

Surplus production model

Pella & Tomlinson (1969):

$$\frac{dB_t}{dt} = \frac{r}{n-1} B_t \left(1 - \left[\frac{B_t}{K} \right]^{n-1} \right) - F_t B_t,$$

Parameters:

- B_t : Exploitable stock biomass.
- F_t : Fishing mortality.
- r : Intrinsic growth rate.
- K : Carrying capacity.
- n : Parameter determining the shape of the production curve.

M.H. Prager / Fisheries Research 58 (2002) 41–57

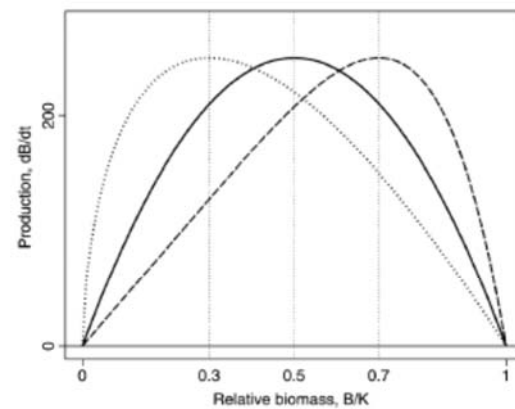
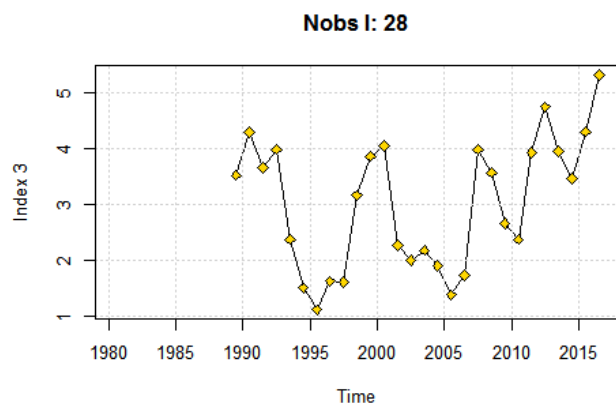
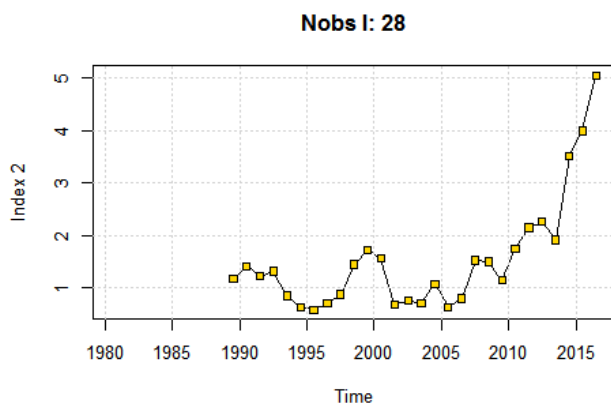
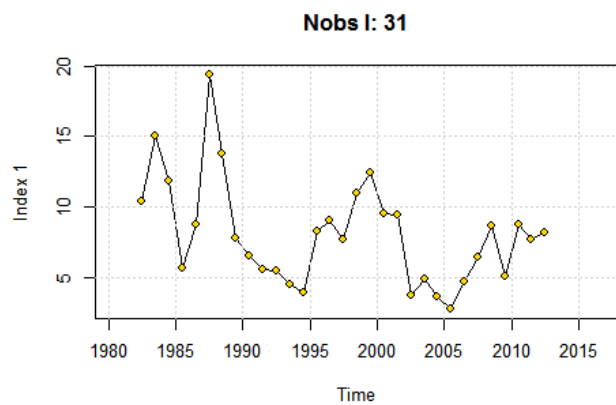
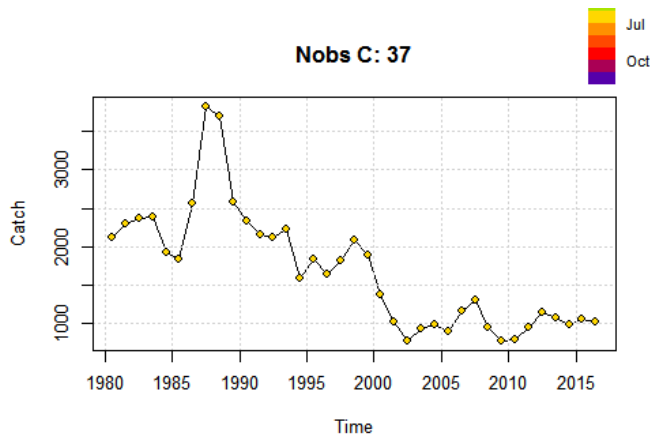
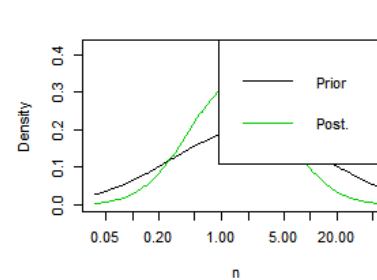
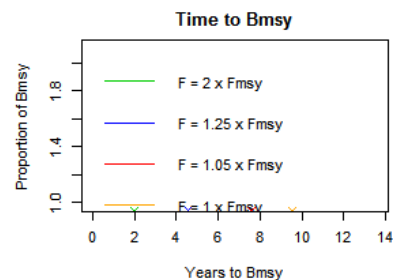
