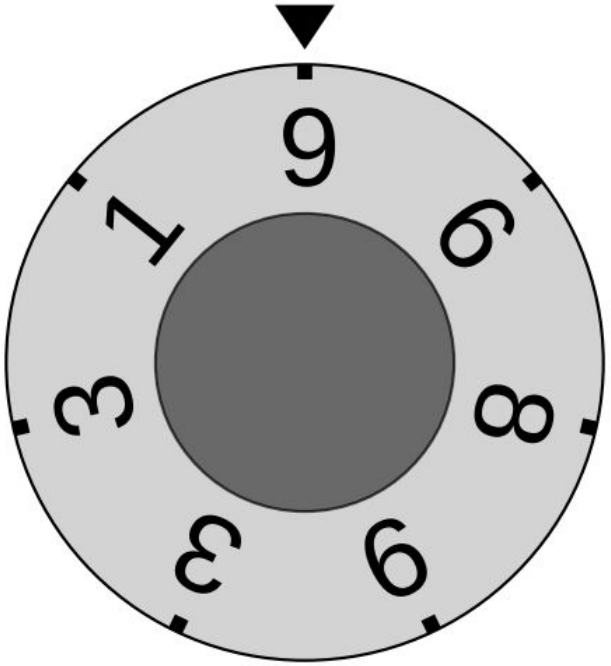
Problem K
Kitchen Knobs

Submits: 52
Accepted: at least 1

First solved by: UW1
University of Warsaw
(Dębowski, Radecki, Sommer)
01:24:54

Author: Goran Žužić, Luka Kalinovčić

Weird kitchen knobs with 7 non-zero digits. The power of a kitchen element is the number you get from reading the digits clockwise starting from the top position.



Power: 9689331

We have a sequence of N kitchen elements, and can rotate any consecutive subsequence of kitchen knobs by an arbitrary degree in a single step.

Find the smallest number of steps to get maximum power on each element.

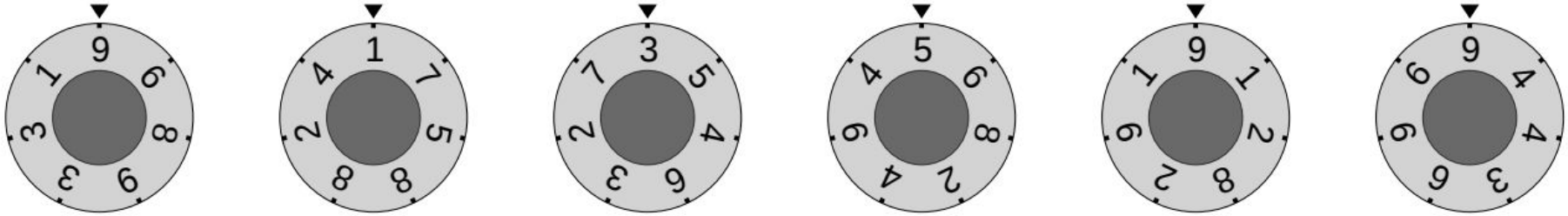
Because we have exactly 7 digits on each knob, every element either has:

a) all digits the same, in which case it's always at maximal power, or

b) exactly one position in which the maximal power is achieved.

We can pretend as if knobs of type a) didn't exist, and simplify the problem statement:

Given a sequence A with elements from [0, 6], find the smallest number of operations to make every element equal to 0. In a single operation we can add k to each number in an arbitrary subsequence of A (modulo 7).



0 3 6 5 5 5

+ 4

0 3 3 2 2 2

+ 4

0 0 0 2 2 2

+ 5

0 0 0 0 0 0

Let define another sequence B: B[i] = A[i] − A[i − 1]

A:		1	5	6	2	2	0	5	2	3
B:	1	4	1	3	0	5	5	4	1	4

+ 2

A:		1	5	**1**	**4**	**4**	**2**	**0**	**4**	3
B:	1	4	**3**	3	0	5	5	4	**6**	4

Observe what happens to sequence B as we apply the operation to sequence A.

Once again we can simplify the problem:

Given a set B with elements from [0, 6], find the smallest number of operations to make every element equal to 0. In a single operation we can add k to any number in the set and subtract k from any other number in the set (modulo 7).

1	4 -2	1	3	0	5	5 +2	4	1	4
1	2 -2	1	3	0	5 +2	0	4	1	4
1	0	1	3 -3	0	0	0	4 +3	1	4
1 +1	0	1 -1	0	0	0	0	0	1	4
2 +1	0	0	0	0	0	0	0	1 -1	4
3 +4	0	0	0	0	0	0	0	0	4 -4
0	0	0	0	0	0	0	0	0	0

Observation: Given any set of N numbers that add up to 0 (modulo 7), we can make all numbers zero in $N - 1$ operations.

In each operation take any two non-zero numbers from the set, and make one of them zero. If there are only two numbers left, it is guaranteed they will both become zero after the last operation.

Simplifying the problem even further:

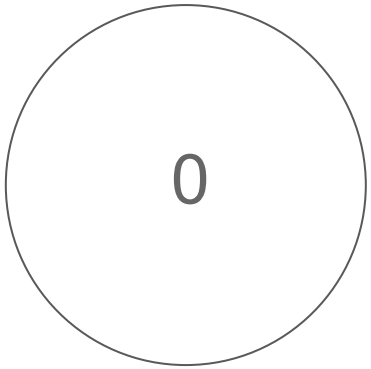
Given a set B with elements from [0, 6], group them into as many groups as possible such that the sum of each group is 0 (modulo 7).

B: 1 4 1 3 0 5 5 4 1 4

Simplifying the problem even further:

Given a set B with elements from [0, 6], group them into as many groups as possible such that the sum of each group is 0 (modulo 7).

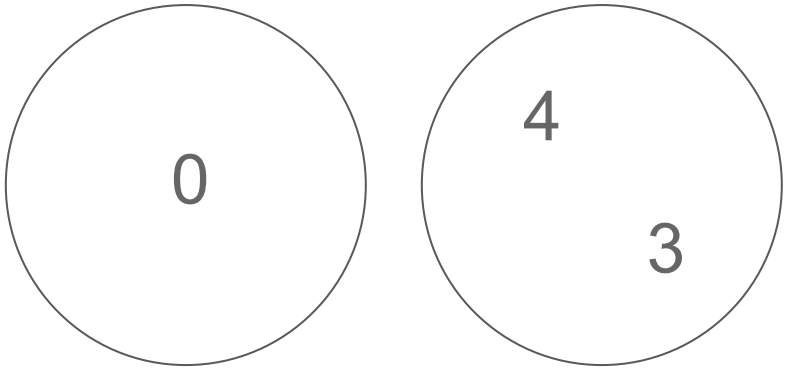
B: 1 4 1 3 5 5 4 1 4



Simplifying the problem even further:

Given a set B with elements from [0, 6], group them into as many groups as possible such that the sum of each group is 0 (modulo 7).

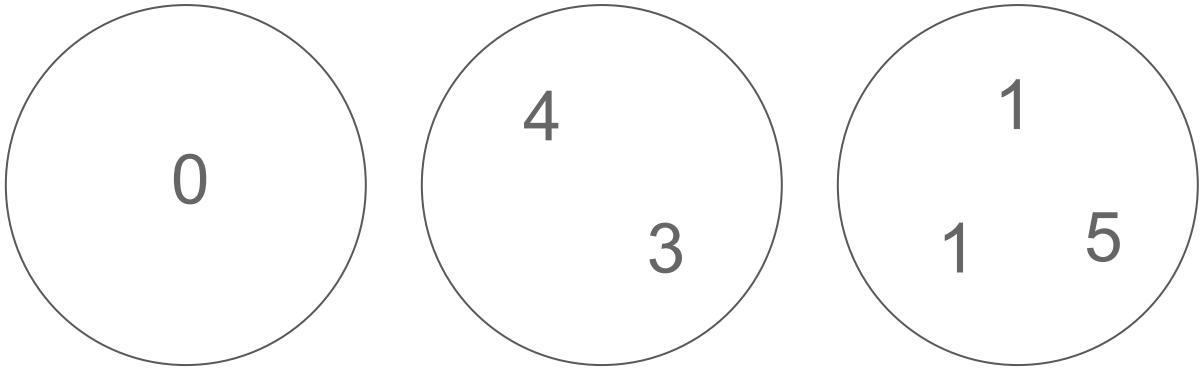
B: 1 1 5 5 4 1 4



Simplifying the problem even further:

Given a set B with elements from [0, 6], group them into as many groups as possible such that the sum of each group is 0 (modulo 7).

