計網概HW1

107061218 謝霖泳

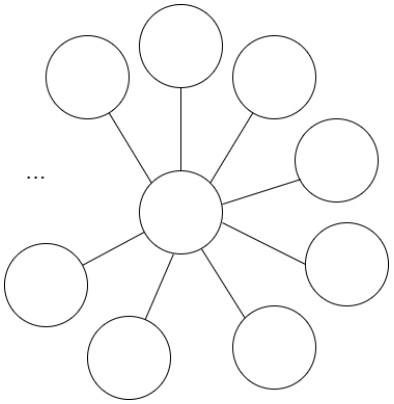
Define the geodesic (short path) distance between two nodes as the minimum number of hops from one node to the other. Define the diameter of a network as the maximum geodesic distance among all the pairs of two nodes. Define the degree of a node as the number of links connected to that node.

1. If the diameter of a network with 100 nodes is 1, what is the minimum number of links in this network?
2. If the diameter of a network with 100 nodes is 2, what is the minimum number of links in this network?
3. For a network of 100 nodes, if the degree of every node is at most 2, what is the minimum diameter of that network?
4. For a network of 100 nodes, if the degree of every node is at most 3, is it possible that the diameter of this network is not greater than 5?

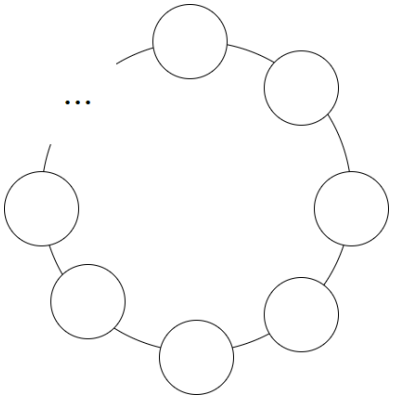
Solution

1. diameter = 1代表每兩個node之間都有彼此連通，所以minimum number of link = $\binom{100}{2} = 4950$。

2. diameter = 2又要達到最少的link，方法是中間擺一個node，分別向外連接剩下的99個node，如下圖。故minimum number of link = 100 - 1 = 99。



3. 如下圖，當所有node繞成一個cycle時，diameter最小。此時的diameter $= 100/2 = 50$。



4. 考慮一個Moore Graph如下圖，要達到diameter<=5，maximum number of nodes $= 1 + 3 * \sum_{i=0}^{4}(3-1)^i = 1 + 3 * 2^0 + 3 * 2^1 + 3 * 2^2 + 3 * 2^3 + 3 * 2^4 = 94$，而$100 > 94$，所以diameter不可能$<= 5$，一定會多用到一層(紅色部分)。