



ECE408/CS483/CSE408 Fall 2021

Applied Parallel Programming

Lecture 9:  
Tiled Convolution Analysis

# Course Reminders

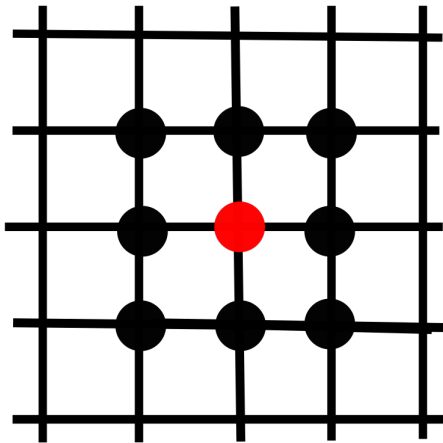
- We are still grading Labs 2 and 3
- Lab 4 out, it is due this week
- Midterm 1 is on Thursday, October 7<sup>th</sup>
  - On-line, everybody will be taking it at the same time
    - Thursday, Oct. 7th 8:00pm-9:20pm US Central time
    - Friday, Oct. 9th 9:00am-10:20am Beijing time
  - Includes materials from Lecture 1 through Lecture 10
- Project Milestone 1: Rai Installation and baseline CPU implementation is due Friday October 15<sup>th</sup>
  - Project details to be posted next week on course wiki

# Objective

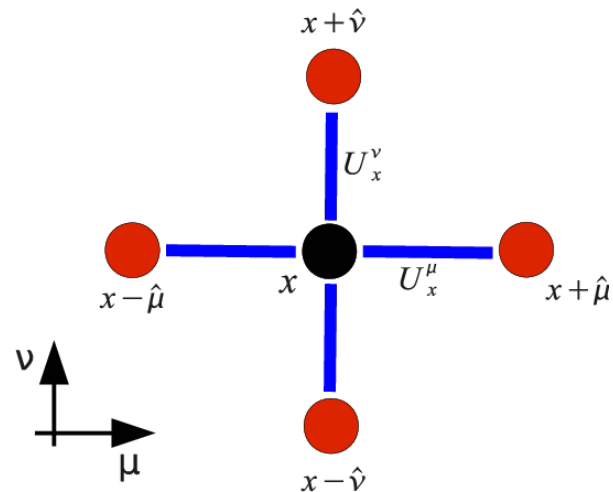
- To learn more about the analysis of tiled convolution/stencil algorithms

# Stencil Algorithms

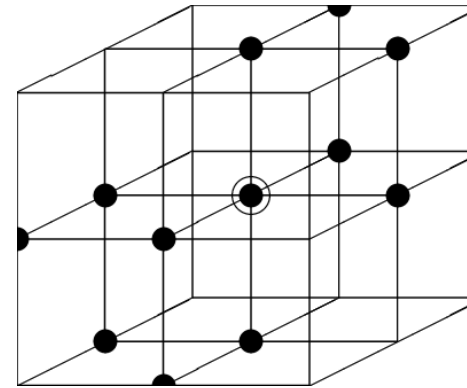
- Numerical data processing algorithms which update array elements according to some fixed pattern, called a *stencil*
  - Convolution is just one such example



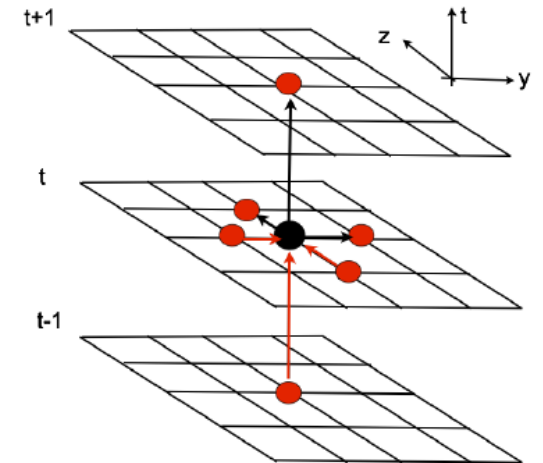
2D convolutional kernel (9-point 2D stencil)



Nearest neighbor lattice Dirac operator (5-point 2D stencil)