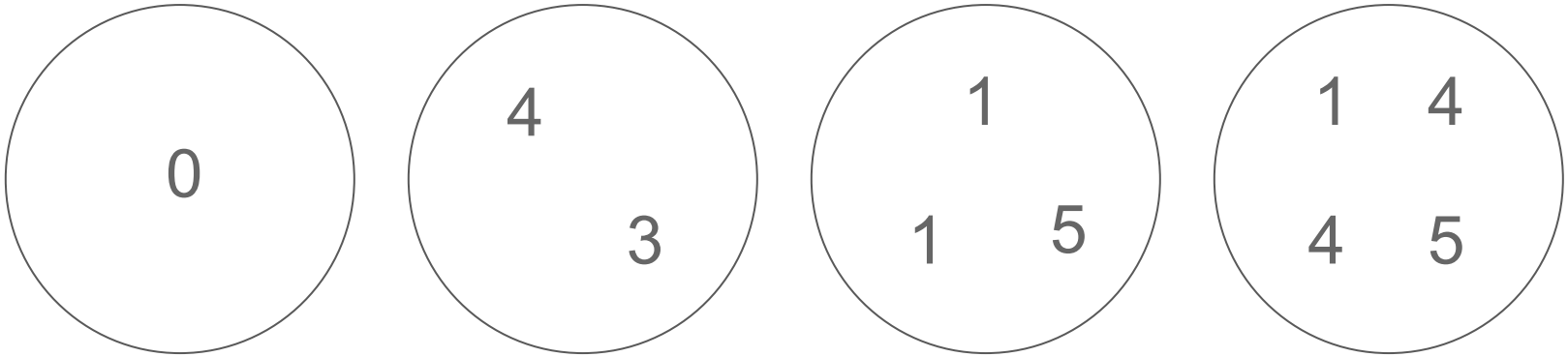
B: 1 5 4 4



Simplifying the problem even further:

Given a set B with elements from [0, 6], group them into as many groups as possible such that the sum of each group is 0 (modulo 7).

The solution is then N − number of groups = 10 - 4 = 6



To find the optimal grouping of numbers we start greedy:

1) As long as we have a zero in the set, make a group with a single zero in it.

2) As long as there is a pair of numbers that add up to 7 (1 and 6, 2 and 5, 3 and 4), make a group with these two numbers in it.

At this point the numbers in our set come from a set of at most three distinct integers: no zeros, either ones or sixes, either twos or fives, either threes or fours.

There exists a greedy O(N) strategy we could follow, but it's rather hard to find. Instead we may use a O(N^3) dynamic programming to complete the assignment.

Problem I
Intrinsic Interval

Submits: 42
Accepted: at least 1

First solved by: Jagiellonian 1
Jagiellonian University in Krakow
(Hlembotskyi, Stokowacki, Zieliński)
02:10:47

Author: Gustav Matula

An interval of the permutation is a consecutive subsequence consisting of consecutive numbers.

An interval of the permutation is a consecutive subsequence consisting of consecutive numbers.

An interval of the permutation is a consecutive subsequence consisting of consecutive numbers.

An interval of the permutation is a consecutive subsequence consisting of consecutive numbers.

2 3 1 6 4 7 5 8

An interval of the permutation is a consecutive subsequence consisting of consecutive numbers.

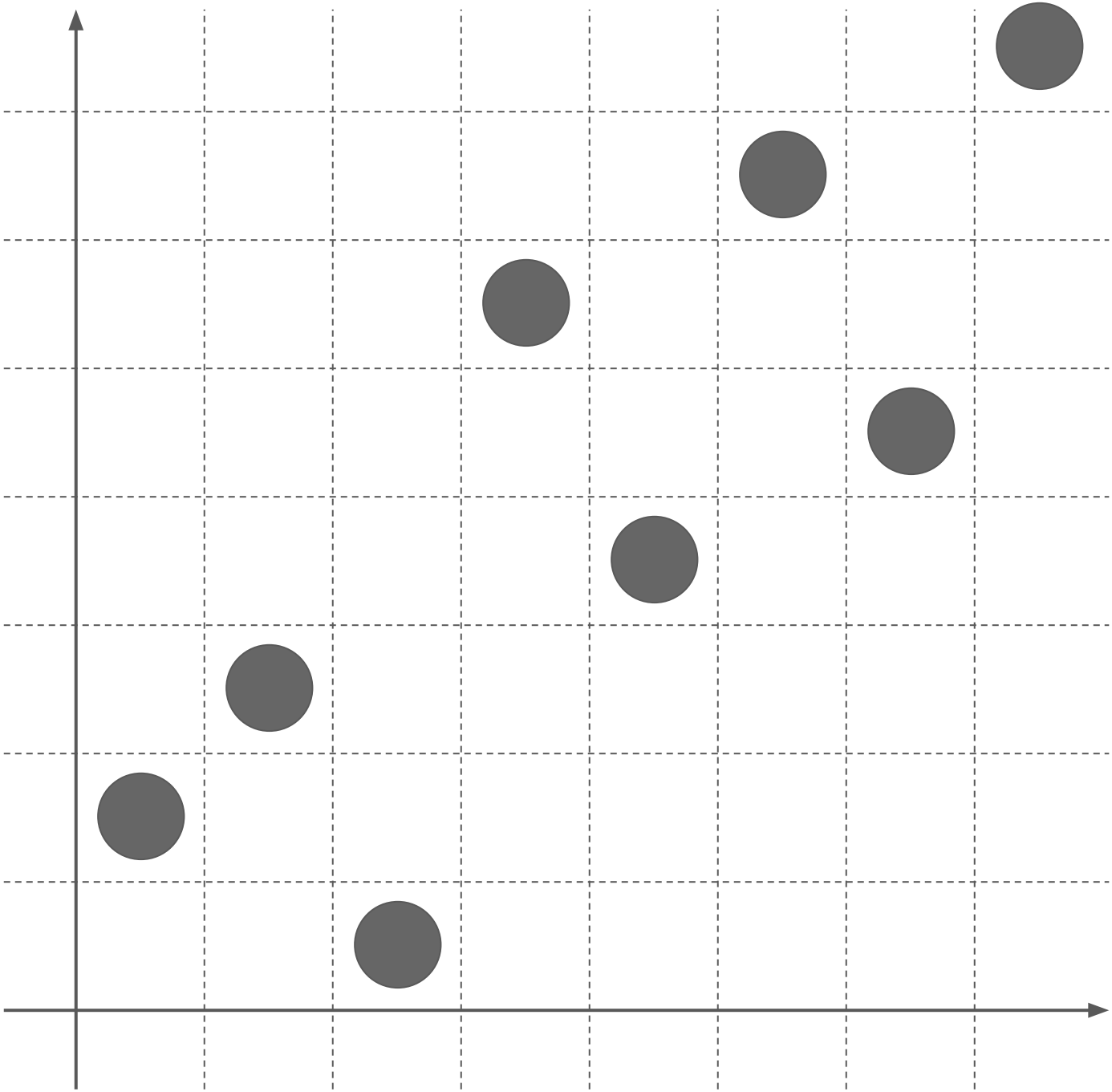
An interval of the permutation is a consecutive subsequence consisting of consecutive numbers.

