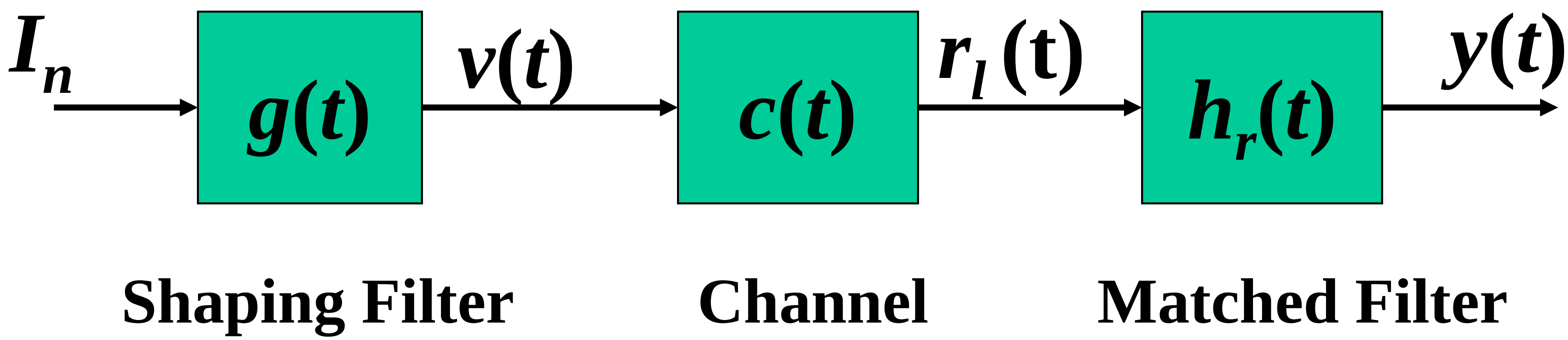
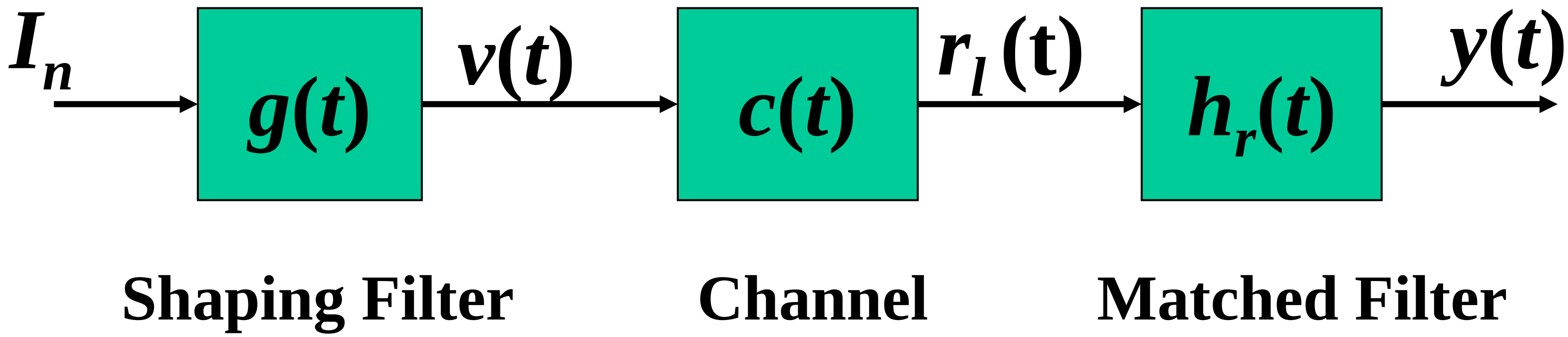
第六章　软件无线电中的信号处理

1. Signal Design for Band-Limited Channels



The equivalent lowpass transmitted signal has the form of

$$v(t) = \sum_{n=0}^{+\infty} I_n g(t - nT)$$



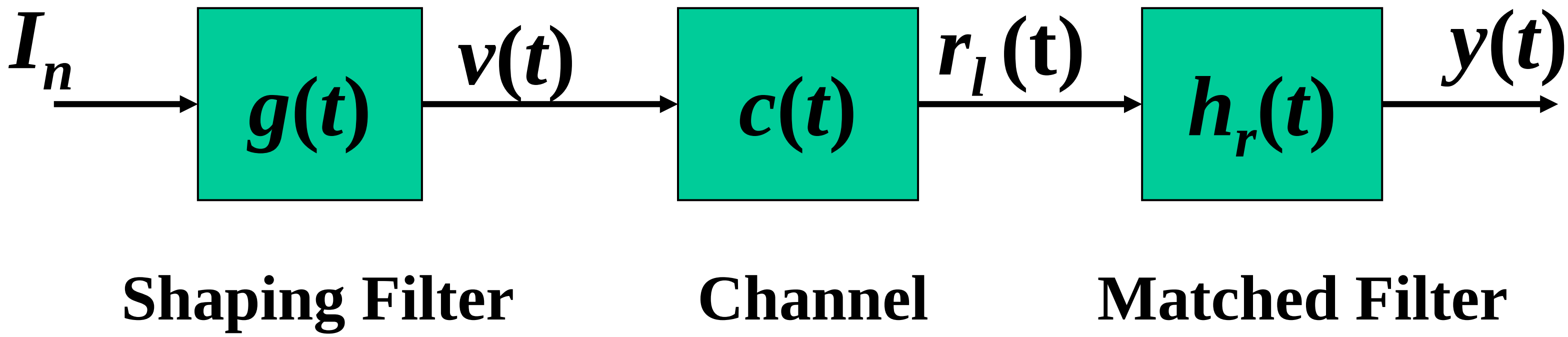
where

$\{I_n\}$ represents the transmitted symbols,

$g(t)$ is a shaping function with a band-limited frequency response $G(f)$, i.e., $G(f)=0$, for $|f|>W$,

