Design example: A factory making ball-bearings (BB). The three sensors

S_0 = gives 0 if a BB is too small

S_1 = " too rough

S_2 = " too light

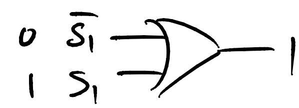
— design reject function (R) gives 1 iff at least two sensors give 0.

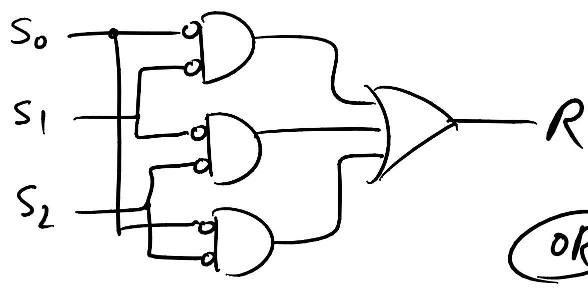
S_0	S_1	S_2	R
0	0	0	1 ←
0	0	1	1 ←
0	1	0	1 ←
0	1	1	0
1	0	0	1 ←
1	0	1	0
1	1	0	0
1	1	1	0

$$R = \bar{S}_0\bar{S}_1\bar{S}_2 + \bar{S}_0\bar{S}_1 S_2 + \bar{S}_0 S_1 \bar{S}_2 + S_0\bar{S}_1\bar{S}_2$$

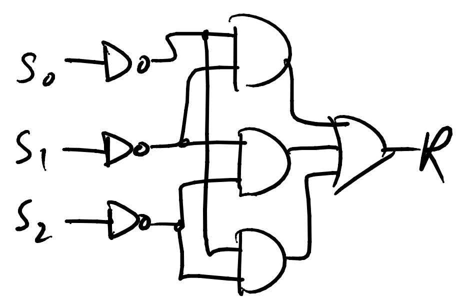
$$= \bar{S}_0\bar{S}_1(\underbrace{\bar{S}_2 + S_2}_{1}) + \bar{S}_1\bar{S}_2(\underbrace{\bar{S}_0 + S_0}_{1}) + \bar{S}_0\bar{S}_2(\underbrace{\bar{S}_1 + S_1}_{1})$$

$$= \bar{S}_0\bar{S}_1 + \bar{S}_1\bar{S}_2 + \bar{S}_0\bar{S}_2$$

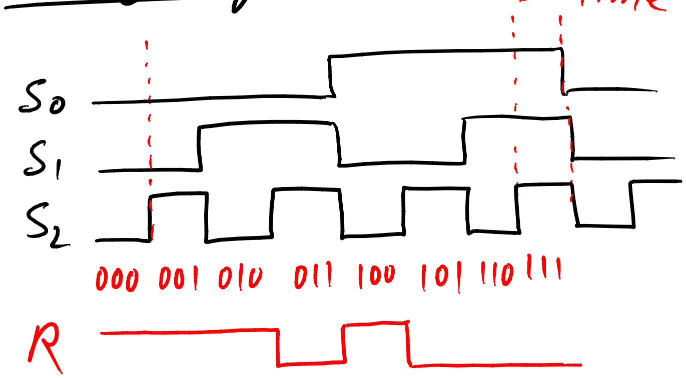




OR



Timing Diagram



Boolean Algebra

Axiom

1. $0 \cdot 0 = 0$ $\qquad 1 + 1 = 1$
2. $1 \cdot 1 = 1$ $\qquad 0 + 0 = 0$
3. $0 \cdot 1 = 1 \cdot 0 = 0$ $\qquad 1 + 0 = 0 + 1 = 1$
4. if $x = 0$, $\bar{x} = 1$ $\qquad$ if $x = 1$, $\bar{x} = 0$

<u>Duality</u>: given a logic expression, if we swap $0 \leftrightarrow 1$ and also $\cdot \leftrightarrow +$, then the expression still holds.

based on these axioms, we can define a set of rules:

5. $x \cdot 0 = 0$ $\quad x+1=1$
6. $x \cdot 1 = x$ $\quad x+0=x$
7. $x \cdot x = x$ $\quad x+x=x$
8. $x \cdot \bar{x} = 0$ $\quad x+\bar{x}=1$
9. $\bar{\bar{x}} = x$

→ some convenient identities or theorems

10. $x \cdot y = y \cdot x$ $\quad x+y=y+x$ (commutative)
11. $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ $\quad x+(y+z)=(x+y)+z$ (associative)
12. $x \cdot (y+z) = x \cdot y + x \cdot z$ $\quad x+(y \cdot z) = (x+y)(x+z)$ (distributive)