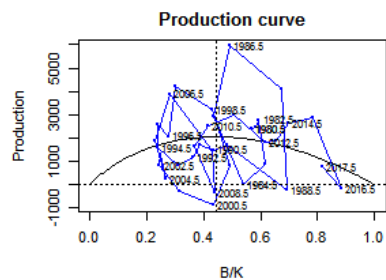
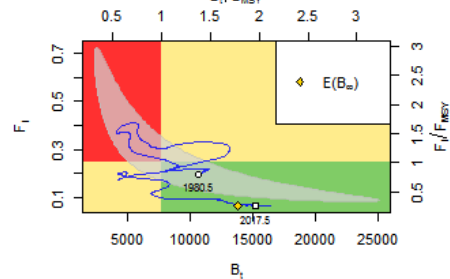
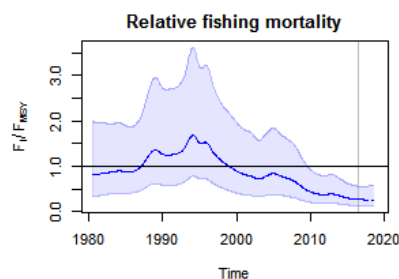
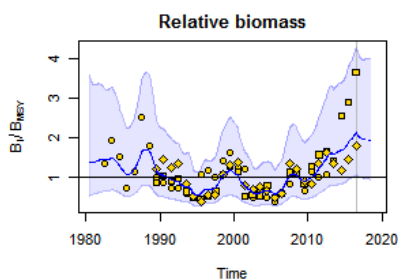
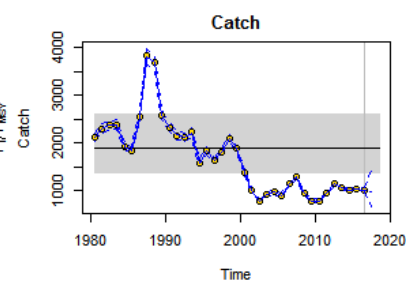
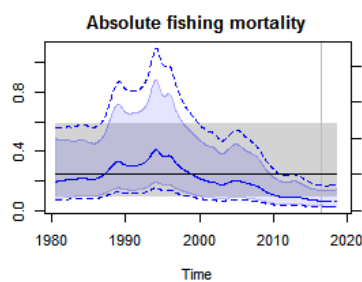
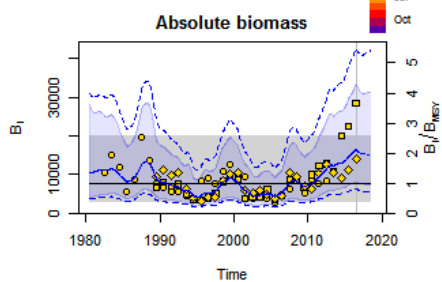
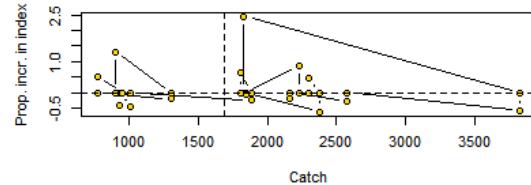
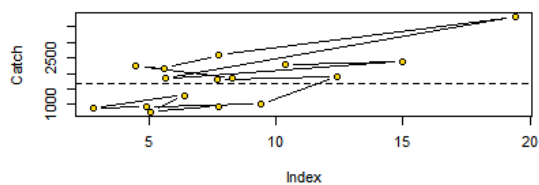
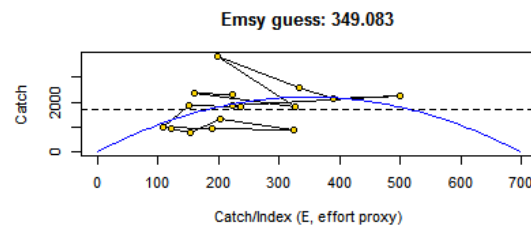
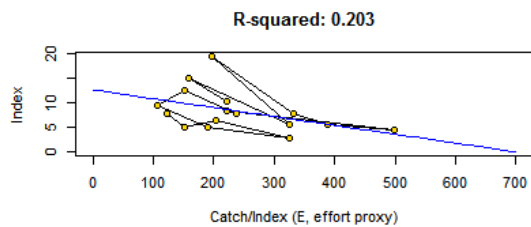
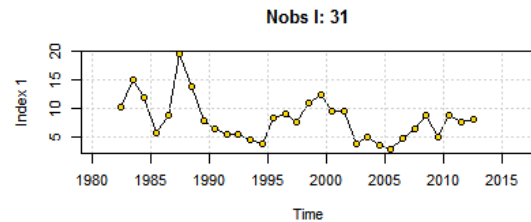
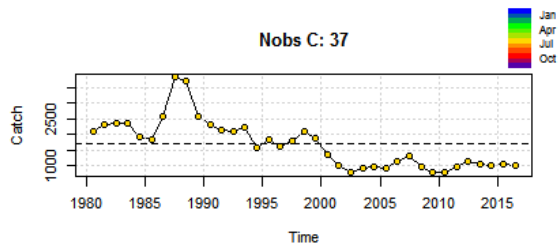
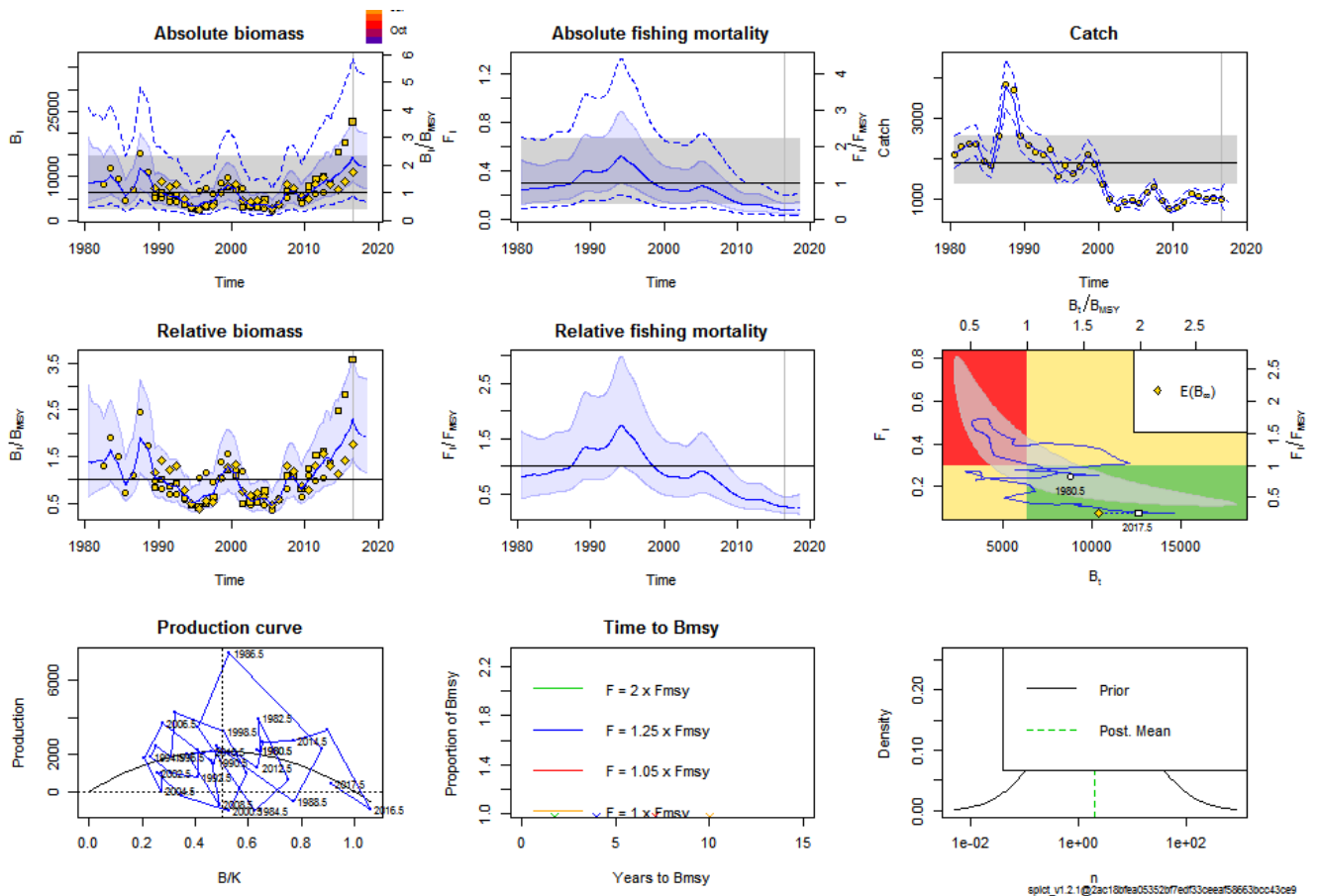
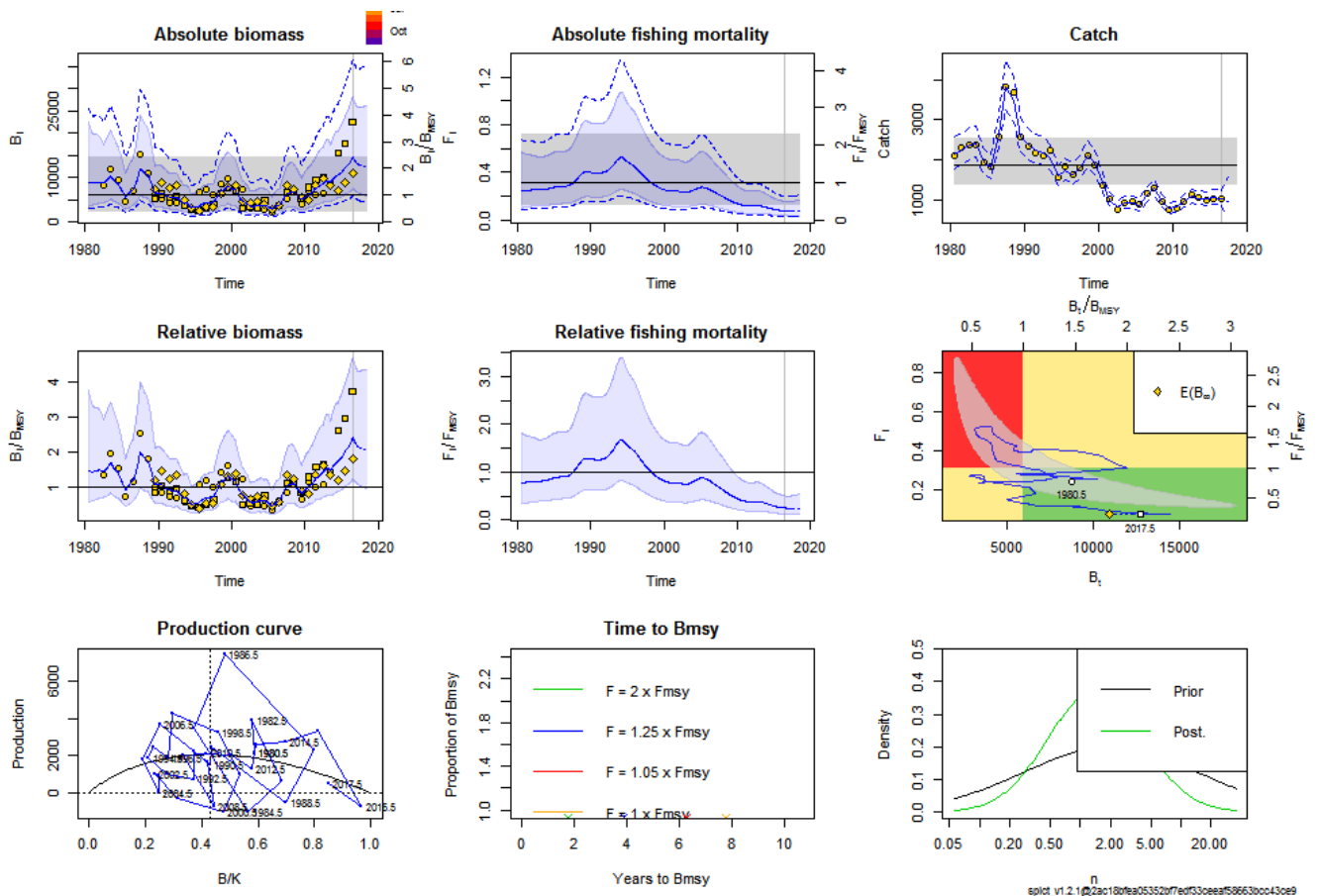
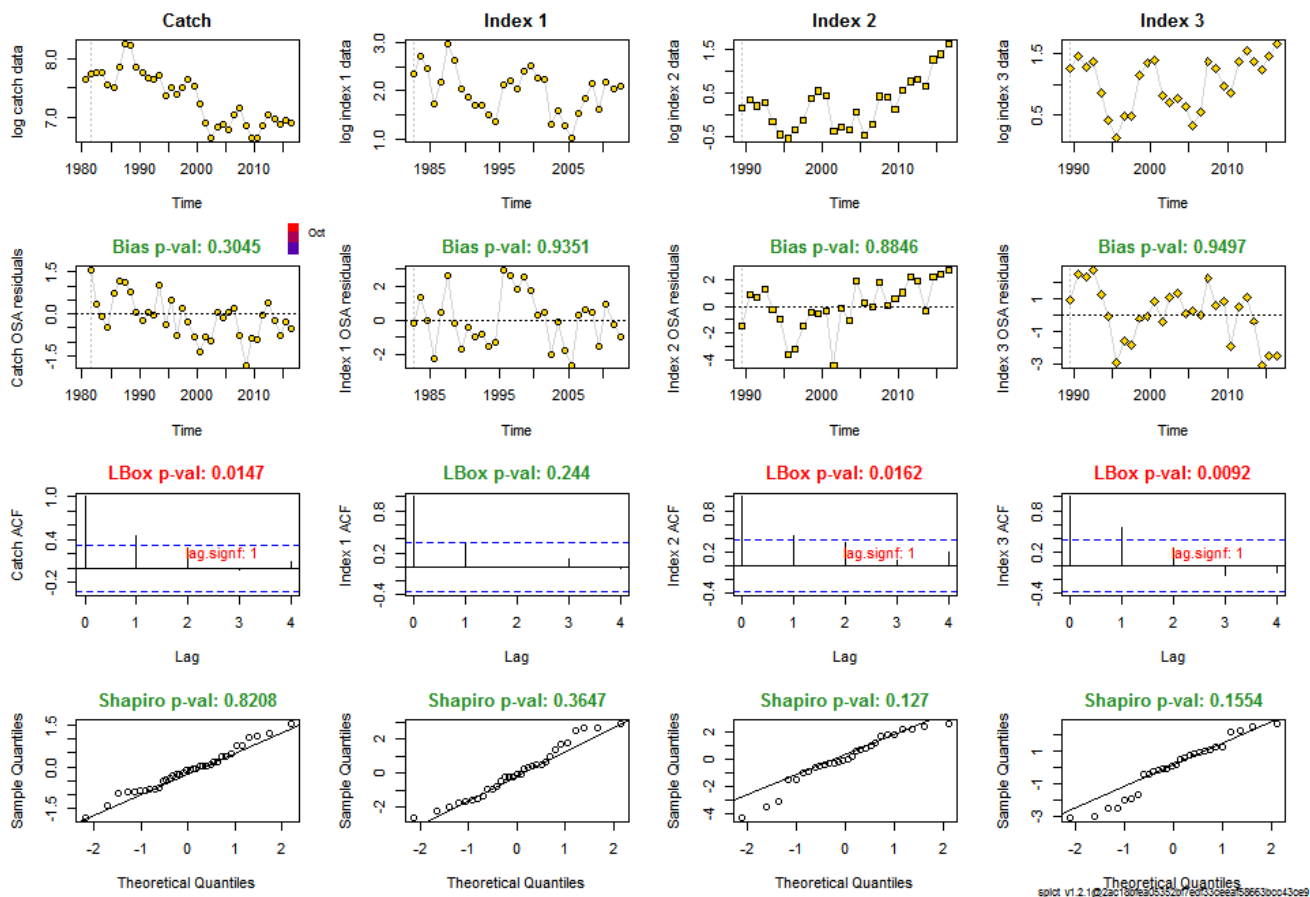
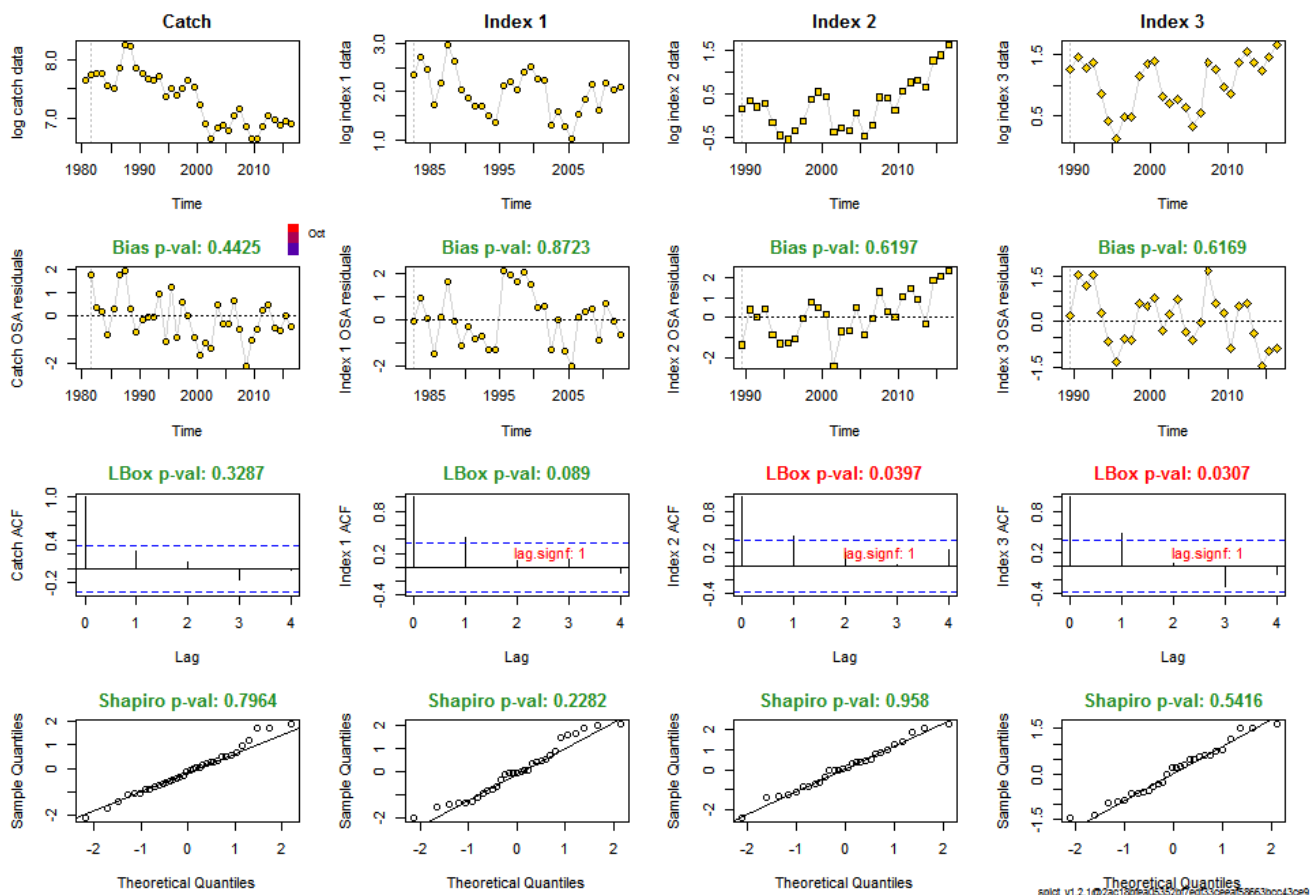


