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WARWICK BUSINESS SCHOOL
THE UNIVERSITY OF WARWICK

For the
Open
Minded

Vladimir Deineko

Introduction to Linear Programming

Part II

1

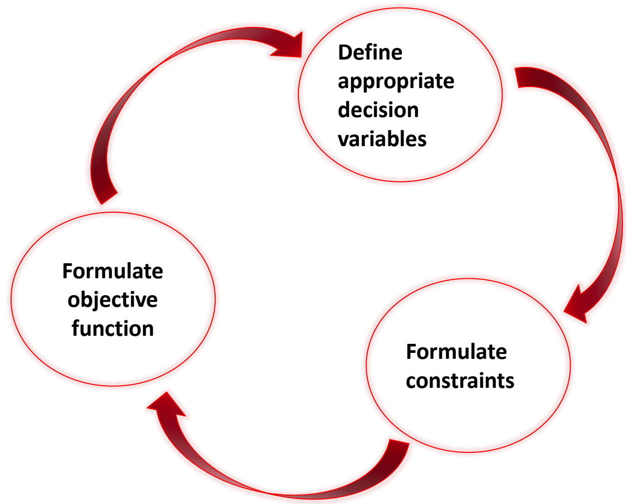
Formulation of Linear Programming Problems, Solving LPs with Spreadsheets

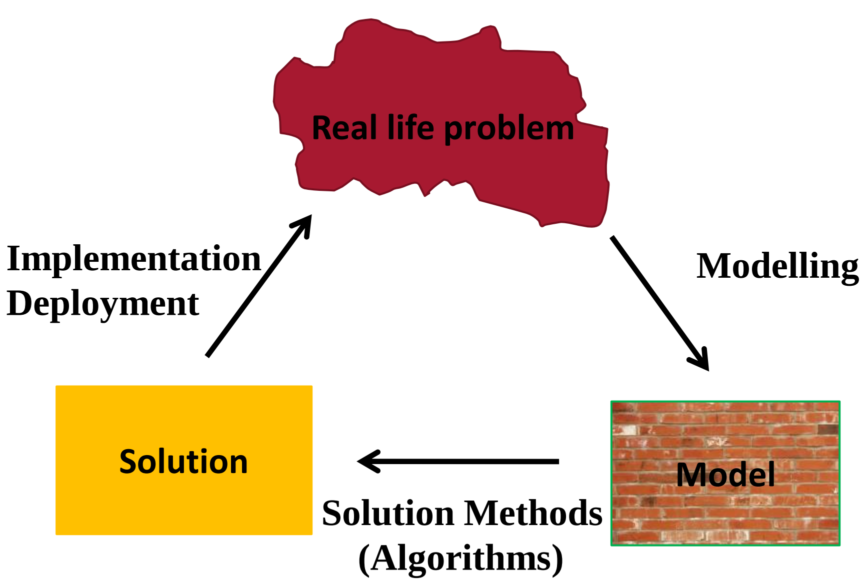
You will have mastered the material discussed at the lecture WHEN, given a practical problem, you can

- identify decision variables
- formulate an objective function
- formulate constraints on the decision variables
- set up the problem in an attractive (spreadsheet) format
- *Solve the problem using a spreadsheet* Solver
- *Check whether the solution found is what you expected*

