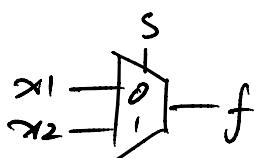


## Verilog: procedural statement "always" Block

Recall 2-to-1 mux



```
module mux(input x1, x2, s, output f);
    assign f = (~s & x1) | (s & x2);
end module
```

→ Alternative code (using if-else):

```
module mux(input x1, x2, s, output f);
    reg f;
    always @(*)
        if (!s)
            f = x1;
        else
            f = x2;
    end module
```

alternative code:

```
f = x1;
if (s == 1)
    f = x2;
```

- ① any signal assigned a value inside a "always" block must be declared as reg
- ② if-else statement must be used inside a "always" block
- ③ sensitivity list = variables that will affect the assigned signal.  
alternatively = always @ (\*)
- ④ if more statements inside the "always" block is needed, use begin  
⋮

end

if-else is considered as one statement

## Case statement (used inside "always" block)

```
case (s)
    1bit → 1'b0: f = x1;
    binary value → 1'b1: f = x2;
    end case
```

✓ cover remaining (unused) cases.

default:

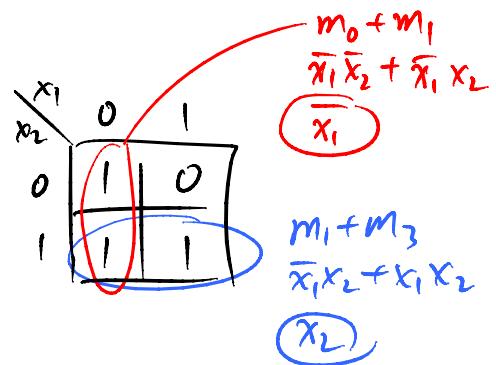
## Karnaugh Map (K-map)

→ minimize logic expressions

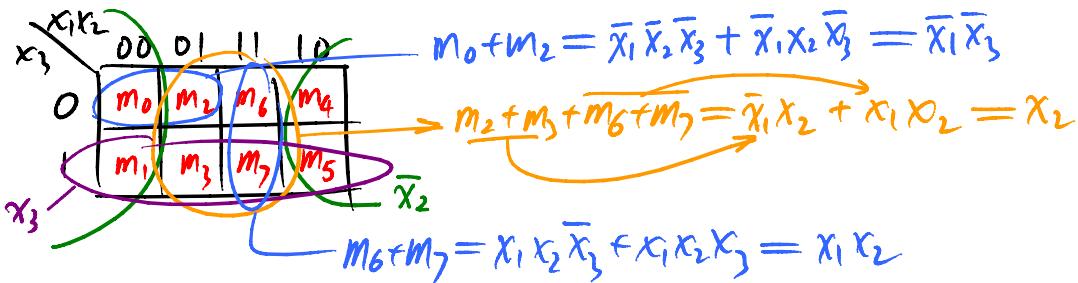
$x_1 \ x_2$	f
0 0	1
0 1	1
1 0	0
1 1	1

$$\begin{aligned} &\leftarrow \bar{x}_1 \bar{x}_2 \ (m_0) \\ &\leftarrow \bar{x}_1 x_2 \ (m_1) \\ &\leftarrow x_1 x_2 \ (m_3) \end{aligned}$$

$x_1$	0	1
0	$m_0$	$m_2$
1	$m_1$	$m_3$



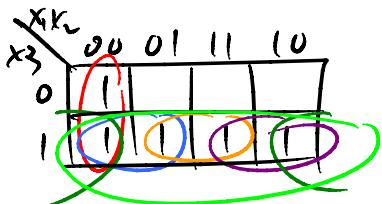
## 3-variable K-map ( $x_1, x_2, x_3$ )



Implicant: any product term for which the function is equal to 1.

