ECE408/CS483/CSE408 Fall 2021

Applied Parallel Programming

Lecture 16
Parallel Computation Patterns – Parallel Scan (Prefix Sum)

Course Reminders

- Project Milestone 1 was due this past Sunday
- Project Milestone 2: Baseline Convolution Kernel
  - Updated repo will be released later this week
- Lab 5.1 – due this week
  - Implement a kernel and associated host code that performs reduction of a 1D list stored in a C array. The reduction should give the sum of the list. You should implement the improved kernel discussed in the lecture. Your kernel should be able to handle input lists of arbitrary length.
  - Note that WebGPU still has an issue with security certificate, you just need to proceed with the non-secure mode
- MT1
  - Regrade requests are due now

Objective

- To learn parallel scan (prefix sum) algorithms based on reductions
- To learn the concept of double buffering
- To understand tradeoffs between work efficiency and latency

Scan Includes all Partial Results

Reductions are a simplified form of scans.

In scan / parallel prefix,

- we need all of the partial sums
- (or whatever the operator might be).

(Inclusive) Scan (Prefix-Sum) Definition

Definition: *The scan operation takes a binary associative operator* $\oplus$, *and an array of n elements*

$$[x_0, x_1, \ldots, x_{n-1}],$$

and returns the prefix-sum array

$$[x_0, (x_0 \oplus x_1), (x_0 \oplus x_1 \oplus x_2), \ldots, (x_0 \oplus x_1 \oplus \ldots \oplus x_{n-1})].$$

Example: If $\oplus$ is addition, the scan operation
on the array [3 1 7 0 4 1 6 3],
returns [3 4 11 11 15 16 22 25].

Example: Sharing a Big Sandwich

You order a 100-inch sandwich to feed 10 people, and you know how much each person wants in inches:

[3 5 2 7 28 4 3 0 8 1] .

How do you cut the bread quickly?

How much of the sandwich is left over?

Method 1: sequentially!

Cut 3 inches, then cut 5 inches, then …

Method 2: **calculate cutting offsets with prefix-sum**

[3, 8, 10, 17, 45, 49, 52, 52, 60, 61] (39 inches left)

Typical Applications of Scan

A simple and useful parallel building block.

Convert sequential recurrences

```
for(j=1;j<n;j++)
    out[j] = out[j-1] + f(j);
```

into parallel:

```
forall(j) { temp[j] = f(j) };
scan(out, temp);
```

Typical Applications of Scan

- Useful for many parallel algorithms:
  - radix sort
  - quicksort
  - String comparison
  - Lexical analysis
  - Stream compaction
  - Polynomial evaluation
  - Solving recurrences
  - Tree operations
  - Histograms
  - Etc.

Other Applications

- Assigning camp slots
- Assigning farmer market space
- Allocating memory to parallel threads
- Allocating memory buffer to communication channels
- …

An Inclusive Sequential Scan

Given a sequence $[x_0, x_1, x_2, \ldots]$

Calculate output $[y_0, y_1, y_2, \ldots]$

Such that

$$y_0 = x_0$$

$$y_1 = x_0 + x_1$$

$$y_2 = x_0 + x_1 + x_2$$

…

Using a recursive definition

$$y_i = y_{i-1} + x_i$$

An Sequential C Implementation

```
y[0] = x[0];
for (i = 1; i < Max_i; i++)
    y[i] = y[i-1] + x[i];
```

Computationally efficient:

N additions needed for N elements - O(N).

A Naïve Inclusive Parallel Scan

- Assign one thread to calculate each y element
- Have every thread to add up all x elements needed for the y element

$$y_0 = x_0$$
$$y_1 = x_0 + x_1$$
$$y_2 = x_0 + x_1 + x_2$$

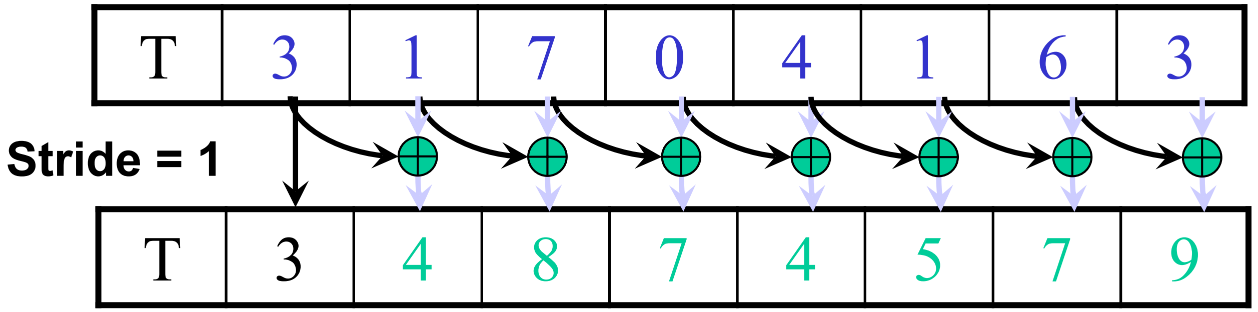
"Parallel programming is easy as long as you do not care about performance."