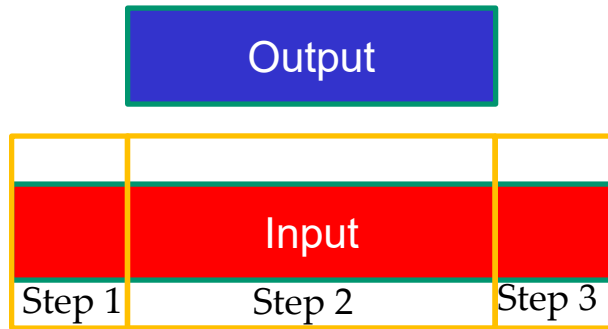


Finite difference stencil for 3D explicit time-marching (13-point 3D stencil)



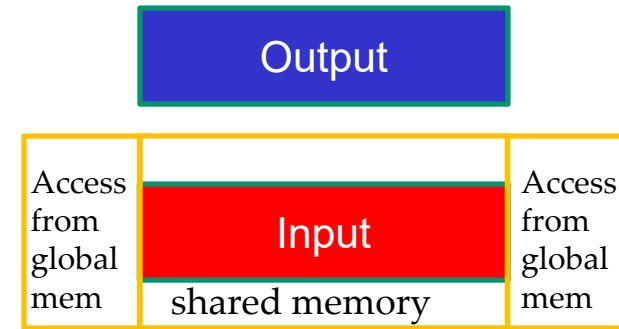
The Wilson-Dslash operator (4D stencil)

