 3 hours to 3 days than to avoid an immediate shock. They were willing to pay substantially more to avoid a shock delayed by one or ten years. In contrast to the patterns of responses for the kiss and shock, the money amounts included in the survey appear to be discounted in the normal fashion.

Why haven't patterns of intertemporal preference such as those exhibited towards the kiss and shock appeared in earlier empirical work on intertemporal choice? Several answers are possible. In some cases, economists have attempted to infer intertemporal preferences from behaviour in which such preferences were irrelevant to behaviour. Hausman's attempt to estimate individual discount rates from air-conditioner purchases provides an example. Air conditioners vary in purchase price and energy efficiency, thus creating a choice between immediate versus deferred payments. Hausman, by observing the price/efficiency rating of a purchased air-conditioner, attempted to impute the discount rate of the purchaser. The problem with this approach is that individual discount rates should be irrelevant to what model is purchased. If consumers are able to save and dissave (or borrow) at established interest rates, they should logically purchase the model of air conditioner that minimises, at the desired level of cooling capacity, the net present value of the time stream of payments.2 Similar considerations may have caused subjects in the current experiment to discount the money amounts in a conventional manner, in contrast to their behaviour towards the kiss and shock.

In other cases, economists have attempted to estimate discount rate from individual saving behaviour, but such attempts are even more problematic since individuals have little influence on interest rates. The interest rate at which an individual saves or borrows gives no information about his or her own discount rate and hence it is necessary to infer discount rates from level or rate of saving, a process that is extremely sensitive to the specification of the model used to represent the savings decision. Furthermore, savings behaviour depends on so many factors other than discount rates (e.g. expected future income streams,

<sup>&</sup>lt;sup>2</sup> His finding of substantial differences in discounting between different income groups suggests either that unobserved economic factors such as liquidity constraints were operative, or that consumers were failing to behave rationally.

projected needs) that it is exceedingly difficult to isolate the effect of time preference on saving.

In this paper it is argued that patterns of preference such as those exhibited towards the kiss and shock, and other DU anomalies discussed below, can be explained by incorporating Jevons' anticipal pleasure and pain into an otherwise standard model of intertemporal choice. In what follows, such a model is developed in the simplest possible terms, and its implications are discussed.

#### II. THE MODEL

The following model explores the question of how an individual values a single future act of consumption under conditions of certainty. The model depicts a consumer at time  $t_0$  who anticipates consuming x at time  $T \ge t_0$ . Consumption is assumed to yield a constant stream of utility, U(x), beginning at time T and continuing for duration L, after which it drops to zero. Formally:

$$\begin{array}{ll} U_t^c(x,\,T,L) = U(x) & \text{for} \quad T \leqslant t \leqslant T + L, \\ = \text{o} \quad \text{otherwise} \end{array} \tag{1}$$

where  $U_t^c$  indicates utility experienced at time t from consumption.

At any time t between  $t_0$  and T (when consumption begins) the individual derives utility from anticipation,  $U_t^A$ . Utility from anticipation is assumed to be proportional to the integral of utility from consumption discounted at a rate of  $\delta$ .  $\delta$  is not the conventional discount rate, but a measure of the degree to which the individual derives immediate utility from anticipated consumption. Thus sayouring or dread at each point t is equal to:

$$U_t^A(x, T, L) = \alpha \int_T^{T+L} e^{-\delta(\tau - t)} U(x) d\tau$$
 (2)

$$= \frac{\alpha}{\delta} U(x) e^{-\delta(T-t)} (I - e^{-\delta L}). \tag{3}$$

This formulation has four desirable properties discussed by Jevons in his enumeration of the laws of anticipal pleasure and pain. Referring to anticipation of a planned vacation, Jevons wrote:

The intensity of the anticipation will be greater the longer the holiday; greater also, the more intensely one expects to enjoy it when the time comes. In other words the amount of pleasure expected is one factor determining the intensity of anticipal pleasure. Again, the nearer the date fixed for leaving home approaches, the greater does the intensity of anticipal pleasure become: at first when the holiday is still many weeks ahead, the intensity increases slowly; then, as the time grows closer, it increases faster and faster, until it culminates on the eve of departure (1905, p. 64).

In the current formulation, as Jevons proposed, utility from anticipation,  $U_t^A$ , is a positive function of L, the duration of consumption, a positive function

of U(x), the utility that will be derived from consumption, and a negative function of (T-t), the time delay prior to consumption. Also, the second derivative of  $U^A$  with respect to (T-t) is positive, yielding the accelerating path of utility from anticipation suggested by Jevons.

Fig. 2 depicts one possible time path of utility from anticipation and consumption.

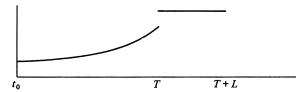


Fig. 2. Utility from anticipation and consumption.

The individual is assumed to evaluate a delayed act of consumption according to the integral of discounted utility from anticipation and consumption that it yields. Thus, the present value Y (measured in dollars) of a delayed act of consumption is defined by:

$$U(Y) = \int_{t_0}^T \frac{\alpha}{\delta} U(x) \, e^{-\delta(T-t)} \left( \mathbf{I} - e^{-\delta L} \right) e^{-r(t-t_0)} \, dt + \int_T^{T+L} U(x) \, e^{-r(t-t_0)} \, dt, \quad (4) = 0$$
utility from anticipation

where r is the conventional discount rate used to discount future utility from all sources and U(Y) is a 'ratio scale' utility function<sup>3</sup> with positive first and negative second derivative. Throughout the following, r is assumed to be positive.

