# ESTIMATION OF CORONAVIRUS EPIDEMIC IN "INDIANA USA" USING THE SIR MODEL

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ABSTRACT

Presently the World is infected by the SARS-COV-2 virus which has created an emergency for public health. For Controlling the spreading of the virus , we have to prepare for precaution and futuristic calculation for infection spreading. The coronavirus affects the population of the world including the U.S.A. which is considered to be one of the best in medical services. Here we study the virus spreading rate on **Indiana** state which is a part of the USA. We are predicting the infected people by the SIR model. SIR model is one of the most effective models which can predict the spreading rate of the virus.

We have validated the model with the current spreading rate with this SIR model.

The SIR Model of Infection
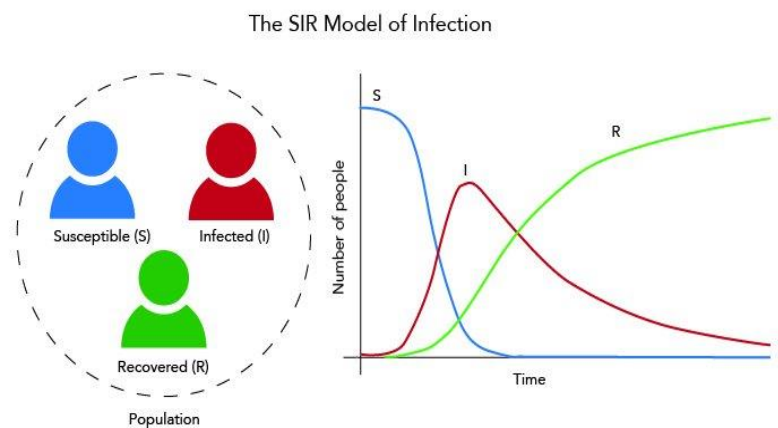
INTRODUCTION

Virus is a micro/nanometer in size which is reproduced inside the living cell of an organism. Presently coronavirus has created a health emergency to the world population and became a pandemic. Initially, this virus is transferred from the bat animal to the human. Further this virus shows the human to human transmission. Covid-19 virus is spreading to people by the respiratory droplets and contact mode. Previously there are several mathematical models reported. The SIR model is a simple and effective model which can give the prediction of different pandemic situation.

Here we are studying the spreading effects of COVID-19 to Indiana state. There are 50 states in the USA and Indiana is one of them. This state has a total population of 67.3 lacs which is ~4.9 % of the overall population of the USA. The first case observed on 6 march 2020 in Indiana was a Marrion County(Indianapolis) resident who had attended the Biogen conference in Boston. On March 12, the world health organization (WHO) announces that COVID-19 is an outbreak of pandemic where the term used by disease experts when epidemics are growing in multiple countries and continents at the same time. In the current situation, after 20 march coronavirus cases are rapidly increasing in Indiana.

This study explains the epidemic growth by using the SIR Model for this state which helps to control this epidemic.

THE SIR MODEL

The **SIR Model** is one of the simplest compartmental models, and many models are derivatives of this basic form. The model consists of three compartments:

- ➢ **S** for the number of Susceptible
- ➢ **I** for the number of Infectious
- ➢ **R** for the number of Recovered or deceased (or immune) individuals.

In the SIR model the idea works as follows I'm going to imagine taking all the people in the region say, Indiana (let it be **N)**, and dividing them into three different categories. The first category is called Susceptible. Susceptible are all of the people that are capable of becoming sick from an infection. In this case, we'll assume we have some sort of infection that is capable of infecting everybody so at the beginning, everybody is going to be susceptible then we have the second category of people those are the people who are infected and when you become infected you leave susceptible category for most diseases after you've gotten it after you've been infected you're not now susceptible. Those who have the infected disease, they are going to transition at some point hopefully into a recovered status, and as long as I assume that the death rate is small compared to the number that is being recovered.

I can write this assumption as an equation by saying that the sum of the susceptible **(S)**, the infected **(I)**, and the recovered **(R)**, people add up to **N.**

MATHEMATICAL MODEL :

Rate of change of Susceptible Population : $\dfrac{dS}{dt} = -\dfrac{\beta SI}{N}$

Rate of change of Infectious Population : $\dfrac{dI}{dt} = \dfrac{\beta SI}{N} - \gamma I$

Rate of change of Recovered Population : $\dfrac{dR}{dt} = \gamma I$

This model was implemented in python using SciPy for solving the differential equations. Key influencers here are the parameters β and γ which are described as :

➢ β: Decides how much the disease can be transmitted due to exposure. An interesting point here is, this β can be different for the same kind of virus in different societies. A society with less socialization will have a lower value of β.
➢ γ: is a parameter expressing how much the disease can be recovered in a specific period

SIR MODEL FOR INDIANA USA:-

Indiana is a USA state in the midwestern and Great Lakes region of North America. It is the 38 largest by the area and 17th most populous state of 50 United States and its capital and largest city is Indianapolis. It has a population of 6,732,219.