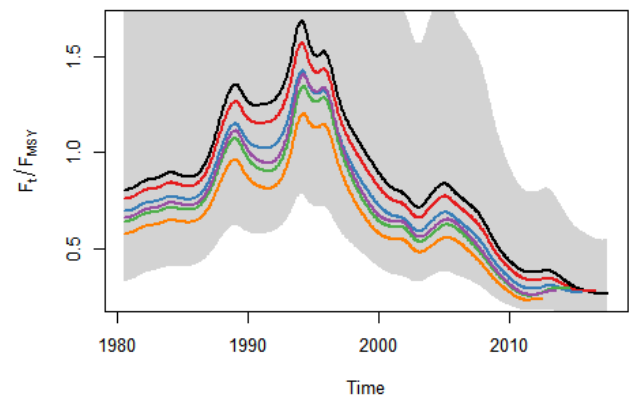
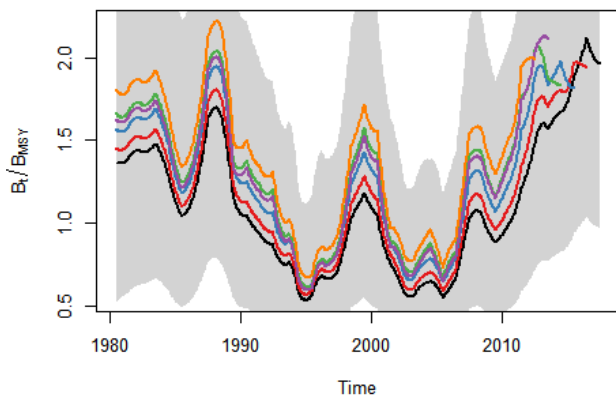
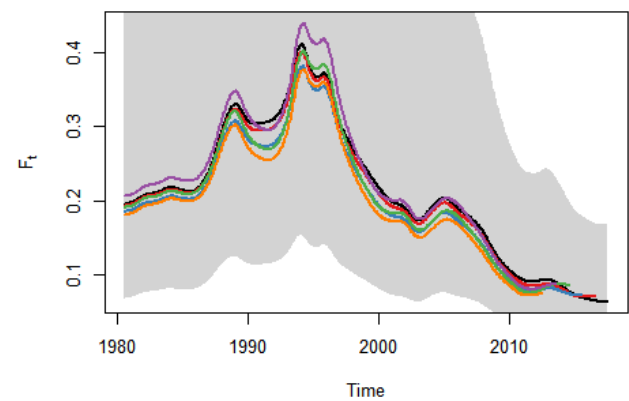
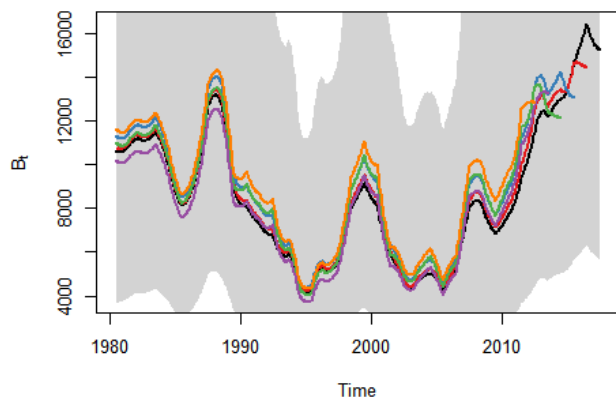
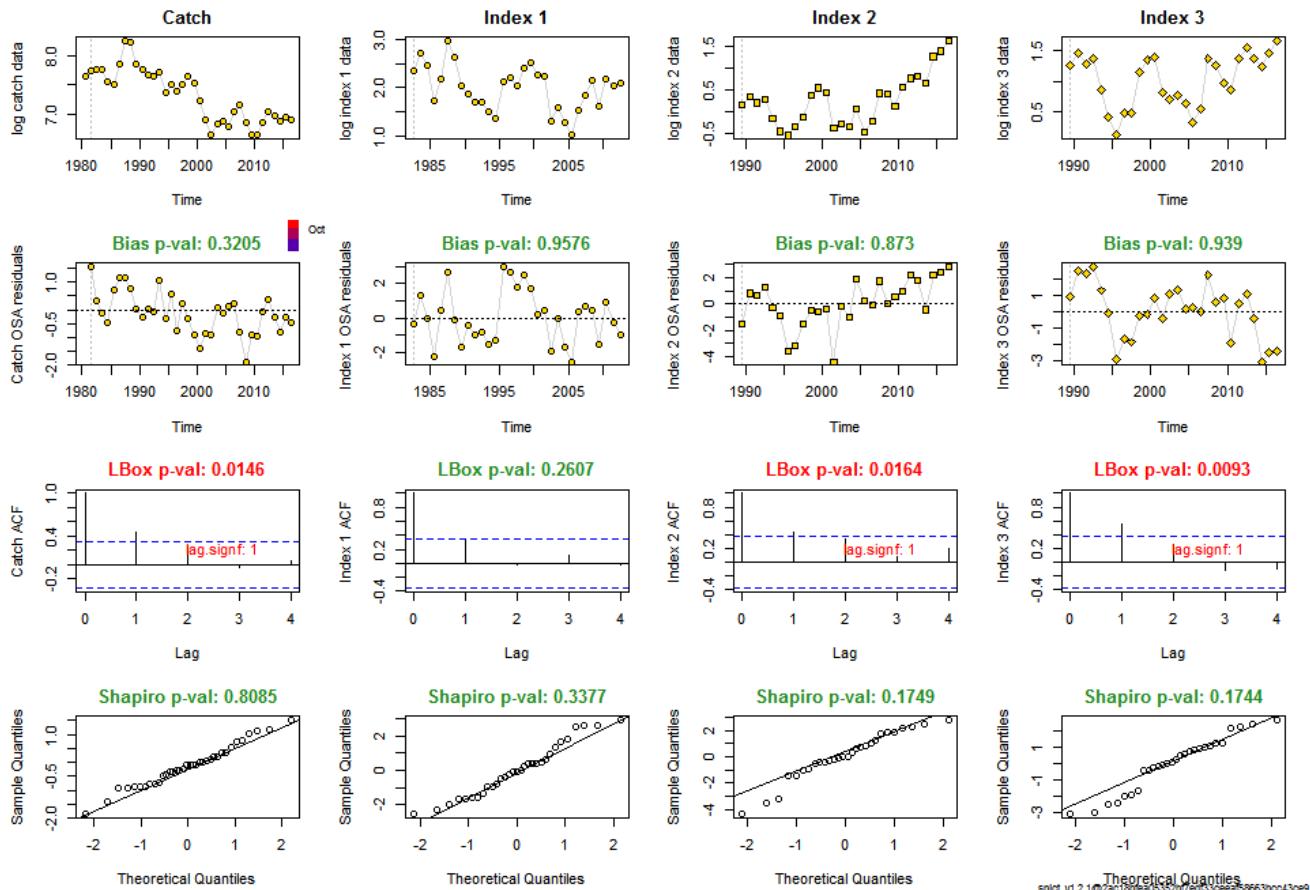
Figure: $n=0.68, 2, \text{ and } 6.04$.





