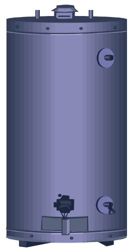
Exercise 1:

Modelling flexibility resources



To effectively utilize flexibility resources in grid planning and operation, there is a need for suitable models of the flexibility resources. In this exercise we will consider electric water heaters (EWH) – also called hot water tanks – as an example of a demand-side flexibility resource. We will use a simple model for quantifying its flexibility potential that was presented in [1] and is reproduced mathematically and as a thermal network diagram below. In this exercise we will ignore hot water use so that the only loss of thermal energy in the hot water tank is the passive heat loss from the tank surface.

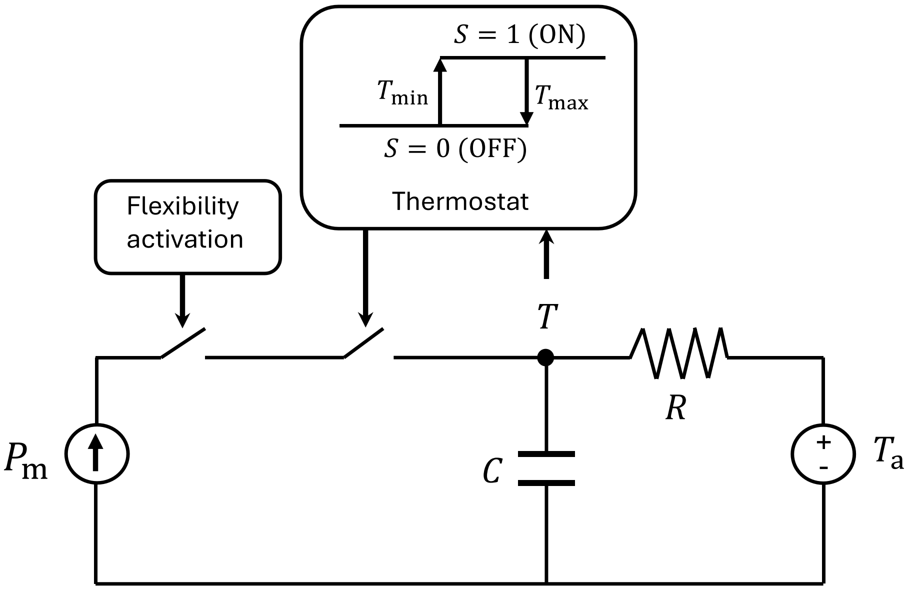


Figure 1: Equivalent network model of the thermal dynamics of an electric water heater as a flexibility resource, based on [1].

Here, $T(t)$ is the time dependent temperature inside the EWH and P_{m}. T_{a} is the ambient temperature outside the EWH that is assumed to be constant. For a given value of the thermostat status $S(t)$, the thermal dynamics of the model in Figure 1 is governed by Equations (1):

$$\frac{\mathrm{d}T(t)}{\mathrm{d}t} = \frac{1}{CR}\left[T_{\mathrm{a}} - T(t) + S(t)RP_{\mathrm{m}}\right] \tag{1}$$

The thermostatic control can be described as follows:

- If $T(t) \leq T_{\mathrm{min}}$ while $S(t) = 0$, then $S(t)$ is set to 1
- If $T(t) \geq T_{\mathrm{max}}$ while $S(t) = 1$, then $S(t)$ is set to 0
- Otherwise, $S(t)$ is kept constant

While $S(t)$ is constant, the time evolution of the temperature is given by the solution:

$$T(t) = \left[T_0 - T_{\mathrm{a}} - S(t)RP_{\mathrm{m}}\right]e^{-\frac{t-t_0}{RC}} + T_{\mathrm{a}} + S(t)RP_{\mathrm{m}} \tag{2}$$

The quantities are defined as follows:

- $T(t)$ is the water temperature inside the EWH at time t in $^\circ$C
- T_a is the ambient temperature outside the EWH in $^\circ$C
- C is the thermal capacitance of the EWH in kWh/$^\circ$C
- R is the thermal resistance of the EWH in $^\circ$C/kW
- P_m is the rated power consumption of the EWH in kW
- $S(t)$ is the EWH thermostat status at time t; when $S = 1$ the EWH is turned ON, and when $S = 0$ the thermostat is turned OFF.
- T_{min} is the minimum allowed EWH water temperature in $^\circ$C
- T_{max} is the maximum allowed EWH water temperature in $^\circ$C
- T_0 is the temperature $T(t_0)$ at some given initial time t_0

Flexibility activation entails overriding the default thermostatic control (normal operation) of the electric water heater by turning it ON ($S = 1$) or OFF ($S = 0$).