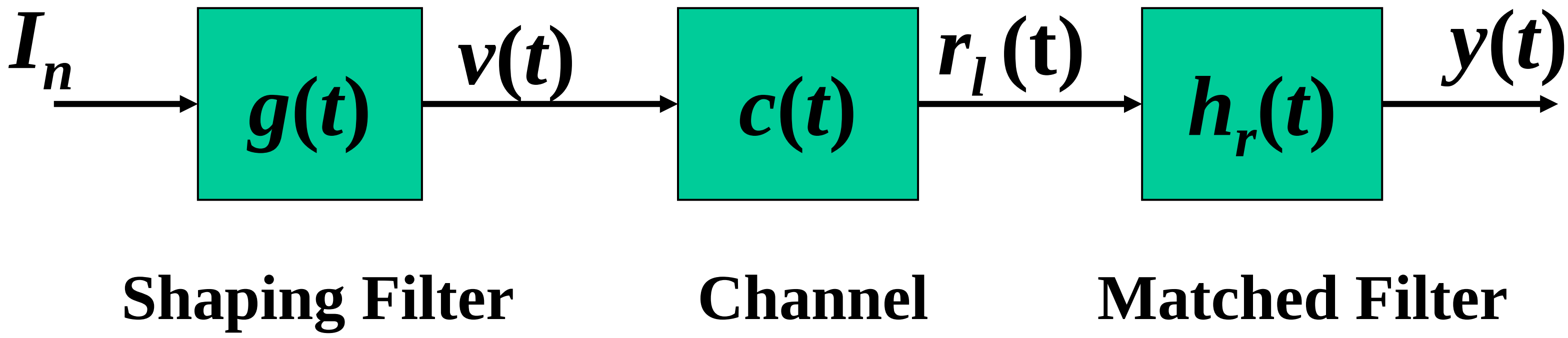
T is the symbol period,

$c(t)$ is the impulse response of channel with a frequency response $C(f)$, $C(f)=0$, for $|f|>W$.



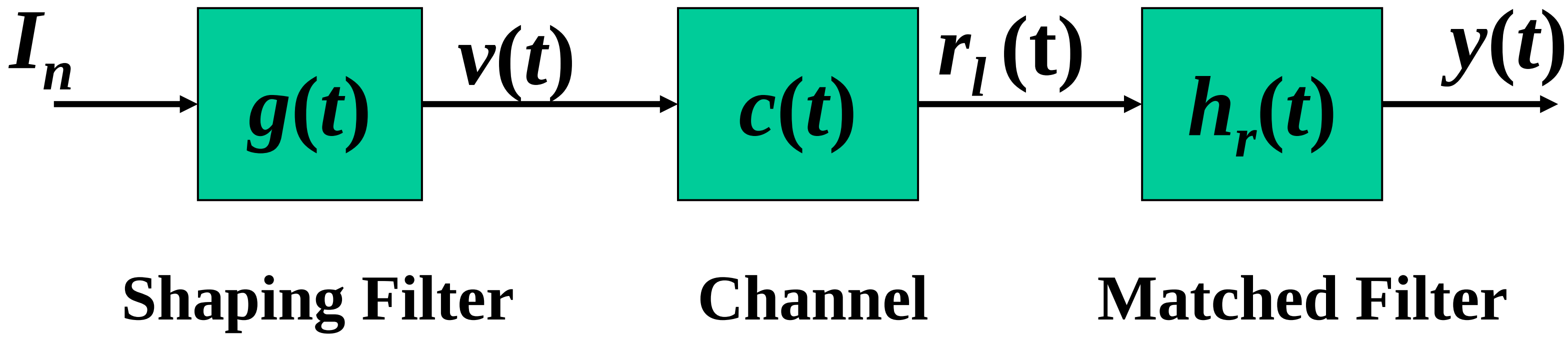
Consequently, the received signal can be represented as

$$r_l(t) = v(t) \otimes c(t) + z(t)$$



$$
\begin{aligned}
r_l(t) &= v(t) \otimes c(t) + z(t) \\
&= \sum_{n=0}^{+\infty} I_n \big(g(t-nT) \otimes c(t) \big) + z(t) \\
&= \sum_{n=0}^{+\infty} I_n \big(g(t) \otimes c(t-nT) \big) + z(t) \\
&= \sum_{n=0}^{+\infty} I_n h(t-nT) + z(t)
\end{aligned}
$$

where $h(t) = g(t) \otimes c(t)$



$$\boxed{r_l(t) = \sum_{n=0}^{+\infty} I_n h(t - nT) + z(t)}$$

where

$$h(t) = g(t) \otimes c(t)$$

represents the response of the channel to the input of the signal pulse $g(t)$, and $z(t)$ represents the additive white Gaussian noise (AWGN).

The received signal is passed through the matched filter that has the impulse response $h_r(t) = h^*(-t)$ (or the frequency response $H^*(f)$). The output of the matched filter can be expressed as

$$y(t) = r_l(t) \otimes h^*(-t)$$

$$= \sum_{n=0}^{+\infty} I_n h(t - nT) \otimes h^*(-t) + z(t) \otimes h^*(-t)$$

$$= \sum_{n=0}^{+\infty} I_n x(t - nT) + n(t)$$

where

$$x(t) = h(t) \otimes h^*(-t)$$

$$n(t) = z(t) \otimes h^*(-t)$$

If $y(t)$ is sampled at times $t = kT$, $k = 0, 1, \ldots$, we have

$$y(kT) \equiv y_k = \sum_{n=0}^{+\infty} I_n x(kT - nT) + n(kT)$$

or, equivalently

$$y_k = \sum_{n=0}^{+\infty} I_n x_{k-n} + n_k, \qquad k = 0, 1, \ldots,$$

$$= x_0 \Big(I_k + \frac{1}{x_0} \sum_{\substack{n=0 \\ n \neq k}}^{+\infty} I_n x_{k-n} \Big) + n_k$$

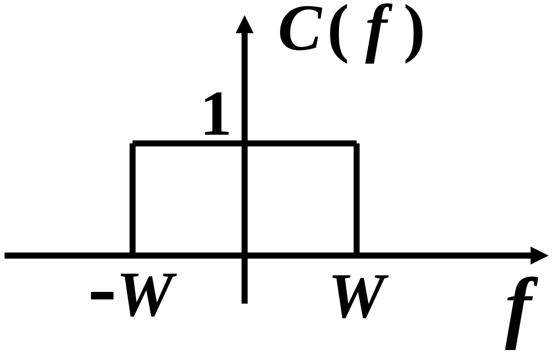
The term I_k represents the desired information symbol at the k^{th} sampling instant, the term

$$\sum_{\substack{n=0 \\ n \neq k}}^{+\infty} I_n x_{k-n}$$ **represents the ISI**

