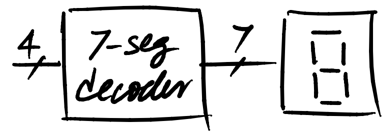
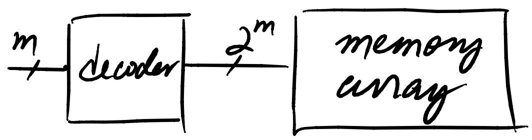
# 4.2.3.5 Arithmetic Circuits

→ Decoder: 7-segment decoder

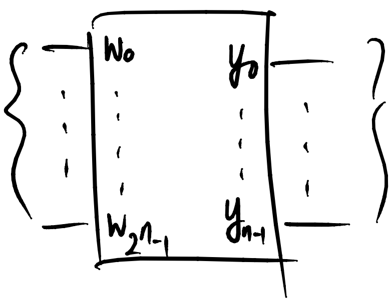



memory address (one-hot) decoder




→ Encoder: performs the opposite function of a decoder
used to reduce the number of bits needed to represent
some given information

$2^n$ inputs




n outputs

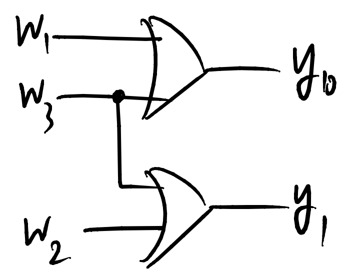
$2^n$-to-$n$ binary encoder

4-to-2 binary encoder

| $W_3$ | $W_2$ | $W_1$ | $W_0$ | $y_1$ | $y_0$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 |

$y_0 = W_1 + W_3$

$y_1 = W_2 + W_3$




→ Priority encoder order $(W_3 > W_2 > W_1 > W_0)$

indicates if any input has been set

| | $W_3$ | $W_2$ | $W_1$ | $W_0$ | $y_1$ | $y_0$ | $Z$ |
|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | d | d | 0 |
| $i_0 \rightarrow$ | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| $i_1 \rightarrow$ | 0 | 0 | 1 | d | 0 | 1 | 1 |
| $i_2 \rightarrow$ | 0 | 1 | d | d | 1 | 0 | 1 |
| $i_3 \rightarrow$ | 1 | d | d | d | 1 | 1 | 1 |

intermediate functions

$i_0 = \overline{W_3}\,\overline{W_2}\,\overline{W_1}\,W_0$

$i_1 = \overline{W_3}\,\overline{W_2}\,W_1$

$i_2 = \overline{W_3}\,W_2$

$i_3 = W_3$

$y_1 = i_2 + i_3$

$y_0 = i_1 + i_3$

$Z = i_0 + i_1 + i_2 + i_3$

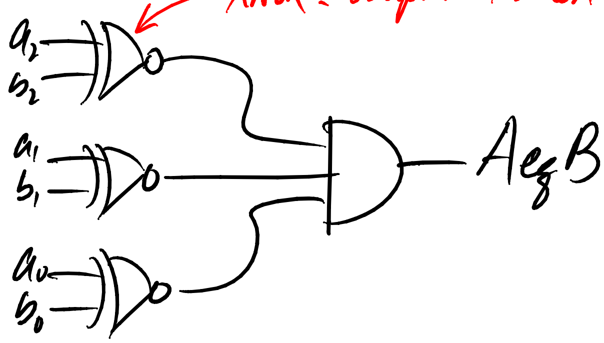
# Comparator Circuit

Two 3-bit numbers $A = a_2 a_1 a_0$ and $B = b_2 b_1 b_0$

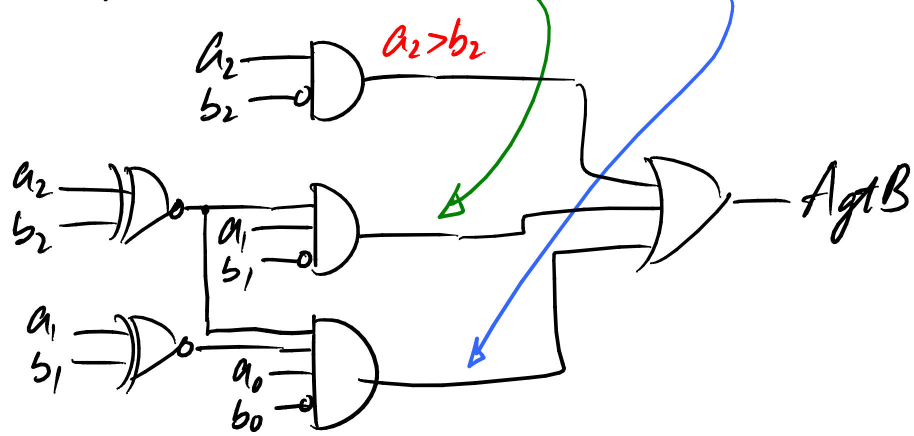
How to compare A and B? $\Rightarrow$ AeqB, AgtB, AltB

AeqB if $a_2 = b_2$ and $a_1 = b_1$ and $a_0 = b_0$

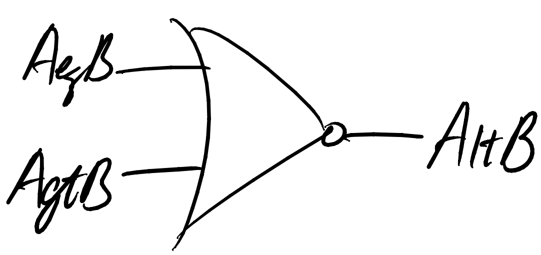
XNOR: output = 1 when the two inputs are the same




AgtB if $a_2 > b_2$ OR $a_2 = b_2$ and $a_1 > b_1$ OR $a_2 = b_2$ and $a_1 = b_1$ and $a_0 > b_0$




AltB $\overline{AeqB + AgtB}$



# 3.6 Multiplication Circuit

(let's use positive numbers)

$$\begin{array}{rrr} & 1 & 2 \\ \times & 1 & 1 \\ \hline & 1 & 2 \\ +1 & 2 & \\ \hline 1 & 3 & 2 \end{array}
\qquad
\begin{array}{rrrrrrrrr} & & & & & 1 & 1 & 0 & 0 \\ & & & & \times & 1 & 0 & 1 & 1 \\ \hline & & & & & 1 & 1 & 0 & 0 \\ & & & & 1 & 1 & 0 & 0 & \\ & & & 0 & 0 & 0 & 0 & & \\ + & & 1 & 1 & 0 & 0 & & & \\ \hline & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}$$

multiply by 1

$$\begin{array}{ccccccccc} & & & & a_3 & a_2 & a_1 & a_0 & \\ & & & \times & b_3 & b_2 & b_1 & b_0 & \\ \hline & & & & b_0a_3 & b_0a_2 & b_0a_1 & b_0a_0 & (\text{row}1) \\ & & & b_1a_3 & b_1a_2 & b_1a_1 & b_1a_0 & & (\text{row}2) \\ & & b_2a_3 & b_2a_2 & b_2a_1 & b_2a_0 & & & \\ +) & b_3a_3 & b_3a_2 & b_3a_1 & b_3a_0 & & & & \\ \hline P_7 & P_6 & P_5 & P_4 & P_3 & P_2 & P_1 & P_0 & \end{array}$$