In the code that is made available with this exercise, the model is implemented by considering time steps of duration $\Delta t = 1$ minute and that $S(t)$ is constant within each time step $[t_0, t_0 + \Delta t]$. The code simulates the time dynamics and flexibility activation of this flexibility resource over a time period of 24 hours (1440 minutes).

In some of the tasks you will be asked to characterize the flexibility potential of the EWH. Here you may refer to the general illustration of flexibility characteristics in Figure 2 (below) and its presentation in lecture 1 and Reference [3].

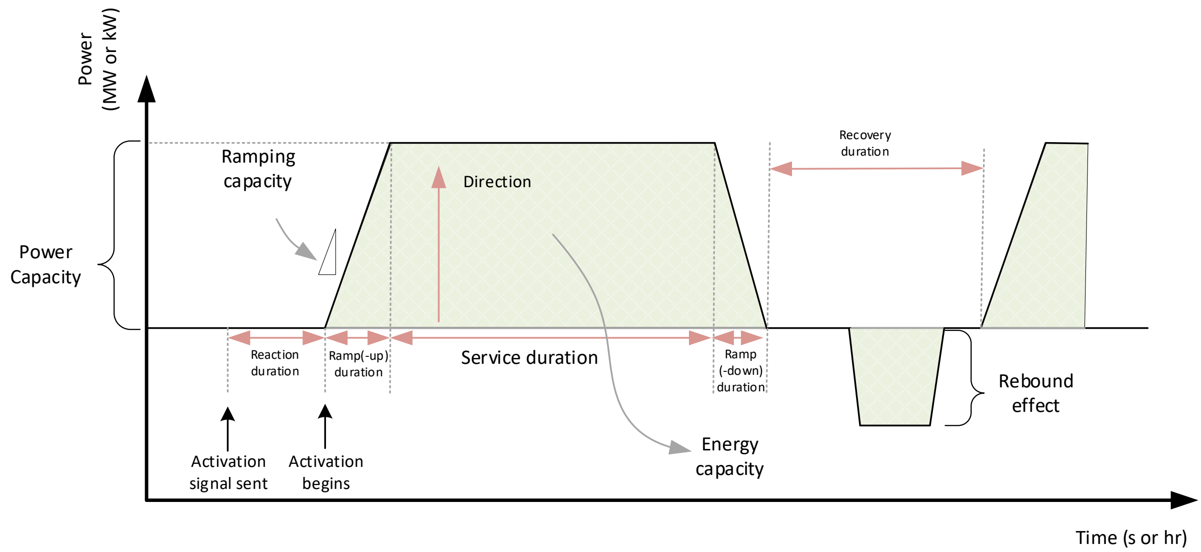


Figure 2: Illustration of flexibility characteristics for a general flexibility resource [3].

Learning outcomes:

- Practical experience with a flexibility modelling approach
- Characterize and quantify flexibility potential

- Characterize and quantify rebound effects

Data and code inputs:

- Script with code used to carry out the exercise: https://github.com/SINTEF-Power-system-asset-management/CINELDI_MV_reference_system/blob/flexibility_course_NTNU_public/exercise_1_flexibility_modelling.py
  - (This code is a simplified version of the code made available online in [2])
- Input data are specified in the script for the following parameters:
  - $P_{\mathrm{m}} = 2$ kW (200 L water heater)
  - $T_{\mathrm{a}} = 24$ °C
  - $T_{\mathrm{min}} = 70$ °C
  - $T_{\mathrm{max}} = 75$ °C
  - $C = 0.335$ kWh/°C
  - $R = 600$ °C/kW

Tasks:

1. **Plot and explain the temperate and power consumption of the EWH:**
   Run the code without activation of flexibility (i.e. without overriding the default thermostatic control of the electric water heater). By inspecting the plot of the temperature and power consumption, choose a time *t_act* to activate flexibility and the activation signal *S_act* so that part of the load is shifted to a later time. Plot the temperature and power consumption and explain what the figure shows. (There is not one single correct value of *t_act* for this task; you are free to choose which *t_act* you select.)