Parallel Inclusive Scan using Reduction Trees

Calculate each output element as the reduction of all previous elements

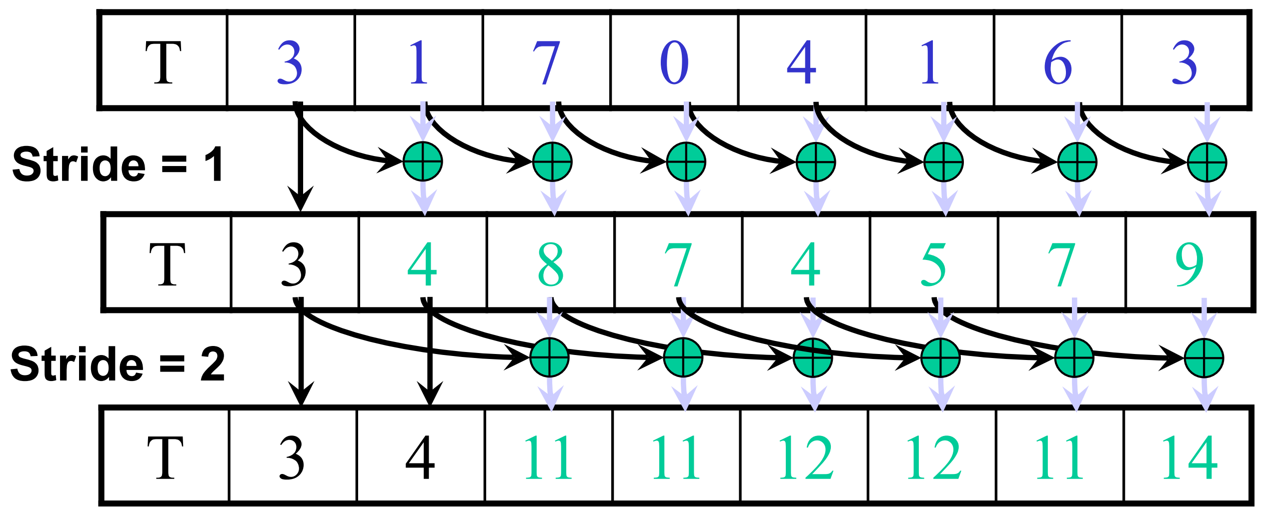
- Some reduction partial sums will be shared among the calculation of output elements
- Based on hardware added design by Peter Kogge and Harold Stone at IBM in the 1970s – Kogge-Stone Trees
- Goal: low latency