VARIABLES USED:-

N: - Total population

S(t): - Number of people susceptible on day t

I(t): - Number of people infected on day t

R(t): - Number of people recovered on day t

β: - The expected amount of people an infected person infects per day

D: - Number of days an infected person has and can spread the disease

γ: - The proportion of infected recovering per day ($\gamma = 1/D$)

We will first see what could happen if No precautions would have taken and no lockdown implemented.

Initial values of unknown parameters:

N=6732219

I(0)=1

R(0)=0

S(0)=N-R(0)-I(0)=6732218

β =0.315055143

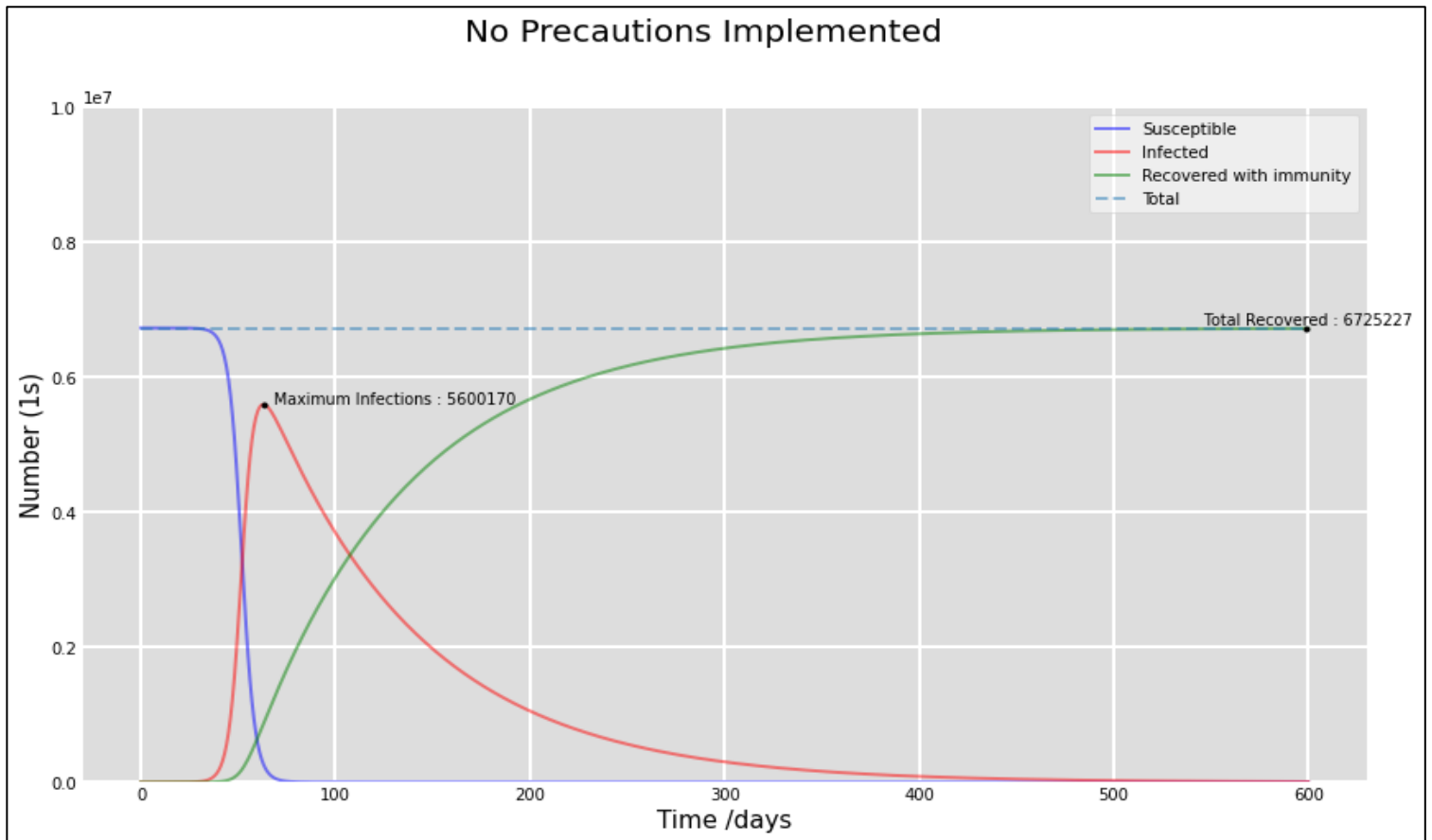γ =0.012541575

β = average(β' for each day before locked down)

