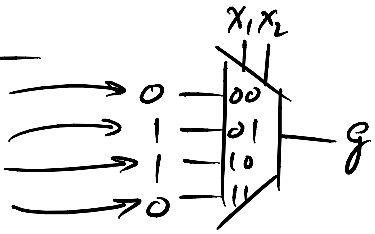
Shannon's Expansion Theorem

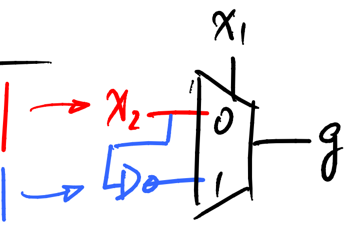
example 1 implement g in 4-to-1 mux:

x_1	x_2	g
0	0	0
0	1	1
1	0	1
1	1	0



Now, use 2-to-1 mux

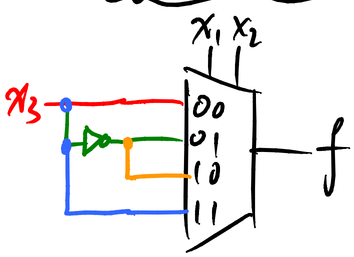
x_1	x_2	g
0	0	0
0	1	1
1	0	1
1	1	0



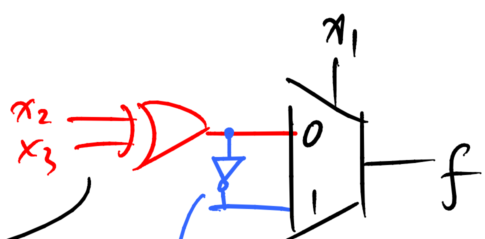
example 2 3-variable logic

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

using 4-to-1 mux

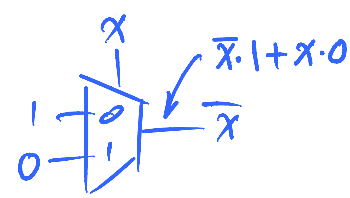


using 2-to-1 mux

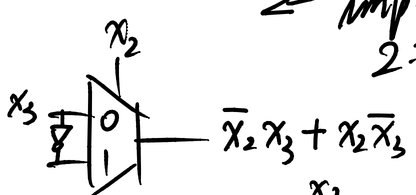


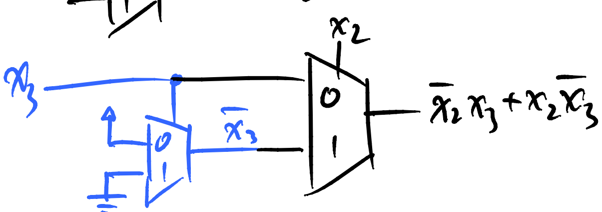
implement using 2-to-1 mux

implement using 2-to-1 mux



$x_2 x_3$	XOR
00	0
01	1
10	1
11	0



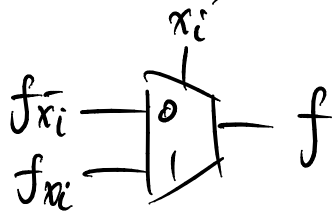


Shannon's Expansion Theorem: Given a function $f(x_1, x_2, \cdots x_n)$ the function can be expressed as $f(x_1, x_2, \cdots x_n) = \bar{x}_i f_{\bar{x}_i} + x_i f_{x_i}$

$f_{\bar{x}_i}$ is the 0-cofactor of f

f_{x_i} is the 1-cofactor of f

$$f(x_1, x_2, \cdots x_n) = \bar{x}_i f(x_1, \cdots \underset{\uparrow x_i}{0}, \cdots x_n) + x_i f(x_1, \cdots \underset{\uparrow x_i}{1} \cdots x_n)$$

