Benchmark问题

算法的性能比较是基于一些称为 Benchmark 的典型问题展开的. 就函数优化问题而言， 目前文献中常用的 Benchmark问题举例如下：

- Sphere Model

$$f(x) = \sum_{i=1}^{30} x_i^2, \quad |x_i| \leq 100$$

其最优状态和最优值为

$$\min f(x^*) = f(0, 0, \cdots, 0) = 0.$$

- Schwefel's Problem

$$f(x) = \sum_{i=1}^{30} |x_i| + \prod_{i=1}^{30} |x_i|, \quad |x_i| \leq 10$$

其最优状态和最优值为

$$\min f(x^*) = f(0, 0, \cdots, 0) = 0.$$

- Generalized Rosenbrock's Function

$$f(x) = \sum_{i=1}^{29} [100(x_{i+1} - x_i^2)^2 + (1 - x_i)^2], \quad |x_i| \le 30$$

其最优状态和最优值为

$$\min f(x^*) = f(1, 1, \cdots, 1) = 0.$$

- Step Function

$$f(x)=\sum_{i=1}^{30}(\lfloor x_i+0.5\rfloor)^2,\quad |x_i|\le 100$$

其最优状态和最优值为

$$\min f(x^*)=f(0,0,\cdots,0)=0.$$

- Quartic Function, i.e. Niose

$$f(x) = \sum_{i=1}^{30} i\, x_i^4 + \mathrm{random}[0,1), \quad |x_i| \le 1.28$$

其最优状态和最优值为

$$\min f(x^*) = f(0, 0, \cdots, 0) = 0.$$

- Generalized Schwefel's Problem

$$f(x) = -\sum_{i=1}^{30}(x_i \sin(\sqrt{|x_i|})), \quad |x_i| \leq 500$$

其最优状态和最优值为

$$\min f(x^*) = f(420.9687, 420.9687, \cdots, 420.9687) = -12569.5.$$

- Goldstein-Price Function

$$
\begin{aligned}
f(x) = & [1 + (x_1 + x_2 + 1)^2 (19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] \\
& \times [30 + (2x_1 - 3x_2)^2 (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 \\
& + 27x_2^2)], \quad |x_i| \le 2
\end{aligned}
$$

其最优状态和最优值为

$$
\min f(x^*) = f(0, -1) = 3.
$$

- Schaffer' Function

$$f(x) = \frac{\sin^2 \sqrt{x_1^2 + x_2^2} - 0.5}{[1 + 0.001(x_1^2 + x_2^2)]^2} - 0.5, \quad |x_i| \leq 100$$

其最优状态和最优值为

$$\min f(x^*) = f(0, 0) = -1.$$

- Problem

$$\begin{cases} \max f(x) = (\sqrt{n})^n \prod_{i=1}^{n} x_i \\ \text{subject to:} \\ \qquad \sum_{i=1}^{n} x_i^2 = 1, \\ \qquad 0 \le x_i \le 1, \quad i = 1, 2, \cdots, n \end{cases}$$

其最优状态和最优值为

$$\max f(x^*) = f\left(\frac{1}{\sqrt{n}}, \cdots, \frac{1}{\sqrt{n}}\right) = 1.$$

- Problem

$$\begin{cases} \min f(x) = 5\sum_{i=1}^{4}(x_i - x_i^2) - \sum_{i=5}^{13} x_i \\ \text{subject to:} \\ \qquad 2x_1 + 2x_2 + x_{10} + x_{11} \le 10 \\ \qquad 2x_1 + 2x_3 + x_{10} + x_{11} \le 10 \\ \qquad 2x_2 + 2x_3 + x_{11} + x_{12} \le 10 \\ \qquad -8x_1 + x_{10} \le 0,\ -8x_2 + x_{11} \le 0,\ -8x_3 + x_{12} \le 0 \\ \qquad -2x_4 - x_5 + x_{10} \le 0,\ -2x_6 - x_7 + x_{11} \le 0 \\ \qquad -2x_8 - x_9 + x_{12} \le 0 \\ \qquad 0 \le x_i \le 1,\ i = 1, 2, \cdots, 9, 13;\ 0 \le x_j \le 100, j = 10, 11, 12 \end{cases}$$

其最优状态和最优值为

$$\min f(x^*) = f(1,1,1,1,1,1,1,1,1,3,3,3,1) = 1.$$

- Problem

$$\begin{cases} \max f(x) = \dfrac{\sin^3(2\pi x_1)\sin(2\pi x_2)}{x_1^3(x_1+x_2)} \\ \text{subject to:} \\ \qquad x_1^2 - x_2 + 1 \le 0 \\ \qquad 1 - x_1^2 + (x_2-4)^2 \le 0 \\ \qquad 0 \le x_i \le 10, \quad i = 1, 2 \end{cases}$$

其全局最优解在原点附近，如

$$\max f(x^*) \ge f(0.00015, 0.0225) > 1552.$$

- Problem

$$\begin{cases} \min f(x) = x_1^2 + (x_2 - 1)^2 \\ \text{subject to:} \\ \qquad x_2 - x_1^2 = 0 \\ \qquad -1 \le x_i \le 1, \quad i = 1, 2 \end{cases}$$

其全局最优解和最优值为

$$\min f(x^*) \ge f(\pm 0.70711, 0.5) = 0.750000455.$$