Experimental Approach to Find the Tunnelling Time of Quantum Particles

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Quantum Tunneling, is the phenomenon that describes quantum particles passing through classically forbidden potentials.

The tunneling time for a quantum particle is hard to measure and is also a controversial subject.[1]

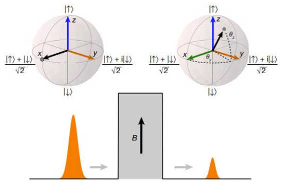
Using a relatively new idea of a "Lamor Clock", a team from Toronto was able to build an experiment with the purpose of studying the time required for quantum tunneling to take place.

Lamor Clock

Lamor Clock is the key idea of the whole experiment.

The researchers use an internal "clock" that ticks only when the particle is in the tunnelling region.[4]

They do this by creating a magnetic field that is localized in the objective region, so that the spin of a particle precesses only when it is tunnelling. One can then tell the time particles spent inside the region by measuring the spin precession.



Experiment Setups

Overall Experiment Setup

A rough sketch is shown below of how the experiment works.

The graph shows 3 beams of laser intersecting each other. 87Rb atoms are first located in the light red beam, when the beam is turned off, sent towards the potential barrier (blue beam) through the red beam connected by a Stern-Gerlach experiment to measure the results.

Optical Dipole Trap:

The beams shown in the graph are called optical dipole trap, a fundamental tool for trapping and guiding particles for the experiment to proceed.

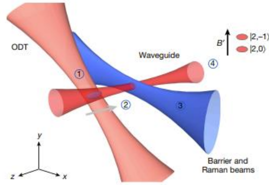
Each beam labeled red or blue in the graph functions differently.

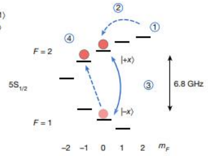
Red Detuned Beam

Beams labeled in the color red represents a frequency tuned below the atomic transition frequency. The dipole force points towards increasing intensity, thus can be used to storage or guide Neutral Atoms.

Blue Detuned Beam

Beams labeled in the color blue represents a frequency above the atomic transition frequency. In this case the beam repulses the Atoms, resulting in a potential barrier.



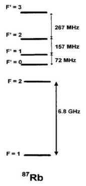


Energy Level of 87Rb Atom

To simulate a pseudo-two-level spin-1/2 system with 87Rb atoms, the atoms are initially transferred to the |2,0⟩ state. Setting the blue beam at 6.8GHz, couples the |2,0⟩ and |1,0⟩ state of the particle.

To make a measurement through the Stern-Gerlach experiment, we must again transfer the particles in |1,0⟩ state to the |2,-1⟩ state.

We can then measure the populations of |2,0⟩ and |1,0⟩ states.

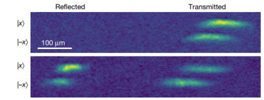


Pseudo-Magnetic Field

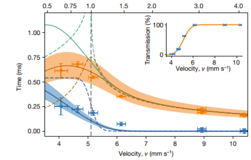
The "magnetic field" in this experiment is also generated from the raman beam, acting on the two level atom analogously to how magnetic field acts on a spin.

Results

After interacting the potential barrier, atoms are either transmitted or reflected in a superposition of |2,0⟩ and |1,0⟩ state.



The researchers performs full-spin tomography to probe the Lamor Times. They can then measure the spin components.



In their latest paper in 2021, they have showed a surprising and counterintuitive fact that tunneling takes less time when tunneling higher potential barriers.

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[1]R. Ramos, D. Spierings, I. Racicot, and A. M. Steinberg, Measurement of the time spent by a tunnelling atom within the barrier region, Nature 583, 529 (2020).

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Acknowledegement

Thank you Dr. O'Dell for guiding me through this project.

Literature Review on Time Required for Quantum Tunneling to Take Place
Physics 4L03

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1 Abstract

One of the most fascinating discoveries in quantum mechanics is the phenomenon quantum tunneling, which reveals the fact that in the quantum regime a classically unapproachable potential barrier for a particle is no longer forbidden. Despite many aspects of quantum tunneling has been well understood for it being discovered and studied over decades, the time it takes for the process to happen remains controversy. The reason is due to no strict consensus of this time has been defined and no definite trajectories can be used to describe quantum particles. A team from the University of Toronto leaded by Dr.Aephraim M. Steinberg has published a series of papers on their experiments about this topic. Since 2019, they have conducted a number of experiments using a "Larmor CLock" to seek for a result. Their most recent experiments have showed that the tunneling of a particle takes less time when the probability is less.