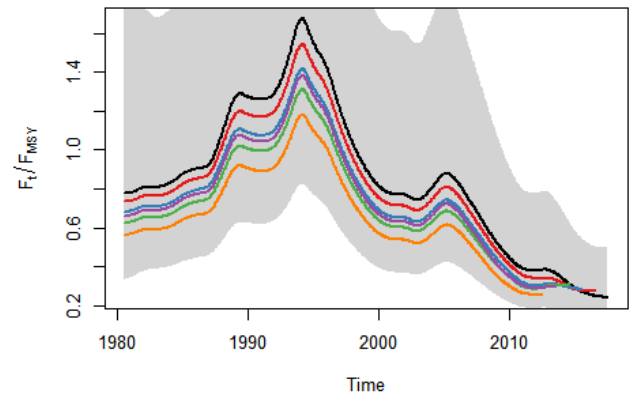
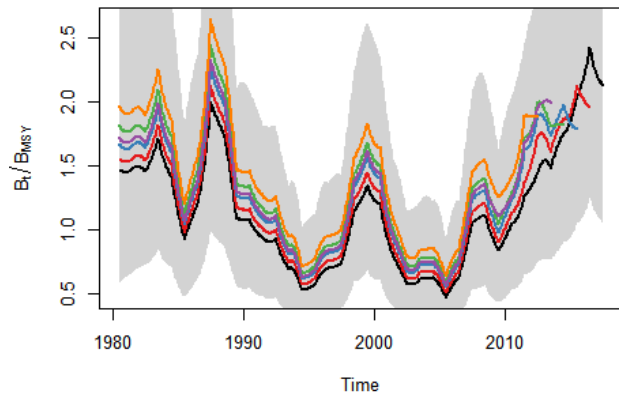
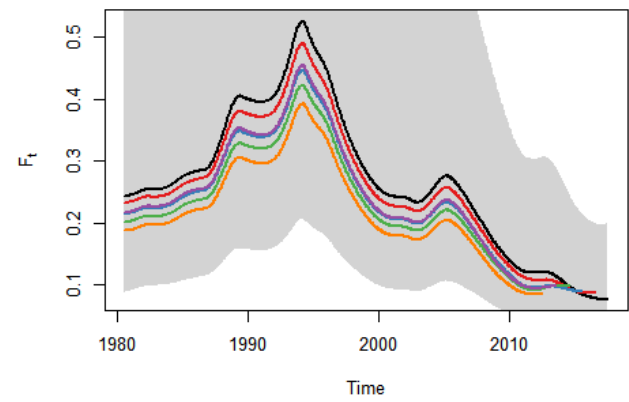
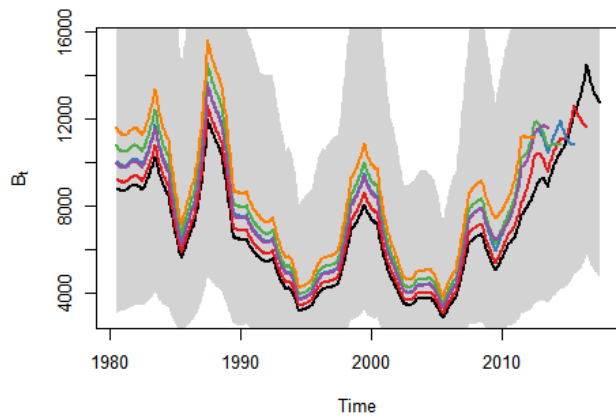




