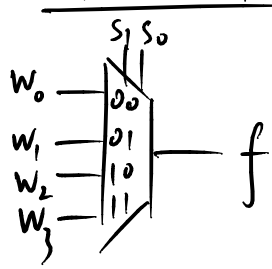
4.1 Shannon's Expansion Theorem

Implementing logic function using mux.

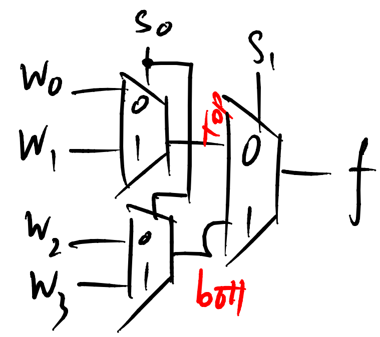
s_1	s_0	f
0	0	w_0
0	1	w_1
1	0	w_2
1	1	w_3

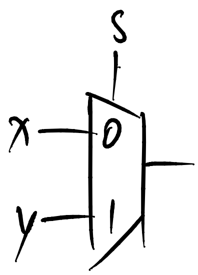
top (rows $s_1 s_0 = 00, 01$), bott (rows $s_1 s_0 = 10, 11$)

implementing using 4-to-1 mux



using 2-to-1 mux





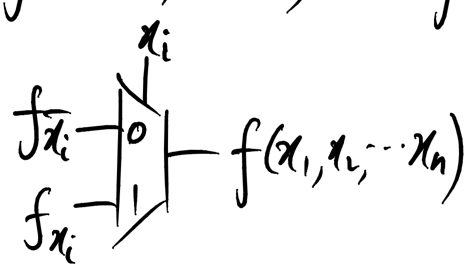
$$f = \bar{s}x + sy$$

given a logic expression $f(x_1, x_2, \cdots x_n) = \bar{x}_i (f_{\bar{x}_i}) + x_i (f_{x_i})$

$f_{\bar{x}_i}$: 0-cofactor of x_i

f_{x_i}: 1-cofactor of x_i

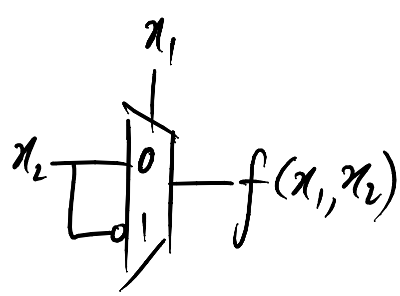
$$f(x_1, x_2, \cdots x_n) = \bar{x}_i f(x_1, x_2, \cdots \underset{\substack{\uparrow \\ x_i}}{0}, \cdots x_n) + x_i f(x_1, x_2, \cdots \underset{\substack{\uparrow \\ x_i}}{1}, \cdots x_n)$$



eg.

x_1	x_2	f
0	0	0
0	1	1
1	0	1
1	1	0

$$f(x_1, x_2) = x_1 \oplus x_2$$
$$= \bar{x}_1 \underset{f_{\bar{x}_1}}{\underset{\uparrow}{x_2}} + x_1 \underset{f_{x_1}}{\underset{\uparrow}{\bar{x}_2}}$$

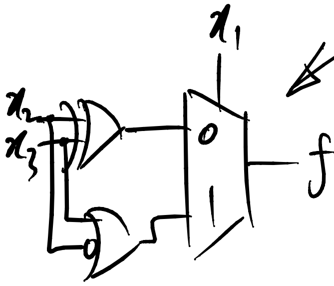


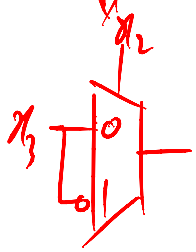
e.g. $f(x_1,x_2,x_3) = x_1\bar{x}_2 + x_1x_2x_3 + \bar{x}_1x_2\bar{x}_3 + \bar{x}_2x_3$

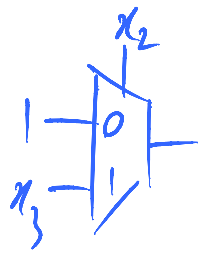
$$= \bar{x}_1 f_{\bar{x}_1} + x_1 f_{x_1}$$

$$= \bar{x}_1(x_2\bar{x}_3+\bar{x}_2x_3) + x_1(\underbrace{\bar{x}_2+x_2x_3+\bar{x}_2x_3}_{\text{simplify}})$$

