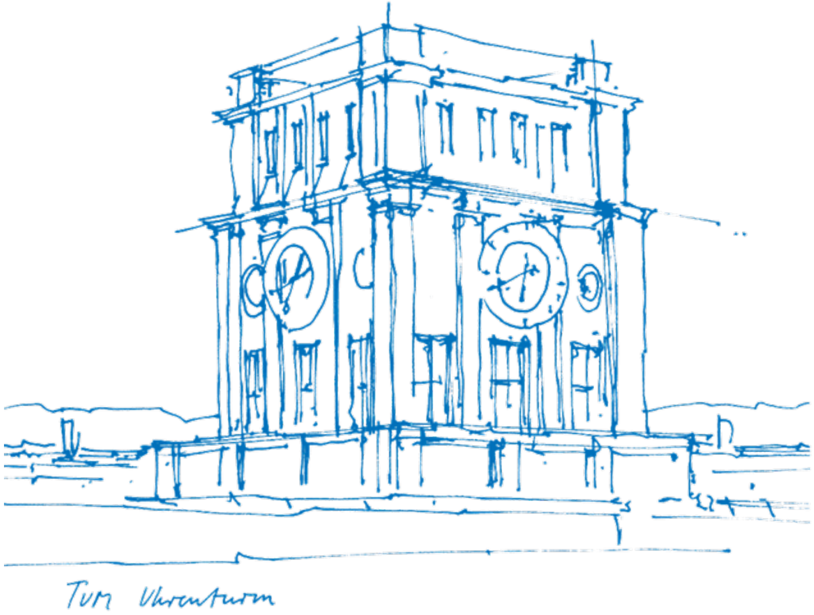
Seminar “Advanced Topics of Quantum Computing”

Prof. Dr. Christian B. Mendl
Technical University of Munich
CIT, Department of Computer Science 5
Chair of Quantum Computing
11. July 2024

Kick-off Meeting



Organization

Time: Fridays 10:00 – 12:00, starting *8. November 2024*
Location: seminar room 02.07.023

6 or 7 blocks of presentations (weekly):

- 3 students per block
- 20 minutes talk, 10 minutes discussion
- All students should attend all presentations

Requirements for passing:

- Presentation
- Implement and document prototype or demonstration
  (Programming language and toolbox are up to you, e.g., Python + NumPy, Julia, Qiskit, Cirq, *your own*, . . .)
  or
  Write a technical report on your topic (5 – 8 pages)
- Meet at least once to discuss topic in detail, and again one week before presentation, hand in prototype/report (at the latest) one week after presentation

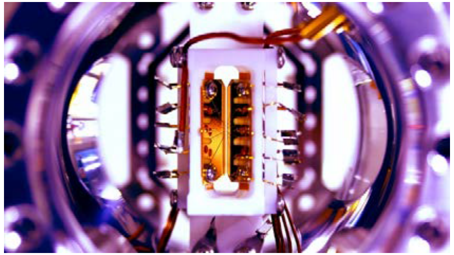
Evaluation: Presentation and prototype/report count equally to final grade.

Topics overview

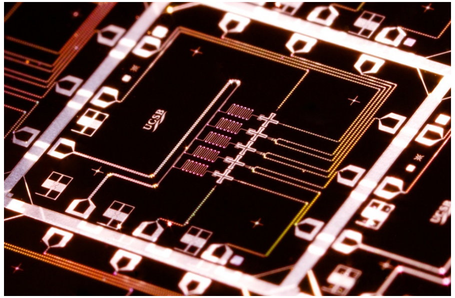
- Quantum hardware and physical realizations
- Quantum circuit simulation and visualization
- Simulating quantum many-body dynamics
- Tensor networks
- Qubitization and quantum eigenvalue transforms
- Classical shadows
- Variational quantum circuits
- Quantum optimization
- Quantum error correction and topological computation
- Quantum cryptography
- *. . . suggest your own topic!*

Most topics will be split into subtopics.