2 Introduction

One way suggested to determine the tunneling time of a particle in the early times is called the "Arrival Time". Researchers at that time intends to find the group delay of wave packets, focusing on defining the moment the peaks of transmitted particles appear on the other side of barrier. Their results turns out being equal or smaller than zero.

Later, scientist proposed that the time of quantum-mechanical collision events can be measured using the Larmor Precession as a clock. In 1980s, the term "interaction time" was first introduced by Büttiker and Landauer. [4] This term describes the amount of time a particle actually spent interacting with the forbidden region in classical Physics. On that purpose, many experiments have been done leading to an idea of the Larmor Clock. The Larmor Clock comes from the thought of a clock that advance exceptionally when the particle is located in the region of interest. This is possible by implementing the Larmor precession as an unique intrinsic clock of particles. The spin of particle will precess only when it's position is within a uniform magnetic field. One may tell the duration particles within an objective region by setting a magnetic field in the region and measure the spin precession of particle after interaction.

On the purpose of studying the interaction time, the authors aimed on building an experiment implementing the Larmor clock. A rough interpretation of the Larmor clock experiment is shown below. By Larmor precession, the spin of particle will precess around the z-axis with a Larmor frequency ω_L when presented in a uniform field. For a population of particles all polarized to a certain direction, the time they spent interacting the region may be studied by looking at their states after interaction.

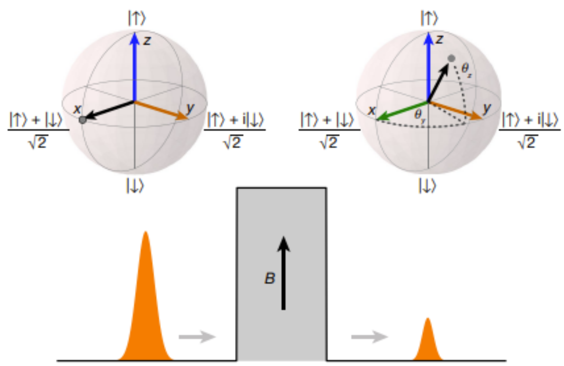

Figure 1: A rough interpretation of the Lamor Clock. The spin of particle precesses around the z-axis after interacting with the potential barrier.

3 Theories

Weak Value Measurement

To fully present the theory behind this experiment, I shall first introduce the weak value measurement. The weak value is a kind of value for quantum variables that weirdly appeared from measurements under certain conditions. This value differs from a weak measurement, where the term "weak" in the weak value actually defines a weakness found in usual measurement procedure with pre- and post-selected ensembles of quantum systems.

In the paper "How the Result of a Measurement of a Component of the Spin of a Spin- 2 Particle Can Turn Out to be 100", Yakir Aharonov, David Albert and Lev Vaidman first discussed weak value found in measurements in the year 1988[5]. In their paper, the authors reported some unusual values nowhere near the range of expected results and defined it as the weak value.

For an arbitary observable A with pre-selected state Φ_i and post selected state Φ_f, the weak value of A is defined by:

$$A_w \equiv \frac{\langle\psi_f|\,A\,|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle} \tag{1}$$

The actual meaning and the natural of weak values are yet too abstract to be fully understood and being controversial. But, it has been successfully instrumented as a useful tool in many fields such as quantum metrology and tomography. It is certain to say that the weak value is a reliable tool for the measurements in this paper.

Larmor Precession as an indicator of Time

In 1983 M. Buttiker intoduced a way of indicating time for quantum particles, later known as Larmor times.

They defined a dwell time τ_d as the average tunneling time for all particles no matter reflected or not, a traversal timeτ_T as the tunneling time of the transmitted particles and a reflection time τ_R the reflected particles.