A Kogge-Stone Parallel Scan Algorithm



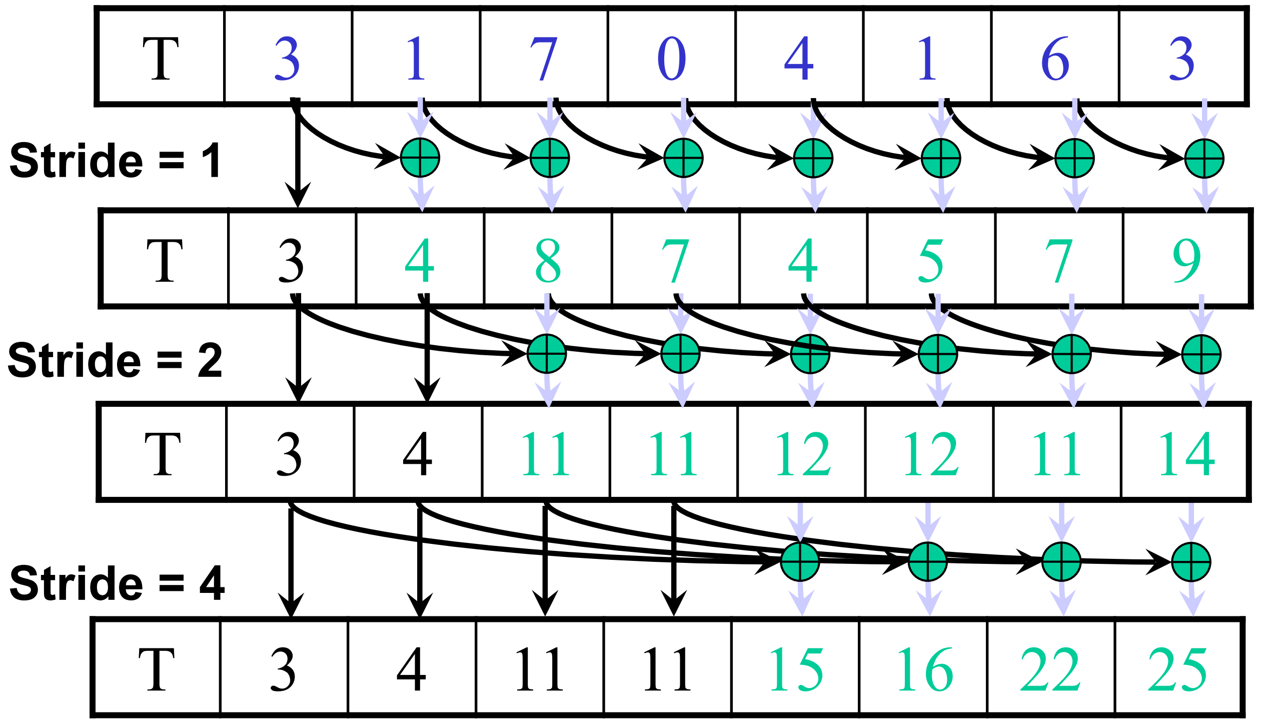
Iteration #1
Stride = 1

A Kogge-Stone Parallel Scan Algorithm



Iteration #2
Stride = 2

A Kogge-Stone Parallel Scan Algorithm



Iteration #3
Stride = 4

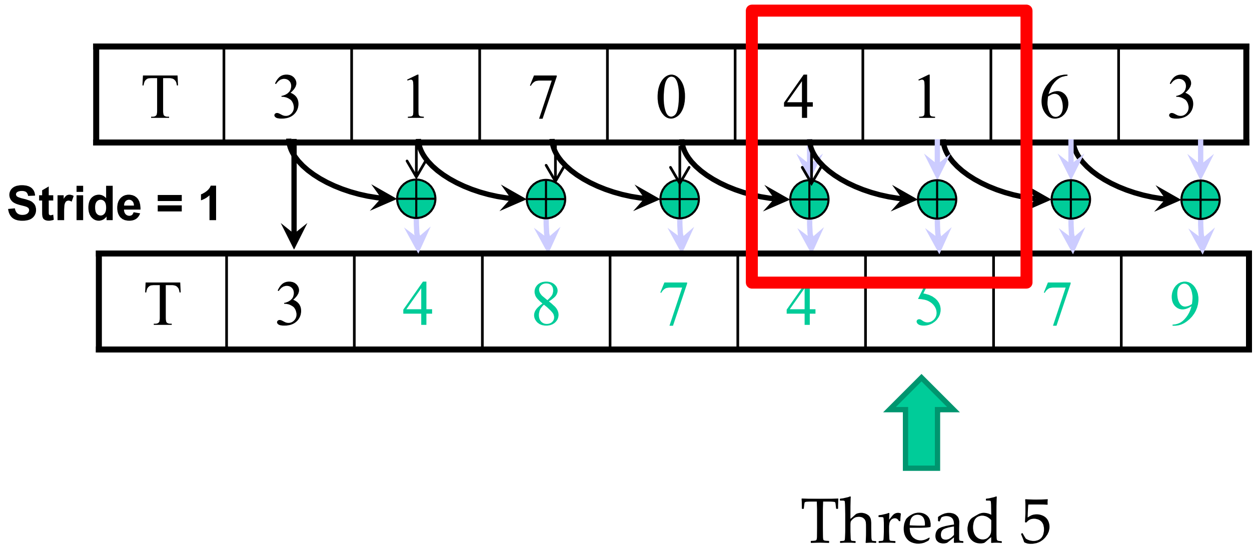
A Kogge-Stone Parallel Scan Algorithm

T	3	1	7	0	4	1	6	3

1. Load input from global memory into shared memory array T

Each thread loads one value from the input (global memory) array into shared memory array T.