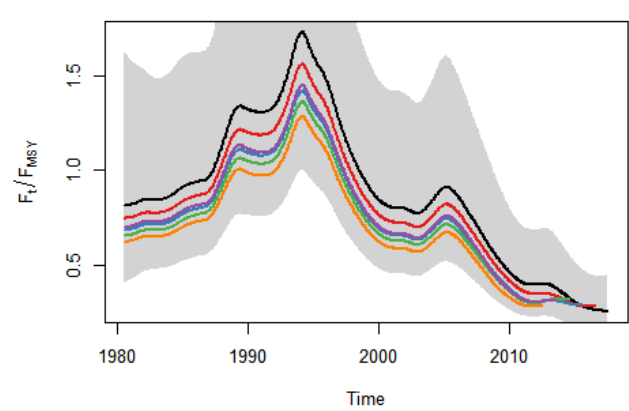
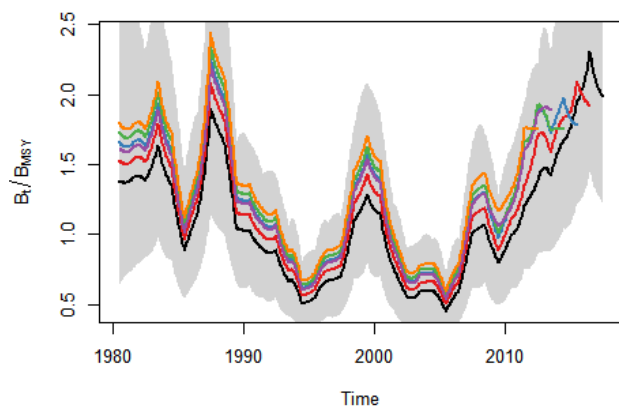
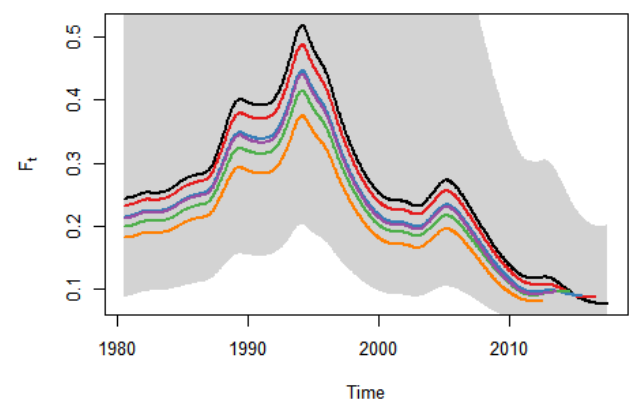
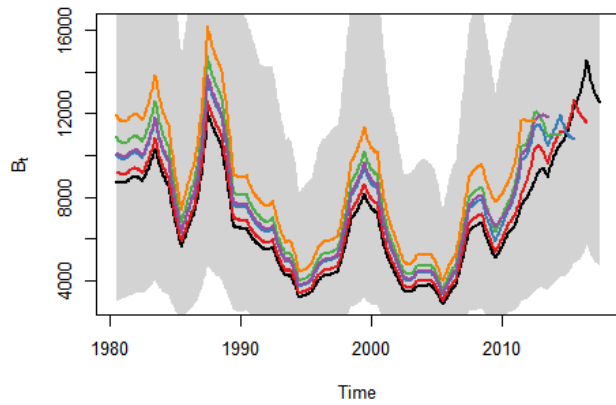