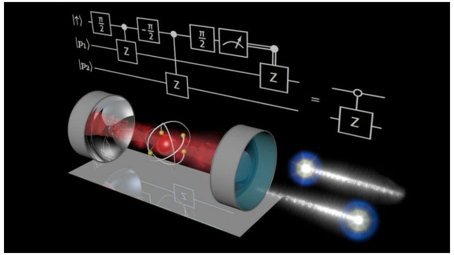
Quantum hardware and physical realizations



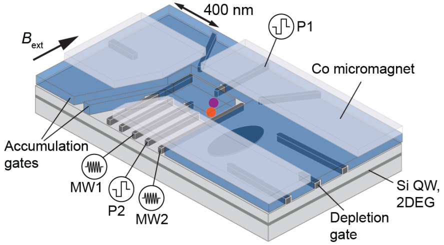
ion traps



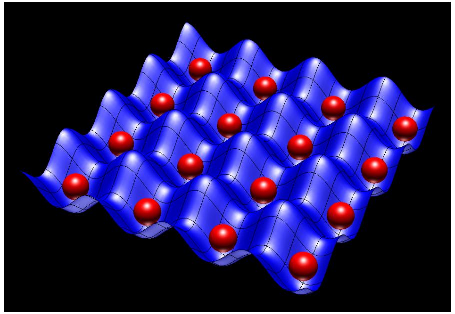
superconducting qubits



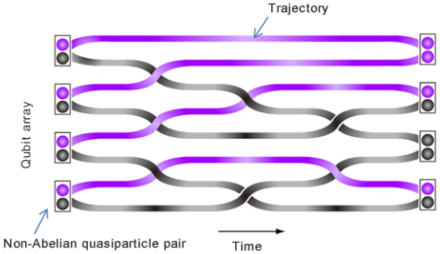
photons



quantum dots



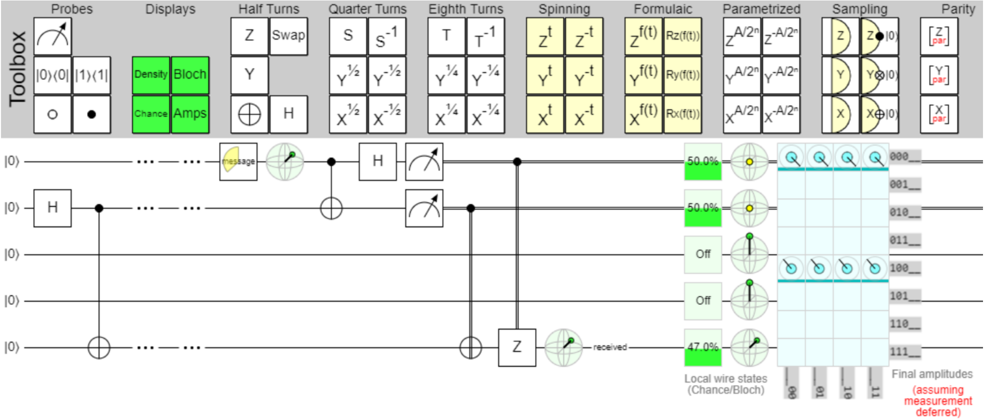
optical lattices



topological

Several hardware platforms thinkable due to “universality” of quantum mechanics

Quantum circuit simulation and visualization



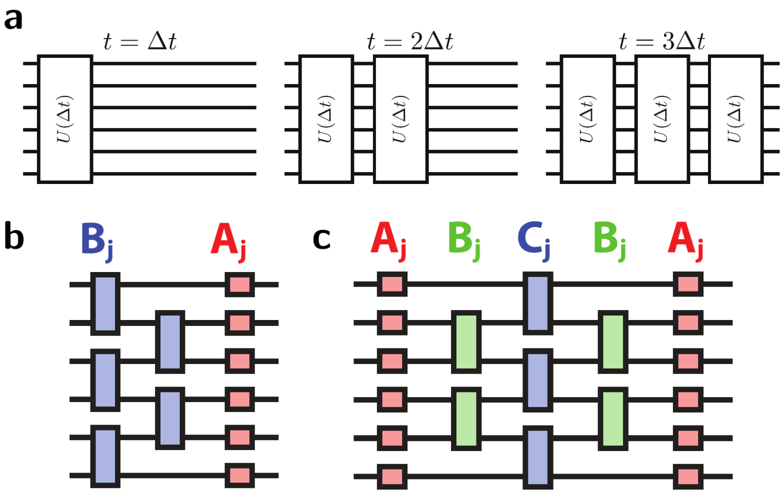
Source: Quirk (https://algassert.com/quirk)