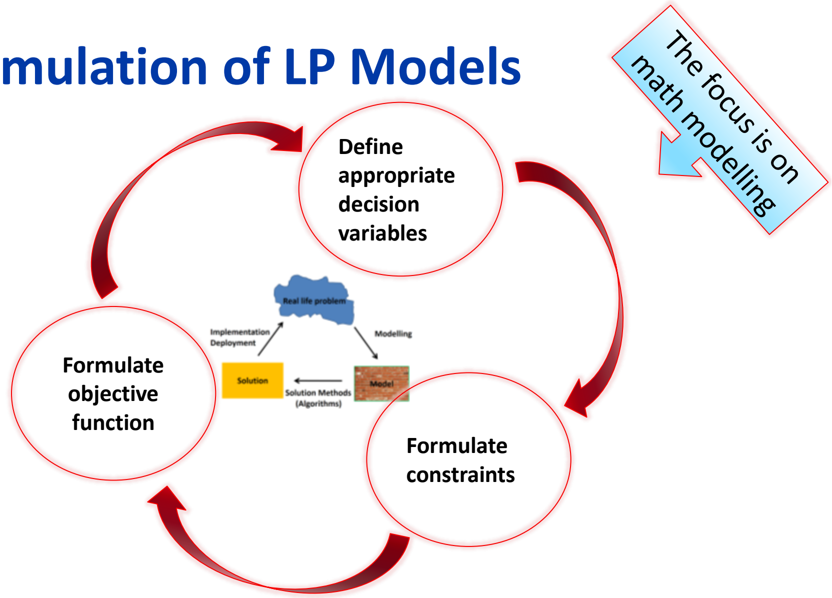
2

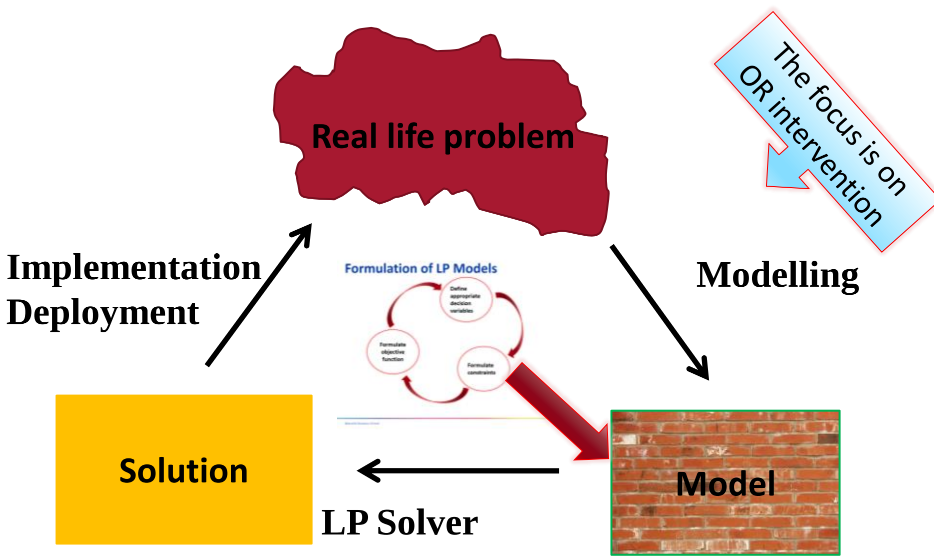
Formulation of LP Models



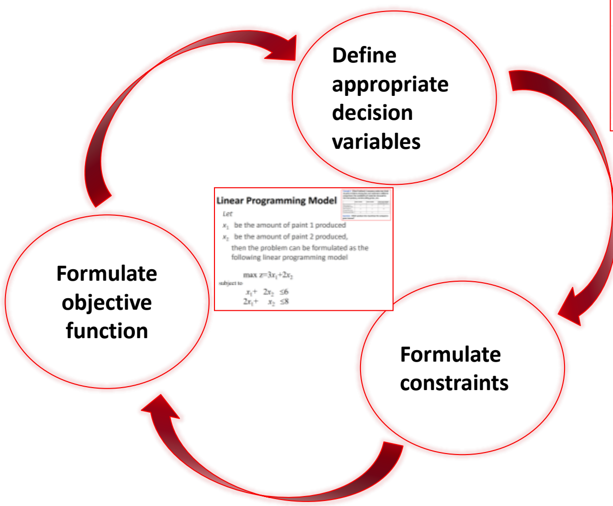


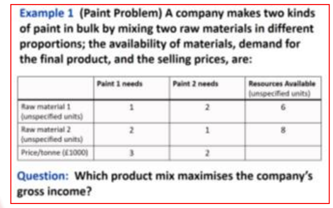
Formulation of LP Models

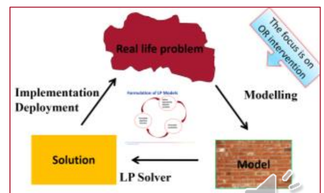




Formulation of LP Models







Example 1 (Paint Problem) A company makes two kinds of paint in bulk by mixing two raw materials in different proportions; the availability of materials, demand for the final product, and the selling prices, are:

	Paint 1 needs	Paint 2 needs	Resources Available (unspecified units)
Raw material 1 (unspecified units)	1	2	6
Raw material 2 (unspecified units)	2	1	8
Price/tonne (£1000)	3	2	

Additional Information: The customer has forgotten to mention the demand!

Research shows that demand for product 2 is no more than 1.9 tonne/day, and in any case will not exceed the demand for product 1 by more than 1 tonne/day.

Which product mix maximises the company's gross income?

Known limits on demand:

Research shows that demand for product 2 is no more than 1.9 tonne/day, and in any case will not exceed the demand for product 1 by more than 1 tonne/day.

$$x_2 \leq 1.9$$

$$x_2 \leq 1 + x_1$$