# Review: Three Tiling Strategies



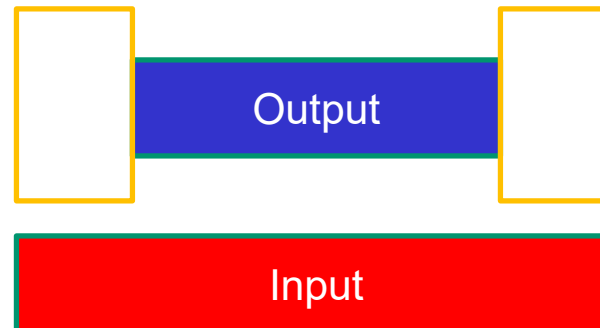
Strategy 1

1. Block size covers **output** tile
2. Use multiple steps to load input tile



Strategy 3

1. Block size covers **output** tile
2. Load only “core” of input tile
3. Access halo cells from global memory

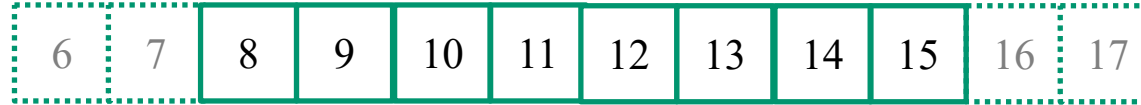


Strategy 2

1. Block size covers **input** tile
2. Load input tile in one step
3. Turn off some threads when calculating output

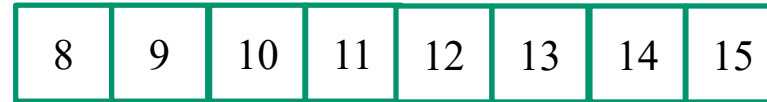
# A Small 1D Convolution Example

tile



MASK\_WIDTH is 5

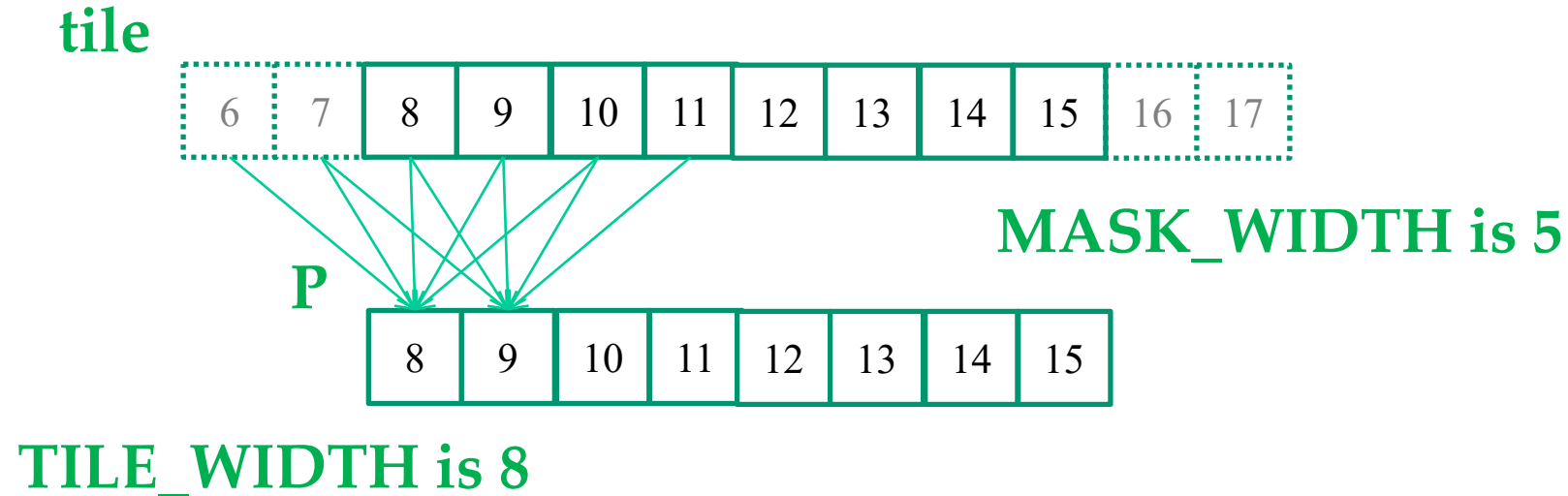
P



TILE\_WIDTH is 8

- output and input tiles for block 1
- For MASK\_WIDTH of 5, each block loads  $8 + (5 - 1) = 12$  elements (**12 memory loads**)

# Each Output Uses MASK\_WIDTH Inputs



- P[8] uses N[6], N[7], N[8], N[9], N[10]
- P[9] uses N[7], N[8], N[9], N[10], N[11]
- ...
- P[15] uses N[13], N[14], N[15], N[16], N[17]

Total of **8 \* 5 values** from **tile** **used for the output**.

# A simple way to calculate tiling benefit

- $8+(5-1) = 12$  unique elements of input array N loaded
- $8*5$  global memory accesses potentially replaced by shared memory accesses
- This gives a bandwidth reduction of  $40/12=3.3$
- This is independent of the size of N

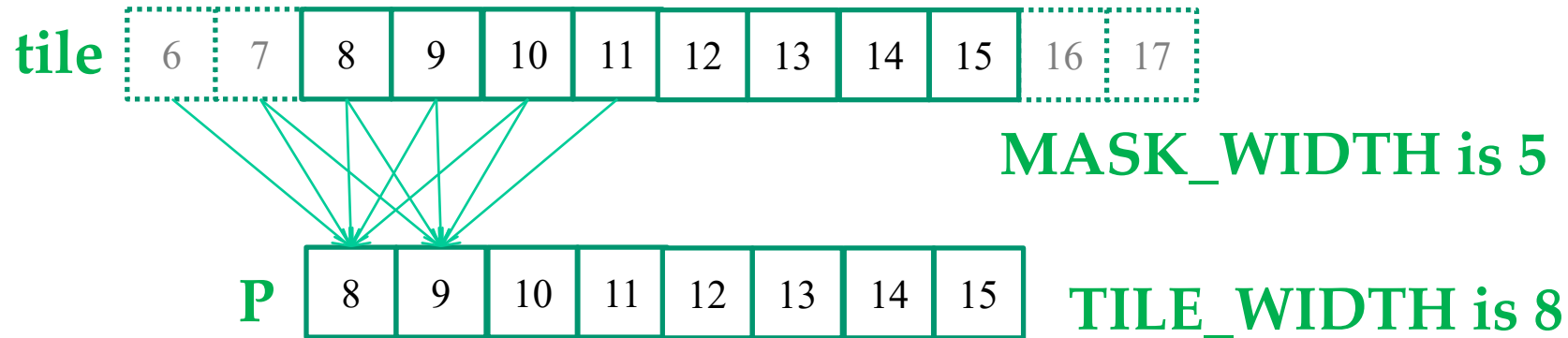


# In General, for 1D convolution kernels

- Load  $(\text{TILE\_WIDTH} + \text{MASK\_WIDTH} - 1)$  elements from global memory to shared memory
- Replace  $(\text{TILE\_WIDTH} * \text{MASK\_WIDTH})$  global memory accesses with shared memory accesses
- This leads to bandwidth reduction of

$$(\text{TILE\_SIZE} * \text{MASK\_WIDTH}) / (\text{TILE\_SIZE} + \text{MASK\_WIDTH} - 1)$$

