example: simplify the logic expression using Boolean algebra

$f = z\bar{x}\bar{y} + \bar{x}y + \bar{z}xy$

$= \bar{x}(z\bar{y} + y) + \bar{z}xy$ (12a) distributive

$= \bar{x}(z + y) + \bar{z}xy$ (13a) absorption

$= \bar{x}z + \bar{x}y + \bar{z}xy$

$= \bar{x}z + (\bar{x} + \bar{z}x)y$ (12a) distributive

$= \bar{x}z + (\bar{x} + \bar{z})y$ (13a) absorption

$= \bar{x}z + \bar{x}y + y\bar{z}$

$f = y\bar{z} + (\bar{x} + \bar{z})(x + y + \bar{z})$

$= y\bar{z} + \bar{z} + (\bar{x})(x + y)$ (12b) distributive

$= \bar{z} + \bar{x}y$ (12a) distributive

(13a absorption)

$f = x\bar{z} + xy + \bar{x}\bar{y} + \bar{y}z$

$= x\bar{y}\bar{z} + xy\bar{z} + \cancel{xyz} + xyz + \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \cancel{x\bar{y}z} + x\bar{y}z$

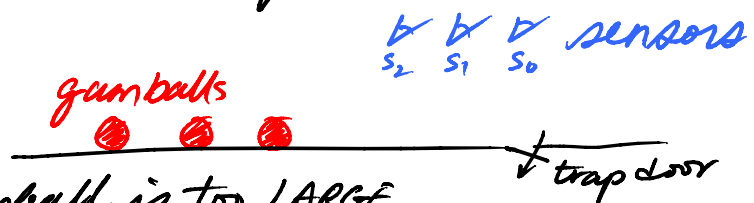
$= \bar{y}\bar{z} + xy + \bar{y}z$ (14a)

$= \bar{y} + xy$ (14a)

$= \bar{y} + x$ (16a)

2-6 Logic representations (SOP/POS)

Design example:



$S_2 = 1$ if a gumball is too LARGE
$S_1 = 1$ " SMALL
$S_0 = 1$ " LIGHT

design a logic function ($t=1$ ie opens the trap door) when a gumball is both too small and too light or too large

$\Rightarrow$ intuitive $t = S_0 S_1 + S_2$

Terminology

product term = any AND term ie. xy, $x_1 \bar{x}_2 x_3$

sum-of-product (SOP) = any sum of product terms i.e. $\underbrace{\bar{a}\bar{b}}_{\text{prod.}} + \underbrace{\bar{c}\bar{d}}_{\text{prod.}}$ ($\bar{a}\bar{b}$ + $\bar{c}\bar{d}$ = sum)

canonical SOP = a unique form of SOP where each product term is a minterm. ie. 3-var. $\bar{a}bc + a\bar{b}\bar{c}$ $\quad ab + cde$

minterm = a product term that includes all of the inputs to a function

$\Rightarrow$ 3-variable truth table

row	x	y	z	minterm	f	
0	0	0	0	$\bar{x}\bar{y}\bar{z}$	1	←
1	0	0	1	$\bar{x}\bar{y}z$	1	←
2	0	1	0	$\bar{x}y\bar{z}$	1	←
3	0	1	1	$\bar{x}yz$	1	←
4	1	0	0	$x\bar{y}\bar{z}$	0	
5	1	0	1	$x\bar{y}z$	0	
6	1	1	0	$xy\bar{z}$	1	←
7	1	1	1	xyz	1	←

$= \sum m(0,1,2,3,6,7)$

$f = \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz + xy\bar{z} + xyz$

Simplify the expression

$f = \bar{x}\bar{y} + \bar{x}y + xy$

$= \bar{x} + y$

Product of sum = any product of sum terms i.e. $(x+\bar{y})(y+z)$
(POS)

canonical POS = when each sum term is a maxterm

maxterm = a sum term that includes all of the inputs of a function
i.e. 3-var. $(x+\bar{y}+z)$

(back to the previous example)

row	x y z	minterm	f	maxterm
0	0 0 0	$\bar{x}\bar{y}\bar{z}$	1	$x+y+z$
1	0 0 1	$\bar{x}\bar{y}z$	1	$x+y+\bar{z}$
2	0 1 0	$\bar{x}y\bar{z}$	1	$x+\bar{y}+z$
3	0 1 1	$\bar{x}yz$	1	$x+\bar{y}+\bar{z}$
4	1 0 0	$x\bar{y}\bar{z}$	0	$\bar{x}+y+z$ ⟸
5	1 0 1	$x\bar{y}z$	0	$\bar{x}+y+\bar{z}$ ⟸
6	1 1 0	$xy\bar{z}$	1	$\bar{x}+\bar{y}+z$
7	1 1 1	xyz	1	$\bar{x}+\bar{y}+\bar{z}$

$\bar{f}(x,y,z) = \sum m(4,5)$

$= x\bar{y}\bar{z} + x\bar{y}z$

$f(x,y,z) = \overline{\bar{f}} = \overline{x\bar{y}\bar{z} + x\bar{y}z}$

$= \overline{x\bar{y}\bar{z}} \cdot \overline{x\bar{y}z}$ (Demorgan's)

$= (\bar{x}+y+z)(\bar{x}+y+\bar{z})$

maxterms

Walk away points

to describe a function in SOP form → sum (+) up all minterms in rows where $f=1$

to describe a function in POS form → AND (·) all maxterms in rows where $f=0$

SOP form ⟷ (Boolean Algebra) POS form