ECE241 Digital Systems

Chip used FPGA - field programmable gated array

Binary Numbers

Base 10 (decimal) = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Base 2 (binary) = 0, 1

Base 16 (hexadecimal) = 0, 1, 2, ··· 9, A, B, C, D, E, F

example: $(17)_{10}$ seventeen

$(10001)_2$

$(11)_{16}$

Notation: $(115)_{10} = 1\times10^2 + 1\times10^1 + 5\times10^0$

$(10001)_2 = 1\times2^4 + 0\times2^3 + 0\times2^2 + 0\times2^1 + 1\times2^0$

Conversion:

binary to decimal $(1011)_2 = 1\times2^3 + 1\times2^1 + 1\times2^0 = 8+2+1 = (11)_{10}$

decimal to binary

$A = b_{n-1}\times2^{n-1} + b_{n-2}\times2^{n-2} \cdots + b_1\times2^1 + \boxed{b_0\times2^0} \div 2$

$= b_{n-1}\times2^{n-2} + b_{n-2}\times2^{n-3} \cdots + \boxed{b_1\times2^0}$ Remainder $\div 2$

Remainder

example = $(9)_{10} \div 2$

4	R=1	÷2
2	R=0	÷2
1	R=0	÷2
0	R=1	

$(1001)_2$

$(35)_{10} = 32+2+1 = 2^5+2^1+2^0 = (100011)_2$

$(100)_{10} = 64+32+4 = 2^6+2^5+2^2 = (1100100)_2$

each base 2 digit is called a bit

eight bits are called a byte

four bits are called a nibble

Hexadecimal: each 4 bits → hexadecimal digit

$(1234)_{16} = 0001\ 0010\ 0011\ 0100$

$(A387)_{16} = 1010\ 0011\ 1000\ 0111$

Adding two bits

b_0	0	0	1	1
$+\ b_1$	+0	+1	+0	+1
$S_1 S_0$	00	01	01	10

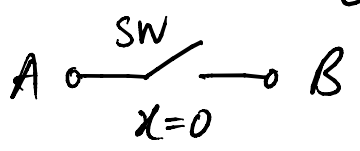
Truth Table

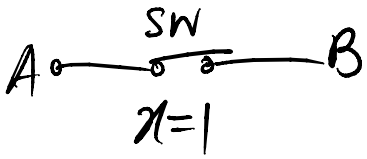
b_1 b_0	S_1 S_0
0 0	0 0
0 1	0 1
1 0	0 1
1 1	1 0

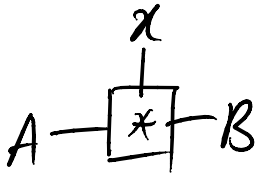
S_0 will be 1 iff $b_1b_0 = 01$ **or** $b_1b_0 = 10$

S_1 will be 1 iff $b_1b_0 = 11$ (b_1 is 1 **and** b_0 is 1)

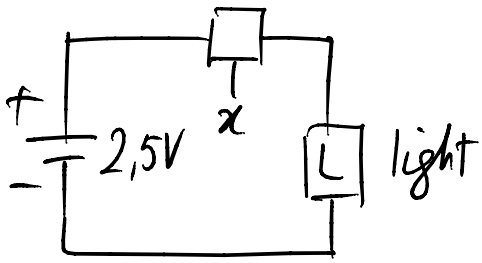
Introduce to logic circuit







* This SW connects A to B if $x=1$, otherwise it does not.

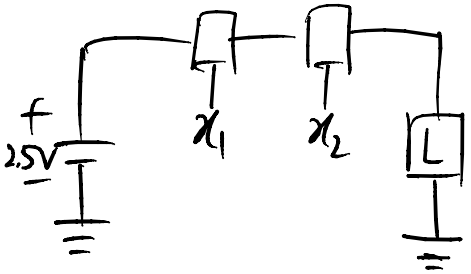


if $x=0$, then $L=0$ which means light's off.

if $x=1$, then $L=1$ which means light's on

$L(x) = x$ or just $L = x$ Logic expression

L is a logic function



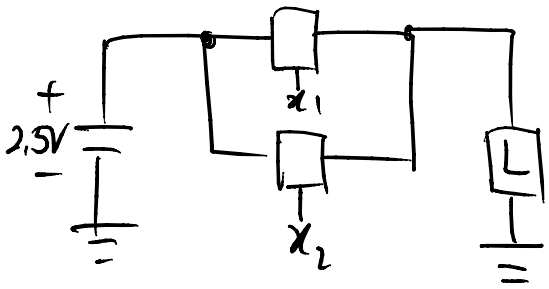
light is on iff x_1 **and** x_2 are closed(on) (1)

$L(x_1, x_2) = x_1 \cdot x_2 \quad (x_1 x_2)$

$= x_1 \,\&\, x_2$

$= x_1$ AND x_2

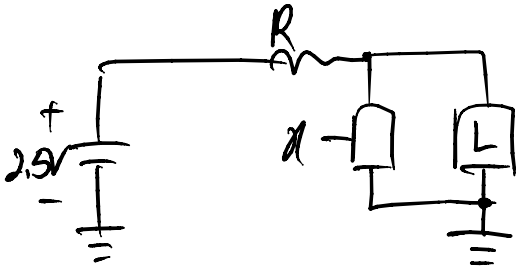
AND means switches in series



light is on iff either x_1 or x_2 is on

$$L(x_1, x_2) = x_1 + x_2$$
$$= x_1 \mid x_2$$
$$= x_1 \text{ OR } x_2$$

OR means switches in parallel



light is on when $x = 0$

$$L = \bar{x}$$
$$= !\, x$$
$$= \text{NOT } x$$
$$= \sim x$$

This system based on AND, OR, NOT with variables that can be 0 or 1 is called Boolean Logic !