Setting  $t_0 = 0$  for simplicity, and integrating:

$$U(Y) = U(x) \left[ \frac{\alpha}{\delta(\delta-r)} \left( e^{-rT} - e^{-\delta T} \right) \left( \mathbf{I} - e^{-\delta L} \right) + \frac{\mathbf{I}}{r} e^{-rT} \left( \mathbf{I} - e^{-rL} \right) \right]. \tag{5}$$

 $\delta$  and  $\alpha$  define to the relationship between  $U^A$  and  $U^c$ .  $\delta$  is a measure of an individual's preoccupation with the future. Someone with a low  $\delta$  savours or dreads even those outcomes that will occur in the distant future.  $\alpha$  is a measure of the 'imaginability' or 'vividness' of a particular outcome. Factors that raise  $\alpha$  or lower  $\delta$  increase utility from savouring or dread. (A later section discusses attempts on the part of marketers and policy makers to influence  $\alpha$ .) Since most people take account of future outcomes (e.g. save for their retirement) even when they do not immediately savour or dread those outcomes, it is assumed that  $\delta > r$ .

## III. IMPLICATIONS OF THE MODEL

The model, as it stands, suggests the conditions under which people will prefer to delay desired consumption or get undesirable consumption over with quickly. These two cases are explored in turn.

<sup>&</sup>lt;sup>3</sup> I.e. invariant with respect to multiplicative transformations.

# Delaying of Desired Consumption

Desired consumption will be delayed when  $(\partial Y/\partial T) > 0$  i.e. when the net present value of consumption, taking account of both savouring and consumption itself, increases as a function of time delay. Since U(Y) is monotonically increasing, this condition is equivalent to  $\partial U(Y)/\partial T > 0$ .

Differentiating (5) with respect to T:

$$\frac{\partial U(Y)}{\partial T} = U(\mathbf{x}) \left[ \frac{\alpha}{\delta(\delta-r)} \left( \delta e^{-\delta T} - r e^{-rT} \right) \left( \mathbf{I} - e^{-\delta L} \right) - e^{-rT} (\mathbf{I} - e^{-rL}) \right] \! . \tag{6}$$

The first term in the brackets is the marginal benefit from savouring, that would be gained from delaying. The second term is the marginal cost of delay, in terms of increased discounting of consumption.

Consumption will be deferred when  $\partial U(Y)/\partial T$  is positive for T=0. Setting T=0:

$$\left.\frac{\partial U(Y)}{\partial T}\right|_{T=0} = U(x)\frac{\alpha}{\delta}\left(\mathbf{1}-e^{-\delta L}\right) - U(x)\left(\mathbf{1}-e^{-rL}\right). \tag{7}$$

A necessary and sufficient condition for delaying desired consumption is, therefore,

 $\frac{\alpha}{\delta} \left( \mathbf{I} - e^{-\delta L} \right) > \mathbf{I} - e^{-rL}. \tag{8}$ 

As would be expected, delaying is more likely when  $\alpha$  is large and  $\delta$  is small. Fig. 3 illustrates the relationship between Y, the net present value of consumption and T, the time at which consumption of x begins, for the case when  $(\partial Y/\partial T)$  is initially positive.

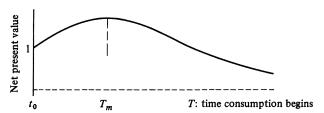


Fig. 3. Value at time  $t_0$  of an outcome to be consumed at time T.

By reading off the net present value of consumption at any point T on Fig. 3, and subtracting it from one, it is possible to derive a crude measure of the value of consumption of x at time T relative to the value of immediate consumption of x. Note that such 'devaluing' (negative or positive) of consumption as a function of delay must be distinguished from the individual's discounting of future utility, which is based only on  $r(\cdot)$ . To avoid confusion between these concepts a distinction is henceforth drawn between 'discounting', which refers to a preference for early over later utility, and 'devaluing', which refers to a decrease in the outcome's value at time  $t_0$  as a function of delay. 'Devaluing' is also synonymous with the rate of time discount which would

be estimated by an observer who did not take account of utility from anticipation. In Fig. 3, it can be seen, devaluing is initially negative and only eventually becomes positive even though *time* preference is always positive – i.e. r > 0.

Point  $T_m$  in Fig. 3 has special significance.  $T_m$  is, for desirable outcomes, the individual's most preferred time of consumption.  $T_m$  will be greater than  $t_0$  if  $(\partial Y/\partial T)$  is positive for T=0 (condition 8). When condition 8 is met it will also be the case that  $(\partial^2 Y/\partial T^2) < 0$ , ensuring that the point  $T_m$  is a maximum (see Appendix 2). Under the standard approach (in the absence of planning or scarcity effects),  $T_m$  is always equal to  $t_0$ .  $T_m > t_0$  thus constitutes a sharp distinction between the predictions of the current model and the standard DU approach.

Conditions Conducive to Delaying of Desired Consumption

By totally differentiating (6) (see Appendix 3), it is possible to derive propositions regarding the effect on  $T_m$  of changes in the different parameters of the model.

