

# **Lab2: 3Ps cod length-at-age 2019 course**

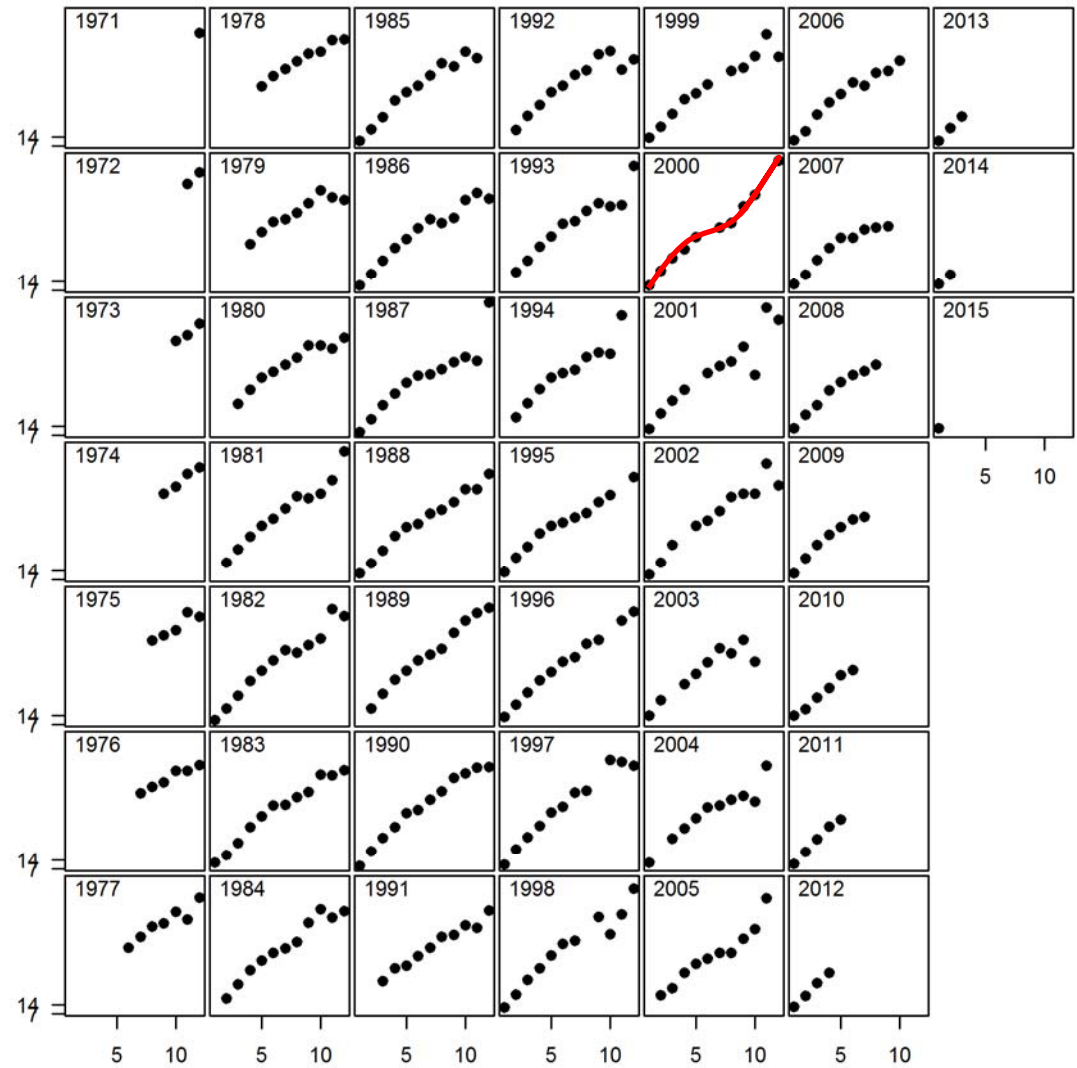
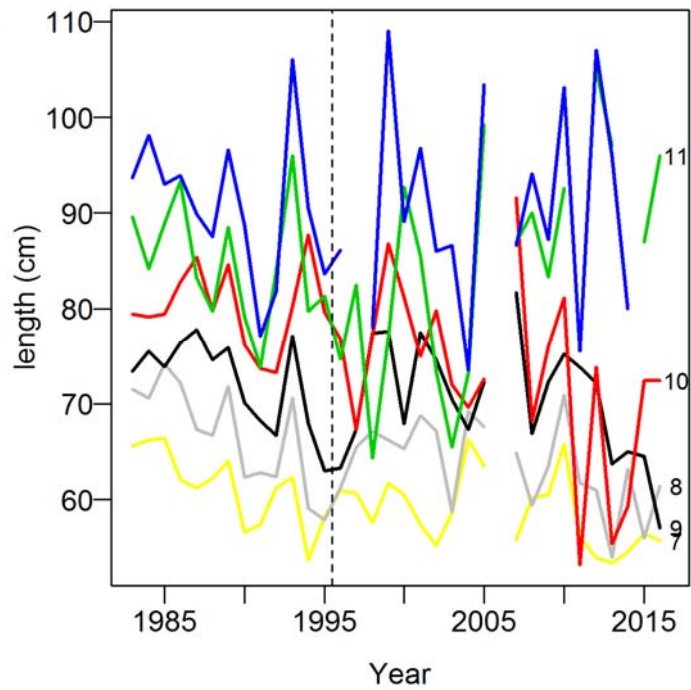
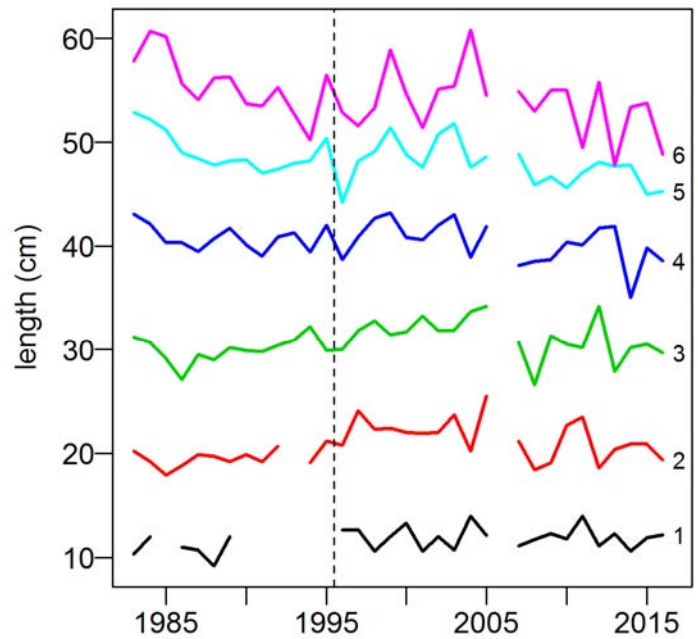
Noel Cadigan