1.1 Signal Design for Channels with Ideal Frequency Response

We assume that the band-limited channel has ideal frequency response characteristics, i.e.,

$$C(f) = 1 \qquad for\ |f| \le W$$



Consequently

$$X(f) = H(f)H^*(f)$$

$$X(f) = |H(f)|^2$$

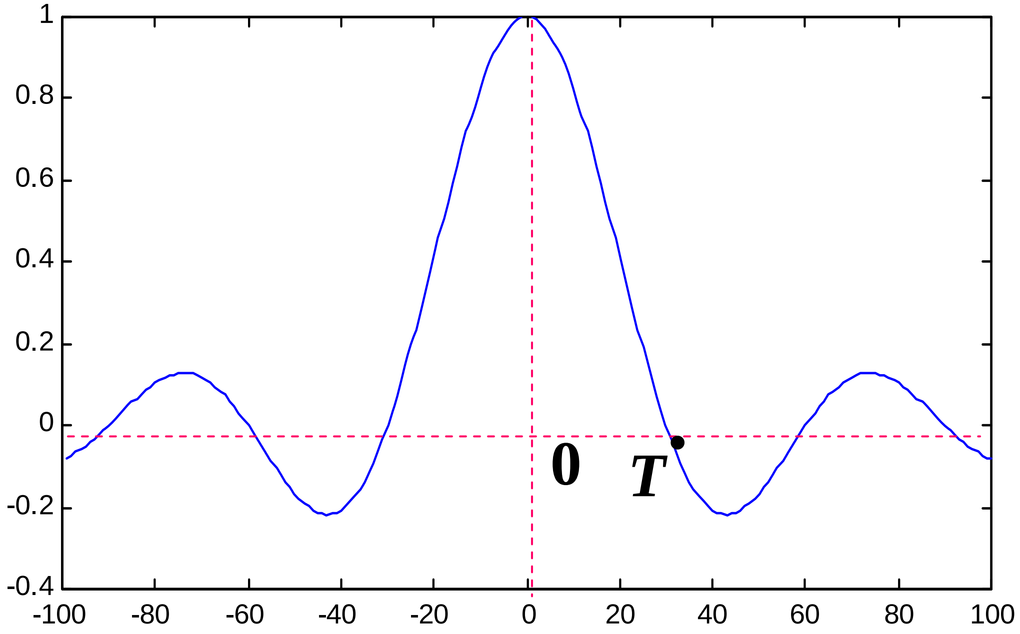
$$= |G(f)|^2 |C(f)|^2$$

$$= \begin{cases} |G(f)|^2 & \textit{for} \ |f| \le W \\ 0 & \textit{for} \ |f| > W \end{cases}$$

We are interested in determining the spectral properties of the pulse $x(t)$ and hence, the transmitted pulse $g(t)$, that results in no ISI.

Since $$y_k = x_0\left(I_k + \frac{1}{x_0}\sum_{\substack{n=0\\ n\neq k}}^{+\infty} I_n x_{k-n}\right) + n_k$$

The condition for no ISI is $$x_k = \begin{cases} 1 & (k=0) \\ 0 & (k\neq 0) \end{cases}$$



Theorem

The necessary and sufficient condition for $x(t)$ to satisfy

$$x(t = kT) \equiv x_k = \begin{cases} 1 & (k = 0) \\ 0 & (k \neq 0) \end{cases}$$

is that its Fourier transform $X(f)$ satisfy

$$\sum_{m=-\infty}^{\infty} X(f + m/T) = T$$

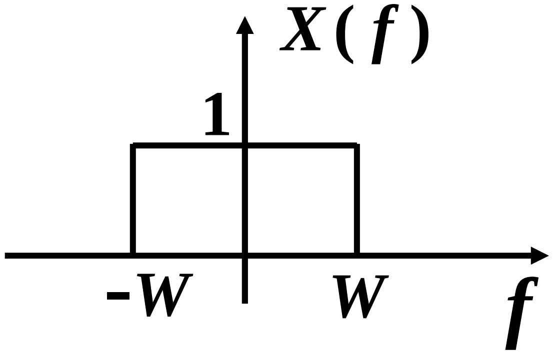
(1) When $1/T = 2W$ (the Nyquist rate)

There is only one choice for $X(f)$ that results in no ISI, namely,

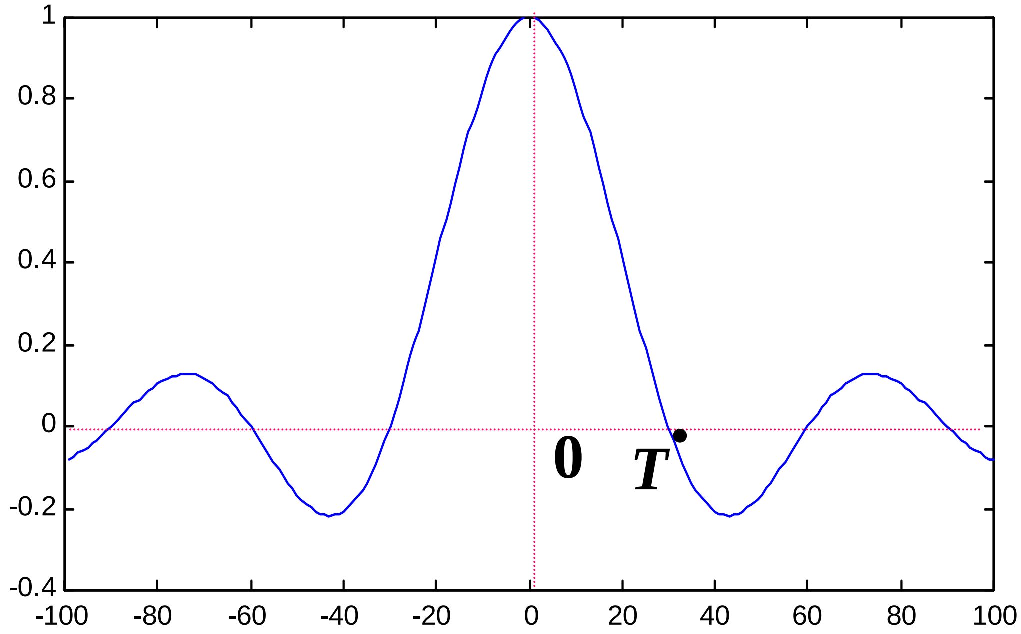
$$X(f) = \begin{cases} T & |f| < W \\ 0 & otherwise \end{cases}$$

which corresponds to the pulse

$$x(t) = \frac{\sin(\pi t / T)}{\pi t / T} \equiv \operatorname{sin} c(\pi t / T)$$



$$W = \frac{1}{2T}$$



(2) When $1/T < 2W$

A spectrum that has been widely used in practice is the raised cosine spectrum, namely,