# Another Way to Look at Reuse



- tile[6] is used by P[8] (1×)
- tile[7] is used by P[8], P[9] (2×)
- tile[8] is used by P[8], P[9], P[10] (3×)
- tile[9] is used by P[8], P[9], P[10], P[11] (4×)
- tile[10] is used by P[8], P[9], P[10], P[11], P[12] (5×)
- ... (5×)
- tile[14] is used by P[12], P[13], P[14], P[15] (4×)
- tile[15] is used by P[13], P[14], P[15] (3×)
- tile[16] is used by P[14], P[15] (2×)
- tile[17] is used by P[15] (1×)

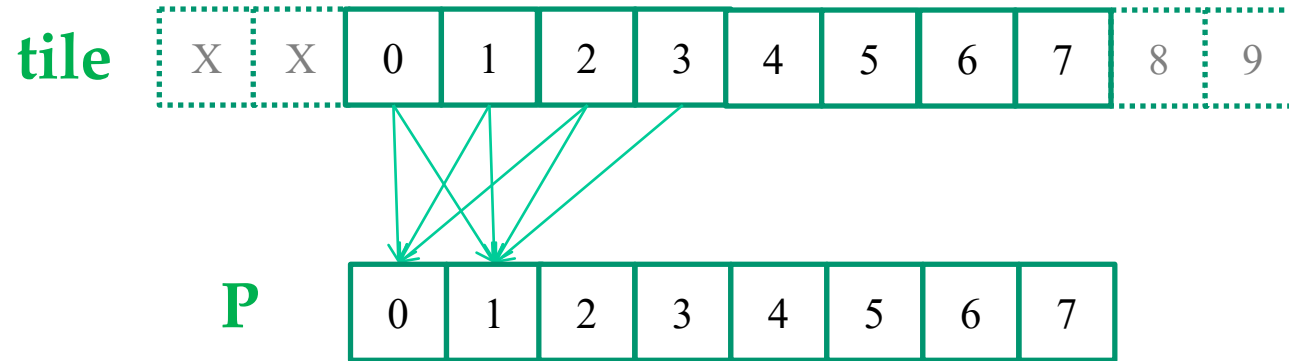
# Another Way to Look at Reuse

- Each access **tile** replaces an access to input N
- The total number of global memory accesses (to the  $(8+5-1)=12$  input elements) replaced by shared memory accesses is

$$\begin{aligned} & 1 + 2 + 3 + 4 + 5 * (8-5+1) + 4 + 3 + 2 + 1 \\ &= 10 + 20 + 10 \\ &= 40 \end{aligned}$$

- There are 12 elements of input N, so the average reduction is  **$40/12 = 3.3$**

# What about Boundary Tiles?



- P[0] uses N[0], N[1], N[2]
- P[1] uses N[0], N[1], N[2], N[3]
- P[2] uses N[0], N[1], N[2], N[3], N[4]

Less than  $8 * 5$  elements of N are used for the output tile

# Ghost elements change ratios

- For a boundary tile, we load  $\text{TILE\_WIDTH} + (\text{MASK\_WIDTH}-1)/2$  elements
  - 10 in our example of  $\text{TILE\_WIDTH}$  of 8 and  $\text{MASK\_WIDTH}$  of 5
- Computing boundary elements do not access global memory for ghost cells
  - Total accesses is  $6*5 + 4 + 3 = 37$  accesses (when computing the P elements)

The reduction is  **$37/10 = 3.7$**

# In General for 1D, internal tiles

- The total number of global memory accesses to the  $(\text{TILE\_WIDTH} + \text{Mask\_Width} - 1)$  elements of input N replaced by shared memory accesses is

$$1 + 2 + \dots + \text{Mask\_Width} - 1 + \text{Mask\_Width} * (\text{TILE\_WIDTH} - \text{Mask\_Width} + 1) + \text{Mask\_Width} - 1 + \dots + 2 + 1$$
$$= ((\text{Mask\_Width} - 1) * \text{Mask\_Width}) / 2 + \text{Mask\_Width} * (\text{TILE\_WIDTH} - \text{Mask\_Width} + 1) + ((\text{Mask\_Width} - 1) * \text{Mask\_Width}) / 2$$