A Kogge-Stone Parallel Scan Algorithm

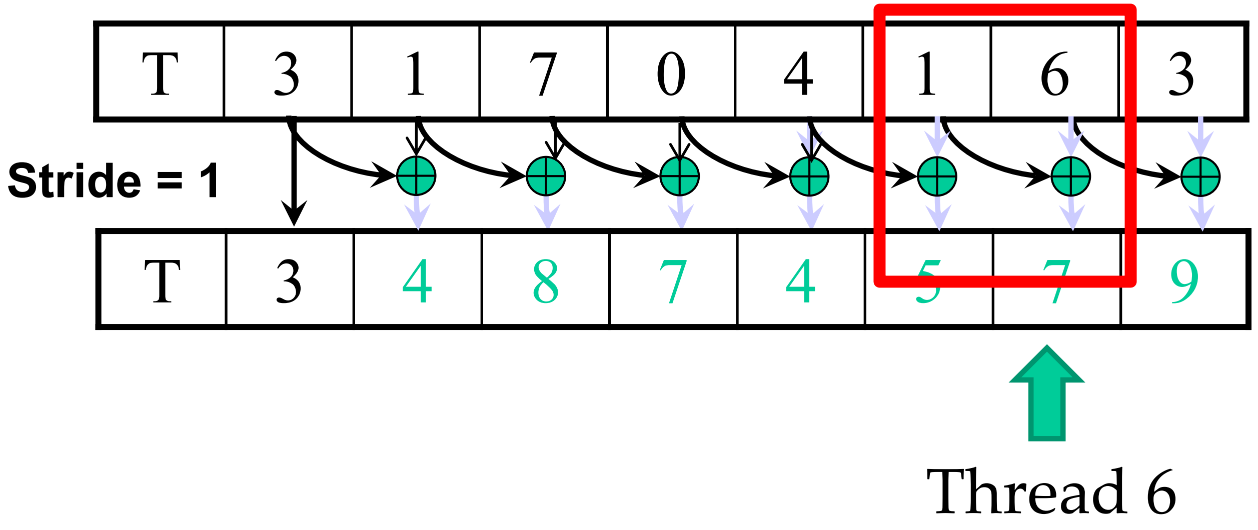


1. (previous slide)
2. Assuming n is a power of 2. Iterate log(n) times, stride from 1 to n/2. Threads *stride* to *n-1 active:* add pairs of elements that are *stride* elements apart.

Iteration #1
Stride = 1

- Active threads: *stride* to *n*-1 (*n* - *stride* active threads)
- Thread *j* adds elements T[*j*] and T[*j-stride*] and writes result into element T[*j*]
- Each iteration requires two syncthreads
  - make sure that input is in place
  - make sure that all input elements have been used

A Kogge-Stone Parallel Scan Algorithm



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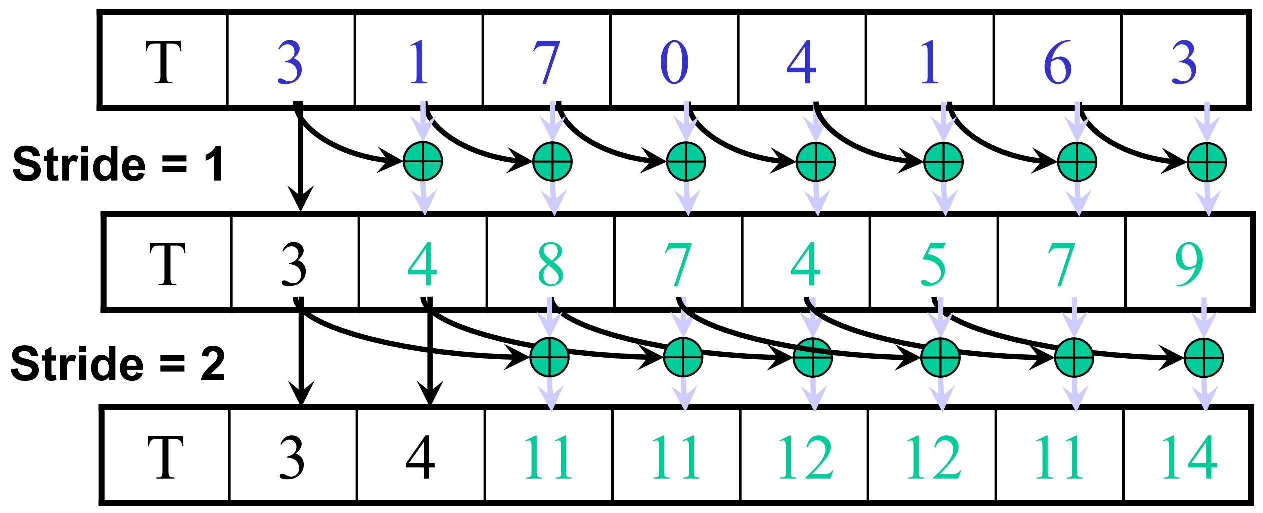
Iteration #1
Stride = 1