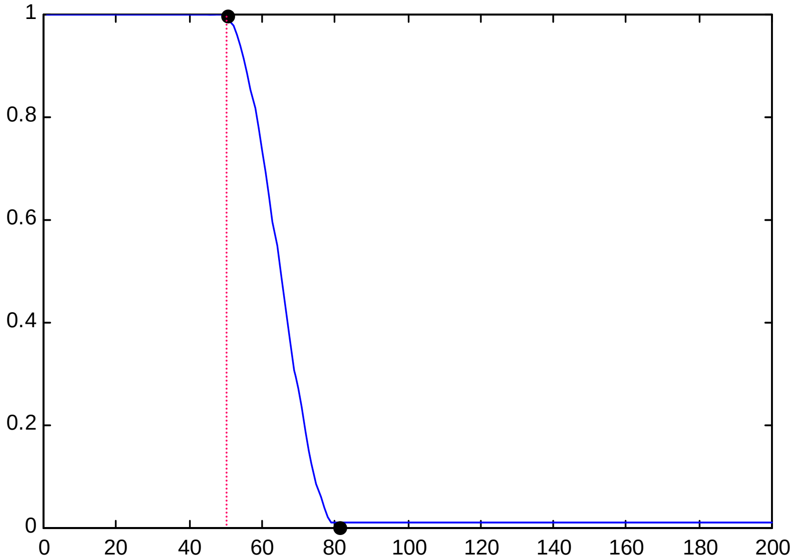
$$X_{rc}(f)=\begin{cases} T & 0\le |f| \le \dfrac{1-\alpha}{2T} \\ \dfrac{T}{2}\left\{1+\cos\left[\dfrac{\pi t}{\alpha}\left(|f|-\dfrac{1-\alpha}{2T}\right)\right]\right\} & \dfrac{1-\alpha}{2T}\le |f| \le \dfrac{1+\alpha}{2T} \\ 0 & |f| > \dfrac{1+\alpha}{2T} \end{cases}$$

where α is called the roll-off factor or the excess bandwidth.

The pulse $x(t)$, having the raised cosine spectrum, is

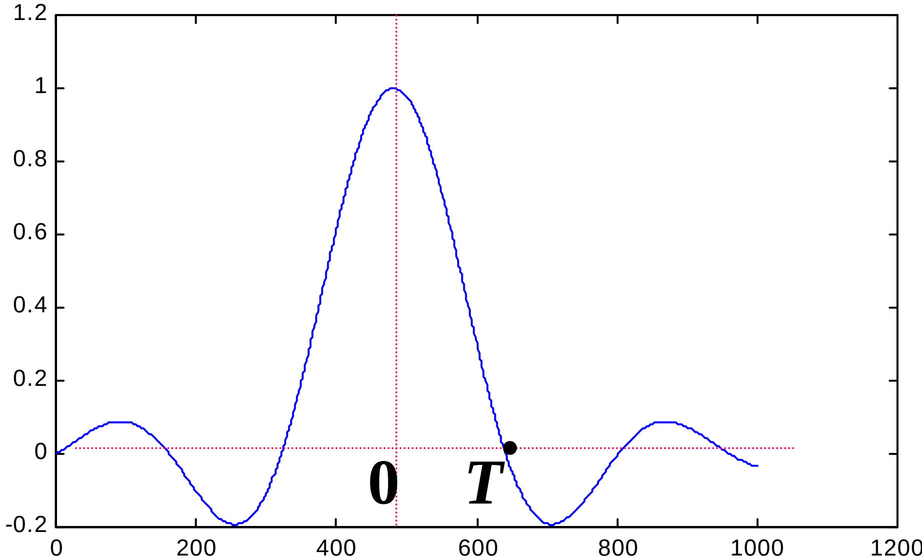
$$x(t) = \frac{\sin(\pi t / T)}{\pi t / T} \frac{\cos(\pi \alpha t / T)}{1 - 4\alpha^2 t^2 / T^2}$$

$X(f)$



$\frac{1-\alpha}{2T}$ $\frac{1+\alpha}{2T}$

$x(t)$



In the special case where the channel is ideal, we have

$$X(f) = \left|G(f)\right|^2 = G(f)G^*(f)$$

$$= G_T(f)G_R(f)$$

where

$G_T(f)$ is the frequency response of the transmitter filter.

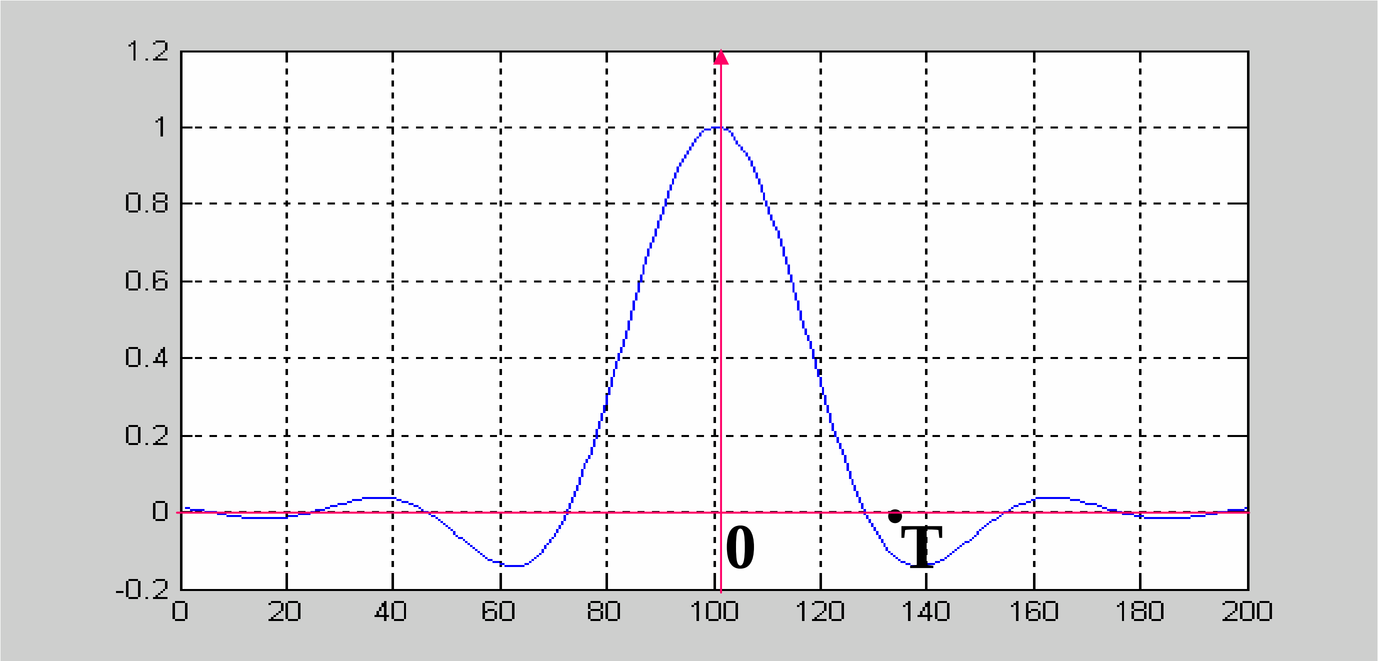
$G_R(f)$ is the frequency response of the receiver filter.