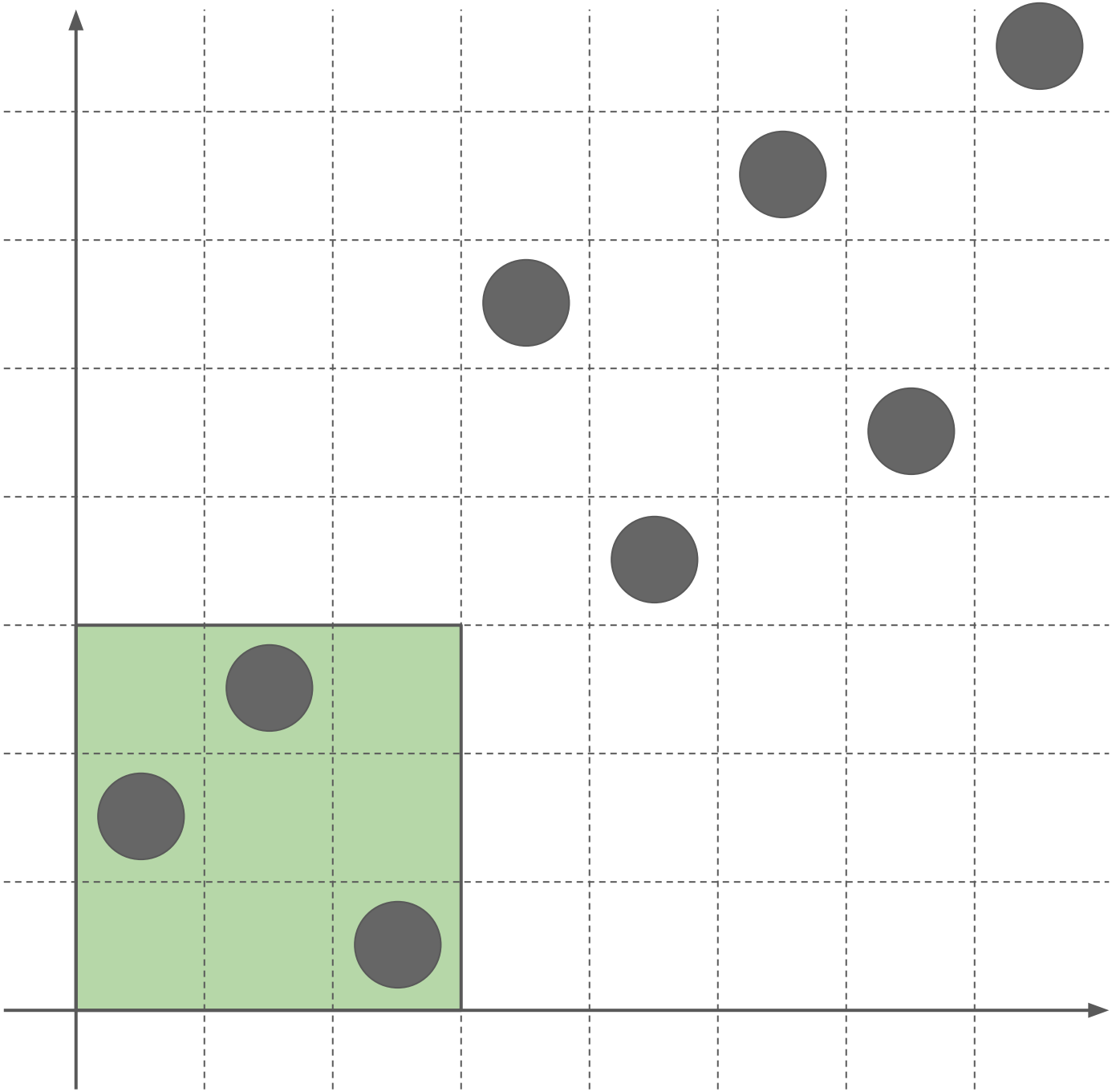
For a given subsequence we need to find the shortest enclosing interval.

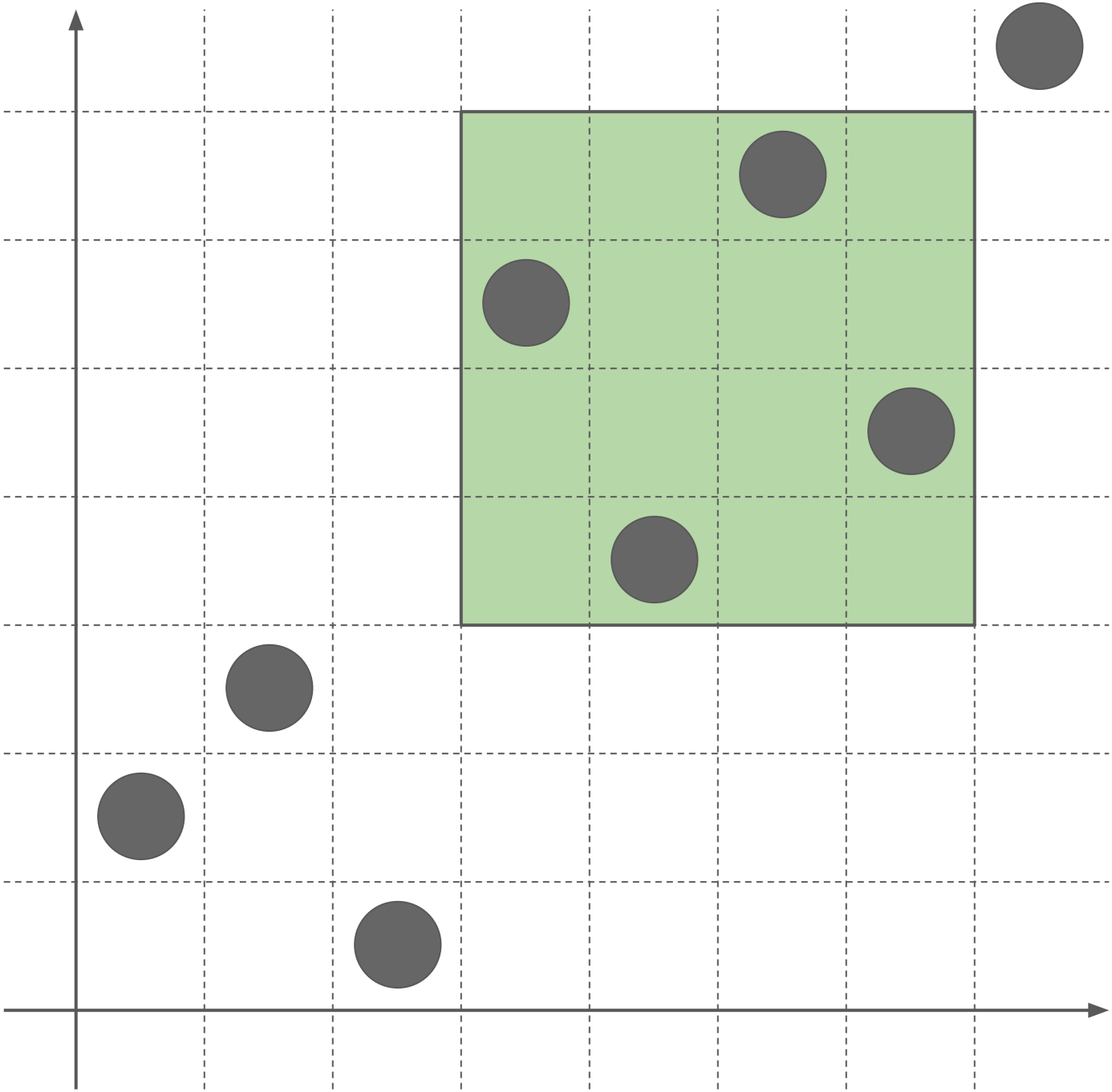
An interval of the permutation is a consecutive subsequence consisting of consecutive numbers.