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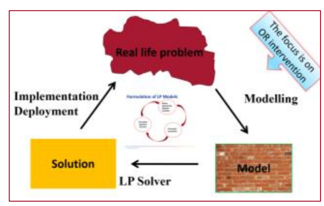
9

Updated Model:



Known limits on demand:

Research shows that demand for product 2 is no more than 1.9 tonne/day, and in any case will not exceed the demand for product 1 by more than 1 tonne/day.

$x_2 \leq 1.9$

$x_2 \leq 1 + x_1$

$$\max\ z = 3x_1 + 2x_2$$

subject to

$$\begin{aligned} x_1 + 2x_2 &\leq 6 \\ 2x_1 + x_2 &\leq 8 \\ x_2 &\leq 1.9 \\ -x_1 + x_2 &\leq 1 \end{aligned}$$

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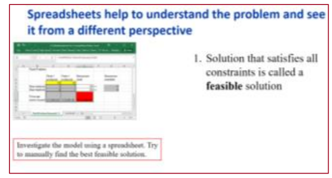
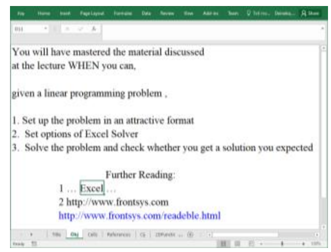
10

Solving LPs with Solver:

We started with setting a Spreadsheet model (see part I) and investigating possible feasible solutions of an LP problem



It was a part of modelling, but NOT YET the process of SOLVING the problem



I have prepared detailed instructions on how to solve LPs by using a Solver (you will find it my.wbs)

Solving LPs with Solver:

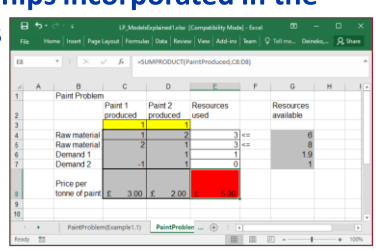
Updated Model:

$$\max z = 3x_1 + 2x_2$$

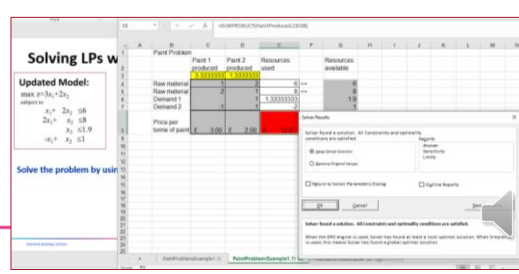
subject to

$$\begin{aligned} x_1 + 2x_2 &\le 6 \\ 2x_1 + x_2 &\le 8 \\ x_2 &\le 1.9 \\ -x_1 + x_2 &\le 1 \end{aligned}$$