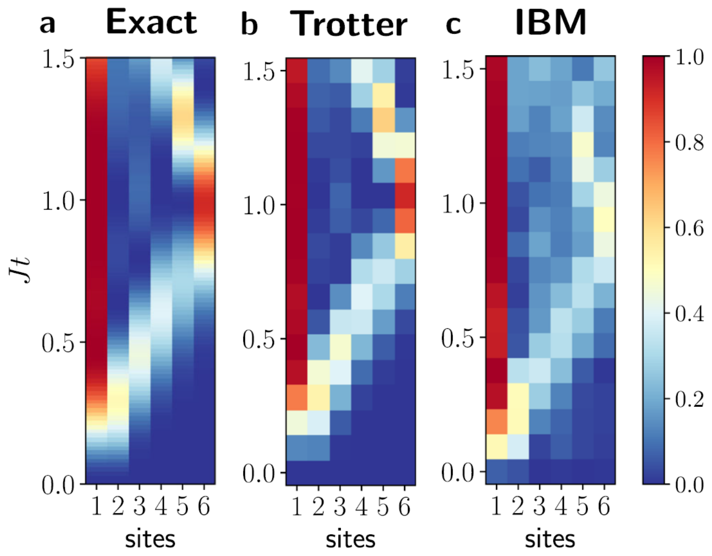
- Statevector simulation
- Tensor network approaches
- Differentiable programming quantum circuits
- Gottesman-Knill theorem:
  Stabilizer circuits (only Hadamard, phase, Pauli, controlled-NOT gates) can be efficiently simulated on a classical computer.

  S. Aaronson and D. Gottesman (2004). “Improved simulation of stabilizer circuits”. In: *Phys. Rev. A* 70, p. 052328. DOI: 10.1103/PhysRevA.70.052328; S. Anders and H. J. Briegel (2006). “Fast simulation of stabilizer circuits using a graph-state representation”. In: *Phys. Rev. A* 73, p. 022334. DOI: 10.1103/PhysRevA.73.022334
- ZX-calculus

Simulating quantum many-body dynamics

Setup: spin-$\frac{1}{2}$ chain





Smith, Kim, Pollmann, Knolle. "Simulating quantum many-body dynamics on a current digital quantum computer". npj Quantum Information 5, 106 (2019)
B. Chiaro et al. "Growth and preservation of entanglement in a many-body localized system". arXiv:1910.06024

Tensor networks (classical simulation, encoding)

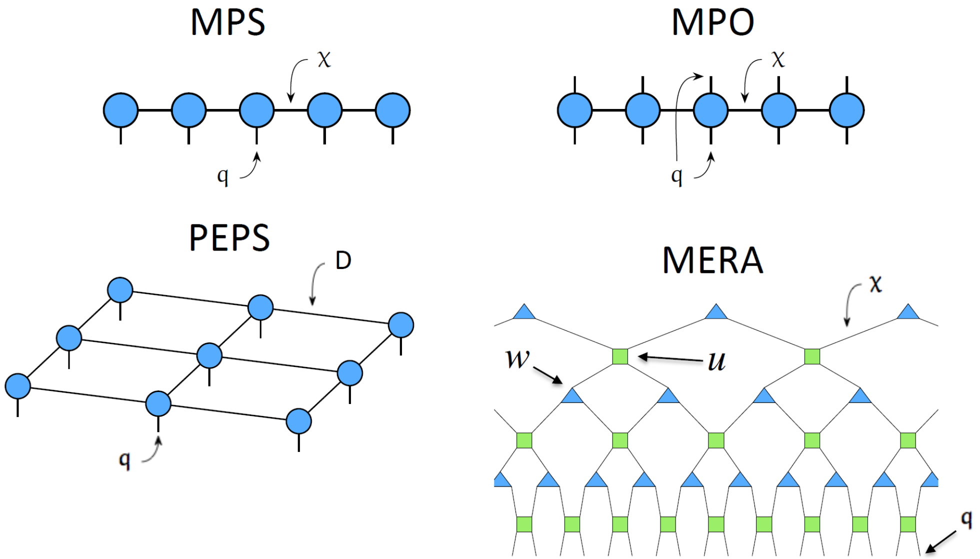


Figure: Tensor network diagrams

Y. Zhou et al. (2020). “What limits the simulation of quantum computers?” In: *Phys. Rev. X* 10 (4), p. 041038. DOI: 10.1103/PhysRevX.10.041038