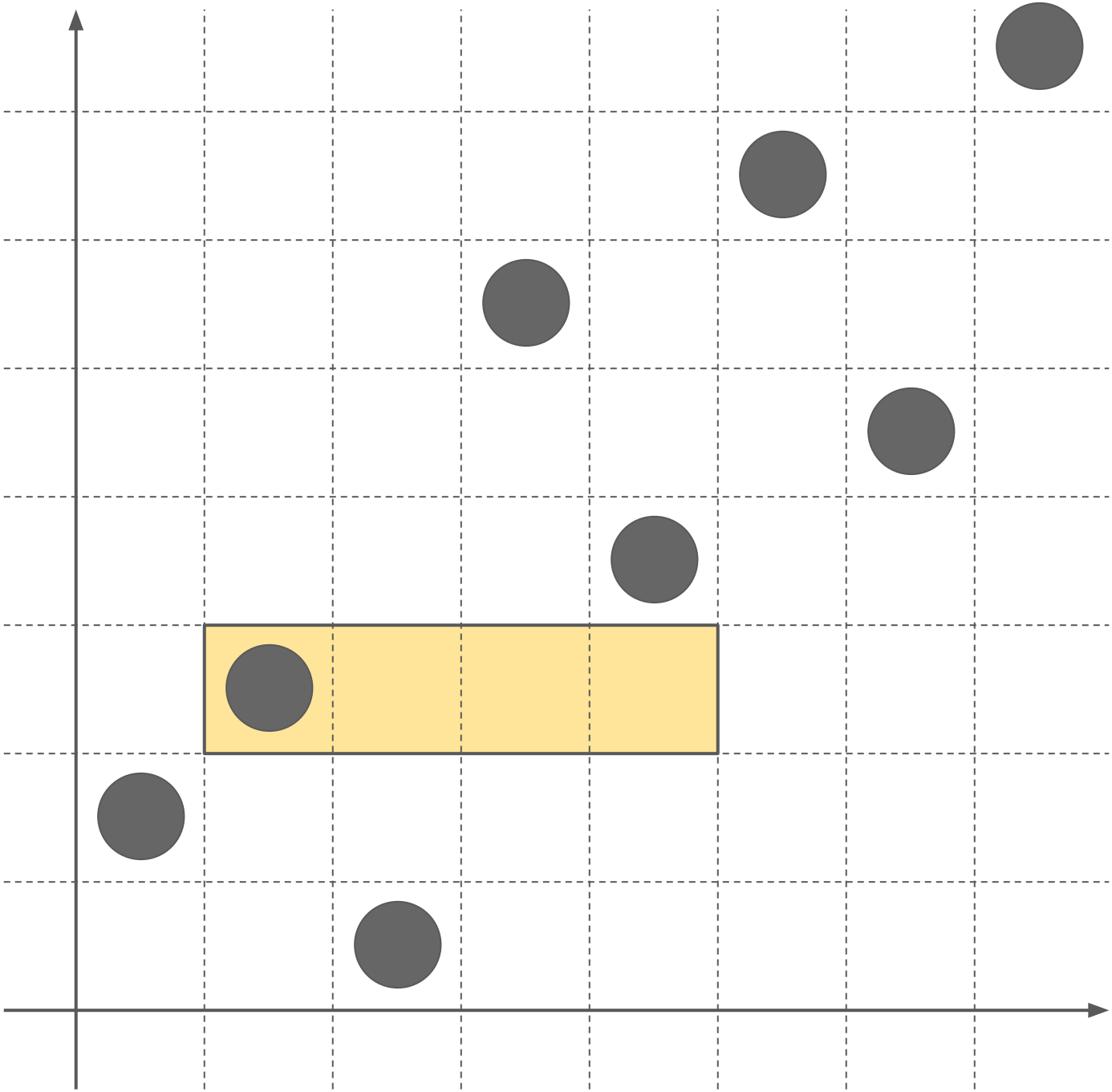
For a given subsequence we need to find the shortest enclosing interval.

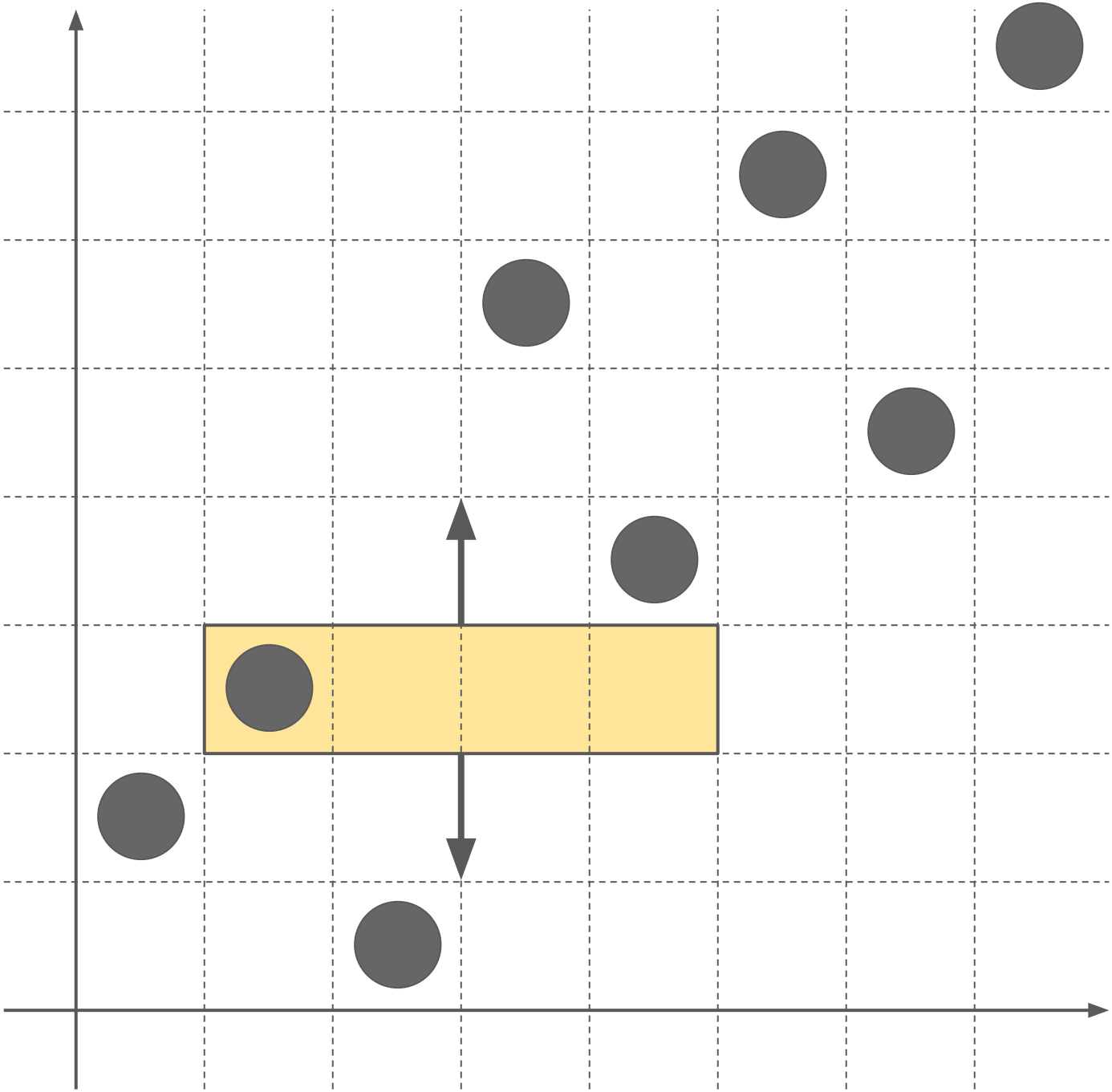
To see how we could expand the subsequence into the shortest enclosing interval, let's visualize the permutation in two dimensions.