$$m_0, m_1, m_3, m_5, m_7$$

$$\bar{x}_1 \bar{x}_2, \bar{x}_1 x_3, x_2 x_3, x_1 x_3, \bar{x}_2 x_3, x_3$$



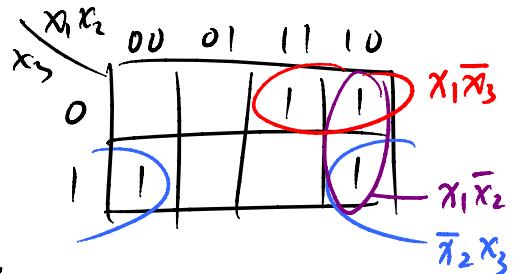
Literal: a variable, or its complement ( $x_1, \bar{x}_1$ )

Prime Implicant (PI): an implicant for which it is not possible to remove any literal & still have a valid implicant  
(draw the biggest circles for each "1" entry)  $\bar{x}_1 \bar{x}_2, x_3$

Cover: any set of implicants that cover all the 1's of a function.

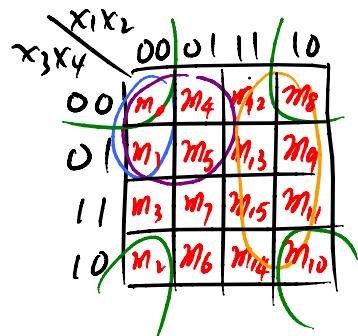
example  $f(x_1, x_2, x_3) = \sum m(1, 4, 5, 6)$

cover:  $x_1 \bar{x}_3 + x_1 \bar{x}_2 + \bar{x}_2 x_3$



Essential PI: a PI that must be included in any cover of the function.  
minimal cover =  $x_1 \bar{x}_3 + \bar{x}_2 x_3$

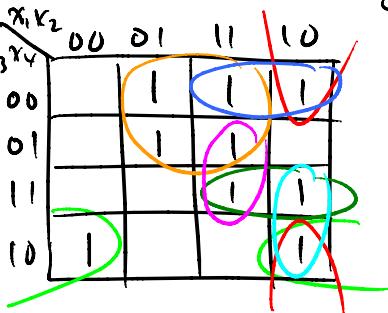
## 4-variable K-map



$$\begin{aligned}
 m_0 + m_1 &= \bar{x}_1 \bar{x}_2 \bar{x}_3 \\
 m_0 + m_1 + m_4 + m_5 &= \bar{x}_1 \bar{x}_3 \\
 m_8 + m_9 + m_{10} + m_{11} + m_{12} + m_{13} + m_{14} + m_{15} &= x_1 \\
 m_0 + m_2 + m_8 + m_{10} &= \bar{x}_2 \bar{x}_4
 \end{aligned}$$

$$f = \sum m(2, 4, 5, 8, 10, 11, 12, 13, 15)$$

example



$$\begin{aligned}
 \text{P.I.s: } & x_2 \bar{x}_3, x_1 x_3 \bar{x}_4, x_1 \bar{x}_2 \bar{x}_4, x_1 \bar{x}_3 \bar{x}_4 \\
 & \bar{x}_2 x_3 \bar{x}_4, x_1 x_2 x_4, x_1 \bar{x}_2 x_3
 \end{aligned}$$

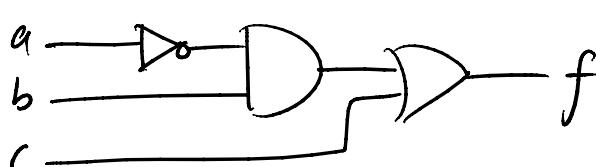
$$\begin{aligned}
 \text{Essential P.I.s: } & x_2 \bar{x}_3 \text{ (due to } m_4, m_5) \\
 & \bar{x}_2 x_3 \bar{x}_4 \text{ (due to } m_2)
 \end{aligned}$$

$$\text{minimal cover } f = x_2 \bar{x}_3 + \bar{x}_2 x_3 \bar{x}_4 + x_1 x_3 \bar{x}_4 + \left\{ \begin{array}{l} x_1 \bar{x}_2 \bar{x}_4 \\ x_1 \bar{x}_3 \bar{x}_4 \end{array} \right\} \text{ or}$$