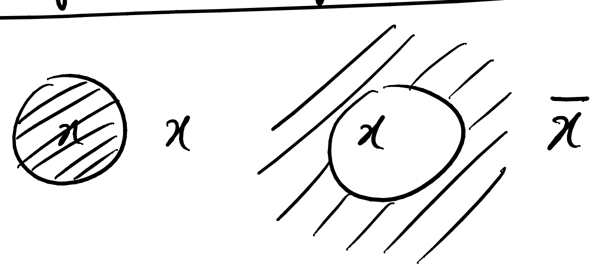
Proof $x+(y \cdot z) = (x+y)(x+z)$ by perfect induction

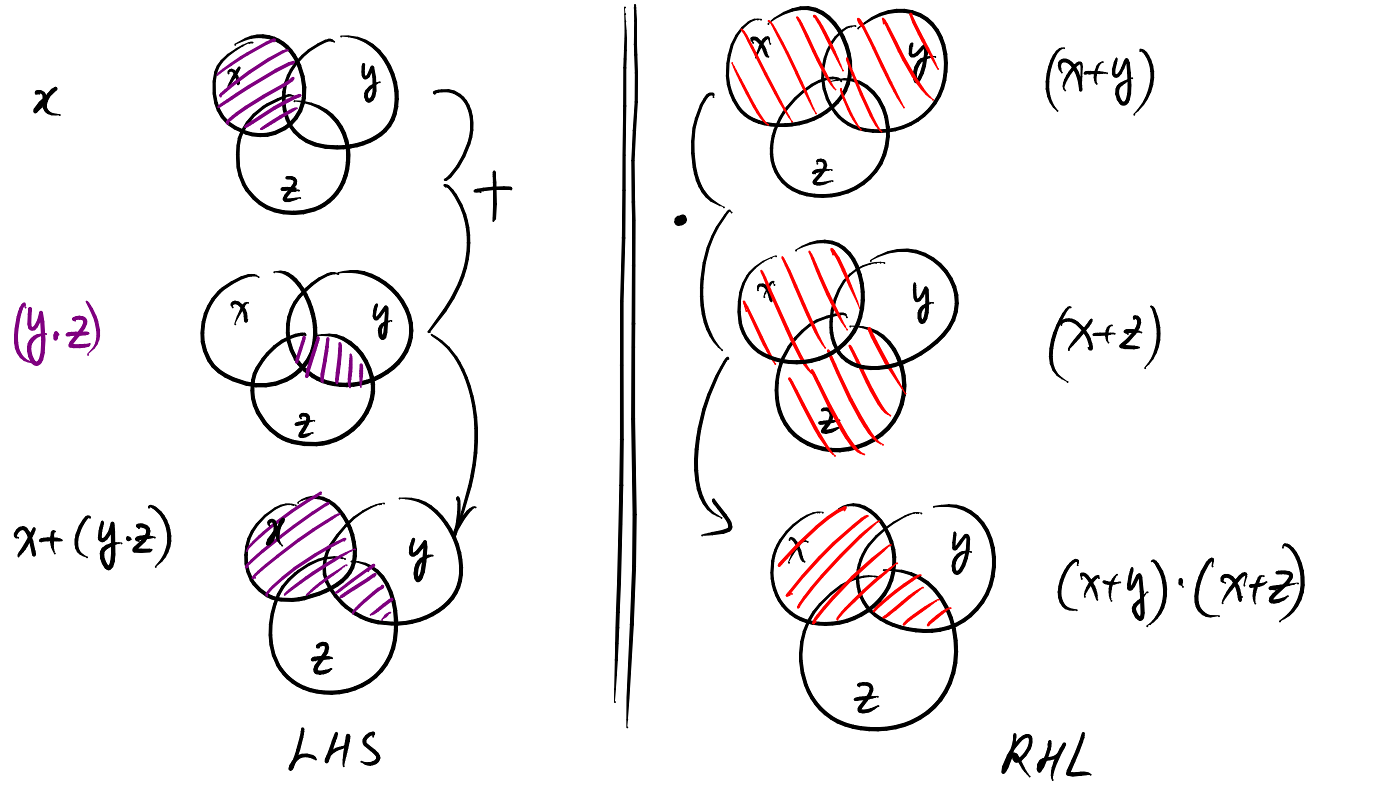
x y z	$(y \cdot z)$	LHS	$(x+y)$	$(x+z)$	RHS
0 0 0	0	0	0	0	0
0 0 1	0	0	0	1	0
0 1 0	0	0	1	0	0
0 1 1	1	1	1	1	1
1 0 0	0	1	1	1	1
1 0 1	0	1	1	1	1
1 1 0	0	1	1	1	1
1 1 1	1	1	1	1	1