## **CFER**

Centre for Fisheries Ecosystems Research

# The Data

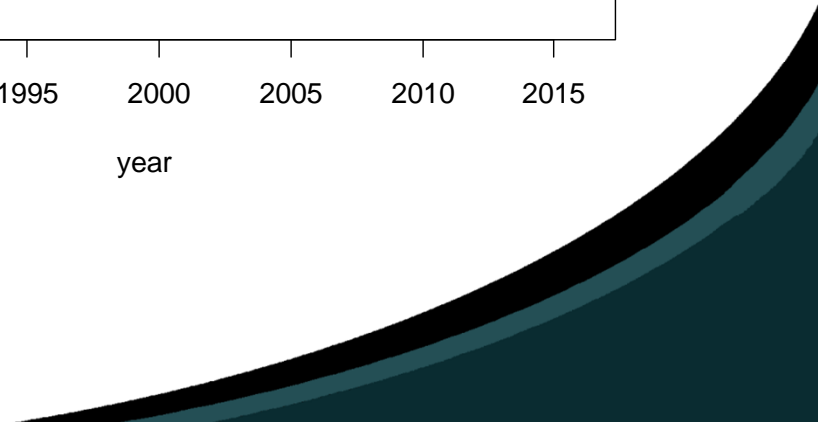
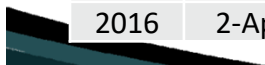
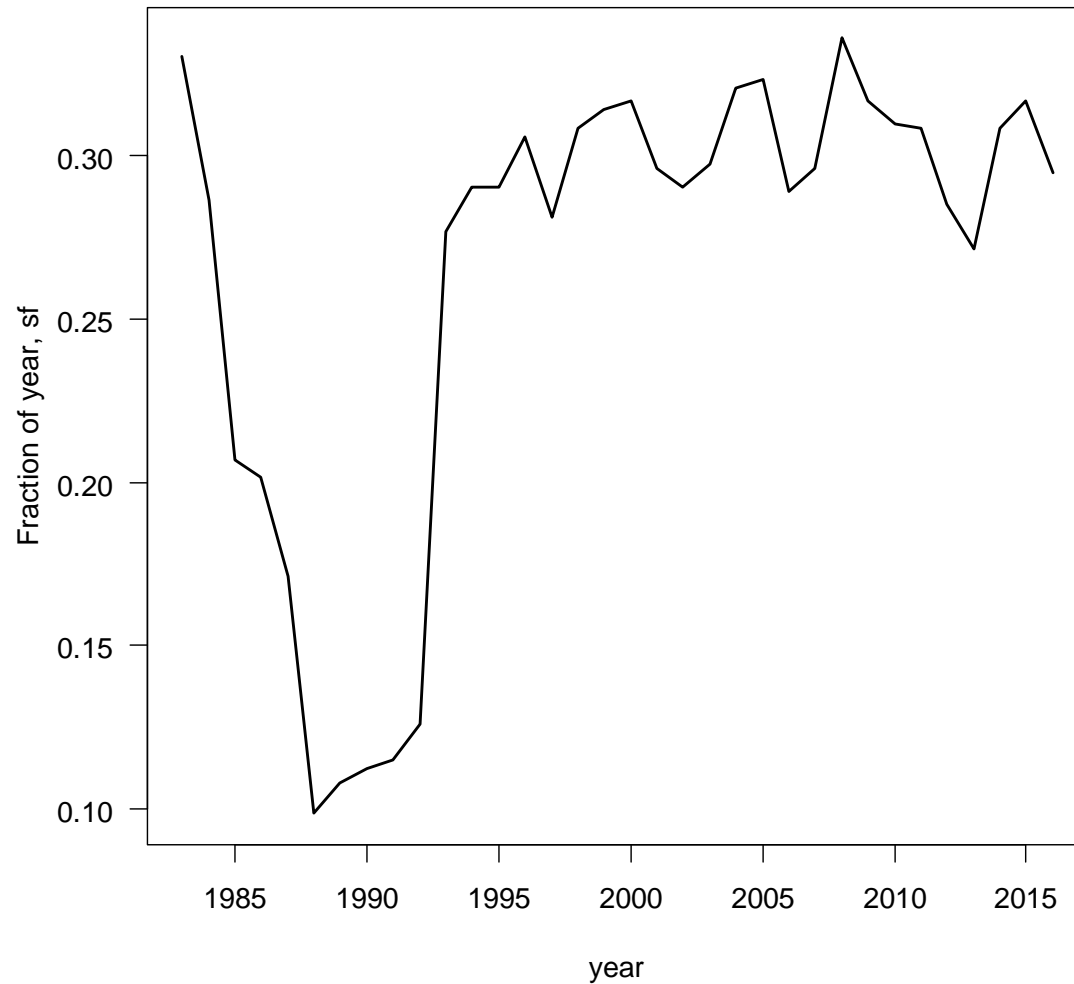
2



# Survey Timing

3

| Year   | Start Date | End Date  |
|--------|------------|-----------|
| 1983   | 23-Apr-83  | 8-May-83  |
| 1984   | 10-Apr-84  | 17-Apr-84 |
| 1985   | 8-Mar-85   | 25-Mar-85 |
| 1986   | 6-Mar-86   | 23-Mar-86 |
| 1987   | 13-Feb-87  | 22-Mar-87 |
| 1988   | 27-Jan-88  | 14-Feb-88 |
| 1989   | 1-Feb-89   | 16-Feb-89 |
| 1990   | 1-Feb-90   | 19-Feb-90 |
| 1991   | 2-Feb-91   | 20-Feb-91 |
| 1992   | 6-Feb-92   | 24-Feb-92 |
| 1993.4 | 2-Apr-93   | 20-Apr-93 |
| 1994   | 6-Apr-94   | 26-Apr-94 |
| 1995   | 4-Apr-95   | 28-Apr-95 |
| 1996   | 10-Apr-96  | 1-May-96  |
| 1997   | 2-Apr-97   | 23-Apr-97 |
| 1998   | 10-Apr-98  | 5-May-98  |
| 1999   | 13-Apr-99  | 6-May-99  |
| 2000   | 8-Apr-00   | 11-May-00 |
| 2001   | 7-Apr-01   | 29-Apr-01 |
| 2002   | 5-Apr-02   | 27-Apr-02 |
| 2003   | 5-Apr-03   | 2-May-03  |
| 2004   | 11-Apr-04  | 11-May-04 |
| 2005   | 17-Apr-05  | 9-May-05  |
| 2006   | 13-Apr-06  | 18-Apr-06 |
| 2007   | 4-Apr-07   | 2-May-07  |
| 2008   | 10-Apr-08  | 23-May-08 |
| 2009   | 8-Apr-09   | 13-May-09 |
| 2010   | 8-Apr-10   | 8-May-10  |
| 2011   | 7-Apr-11   | 8-May-11  |
| 2012   | 31-Mar-12  | 26-Apr-12 |
| 2013   | 26-Mar-13  | 23-Apr-13 |
| 2014   | 5-Apr-14   | 10-May-14 |
| 2015   | 11-Apr-15  | 10-May-15 |
| 2016   | 2-Apr-16   | 1-May-16  |



# Time-varying SR

4

- I fit models with various combinations of time-varying (i.e. RW)

Linf, k, and po,  $l_c(a) = L_{c,\infty}\{1 - (1 - \rho_{c,o})\exp(-k_c a)\}$

|          | k+Linf+po | (k+Linf+po) C | k+po+(Linf C)   | Linf+po+(k C) | Linf+(po+k C) |
|----------|-----------|---------------|-----------------|---------------|---------------|
| Dev      | -777.161  | -803.080      | -803.080        | -802.098      | -802.098      |
| Nparm    | 4.000     | 7.000         | 5.000           | 5.000         | 6.000         |
| AIC      | -769.161  | -789.080      | <b>-793.080</b> | -792.098      | -790.098      |
| BIC      | -753.358  | -761.425      | <b>-773.327</b> | -772.345      | -766.394      |
| MSE      | 0.008     | 0.007         | 0.007           | 0.007         | 0.007         |
| Linf     | 176.359   | 170.268       | 170.268         | 161.020       | 161.020       |
| k        | 0.057     | 0.064         | 0.064           | 0.069         | 0.069         |
| logit_po | -24.638   | -33.391       | -24.409         | -24.503       | -25.228       |
| std_Linf |           | 0.014         | 0.014           | 0.017         | 0.017         |
| std_k    |           | 0.000         |                 | 0.083         | 0.275         |
| std_lpo  |           | 0.000         |                 |               | 0.083         |
| std_me   | 0.088     | 0.083         | 0.083           |               |               |