Increasing the duration of consumption, L, raises the marginal cost of deferring at  $T_m$  more than it increases the marginal benefit of savouring. Therefore, increasing L discourages delaying behaviour  $(\partial T_m/\partial L \leq 0)$ . Conversely, delaying is more likely when consumption is fleeting. Intuitively this seems plausible. Those forms of consumption that are commonly delayed typically provide brief but intense pleasure. In such cases anticipation (and sometimes memory) serves to extend the otherwise fleeting benefit provided by consumption.

Raising  $\alpha$ , as would be expected, also encourages delaying behaviour.  $\alpha$ , for desirable outcomes, can be viewed as a measure of the 'savourability' of consumption. Outcomes that can be readily imagined and that are pleasurable to contemplate are therefore more likely to be delayed. The effect of changes in  $\delta$  on  $T_m$  is ambiguous.

The 'kiss from the movie star of your choice' in the study was chosen for its fleeting quality and high degree of 'savourability', characteristics which are predicted to promote delay. It was also chosen to rule out, as much as possible, alternative explanations for delaying behaviour. Two such explanations are worthy of note. The first, the 'planning effect', provides an incentive for delay when, by delaying a desired outcome, preparations can be made that will enhance utility from consumption. This is certainly true of food consumption, in which case fasting (within limits) enhances pleasure from subsequent eating. In the case of the kiss it also seems reasonable to assume that some preparations (such as gargling, or moving to a dimly lit room) could intensify the experience. But it is difficult to see how a delay of three days, which was generally preferred to a single-day delay, could add much to such preparations.

The second alternative explanation for delaying is the 'scarcity effect'. The basis of this argument is that some items are scarcer in the future (e.g. raspberries in winter) than they are in the present and thus should be more highly valued. The kiss from a movie star of your choice, which is a scarce item

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at any time and certainly as scarce in three days as at the present moment, was chosen to avoid this alternative explanation.

# Accelerating Undesirable Consumption

The shape of  $U_{t_0}(T)$  for an undesirable outcome, which is a negative reflection of its shape for desirable outcomes, is shown in Fig. 4. In the case of undesirable outcomes a different point,  $T_i$ , has behavioural significance.  $T_i$  is the point at which an individual is indifferent between immediate and deferred consumption. If allowed to defer a negative outcome beyond  $T_i$ , the individual will do so. If constrained to consume x prior to  $T_i$ , however, his preferences will be reversed, and he will prefer to get x over with immediately. A common example of such behaviour is associated with medical or dental treatment. One puts off visiting the doctor or dentist as long as possible, but once a maximum delay is imposed – e.g., when one receives a card in the mail urging a visit – typically one asks for the first available appointment. Under the standard approach  $T_i$  always equals  $t_0$ .

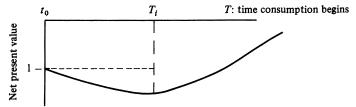


Fig. 4. Value at time  $t_0$  of consumption at time T.

Equation (9) defines  $T_i$ .

$$U(x)\frac{\mathbf{I}}{r}(\mathbf{I} - e^{-rL}) = U(x)\frac{\alpha}{\delta(\delta - r)}\left(e^{-rT_i} - e^{-\delta T_i}\right)\left(\mathbf{I} - e^{-\delta L}\right) + U(x)\frac{\mathbf{I}}{r}e^{-rT_i}(\mathbf{I} - e^{-rL}). \tag{9}$$

The left-hand side of (9) represents the (negative) utility that would be experienced if x were consumed beginning immediately. The right-hand side expresses discounted utility (from dread and consumption) when consumption is delayed until  $T_i > t_0$ . When these expressions are equal, the individual at  $t_0$  is indifferent between consuming immediately or at  $T_i$ . If constrained to consume at any point prior to  $T_i$ , he would prefer to consume immediately.

Again we are interested in the specific conditions that encourage individuals to get unpleasant outcomes over with quickly—i.e. make  $T_i \gg t_0$ . These conditions turn out to be analogous to those for  $T_m$  (see Appendix 4). The more fleeting an undesirable outcome—i.e. the shorter L—the greater is the tendency to get consumption over with quickly. People should therefore be especially likely to get fleeting outcomes over with quickly and defer those for which consumption is prolonged. This has the sensible implication that people will always defer outcomes whose effects are prolonged or permanent, e.g. loss of

a leg. It is also the case that  $\partial T_i/\partial \alpha > 0$ ; people are likely to get those outcomes over with quickly that can be vividly imagined beforehand.

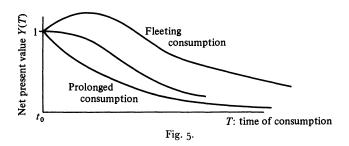
Thus we should expect that the tendency to get unpleasant outcomes over with quickly will be greatest for outcomes that are fleeting and vivid. The shock item in the study was chosen to have these characteristics. Lest the survey results, which involve hypothetical responses, be questioned, it should be noted that several other studies involving real shocks produced similar results (Carlsmith, 1962; Barnes and Barnes, 1964). In fact even rats, in one experiment, when faced with a choice between an immediate or moderately delayed shock, tended to choose the immediate shock (Knapp et al. 1959). Furthermore, when rats are exposed to delayed shocks they typically exhibit physiological signs of fear such as elevated heartbeat and blood pressure while they wait, suggesting that fear may be a motivating factor in the choice of the immediate shock.

