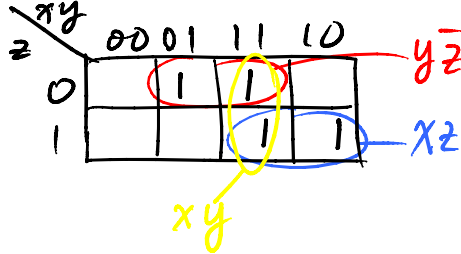


## example of using K-map

Q. Is this a minimum-cost SOP expression?

$$f = xz + y\bar{z} + xy$$

(Note: each of the prod. term represents 2 minterms (i.e. two "1" entries)



minimum-cost SOP:  $f = y\bar{z} + xz$

(Consensus rule)

use Boolean algebra.  $f = xz + y\bar{z} + xy$

$$= xz + y\bar{z} + xy(z + \bar{z})$$

$$= xz + y\bar{z} + xyz + xy\bar{z}$$

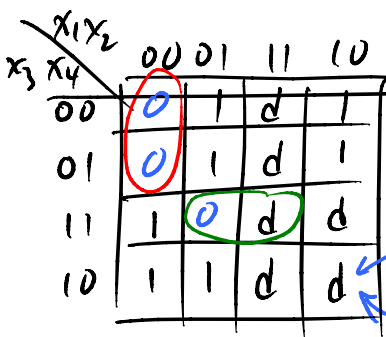
$$\begin{matrix} \xrightarrow{\text{red}} xz(1+y) & \xrightarrow{\text{blue}} (1+x)y\bar{z} \end{matrix}$$

absorption rule.

$$= xz + y\bar{z}$$

## K-map POS

$$f = \sum m(2, 3, 4, 5, 6, 8, 9) + \sum d(10-15)$$



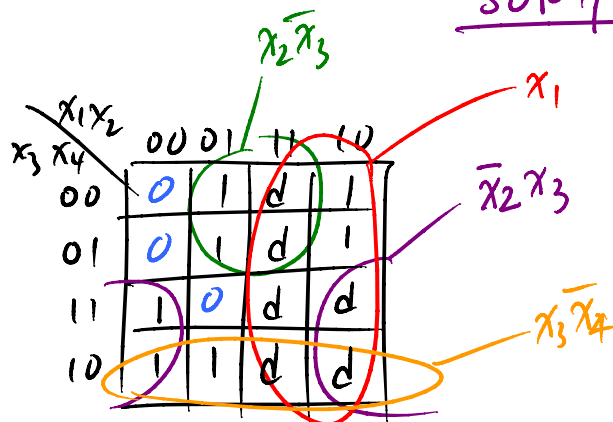
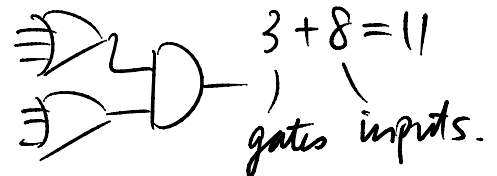
$$f = (x_1 + x_2 + x_3)(\bar{x}_2 + \bar{x}_3 + \bar{x}_4)$$

minterm  $x_1\bar{x}_2x_3\bar{x}_4$

Maxterm  $(\bar{x}_1 + x_2 + \bar{x}_3 + x_4)$

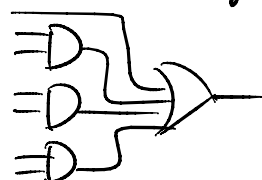
SOP form:

Cost: ignore inverter.



$$f = x_1 + x_2\bar{x}_3 + \bar{x}_2x_3 + x_3\bar{x}_4$$

$$\text{Cost} = 4 \text{ gates} + 10 = 14$$

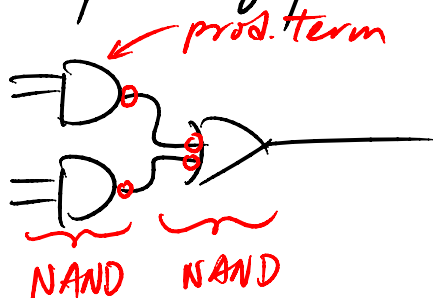


## NAND and NOR (cheaper)

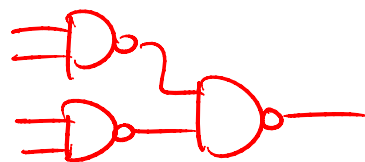
Recall DeMorgan's  $\overline{xy} = \overline{x} + \overline{y}$