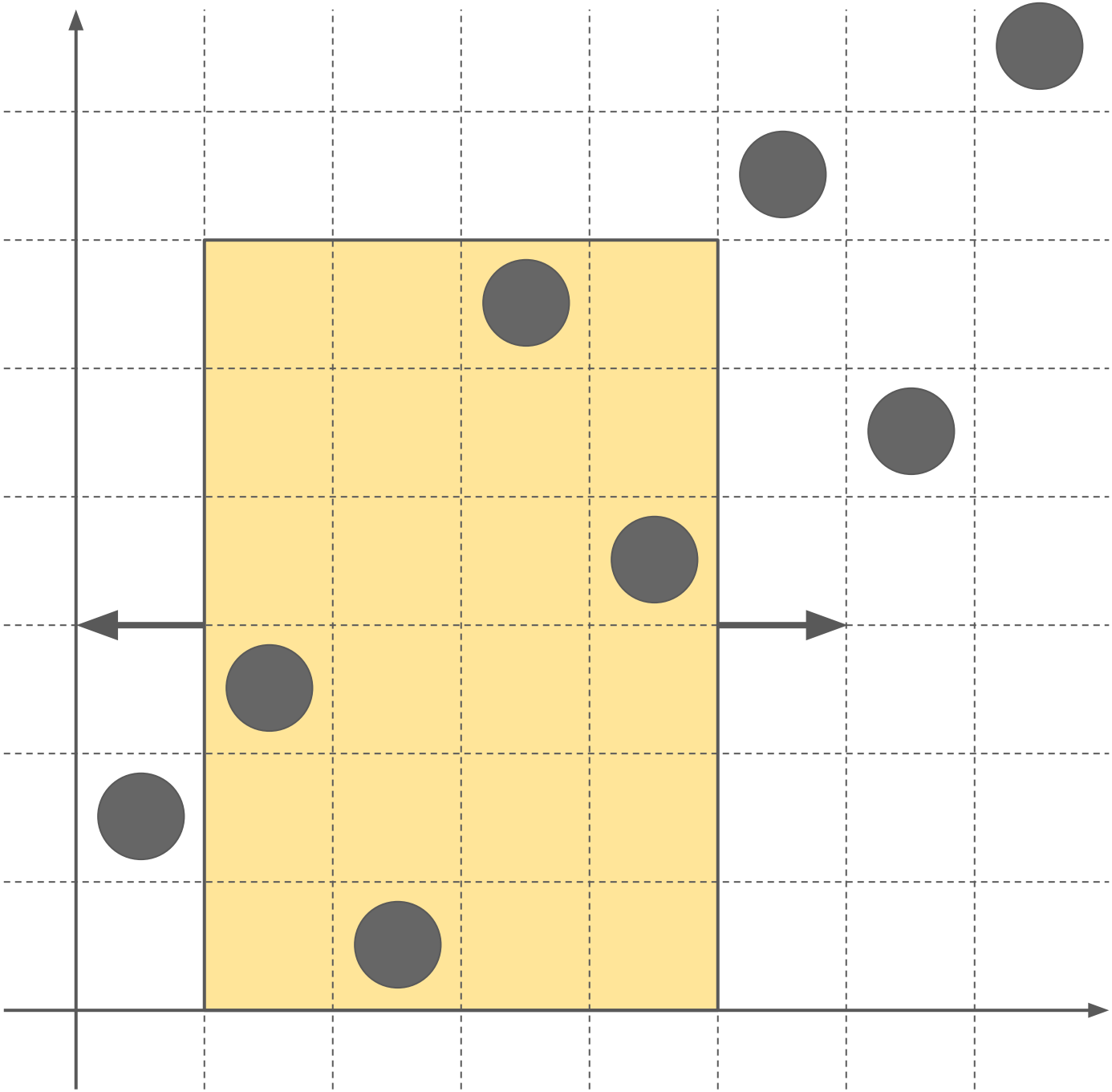


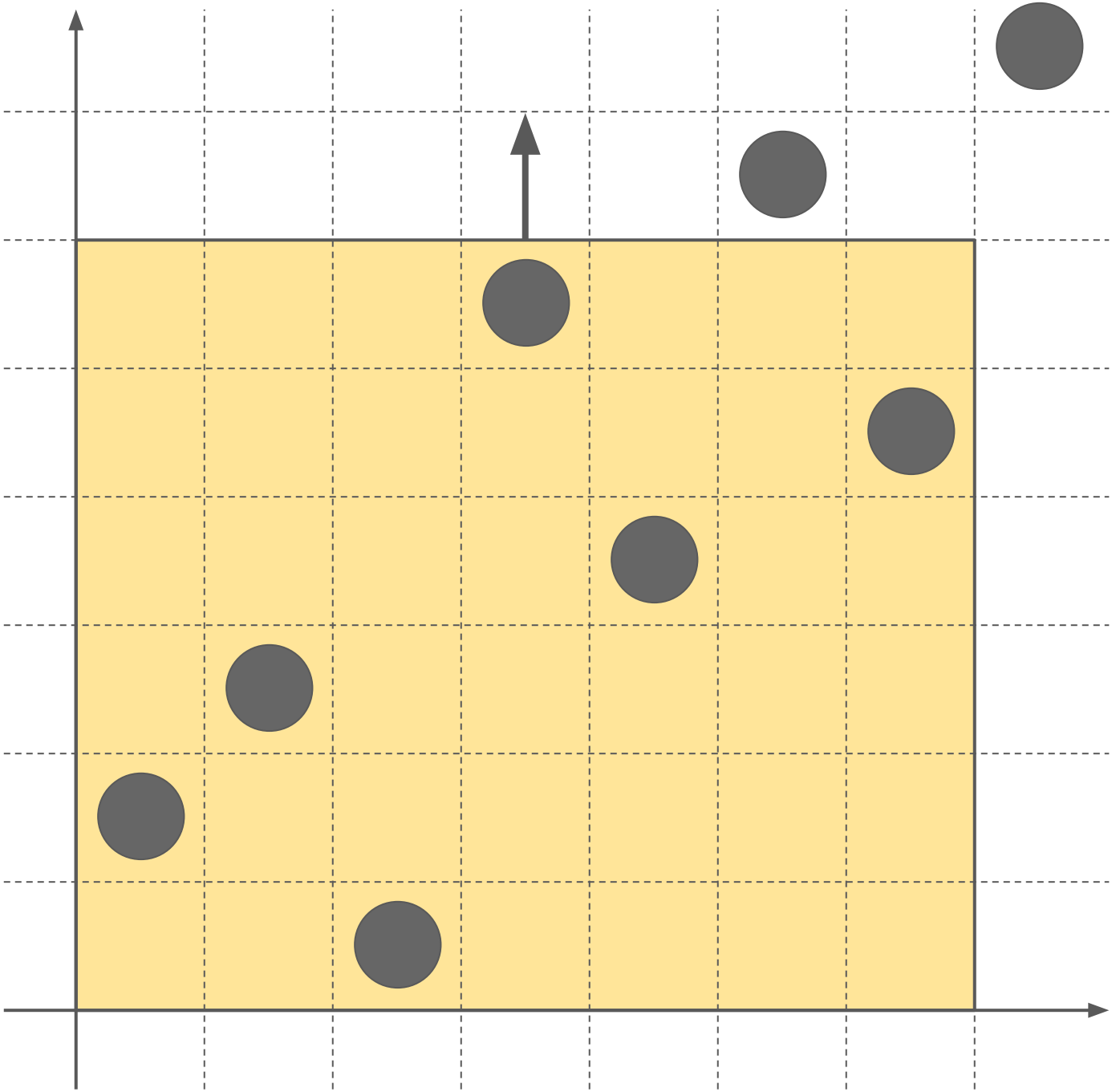


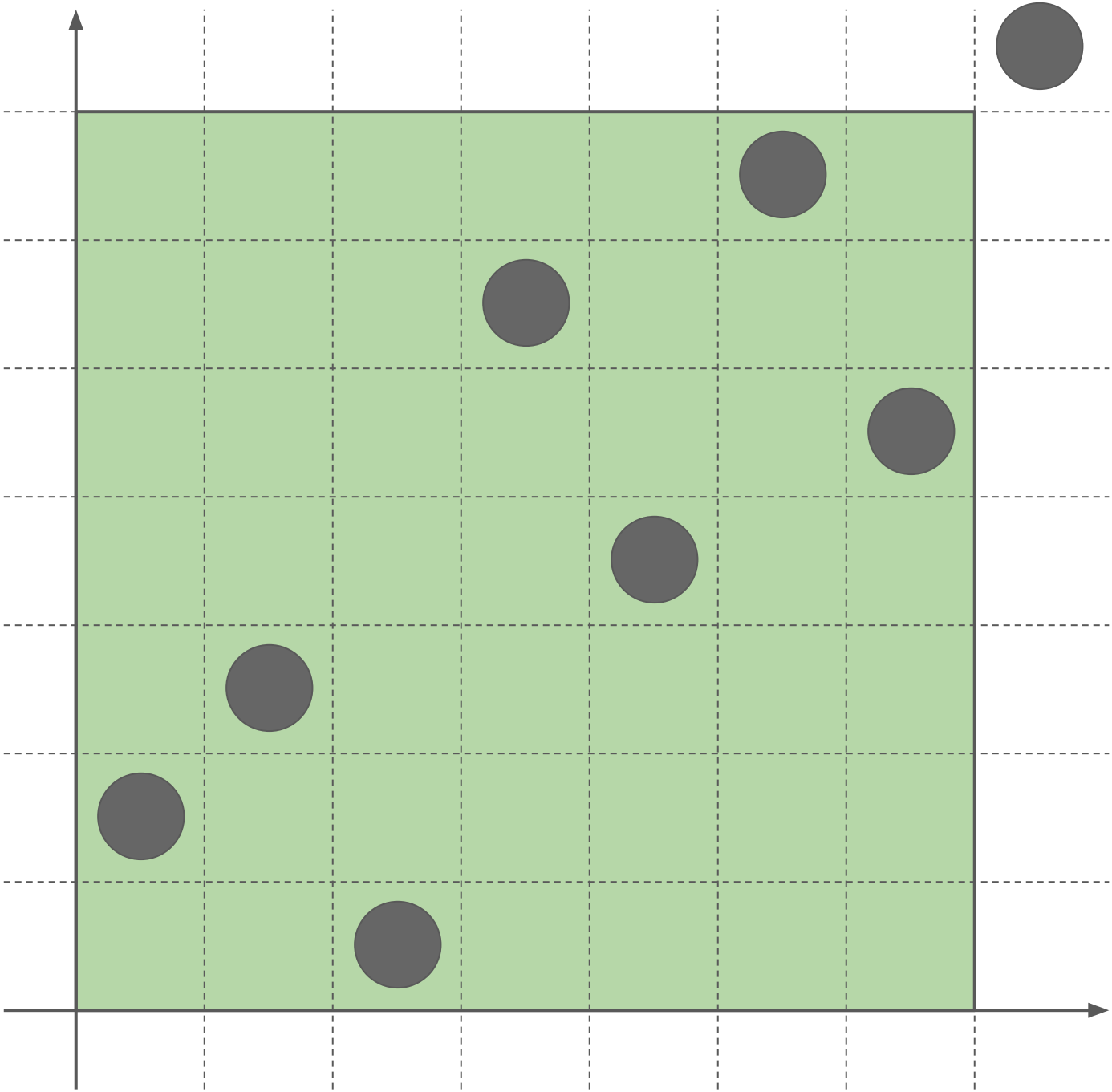












With careful implementation of the algorithm, it is possible to expand a subsequence [a, b] to an enclosing interval [x, y] in O(|y - x| - |b - a|).

However, that's too slow for this problem.

Instead, we'll develop divide and conquer algorithm to answer all queries at once.

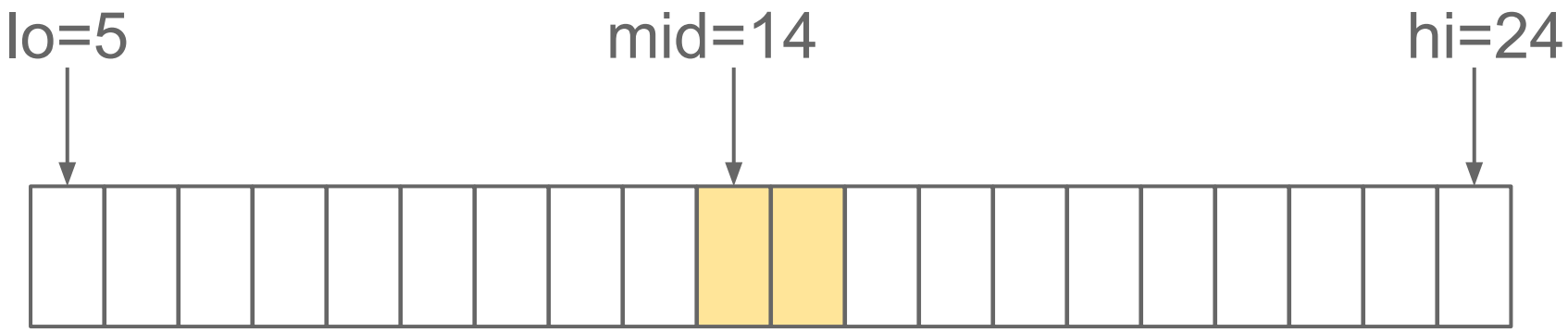
We initialize the result for each query with interval [1, n] and then we'll try to improve it.

