Gambler's Ruin

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1 Introduction

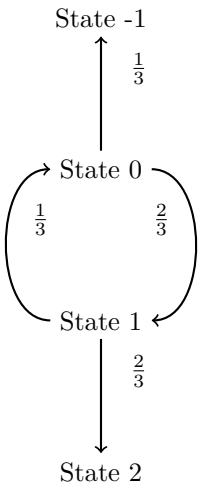
Question 1.1. *Angelo has 1$ and Leonardo has 2$. They decide to play a game against each other. At each turn of the game, the losers gives the winner 1$. Angelo is a stronger player than Leonardo and wins each turn with probability* $\frac{2}{3}$*. The game ends when one of the player goes bankrupt. What is the probability that Angelo wins the game?*

2 The solution

We can model the problem with Markov Chains. In particular, the game is made of 4 states:

- State 0: start of the Game: Leonardo has 2$ and Angelo has 1$.
- State -1: Leonardo wins: Leonardo has 2$ and Angelo has 0$.
- State 1: Leonardo has 1$ and Angelo has 2$.
- State 2: Angelo wins: Leonardo has 0$ and Angelo has 3$.

The Markov Chain looks as follows:



We can compute the probability that Angelo wins as follows:

$\mathbb{P}[State2|State0] = \frac{2}{3}(\frac{2}{3} + \frac{1}{3} \cdot \mathbb{P}[State2|State0])$

We can solve the equation and obtain $\mathbb{P}[State2|State0] = \frac{4}{7}$.
As you see it is better to be twice as good than twice as wealthy.

In the simulations file, we generalize the problem for an arbitrary amount of starting money M and N, respectively for Angelo and Leonardo. The analytical solution to that problem will be coming out soon.