$$\overline{xy} = \overline{xy} = \overline{x} + \overline{y}$$

eg. SOP form of function

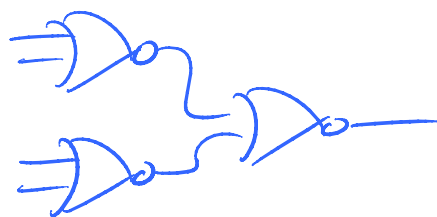
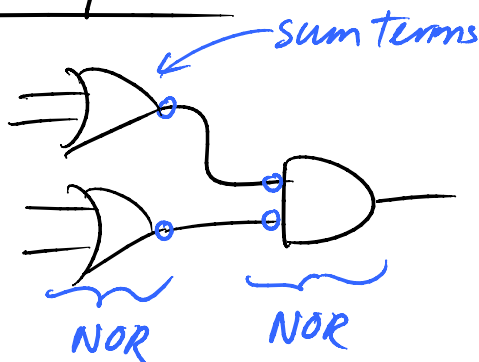


all NAND gate



NAND gates for SOP forms!

POS form



NOR gates for POS forms!

example draw cct. diagram using only 2-input NOR gates.  
for  $f = \overline{x_1}x_2 + x_1x_3 + x_1\overline{x_3}$  (as few gates as possible)  
assume only non-comp. inputs are available.

$$\overline{xy} = \overline{xy} = \overline{x} + \overline{y}$$

Option 1 SOP form

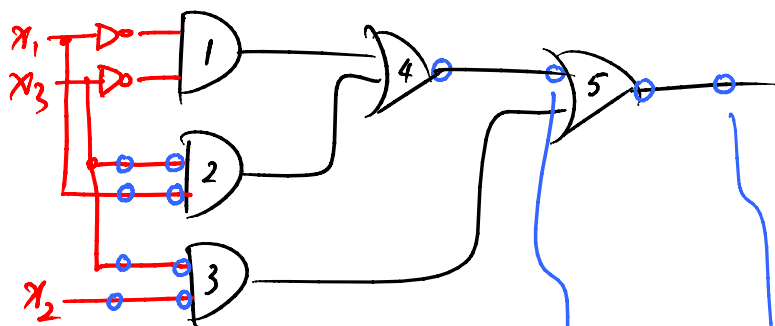
$$f = \overline{x_1}x_2 + x_1x_3 + x_2x_3$$

one of the two possible SOPs.