Let's first recall Larmor Precession.[Cite Griffiths] The Hamiltonian of a charged particle in a magnetic field:

$$H = -\mu B = -\gamma BS \tag{2}$$

In a uniform field pointing in z direction:

$$B = B_0 \hat{k} \tag{3}$$

As a result, the spin of particle precesses around the z-axis with a frequency:

$$\omega_L = \gamma B_0 \tag{4}$$

Define τ_y the time a particle takes to traverse, θ_y the precession angle around z-axis on the bloch sphere:

$$\theta_y = \tau_y \omega_L \tag{5}$$

The time could be measured not perturbing the particle in the limit of a weak magnetic field where $\omega_L \longrightarrow 0$, with a measurement back action:

$$\theta_z = \tau_z \omega_L \tag{6}$$

The spin in the infinitesimal field:

$$\langle S_y \rangle = -\frac{\hbar}{2}\omega_L \tau_y \quad \langle S_z \rangle = \frac{\hbar}{2}\omega_L \tau_z \tag{7}$$

Existence Operator Θ_B

With no background information, one may find it confusing to read the papers published by Dr. Steinberg, specifically around a projector Θ_B that is not much defined in the papers. This operator was introduced by him about 25 years ago, where he discuss little about it since then. Here, we provide a rough explanation to this operator.

In 1995, Aephraim M. Steinberg published one of his earliest paper in the field of tunneling time. He argued that the greatest reason the topic being controversial is due to a lack of quantum mechanical time operator. With the results of the weak values, he intelligently proposed an operator Θ_B that measures the existence of particles.[3] This operator has eigenvalues only 0 or 1, depending on if the particle exists within the barrier region

$$\Theta_B = \int_{y_1}^{y_2} dy \left|y\right\rangle \left\langle y\right| \tag{8}$$

This automatically satisfied the equation where they considered a main criteria of tunneling times when defined in terms of weak values:

$$|t|^2 \tau_T + |r|^2 \tau_R = \tau_d \tag{9}$$

By definition it can be easily understood that the dwell time is the integral over all time of this operator acting on the state:

$$\tau_d = \int_{-\infty}^{\infty} \left\langle \psi\right| \Theta_d \left|\psi\right\rangle \, dt \tag{10}$$

The spin of each particle can be used as a pointer that determines the existence of the particle, by coupling with the projector in the interaction Hamiltonian:

$$H_{int} \sim SB \sim S_z B_0 \Theta_d, \tag{11}$$

Also, the weak value of this operator is equivalent to the Larmor times.

4 Experiment Apparatus

A plot of the experiment apparatus is shown below, the authors built an pseudo-two-level spin-1/2 system using Bose-Einstein condensate 87Rb atoms of the state $|2,0\rangle$ and $|1,0\rangle$ to modify a two-level spin system with spin states $|+x\rangle$ and $|-x\rangle$. 87Rb atoms are initially transferred to the $|2,0\rangle$ state in the light red beam, then send towards to blue beam throgh the wave guide. Setting the blue beam at 6.8GHz, couples the $|2,0\rangle$ and $|1,0\rangle$ state of the particle. After the atoms interact with the blue beam, a Stern-Gerlach is connected afterwards to separate the spin states. But before that, the $|1,0\rangle$ states must be one more time transformed to $|2,-1\rangle$ for the Stern-Gerlach to be able to function. As a result, populations of $|+x\rangle$ states and $|-x\rangle$ states are collected from the experiment.