- Active threads: *stride* to *n*-1 (*n* - *stride* active threads)
- Thread *j* adds elements T[*j*] and T[*j-stride*] and writes result into element T[*j*]
- Each iteration requires two syncthreads
  - syncthreads(); // make sure that input is in place
  - float temp = T[*j*] + T[*j-stride*];
  - syncthreads(); // make sure that previous output has been consumed
  - T[*j*] = temp;

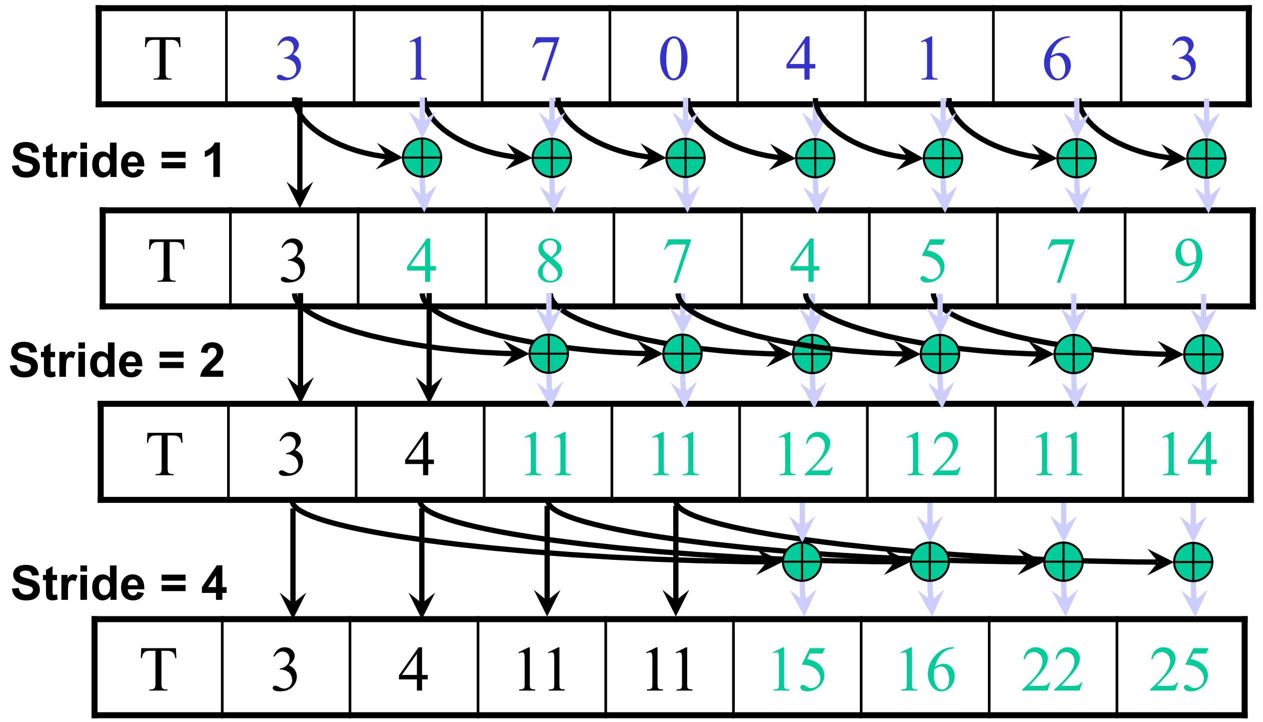
A Kogge-Stone Parallel Scan Algorithm



1. …
2. Assuming n is a power of 2. Iterate log(n) times, stride from 1 to n/2. Threads *stride* to *n-1* *active:* add pairs of elements that are *stride* elements apart.