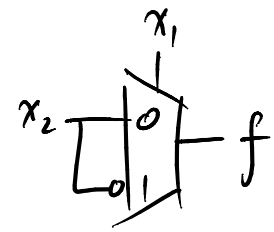


example

x_1	x_2	f
0	0	0
0	1	1
1	0	1
1	1	0

$$f(x_1, x_2) = \bar{x}_1 x_2 + x_1 \bar{x}_2$$
$$= \bar{x}_1 f_{\bar{x}_1} + x_1 f_{x_1}$$

$$f_{\bar{x}_1} = x_2 \ , \quad f_{x_1} = \bar{x}_2$$



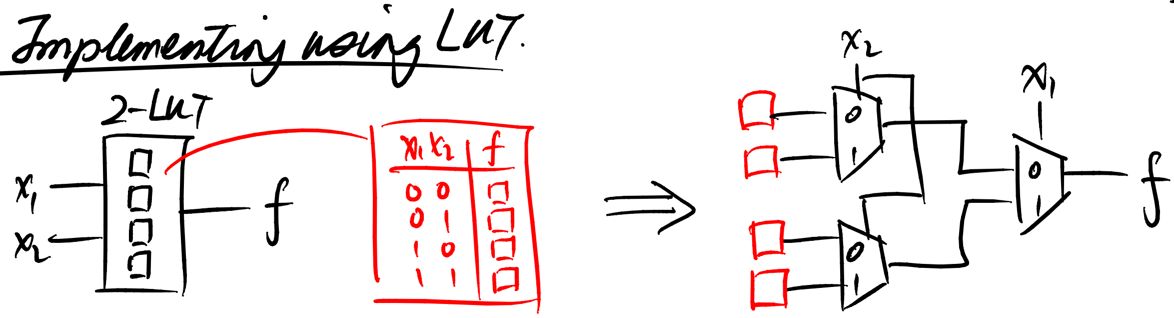
3-variable example: $f(x_1, x_2, x_3) = x_1 \bar{x}_2 + x_1 x_2 x_3 + \bar{x}_1 x_2 \bar{x}_3 + \bar{x}_2 x_3$
$$= \bar{x}_1 f_{\bar{x}_1} + x_1 f_{x_1}$$
$$= \bar{x}_1 (x_2 \bar{x}_3 + \bar{x}_2 x_3) + x_1 (\bar{x}_2 + x_2 x_3 + \bar{x}_2 x_3)$$

Shannon's theorem again

$$f(x_1, \cdots x_n) = \bar{x}_1 \bar{x}_2 f_{\bar{x}_1 \bar{x}_2} + \bar{x}_1 x_2 f_{\bar{x}_1 x_2} + x_1 \bar{x}_2 f_{x_1 \bar{x}_2} + x_1 x_2 f_{x_1 x_2}$$

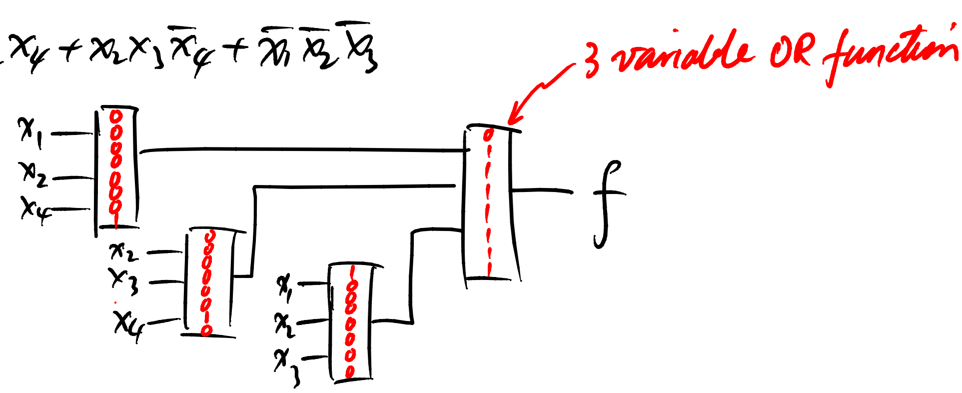
2-to-1 mux $\Leftarrow \bar{x}_2 (x_3) + x_2 (\bar{x}_3)$

Implementing using LUT.