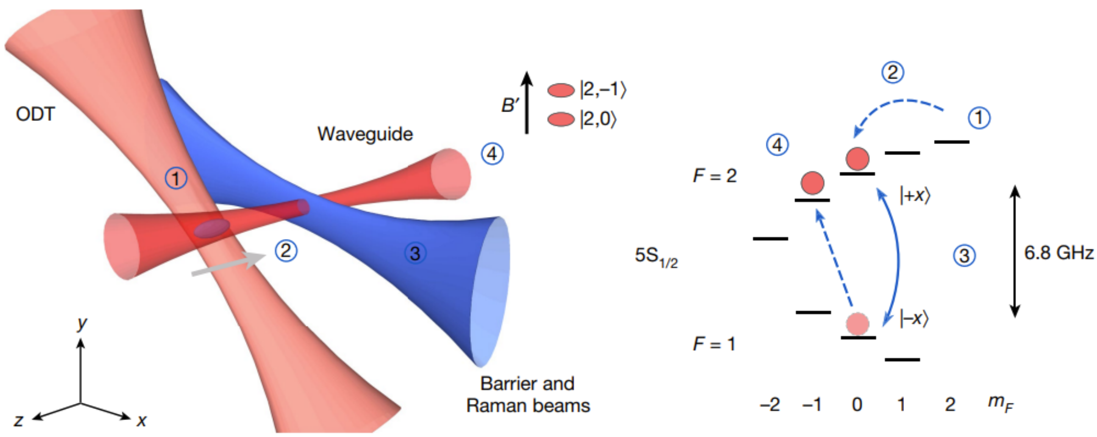

Figure 2: (a) Cross Dipole Trap. Atoms are pushed along the waveguide towards the blue-detuned beam which generates the potential barrier. (b) Energy level of 87Rb atoms. Atoms were first all transformed to $|2,0\rangle$ state before entering the system.

Dipole Trap

The beams shown in the graph are called optical dipole trap, a fundamental tool for trapping and guiding particles for the experiment to proceed. Each beam in the graph is labeled in different colors due to the frequency they are set to, and functions differently.

–Red-Detuned Beams:

Beams labeled in the color red represents a frequency tuned below the atomic transition frequency. The dipole force points towards increasing intensity, thus can be used to storage or guide Neutral Atom.

–Blue-Detuned Beams:

Beams labeled in the color blue represents a frequency above the atomic transition frequency. In this case the beam repulses the Atoms, resulting in a potential barrier.

Barrier Beam

The barrier beam plays a vital role in this experiment. Before explaining the details of the barrier, let me introduce the AC Stark effect.

Similar to the Zeeman effect, where under a magnetic field the spectral lines of atoms splits and shifts into many components, the spectral lines of atoms undergoes similar process when presented in a electronic field. This is known as the stark effect. The Autler–Townes effect or AC Stark shift is one stark effect, that when electric field is in resonance to the transition frequency changes the absorption spectra.

The barrier beam used in this experiment is set to be 421.38 nm. This wavelength is about 1.8 THz below (red-detuned) the 6P3/2 transition, 0.5 THz above (blue-detuned) the 6P1/2 transition, and 330 THz above the 5P3/2 transition. The AC Stark shift of these together formed a repulsive interaction, making a potential barrier.

The $|+x\rangle$ state ($|2,0\rangle$ state of 87Rb atom) and $|-x\rangle$ state ($|1,0\rangle$ state of 87Rb atom) are coupled together by phase-modulating the barrier beam at the 6.8-GHz, also generating the pseudo-magnetic field.

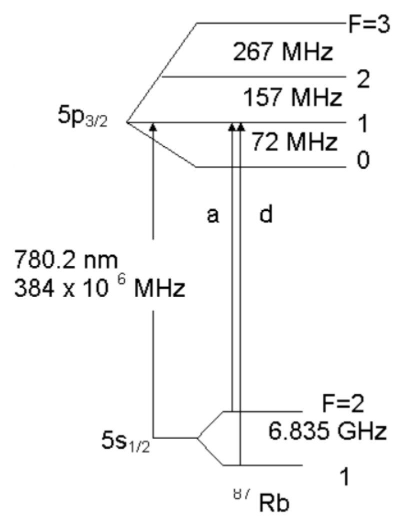

Figure 3: Energy level diagram of 87Rb atom

5 Results and Data Analysis

As a result, the authors measure from the experiment either $\langle S_x \rangle$, $\langle S_y \rangle$, or $\langle S_z \rangle$ of the net magnetization vector of the transmitted atomic cloud. Counting the populations of $|+x\rangle$ states and $|-x\rangle$ states gives $\langle S_x \rangle$ directly. Measurements of $\langle S_y \rangle$ and $\langle S_z \rangle$ requires a rotation by the z-axis and y-axis by 90 degrees before measuring.