<u>LHS = RHS</u>

<u>Proof by Venn Diagram</u>

→ each variable is represented as a circle





13. $x + xy = x$ $\quad x\cdot(x+y) = x$ (covering)

$LHS = x\cdot 1 + xy$

$= x(\underbrace{1+y}_{1})$ (distributive)

$= x$

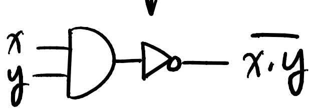
14. $xy + x\bar{y} = x$ $\quad (x+y)\cdot(x+\bar{y}) = x$ (combining)

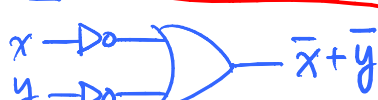
$LHS = x(y+\bar{y})$

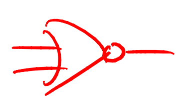
$= x\cdot 1$

$= x$

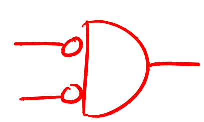
15. $\overline{x\cdot y} = \bar{x} + \bar{y}$ $\quad \overline{x+y} = \bar{x}\cdot\bar{y}$ (Demorgan's theorem)

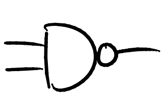




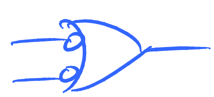


NOR





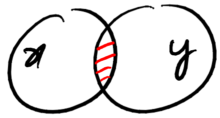
equivalent



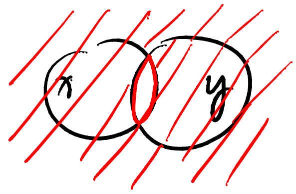
NAND

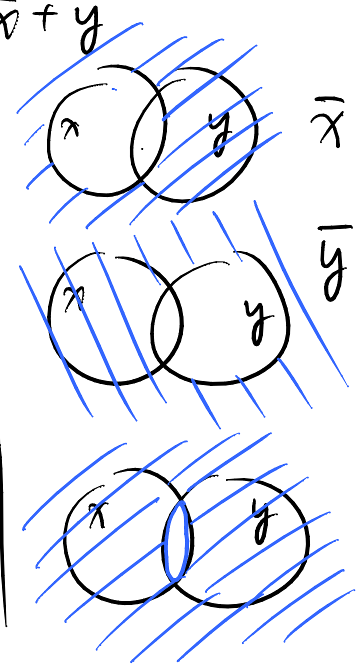
Venn Diagram Proof $\quad \overline{x\cdot y} = \bar{x} + \bar{y}$

LHS. $\quad x\cdot y$



$\overline{x\cdot y}$





$\bar{x}$

$\bar{y}$

$\bar{x}+\bar{y}$

RHS

16. $x+\bar{x}y = x+y \qquad x\cdot(\bar{x}+y) = x\cdot y \quad$ (absorption)

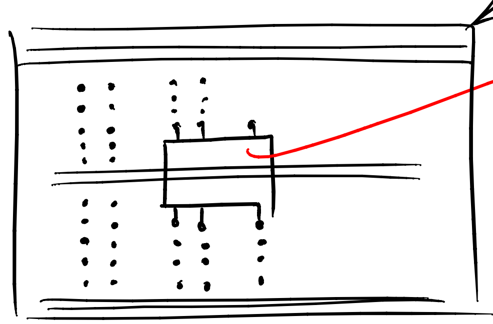
Proof:

$LHS = (\overset{1}{\cancel{x+\bar{x}}})(x+y)$ (distributive)

$= x+y$

$= RHS$

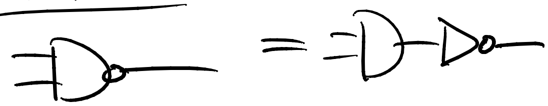
Lab #1



protoboard

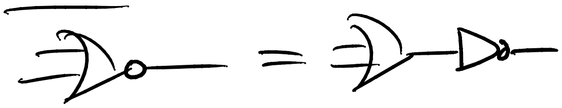
7400 Series of logic gate chips.

NAND



x_1	x_2	NAND
0	0	1
0	1	1
1	0	1
1	1	0

NOR



x_1	x_2	NOR
0	0	1
0	1	0
1	0	0
1	1	0