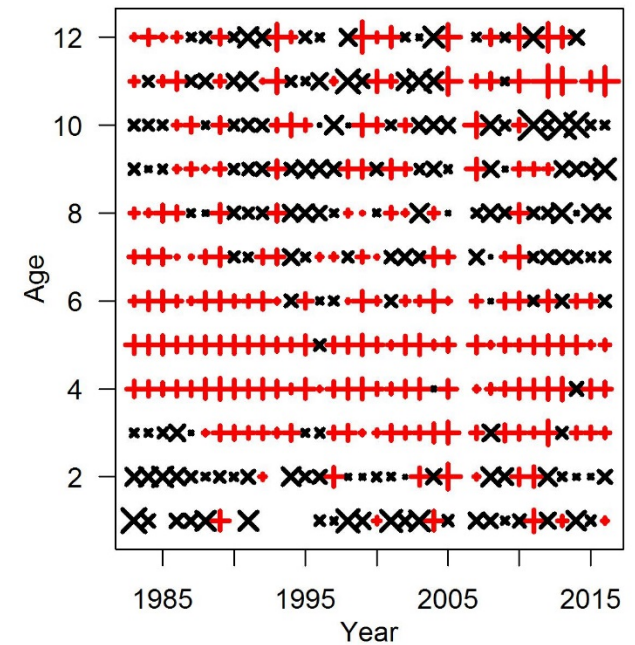
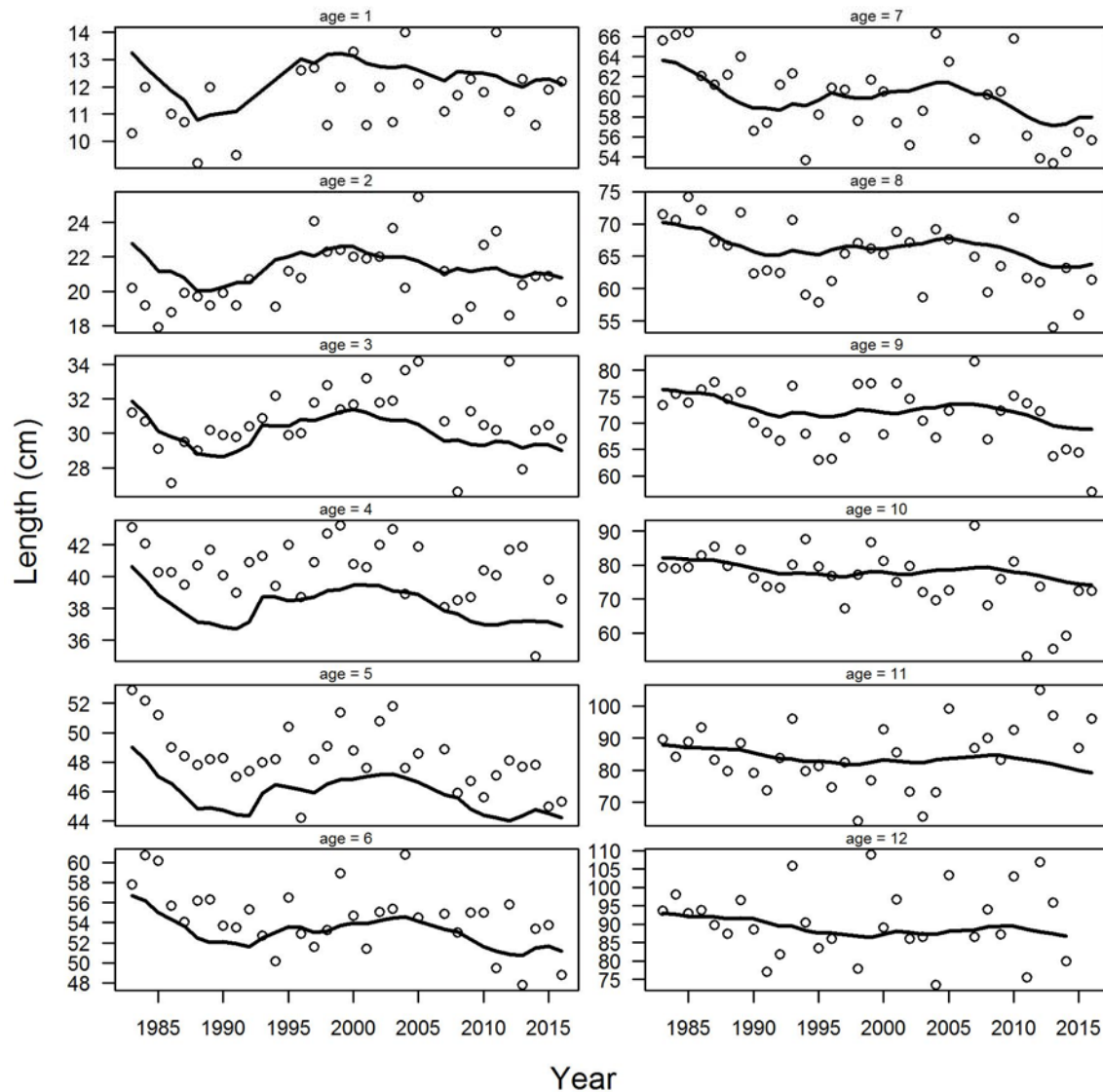
Fit using age+sf

best

close

# 3Ps cod RW VonB

5



VonB2 model did not help much!

None of these models fit the data well – under-estimate size at ages 3-6

# Time-Varying Bi-phasic Growth Model

6

//Fixed effects

```
PARAMETER(log_Linf);  
PARAMETER(log_k1);  
PARAMETER(log_k2);  
PARAMETER(theta0);  
PARAMETER(theta1);  
PARAMETER(log_po);  
PARAMETER(log_std_log_Linf);  
PARAMETER(log_std_log_k1);  
PARAMETER(log_std_log_k2);  
PARAMETER(log_std_me);
```

//Random Effects;

```
PARAMETER_VECTOR(rlog_Linf);  
PARAMETER_VECTOR(rlog_k1);  
PARAMETER_VECTOR(rlog_k2);
```

```
vector<Type> k1 = k1_c(ic);  
vector<Type> k2 = k2_c(ic);  
vector<Type> logit_p = theta0 + theta1*age;  
vector<Type> pmat = exp(logit_p)/(one + exp(logit_p));  
vector<Type> k = k1*pmat + k2*(one-pmat);  
log_pred = log_Linf_c(ic) + log(one - (one - po)*exp(-k*age));
```

TMB

R

```
Its = data.frame(  
  len=as.numeric(unlist(len[,2:(n.age+1)])),  
  age = as.numeric(unlist(agem)),  
  year = as.numeric(unlist(yearm)),  
  sf=as.numeric(unlist(sfm)))
```

```
Its$cohort = Its$year - Its$age
```

```
Its$age = Its$age + Its$sf
```

```
Its = subset(Its,!is.na(len))
```

```
tmb.data = list(  
  nobs = nrow(Its),  
  nc = length(unique(Its$cohort)),  
  y = Its$len,  
  cohort = Its$cohort,  
  age = Its$age,  
  ic = as.numeric(as.factor(Its$cohort))-1 ,  
  REk = 0, #0 for no, 1 for yes;  
  RELinf = 0 #0 for no, 1 for yes;  
)
```