F. Pan et al. (2020). “Contracting arbitrary tensor networks: General approximate algorithm and applications in graphical models and quantum circuit simulations”. In: *Phys. Rev. Lett.* 125, p. 060503. DOI: 10.1103/PhysRevLett.125.060503

Qubitization and quantum eigenvalue transforms

Quantum signal processing (QSP)

Goal: realize a function $f(a)$
using an alternating sequence of single-qubit gates:

- "signal rotation operator", for $a \in [-1, 1]$:

$$W(a) = \begin{pmatrix} a & i\sqrt{1-a^2} \\ i\sqrt{1-a^2} & a \end{pmatrix}$$

- "signal-processing rotation operator", with $Z = \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)$:

$$S(\varphi) = e^{i\varphi Z}, \quad \varphi \in \mathbb{R}$$

For phases $\vec{\varphi} = (\varphi_0, \dots, \varphi_d) \in \mathbb{R}^{d+1}$, define

$$U_{\vec{\varphi}} = e^{i\varphi_0 Z} W(a) e^{i\varphi_1 Z} W(a) \cdots e^{i\varphi_d Z} = e^{i\varphi_0 Z} \prod_{k=1}^{d} W(a) e^{i\varphi_k Z}$$

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Theorem (Quantum signal processing)

The QSP sequence $U_{\vec{\varphi}}$ produces a matrix that maybe be expressed as polynomial function of a:

$$U_{\vec{\varphi}} = e^{i\varphi_0 Z} \prod_{k=1}^{d} W(a)\, e^{i\varphi_k Z} = \begin{pmatrix} P(a) & iQ(a)\sqrt{1-a^2} \\ iQ^*(a)\sqrt{1-a^2} & P^*(a) \end{pmatrix}$$

for $a \in [-1,1]$, and a $\vec{\varphi}$ exists for any polynomials P, Q such that:

(i) $\deg(P) \le d$, $\deg(Q) \le d-1$

(ii) *P has parity $d \mod 2$ and Q has parity $(d-1) \mod 2$*

(iii) $|P(a)|^2 + (1-a^2)|Q(a)|^2 = 1$

J. M. Martyn et al. "Grand unification of quantum algorithms". PRX Quantum, 040203 (2021)
G. H. Low and I. L. Chuang. "Hamiltonian simulation by qubitization". Quantum 3, 163 (2019)

Qubitization and quantum eigenvalue transforms (cont.)

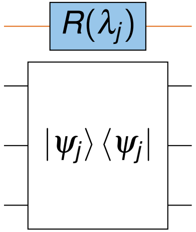
Goal: apply QSP to all eigenvalues $\{\lambda_j\}$ of a Hermitian matrix H simultaneously, with "$a = \lambda_j$"

Strategy: block-encode H into a larger unitary V:

$$V = \begin{pmatrix} H & \sqrt{I - H^2} \\ \sqrt{I - H^2} & -H \end{pmatrix}$$

Inserting $H = \sum_j \lambda_j |\psi_j\rangle\langle\psi_j|$:

$$V = \sum_j$$



with $R(a) = \begin{pmatrix} a & \sqrt{1-a^2} \\ \sqrt{1-a^2} & -a \end{pmatrix}$

Qubitization and quantum eigenvalue transforms (cont.)