Iteration #2
Stride = 2

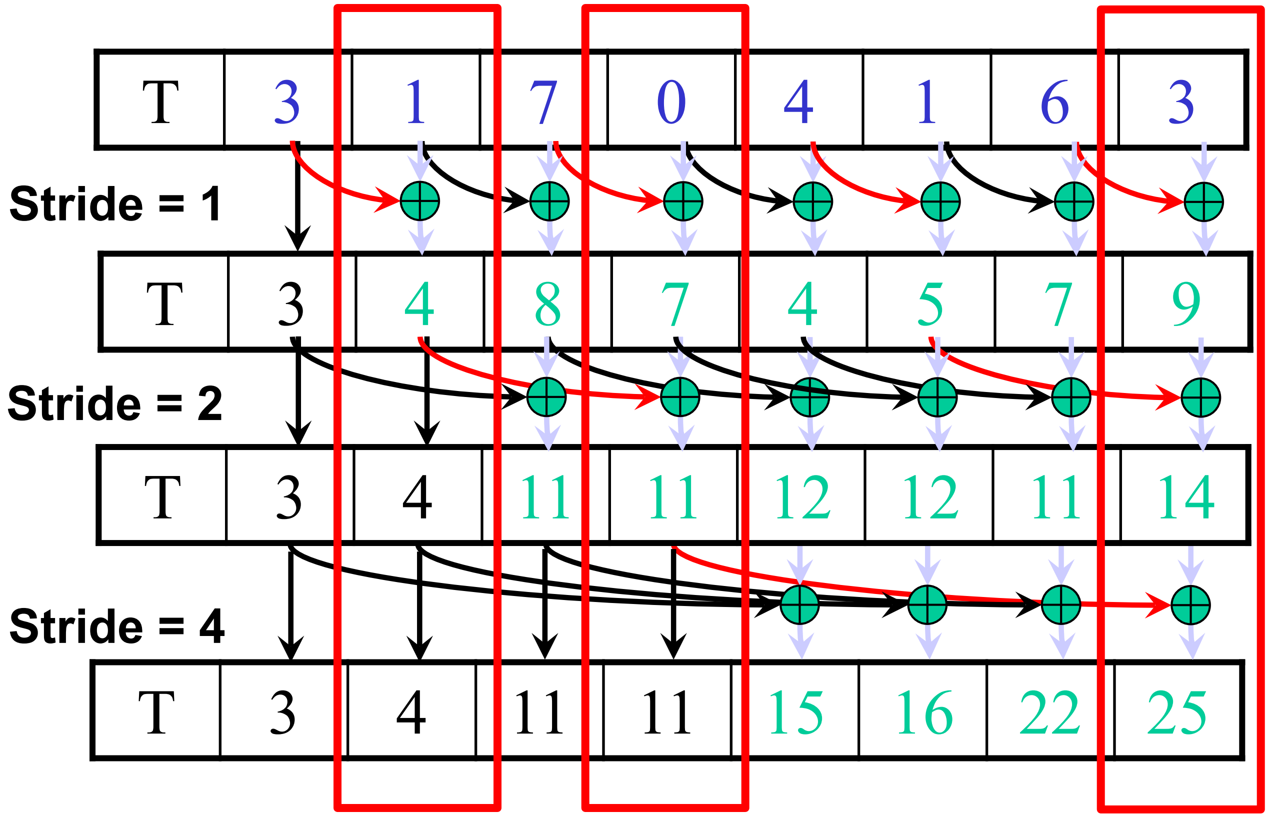
A Kogge-Stone Parallel Scan Algorithm



1. ...
2. …
3. Write output from shared memory to device memory

Iteration #3
Stride = 4

Sharing Computation in Kogge-Stone



Iteration #3
Stride = 4

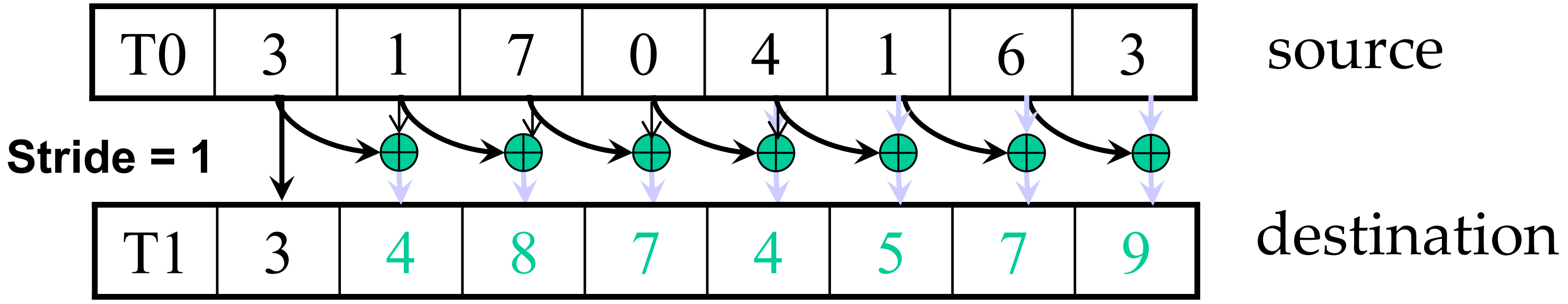
(*Incomplete*) Implementation

```
__global__
void Kogge_Stone_scan_kernel(float *X, float *Y, int InputSize)
{
 __shared__ float XY[SECTION_SIZE];
 int i = blockIdx.x*blockDim.x + threadIdx.x;
 if (i < InputSize) XY[threadIdx.x] = X[i];

 for (unsigned int stride = 1; stride < blockDim.x; stride *= 2) {
   __syncthreads();
   if (threadIdx.x >= stride) XY[threadIdx.x] += XY[threadIdx.x-stride];
 }

 Y[i] = XY[threadIdx.x];
}
```