γ = average(γ 'gamma for each day before locked down)

On day t :,

$$\beta' = -(S(t)-S(t-1))*N/ (S(t)*I(t)$$

$$\gamma' =(R(t)-R(t-1))/I(t)$$

Calculations are shown in file 'Initial beeta ,gamma.xlsx'

No Precautions Implemented

Initially, the entire population is susceptible to the infection, and a very negligible amount of people bring this infection into the country. As the disease spreads, the infectious section starts increasing, along with the number of recovered cases.

PREDICTIONS   (WITHOUT LOCKDOWN)   :

- ✓ Highest Number of Infected that may be Observed = 5,600,170 on 63rd day from the start of the Epidemic.
- ✓ Total Number of Recovered that may be Observed = 6,725,227 at the end of the Epidemic
- ✓ Duration of Epidemic ≈ 600 days

Now we will use Actual data to fit in our SIR with certain assumptions as follows:

ASSUMPTIONS   .

- ▪ The Total Population, N is fixed. In other words, we will assumes a time scale short enough that births and deaths (other than deaths from this disease) can be neglected.

- The only way a person can leave the susceptible group is to become Infected. The only way a person can leave the infected group is to recover(be immune or die) from the disease.
- Age, Sex, social status, and race do not affect the probability of being Infected.
- There is no inherited immunity.
- The member of the population mix homogenously(have the same interactions with one another to the same degree).
- A person become Infectious as soon as he/she gets the disease.
- No vaccine is available.

With These assumptions we can Fit our model to following equations obtained after Integrating set of first-order ODE's defined above:

$$
\begin{aligned}
&\text{i.} && S(t) = s0 * e^{-\frac{\beta R(t)}{\gamma}} \approx s0\left(1 - \frac{\beta R(t)}{\gamma} + \frac{\beta^2 R(t)^2}{\gamma^2} * \frac{1}{2}\right) \\
&\text{ii.} && R(t) = \rho^2 * \frac{\frac{s0}{\rho} - 1}{s0} + \frac{\rho^2}{s0} * \alpha * \tanh\left(\frac{\gamma\alpha t}{2} - \emptyset\right) \\
&\text{iii.} && \frac{dR}{dt} = \frac{\gamma\rho^2\alpha^2}{2s0} * \text{sech}^2\left(\frac{\gamma\alpha t}{2} - \emptyset\right) = \gamma I(t) \\
&\text{iv.} && I(t) = \frac{\rho^2\alpha^2}{2s0} * \text{sech}^2\left(\frac{\gamma\alpha t}{2} - \emptyset\right)
\end{aligned}
$$

Here S(t), R(t), I(t) denotes the Number of Susceptible, Recovered, Infected People on day t.

And constants $\gamma, \beta, \rho, \alpha, \emptyset$ are related by the following expressions:

➢ $\rho = \frac{\gamma}{\beta}$

➢ $\alpha = \sqrt{\left(\frac{s0}{\rho} - 1\right)^2 + 2s0 * \frac{(N - s0)}{\rho^2}}$

➢ $\emptyset = \tanh^{-1}\left[\frac{\left(\frac{s0}{\rho} - 1\right)}{\alpha}\right]$

CURVE FITTING PROCEDURE :

- For Curve fitting and error minimization, I used curve_fit() function from scipy. optimize library of python.

- On searching the various databases for Indiana USA the data available was only total COVID cases (=N-S(t)) and death cases. Recovered People data was not available day by day until 1 June. So very first I use the Susceptible People (which is actual data) to Fit it to curve S(t).

- Then I Used its result to predict R(t) such that It matches the actual recovered people data that I have after 1 June. This ensures that I am doing my modeling so precisely as I have data to crosscheck. (!!!!!!!)