- Summary:
1. Find all P.I.s (largest groups of 1's)
  2. Identify Essential P.I.s. If essential P.I.s cover all the 1's of the function, then we are done!
  3. Choose non-essential P.I.s to obtain a min-cost ~~cost~~  $\oplus$  cover

~~Cost~~ cost is measured as total # of gates + total # of inputs

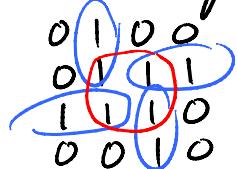
i.e.



$$\begin{array}{rcl}
 \text{Cost} & = & 3 + 5 = 8 \\
 & & (\text{gates}) (\text{inputs})
 \end{array}$$

If  $\oplus$  are not counted, then Cost = 2 + 4 = 6

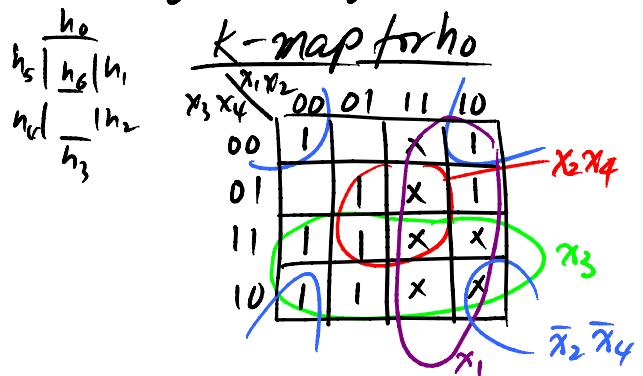
Back to K-map example (4-variable)



Note: in this case, we don't always need the largest groups of 1's. because all  $\oplus$  would make a cover

Don't-care minterms: sometimes we know that certain combinations of inputs to a function either **can't occur** or we **don't care** about them. For example, we wish to display decimal digit (only) 0-9 on a 7-seg. display

$x_1$	$x_2$	$x_3$	$x_4$	$h_0$	$h_1$	$h_2$	$h_3$	$h_4$	$h_5$	$h_6$
0	0	0	0	1	1	1	1	1	1	0
1	0	0	1	0	1	1	0	0	0	0
0	0	1	0	-	-	-	-	-	-	-
0	0	1	1	-	-	-	-	-	-	-
0	1	0	0	0	-	-	-	-	-	-
0	1	0	1	-	-	-	-	-	-	-
0	1	1	0	-	-	-	-	-	-	-
0	1	1	1	-	-	-	-	-	-	-
1	0	0	0	-	-	-	-	-	-	-
1	0	0	1	-	-	-	-	-	-	-
1	0	1	0	x	x	x	x	x	x	x
1	0	1	1	x	-	-	-	-	-	x
1	1	0	0	x	-	-	-	-	x	x
1	1	0	1	x	-	-	-	-	x	x
1	1	1	0	x	-	-	-	-	x	x
1	1	1	1	x	x	x	x	x	x	x



$$h_0 = x_3 + x_1 + x_2x_4 + \bar{x}_2\bar{x}_4$$

Note: in this K-map, we choose all "x" to be "1". On other K-maps, we may pick some "x" to be "0" to help us get minimal-cost cover.

5-variable K-map ( $x_1x_2x_3x_4x_5$ )

$x_1x_2$	00	01	11	10
$x_3x_4$	$m_0$	8	24	16
$x_5=0$	10	26	18	
$x_5=1$	14	30	22	
$x_5=0$	$m_4$	12	28	20

$x_1x_2$	00	01	11	10
$x_3x_4$	$m_1$	9	25	17
$x_5=0$	11	27	19	
$x_5=1$	$m_7$	15	31	23
$x_5=0$	$m_5$	13	29	21

all odd numbered minterms are represented in this map.

$$x_1\bar{x}_2x_3\bar{x}_4x_5$$

all even # minterms