$$= (\text{Mask\_Width} - 1) * \text{Mask\_Width} + \text{Mask\_Width} * (\text{TILE\_WIDTH} - \text{Mask\_Width} + 1)$$

$$= \text{Mask\_Width} * \text{TILE\_WIDTH}$$

# Bandwidth Reduction for 1D

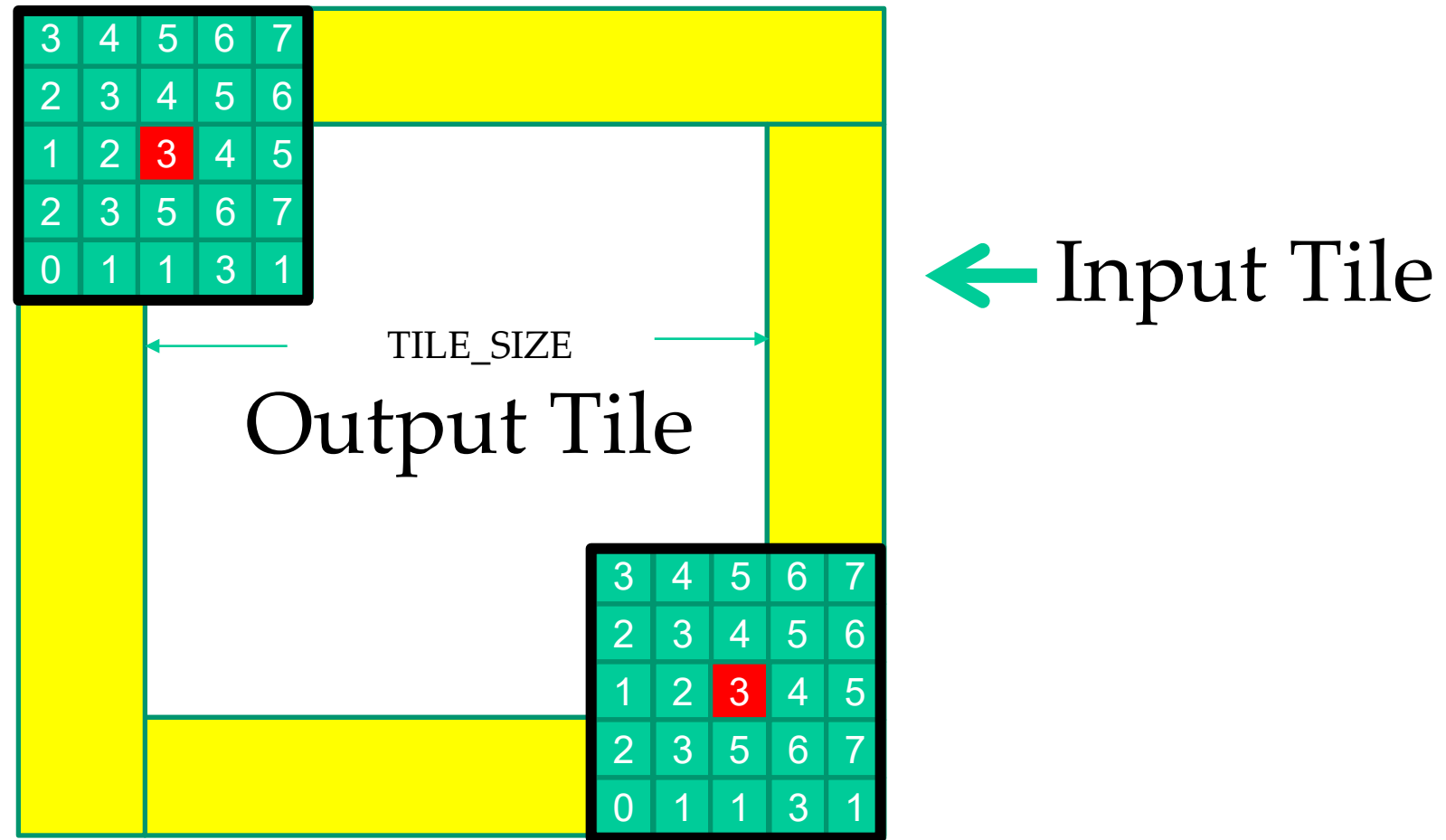
- The reduction is

$$(TILE\_SIZE * MASK\_WIDTH) / (TILE\_SIZE + MASK\_WIDTH - 1)$$

TILE_WIDTH	16	32	64	128	256
Reduction MASK_WIDTH = 5	4.0	4.4	4.7	4.9	4.9
Reduction MASK_WIDTH = 9	6.0	7.2	8.0	8.5	8.7

