Quantum Analogs Lab Report
3H03

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1 Abstract

The wave equation of a Quantum Particle in a one dimensional box behaves alike to a standing wave in a tube. Similarly, a wave equation of 1 D solid can also be seen as wave equation of standing waves with band gaps. This experiment aims to study a sound wave in a tube, to measure some characteristics of a sound wave and simulate an quantum object. The speed of sound as a result has been calculated to be 342.7(1) m/s which agrees with the fact that sound waves travels at the speed of approximately 343m/s.

2 Introduction

A particle in a box, refers to the problem in quantum mechanics of an particle trapped in a 1-d potential well. The possibility distribution of the position of particle is defined by the Schrödinger's equation, similar to the wave function of sound wave in a tube.

The goal of this experiment is to simulate particle in a box and a 1-d solid with a sound wave and iris of different diameter. In the experiment a speaker will be used to produce the sound waves in a tube where we can adjust the length and add iris to it to create band gaps, then out put to a microphone connected to the computer. The program SpectrumSLC will be used to identify the waves and picking peak frequencies precisely with an estimated error of 0.001. Apart the excellent accuracy of equipment, it has been tested that direct contact with the tube, table may influence the wave in the tube. To maintain the 0.001 estimated error no direct contact or noises are made during the experiment. We will first use a tube of different lengths to test that the speed of sound is constant for all trials, then add iris of different diameters to check the dependency of that.

3 Data Analysis

Speed of Sound

The speed of sound can be calculated from the formula:

$$2L = n\frac{c}{f} \tag{1}$$

$$c = 2L\frac{f}{n} \tag{2}$$

The Spectrum of one trial is shown below:

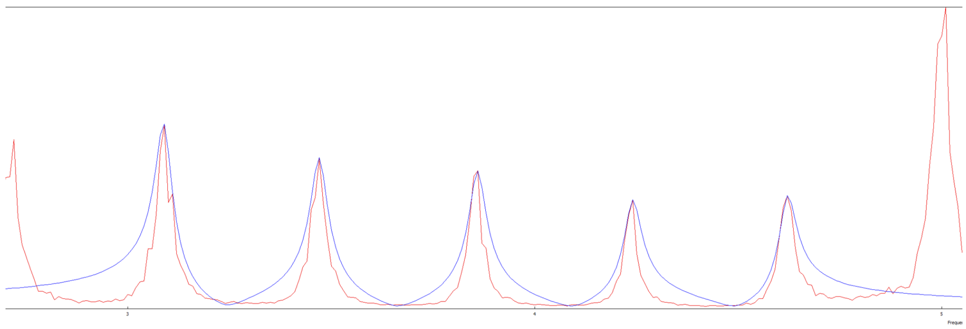

Figure 1: Spectrum of Sound Wave in Tube

The Chosen Peaks are:

Wave Number	Frequency (Hz)
1	2900.786(1)
2	3180.214(1)
3	3473.613(1)
4	3760.026(1)
5	4041.201(1)
6	4325.868(1)

Table 1: Chosen Peaks

An 8 x 7.5cm = 600mm tube is used and the result is f/n = 285.6(9) thus c = 342.7(1)m/s:

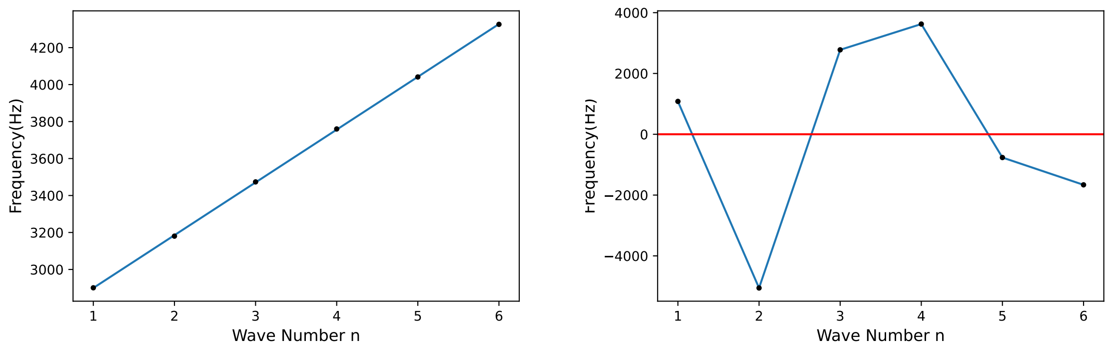

Figure 2: Frequency/wave number plot for 600mm

To make sure the result is accurate, another trial is done the same with tube of 6 x 7.5cm = 450mm.