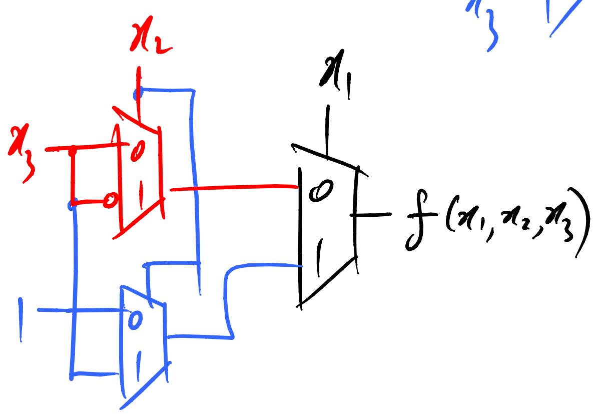
$$\bar{x}_2+x_3 = \bar{x}_2(1+x_3)+x_2(x_3)$$
$$= \bar{x}_2(1)+x_2(x_3)$$

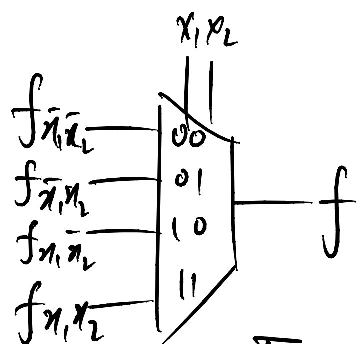






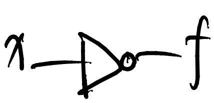
Combine all three circuits together $\Longrightarrow$

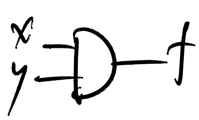


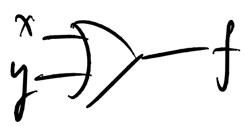


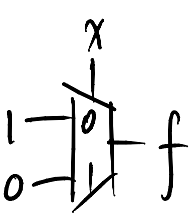
if 4-to-1 mux is available,

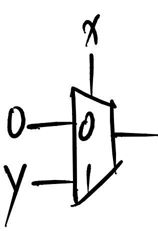
$$f(x_1,x_2,\cdots x_n) = \bar{x}_1\bar{x}_2 f_{\bar{x}_1\bar{x}_2} + \bar{x}_1x_2 f_{\bar{x}_1x_2} + x_1\bar{x}_2 f_{x_1\bar{x}_2} + x_1x_2 f_{x_1x_2}$$