Goal: apply QSP to all eigenvalues $\{\lambda_j\}$ of a Hermitian matrix H simultaneously, with "$a = \lambda_j$"

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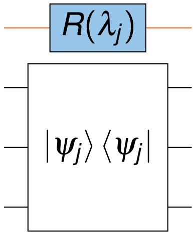
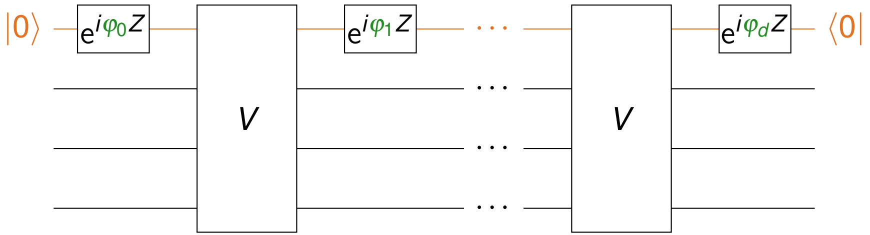


with $R(a) = \begin{pmatrix} a & \sqrt{1-a^2} \\ \sqrt{1-a^2} & -a \end{pmatrix}$

QSP on auxiliary qubit transforms all eigenvalues simultaneously!

$$U_{\vec{\varphi},\mathrm{eig}} = e^{i\varphi_0 Z} \prod_{k=1}^{d} V e^{i\varphi_k Z} = \begin{pmatrix} \mathrm{Poly}(H) & \cdot \\ \cdot & \cdot \end{pmatrix}$$



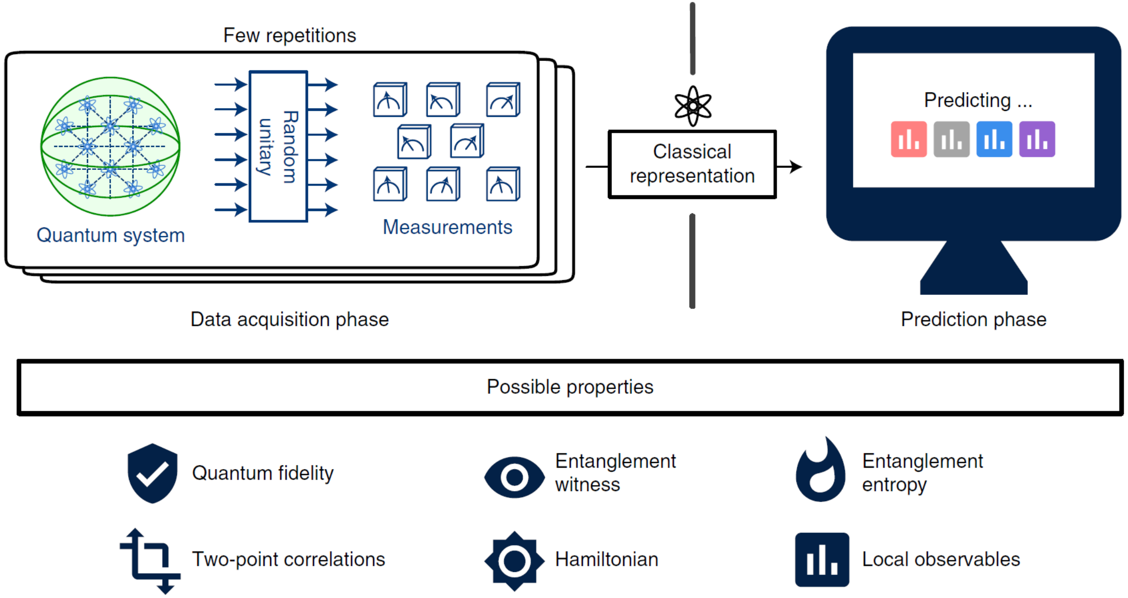
$= \mathrm{Poly}(H)$

$\rightsquigarrow$ broad usefulness for designing matrix functions:

- time evolution e^{-iHt}
- thermal Gibbs ensemble $\rho(\beta) = \frac{1}{Z(\beta)} e^{-\beta H}$
- spectral filtering
- $\ldots$