- Finally, I find the No. of Infected I(t) by using the expression I(t)= N-S(t)-R(t)

Methods/functions used in Coding :

❖ Function _1

```
def susceptible(t,c0,c1,c2,c3,c4): # function for curve-fit susceptible people
    S0=6732219                     # No of susceptible on day 1
    return S0*(1-c4*(c0+c1*np.tanh(c2*t-c3)) + (c4*(c0+c1*np.tanh(c2*t-c3)))**2)
```

Here constants c0,c1,c2,c3,c4 are defined as :

One important note here is that c0,c1 values that we obtain here may or may not be the one that is defined by given expression as what this function calculates exactly is c4*c1,c4*c1 but since c2, c3 are inside the tanh function so values obtained after proper curve fitting are very good approximate values of parameters as defined by formulas. So to find the exact values of c0,c1 we need recovery data such that after performing curve_fitting on the recovered people data it gives us the correct result.

$$C0=\rho^2 * \frac{\frac{s0}{\rho}-1}{s0} \qquad C1=\frac{\rho^2}{s0} * \alpha$$

$$C2=\frac{\gamma\alpha}{2} \qquad C3 = \emptyset$$
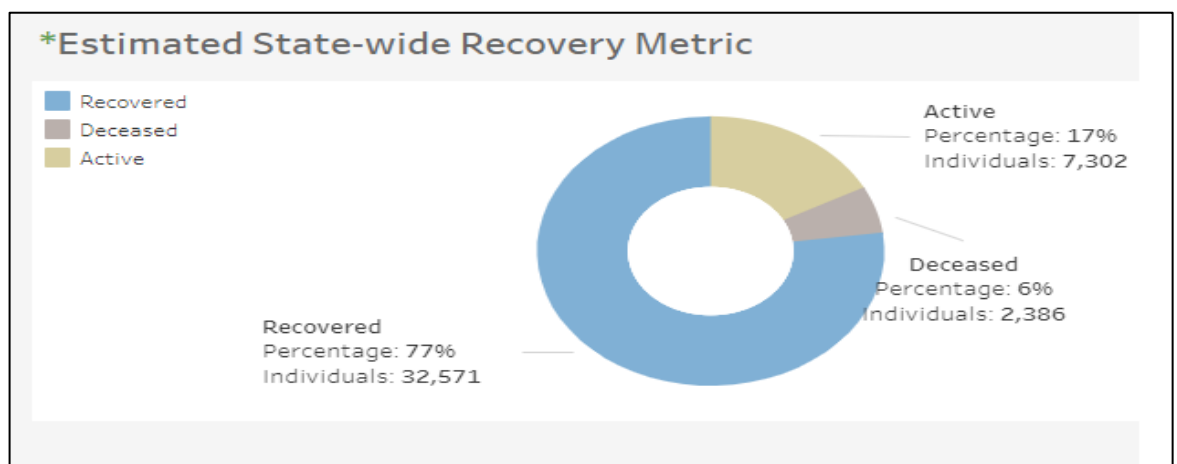
$$d = \frac{\beta}{\gamma}$$

This lead to the prediction of S(t) curve ....

For the estimation of recovered People, I assume that the Recovery rate is proportional to death rate and is constant over the time and then find the constant of proportionality using Recovered data already available. This is a big but necessary assumption .

*Estimated State-wide Recovery Metric

- Recovered
- Deceased
- Active

Active
Percentage: 17%
Individuals: 7,302

Deceased
Percentage: 6%
Individuals: 2,386

Recovered
Percentage: 77%
Individuals: 32,571

Let d_rate=constant*R_rate=n(deaths)/(n(deaths)+n(recovered but not died))

   R_rate= n(recovered)/(n(deaths)+n(recovered but not died))

On performing calculations I found that **constant**= **0.07325534985**

For more details see the file 'Indiana_USA_SIR_Model.xlsx'.

❖Function_2

```python
import numpy as np
def recovered(t,d):  #function for R(t)
    d0=(0.003127806020770324)/d
    d1=(0.0036799147589641344)/d
    d2=2.09301098e-02
    d3=1.55054122e+00
    return (d0+d1*np.tanh(d2*t-d3))
```

Here d0,d1 are the actual values of co,c1 that will fit the curve R(t) s.t.
,d0=c0*c4/d   d1=c1*c4/d   d2=c2   d3=c3
So recovered function will optimize only one parameter d.
So in this way we find the R(t) Curve.....
As Total Population remain constant so Infected population should be
I(t)=N-S(t)-R(t)

Hence Our SIR modeling completes here.

