# Game Theory & Control
# Assignment 2

S. Bolognani

23 April 2022

---

Please submit your solutions on Moodle.

You can either type them (in LaTeX, for example), or scan your handwritten solutions.

If you are scanning your handwritten solutions, please create a PDF that is readable and small in size. You can use any smartphone app that allows to scan documents to PDF (this feature is present in both the Dropbox app and the Google Drive app, plus many others). Do not send plain photos of the pages.

**The deadline for this assignment is 10 May 2022, at 23:59.**

Please write your solutions in a legible form, include all the steps of your reasoning, indicate what question your are answering, and don't include any solution attempt that you don't want us to grade.

Verify your answers by trying to get to the same results in different ways, checking if they are compatible with the theoretical results that you know and with your intuition. You have plenty of time. Typos and errors by distraction are not acceptable.

This assignment contributes towards 10% of the final score for the course, and must be done independently by each student. No group work is allowed.

**Each "letter" question is worth 1 point, unless stated otherwise.**
**Make sure you answer everything to get the full point for each question. No fraction of points are awarded.**
**Take your time to double-check that you have not missed any part of each question.**

---

**Exercise 1    Demand-Side Management game**

Consider a Demand-Side Management (DSM) game in which $N$ energy consumers (players), labelled by $i \in \mathcal{I} := \{1, 2, \dots, N\}$, aim at minimizing their local energy consumption costs. Each player shall choose its energy consumption profile over the following 24 hours, i.e., $x_i = (x_i^1, \dots, x_i^{24}) \in \mathbb{R}^{24}_{\geq 0}$. The energy consumption at each hour $h$ has an upper limit $\bar{x}_i^h \in \mathbb{R}_{\geq 0}$, imposed, for example, by the network operator and/or by the player's consumption preferences, i.e.,

$$x_i^h \in \left[0,\ \bar{x}_i^h\right], \quad \forall h \in \{1, \dots, 24\}. \tag{1}$$

Moreover, the cumulative consumption over the 24 hours must satisfy the player's (cumulative) nominal energy need:

$$\sum_{h=1}^{24} x_i^h = \sum_{h=1}^{24} e_i^h, \tag{2}$$

where the parameter $e_i^h \in \mathbb{R}_{>0}$ represents the nominal energy consumption at time $h$, namely, the amount of energy player $i$ would normally consume at time $h$ if it was not participating in the DSM game. The goal of each player $i \in \mathcal{I}$ is to choose a feasible energy consumption profile $x_i$ that minimizes the cumulative (over the 24 hours) energy bill, i.e.,

$$J_i(x) = \varphi_i \left( \sum_{j \in \mathcal{I}} x_j \right)^\top x_i, \tag{3}$$

where $x = (x_1, \dots, x_N)$ is the stacked vector of all player's consumption profiles, and $\varphi_i : \mathbb{R}^{24} \to \mathbb{R}^{24}$ is a personalized energy price function defined as

$$\varphi_i(y) = p_{i,1} y + p_{i,2} \mathbf{1}_{24}, \tag{4}$$

with $p_{i,1} \in \mathbb{R}$ pricing the aggregate energy consumption and $p_{i,2} \in \mathbb{R}$ representing a baseline energy price.

a) Find the ranges of the parameters $p_{i,1}$ and $p_{i,2}$ for which the DSM game is a convex game. Is the Nash equilibrium unique for all the price parameters in such ranges? If yes, prove it formally. If not, construct a counter-example.

b) Find some ranges of the parameters $p_{i,1}$ and $p_{i,2}$ for which the DSM game is an exact potential game. Give the expression of the exact potential function.