A problem with the use of the shock item is that it is difficult to generalise the findings to economic behaviour in naturalistic settings. This particular problem is avoided in subsequent research in which subjects were asked among other questions: 'What is the least amount of money you would accept for cleaning 100 hamster cages at the Psychology Department's animal laboratory. You will be paid the money immediately. You should be willing to do the job for the amount you specify but not if you were offered only one dollar less. The job is unpleasant but takes only 3 hours. How much would you need to be paid to clean the cages: (1) once during the next 7 days; (2) once during the week beginning two months from today; (3) once during the week beginning one year from now?'

The mean reservation wage for cage cleaning in the following week was \$30 (standard error  $3 \cdot 1$ ). For cleaning the cage in a year respondents required an average of \$37 in immediate payment (standard error  $3 \cdot 8$ ). In fact, of 37 respondents only 2 gave a smaller response to question (3) than to question (1)!

## Positive Devaluation

When condition (8) is not met – i.e. for consumption that is lasting, or if  $\alpha$  is small – we should not expect to observe negative devaluing. Nevertheless, utility from anticipation will still affect the devaluation of consumption. Fig. 5 illustrates the effect of variations in L on devaluation of future consumption. Decreasing L will attenuate devaluing, flatten the slope  $\partial U_{t_0}/\partial T$  and thus  $\partial Y/\partial T$  (see Appendix 5, which demonstrates that  $\partial^2 Y/\partial T \partial L < 0$ ). As L increases, therefore, Y(T) will lose its hump and will begin to take on the



reversed '\$' shape depicted in Fig. 5. Savouring, even when it does not cause devaluing to be negative, attenuates devaluing for small delays, bridging the gap between sporadic or short-lived consumption and pleasure of ownership.

## IV. ECONOMIC IMPLICATIONS

# Attempts to Manipulate $\alpha$

The concept of 'vividness' or imaginability embodied in the  $\alpha$  term played a central role in historical formulations of intertemporal choice. For example, Böhm-Bawerk cited 'the fragmentary nature of the imaginary picture that we construct of the future state of our works' (1889, p. 269) first in his list of causes for the systematic tendency to undervalue future wants. Marshall noted that a person's willingness... to wait depends on his habit of vividly realising the future' (1891, p. 293). Although modern accounts of intertemporal choice have not included discussions of vividness, perhaps because vividness is not directly observable, its meaning is nevertheless intuitively comprehensible, and there are numerous examples of attempts to manipulate it.

Drivers' education films depict the gruesome consequences of car accidents in an effort to make those consequences more vivid to young (and presumably myopic) drivers. But facts and statistics and even photographs of accidents have only a limited impact. Faced with continuing high accident rates among young drivers, several school districts have taken more drastic measures. A device intended to demonstrate the violence of even a low-speed collision has been put into use in several school districts. Students are accelerated down a ramp on a wheeled chair which, after gaining a speed of 5 miles per hour, is abruptly halted. It is difficult to understand this device in purely informational terms. Rather, the intention is to give immediate emotional significance to an otherwise abstract outcome. An analogous attempt to counter youthful myopia through manipulation of vividness is depicted in the documentary movie 'Scared Straight', in which high-school students thought to be at risk of future delinquency toured a maximum security prison, were harassed by inmates, and were given as vivid an image of prison life as is possible without actually being locked up.

While the accident simulator and the visit to the prison were aimed at youth, such efforts are not limited to teenagers. For example, a recent television advertisement for a supplementary retirement plan showed a young couple in the first frame opting not to participate in the plan, and then showed them physically ageing against a backdrop of increasing poverty. Each of these efforts at persuasion can be viewed as attempts to decrease  $\alpha$  so as to raise dread and enhance evaluation of the future consequences of current actions.

## Estimation of Discount Rates

The sensitivity of intertemporal choice behaviour to savouring and dread suggests that the standard model may be mis-specified, leading to systematic bias in the estimation of discount rates. Since DU does not ordinarily incorporate savouring or dread, and since both of these factors attenuate

devaluation, conventional estimates of discount rates should be biased downward, especially in situations where savouring or dread significantly affect devaluation. As demonstrated, these include situations in which future outcomes are fleeting or can be vividly imagined. Also, savouring and dread exert their greatest impact at short and moderate time delays, so discount rates estimated from short-term trade-offs will tend to be more biased than rates estimated from long-term trade-offs.

The bias in estimation of discount rates will be especially serious if savouring and dread are different for different categories of consumption. If this were the case, then the general assumption that 'the discount rate is independent of the category of consumption goods for which it is calculated' (Landsberger, 1971, p. 1351) would be invalid. Thus, for example, Hausman, in his 1979 study, implicitly assumed that discount rates imputed from air-conditioner purchases would also apply to other intertemporal trade-offs. What he failed to consider was the possibility that air conditioners or other energy-using appliances have special characteristics that promote steep discounting. Monthly payments for electricity are spread over time and made less salient by being combined with pre-existing electrical charges, leading to the prediction of high devaluing even when time discounting is low. In contrast, purchase cost is immediate and lump-sum. This myopia-inducing quality of fuel- or electricity-consuming durables seems to have been recognised by various agencies of government that require efficiency ratings for appliances, and estimates of fuel consumption and yearly operating costs, to be affixed to new cars.