RESULTS:-
    S0=6732218

   By curve fiitng we obtain d0,d1,c2,c3,d as :

    d= 1.48679146e-07
. d0=c0*c4/d= 21037.288045563055
. d1=c1*c4/d= 24750.71224154149
   d2=c2=0.0214281098
   d3=c3=1.55054122

Values of unknown parameters obtained using equations shown here are:

$\checkmark \ \rho \approx 6723559.746$

$\checkmark \ \alpha \approx 1.405 * 10^{-3}$

$\checkmark \ \gamma \approx 32.835$

$\checkmark \ \beta \approx 4.88 * 10^{-6}$

$\checkmark \ R0 = \dfrac{\beta s0}{\gamma} \approx 1.001$
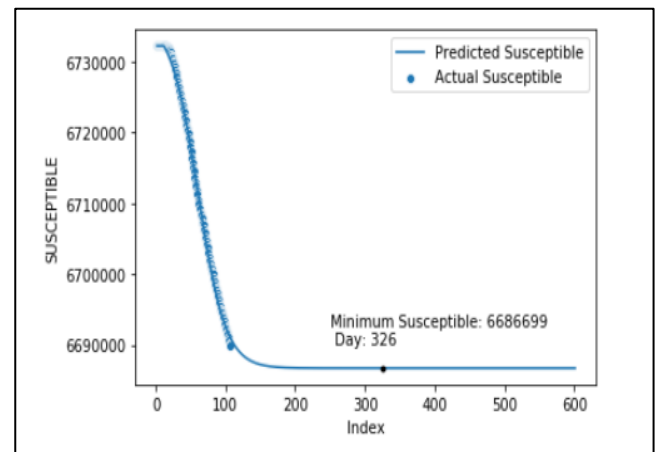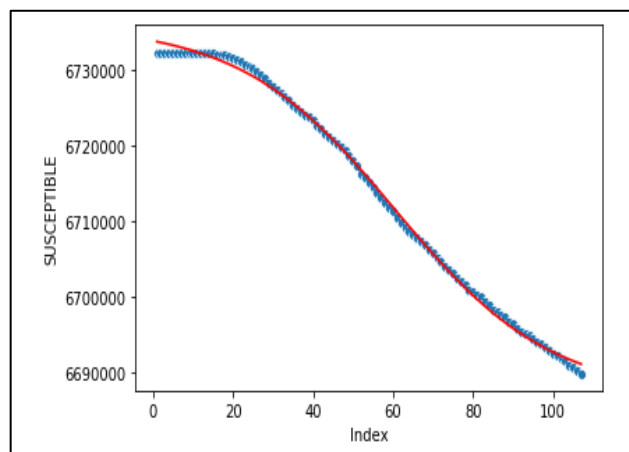
$$d0 = \rho^2 * \frac{\frac{s0}{\rho} - 1}{s0} \qquad d1 = \frac{\rho^2}{s0} * \alpha$$

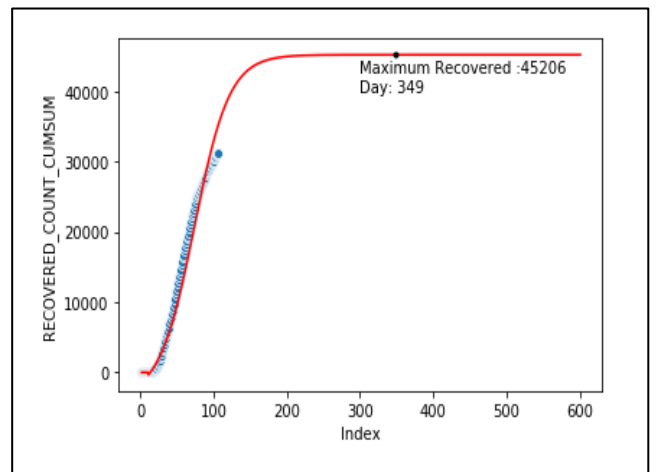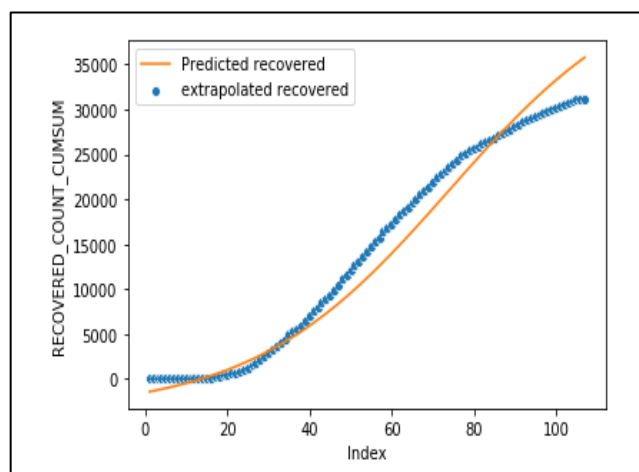$$C2 = \frac{\gamma \alpha}{2} \qquad C3 = \emptyset$$