Now, consider an instance of the DSM game with 100 players whose consumption bounds, nominal consumption, and price parameters are given in the data structure `DSM_game.mat`.

c) Assess the existence and uniqueness of Nash equilibria for this game.

d) Design an iterative algorithm with global convergence guarantee to a Nash equilibrium $x^*$. Provide a self-contained and ready-to-run MATLAB script, i.e., an m file named <`your surname`>`_NE.m`, that

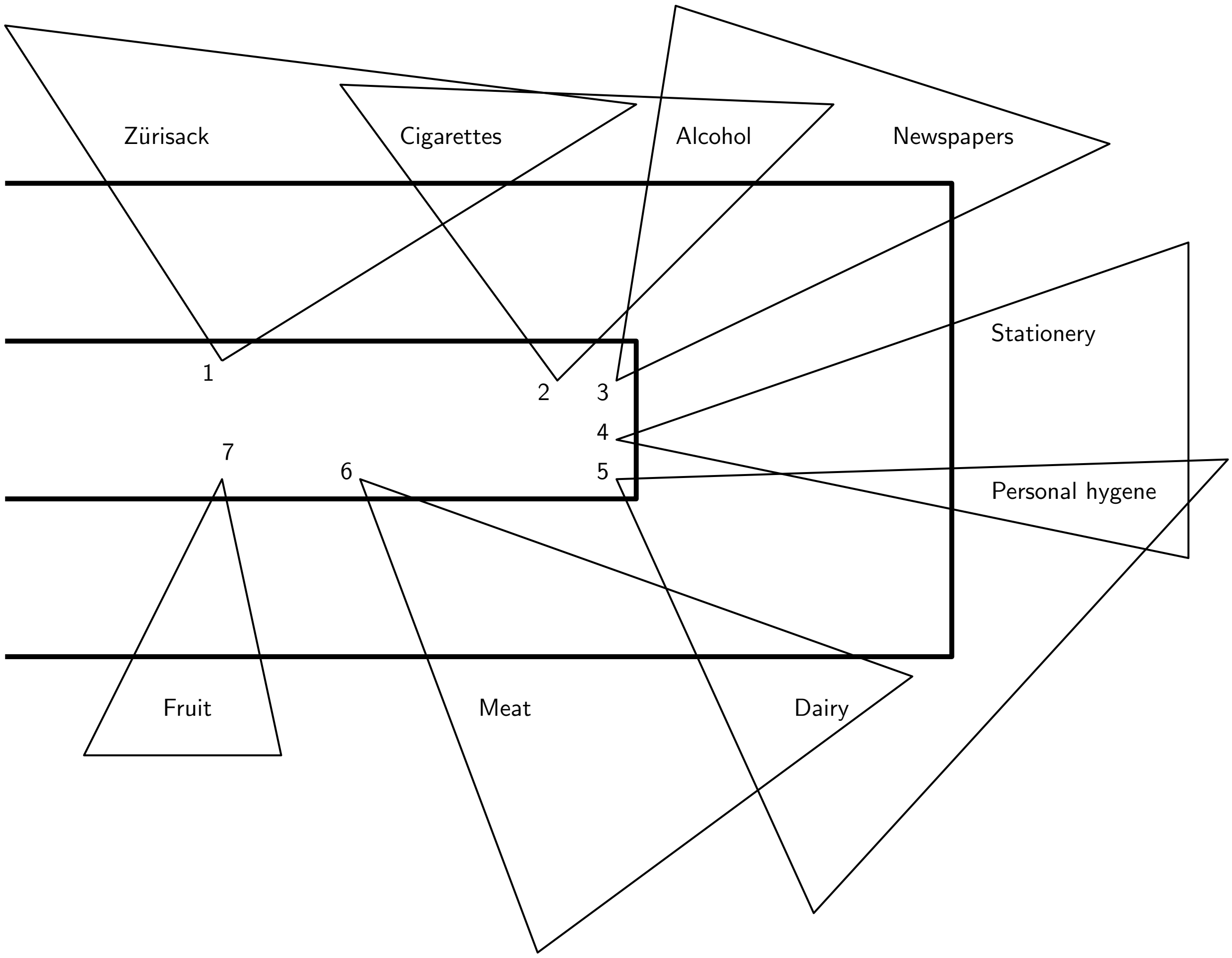
   (i) loads the DSM game data `DSMgame_data.mat`;
   
   (ii) runs your iterative algorithm, with a random initialization;
   
   (iii) generates a column vector `x_star`, of length $24 \cdot 100$, containing a Nash equilibrium;
   
   (iv) terminates in less than 2 minutes.

e) Plot the aggregate nominal energy consumption $\sum_{i \in \mathcal{I}} e_i^h$ and the aggregate energy consumption at the Nash equilibrium $\sum_{i \in \mathcal{I}} x_i^{h*}$ in one Figure over the 24 hours. Compare the two plots and comment on their differences.

