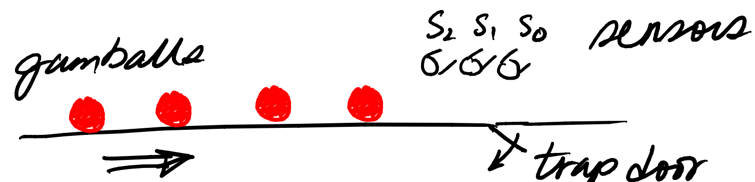
Design example:



$S_2 = 1$ iff a gumball is too large
$S_1 = 1$ " small
$S_0 = 1$ " light

Design a logic function $t=1$ when a gumball is both (too small & too light) or (too large)

$\Rightarrow$ intuitive : $t = S_1 S_0 + S_2$

$\Rightarrow$ derivation : by truth table, Boolean algebra

Terminology

<u>product term</u> : any <u>AND</u> term : xy, $\bar{x}_1 x_2 x_3$

<u>sum-of-product form</u> : any sum of product terms : $a\bar{b} + ab$

<u>Canonical SOP form</u> : a unique form of expression of a function where each product term is a <u>minterm</u>.

<u>minterm</u> : a product term that includes all of the inputs to a function

i.e. a 3-input function

row	x	y	z	minterm	f	maxterm
0	0	0	0	$\bar{x}\bar{y}\bar{z}$	1	$x+y+z$
1	0	0	1	$\bar{x}\bar{y}z$	1	$x+y+\bar{z}$
2	0	1	0	$\bar{x}y\bar{z}$	1	$x+\bar{y}+z$
3	0	1	1	$\bar{x}yz$	1	$x+\bar{y}+\bar{z}$
4	1	0	0	$x\bar{y}\bar{z}$	0	$\bar{x}+y+z$ $\Leftarrow$
5	1	0	1	$x\bar{y}z$	0	$\bar{x}+y+\bar{z}$ $\Leftarrow$
6	1	1	0	$xy\bar{z}$	1	$\bar{x}+\bar{y}+z$
7	1	1	1	xyz	1	$\bar{x}+\bar{y}+\bar{z}$

$$f(x,y,z) = \sum m(0,1,2,3,6,7)$$

<u>Canonical SOP</u> $\rightarrow$ $= \underbrace{\bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z}_{} + \underbrace{\bar{x}y\bar{z} + \bar{x}yz}_{} + \underbrace{xy\bar{z} + xyz}_{}$

simplified $\rightarrow$ $= \bar{x}\bar{y} + \bar{x}y + xy$

$= \bar{x} + y$

Product-of-sum form: any product of sum terms: $(x+\bar{y})(x+y)$

Canonical POS: when each sum term is a **maxterm**

Maxterm: a sum term that includes all of the inputs of a function

back to the previous example, the function can also be fully described by covering all rows in the truth table where $f=0$

$$\bar{f}(x,y,z) = \sum m(4,5) = x\bar{y}\bar{z} + x\bar{y}z$$

$$f(x,y,z) = \bar{\bar{f}} = \overline{x\bar{y}\bar{z} + x\bar{y}z} = (\overline{x\bar{y}\bar{z}})(\overline{x\bar{y}z})$$ by Demorgan's theorem

$$f(x,y,z) = \underbrace{(\bar{x}+y+z)}_{\text{maxterms}}\underbrace{(\bar{x}+y+\bar{z})}_{\text{maxterms}}$$ ⇐ Canonical POS form

let's use Boolean algebra to simplify this POS form

$$f(x,y,z) = \bar{x} + y \quad \text{(combining rule)}$$

Walk away points:

to describe a function in SOP form → sum up (+) all minterms in a truth table where the function is equal to "1"

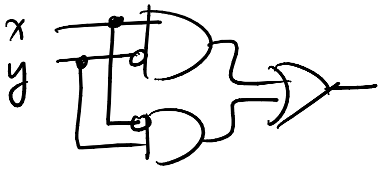
to describe the same function in POS form → AND (•) all maxterms in a truth table where the function is equal to "0"

SOP form $\xleftrightarrow{\text{Boolean algebra}}$ POS form

Intro to Verilog

Verilog : a hardware descriptive language

recall



$z = x\bar{y} + \bar{x}y$ (SOP form)

keyword for declaration

```
module zlogic (input x, y, output z);
  assign z = (x & !y) + (!x & y)
end module
```

Verilog keyword