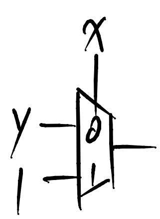








$= \bar{x}\cdot 0 + xy$
$= xy$



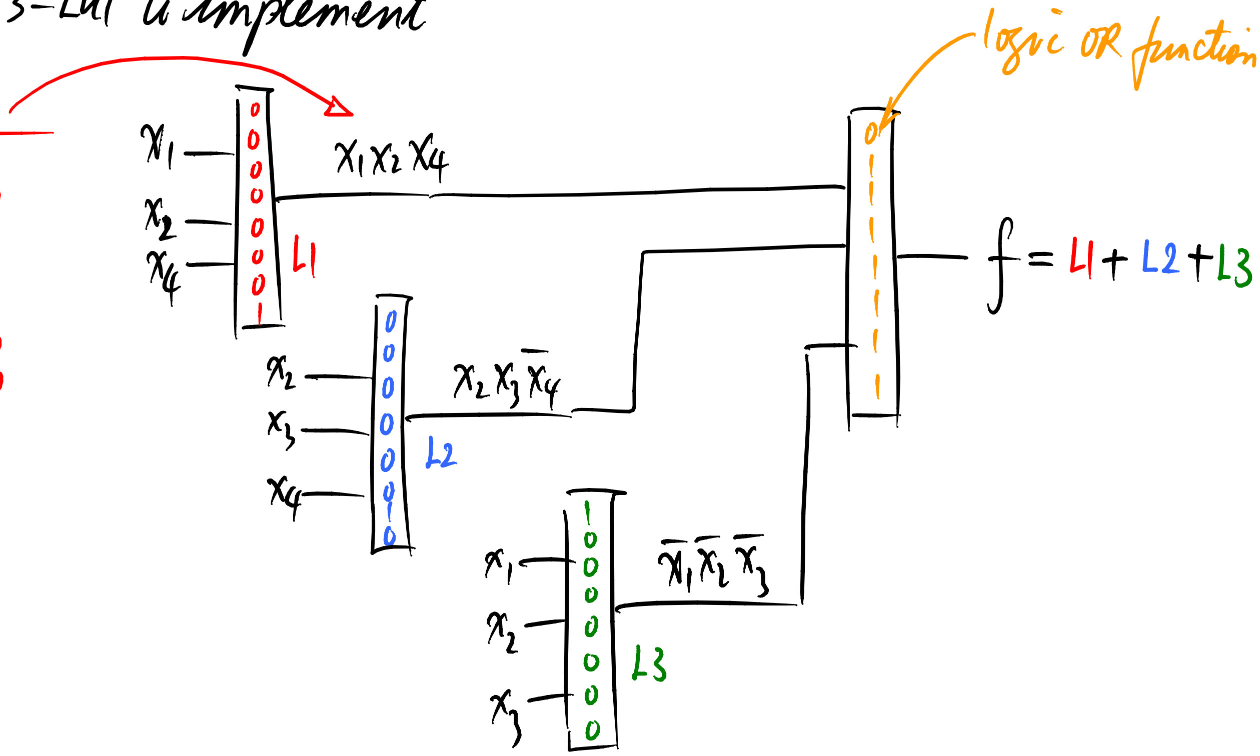
$= \bar{x}y + x = x+y$

Implementing logic function using look-up tables (LUTs)

example: $f = x_1x_2x_4 + x_2x_3\bar{x}_4 + \bar{x}_1\bar{x}_2\bar{x}_3$

Use 3-LUT to implement

x_1	x_2	x_4	
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1



Can you use only 3 – 3LUTs to implement this function?

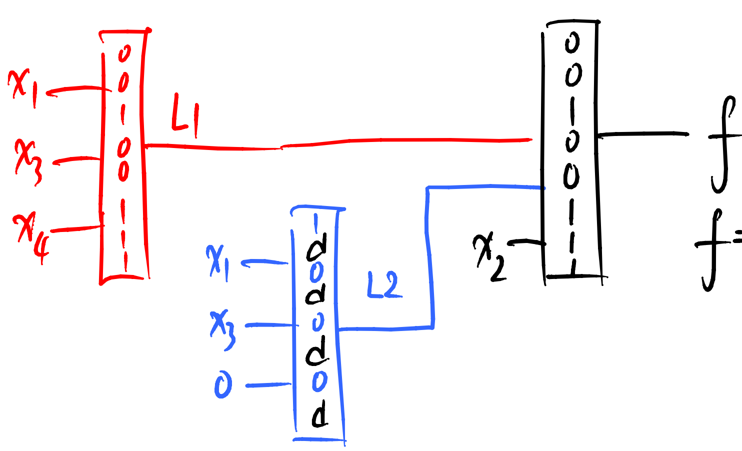
$f = x_1x_2x_4 + x_2x_3\bar{x}_4 + \bar{x}_1\bar{x}_2\bar{x}_3$ (think about factoring)

$= x_2(\underbrace{x_1x_4 + x_3\bar{x}_4}_{L1}) + \bar{x}_2(\underbrace{\bar{x}_1\bar{x}_3}_{L2})$

x_1	x_3	x_4	L1
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$x_3\bar{x}_4$

x_1x_4



x_1	x_3	0	L2
0	0	0	1
0	0	1	d
0	1	0	0
0	1	1	d
1	0	0	0
1	0	1	d
1	1	0	0
1	1	1	d

$f = L1\,x_2 + L2\,\bar{x}_2$

L1	L2	x_2	f

use only 2-LUTs to implement the same function.

$$f=\left\{\left[\left(\left(x_1 x_2\right)_{L1} x_4\right)_{L2}+\left(\left(x_2 x_3\right)_{L3} \overline{x_4}\right)_{L4}\right]_{L7}+\left(\left(\overline{x_1}\,\overline{x_2}\right)_{L5} \overline{x_3}\right)_{L6}\right\}_{L8}$$