If the receiver filter is matched to the transmitter filter, we have

$$G_T(f) = G_R^*(f) = \sqrt{|X(f)|}$$

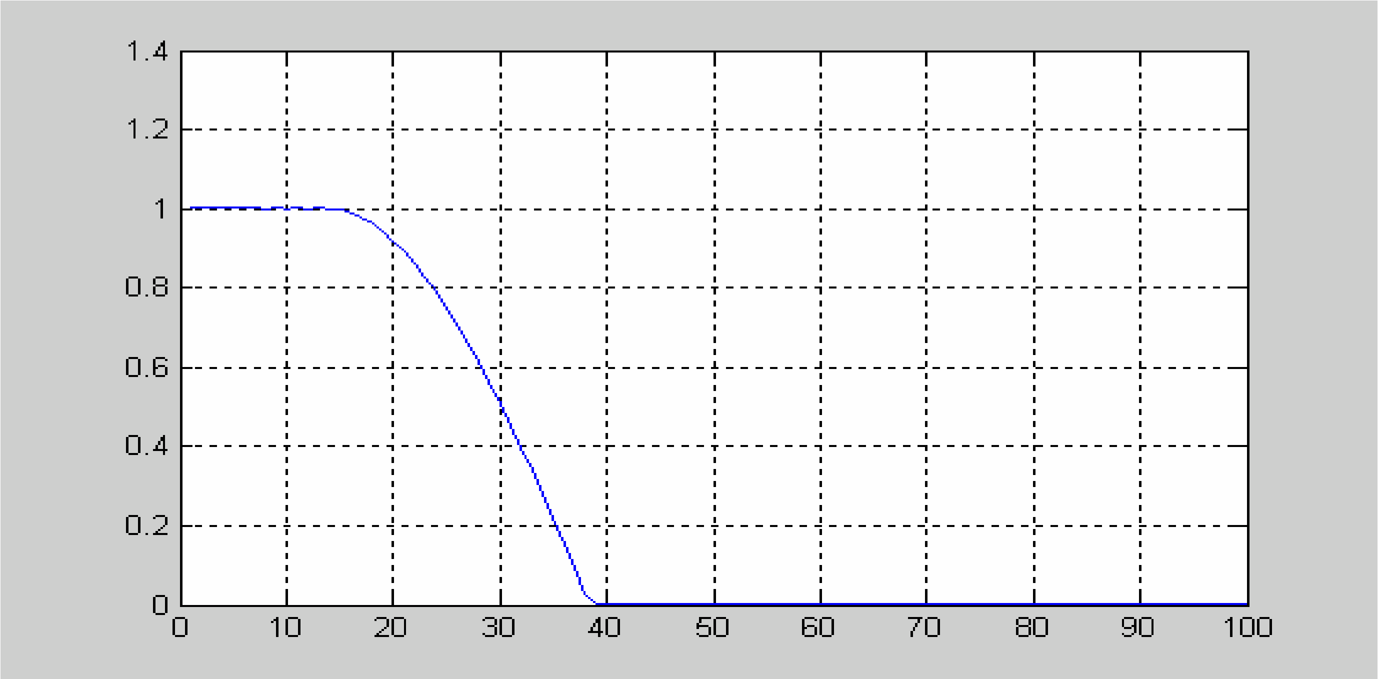
which is the square root raised cosine spectrum.

$h_T(t)$



1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
20
40
60
80
100
120
140
160
180
200
0
T

$G_T(f)$



1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
70
80
90
100

1.2 Signal Design for Channel with Unideal Frequency Response

We assume that the channel frequency response $C(f)$ is

$$C(f)=\begin{cases} known & |f|\le W \\ 0 & |f|>W \end{cases}$$

In the special case where the additive noise at the input to the demodulator is White Gaussian with spectral density $N_0/2$, we have

$$|G_R(f)|=\frac{|X_{rc}(f)|^{1/2}}{|C(f)|^{1/2}} \qquad |G_T(f)|=\frac{|X_{rc}(f)|^{1/2}}{|C(f)|^{1/2}} \qquad |f|\le W$$