Set spreadsheet model with all relationships incorporated in the formulas

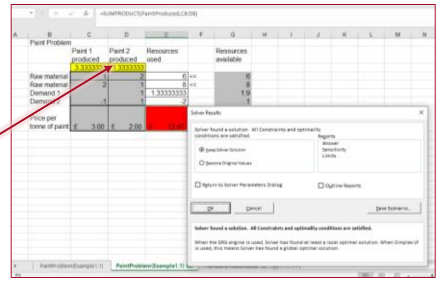


Solve the problem by using LP Solver

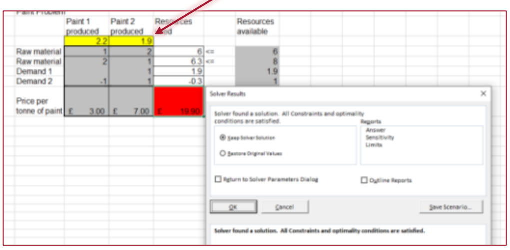


Solving LPs with Solver:

I want to check the behaviour of my model.



What happened if the price for Paint 2 is increased up to £7000?



That is what we expected: an increase in production of Paint 2 is recommended by the model.

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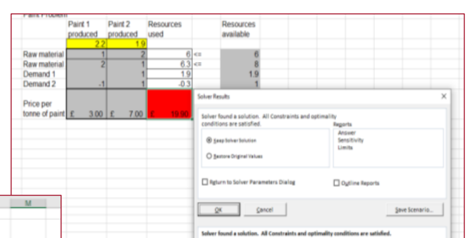
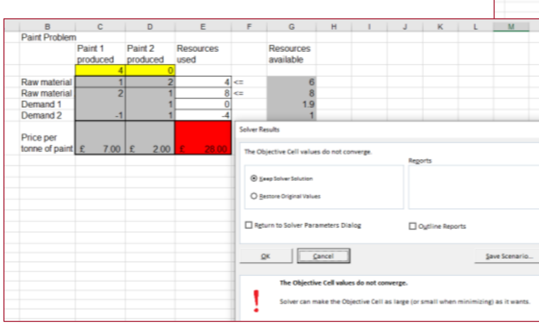
13

Solving LPs with Solver:

I still have doubts.



What happened if the price for Paint 1 is increased?



That is what we did not expect**: the value of the objective function is unbounded!**

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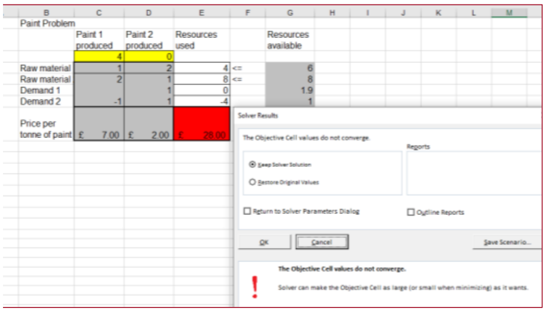
14

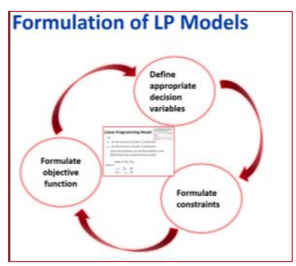
Solving LPs with Solver:



That is what we did not expected**: the value of the objective function is unbounded!**

If the value of the objective function is unbounded, then …

It means that we have forgotten a constraint (or two?)