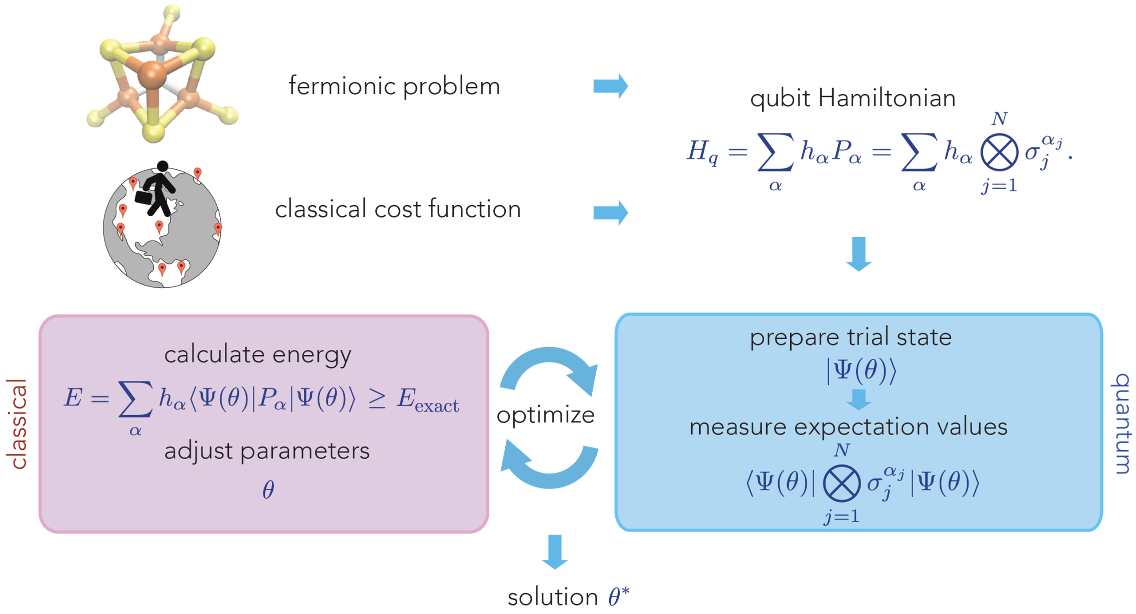
J. M. Martyn et al. "Grand unification of quantum algorithms". PRX Quantum, 040203 (2021)
A. Gilyén et al. "Quantum singular value transformation and beyond: exponential improvements for quantum matrix arithmetics". STOC 2019

Randomized measurements and classical shadows



H.-Y. Huang et al. (2020). "Predicting many properties of a quantum system from very few measurements". In: *Nat. Phys.* 16, pp. 1050–1057. DOI: 10.1038/s41567-020-0932-7;
H.-Y. Huang et al. (2021). "Provably efficient machine learning for quantum many-body problems". In: *arXiv:2106.12627*. URL: https://arxiv.org/abs/2106.12627

Hybrid classical-quantum computing



Source: Moll et al. (2018)

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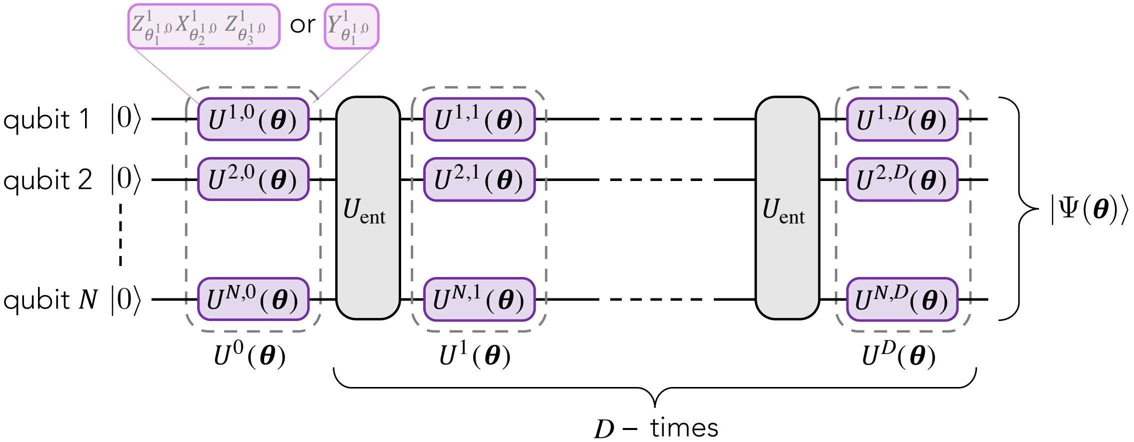