2. Communication Through Band-limited Linear Filter Channels

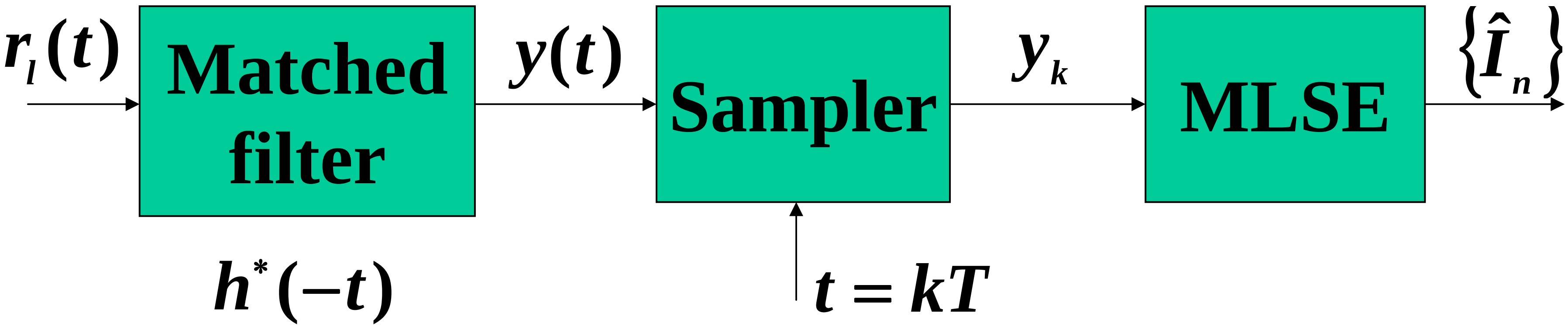
In practical digital communications systems, the frequency response $C(f)$ of the channel is not known a priori to design optimum fixed filters for the modulator and demodulator.

The channel distortion results in inter-symbol interference (ISI). The solution to the ISI problem is to employ an equalizer to compensate or reduce the ISI.

2.1 Optimum Receiver for Channel with ISI and AWGN

(1) Optimum Maximum-Likelihood Receiver I

It can be demonstrated that the optimum demodulator can be realized as a filter matched to $h(t)$, followed by a sampler operating at the symbol rate 1/T and a subsequent processing algorithm for estimating the information sequence $\{ I_n \}$ from the sample values.



As described before, the received (equivalent low-pass) signal is expressed as

$$r_l(t) = \sum_{n=0}^{+\infty} I_n h(t - nT) + z(t)$$

where

$$h(t) = g(t) \otimes c(t)$$

$z(t)$ represents the additive white Gaussian noise.

The output of the matched filter can be written as

$$y(t) = r_l(t) \otimes h^*(-t)$$

$$= \sum_{n=0}^{+\infty} I_n x(t - nT) + n(t)$$

where

$$x(t) = h(t) \otimes h^*(-t)$$

$$n(t) = z(t) \otimes h^*(-t)$$

The output of the sampler can be written as

$$y(kT) \equiv y_k = \sum_{n=0}^{+\infty} I_n x_{k-n} + n_k$$

where

$$x_k = \int_{-\infty}^{+\infty} h^*(\tau) h(\tau + kT) d\tau$$

$$n_k = \int_{-\infty}^{+\infty} z(\tau) h^*(\tau - kT) d\tau$$

The maximum-likelihood estimates of the symbols $I_1, I_2, \ldots, I_P$ are those that maximize the metrics

$$\boxed{CM(I_1, I_2, \ldots, I_P) = 2\,\mathrm{Re}\left(\sum_k I_k^* y_k\right) - \sum_k \sum_m I_k^* I_m x_{k-m}}$$

Ungerboeck Receiver

(2) Optimum Maximum-Likelihood Receiver II

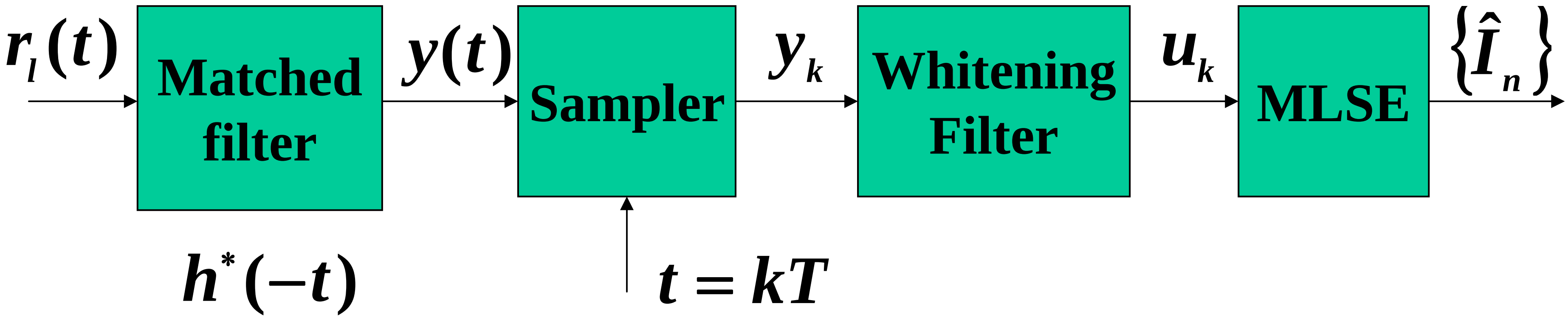
The noise added to the MLSE is not white because of its auto-correlation function expressed as following

$$E\left(n_k^* n_j\right) = E\left\{\int_{-\infty}^{+\infty} z(t)h^*(t-kT)dt \int_{-\infty}^{+\infty} z(\tau)h^*(\tau-jT)d\tau\right\}$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E\left[z(t)z(\tau)\right]h^*(t-kT)h^*(\tau-jT)dtd\tau$$

$$= 2N_0\int_{-\infty}^{+\infty} h^*(t-kT)h^*(t-jT)dt$$

$$= \begin{cases} 2N_0 x_{k-j} & |k-j| \le L \\ 0 & otherwise \end{cases}$$



To design the whitening filter, we determine the power spectrum of the noise, or equivalently, determine the spectrum of x_k.