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[1] "Convergence: 0 MSG: relative convergence (4)"
[2] "Objective function at optimum: 33.0054275"
[3] "Euler time step (years): 1/16 or 0.0625"
[4] "Nobs C: 37, Nobs I1: 31, Nobs I2: 28, Nobs I3: 28"
[5] ""
[6] "Residual diagnostics (p-values)"
[7] "      shapiro bias acf LBox shapiro bias acf LBox  "
[8] " C    0.7964 0.4425 0.1342 0.3287      -      -      -      -  "
[9] " I1   0.2282 0.8723 0.0174 0.0890      -      -      *      .  "
[10] " I2   0.9580 0.6197 0.0184 0.0397      -      -      *      *  "
[11] " I3   0.5416 0.6169 0.0133 0.0307      -      -      *      *  "
[12] ""
[13] "Priors"
[14] "      logn ~ dnorm[log(2), 2^2]"
[15] "      logalpha ~ dnorm[log(1), 2^2]"
[16] "      logbeta ~ dnorm[log(1), 2^2]"
[17] ""
[18] "Model parameter estimates w 95% CI "
[19] "      estimate      ci low      ci upp      log.est  "
[20] " alpha1 1.434218e+00 0.9229070 2.228807e+00 0.3606199  "
[21] " alpha2 1.159846e+00 0.6922013 1.943425e+00 0.1482868  "
[22] " alpha3 1.094407e+00 0.6472023 1.850623e+00 0.0902131  "
[23] " beta    1.413639e-01 0.0233813 8.546875e-01 -1.9564181  "
[24] " r        3.701894e-01 0.0288107 4.756570e+00 -0.9937406  "
[25] " rc       4.984816e-01 0.2144635 1.158630e+00 -0.6961886  "
[26] " rold     7.628556e-01 0.0049739 1.169997e+02 -0.2706866  "
[27] " m        2.055839e+03 1367.4663233 3.090735e+03 7.6284396  "
[28] " K        1.863851e+04 5455.6634291 6.367586e+04 9.8329852  "
[29] " q1       9.983000e-04 0.0003865 2.578200e-03 -6.9094590  "
[30] " q2       1.768000e-04 0.0000684 4.568000e-04 -8.6404828  "
[31] " q3       3.803000e-04 0.0001473 9.821000e-04 -7.8745130  "
[32] " n        1.485268e+00 0.1336051 1.651151e+01 0.3955952  "
[33] " sdb      2.184630e-01 0.1587367 3.006621e-01 -1.5211384  "
[34] " sdf      1.529440e-01 0.1049178 2.229540e-01 -1.8776838  "
[35] " sdi1     3.133237e-01 0.2320627 4.230396e-01 -1.1605185  "
[36] " sdi2     2.533834e-01 0.1709836 3.754929e-01 -1.3728516  "
[37] " sdi3     2.390876e-01 0.1642025 3.481242e-01 -1.4309253  "
[38] " sdc      2.162070e-02 0.0036892 1.267078e-01 -3.8341019  "
[39] ""
[40] "Deterministic reference points (Drp)"
[41] "      estimate      ci low      ci upp      log.est  "
[42] " Bmsyd 8248.4070505 3135.6545457 2.169761e+04 9.017775  "
[43] " Fmsyd 0.2492408 0.1072318 5.793151e-01 -1.389336  "
[44] " MSYd 2055.8394873 1367.4663233 3.090735e+03 7.628440  "
[45] "Stochastic reference points (Srp)"
[46] "      estimate      ci low      ci upp      log.est rel.diff.Drp  "
[47] " Bmsys 7766.1682414 3038.3347196 1.985080e+04 8.957532 -0.06209482  "
[48] " Fmsys 0.2435681 0.1016636 5.835464e-01 -1.412359 -0.02328995  "
[49] " MSYs 1888.8556394 1374.8907022 2.594952e+03 7.543726 -0.08840477  "
[50] ""
[51] "States w 95% CI (inp$msytype: s)"
[52] "      estimate      ci low      ci upp      log.est  "
[53] " B_2016.50 1.645024e+04 6355.2890188 4.258034e+04 9.7080953  "
[54] " F_2016.50 6.548060e-02 0.0252171 1.700322e-01 -2.7260007  "
[55] " B_2016.50/Bmsy 2.118193e+00 1.0433480 4.300329e+00 0.7505631  "
[56] " F_2016.50/Fmsy 2.688391e-01 0.1301010 5.555260e-01 -1.3136422  "
[57] ""
[58] "Predictions w 95% CI (inp$msytype: s)"
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[61] " F_2017.50 6.382420e-02 0.0240076 1.696765e-01 -2.7516223  "
[62] " B_2017.50/Bmsy 1.963484e+00 0.9678208 3.983453e+00 0.6747204  "
[63] " F_2017.50/Fmsy 2.620385e-01 0.1240519 5.535117e-01 -1.3392638  "
[64] " Catch_2017.50 9.662766e+02 649.6201307 1.437287e+03 6.8734501  "
[65] " E(B_inf) 1.383856e+04 NA NA 9.5352145  "

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[2] "Objective function at optimum: 70.2513443"
[3] "Euler time step (years): 1/16 or 0.0625"
[4] "Nobs C: 37, Nobs I1: 31, Nobs I2: 28, Nobs I3: 28"
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[6] "Residual diagnostics (p-values)"
[7] "      Shapiro bias      acf      LBox      Shapiro bias      acf      LBox      "
[8] " C      0.8208 0.3045 0.0062 0.0147      -      -      **      *      "
[9] " I1     0.3647 0.9351 0.0517 0.2440      -      -      .      -      "
[10] " I2     0.1270 0.8846 0.0181 0.0162      -      -      *      *      "
[11] " I3     0.1554 0.9497 0.0029 0.0092      -      -      **      **      "
[12] ""
[13] "Priors"
[14] "      logn ~ dnorm[log(2), 2^2]"
[15] "      logalpha ~ dnorm[log(1), 2^2]"
[16] "      logbeta ~ dnorm[log(1), 2^2]"
[17] ""
[18] "Fixed parameters"
[19] "      fixed. value      "
[20] "      sdc      0.10      "
[21] "      sdi      0.15      "
[22] ""
[23] "Model parameter estimates w 95% CI "
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[26] " r          4.492040e-01      0.0477647 4.224544e+00 -0.8002781      "
[27] " rc         6.421626e-01      0.2908193 1.417969e+00 -0.4429137      "
[28] " rold       1.125726e+00      0.0053420 2.372235e+02 0.1184279      "
[29] " m          2.075938e+03 1394.6308149 3.090078e+03 7.6381683      "
[30] " K          1.499866e+04 4902.4522158 4.588719e+04 9.6157161      "
[31] " q1         1.267900e-03      0.0005090 3.158400e-03 -6.6704022      "
[32] " q2         2.255000e-04      0.0000904 5.626000e-04 -8.3972577      "
[33] " q3         4.850000e-04      0.0001944 1.210200e-03 -7.6312879      "
[34] " n          1.399035e+00      0.1700799 1.150811e+01 0.3357827      "
[35] " sdb        2.686618e-01      0.2015253 3.581642e-01 -1.3143021      "
[36] " sdf        1.643215e-01      0.1124538 2.401125e-01 -1.8059301      "
[37] ""
[38] "Deterministic reference points (Drp)"
[39] "      estimate      ci low      ci upp      log. est      "
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[41] " Fmsyd 0.3210813 0.1454096 7.089847e-01 -1.136061      "
[42] " MSYd 2075.9379129 1394.6308149 3.090078e+03 7.638168      "
[43] "Stochastic reference points (Srp)"
[44] "      estimate      ci low      ci upp      log. est      rel. diff. Drp      "
[45] " Bmsys 5999.5777618 2457.3526236 1.464785e+04 8.699444 -0.07765226      "
[46] " Fmsys 0.3141442 0.1378593 7.158497e-01 -1.157903 -0.02208257      "
[47] " MSYs 1881.5004349 1386.9836363 2.552333e+03 7.539825 -0.10334171      "
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[49] "States w 95% CI (inp$msytype: s)"
[50] "      estimate      ci low      ci upp      log. est      "
[51] " B_2016.50 1.451930e+04 5776.6763514 3.649334e+04 9.583234      "
[52] " F_2016.50 7.801300e-02 0.0304804 1.996702e-01 -2.550880      "
[53] " B_2016.50/Bmsy 2.420054e+00 1.2503472 4.684029e+00 0.883790      "
[54] " F_2016.50/Fmsy 2.483349e-01 0.1227927 5.022304e-01 -1.392977      "
[55] ""
[56] "Predictions w 95% CI (inp$msytype: s)"
[57] "      prediction      ci low      ci upp      log. est      "
[58] " B_2017.50 1.276680e+04 4725.380377 3.449273e+04 9.4546036      "
[59] " F_2017.50 7.599380e-02 0.028806 2.004808e-01 -2.5771034      "
[60] " B_2017.50/Bmsy 2.127950e+00 1.055429 4.290362e+00 0.7551592      "
[61] " F_2017.50/Fmsy 2.419074e-01 0.115692 5.058188e-01 -1.4192001      "
[62] " Catch_2017.50 9.534497e+02 577.654106 1.573721e+03 6.8600867      "
[63] " E(B_inf) 1.099121e+04 NA NA 9.3048507      "