Larmor Time Calculation

In an infinitesimal field the Larmor times τ_y and τ_x can be written as follows, for V_0 the energy of barrier and ϕ the phase of transition amplitude. [6] :

$$\tau_y + i\tau_z = -\hbar \frac{\delta \phi}{\delta V_0} + i\hbar \frac{\delta lnt}{\delta V_0} = i\hbar \frac{\delta lnt}{\delta V_0} \tag{12}$$

$$\theta_y = arctan(\langle S_y \rangle \,/\, \langle S_x \rangle) \tag{13}$$

In the limit of infinitesimal perturbations:

$$\theta_z \sim \alpha_z = arctanh(\langle S_z \rangle) \tag{14}$$

An effective Larmor frequency Ω calibrated from rotation angles experienced by high-velocity atomic clouds is defined by:

$$\theta = \Omega \int_{-\infty}^{\infty} G(y) / \sqrt{v^2 - v_b^2 G(y)}\, dy \qquad G(y) = e^{-2y^2/\sigma^2} \tag{15}$$

for θ the rotation angle, $G(y)$ the Gaussian profile of the barrier along the waveguide direction, $v$the velocity of the cloud, and v_b the velocity that matches height of barrier.

Experiment Results

The figure below shows the data of one experimental run.

Figure.a shows the transmission probability over barrier height for a wave packet with incident velocity $v^* = 4.26(6)$mm/s measured by varying the height of potential barrier. Velocity is selected lower than barrier height to ensure that transmission is dominated by tunneling for the least energies measured. This graph also calibrates the height and characterized the velocity width.

Figure.b shows the population of rotation angles after interaction. The plots on the left and right represents 2D projections of the 3D Bloch sphere. Velocity of particles are marked green and red from fast to slow. We can observe from the left plot that particles with higher speed precess less in angle towards the y direction, as faster particles would spend less time interacting the region.

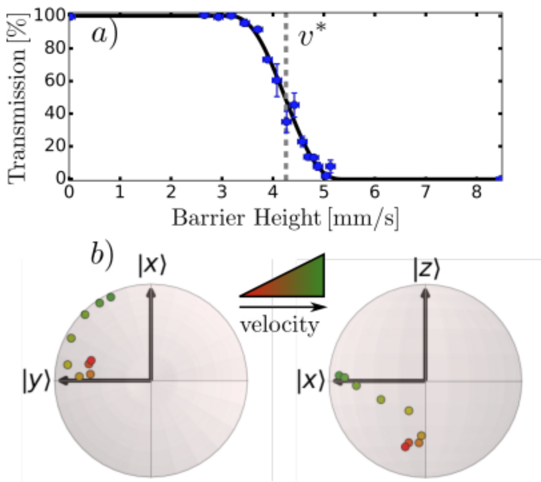

Figure 4: Experimental results for one run. (a) The transimission profile over a range of barrier heights for particles with incident velocity 4.26(6)mm/s. (b) Populations of Bloch vectors after interaction. Velocity are marked green for higher incident velocities and red for lower incident velocities.

The Larmor times over incident velocity for 2 different heights is shown in the graph below. The plot is represented in a very complex way, containing many information. Here I will briefly explain the plot in a few words. The vertical dash lines represents the barriers of different heights: a lower barrier of 4.13(8) mm/s in blue, and a higher barrier of 4.71(5) mm/s in red. Two sets of experiment are done for different velocity interacting the 2 barriers shown in red dots and blue dots for each barrier. The tunneling time τ_y is represented in solid markers and measurement back action τ_z is represented in faded markers. The color bands corresponds to GP simulations for the corresponding curves and colors.