# Savings Behaviour

The concepts of savouring and dread may help to explain the anomalous observation that – rather than dissaving following retirement (as is predicted by the permanent income and life-cycle theories of saving) – individuals typically increase their rate of saving following retirement and continue to amass increasing amounts of wealth until they die. A number of explanations have been offered for this phenomenon, including uncertain life expectancy and bequest motives.<sup>4</sup> Another explanation offered by Moore (1978) is that individuals derive utility from wealth itself. The current model bolsters Moore's account by providing a reason why wealth may be a source of utility. Accumulations of wealth can constitute, in effect, a 'licence to savour' – i.e. to imagine and thus derive immediate pleasure from the consumption which the wealth could finance.

But the theory also provides a second, perhaps even more plausible, explanation for the observed failure to dissave following retirement. Retirement for the young is a non-vivid event – perhaps partly because thinking about old age is aversive and tends to be avoided (resulting in a small  $\alpha$ ). Young middle-aged couples and individuals, possibly for this reason, often 'live like there's no tomorrow'. As retirement approaches, however, the prospect of having inadequate funds for retirement becomes increasingly vivid and causes

<sup>&</sup>lt;sup>4</sup> For a recent summary of this debate see Modigliani (1986).

anxiety, anxiety that can be allayed in part by stepping up savings. The onset of retirement itself, and the sudden loss of wage income, of course, greatly increases this anxiety for the future. This anxiety raises the returns, in terms of anxiety reduction, of saving, and counteracts the savings-discouraging effect of the loss of income upon retirement.

The concept of savouring can also shed light on other anomalous savings phenomena. In a recent series of interviews with family heads that aimed at a preliminary understanding of saving behaviour, one theme that emerged was the apparent need of many individuals for highly specific goals to motivate savings. Such individuals seem incapable of saving for a future retirement in the abstract; they can, however, save for a retirement condominium or other concrete goal that, it is plausible to assume, brings them anticipal pleasure. Also comprehensible in terms of savouring is the behaviour of misers. In this extreme case, anticipated consumption exerts so strong a pull that no present consumption can compete with it.

# Reverse Time Inconsistency

Since the publication of Strotz's well-known 1955 article, the phenomenon of time inconsistency has attracted the attention of social scientists in a number of disciplines. Time inconsistency occurs when an individual makes a long-term plan at time t but systematically departs from that plan in later periods. As has been said, subsequent departures tend to be myopic in character – i.e. to allocate greater consumption to the immediate period than was originally planned. While not ruling out such forms of time inconsistency, the current model predicts, under certain conditions, a related phenomenon, which could be termed 'reverse time inconsistency'.

Suppose that an individual at time  $t_0$ , in the light of the characteristics of a prospective act of consumption, chooses to defer consumption until time  $T_m > t_0$  so as to savour his expectations. Logically, it would seem that when he arrives at  $T_m$  he has every incentive to defer consumption once again, since he is in the identical situation he was in earlier. One could imagine this situation repeating itself indefinitely. Several factors, however, limit such repeated delaying and savouring of the same object of consumption.

First, concerns about self-credibility may make plans self-reinforcing. Suppose the individual at  $T_M$  decides to delay again, until  $T_{M2}$ . The problem is, having planned to consume at  $T_M$  and then failed to do so, he may not be able to convince himself at  $T_M$  that he will follow through on his plan to consume at  $T_{M2}$ . Failing to believe that he will consume at  $T_{M2}$ , he will be unable to savour the prospect of consumption at  $T_{M2}$  and he will have no incentive to defer past  $T_M$ . Ultimately, therefore, he will consume at  $T_M$  as he had originally planned, without binding himself in any way. Failure to consume as planned may interfere not only with the individual's ability to savour the re-deferred object but also with his ability to savour other forms of delayed consumption, thus adding an additional cost to any act of reverse time inconsistency.

Second, many decisions concerning future consumption carry with them their own enforcement, or at least reinforcement mechanisms – for example,

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when reservations are made, items are ordered, or plans are made with other people. In addition there are well-known self-control techniques such as the strategy of making 'side bets' that alter future incentives to encourage conformity with current plans. These have received extensive coverage in the literature on self-control and time consistency (Strotz, 1956; Ainslile, 1975; Elster, 1979; Schelling, 1978).

Nevertheless, despite the self-reinforcing character of plans, psychological incentives for adherence to plans, and various types of self-control techniques, people often do exhibit time inconsistency of this reverse form – i.e. repeated deferral for the purpose of savouring. This pattern is often observed among children at Hallowe'en when some trick-or-treaters collect then hoard their candy rather than consuming it. Apparently the pleasure derived from savouring future consumption of the candy in some cases outweighs what can be obtained from immediate consumption. Such behaviour often ultimately results in the stale candy being thrown out without having been even partially consumed. Similar behaviour can be seen among adults towards bottles of wine and holidays. Often people profess to need a vacation desperately, yet they keep putting off actually taking one, preferring instead to savour their accumulated vacation days.

# Violations of Independence

Like axiomatic derivations of the Expected Utility Model, derivations of the Discounted Utility Model (Koopmans, 1960; Meyer, 1976) have centred on some type of independence axiom. Independence, in the context of intertemporal choice, posits that an individual's preference between two time streams or sequences of consumption should not be affected by periods in which the streams share identically valued consumption.

The following study illustrates a common violation of independence in an intertemporal context. Thirty-seven undergraduates were given the following paired choices. They were asked to ignore scheduling considerations.