$$d = \frac{\beta}{\gamma}$$

CURVE FITTINGS:-

1. a) Susceptible Population curve fitting   b) Prediction on basis of fiiting

2. a)Recovered Population curve fiiting   b)Prediction on basis of fitting

- Minimum Susceptible of susceptible population observed =6686699 on 326th day of epidemic.
- Total Number of Recovered to be Observed =45206 at the end of epidemic i.e. after 349 days from 6 march 2020.
- At end of epidemic Error in recovered Observed : (N-Smin)/Total recovered*100≈0.7%

DISCUSSION:-

➢ Lets First look at how lockdown affected the death rate and compare it with our model:(This is Just to see If our model is close to reality or not)

➢ Indiana reported it's first case of COVID-19 on March, 6. On March, 16 Indiana reported it's first death due to COVID-19. Looking at the risisng cases Goverener implemented "Stay At Home" order on March 23rd. This greatly helped in reducing mobility to -53%.
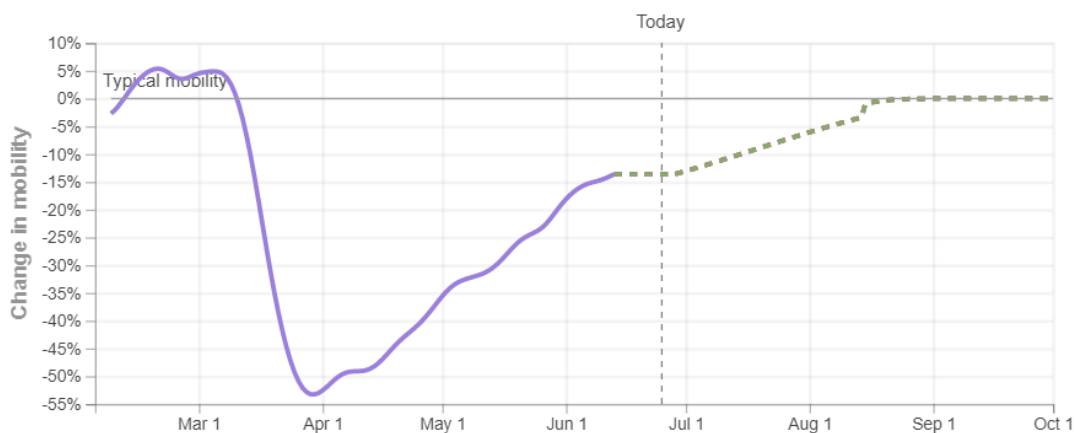

➢ Social distancing is the only proven solution for halting corona virus growth as of now. Nation practices strict social distancing in the lockdown, but still due to several factors, we see the failures in some lockdowns. Parameter β governs the transmission efficiency of the virus in given society, since lockdowns and social distancing measures focuses on reducing the transmission, reduction in β for different stages will reflect the how better the lockdown was performed.

➢ The β for a complete shutdown in Indiana comes out to be $4.88 * 10^{-6}$

➢ As expected of a lockdown, β decreases .In Indiana according to our modeling β will decreas from 0.315055143 to $4.88 * 10^{-6}$.

Effect on death rate:-

➤ This is defined as the number of deaths / confirmed cases. this metric is of importance as the death rate is merely 5.5% for the virus, but more fatal for elderly people and those having other complications. . Indiana has 706 ICU beds available and total of 8,485 beds.Daily deaths had been increasing till April, 29 where it recorded highest single deaths in a day. After, that had been decreasing rapidly.