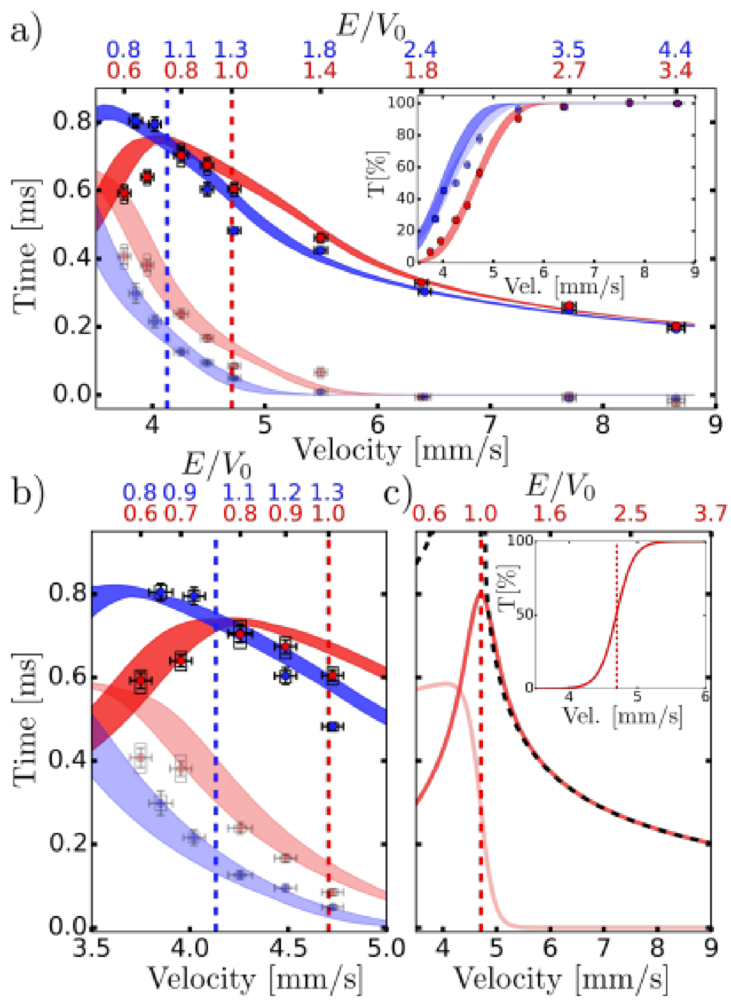

Figure 5: (a) The Larmor times over incident velocity (or E/V_0). (b) The low energy region of a. (c) Larmor times for the 4.71 mm/s barrier by Monochromatic weak-value calculations.

The Gross–Pitaevskii (GP) named after Eugene P. Gross and Lev Petrovich Pitaevskii simulations is a theoretical calculation used to compare with the experiment results. The equations they used to perform the GP simulation is provided in the paper[2].

In the graph, the GP simulation is represented in the color bands. We can see a clear agreement of the GP bands with the experiment data. The important conclusions one can draw from this is that τ_z does not go to zero when velocity is above the barrier. This is contradictory to the Monochromatic weak-value calculations shown in the Figure 5.c.

Now, we look at the plot 5.b, which is the cut off of 5.a in the low energy region. A counter intuitive conclusion could be drawn from this part of the graph that in the region of very low incident velocity, τ_y tends to be smaller for lower velocity, or in other words higher potential barrier. This is a very surprising result as one would expect the particle to spend more time within the region when it has less energy to tunnel through the barrier. The meaning of this result in the real world may be difficult to comprehend, due to the limitations of human beings to observe the quantum world. It is not able for us to obtain an transparent picture of what is actually going on in the system. We can only rely on observable that interacts the system and feed backs certain information to make a judgement on what is happening. Thus, I suppose a deeper investigation of this trait may contribute to our understandings to the quantum system.

6 Conclusion

To sum up, with the idea of Larmor Clock and weak value measurements Dr. Steinberg and his team has successfully built an apparatus to simulate a spin-1/2 particle system interacting a potential barrier where they can study the time taken for the particles to interact the classically forbidden region, without having to determine the trajectories of quantum particle or where they actually are. Instead of actual spin-1/2 particles the system is simulated by confining 87Rb atoms into 2 states, but we need not to worry about that as to our understanding so far both systems behaves in the same way under the conditions.

In the experiment they have observed that τ_y decreases below the potential barrier, indicating transmissions are dominated by tunneling below the barrier for a Gaussian barrier. From increase of τ_y in lower barriers, they conclude that atomic interactions effect little to their measurement. Most importantly, their experiment has proved the surprising fact that tunneling takes less time when it is probable.

Their measurement is the first success in history to apply the Larmor clock in real world experiment and first direct measurement to time of populations of particles interacting potential barrier. In their paper they have stated that they have not yet stop their investigations. More experiments could be done with the same apparatus or based on the same idea, and may even contribute towards understanding the fundamental problems of quantum mechanics unsolved.

7 References

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