# Time-Varying Bi-phasic Growth Model 7

|          | Const. k and Linf | Cohort k and Linf | Cohort k and const. Linf |
|----------|-------------------|-------------------|--------------------------|
| Dev      | -820.640          | -857.135          | -859.587                 |
| Nparm    | 6.000             | 9.000             | 8.000                    |
| AIC      | -808.640          | -839.135          | <b>-843.587</b>          |
| BIC      | -784.936          | -803.579          | <b>-811.982</b>          |
| MSE      | 0.007             | 0.006             | 0.006                    |
| Linf     | 145.286           | 141.534           | 133.004                  |
| k1       | 0.074             | 0.085             | 0.094                    |
| k2       | 0.061             | 0.064             | 0.067                    |
| theta0   | -5.536            | -5.396            | -5.583                   |
| theta1   | 2.224             | 2.106             | 2.199                    |
| std_Linf |                   | 0.015             |                          |
| std_k1   |                   | 0.000             | 0.023                    |
| std_k2   |                   | 0.015             | 0.000                    |
| std_me   | 0.083             | 0.076             | 0.077                    |

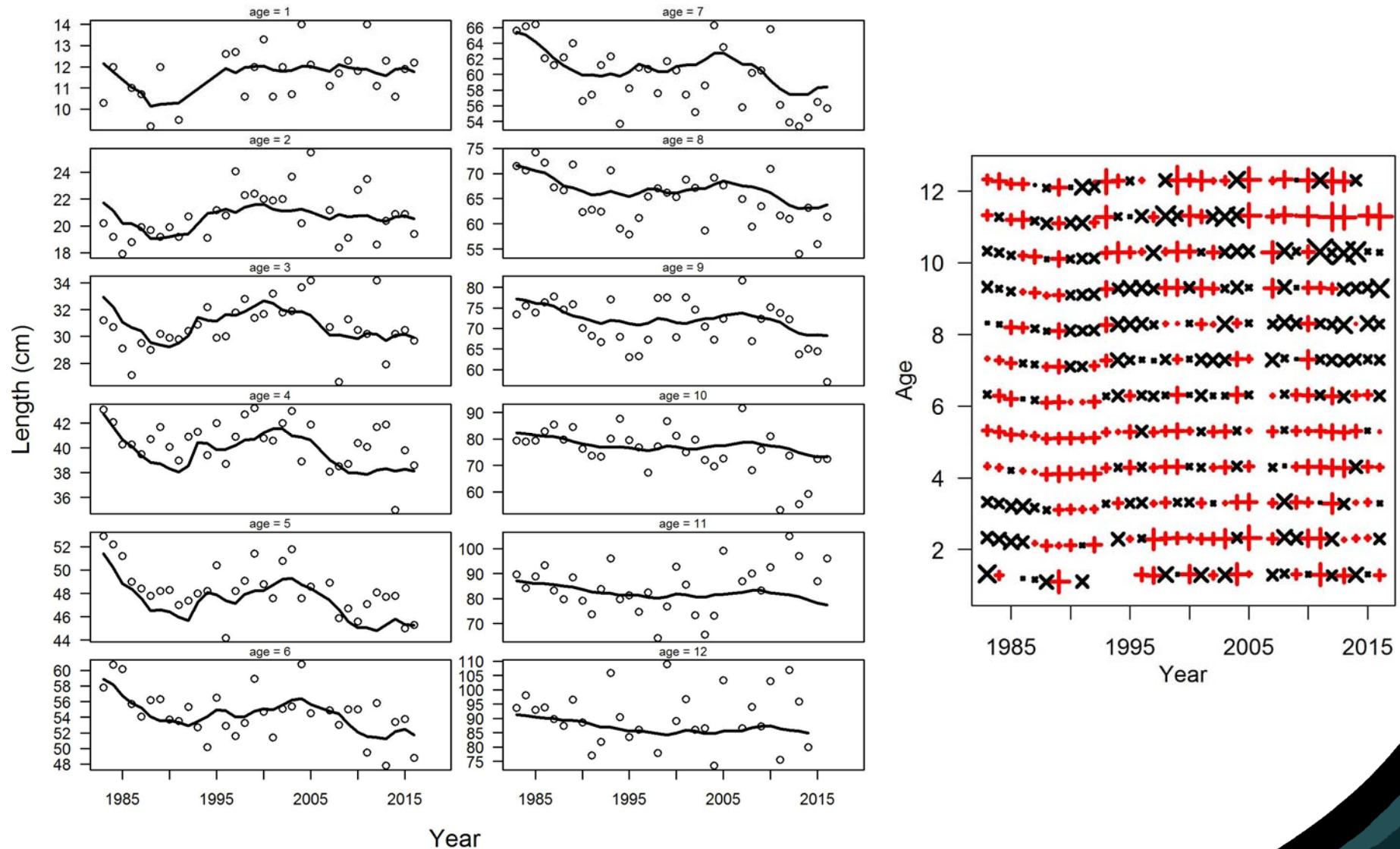
Bi-phasic model fit much better

best



# 3Ps cod RW Bi-Phasic VonB

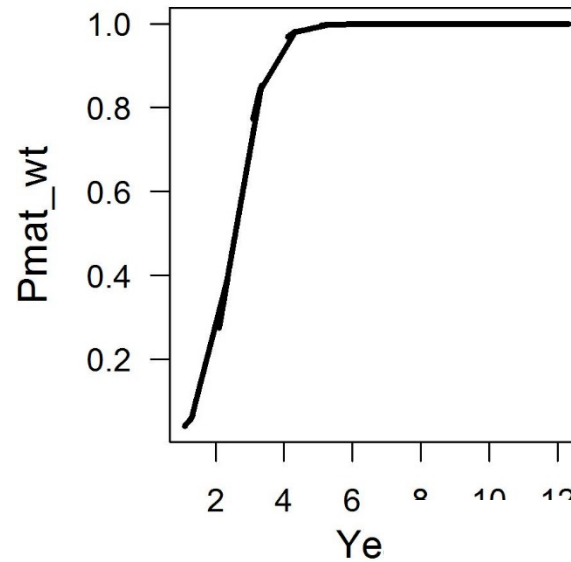
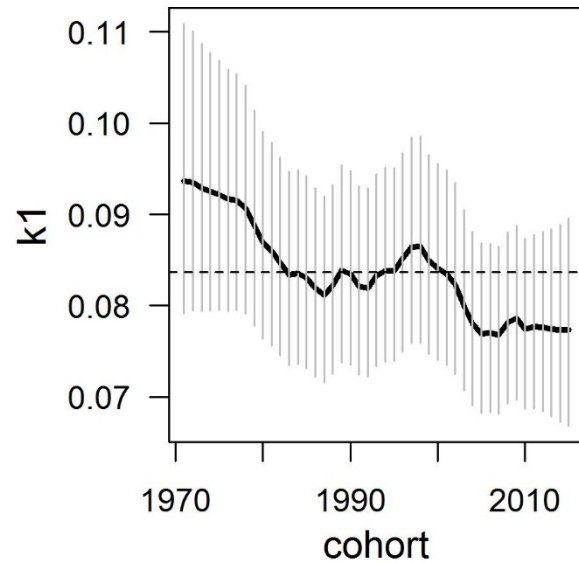
8