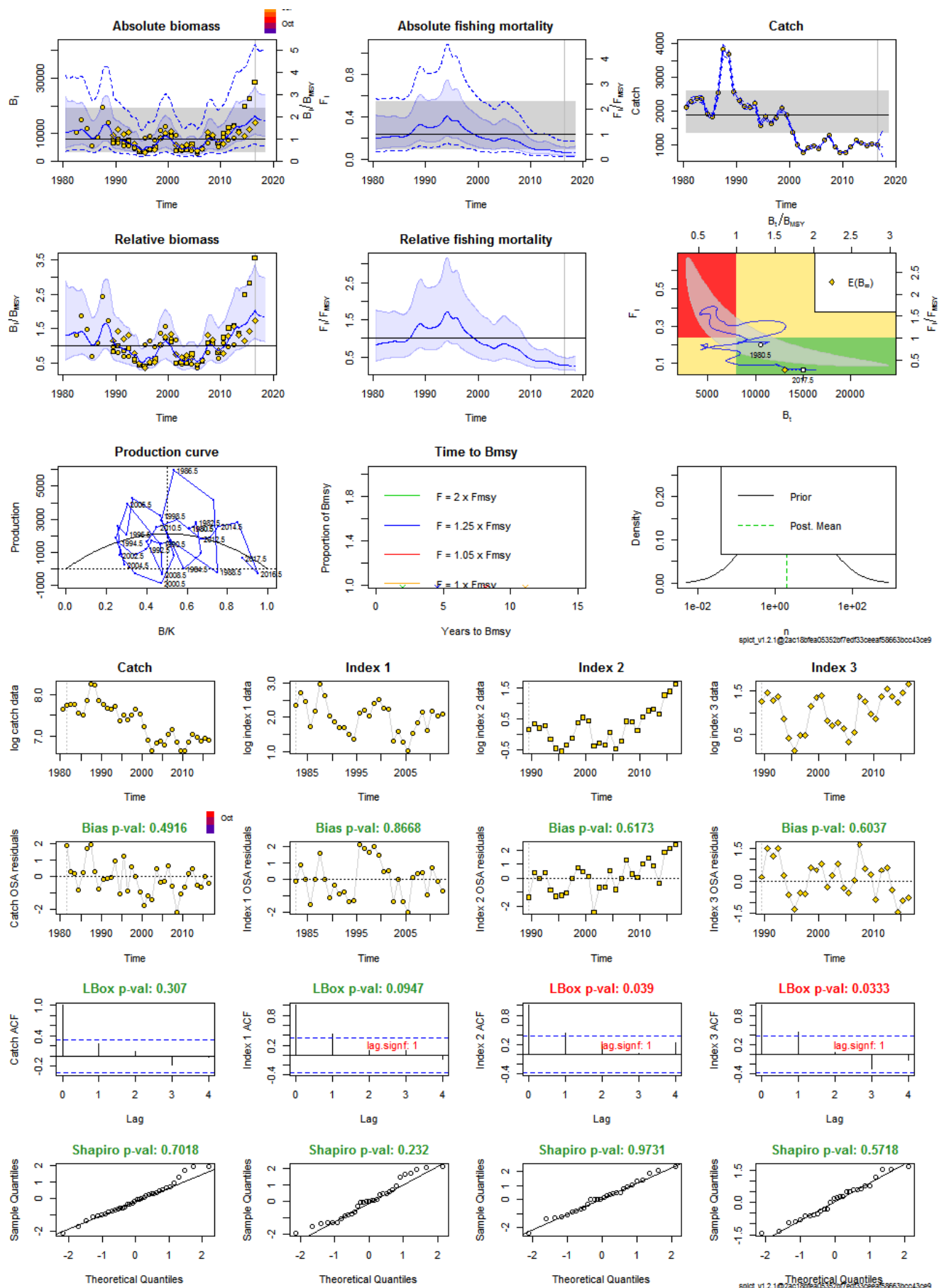
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[1] "Convergence: 0 MSG: relative convergence (4)"
[2] "Objective function at optimum: 70.3212418"
[3] "Euler time step (years): 1/16 or 0.0625"
[4] "Nobs C: 37, Nobs I1: 31, Nobs I2: 28, Nobs I3: 28"
[5] ""
[6] "Residual diagnostics (p-values)"
[7] "      shapiro bias      acf      LBox      shapiro bias      acf      LBox  "
[8] " C      0.8085 0.3205 0.0055 0.0146      -      -      **      *  "
[9] " I1     0.3377 0.9576 0.0568 0.2607      -      -      .      -  "
[10] " I2     0.1749 0.8730 0.0182 0.0164      -      -      *      *  "
[11] " I3     0.1744 0.9390 0.0029 0.0093      -      -      **     **  "
[12] ""
[13] "Priors"
[14] "      logn ~ dnorm[log(2), 2^2]"
[15] "      logalpha ~ dnorm[log(1), 2^2]"
[16] "      logbeta ~ dnorm[log(1), 2^2]"
[17] ""
[18] "Fixed parameters"
[19] "      fixed. value  "
[20] " n                  2.00  "
[21] " sdc                 0.10  "
[22] " sdi                 0.15  "
[23] ""
[24] "Model parameter estimates w 95% CI  "
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[26] " beta      6.119885e-01      0.4192234 8.933897e-01 -0.4910419  "
[27] " r          6.343565e-01      0.2961862 1.358633e+00 -0.4551442  "
[28] " rc         6.343565e-01      0.2961862 1.358633e+00 -0.4551442  "
[29] " rold       6.343565e-01      0.2961862 1.358633e+00 -0.4551442  "
[30] " m          2.183675e+03 1592.5646712 2.994186e+03 7.6887644  "
[31] " K          1.376938e+04 5835.5229208 3.248997e+04 9.5302029  "
[32] " q1         1.252800e-03      0.0004991 3.144900e-03 -6.6823731  "
[33] " q2         2.229000e-04      0.0000887 5.601000e-04 -8.4089953  "
[34] " q3         4.794000e-04      0.0001908 1.204700e-03 -7.6430255  "
[35] " sdb        2.682200e-01      0.2015028 3.570271e-01 -1.3159477  "
[36] " sdf        1.634018e-01      0.1119332 2.385363e-01 -1.8115432  "
[37] " "
[38] "Deterministic reference points (Drp)"
[39] "      estimate      ci low      ci upp      log. est  "
[40] " Bmsyd 6884.6922627 2917.7614604 1.624498e+04 8.837056  "
[41] " Fmsyd 0.3171783 0.1480931 6.793163e-01 -1.148291  "
[42] " MSYd 2183.6746679 1592.5646712 2.994186e+03 7.688764  "
[43] "Stochastic reference points (Srp)"
[44] "      estimate      ci low      ci upp      log. est rel. diff. Drp  "
[45] " Bmsys 6333.2662450 2736.6778697 1.465655e+04 8.753571 -0.08706819  "
[46] " Fmsys 0.2998317 0.1353664 6.641162e-01 -1.204534 -0.05785430  "
[47] " MSYs 1889.3485702 1386.0020179 2.575493e+03 7.543987 -0.15578179  "
[48] ""
[49] "States w 95% CI (inp$msytype: s)"
[50] "      estimate      ci low      ci upp      log. est  "
[51] " B_2016.50 1.459331e+04 5756.8030854 3.699358e+04 9.5883186  "
[52] " F_2016.50 7.804630e-02 0.0302511 2.013555e-01 -2.5504526  "
[53] " B_2016.50/Bmsy 2.304232e+00 1.4561676 3.646203e+00 0.8347472  "
[54] " F_2016.50/Fmsy 2.603005e-01 0.1535807 4.411776e-01 -1.3459186  "
[55] ""
[56] "Predictions w 95% CI (inp$msytype: s)"
[57] "      prediction      ci low      ci upp      log. est  "
[58] " B_2017.50 1.257741e+04 4673.8229984 3.384620e+04 9.439657  "
[59] " F_2017.50 7.628410e-02 0.0287226 2.026023e-01 -2.573291  "
[60] " B_2017.50/Bmsy 1.985927e+00 1.2296306 3.207392e+00 0.686086  "
[61] " F_2017.50/Fmsy 2.544231e-01 0.1441383 4.490905e-01 -1.368756  "
[62] " Catch_2017.50 9.378434e+02 586.6805172 1.499198e+03 6.843583  "
[63] " E(B_inf) 1.036583e+04 NA NA 9.246270  "