➤ According to our modeling We have seen that maximum infectives are obtained after 77 days which is after 17 days from April ,29 from where deaths per day start decreasing and we can see as our modeling it will take about 17 days to reach the peak after 29 April.
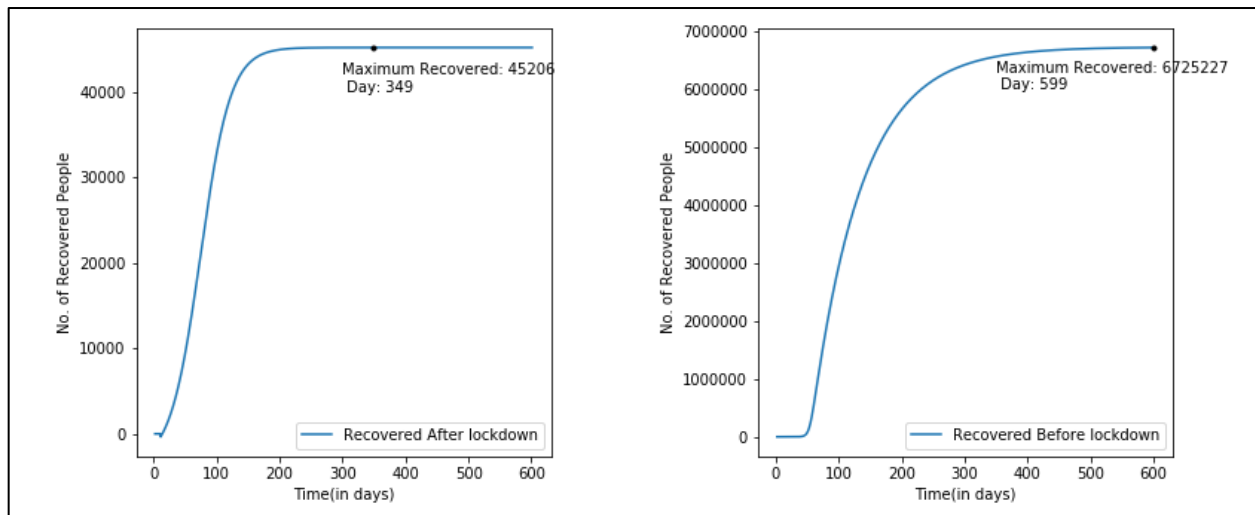
➤ EFFECT OF LOCKDOWN ON SPREADING OF DISEASE:-

FIGURE 1
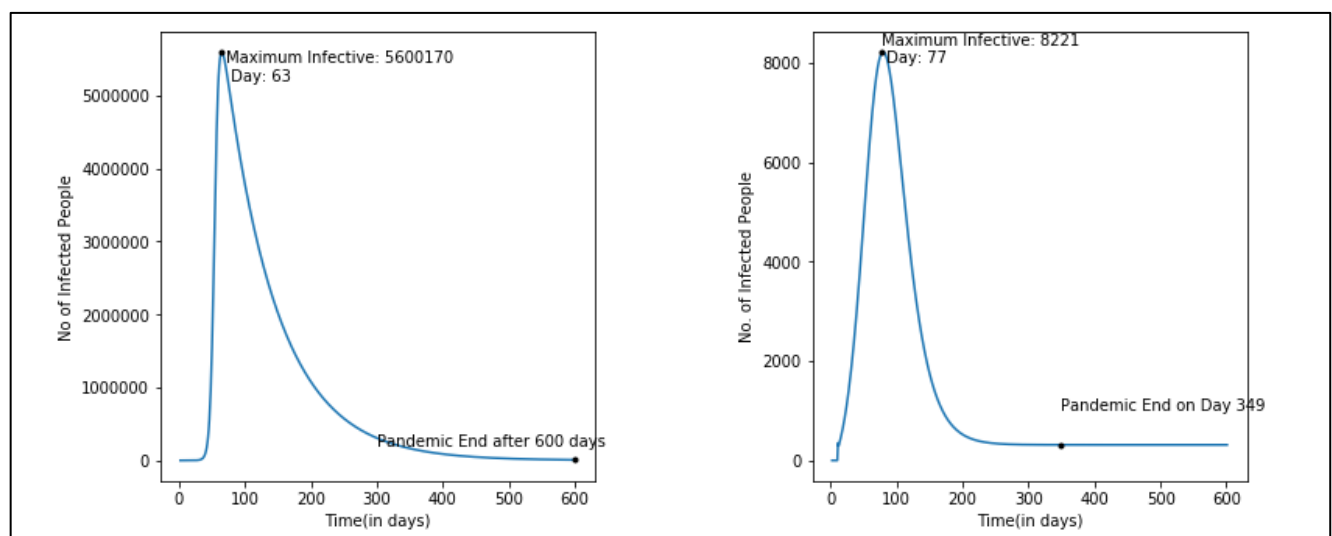
➤ Lockdown has very positive effect on controlling the spread of coronavirus. As you can see Maximum Infected People reduce from 5600170 to just 8221.