Let $X(z)$ denote the z transform of x_k , namely,

$$X(z) = \sum_{k=-L}^{L} x_k z^{-k}$$

Since $x_k = x_{-k}^*$, it follows that $X(z) = X^*(1/z^*)$ and the $2L$ roots of $X(z)$ have the symmetry, i.e.,

$\boxed{\text{if } \rho \text{ is a root, } 1/\rho^* \text{ is also a root.}}$

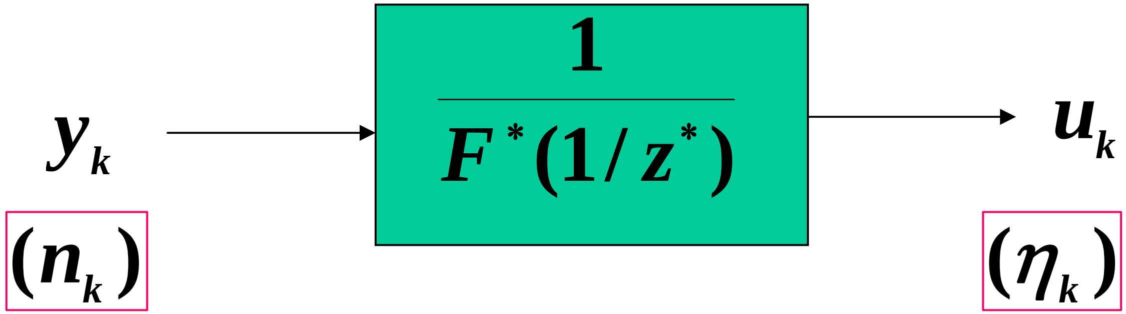
Hence, X(z) can be factored and expressed as

$$X(z) = F(z)F^*(1/z^*)$$

where $F(z)$ is a polynomial of degree L having the roots $\rho_1, \rho_2, ..., \rho_L$, and $F^*(1/z^*)$ is a polynomial of degree L having the roots $1/\rho_1^*, 1/\rho_2^* ..., 1/\rho_L^*$

We choose $F^*(1/z^*)$ so that it is a minimum phase system, i.e., $F^*(1/z^*)$ has its roots inside the unit circle.

Thus $1/F^*(1/z^*)$ is a physically realizable, stable, recursive discrete time filter.



$$y_k = \sum_{n=0}^{+\infty} I_n x_{k-n} + n_k$$

$$= I_k \otimes x_k + n_k$$

$$u_k = \sum_{n=0}^{L} f_n I_{k-n} + \eta_k$$

where $f_n \leftrightarrow F(z)$

η_k **is a white Gaussin noise sequence.**

The power spectral density of $\boxed{\eta_k}$ after the whitening filter can be written as

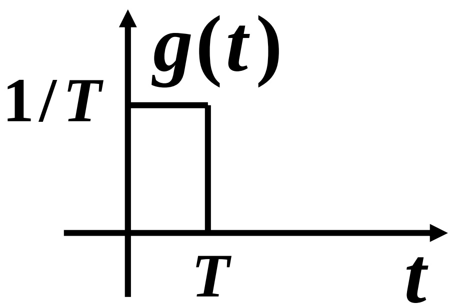
$$p_\eta(e^{j\omega}) = p_n(e^{j\omega})\left|\frac{1}{F^*(1/z^*)}\right|^2_{z=e^{j\omega}} = p_n(e^{j\omega})\left|\frac{1}{F^*(e^{j\omega})}\right|^2$$

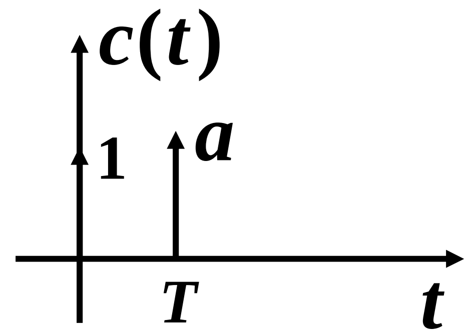
where $p_n(e^{j\omega})$ is the power spectral density of n_k

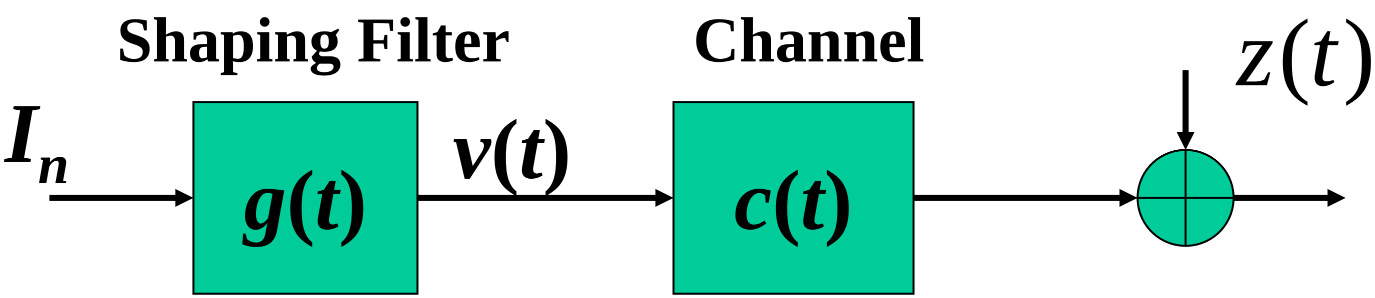
$$p_n(e^{j\omega}) = 2N_0 X(z)\big|_{z=e^{j\omega}} = 2N_0 F(e^{j\omega})F^*(e^{j\omega})$$