Improve(queries, lo, hi) will try to improve each query in queries by considering intervals completely within [lo, hi] window.

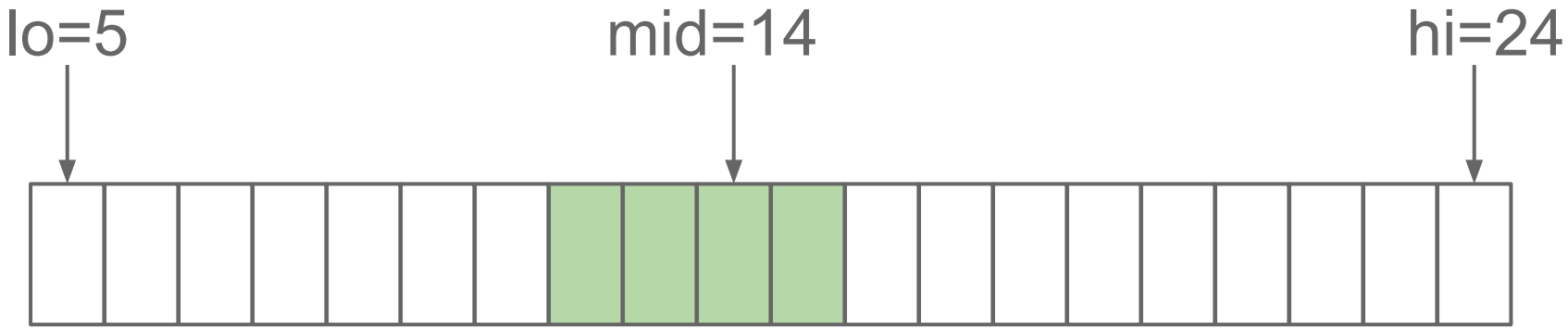
```
Improve(queries, lo, hi):
  if lo == hi: return
  mid = (lo + hi) / 2
  Improve([q in queries where q.b <= mid], lo, mid)
  Improve([q in queries where q.a > mid], mid + 1, hi)

  ImproveViaMid(queries, lo, mid, hi)
```

ImproveViaMid considers all intervals that contain [mid, mid + 1], and are within the [lo, hi] to improve provided queries.