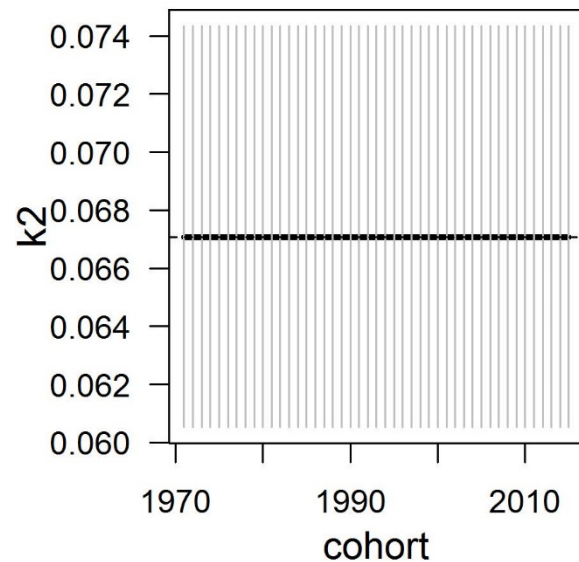
# 3Ps cod RW Bi-Phasic VonB

9

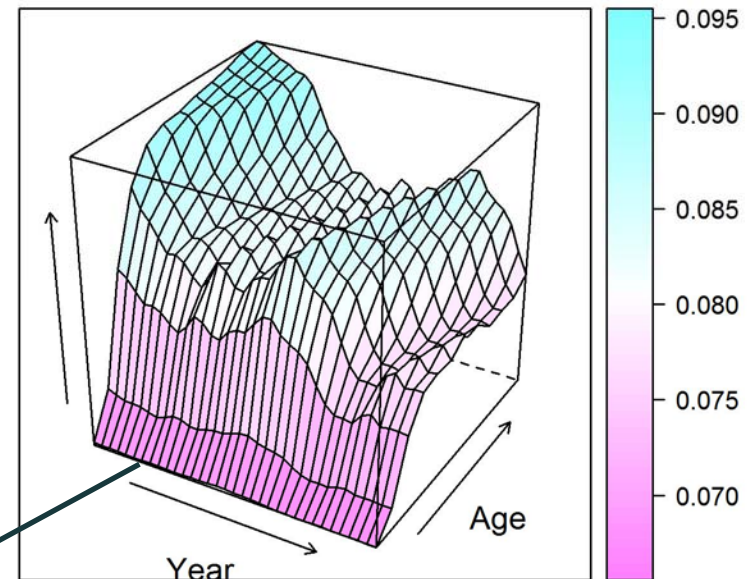


Bi-Phasic VonB  
better, but still some  
bias.

Lets go more semi-  
parametric to see  
what data want



Using age+sf got  
rid of change in  
 $k$  at young ages



# 3Ps cod agexyear RW K VonB

10

A nonparametric Von Bertalanffy model for estimating growth curves of Atlantic cod

By N. G. Cadigan and J. Bratney<sup>1</sup>

The VONB equation (2) can be developed from the differential equation

$$\frac{\partial L(t)}{\partial t} = k\{\lambda_{\infty} - L(t)\}.$$

We modify this simple model to accommodate annual/seasonal variations in growth rates by allowing  $k$  to be a smooth function of  $t$ ; that is,

$$\frac{\partial L(t)}{\partial t} = k(t)\{\lambda_{\infty} - L(t)\}. \quad (3)$$

Given that a fish is known to be length  $l_r$  at the time-of-release  $t_r$ , then the solution to (3) is

$$L(t) = l_r + (\lambda_{\infty} - l_r) \{1 - e^{-K(t,t_r)}\}, \quad (4)$$

where  $K(t, t_r) = \int_{t_r}^t k(u)du$ . By subtracting  $l_r$  from both sides of this equation we get

# 3Ps cod agexyear RW K VonB

11

- $K_c(a) = \sum_{i=1}^a \delta_{c,i}$
- I use a bi-variate RW for  $\delta_{c,a}$ , over cohorts ( $c$ ) and ages ( $a$ )

If  $K(a) = k \times a$  then  $K(a+1) - K(a) = k$

//Fixed effects

```
PARAMETER(log_Linf);  
PARAMETER(log_k_main);  
PARAMETER(logit_po);  
PARAMETER(log_std_log_Linf);  
PARAMETER(log_std_log_k);  
PARAMETER(log_std_lpo);  
PARAMETER(log_std_me);
```

//Random Effects;

```
PARAMETER_VECTOR(rlog_Linf);  
PARAMETER_MATRIX(rlog_k_dev);  
PARAMETER_VECTOR(rlogit_po);
```

```
matrix<Type> log_k(na,nc);  
matrix<Type> ka(na,nc);  
for (int j = 0; j < nc; ++j){  
  log_k(0,j) = log_k_main + rlog_k_dev(0,j);  
  ka(0,j) = exp(log_k(0,j));  
  for (int i = 1; i < na; ++i){  
    log_k(i,j) = log_k_main + rlog_k_dev(i,j);  
    ka(i,j) = ka(i-1,j) + exp(log_k(i,j));  
  }  
}
```

# 3Ps cod agexyear RW K VonB

12

- Bi-variate RW for  $\delta_{c,a}$ , over cohorts ( $c$ ) and ages ( $a$ )

```
for(int j = 0;j < nc;++j){  
  for(int i = 0;i < na;++i){  
  
    if((i==0)&(j == 0)){mZ = zero;}  
    if((i>0)&(j == 0)){mZ = rlog_k_dev(i-1,j);}   
    if((i == 0)&(j > 0)){mZ = rlog_k_dev(i,j-1);}   
    if((i > 0)&(j > 0)){mZ = rlog_k_dev(i,j-1) + rlog_k_dev(i-1,j) - rlog_k_dev(i-1,j-1);}   
  
    nll -= dnorm(rlog_k_dev(i,j),mZ,std_log_k,true);  
  }  
}
```



# agexyear RW K VonB

13

Exactly the same fit  
as slide 4, because it  
is the same model

|          | k+Linf+po | po+(k+Linf)   C | k+po+(Linf   C) | Linf+po+(k   C) |
|----------|-----------|-----------------|-----------------|-----------------|
| Dev      | -777.161  | -854.655        | -803.080        | -854.655        |
| Nparm    | 4.000     | 6.000           | 5.000           | 5.000           |
| AIC      | -769.161  | -842.655        | -793.080        | <b>-844.655</b> |
| BIC      | -753.358  | -818.951        | -773.327        | <b>-824.902</b> |
| MSE      | 0.008     | 0.005           | 0.007           | 0.005           |
| Linf     | 176.359   | 192.101         | 170.268         | 192.101         |
| k_main   | 0.057     | 0.052           | 0.064           | 0.052           |
| logit_po | -24.747   | -20.118         | -24.529         | -22.908         |
| std_Linf |           | 0.000           | 0.014           | 0.021           |
| std_k    |           | 0.021           |                 | 0.073           |
| std_me   | 0.088     | 0.073           | 0.083           |                 |

