Laboration 2

Due 2024-12-03

1. Given an array $A = \langle a_1, a_2, \cdots, a_n \rangle$ of non-zero real numbers, the problem is to find a subarray $\langle a_i, a_{i+1}, \cdots, a_j \rangle$ (of consecutive elements) such that the sum of all the numbers in this subarray is maximum over all possible consecutive subarrays. Design a divide and conquer algorithm to compute such a maximum sum. You do not need to actually output such a subarray; only returning the maximum sum. Your algorithm should run in $O(n)$ time in the worst case. You may assume that n is a power of 2.

   - Give a complete and unambiguous high-level description (step-wise) of your algorithm in plain English/Swedish; and
   - Implement your algorithm (using Python) as *one* recursive function. Built-in functions or methods for strings or lists must not be used.

2. Given an array $A = \langle a_1, a_2, \cdots, a_n \rangle$ of n elements (you may assume that n is a power of 4), consider the following comparison-based sorting algorithm:

   def $sortR(\langle a_1, a_2, \cdots, a_n \rangle)$:

   0) If $n \leq 4$, then sort the input with insertion sort and return the sorted array.
   1) Sort $< a_1, \cdots, a_{\frac{3n}{4}} >$ recursively and let $< b_1, \cdots, b_{\frac{3n}{4}} >$ be the sorted output.
   2) Sort $< b_{\frac{n}{4}+1}, \cdots, b_{\frac{3n}{4}}, a_{\frac{3n}{4}+1}, \cdots, a_n >$ recursively and let $< c_{\frac{n}{4}+1}, \cdots, c_n >$ be the sorted output.
   3) Sort $< b_1, \cdots, b_{\frac{n}{4}}, c_{\frac{n}{4}+1}, \cdots, c_{\frac{3n}{4}} >$ recursively and let $< d_1, \cdots, d_{\frac{3n}{4}} >$ be the sorted output.
   4) return $< d_1, \cdots, d_{\frac{3n}{4}}, c_{\frac{3n}{4}+1}, \cdots, c_n >$

   Analyze the above algorithm (namely, show that the algorithm is correct and give a *recurrence equation* for the number of comparisons used by the algorithm in the worst case). No implementation of the algorithm is needed. *Hint: One can use the induction technique to show the correctness. Check Chapter 4 for more examples of performance analyses.*

Report

Each group must hand in one report for each part. The report can be written in either Swedish or English and should not be handwritten.

Before submitting your report, you should discuss your solution to the laboration (design, implementation, and report) with your lab-assistant.