2.5 Boolean Algebra

Axiom

1a. $0\cdot 0=0$	1b. $1+1=1$
2a $1\cdot 1=1$	2b $0+0=0$
3a $0\cdot 1=1\cdot 0=0$	3b $1+0=0+1=1$
4a if $x=0$, $\bar{x}=1$	4b if $x=1$, $\bar{x}=0$

Duality: given a logic expression, if we swap $0\leftrightarrow 1$ and $\cdot \leftrightarrow +$ then the resulting expression still holds.

5a $x\cdot 0=0$	5b $x+1=1$
6a $x\cdot 1=x$	6b $x+0=x$
7a $x\cdot x=x$	7b $x+x=x$
8a $x\cdot\bar{x}=0$	8b $x+\bar{x}=1$
9 $\bar{\bar{x}}=x$	

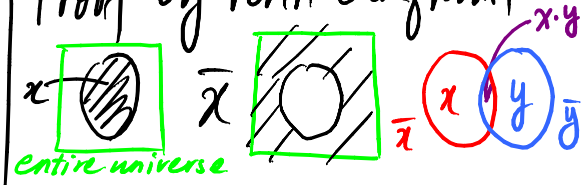
10a $x\cdot y=y\cdot x$ 10b $x+y=y+x$ (commutative)

11a $x(y\cdot z)=(x\cdot y)\cdot z$ 11b $x+(y+z)=(x+y)+z$ (associative)

12a $x(y+z)=(x\cdot y)+(xz)$ 12b $x+(y\cdot z)=(x+y)\cdot(x+z)$ (distributive)

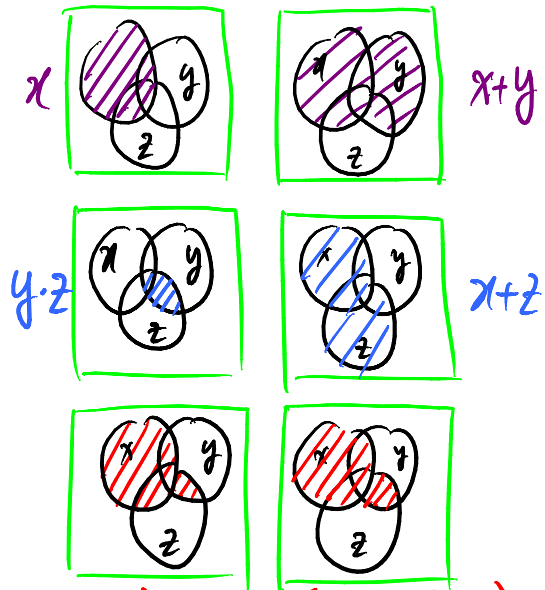
Proof for 12b. by perfect induction

Proof by Venn diagram



x	y	z	$(y\cdot z)$	LHS	$(x+y)$	$(x+z)$	RHS
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

LHS = RHS



$x+(y\cdot z) = (x+y)\cdot(x+z)$

13a. $x + xy = x$

$LHS = x\cdot 1 + x\cdot y$ (6a)

$= x(1+\cancel{y})$ (12a)

$\quad\;\; 1$ (5b)

$= x$

13b. $x\cdot(x+y) = x$ (absorption)

14a $xy + x\bar{y} = x$

$LHS = x(\cancel{y+\bar{y}})$ (12a)

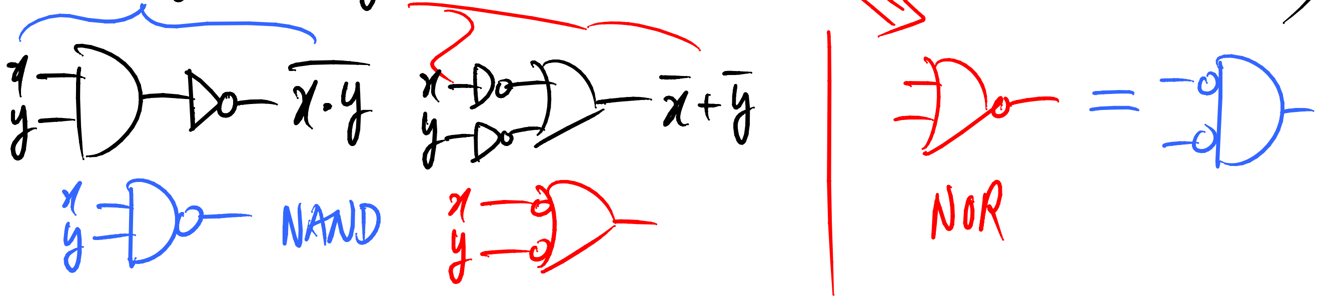
$\quad\;\; 1$ (8b)