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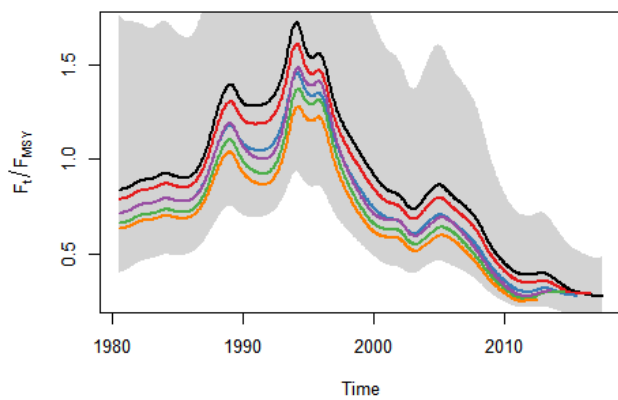
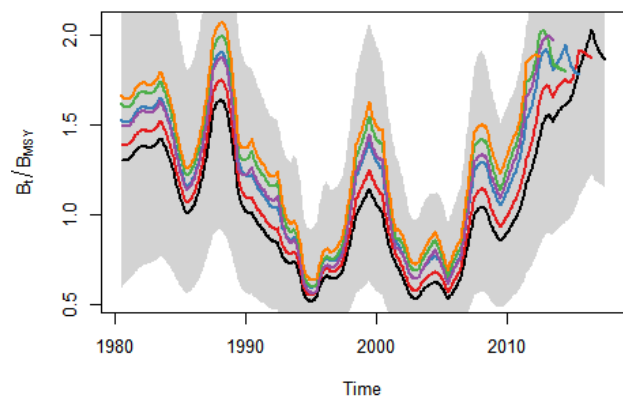
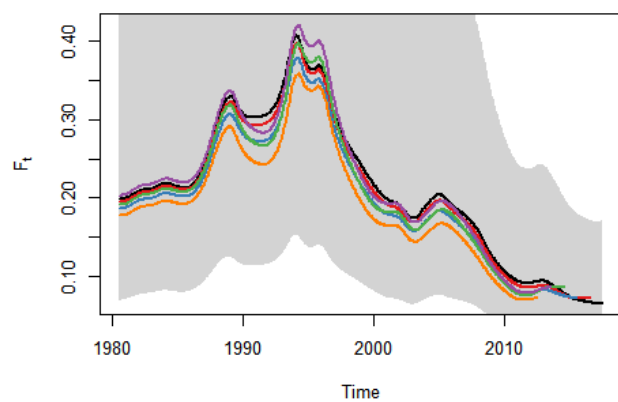
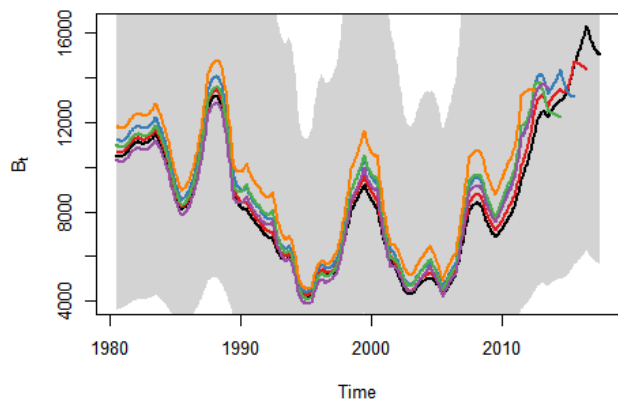


- [1] "Convergence: 0 MSG: relative convergence (4)"
- [2] "Objective function at optimum: 33.0394945"
- [3] "Euler time step (years): 1/16 or 0.0625"

```

[4] "Nobs C: 37, Nobs I1: 31, Nobs I2: 28, Nobs I3: 28"
[5] ""
[6] "Residual diagnostics (p-values)"
[7] "      shapiro bias      acf      LBox shapiro bias      acf      LBox  "
[8] " C      0.7018 0.4916 0.1291 0.3070      -      -      -      -  "
[9] " I1      0.2320 0.8668 0.0184 0.0947      -      -      *      .  "
[10] " I2      0.9731 0.6173 0.0184 0.0390      -      -      *      *  "
[11] " I3      0.5718 0.6037 0.0142 0.0333      -      -      *      *  "
[12] ""
[13] "Priors"
[14] "      logn ~ dnorm(log(2), 2^2)"
[15] "      logalpha ~ dnorm(log(1), 2^2)"
[16] "      logbeta ~ dnorm(log(1), 2^2)"
[17] ""
[18] "Fixed parameters"
[19] "      fixed.value  "
[20] "      n            2  "
[21] ""
[22] "Model parameter estimates w 95% CI  "
[23] "      estimate      ci low      ci upp      log.est  "
[24] " alpha1 1.433838e+00      0.9230506 2.227278e+00 0.3603545  "
[25] " alpha2 1.169689e+00      0.7037572 1.944097e+00 0.1567379  "
[26] " alpha3 1.093044e+00      0.6466311 1.847646e+00 0.0889667  "
[27] " beta    1.419067e-01      0.0234410 8.590730e-01 -1.9525853  "
[28] " r        4.964333e-01      0.2216886 1.111677e+00 -0.7003061  "
[29] " rc       4.964333e-01      0.2216886 1.111677e+00 -0.7003061  "
[30] " rold    4.964333e-01      0.2216886 1.111677e+00 -0.7003061  "
[31] " m        2.130578e+03 1541.9229100 2.943962e+03 7.6641485  "
[32] " K        1.716708e+04 7032.6212229 4.190595e+04 9.7507489  "
[33] " q1       9.930000e-04      0.0003825 2.577900e-03 -6.9148088  "
[34] " q2       1.760000e-04      0.0000678 4.567000e-04 -8.6449466  "
[35] " q3       3.786000e-04      0.0001460 9.819000e-04 -7.8789768  "
[36] " sdb      2.181234e-01      0.1586220 2.999447e-01 -1.5226941  "
[37] " sdf      1.523356e-01      0.1046241 2.218050e-01 -1.8816690  "
[38] " sdi1     3.127536e-01      0.2317206 4.221239e-01 -1.1623396  "
[39] " sdi2     2.551366e-01      0.1736210 3.749239e-01 -1.3659563  "
[40] " sdi3     2.384186e-01      0.1639869 3.466339e-01 -1.4337275  "
[41] " sdc      2.161750e-02      0.0036843 1.268397e-01 -3.8342543  "
[42] ""
[43] "Deterministic reference points (Drp)"
[44] "      estimate      ci low      ci upp      log.est  "
[45] " Bmsyd 8583.5404397 3516.3106114 2.095297e+04 9.057602  "
[46] " Fmsyd 0.2482167 0.1108443 5.558384e-01 -1.393453  "
[47] " MSYd 2130.5778307 1541.9229100 2.943962e+03 7.664148  "
[48] "Stochastic reference points (Srp)"
[49] "      estimate      ci low      ci upp      log.est rel.diff.Drp  "
[50] " Bmsys 8047.398927 3362.856162 1.925763e+04 8.993104 -0.06662296  "
[51] " Fmsys 0.236561 0.102698 5.449096e-01 -1.441549 -0.04927131  "
[52] " MSYs 1897.451799 1378.430203 2.611901e+03 7.548267 -0.12286269  "
[53] ""
[54] "States w 95% CI (inp$msytype: s)"
[55] "      estimate      ci low      ci upp      log.est  "
[56] " B_2016.50 1.634228e+04 6311.4574427 4.231514e+04 9.7015111  "
[57] " F_2016.50 6.604870e-02 0.0254654 1.713078e-01 -2.7173632  "
[58] " B_2016.50/Bmsy 2.030753e+00 1.2287687 3.356172e+00 0.7084069  "
[59] " F_2016.50/Fmsy 2.792036e-01 0.1607968 4.848023e-01 -1.2758141  "
[60] ""
[61] "Predictions w 95% CI (inp$msytype: s)"
[62] "      prediction      ci low      ci upp      log.est  "
[63] " B_2017.50 1.501989e+04 5646.2460582 3.995522e+04 9.6171303  "
[64] " F_2017.50 6.454610e-02 0.0243642 1.709971e-01 -2.7403753  "
[65] " B_2017.50/Bmsy 1.866427e+00 1.1532734 3.020577e+00 0.6240261  "
[66] " F_2017.50/Fmsy 2.728519e-01 0.1532719 4.857259e-01 -1.2988262  "
[67] " Catch_2017.50 9.590054e+02 654.8407203 1.404451e+03 6.8658967  "
[68] " E(B_inf) 1.309869e+04 NA NA 9.4802678  "

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