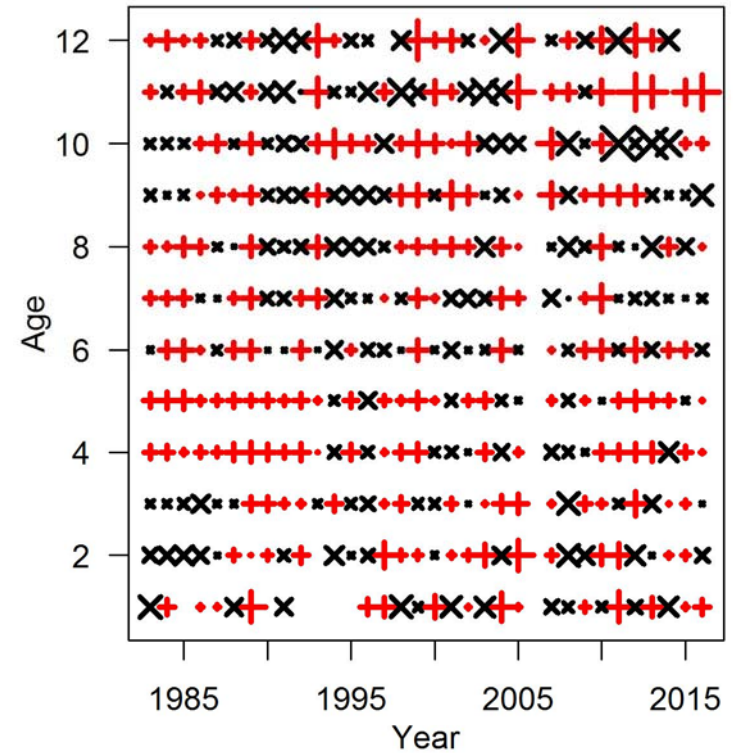
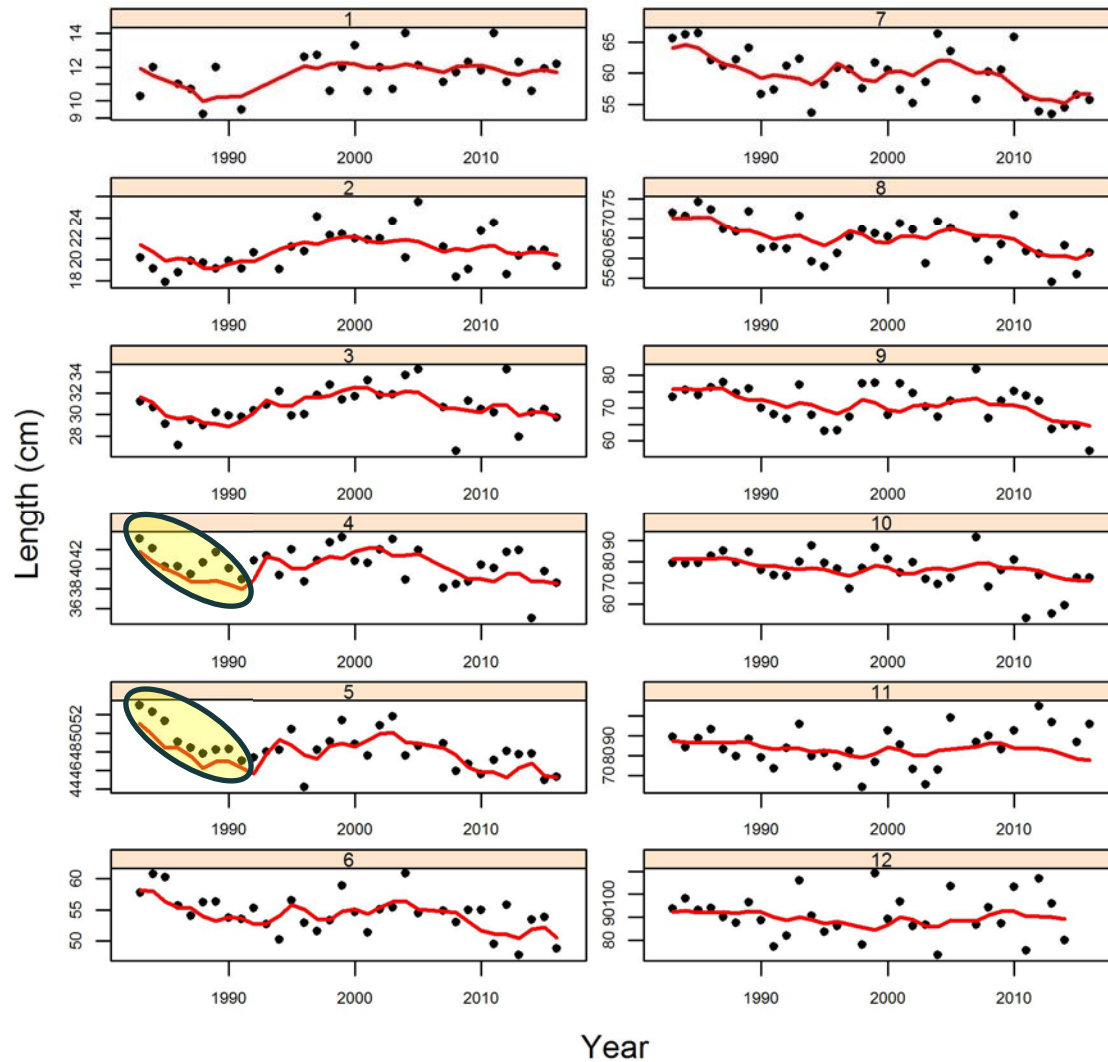
This model fits good!  
AIC and BIC a little  
lower than bi-phasic

