2.-

Considere el modelo de regresion

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 \left(3x_i^2 - 2\right) + \xi_i \quad i = 1, 2, 3$$

donde

$$x_1 = -1, x_2 = 0, x_3 = 1$$

I) Matriz de diseño

$$X = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & -2 \\ 1 & 1 & 1 \end{pmatrix} \text{ pues } \begin{array}{c} 3x_1^2 - 2 = 1 \\ 3x_2^2 - 2 = -2 \\ 3x_3^2 - 2 = 1 \end{array}$$

Asi

$$X^t X = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & -2 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

y

$$\left(X^t X\right)^{-1} = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/6 \end{pmatrix}$$

II)

$$\hat{\beta} = (X^t X)^{-1} X^t y$$

asi:

$$\hat{\beta} = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/6 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ -1/2 & 0 & 1/2 \\ 1/6 & -1/3 & 1/6 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$\hat{\beta} = \begin{pmatrix} \frac{1}{3}\left(y_1 + y_2 + y_3\right) \\ \frac{1}{2}\left(y_3 - y_1\right) \\ \frac{1}{6}\left(y_1 - 2y_2 + y_3\right) \end{pmatrix}$$

Por lo tanto

$$\hat{\beta}_0 = \frac{1}{3}\left(y_1 + y_2 + y_3\right) \quad \hat{\beta}_1 = \frac{1}{2}\left(y_3 - y_1\right) \quad \hat{\beta}_2 = \frac{1}{6}\left(y_1 - 2y_2 + y_3\right)$$

III) Obtenemos los estimadores del modelo reducido:

$$y_i = \beta_0^* + \beta_1^* x_i + \xi_i^* \quad i = 1, 2, 3$$

Obtenemos

$$\left(X^t X\right)^{-1} = \begin{pmatrix} 1/3 & 0 \\ 0 & 1/2 \end{pmatrix}$$

$$\rightarrow \hat{\beta}^* = \begin{pmatrix} 1/3 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ -1/2 & 0 & 1/2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$\hat{\beta}^* = \begin{pmatrix} \frac{1}{3}\left(y_1 + y_2 + y_3\right) \\ \frac{1}{2}\left(y_3 - y_1\right) \end{pmatrix}$$

$$\text{Asi } \hat{\beta}_0^* = \frac{1}{3}\left(y_1 + y_2 + y_3\right) \text{ y } \hat{\beta}_1^* = \frac{1}{2}\left(y_3 - y_1\right)$$

$\therefore \hat{\beta}_0^* = \hat{\beta}_0$ y $\hat{\beta}_1^* = \hat{\beta}_1$