A query participates in O(log(N)) ImproveViaMid calls.

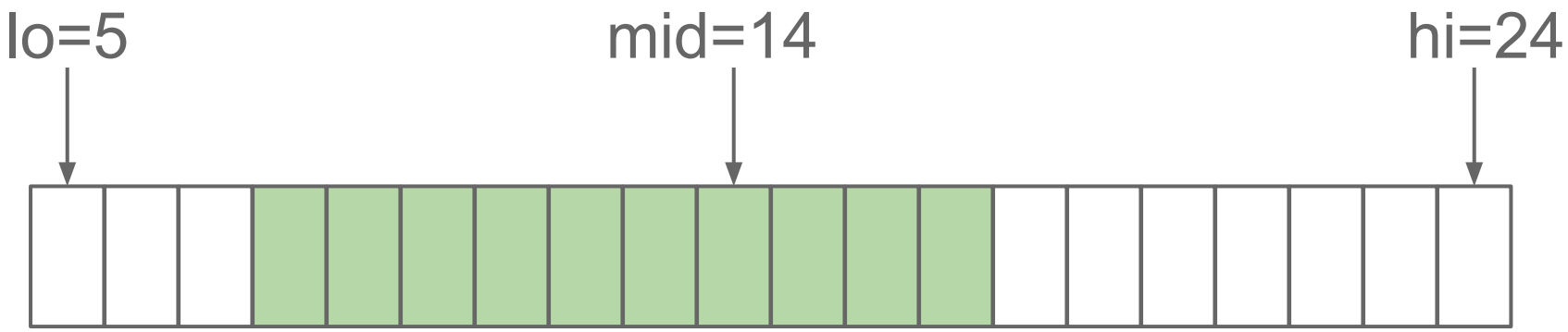


Starting from subsequence [mid, mid + 1], we expand it to the left, storing all intervals we encounter until we exit the [lo, hi] window.



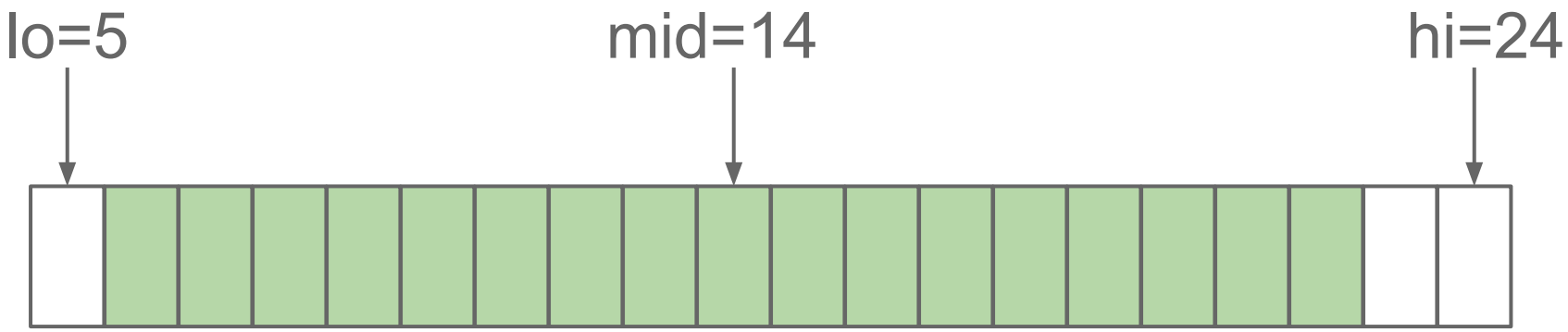
Left intervals: [12, 15]

Starting from subsequence [mid, mid + 1], we expand it to the left, storing all intervals we encounter until we exit the [lo, hi] window.



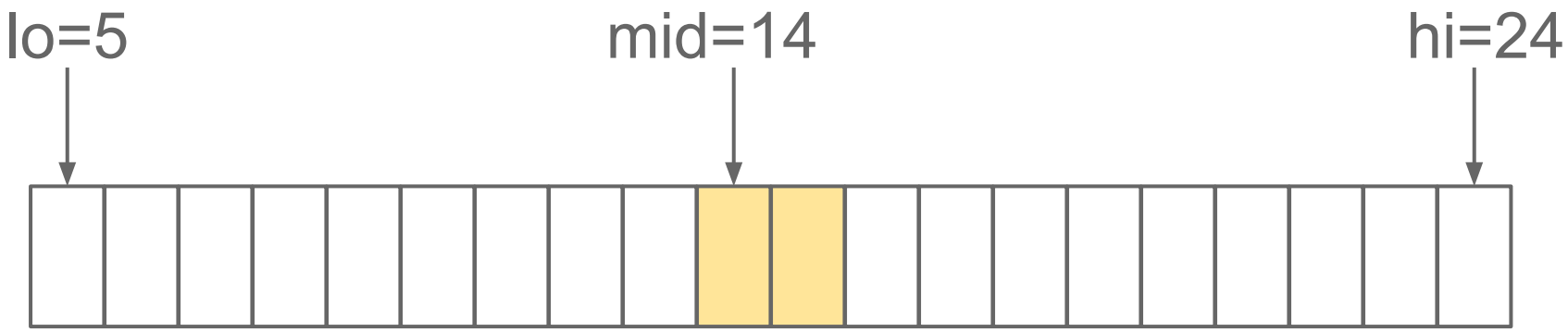
Left intervals: [12, 15], [8, 17]

Starting from subsequence [mid, mid + 1], we expand it to the left, storing all intervals we encounter until we exit the [lo, hi] window.



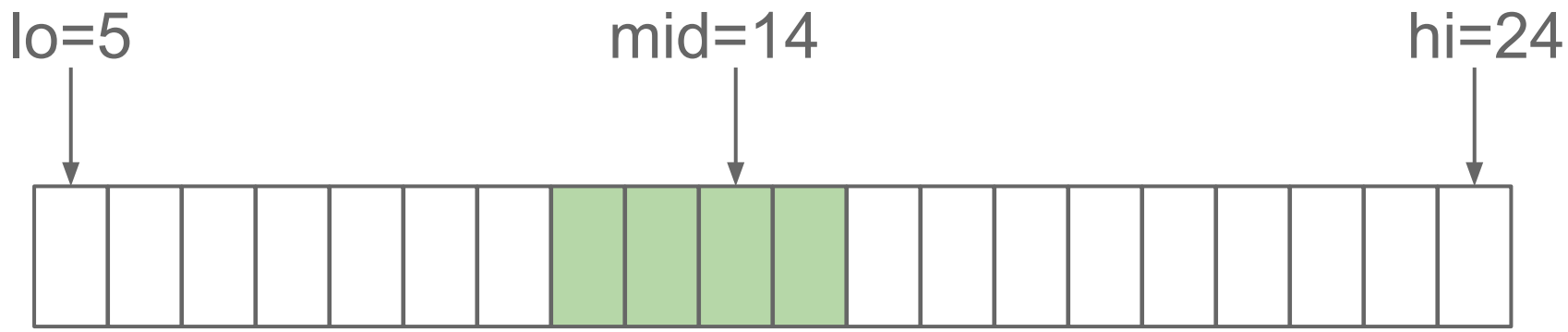
Left intervals: [12, 15], [8, 17], [6, 22]

Starting from subsequence [mid, mid + 1], we expand it to the left, storing all intervals we encounter until we exit the [lo, hi] window.