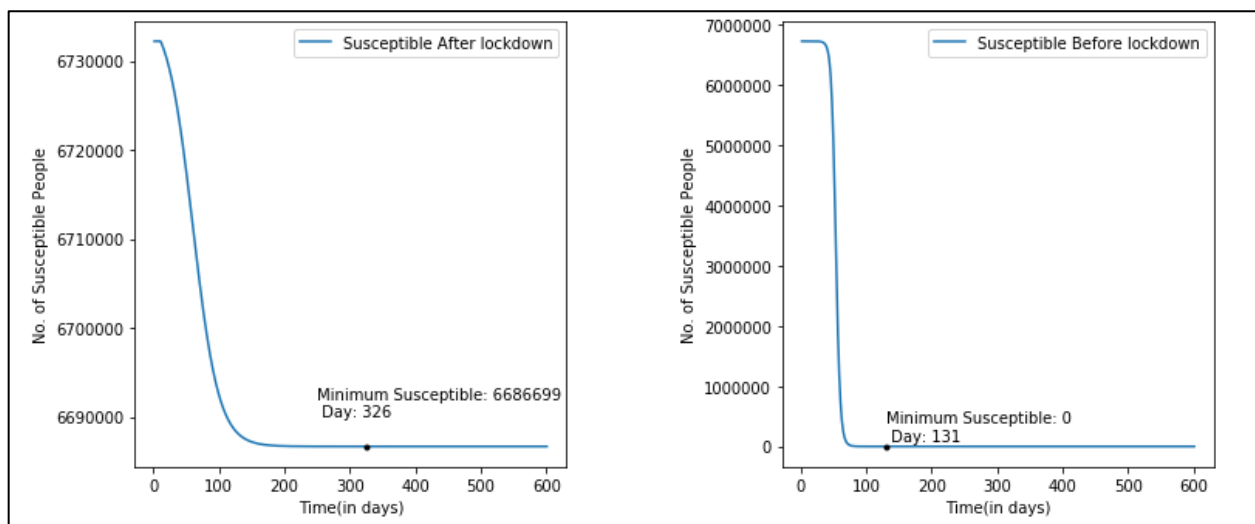➤ This shows the importance of lockdown.!!!!!!!!!

FIGURE 2

➤ Similar Effect also observed for Recovered People

   ❖ According to the Plot of Infected and Susceptible and Infected we conclude that the epidemic is expected to end after 349 days from the start of epidemic (i.e. 6 march 2020).

FIGURE 3

➤ As Expected , If no lockdown happen as people use to go to their work as they were then very soon , as per model after 131 everyone will get infected .

What we did was a simple SIR modeling . We have made certain assumptions like strict lockdown , etc. .These are some suggestion that we have to extend our SIR model if sufficient data provided:-

- We can add **a "Dead" state** for individuals that passed away from the disease as separate compartment if actual recovery data is made available which is not there in case of Indiana.
- We can add **an "Exposed" state** for individuals that have contracted the disease but are not yet infectious (this is known as the *SEIR*-model)
- In our model we have assumed a single lockdown which last until epidemic is not over. But in reality there are multiple lockdowns with different level of strictness of implementation. In our case i.e. Indiana US we have first 2 lockdown from 23 march to 7 april followed by 8 april to 1 may.On 1 On May 1, Governor Holcomb announced a five-stage plan detailing the gradual reopening of business sectors in Indiana, with the final stage, completely reopening the state without restrictions, culminating on July 4.We could have seen effect of these five stage plan with how really infection spread.

  In short, **time-dependent R_0-values** that will allow us to model quarantines, lockdowns more precisely
- We can see the effects of **resource- and age-dependent fatality rates** by adding extra compartments that will enable us to model overcrowded hospitals, populations with lots of young people, ... And many more.

RESOURCES/CITATIONS:

1. John   Hopkins   Coronavirus   Resource   centre, https://coronavirus.jhu.edu/map.html,20/06/20
2. COVID-19 Pandemic in Indiana, https://en.wikipedia.org/wiki/COVID-19_pandemic_in_Indiana,19/06/20
3. Indiana State Department of Health,COVID-19 Statewide Test, Case, and Death Trends, https://hub.mph.in.gov/dataset/covid-19-case-trend,19/06/20
4. COVID-19 Data Repository by the Center for Systems Science and Engineering (CSSE) at Johns Hopkins University, https://github.com/CSSEGISandData/COVID-19,19/06/20
5. Regenstrief   COVID-19   Dashboard,https://www.regenstrief.org/covid-dashboard/?fbclid=IwAR39JFPW2mvPI6Hlu7n3ZS97OJG-YyfKcJ6n8LkuPzLmVFGyvZxpI2vlaqI,16/06/20