It is obvious that $p_\eta(e^{j\omega}) = 2N_0$

例子　发射信号脉冲 $g(t)$ 持续时间为T，能量为1, 信道冲击响应为 $c(t)=\delta(t)+a\delta(t-T)$。

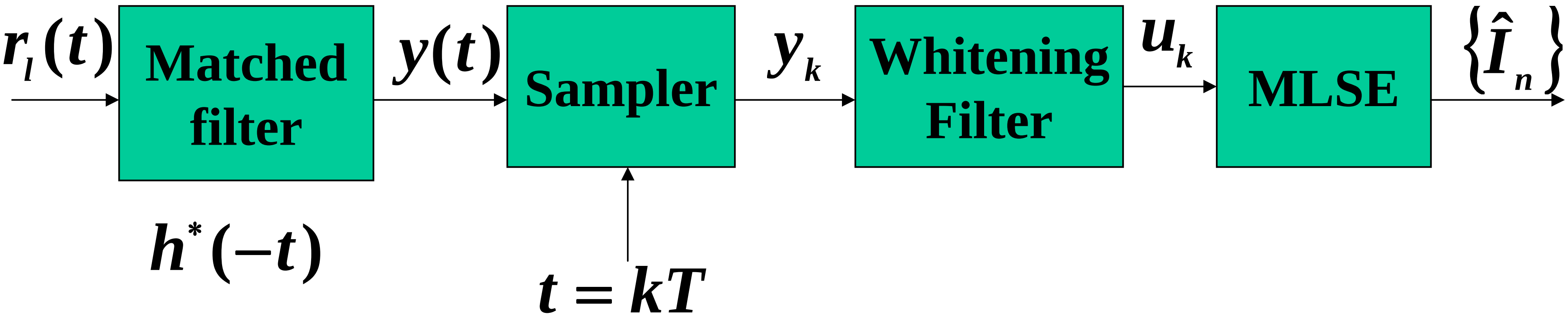






$$v(t)=\sum_{n=0}^{+\infty} I_n g(t-nT)$$

$$h(t)=g(t)\otimes c(t)$$
$$=g(t)+ag(t-T)$$



$$r_l(t) = \sum_{n=0}^{+\infty} I_n h(t - nT) + z(t)$$

$$y(t) = \sum_{n=0}^{+\infty} I_n x(t - nT) + n(t)$$

$$y_k = \sum_{n=0}^{+\infty} I_n x_{k-n} + n_k$$

where $$x_k = \int_{-\infty}^{+\infty} h^*(\tau) h(\tau + kT) d\tau$$

$$n_k = \int_{-\infty}^{+\infty} z(\tau) h^*(\tau - kT) d\tau$$

$$x_{-1}=\int_{-\infty}^{+\infty}h^*(t)h(t-T)dt=a^*$$

$$x_0=1+|a|^2$$

$$x_1=a$$

$$X(z)=\sum_{k=-1}^{1}x_k z^{-k}$$

$$=(az^{-1}+1)(a^*z+1)$$

假定 $|a|>1$　选择

$$\boxed{F(z)=az^{-1}+1,}\qquad \boxed{F^*(1/z^*)=a^*z+1}$$

$$u_k=\sum_{n=0}^{1}f_n I_{k-n}+\eta_k \qquad f_n=\{1,a\}$$

$$u_k = I_k + aI_{k-1} + \eta_k$$

令 $a = 2$, 则 $\quad u_k = I_k + 2I_{k-1} + \eta_k$

现有观察序列 $u_1 = 0.2, u_2 = 0.6, u_3 = 1.6, u_4 = 0.1$

要估计$I_1, I_2, I_3, I_4 \in (0, 1)$

$\boxed{\text{MLSE:}}$

$$\{\hat{I}_1, \hat{I}_2, \hat{I}_3, \hat{I}_4\} = \min \sum_{k=1}^{4} \left| u_k - (I_k + 2I_{k-1}) \right|^2$$

$[0, 0, 0, 0] \longrightarrow 2.97$

$[0, 0, 0, 1] \longrightarrow 3.77$

$[0, 0, 1, 0] \longrightarrow 4.37$

$[0, 0, 1, 1] \longrightarrow 9.17$

$\boxed{[0, 1, 0, 0]} \longrightarrow \boxed{0.37}$

$[0, 1, 0, 1] \longrightarrow 1.17$

$[0, 1, 1, 0] \longrightarrow 5.77$

$[0, 1, 1, 1] \longrightarrow 10.57$

$[1, 0, 0, 0] \longrightarrow 5.17$

$[1, 0, 0, 1] \longrightarrow 5.97$

$[1, 0, 1, 0] \longrightarrow 6.57$

$[1, 0, 1, 1] \longrightarrow 11.37$

$[1, 1, 0, 0] \longrightarrow 6.57$

$[1, 1, 0, 1] \longrightarrow 7.37$

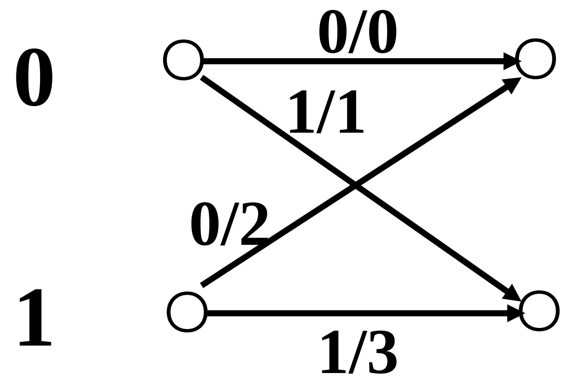
$[1, 1, 1, 0] \longrightarrow 11.97$

$[1, 1, 1, 1] \longrightarrow 16.97$

Viterbi Algorithm:

★状态转换图

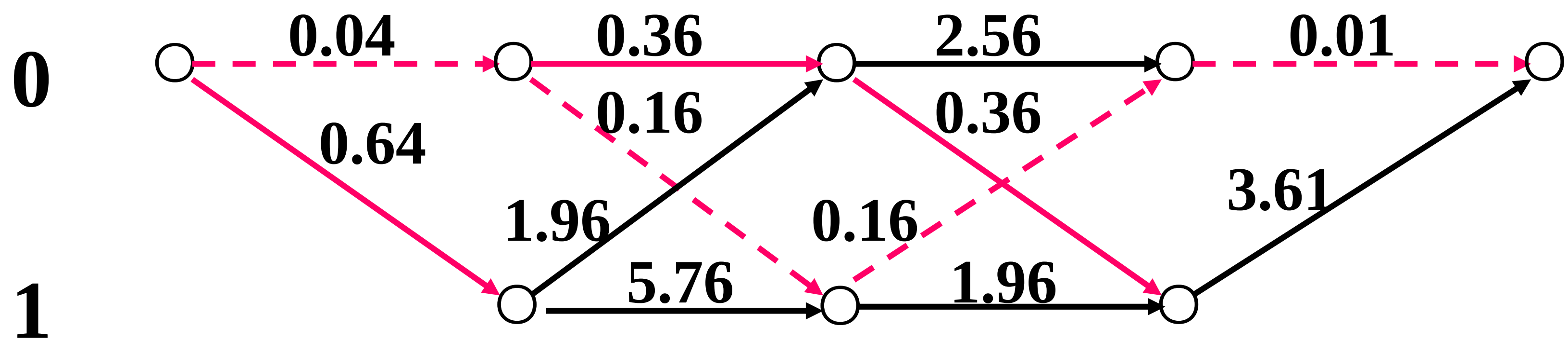
$$u_k = I_k + 2I_{k-1} + \eta_k$$



I_k / u_k

★Trellis 图

$$\left| u_k - (I_k + 2I_{k-1}) \right|^2$$



$$u_1 = 0.2,\ \ u_2 = 0.6,\ \ u_3 = 1.6,\ \ u_4 = 0.1$$

★译码输出

0	1	0	0

2.2 Linear Equalization

2.3 Decision-Feedback Equalization