best

# agexyear RW K VonB

14

Length at age by year

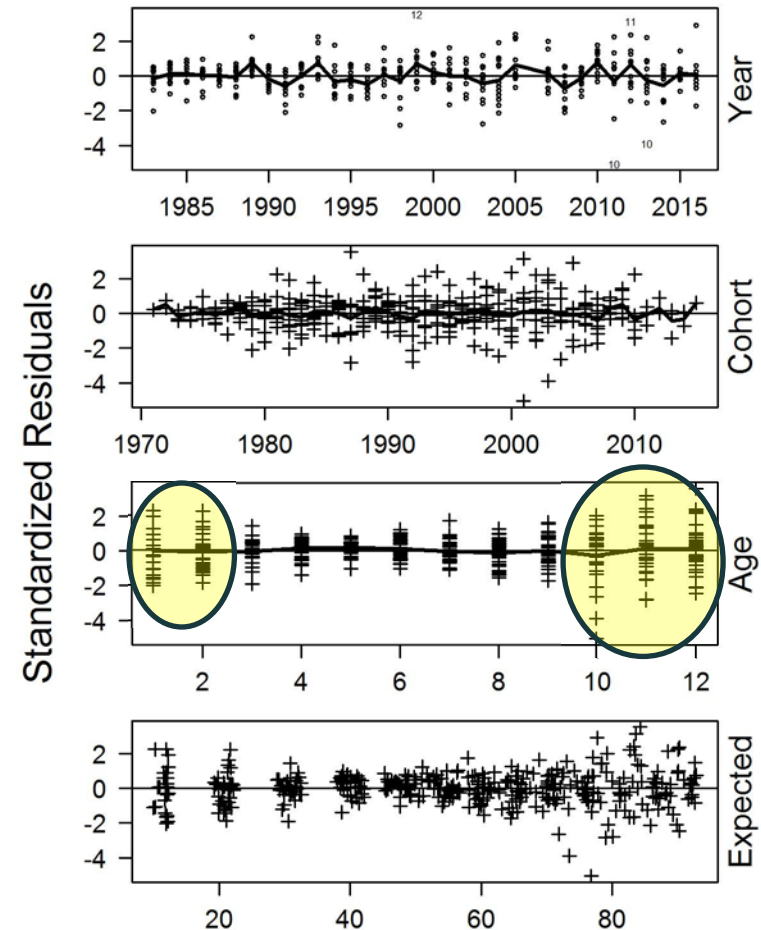
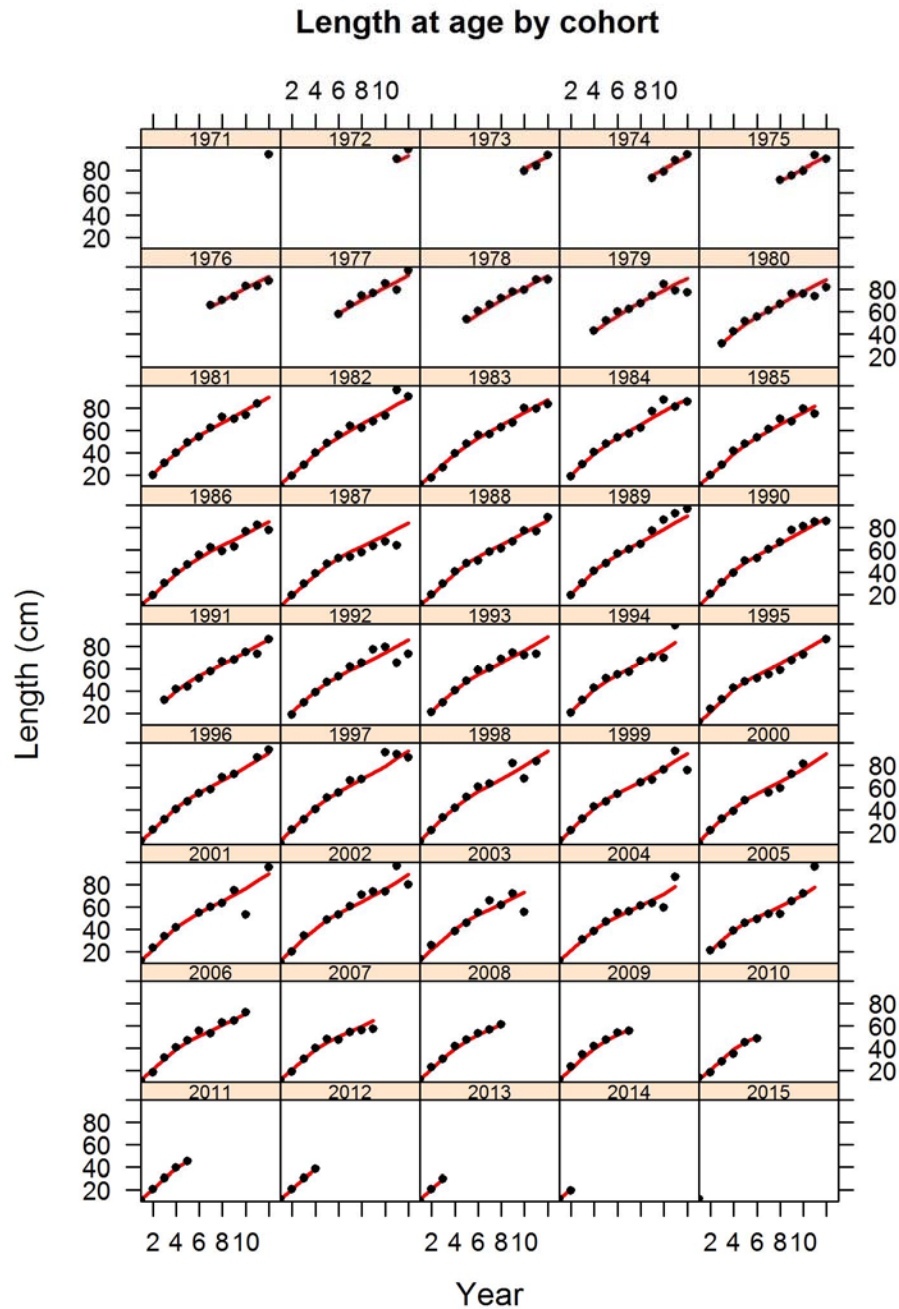


Some “bias” at ages 4+5



# agexyear RW K VonB

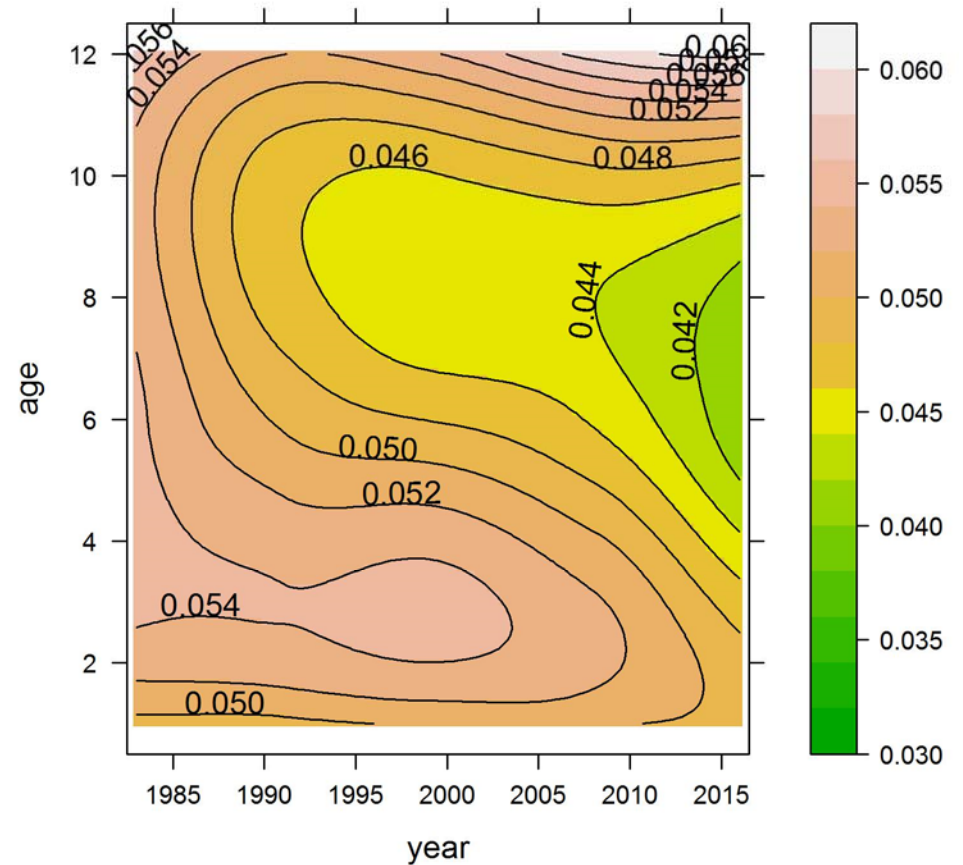
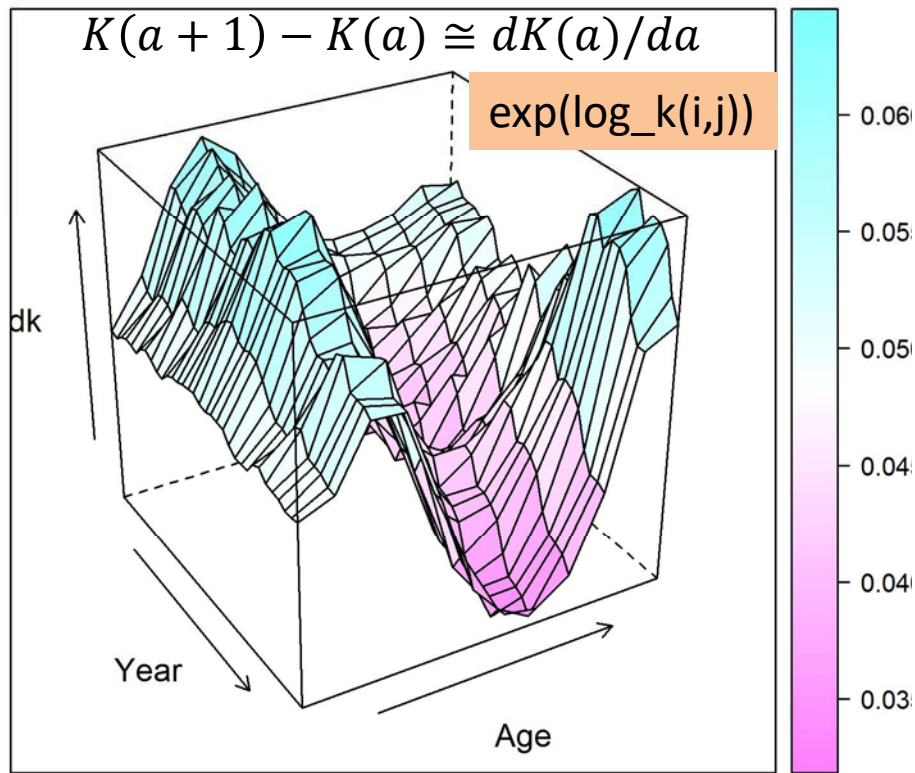
15