Double Buffering

- Use two copies of data T0 and T1
- Start by using T0 as input and T1 as output
- Switch input/output roles after each iteration
  - Iteration 0: T0 as input and T1 as output
  - Iteration 1: T1 as input and T0 and output
  - Iteration 2: T0 as input and T1 as output
- This is typically implemented with two pointers, source and destination that swap their contents from one iteration to the next
- This eliminates the need for the second __syncthreads() call

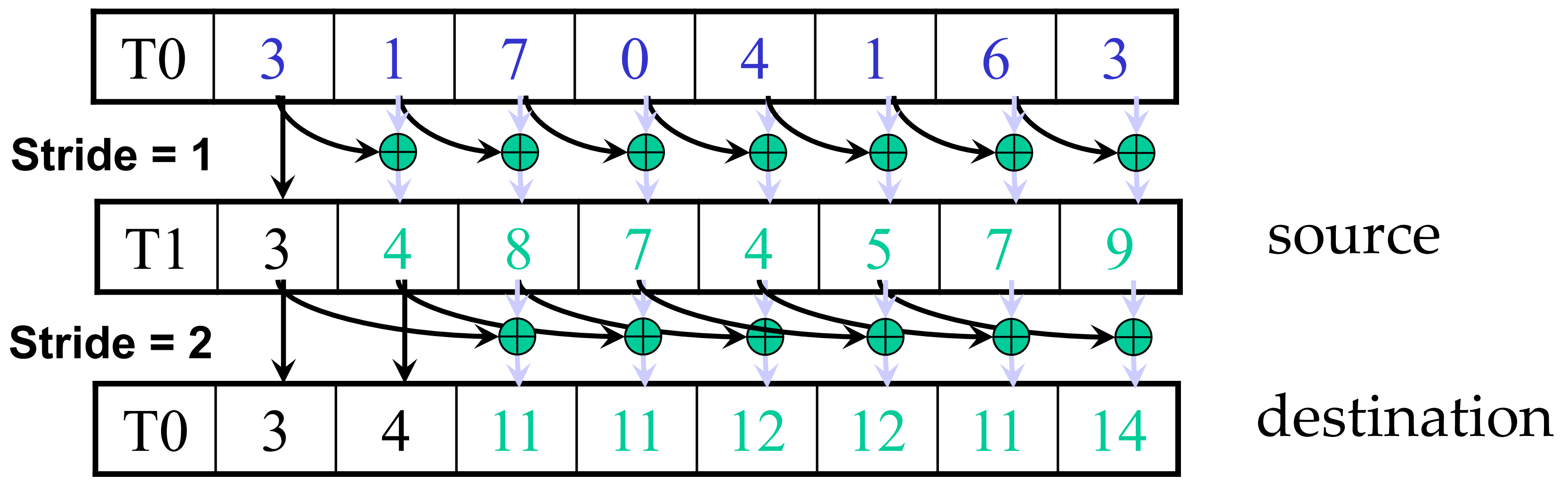
A Double-Buffered Kogge-Stone Parallel Scan Algorithm