The first finding of note is that 84% of the respondents chose option B over option A in question 1. This finding conforms to the prediction made earlier that people will tend to delay desirable and fleeting consumption for purposes of savouring. More interesting is the fact that 57% of the respondents chose option C over option D in question 2. Note that consumption is equivalent in the third weekend for A and B and for C and D. Thus this pattern of responses

Question 1. Which would you prefer? (circle A or B)

Alternative	This weekend	Next weekend	Two weekends from now
A	Dinner at a fancy French restaurant	Eat at home	Eat at home
В	Eat at home	Dinner at a fancy French restaurant	Eat at home

Question 2. Which would you prefer? (circle C or D)

Alternative	This weekend	Next weekend	Two weekends from now
$\overline{C}$	Dinner at a fancy French restaurant	Eat at home	Fancy lobster dinner
<i>D</i>	Eat at home	Dinner at a fancy French restaurant	Fancy lobster dinner

violates the assumption of independence that has formed the core of axiomatic derivations of DU.

These results point to a problem in applying the notion of independence to intertemporal choice. When outcomes are conditional on mutually exclusive and comprehensive states of nature, as is true in decision-making under uncertainty, it follows that one and only one outcome will occur. In this situation the independence axiom seems valid, at least as a normative rule. If the state in which pay-offs are equal is revealed, the individual receives the same pay-off regardless of his choice. Thus the common pay-off should not affect his preference between these alternatives. If any other state occurs, the common pay-off in the state that did not occur would appear to be irrelevant.

In the case of intertemporal choice, however, typically all periods will occur. Hence if consumption of one outcome affects the usefulness or satisfaction associated with another there is no reason to expect independence to hold. The current theory, generalised to accommodate complex patterns of consumption, provides a reason why consumption in a later period may affect utility from consumption in an earlier period, even when there are no interperiod consumption externalities. According to our assumptions, the lobster dinner in week provides greater anticipal pleasure in week 2 than in week 1. This increases the marginal utility of consuming the French dinner in the first week, when utility is low, even at the cost of forestalling the savouring of the French dinner. When no lobster dinner follows, marginal utility in the first two periods is comparable, and the anticipal pleasure preceding the French dinner is sufficient to motivate delay. This effect can account for the general preference for spreading discrete acts of consumption over time, even in the absence of consumption externalities.

## V. DISCUSSION

The model discussed in this paper modifies DU by recognising that anticipation, like consumption itself, is a source of utility. That waiting for consumption to occur can often be pleasurable or painful – indeed, that much of our feeling of

<sup>5</sup> The model can be generalised to deal with complex temporal consumption streams or sequences as follows:  $\max \int_0^\infty V(U_t^c + U_t^A) \, r(t) \, dt, \tag{6}$ 

where V' > 0, V'' < 0.  $U^c$  and  $U^A$  represent the sum of utility from consumption and anticipal pleasure from all sources.

well-being and despair arise from emotions associated with anticipation – seems self-evident. The behavioural consequences of these feelings have been the subject of this paper.

When anticipation is a source of utility, the effect of delay on the value of an object can diverge from the predictions of existing theories. For consumption that is fleeting and easy to imagine devaluing will often be initially negative. Even if it is not initially negative, it may display a reversed 'S' pattern rather than the convex shape suggested by DU.

The dependence of discounting on the characteristics of awaited consumption means that discount rates estimated in specific contexts, such as studies of consumer durable purchases (Hausman, 1979) or of savings behaviour (Landsberger, 1971), cannot be generalised beyond the domain of behaviour in which they were derived. The concepts of anticipal pleasure and pain also provide an explanation for violations of the independence axiom used in axiomatic derivations of DU. Finally, the model draws attention to the phenomenon of 'reverse time inconsistency' that many people at times exhibit.

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Date of receipt of final typescript: September 1986

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## APPENDIX I

The following table presents, for each of the five survey outcomes the fraction of the amount that people would pay to obtain or avoid the outcome immediately, that they would pay to obtain or avoid the outcome after each of the five time delays. Numbers are means across subjects. Standard errors are in parentheses.

Engalian	~ 6	Initial	Valera
Fraction	vj	Intitat	v aiue

Time delays	Obtain four dollars	Avoid losing four dollars	Avoid losing one thousand dollars	Kiss from movie star	120-volt electric shock
3 hours	o·93	0·97	o·97	1·30	1.0
	(o·o28)	(0·023)	(o·016)	(0·155)	(0.012)
24 hours	o·82	o·91	0·96	1·59	o·99
	(o·045)	(o·044)	(0·021)	(0·280)	(o·053)
3 days	o·74	o·75	o·94	1·78	(o.11)
	(o·o57)	(o·072)	(o·031)	(0·487)	1.01
ı year	o·46	o·54	o·91	1·31	1·23
	(o·069)	(o·083)	(o·075)	(0·295)	(0·20)
10 years	o·21	o·32	o·68	0·639	1·84
	(o·065)	(o·081)	(o·153)	(0·156)	(0·4)

## APPENDIX 2

This demonstrates that

$$\frac{\partial^2 Y}{\partial T^2}$$
 < 0 at point  $T_m$ .