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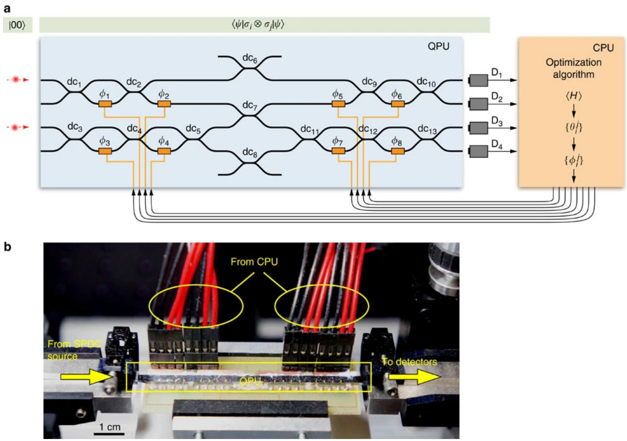
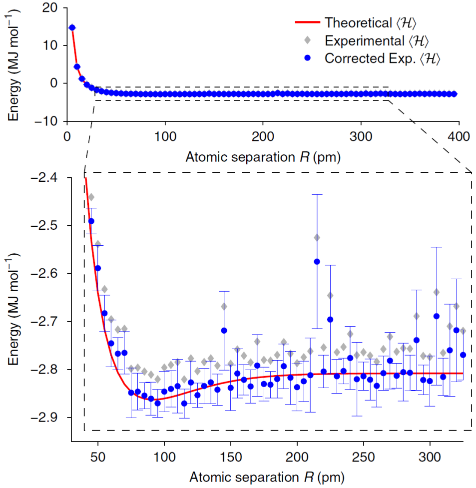
Variational quantum eigensolver (VQE)

Minimal model Hamiltonian (dimension 4×4), for bond dissociation of He-H$^+$

$$H(R) = \sum_{i\alpha} h^i_\alpha(R)\sigma^i_\alpha + \sum_{ij\alpha\beta} h^{ij}_{\alpha\beta}(R)\sigma^i_\alpha\sigma^j_\beta$$

expectation values $\langle\psi|\,\sigma^i_\alpha\sigma^j_\beta\,|\psi\rangle$





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J.-G. Liu et al. “Variational quantum eigensolver with fewer qubits”. Phys. Rev. Research 1, 023025 (2019)

Quantum optimization

- Quantum Approximate Optimization Algorithm (QAOA)
  Maximize

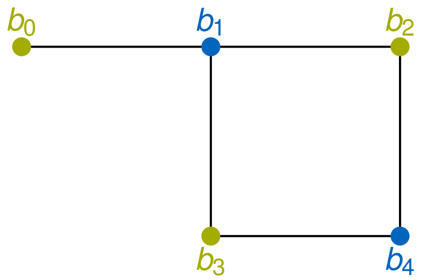
$$C(z) = \sum_{\alpha=1}^{m} C_\alpha(z), \quad z = z_1 z_2 \dots z_n$$

  Ansatz: maximize $\langle \gamma, \beta | C | \gamma, \beta \rangle$ w.r.t. angles γ, β in

$$|\gamma, \beta\rangle = U(B, \beta_p) U(C, \gamma_p) \cdots U(B, \beta_1) U(C, \gamma_1) |s\rangle, \quad U(C, \gamma) = e^{-i\gamma C}$$

  E. Farhi et al. (2014). "A quantum approximate optimization algorithm". In: *arXiv:1411.4028*. URL: https://arxiv.org/abs/1411.4028