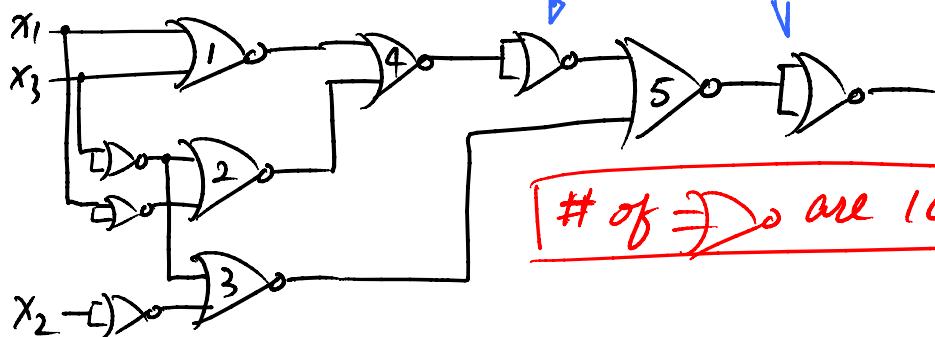
or  $\overline{x_1}x_2$

$x_1 \backslash x_2$	00	01	11	10
0	1	1	0	0
1	0	1	1	1

$\overline{x_1}x_3$   
 $x_2x_3$   
 $x_1x_3$



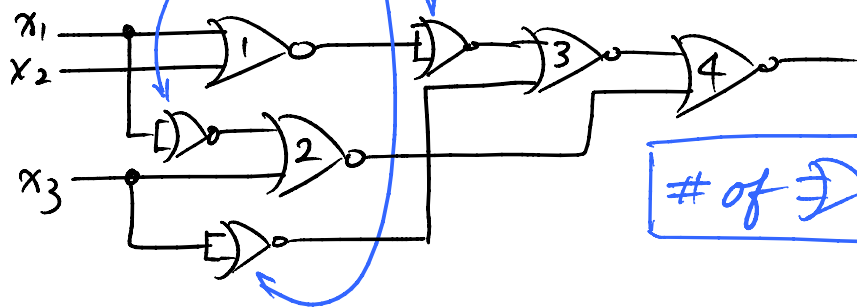
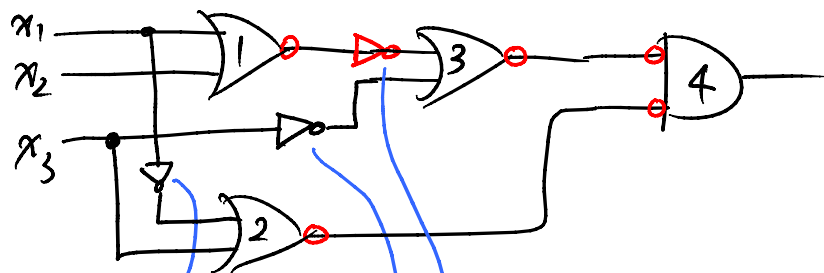
We are trying to create either  $\Rightarrow$  or  $\Leftarrow$



# of  $\Rightarrow$  are 10

Option 2 POS form  $f = (x_1 + x_2 + \bar{x}_3)(\bar{x}_1 + x_3)$

$x_1 x_2$	00	01	11	10
$x_3$				
0	1	1	0	0
1	0	1	1	1



# of  $\Rightarrow$  gates are 7

Option 3 (Clever thinking!)

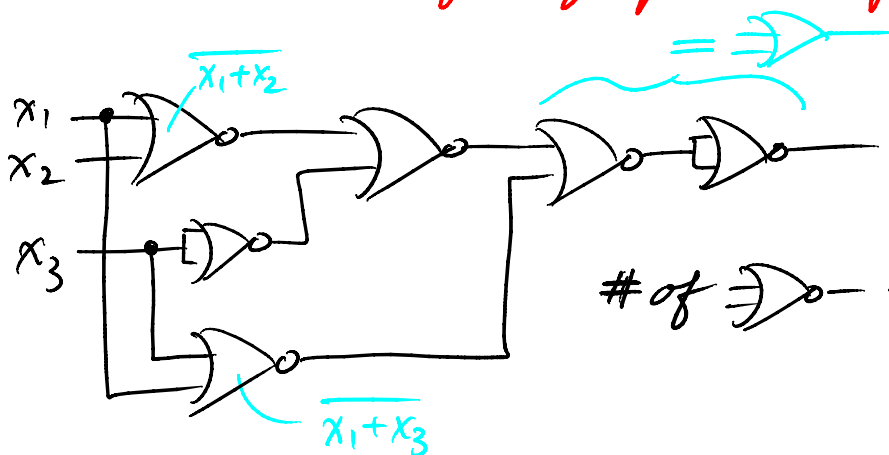
function in SOP form

$$\begin{aligned}
 f &= \underline{x_1 x_3} + \underline{x_2 x_3} + \overline{x_1} \overline{x_3} \\
 &= \underline{(x_1 + x_2) x_3} + \overline{x_1} \overline{x_3} \\
 &= \underline{(x_1 + x_2) + x_3} + \underline{\overline{x_1 + x_3}}
 \end{aligned}$$

→ This form of expression represents a  $\Rightarrow$  —

$x_1 x_2$	00	01	11	10
$x_3$				
0	1	1	0	0
1	0	1	1	1

Annotations:  $\overline{x_1} \overline{x_3}$  (top-left),  $x_2 x_3$  (middle-right),  $x_1 x_3$  (bottom-right),  $\overline{x_1} x_2$  (middle-left)



# of  $\Rightarrow$  used = 6