Differentiating (6) a second time with respect to T:

$$\frac{\partial^2 U}{\partial T^2} = U(x) \left[ \frac{\alpha}{\delta(\delta - r)} \left( r^2 e^{-rT} - \delta^2 e^{-\delta T} \right) \left( \mathbf{I} - e^{-\delta L} \right) + r e^{-rT} \left( \mathbf{I} - e^{-rL} \right) \right] \tag{A I}$$

$$T_m, \frac{\partial U}{\partial T} = 0$$
 so  $\frac{\alpha}{\delta(\delta - r)} (\delta e^{-\delta T} - re^{-rT}) (\mathbf{I} - e^{-\delta L}) = e^{-rT} (\mathbf{I} - e^{-rL}).$  (A 2)

Substituting (A 2) into (A 1):

at

$$\begin{split} \frac{\partial^{2} U}{\partial T^{2}} &= U(x) \left[ \frac{\alpha}{\delta(\delta - r)} \left( r^{2} e^{-rT} - \delta^{2} e^{-\delta T} \right) \left( \mathbf{1} - e^{-\delta L} \right) \right. \\ &\left. + \frac{r\alpha}{\delta(\delta - r)} \left( \delta e^{-\delta T} - r e^{-rT} \right) \left( \mathbf{1} - e^{-\delta L} \right) \right] \quad (A \ 3) \end{split}$$

$$\begin{split} &=U(x)\frac{\alpha}{\delta(\delta-r)}\left(\mathbf{1}-e^{-\delta L}\right)\left(r^2e^{-rT}-\delta^2e^{-\delta T}+r\delta e^{-\delta T}-r^2e^{-rT}\right)\\ &=U(x)\frac{\alpha}{\delta(\delta-r)}\left(\mathbf{1}-e^{-\delta L}\right)e^{-\delta T}(r\delta-\delta^2). \end{split} \tag{A 4}$$

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Since by assumption,  $\delta > r$  it follows that  $(\partial^2 U/\partial T^2) < 0$ . It remains to be demonstrated that  $(\partial^2 U/\partial T^2) < 0$  implies  $(\partial^2 Y/\partial T^2) < 0$ .

Proof: 
$$Y = U^{-1}[U(Y)]$$

so

$$\frac{\partial^2 Y}{\partial T^2} = \frac{\partial^2 U^{-1}[U(Y)]}{\partial U^2(Y)} \left\{ \frac{\partial [U(Y)]}{\partial T} \right\}^2 + \frac{\partial U^{-1}[U(Y)]}{\partial U(Y)} \frac{\partial^2 U(Y)}{\partial T^2}. \tag{A} \ 5)$$

But  $\partial U(Y)/\partial T$  is equal to zero, by the definition of  $T_m$ , so the first term is zero, and  $\partial U^{-1}[U(Y)]/\partial U(Y) > 0$  since U(Y) is a motonically increasing function. Since  $\partial^2 U(Y)/\partial T^2 < 0$ , it follows that  $(\partial^2 Y/\partial T^2)$  is also < 0.

## APPENDIX 3

Derivation of the partial derivatives of  $T_m$  with respect to L,  $\alpha$ ,  $\delta$  and r:  $T_m$  is defined by

$$\mathbf{o} = \frac{\partial U(Y)}{\partial T} = U(x) \left[ \frac{\alpha}{\delta(\delta - r)} \left( \delta e^{-\delta T} \mathbf{m} - r e^{-rT} \mathbf{m} \right) \left( \mathbf{I} - e^{-\delta L} \right) - e^{-rT} \mathbf{m} \left( \mathbf{I} - e^{-rL} \right) \right]. \tag{A 6}$$

Totally differentiation with respect to  $T_m$ , L,  $\alpha$  and  $\delta$ :

$$o = dT_{m} \left[ \frac{\alpha}{\delta(\delta - r)} \left( r^{2}e^{-rT_{m}} - \delta^{2}e^{-\delta T_{m}} \right) \left( \mathbf{I} - e^{-\delta L} \right) + re^{-rT_{m}} \left( \mathbf{I} - e^{-rL} \right) \right]$$

$$< o$$

$$+ dL \left[ \frac{\alpha}{\delta - r} \left( \delta e^{-\delta T_{m}} - re^{-rT_{m}} \right) e^{-\delta L} - re^{-r(T_{m} + L)} \right]$$

$$> o$$

$$+ d\alpha \left[ \frac{\mathbf{I}}{\delta(\delta - r)} \left( \delta e^{-\delta T_{m}} - re^{-rT_{m}} \right) \left( \mathbf{I} - e^{-\delta L} \right) \right].$$
e,
$$\frac{\partial T_{m}}{\partial L} < 0 \quad \text{and} \quad \frac{\partial T_{m}}{\partial \alpha} > o.$$

Therefore,

The sign of the first term is derived in Appendix 2. The second term is equal to  $(\partial^2 U/\partial T\partial L)$ , the sign of which is derived in Appendix 5. The sign of the third term is positive since it is equal to the first term of  $(A \ 6)$  divided by  $\alpha$ ; the second term of  $(A \ 6)$  is negative, and the first and second term sum to zero.