f/n = 382.9(9) thus c = 344.6(1)m/s

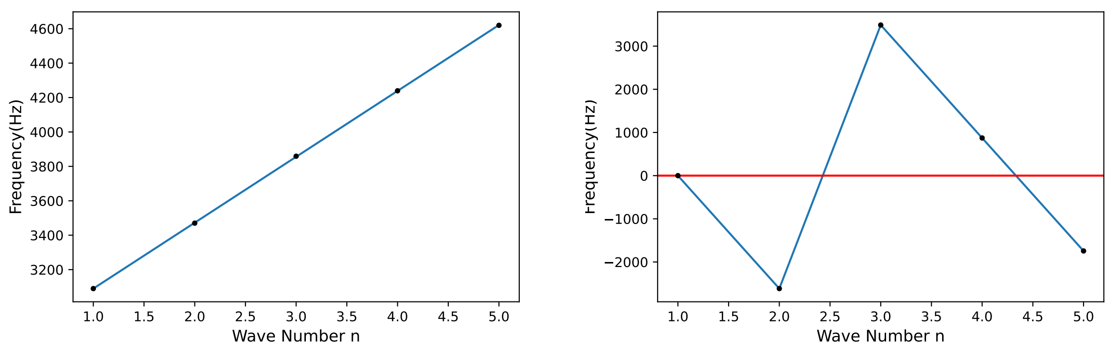

Figure 3: Frequency/wave number plot for 450mm

Dispersion Relationship

The Dispersion relation for the 600mm tube sound wave can be obtained by plotting Angular Frequency $\omega = 2\pi f$ against $k = \frac{n\pi}{L}$:

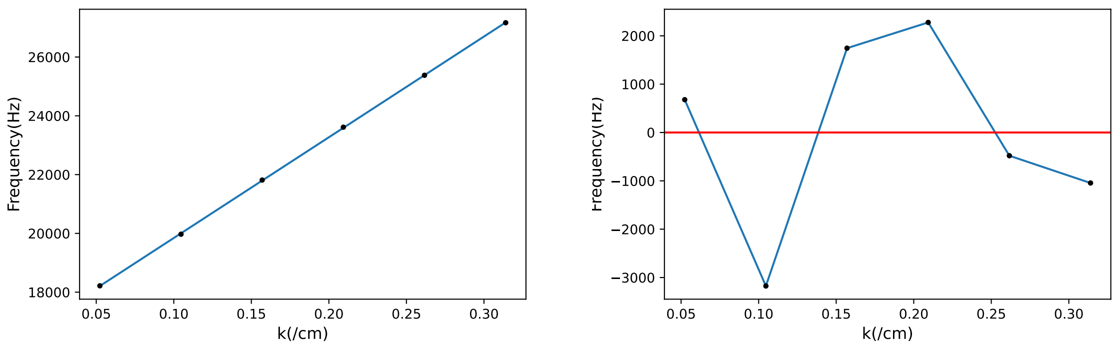

Figure 4: Dispersion Relation plot for 600mm

Modeling 1D Solid

We first plot a comparison of sound waves with 11 Iris of 16mm, 13mm and 10mm in a 12x5cm tube.

It can be observed from the graph that a larger gap exist between the peaks as the Iris narrows. Also by removing the microphone and replace it with a ear, one may hear a high pitch after each gap.:

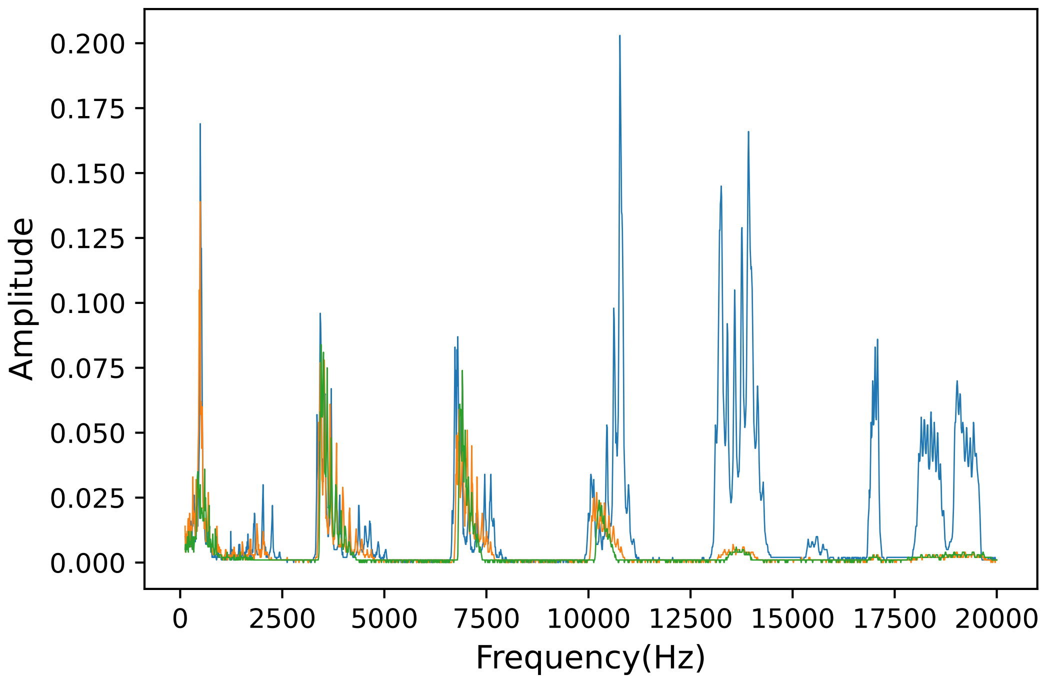

Figure 5: Iris of 16mm(Blue) 13mm(orange) 10mm(green) Comparison

Selecting the gaps from each spectrum, the table below shows the Gap for each:

Gap Number	Initial Frequency (Hz)	Final Frequency (Hz)	Difference (Hz)
1	1780	3380	1600
2	4310	6790	2480
3	7400	10130	2730
4	10680	13360	2680
5	14000	16860	2860

Gap Number	Initial Frequency (Hz)	Final Frequency (Hz)	Difference (Hz)
1	2180	3310	1130
2	4760	6710	1950
3	7700	10020	2320
4	10890	13160	2270
5	14150	16890	2740

Gap Number	Initial Frequency (Hz)	Final Frequency (Hz)	Difference (Hz)
1	2470	3260	790
2	5070	6610	1540
3	7990	9900	1910
4	11240	12960	1720
5	14470	16790	2320

Table 2: 10mm,13mm,16mm from top to below

Plot the gaps against gap number. Now we can clearly see that the gap shifts upward as diameter decreases, and each of them follows a curve of the same shape.

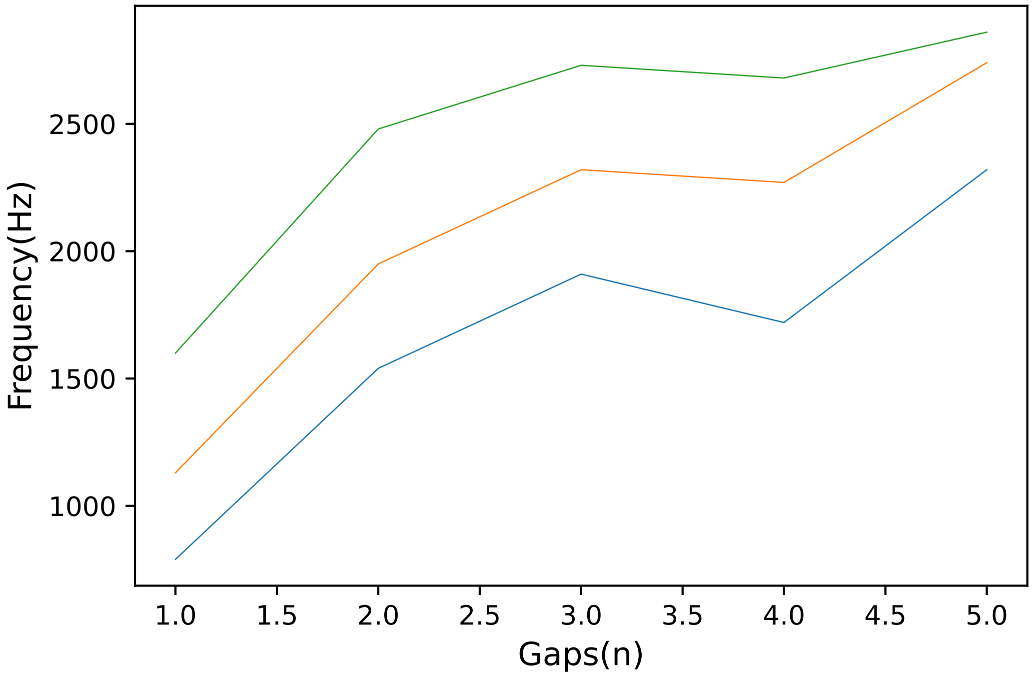

Figure 6: Iris of 16mm(Blue) 13mm(orange) 10mm(green)

4 Discussion

The speed of sound wave determined from peaks and wave numbers is c=342.7(1) m/s. It can be seen that this result is very accurate comparing to the value of speed of sound in 20^oC which is approximately 343m/s.

The solution of a standing sound wave acts very close to a wave function of particle in a 1d box with $\frac{n\pi}{L}$,where n is the wave number for the classical wave and energy level in the quantum case. Both wave functions describes the amplitude or probability of finding the particle at the position at the location.

In the second part. When iris are inserted to the tube, gaps appeared between each peak due to reflection of waves in the tube. Notice that each iris added to the system will create one additional gap. These gaps are observable even by replacing the microphone with ears. The band-gap structure is similar to a solid with lattice structure in 1-D. We study the dependency on diameter of Iris. As a result,when changing the Iris with ones with decreased diameter the frequency of gaps shifts upward. The Iris are analogues to the atomic cores in lattice. And the diameter of Iris represents the potential energy of each where smaller diameter corresponds to larger potential energy. Therefore, explains the increasing band gap lengths between each wave.

5 Conclusion

In conclusion, the experiment has precisely calculated the speed of sound to be 342.7(1) m/s by measuring the peak frequencies. It has also successfully modeled the situation of a particle in a box and a one dimensional solid and showed that the gaps have a positive dependency on the narrowness of Iris.