2. **Quantify the flexibility potential:**
   Quantify how much flexibility the EWH can provide when the activation time is set at the time *t_act* you chose for the previous question. Values for the following flexibility characteristics should be provided: i) power capacity, ii) service duration, and iii) energy capacity. (You may quantify this approximately by inspecting the plots; making code for exact quantification is optional.)

3. **Plot and explain the amount of flexibility activation:**
   Add code for plotting how much flexibility is activated (power) in kW. This should be a similar plot as in Figure 2 that shows how much the consumption pattern is modified by flexibility activation compared to the baseline consumption pattern that one would have without flexibility activation. (Assume that the horizontal line in Figure 2 represents the baseline consumption pattern where the value of the activated power is zero.) Run the model again for the same case as in task 1 and 2 and explain what the new plot shows.

4. **Describe the direction and rebound effect characteristics of the flexibility activation:**
   Comment on what the plot from task 3 shows about a) the direction of flexibility activation and b) potential rebound effects.

5. **Plot and explain load shifting to an earlier time:**
   Choose a time *t_act* to activate flexibility and a value of the activation signal *S_act* so that

load is shifted to an *earlier* time (advancing the load demand) and rerun the model. Explain what the plots show and comment on the direction of flexibility activation.

6. **Quantify the power capacity flexibility characteristics of 100 EWHs:**
   Set the number of EWHs to 100 and set the activation signal *S_act* to turn the EWHs off. Run the model and plot the resulting load time series for a few selections of the activation time *t_act* (that you are free to choose yourself). Quantify the flexibility potential (given the activation times you have selected) of the EWHs in terms of the power capacity.

7. **Explain the variability of the power capacity of 100 EWHs:**
   Explain how the flexibility potential quantified in task 6 depends on the choice of activation time *t_act*.

8. **Explain the reduction in power capacity for 100 EWHs:**
   Explain why the EWH flexibility potential quantified in task 6 (for 100 EWHs) is lower than the flexibility potential quantified in task 2 (for 1 EWH) when measured per EWH.

9. **Prove and explain mathematically the thermal dynamics of the EWH:**
   The code implements Eq. (2) to model the dynamics (time evolution) of the temperature in the EWH.
   a) Show that Eq. (2) is a solution to the differential equation (1).
   b) Show that $T(t = t_0) = T_0$.
   c) Explain the factor 1/60 in the code implementation of Eq. (2).

10. **Describe model simplifications:**
    Comment on some of the simplifications in the model you have been using. You may compare it with the slightly more detailed model presented in [1]. (The most relevant parts of [1] for this purpose are Sec. 1 and Sec. 2.1–2.3; the rest of Sec. 2 and Sec. 4 are also partly relevant.)

Relevant references:

[1] V. Lakshmanan, H. Sæle, and M. Z. Degefa, 'Electric water heater flexibility potential and activation impact in system operator perspective – Norwegian scenario case study', *Energy*, vol. 236, p. 121490, Dec. 2021, doi: 10.1016/j.energy.2021.121490.

[2] V. Lakshmanan, H. Sæle, and M. Z. Degefa, 'Domestic direct electric water heater model for flexibility and impact study', Available: https://zenodo.org/record/4382096

[3] M. Z. Degefa, I. B. Sperstad, and H. Sæle, 'Comprehensive classifications and characterizations of power system flexibility resources', *Electric Power Systems Research*, vol. 194. p. 107022, 2021. doi: https://doi.org/10.1016/j.epsr.2021.107022.

[4] E. F. Bødal *et al.*, 'Demand flexibility modelling for long term optimal distribution grid planning', *IET Generation, Transmission & Distribution*, vol. 16, no. 24, pp. 5002–5014, 2022, doi: 10.1049/gtd2.12651.

Changelog:

- *2025-09-08*: Initial version.