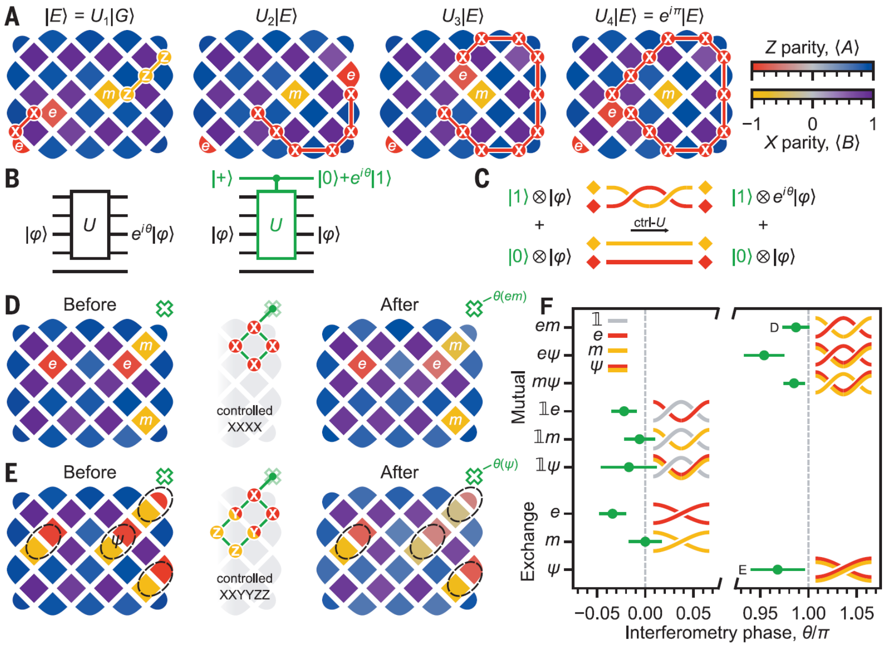
  Application to Max-Cut:



- Quantum annealing ($\rightsquigarrow$ in particular D-Wave)

  V. S. Denchev et al. (2016). "What is the computational value of finite-range tunneling?" In: *Phys. Rev. X* 6, p. 031015. DOI: 10.1103/PhysRevX.6.031015

Quantum error correction and topological computing



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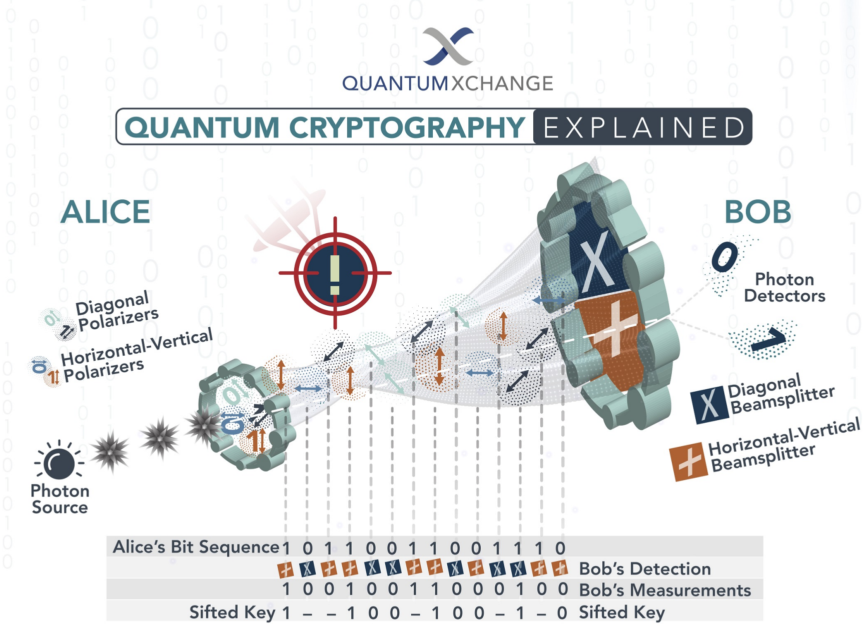
Quantum cryptography



Source: https://quantumxc.com/quantum-cryptography-explained/

N. Gisin et al. (2002). "Quantum cryptography". In: *Rev. Mod. Phys.* 74, pp. 145–195. DOI: 10.1103/RevModPhys.74.145

Practical next steps

- Moodle page: `https://www.moodle.tum.de/course/view.php?id=100667`
  We will upload the kick-off slides and mentioned publications there, too.
- Register at matching platform `https://matching.in.tum.de`

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- Give presentation

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References I

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- Gisin, N. et al. (2002). “Quantum cryptography”. In: *Rev. Mod. Phys.* 74, pp. 145–195. DOI: 10.1103/RevModPhys.74.145.
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References II

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References III

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