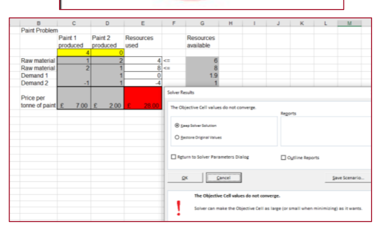


Corrected Model:

$$\max\ z = 3x_1 + 2x_2$$

subject to

$$\begin{aligned} x_1 + 2x_2 &\le 6 \\ 2x_1 + x_2 &\le 8 \\ x_2 &\le 1.9 \\ -x_1 + x_2 &\le 1 \end{aligned}$$



$$x_1, x_2 \ge 0$$

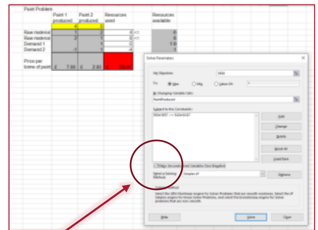
We have forgotten "obvious" non-negativity constraint: the amount of paint produced cannot be negative!

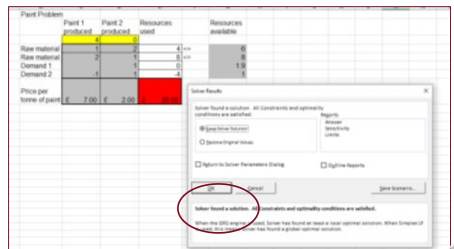
Corrected Model:

max $z=3x_1+2x_2$

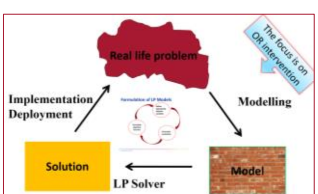
subject to

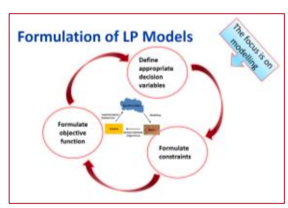
$$\begin{aligned} x_1 + \ 2x_2 &\le 6 \\ 2x_1 + \ \ x_2 &\le 8 \\ x_2 &\le 1.9 \\ -x_1 + \ \ x_2 &\le 1 \\ x_1, x_2 &\ge 0 \end{aligned}$$





Comments on Modelling





Model testing: Non-negativity constraint

Customer: demand constraint

Updated Model:

max $z=3x_1+2x_2$

subject to

$$\begin{aligned} x_1 + \ 2x_2 &\le 6 \\ 2x_1 + \ \ x_2 &\le 8 \\ x_2 &\le 1.9 \\ -x_1 + \ \ x_2 &\le 1 \end{aligned}$$

Final Model:

max $z=3x_1+2x_2$

subject to

$$\begin{aligned} x_1 + \ 2x_2 &\le 6 \\ 2x_1 + \ \ x_2 &\le 8 \\ x_2 &\le 1.9 \\ -x_1 + \ \ x_2 &\le 1 \\ x_1, x_2 &\ge 0 \end{aligned}$$

A **linear programming problem** (LP) is an optimisation problem for which we do the following:

1. We attempt to maximise (or minimise) a *linear* function of the *decision variables*. The function that is to be maximised or minimised is called the *objective function*.

$$F(x_1,x_2, x_3,x_4\ldots)= c_1x_1+ c_2x_2 + c_3x_3 + \ldots$$

19

A **linear programming problem** (LP) …

2. The value of the decision variables must satisfy a set of *constraints*. Each constraint must be a linear equation or linear inequality:

$$a_{11}x_1+ a_{12}x_2 + a_{13}x_3 + \ldots \le(\ge =)\ b_1$$
$$a_{21}x_1+ a_{22}x_2 + a_{23}x_3 + \ldots \le(\ge =)\ b_2$$

20

A **linear programming problem** (LP) **…**

3. A *sign restriction* is associated with each variable. For any variable x_i, the sign restriction specifies either that x_i must be nonnegative or that x_i may be unrestricted in sign:

$$x_i \geq 0$$

21

A **linear programming problem** (LP) is an optimisation problem for which we do the following:

1. We attempt to maximise (or minimise) a *linear* function of the *decision variables*.

2. The value of the decision variables must satisfy a set of *constraints*.

3. A *sign restriction* is associated with each variable.

22

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Your tasks: Model the problems 1.2 – 1.6 (see below) and solve them by using an LP Solver

23

*Example*_1.2 (Furniture problem)   A company makes desks, tables, and chairs from lumber, using both skilled and unskilled labour. The requirements for each product are:

	Desk	Table	Chair	Available
Lumber	8	6	1	48
Skilled	4	2	1.5	20
Unskilled	2	1.5	0.5	8
Price	60	30	20	

24

*Example*_1.2 A company makes desks, tables, and chairs from lumber, using both skilled and unskilled labour. The requirements for each product are:

	Desk	Table	Chair	Available
Lumber	8	6	1	48
Skilled	4	2	1.5	20
Unskilled	2	1.5	0.5	8
Price	60	30	20	

Demand for desks and chairs is known to be more than could possibly be made, but no more than 5 tables are likely to be sold. What production schedule maximises total revenue?

25

What production schedule maximises total revenue?

- **<u>Decision Variables</u>**

x_1 number of desks produced

x_2 number of tables produced

x_3 number of chairs produced

26

*Example*_1.3

A firm has 3 workshops each working 40 hours per week.

It can produce two products, the first selling at £108 and the second at £84. The only direct cost is labour at £5 per hour.

Product 1 requires 5 hours in workshop 1, 9 hours in workshop 2, 7 hours in workshop 3 and 10 person-hours per unit.

Product 2 requires 10 hours in workshop 1, 2 hours in workshop 2, 5 hours in workshop 3 and 8 person-hours per unit.

Google for a definition of a "person-hour" (the same as "man-hour")

27

Formulate this problem as an LP model, assuming the firm wishes to maximise profits.

	Product 1	Product 2	Available
Workshop 1	5 hours	10 hours	40 hours
Workshop 2	9 hours	2 hours	40 hours
Workshop 3	7 hours	5 hours	40 hours
Labour	10 person·h	8 person·h	Cost £5/h
Price	£108	£84	

28

Example_1.4
(refresh % -> https://my.wbs.ac.uk/-/academic/212627/home/ Lesson 4)

The National Coal Board has unlimited supplies of three grades of coal: A,B,C, which contain ash and phosphorus as impurities. A firm requires supplies containing not more than 3% ash and 0.03% phosphorus. The available coal satisfies the following specifications:

	% Phosphorus	%Ash	Profit(£/ton)
A	0.02	3	60
B	0.04	2	75
C	0.03	5	70

Formulate an LP model to determine how the Board should meet the firm’s requirements and maximise its own profit.

29

Example_1.5

A ship has three cargo holds: forward, aft and centre. The capacity limits are:

Forward	2000 tons	100,000 cu ft
Centre	3000 tons	135,000 cu ft
Aft	1500 tons	30,000 cu ft

The following cargoes are offered and the shipowners may accept all or any part of each commodity

Commodity	Amount	Vol per ton	Profit(£/ton)
A	6,000	60	6
B	4,000	50	8
C	2,000	25	5

Formulate an LP model to determine how the cargo should be distributed so as to maximise profits.

30

Example_1.6

Transportation Problem

DryPants wants to deliver 10 truckloads of diapers from its three factories (A, B, C) to three distribution centres (X, Y, Z). Factory A can supply 2 truckloads, factory B and C 4 truckloads each. The demand at distribution centre Y is 4 truckloads, and 3 truckloads at X and Z. Transportation cost per truckload are given in the following table:

	X	Y	Z	Supply
A	1	5	1	*2*
B	2	7	1	*4*
C	5	8	2	*4*
	3	*4*	*3*	

What factory should deliver how many truckloads to which distribution centre to minimise transportation cost?

31