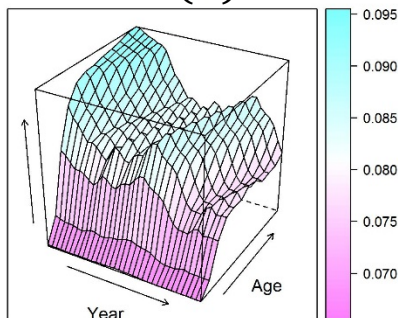
Error at young and old ages seems higher

# agexyear RW K VonB - the fix?

16

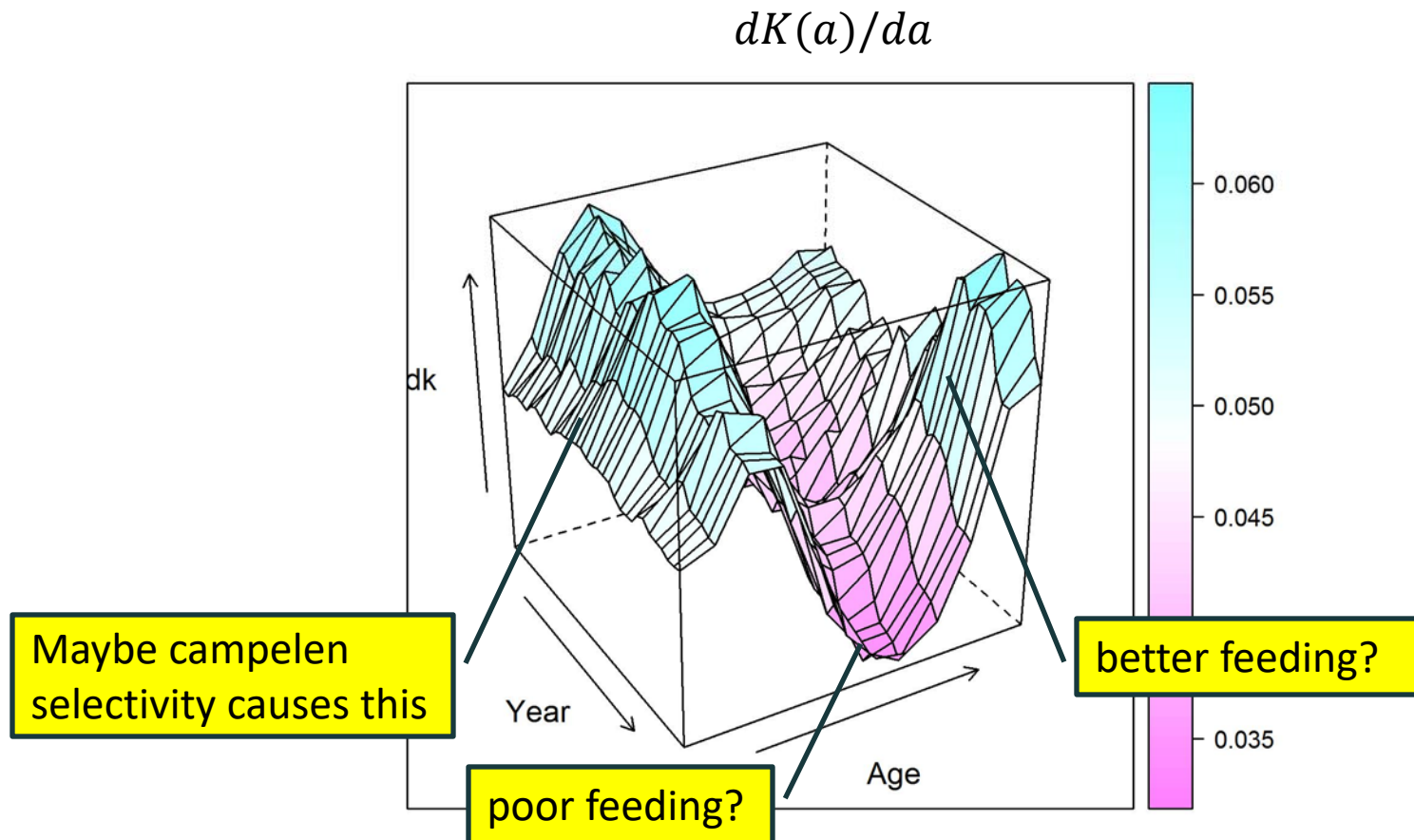


$K(a)$



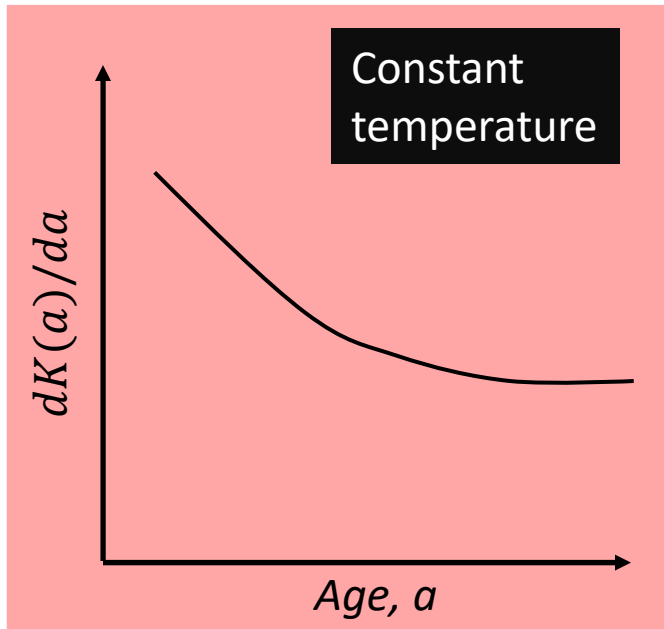
# agexyear RW K VonB -the fix?

17



Cod in 3Ps have ontogenetic? change in spatial distribution, and tend to occupy water with different temperatures. Older ages may be more on warm water slopes?

# A Hypothesis



Could also be explained by:

- 1) Diet
- 2) Stock mixing
- 3) etc

RV selectivity