## Appendix 4

Derivation of the partial derivatives of  $T_i$  with respect to  $\alpha$  and L.  $T_i$  is defined by:

$$\mathbf{o} = \frac{\alpha}{\delta(\delta-r)} \left( e^{-rT_i} - e^{-\delta T_i} \right) \left( \mathbf{I} - e^{-\delta L} \right) - \frac{\mathbf{I}}{r} \left( \mathbf{I} - e^{-rL} \right) \left( \mathbf{I} - e^{-rT_i} \right). \tag{A 8}$$

Totally differentiating with respect to  $T_i$ ,  $\alpha$ , L:

$$\begin{aligned} &< o \\ o &= dT_i \bigg[ \frac{\alpha}{\delta(\delta-r)} \left( \mathbf{I} - e^{-\delta L} \right) \left( \delta e^{-\delta T_i} - r e^{-rT_i} \right) - e^{-rT_i} (\mathbf{I} - e^{-rL}) \bigg] \\ &> o \\ &+ d\alpha \bigg[ \frac{\mathbf{I}}{\delta(\delta-r)} \left( e^{-rT_i} - e^{-\delta T_i} \right) \left( \mathbf{I} - e^{-\delta L} \right) \bigg] \\ &< o \\ &+ dL \bigg[ \frac{\alpha}{\delta-r} \left( e^{-rT_i} - e^{-\delta T_i} \right) e^{-\delta L} - e^{-rL} (\mathbf{I} - e^{-rT_i}) \bigg]. \end{aligned} \tag{A 9}$$

The first term is simply the slope  $(\partial U/\partial T)$  at  $T_i$ , which will be negative. The sign of the second term is straightforward. The sign of the third term can be derived as follows: from (A 8),

$$\frac{\alpha}{\delta - r} \left( e^{-rT_i} - e^{-\delta T_i} \right) = \frac{\delta}{r} \frac{\left( \mathbf{I} - e^{-rL} \right) \left( \mathbf{I} - e^{-rT_i} \right)}{\left( \mathbf{I} - e^{-\delta L} \right)} \tag{A 10}$$

substituting into the third term of (A 9):

$$\frac{\delta}{r}e^{-\delta L}\frac{(\mathbf{I}-e^{-rL})(\mathbf{I}-e^{-rT_i})}{(\mathbf{I}-e^{-\delta L})}-e^{-rL}(\mathbf{I}-e^{-rT_i}) \tag{A II}$$

$$= \left[ \frac{\delta}{r} e^{-\delta L} \frac{(\mathbf{I} - e^{-rL})}{(\mathbf{I} - e^{-\delta L})} - e^{-rL} \right] (\mathbf{I} - e^{-rT}i). \tag{A 12}$$

Multiplying both sides by  $r(1-e^{-\delta L})$  and dividing by  $e^{-rL-\delta L}(1-e^{-rT_i})$  will not change the sign.

$$\operatorname{Sign}\left[\left(\frac{\delta}{r}e^{-\delta L}\frac{(\mathbf{I}-e^{-rL})}{(\mathbf{I}-e^{-\delta L})}-e^{-rL}\right)(\mathbf{I}-e^{-rT_i})\right] = \operatorname{sign}\left[r(\mathbf{I}-e^{\delta L})-\delta(\mathbf{I}-e^{rL})\right] < o \quad (\mathbf{A} \ \mathbf{I} \ \mathbf{3})$$

for all  $\delta > r > 0$ .

APPENDIX 5

Derivation of

$$\frac{\partial^2 Y}{\partial T \partial L} = \frac{\partial}{\partial L} \left( \partial Y / \partial T \right)$$

when  $\partial U/\partial T$  and  $\partial Y/\partial T$  are both negative.

$$\frac{\partial Y}{\partial T} = \frac{\partial U^{-1}[U(Y)]}{\partial T}$$

$$\frac{\partial^2 Y}{\partial T \partial L} = \frac{\partial^2 U^{-1}[U(Y)]}{\partial U^2(Y)} \frac{\partial U(Y)}{\partial T} \frac{\partial U(Y)}{\partial L} + \frac{\partial U^{-1}[U(Y)]}{\partial U(Y)} \frac{\partial^2 U(Y)}{\partial T \partial L}. \tag{A 14}$$

The signs of each of the terms except the last are self-evident. To demonstrate that  $\partial^2 Y/\partial T \partial L < 0$  it is simply necessary to prove that  $\partial^2 U(Y)/\partial T \partial L < 0$ . Differentiating (6) with respect to L:

$$\frac{\partial^2 U(Y)}{\partial T \partial L} = U(x) \left[ \frac{\alpha}{(\delta - r)} \left( \delta e^{-\delta T} - r e^{-rT} \right) e^{-\delta L} - r e^{-r(T+L)} \right]. \tag{A 15}$$

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Since  $\partial U(Y)/\partial T < 0$ , from (6):

$$\frac{\alpha}{\delta - r} \left( \delta e^{-\delta T} - r e^{-rT} \right) < \frac{\delta e^{-rT} \left( \mathbf{I} - e^{-rL} \right)}{\left( \mathbf{I} - e^{-\delta L} \right)}. \tag{A 16}$$

Substituting (A 16) into (A 15).

$$\frac{\partial^2 U(Y)}{\partial T \partial L} < \frac{\delta e^{-\delta L} e^{-rT} (\mathbf{1} - e^{-rL})}{(\mathbf{1} - e^{-\delta L})} - r e^{-r(T+L)}$$

multiplying by  $(1 - e^{-\delta L})$  and dividing by  $e^{-r(T+L)-\delta L}$  leaves the sign unchanged

$$\mathrm{sign}\left[\frac{\delta e^{-\delta L}e^{-rT}(\mathbf{1}-e^{-rL})}{(\mathbf{1}-e^{-\delta L})}-re^{-r(T+L)}\right]=\mathrm{sign}\left[r(\mathbf{1}-e^{\delta L})-\delta(\mathbf{1}-e^{rL})\right]<\mathrm{o.}\quad (\mathrm{Q.E.D.})$$