Iteration #1
Stride = 1

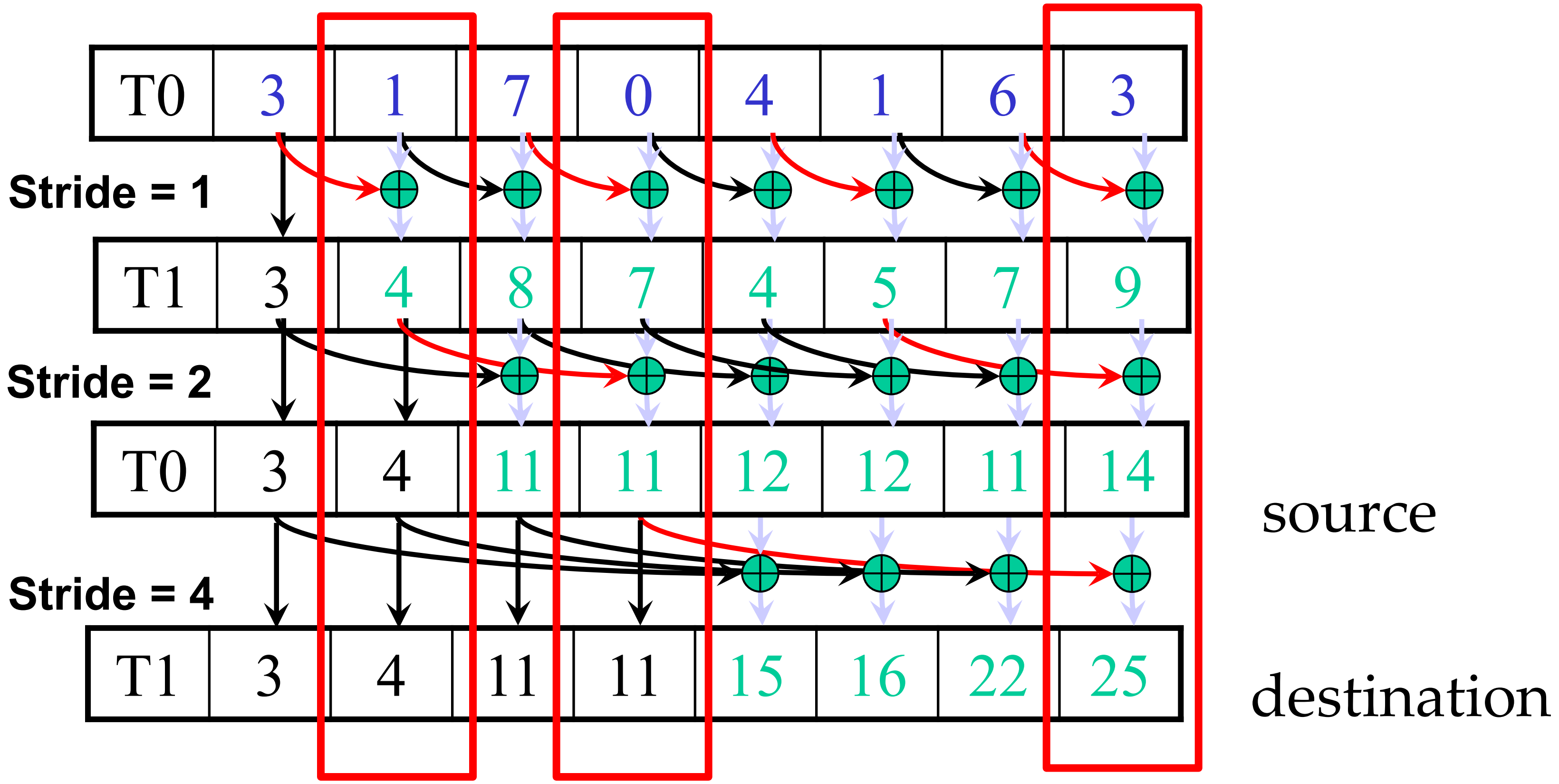
- source = &T0[0]; destination = &T1[0];
- Each iteration requires only one syncthreads()
  - syncthreads(); // make sure that input is in place
  - float destination[*j*] = source[*j*] + source[*j-stride*];
  - temp = destination; destination = source; source = temp;
- After the loop, write destination contents to global memory

A Double-Buffered Kogge-Stone Parallel Scan Algorithm



Iteration #2
Stride = 2

Sharing Computation in a Double-Buffered Kogge-Stone



Iteration #3
Stride = 4

Work Efficiency Analysis

- A Kogge-Stone scan kernel executes log(n) parallel iterations
  - The steps do (n-1), (n-2), (n-4),..(n- n/2) add operations each
  - Total # of add operations: n * log(n) - (n-1) → O(n*log(n)) work

- This scan algorithm is not very work efficient
  - Sequential scan algorithm does *n* adds
  - A factor of log(n) hurts: 20x for 1,000,000 elements!
  - Typically used within each block, where n ≤ 1,024

- A parallel algorithm can be slow when execution resources are saturated due to low work efficiency

ANY MORE QUESTIONS?
READ CHAPTER 8