$= x$

14b $(x+y)\cdot(x+\bar{y}) = x$ (combining)

15a $\overline{x\cdot y} = \bar{x}+\bar{y}$

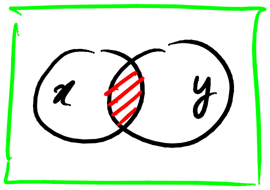
15b $\overline{x+y} = \bar{x}\cdot\bar{y}$ (Demorgan's theorem)



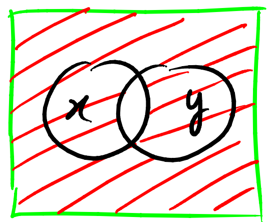
Venn Diagram for 15a

LHS

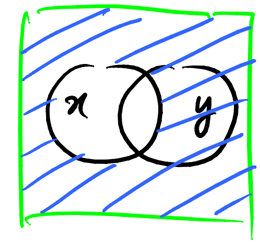
$x \cdot y$



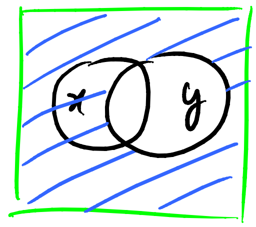
$\overline{x \cdot y}$



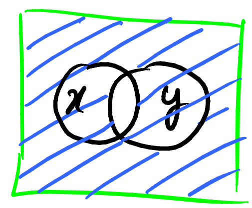
RHS



$\overline{x}$



$\overline{y}$



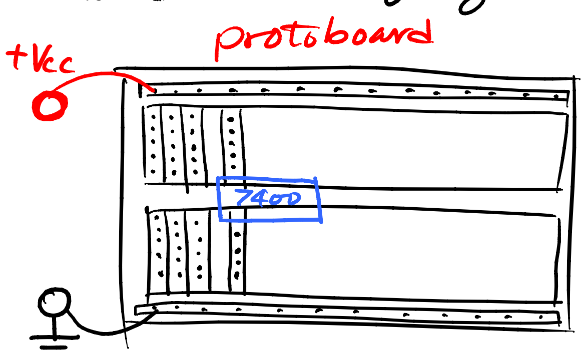
$\overline{x} + \overline{y}$

16a $x + \overline{x}y = x + y$ 16b $x \cdot (\overline{x} + y) = x \cdot y$ (Covering)

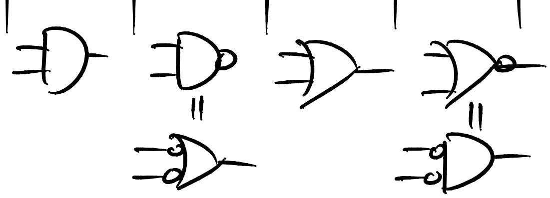
17a $x \cdot y + y \cdot z + \overline{x}z$
$= x \cdot y + \overline{x}z$ included

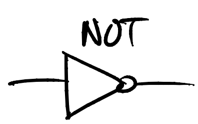
17b $(x+y)(y+z)(\overline{x}+z)$
$= (x+y)(\overline{x}+z)$ (Consensus)

Lab #1 : building logic circuits using 7400 series of logic gates



x	y	AND	NAND	OR	NOR
0	0	0	1	0	1
0	1	0	1	1	0
1	0	0	1	1	0
1	1	1	0	1	0





x	NOT
0	1
1	0