UCB CS70 HW 00

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unique

Propositional Practice

exists

(a) $(\forall k \in \mathbb{R})[(\exists x \in \mathbb{R})(x^3 = k)$ $\wedge (\forall y, z \in R)(((y^3=k) \wedge (z^3=k)) \Rightarrow (y=z))]$

(b) $(\forall p \in \mathbb{N}) \rightarrow$ $P(p) \implies [(\forall a \in \mathbb{N})(\forall b \in \mathbb{N})\,(p \nmid a \wedge p \mid ab) \implies (p \mid b)]$ $(P \nmid a) \equiv \neg(P \mid a)$

$\rightarrow (\forall a, b \in N)$

(c) For every real numbers x,y if xy = 0 then x = 0 or y = 0.

(d) It's not that there exists a natural number y, for all natural number x, if x > y, then y divides x or x is a prime number. $\rightarrow$ understand

too "math" $\rightarrow$ more oral

Solution:

(a) The trickiest part of this problem is the word 'unique'. We can express the existence of a unique solution in propositional logic with two statements connected with an 'and': (1) A solution exists, and (2) Any two solutions have to be the same. Hence, we can rewrite this statement as "For every real number k, there exists a real number x such that $x^3 = k$ and for all reals y and z, if both $y^3 = k$ and $z^3 = k$, then $y = z$." This, in propositional logic, is below:

$$(\forall k \in \mathbb{R})\left[(\exists x \in \mathbb{R})(x^3 = k) \wedge (\forall y, z \in \mathbb{R})(((y^3 = k) \wedge (z^3 = k)) \implies (y = z))\right].$$

(b) This sentence can be written in propositional logic as

$$(\forall p \in \mathbb{N})\,[(P(p)) \implies ((\forall a, b \in \mathbb{N})\,[((p \mid ab) \wedge \neg(p \mid a)) \implies (p \mid b)])].$$

(c) If the product of two real numbers is 0, then one of them must be 0.

(d) There is no natural number that divides every composite number greater than it.