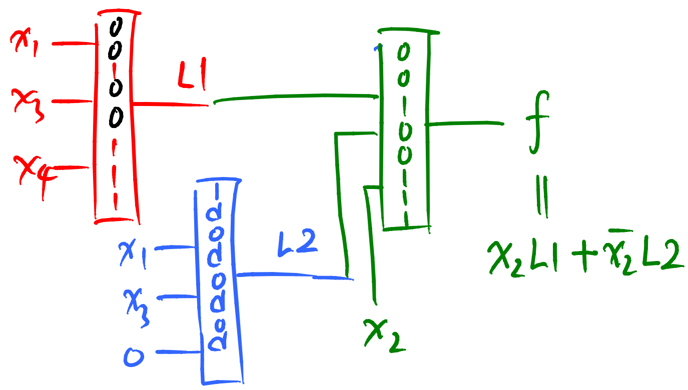
example 1 $f = x_1 x_2 x_4 + x_2 x_3 \bar{x}_4 + \bar{x}_1 \bar{x}_2 \bar{x}_3$

3 LUT available



$$f = x_1x_2x_4 + x_2x_3\bar{x}_4 + \bar{x}_1\bar{x}_2\bar{x}_3$$ this time use factoring

$$= \underbrace{\left(x_2\underbrace{(x_1x_4 + x_3\bar{x}_4)}_{L1} + \bar{x}_2\underbrace{(\bar{x}_1\bar{x}_3)}_{L2}\right)}_{L3}$$



x_1	x_3	0	
0	0	0	1
0	0	1	d
0	1	0	0
0	1	1	d
1	0	0	0
1	0	1	d
1	1	0	0
1	1	1	d

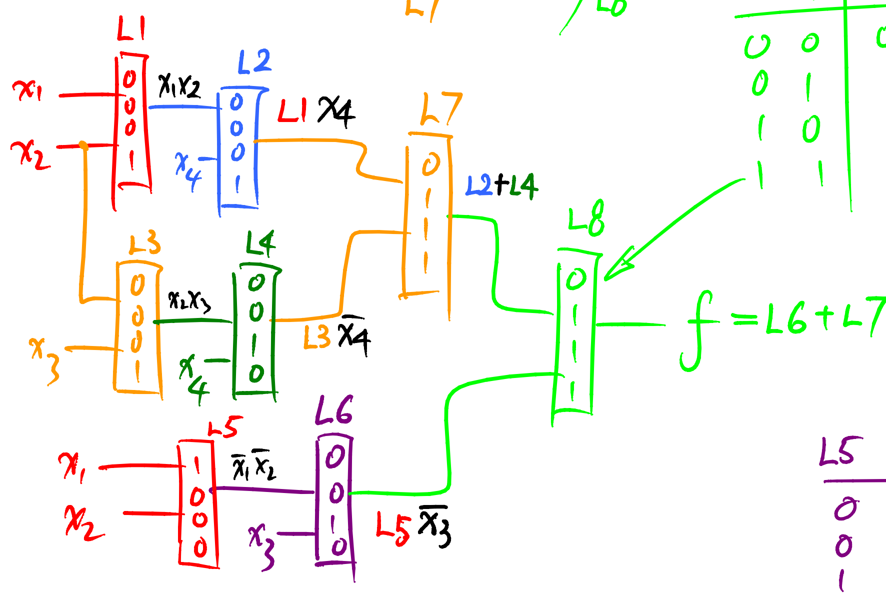
x_1	x_3	x_4	L1
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

now using only 2-LUT to implement

$$f = \underbrace{\left[\underbrace{\left(\underbrace{(x_1x_2)}_{L1}x_4\right)}_{L2} + \underbrace{\left(\underbrace{(x_2x_3)}_{L3}\bar{x}_4\right)}_{L4}\right]}_{L7} + \underbrace{\left(\underbrace{(\bar{x}_1\bar{x}_2)}_{L5}\bar{x}_3\right)}_{L6}$$ L8

OR function

L6	L7	L8
0	0	0
0	1	1
1	0	1
1	1	1



L3	x_4	L4
0	0	0
0	1	0
1	0	1
1	1	0

L5	x_3	L6
0	0	0
0	1	0
1	0	1
1	1	0