2.11 Karnaugh map (K-map)

→ minimizing logic expressions

e.g.

x_1	x_2	f
0	0	1
0	1	1
1	0	0
1	1	1

$x_2 \backslash x_1$	0	1
0	1	0
1	1	1

$$f = \bar{x}_1 + x_2$$

$x_2 \backslash x_1$	0	1
0	m_0	m_2
1	m_1	m_3

$\bar{x}_1$ $\bar{x}_2$

$m_0 = \bar{x}_1 \bar{x}_2$

$m_1 = \bar{x}_1 x_2$

$m_2 = x_1 \bar{x}_2$

$m_3 = x_1 x_2$

x_2 x_1

3-variable K-map (x_1, x_2, x_3)

$x_3 \backslash x_1 x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

x_3 x_1 $\bar{x}_2$

x_1	x_2	x_3	f
0	0	0	:
0	0	1	:
0	1	0	:

$x_3 \backslash x_1 x_2$	00	01	11	10
0	1 m_0	0 m_2	0 m_6	0 m_4
1	1 m_1	1 m_3	1 m_7	1 m_5

Implicant (I): any product term for which the function is equal to 1
i.e. $(m_7 \& m_5)$ $x_1 x_3$

Literal: a variable or it's complement i.e. $x_1, \bar{x}_1$

Prime Implicant (PI): an implicant for which it's not possible to remove any literal and still have a valid implicant
(draw the BIGGEST circling for each $f=1$ entry in K-map)
i.e. x_3 $(m_1, m_3, m_7 \& m_5)$

Cover: any set of implicants that cover all the 1's of a function
i.e.

$$x_1\bar{x}_3 + x_1\bar{x}_2 + \bar{x}_2 x_3$$

or $x_1 x_2 \bar{x}_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 \bar{x}_2 x_3 + \bar{x}_1 \bar{x}_2 x_3$

$x_3 \backslash x_1 x_2$	00	01	11	10
0			1	1
1	1			1

Essential PI: a PI that *must* be included in any cover of the function. ie. $x_1\bar{x}_3$, $\bar{x}_2x_3$

minimal cover: all essential PI's + other PI's necessary to form a cover

$$f = x_1\bar{x}_3 + \bar{x}_2x_3$$

4-variable K-map

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

→ a minterm eg. $m_8 = x_1\bar{x}_2\bar{x}_3\bar{x}_4$, $m_9 = x_1\bar{x}_2\bar{x}_3x_4$

→ Two adjacent minterms combined

eg. m_8 & m_9 → $x_1\bar{x}_2\bar{x}_3$ (3 literals)

→ four adjacent minterms combined

eg. m_{12}, m_{13}, m_8 & m_9 → $x_1\bar{x}_3$ (2 literals)

→ eight adjacent minterms combined

eg. $m_0, m_4, m_{12}, m_8, m_1, m_5, m_{13}$ & m_9 → $\bar{x}_3$ (1 literal)

example: $f(x_1,x_2,x_3,x_4) = \sum m(2,4,5,8,10,11,12,13,15)$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00		1	1	1
01		1	1	
11			1	1
10	1			1

P.I. $x_2\bar{x}_3$, $x_1\bar{x}_3\bar{x}_4$, $x_1\bar{x}_2\bar{x}_4$, $x_1x_2x_4$, $x_1x_3x_4$, $\bar{x}_2x_3\bar{x}_4$, $x_1\bar{x}_2x_3$

Essential P.I. $x_2\bar{x}_3 + \bar{x}_2x_3\bar{x}_4$

minimal cover $f = x_2\bar{x}_3 + \bar{x}_2x_3\bar{x}_4 + x_1x_3x_4 + x_1\bar{x}_2\bar{x}_4$

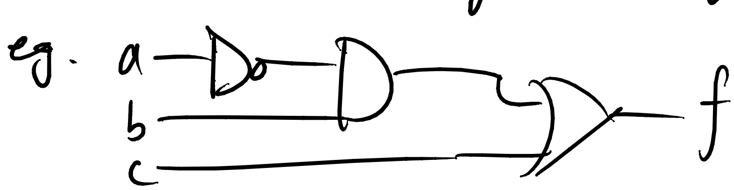
(cost *)

or $f = x_2\bar{x}_3 + \bar{x}_2x_3\bar{x}_4 + x_1x_3x_4 + x_1\bar{x}_3\bar{x}_4$

SOP form

* **cost**: a measure by total # of gates + total # of inputs to gates

eg.



cost = 3 gates + 5 inputs
= 8

if not counting ▷○, then cost = 6

e.g.

