Reflection on the Barabási-Albert model

Béatrice MARKHOFF, Arnaud SOULET, Léo-Paul RENARD

Outlines

- **Context and motivations**
- **Generic aspects incorporated**
- **Opening**
- **Link with the Semantic Web**

Difficulty to describe systems composed of **non identical elements with various interactions** because of their topology

Living systems, nervous systems, social sciences, World Wide Web

With the acquisition of data → Possibility to understand the dynamic and topological stability of large networks

Shows that the probability P(k) that a vertex interacts with k others decays as a power law

$$P(k) \sim k^{-\gamma}$$

Self-organization of large networks in a scale-free state

The random graph theory of Erdos and Rényi (ER)

- Description of networks with complex topology
- N vertices
- Connection of each pair of vertices with probability p
- Probability that a vertex has k edges follows Poisson distribution

Small world model by Watts and Strogatz (WS)

- N vertices form a one-dimensional lattice
- Each vertex is connected to its two nearest and next-nearest neighbors
- Each edge is reconnected to a randomly chosen vertex with probability p
- Six degrees of separation

Common characteristic → Probability of finding a highly connected vertex decreases exponentially with k

2 generic aspects of real networks not incorporated in these models

Fixed number of connected (ER) or reconnected (WS) vertices

Continuous addition of vertices in real-world networks

Probability that 2 vertices are connected is random and uniform

Preferential connectivity in real networks. Probability that a vertex connects to another is not uniform

Incorporation of the growing nature of the network

- We start with a small number of vertices (m0)
- At each time step we add a vertex with m edges connecting to m vertices

Incorporation of preferential attachment

- Connecting a new vertex with vertex i depends on its connectivity

$$\Pi(k_i) = k_i / \Sigma_j k_j$$

After t time step →

- Random network with t +m0 vertices and mt edges, evolves to a scale invariant state
- Probability that a vertex has k edges following a power law $\gamma = 2.9$

System is organized into a scale-free stationary state

Growth and preferential attachment are necessary for the development of the stationary power-law distribution

Older vertices (small t) increase their connectivity at the expense of the younger ones (large t)

↓

"rich-get-richer"

Edge acquisition rate

$$\partial k_i / \partial t = k_i / 2t$$

$$k_i(t) = m(t/t_i)^{0.5}$$

Probability that a vertex i has a connectivity lower than k

$$P[k_i(t) < k] \longrightarrow P(t_i > m^2t/k^2)$$

Adding the vertices at equal time intervals, we obtain

$$P(t_i > m^2t/k^2) = 1 - P(t_i \leq m^2t/k^2) = 1 - m^2t/k^2(t + m_0)$$

The probability density P(k)

$$P(k) = \partial P[k_i(t) < k]/\partial k$$

Long-term stationary solution

$$P(k) = \frac{2m^2}{k^3} \qquad \gamma = 3$$

Finally

The exponents obtained for the different networks **are scattered** between 2.1 and 4

Some networks evolve by **adding/removing** connections between existing vertices

This non-linearity is not taken into account in this model

This model provides the first successful mechanism of the scale invariant nature of real networks

/? With progress, the study of biological systems could help understand the evolutionary history of networks **/?**

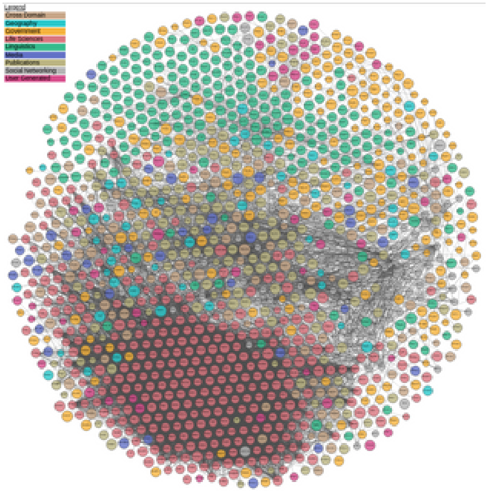
What about the Semantic Web ?

From a global point of view

Each new knowledge base is connected to those already available

/! But this is not the case **/!**

Links are made according to relevance and concordance of the data



DBPedia (created in **2007**) linked to **30** namespaces

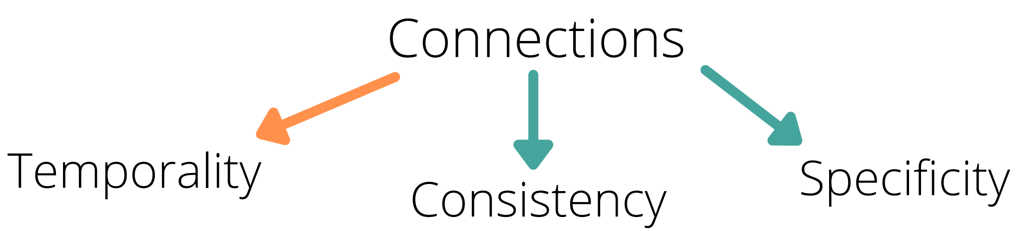
Wikidata (available in **2012**) linked to **54** namespaces

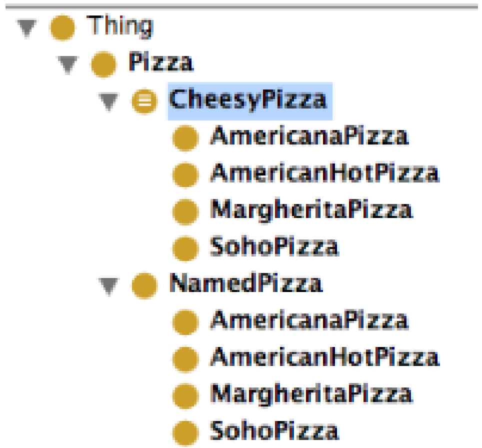
What about the Semantic Web ?

From an intra ontological point of view
TBox

Classes released in early versions should have more connectivity than those added after

/!\ But this is not the case /!\





From an intra ontological point of view

ABox

- Bias due to inconsistency, incompleteness of data within the Web
- Representation of reality that does not have preferential attachment as an axiom
- The connected vertex are diverse and varied in nature

Bibliography

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