Left intervals: [12, 15], [8, 17], [6, 22]

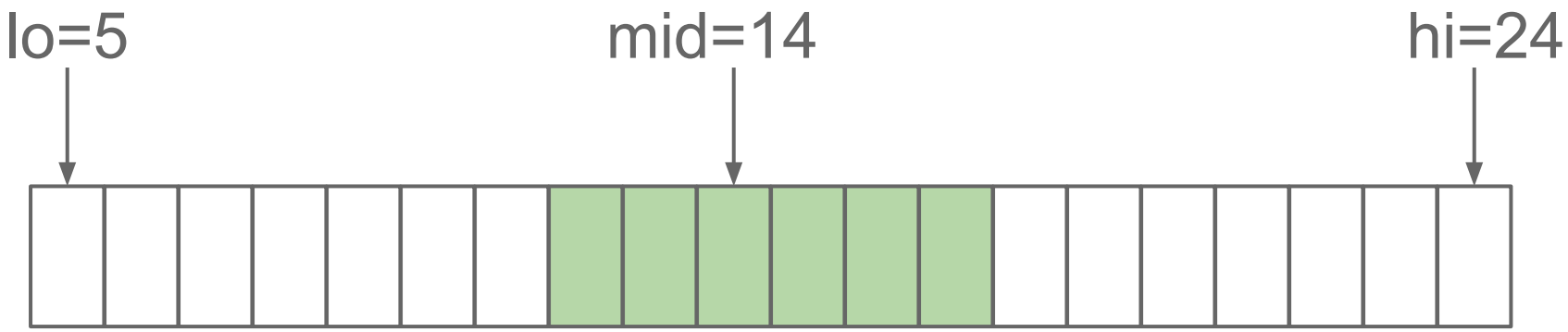
Again, starting from subsequence [mid, mid + 1], we expand it to the right, storing all intervals we encounter until we exit the [lo, hi] window.



Left intervals: [12, 15], [8, 17], [6, 22]

Right intervals: [12, 15]

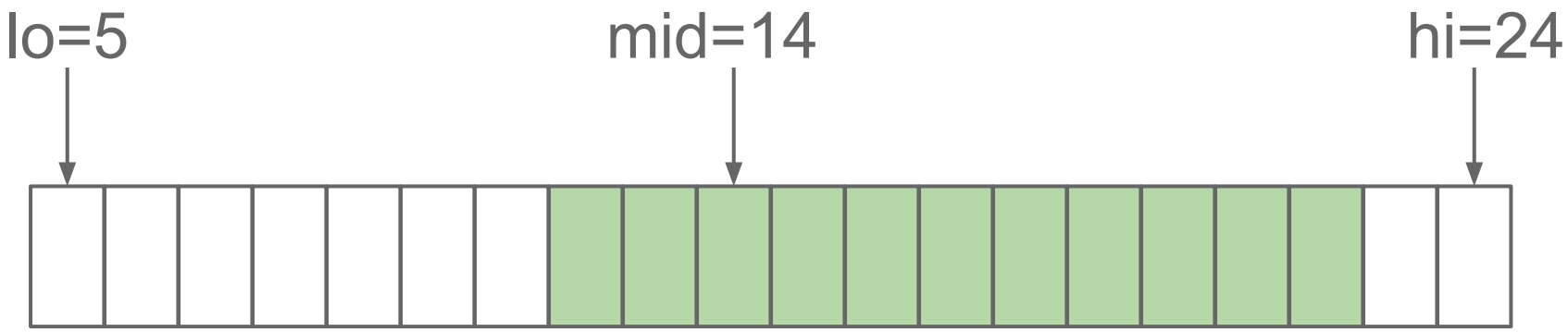
Again, starting from subsequence [mid, mid + 1], we expand it to the right, storing all intervals we encounter until we exit the [lo, hi] window.



Left intervals: [12, 15], [8, 17], [6, 22]

Right intervals: [12, 15], [12, 17]

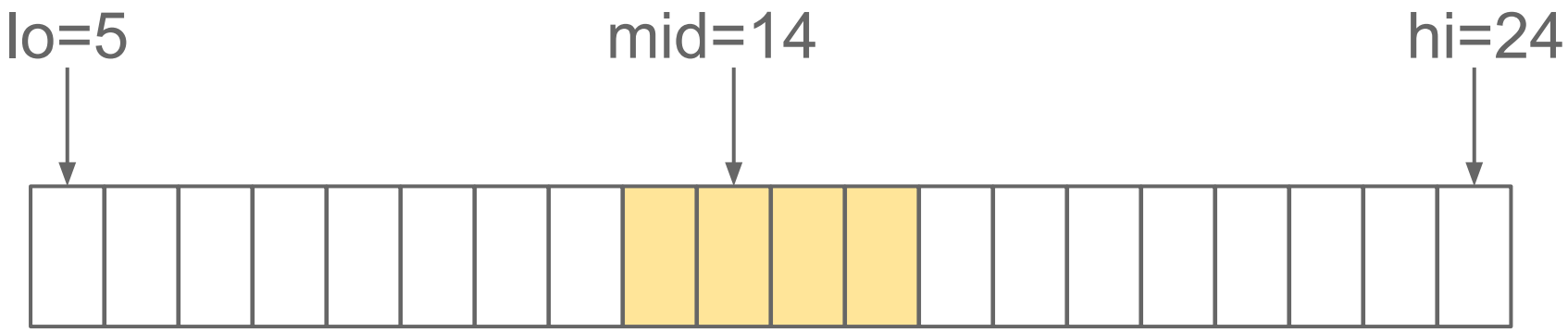
Again, starting from subsequence [mid, mid + 1], we expand it to the right, storing all intervals we encounter until we exit the [lo, hi] window.



Left intervals: [12, 15], [8, 17], [6, 22]

Right intervals: [12, 15], [12, 17], [12, 22]

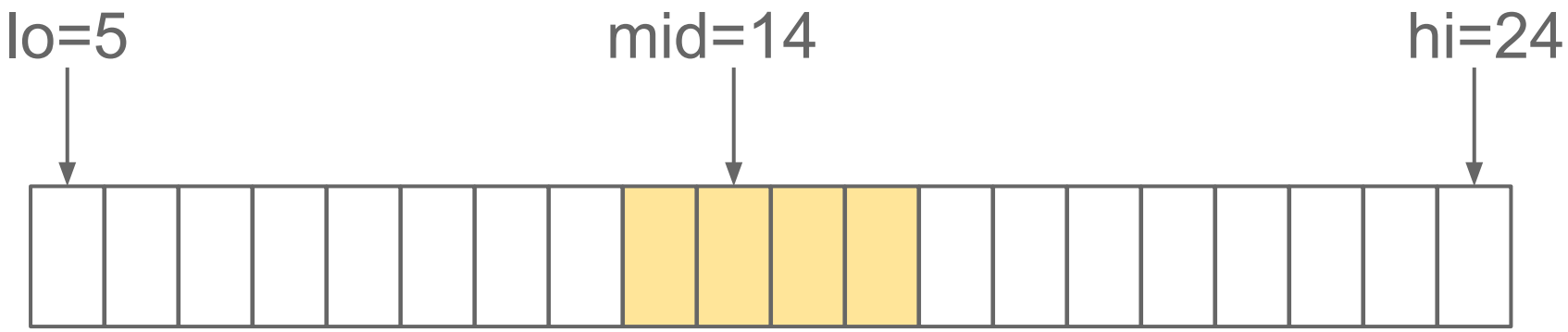
Again, starting from subsequence [mid, mid + 1], we expand it to the right, storing all intervals we encounter until we exit the [lo, hi] window.



Left intervals: [12, 15], **[8, 17]**, [6, 22]

Right intervals: [12, 15], **[12, 17]**, [12, 22]

Finally, for each query [a, b] we find the smallest left interval that contains it and the smallest right interval that contains it. The union of these two intervals is the smallest interval within [lo, hi] that contains the query.



Left intervals: [12, 15], **[8, 17]**, [6, 22]

Right intervals: [12, 15], **[12, 17]**, [12, 22]

We can implement ImproveViaMid(queries, lo, mid, hi) in O(|hi - lo| + queries.size()), for overall complexity of O((N + Q) log N).