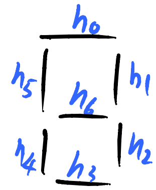
0	1	0	0
0	1	1	1
1	1	1	0
0	0	1	0

In case like this one, we don't always need the largest grouping of 1's (1 1 / 1 1) because all (blue groupings) would make a cover!

<u>Don't care minterms</u> sometimes we know that certain combinations of inputs to a function either can't occur or we don't care about them

example: decimal display (0–9) on a 7-segment display unit

Display	x_1	x_2	x_3	x_4	h_0	h_1	h_2	h_3	h_4	h_5	h_6
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1						
3	0	0	1	1	1						
4	0	1	0	0	0						
5	0	1	0	1	1						
6	0	1	1	0	1						
7	0	1	1	1	1						
8	1	0	0	0	1						
9	1	0	0	1	1						
	1	0	1	0	d	d	d	d	d	d	d
	1	0	1	1	d	d	d	d	d	d	d
	1	1	0	0	d	d	d	d	d	d	d
	1	1	0	1	d	d	d	d	d	d	d
	1	1	1	0	d	d	d	d	d	d	d
	1	1	1	1	d	d	d	d	d	d	d



K-map for h_0

x_3x_4 \ x_1x_2	00	01	11	10
00	1		d	1
01		1	d	1
11	1	1	d	d
10	1	1	d	d

x_2x_4

$\bar{x}_2\bar{x}_4$ x_3 x_1

$h_0 = x_1 + x_3 + \bar{x}_2\bar{x}_4 + x_2x_4$

simpliest SOP form

5-variable K-map (x_1, x_2, x_3, x_4, x_5)

x_3x_4 \ x_1x_2	00	01	11	10
00	0	8	24	16
01	2	10	26	18
11	6	14	30	22
10	4	12	28	20

$x_1\bar{x}_2\bar{x}_3\bar{x}_4\bar{x}_5$

combined

$\bar{x}_1x_2x_4$

x_3x_4 \ x_1x_2	00	01	11	10
00	1	9	25	17
01	3	11	27	19
11	7	15	31	23
10	5	13	29	21

$x_1\bar{x}_2\bar{x}_3\bar{x}_4x_5$

can be combined into $x_1\bar{x}_2\bar{x}_3\bar{x}_4$

$x_5 = 0$ — all even # minterms

$x_5 = 1$ — all odd # minterms

example of using K-map

Q. Is $f = xz + y\bar{z} + xy$ a minimum-cost SOP form?

A.

$z \backslash xy$	00	01	11	10
0		1	1	
1			1	1

xy

$y\bar{z}$ $\quad$ xz

$$f = y\bar{z} + xz$$

Note: this is a 3-variable function, hence each product term with 2 literals maps into two "1" entries in the K-map.

boolean

$$\begin{aligned} f &= xz + y\bar{z} + xy \\ &= xz + y\bar{z} + xy(z+\bar{z}) \\ &= \underline{xz} + \underline{y\bar{z}} + \underline{xyz} + \underline{xy\bar{z}} \\ &= xz + y\bar{z} \end{aligned}$$

example of describing f in POS form

$$f(x_1, x_2, x_3, x_4) = \sum m(2,3,4,5,6,8,9) + \sum d(10,11,12,13,14,15)$$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	1	d	1
01	0	1	d	1
11	1	0	d	d
10	1	1	d	d

x_1 $\quad$ $x_2\bar{x}_3$ $\quad$ $x_3\bar{x}_4$ $\quad$ $\bar{x}_2x_3$

SOP form first $\quad f = x_1 + x_2\bar{x}_3 + x_3\bar{x}_4 + \bar{x}_2x_3$

ignore-Do

cost = 3 AND + 1 OR + 6 inputs + 4 inputs = 14

POS form $\quad$ looking at "0" and use "d"

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	1	d	1
01	0	1	d	1
11	1	0	d	d
10	1	1	d	d

$(\bar{x}_2 + \bar{x}_3 + \bar{x}_4)$

$(x_1 + x_2 + x_3)$

$$f = (\bar{x}_2 + \bar{x}_3 + \bar{x}_4)(x_1 + x_2 + x_3)$$

cost = 2 OR + 1 AND

6 inputs + 2 inputs

= 11