# Review: Parallelization of Tile Load

MASK\_WIDTH is 5





# Analysis for an 8×8 Output Tile, MASK\_WIDTH of 5

- Loading input tile requires  $(8+5-1)^2 = 144$  reads
- Calculation of each output requires  $5^2 = 25$  input elements
- $8 \times 8 \times 25 = 1,600$  global memory accesses for computing output tile are converted to shared memory accesses
- Bandwidth reduction of  $1,600/144 = 11.1\times$

# In General

- $(\text{TILE\_WIDTH} + \text{MASK\_WIDTH} - 1)^2$  elements need to be loaded from  $N$  into shared memory
- The calculation of each  $P$  element needs to access  $\text{MASK\_WIDTH}^2$  elements of  $N$
- $(\text{TILE\_WIDTH} * \text{MASK\_WIDTH})^2$  global memory accesses converted into shared memory accesses
- Bandwidth reduction of  $(\text{TILE\_WIDTH} * \text{MASK\_WIDTH})^2 / (\text{TILE\_WIDTH} + \text{MASK\_WIDTH} - 1)^2$

# Bandwidth Reduction for 2D Convolution Kernel

- The reduction is

$$(\text{TILE\_WIDTH} * \text{MASK\_WIDTH})^2 / (\text{TILE\_WIDTH} + \text{MASK\_WIDTH} - 1)^2$$

TILE_WIDTH	8	16	32	64
Reduction MASK_WIDTH = 5	11.1	16	19.7	22.1
Reduction MASK_WIDTH= 9	20.3	36	51.8	64

# Ghost elements change ratios

- 2D calculation left for your enjoyment

# 2B/FLOP for Untiled Convolution

- How much global memory per FLOP is in untiled convolution?
- In untiled convolution,
  - each value from **N** (**4B** from global memory)
  - is multiplied by a value from **M** (**4B** from constant cache, **1 FLOP**),
  - then added to a running sum (**1 FLOP**)
- That gives **2B / FLOP**

# Full Use of Compute Requires 13.3× Reuse

- Recall our reuse discussion from matrix multiply:
  - **1,000 GFLOP/s** for GPU from ~2010, and
  - **150 GB/s** memory bandwidth.

- Dividing memory bandwidth by **2B/FLOP**,

$$\frac{150 \text{ GB/s}}{2 \text{ B/FLOP}} = \mathbf{75 \text{ GFLOP/s} = 7.50\% \text{ of peak.}}$$

- **Need** at least **100/7.50 = 13.3× reuse** to make full use of compute resources

# In 2020, Need 52.1× Reuse

- In 2020, the **GRID K520** offers
  - nearly **5,000 GFLOP/s**, but only
  - **192 GB/s** memory bandwidth

- Dividing memory bandwidth by **2B/FLOP**,

$$\frac{192 \text{ GB/s}}{2 \text{ B/FLOP}} = \mathbf{96 \text{ GFLOP/s} = 1.92\% \text{ of peak}}$$

- **Need** at least **100/1.92 = 52.1× reuse** to make full use of compute resources

# Need Really Big Mask to Balance Resources

- Let's make another table: **% of peak compute**
  - for **1D** tiled convolution,
  - with **TILE\_WIDTH 1024**

MASK_WIDTH	2010	2020
5	37%	9.6%
9	67%	17%
15	100%	28%
55	100%	100%



# Need Really Big Mask to Balance Resources

- And one more: **% of peak compute**
  - for **2D** tiled convolution,
  - with **TILE\_WIDTH 32×32**

MASK_WIDTH	2010	2020
3	60%	15%
5	100%	37%
7	100%	67%
9	100%	almost 100%

# Food for Thought

- Ratios are different for tiles on boundaries
- More importantly,
  - Each thread loads 4B to shared memory
  - 2,048 threads load only 8kB
  - Shared memory is usually 64kB or larger
  - What can one do with the rest?

**Improved approach left as homework**

(For example, can raise MW=7 from 67% to 81%)

Two vertical lines, one blue and one orange, are positioned on the left side of the slide.

**ANY MORE QUESTIONS?  
READ CHAPTER 7**