**Exercise 2 Security cameras**

Consider the problem of deciding how to cycle through surveillance cameras in a supermarket in order to minimize theft.

Suppose that there is only one monitor, and a security guard observes the monitor closely. There are however 7 cameras in the store, and only one video feed can be shown on the monitor at the same time. The following diagram shows how different cameras point at different kind of merchandise.




The camera system loops over the same sequence of viewpoints every 1 minute, and during this minute it can switch from a camera to a different one at any time. Every minute, the same sequence is repeated identically.

To simplify things, we assume that the event to be detected (theft) happens in a single instant, and it is detected if the location where the theft happens is visible on the monitor at that instant. Of course, thieves do not know which video feed is shown on the monitor at a given moment.

You have been hired in order to provide guidance on how to design the best sequence of camera feeds. Answer the following questions in order to provide a scientific piece of advice on this matter. Throughout the exercise, feel free to use the computer for the computations, but make sure you explain all steps in detail. You do not need to submit any code.

a) Show how this design problem can be formulated as a *Security Game*, a special form of Stackelberg game. What are the targets? What are the coverage choices? What are the coverage vectors? How are defender strategies mapped to sequence of camera feeds?

**Hint.** *In class, we assumed that the defender randomizes their strategy. Technically speaking, the setup in this exercise does not include any randomization. However, from the point of view of the thief, the probability of the camera pointing at a given item corresponds to the fraction of the time in which the camera points at that item, even if no randomization happens.*

b) Suppose that the following estimates are available to you.

| **Merchandise** | **Cost for the store** | **Resale value outside the supermarket** |
|---|---|---|
| Zürisack | CHF 20 | CHF 20 |
| Cigarettes | CHF 15 | CHF 15 |
| Alcohol | CHF 10 | CHF 5 |
| Magazines | CHF 2 | CHF 1 |
| Stationery | CHF 3 | CHF 2 |
| Personal hygene products | CHF 5 | CHF 3 |
| Dairy | CHF 4 | CHF 0 |
| Meat | CHF 10 | CHF 0 |
| Fruit | CHF 8 | CHF 1 |

To simplify the problem, we assume that nothing happens to the thieves if they are caught (i.e., no fine), but the theft is averted.

- What is the resulting Stackelberg game (i.e., what are the $A$ and $B$ matrices)?
- What is the best response of the attacker (thief) if only camera 1 is fed to the monitor, all the time?

c) Consider the best response that you determined in the previous step. Suppose that we want that item to be the best response by the attacker, i.e., the item targeted by the thieves. What is the best strategy by the defender under this condition?

d) Solve the Stackelberg game. Notice that this would correspond to the supermarket disclosing their camera sequence (but not instantaneously: for example by showing a delayed version on a screen).

- What is the cost for the supermarket at the Stackelberg equilibrium?
- What is a possible sequence of camera feed that implements the equilibrium?
- What is the target of the thieves at the Stackelberg equilibrium?

e) Compute a Nash Equilibrium for the same game, corresponding to the scenario in which the supermarket does not disclose their surveillance strategy.

- Compare the Nash equilibrium cost with the Stackelberg cost (for both the attacker and the defender).
- Is the defender Stackelberg-equilibrium strategy a Nash-equilibrium strategy?

**Hint.** *As this is a non-zero sum game, you need to exploit the fact that it is a security game in order to make the computation of the Nash equilibrium a tractable task.*