3.1 Binary number system

Binary Numbers

Base 10 (decimal): 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Base 2 (binary): 0, 1

Base 16 (hexadecimal): 0, 1, ⋯ 9, $\overset{10}{A}$, $\overset{11}{B}$, $\overset{12}{C}$, $\overset{13}{D}$, $\overset{14}{E}$, $\overset{15}{F}$

example: $(17)_{10}$ $(1001)_2$ $(11)_{16}$

notation: $(1\overset{1}{1}\overset{0}{5})_{10} = 1\times10^2 + 1\times10^1 + 5\times10^0$

$$(\overset{4}{1}\overset{3}{0}\overset{2}{0}\overset{1}{0}\overset{0}{1})_2 = 1\times2^4 + 0\times2^3 + 0\times2^2 + 0\times2^1 + 1\times2^0$$

Conversion:

binary to decimal $(\overset{3}{1}\overset{2}{0}\overset{1}{1}\overset{0}{1})_2 = 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0 = (11)_{10}$

decimal to binary: $A \Rightarrow (b_{n-1}\times2^{n-1} + b_{n-2}\times2^{n-2} + \cdots + b_1\times2^1 + b_0\times2^0) \div 2$

$(b_{n-1}\times2^{n-2} + b_{n-2}\times2^{n-3} + \cdots + b_1 2^0$ Remainder$) \div 2$

Remainder

example $(9)_{10} \div 2$

(4)	R=1	÷2
(2)	R=0	÷2
(1)	R=0	÷2
0	R=1	

$(1001)_2$

good to remember

$2^2=4$	$2^7=128$
$2^3=8$	$2^8=256$
$2^4=16$	$2^9=512$
$2^5=32$	$2^{10}=1024$
$2^6=64$	

$(35)_{10} = 32 + 2 + 1 = 2^5 + 2^1 + 2^0 = (\overset{5}{1}\overset{4}{0}\overset{3}{0}\overset{2}{0}\overset{1}{1}\overset{0}{1})_2$

each binary digit is a "bit"

eight bits → "byte"

four bits → "nibble"

Hexadecimal : each 4 bits of a binary into one digit in hex.

$(1234)_{16} = (\underbrace{0001}\ \underbrace{0010}\ \underbrace{0011}\ \underbrace{0100})_2$

$(A387)_{16} = (\underbrace{1010}\ \underbrace{0011}\ \underbrace{1000}\ \underbrace{0111})_2$

Add two bits

$$\begin{array}{r} b_0 \\ +\ b_1 \\ \hline S_1 S_0 \end{array} \qquad \begin{array}{r} 0 \\ +0 \\ \hline 00 \end{array} \qquad \begin{array}{r} 0 \\ +1 \\ \hline 01 \end{array} \qquad \begin{array}{r} 1 \\ +0 \\ \hline 01 \end{array} \qquad \begin{array}{r} 1 \\ +1 \\ \hline 10 \end{array}$$

Truth Table

$b_1 b_0$	$S_1 S_0$
0 0	0 0
0 1	0 1
1 0	0 1
1 1	1 0

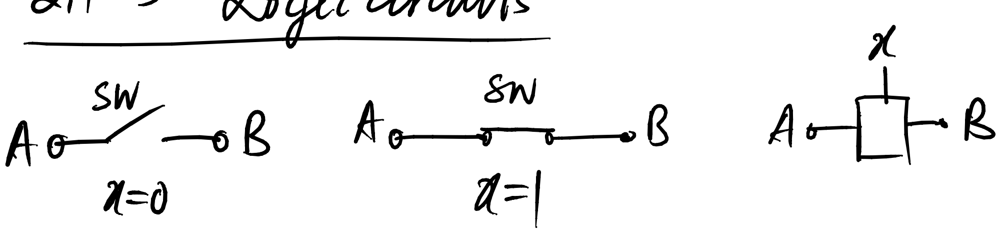
"0" → false "1" → true

S_0 will be "1" if $b_1=0$ and $b_0=1$

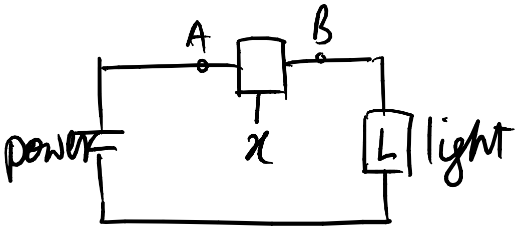
S_0 will be "1" if $b_1=1$ and $b_0=0$

S_0 will be "1" iff $b_1b_0=01$ or $b_1b_0=10$

2.1-3 Logic circuits



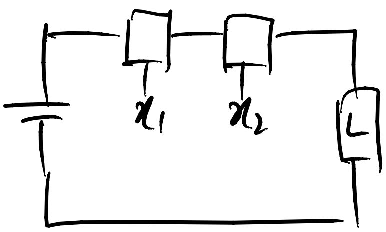
$x=1$, connecting A to B ; $x=0$ no connection between A and B



if $x=1$, then light's on $(L=1)$

if $x=0$, then light's off $(L=0)$

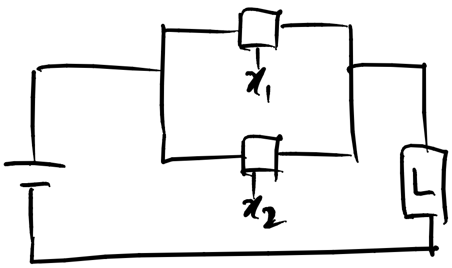
$L(x)=x$ Logic expression or Logic function



"AND"
switches in series

light's on if $x_1=1$ and $x_2=1$
otherwise off.

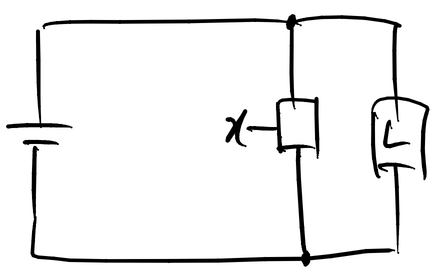
$$L(x_1,x_2)=\underset{x_1=1}{x_1} \text{ and } \underset{x_2=1}{x_2} = x_1 \cdot x_2 \quad (\text{e.g. } x_1x_2)$$
$$= x_1 \,\&\, x_2$$



Light's on if $x_1=1$ or $x_2=1$ or both

$$L(x_1,x_2)=x_1 \text{ OR } x_2 = x_1+x_2$$
$$= x_1 \mid x_2$$

"OR" → switches in parallel



Light's on if $x=0$

$$L(x)=\underset{x=0}{\bar{x}} = !x = \text{NOT } x = \sim x$$

The system based on AND, OR, NOT with variables that can be "0" or "1" is called BOOLEAN LOGIC