# Ejercicio 1

Equipo

2024-03-26

## 1. Regresión a través del origen.

$$y_i = \beta x_i + \xi_i \quad i = 1 \dots n$$

donde $\xi_1, \dots, \xi_n$ son v.a.i. talque $\xi_i \sim N\left(0, \frac{\sigma^2}{w_i}\right)$

$$\forall i = 1 \dots n$$

Suponiendo $\sigma^2$ conocida y $\omega_i = \frac{1}{x_i^2} \quad i = 1, \dots, n$ I) Como las $\xi_i$ son normales, entonces $y_i \sim N\left(\beta x_i, x_i^2 \sigma^2\right)$ y son independientes, entonces la funcion de verosimilitud nos queda:

$$\prod_{i=1}^{n} \frac{1}{\sqrt{2\pi} x_i \sigma} e^{-\frac{(y_i - \beta x_i)^2}{2 x_i^2 \sigma^2}}$$

es decir:

$$\frac{1}{(2\pi)^{n/2} x_i^n \sigma^n} e^{\sum_{i=1}^{n} \frac{-(y_i - \beta x_i)^2}{2 x_i^2 \sigma^2}}$$

Aplicando logaritmo:

$$\ln(1) - \frac{n}{2} \ln(2\pi) - n \ln\left(x_i\right) - n \ln(\sigma) + \sum_{i=1}^{n} \frac{-\left(y_i - \beta x_i\right)^2}{2 x_i^2 \sigma^2}$$

derivando e igualando a cero obtenemos:

$$\frac{d}{d\beta} \ln(f) = -\sum_{i=1}^{n} \frac{\left(y_i - \beta x_i\right)}{x_i^2 \sigma^2} \left(-x_i\right) = 0$$

$$\rightarrow \sum_{i=1}^{n} \frac{\left(y_i - \beta x_i\right)}{x_i \sigma^2} = 0 \rightarrow \sum_{i=1}^{n} \frac{y_i}{x_i \sigma^2} - \sum_{i=1}^{n} \frac{\beta}{\sigma^2} = 0$$

Asi

$$\sum_{i=1}^{n} \frac{y_i}{x_i \sigma^2} = \frac{n\beta}{\sigma^2} \rightarrow \hat{\beta} = \sum_{i=1}^{n} \frac{y_i}{x_i n}$$

II)

$$
\begin{aligned}
&\operatorname{Var}(\hat{\beta}) \\
&= \operatorname{Var}\left(\sum_{i=1}^{n} \frac{y_i}{x_i n}\right) = \frac{1}{n^2} \cdot \operatorname{Var}\left(\sum_{i=1}^{n} \frac{y_i}{x_i}\right) \\
&= \frac{1}{n^2} \sum_{i=1}^{n} \operatorname{Var}\left(\frac{y_i}{x_i}\right) = \frac{1}{n^2} \sum_{i=1}^{n} \frac{1}{{x_i}^2} \operatorname{Var}\left(y_i\right) \\
&= \frac{1}{n^2} \sum_{i=1}^{n} \frac{1^2}{x_i^2}\left(x_i^2 \sigma^2\right) = \frac{1}{n^2} \sum_{i=1}^{n} \sigma^2 = \frac{\sigma^2}{n} \\
&\therefore \operatorname{Var}(\hat{\beta}) = \frac{\sigma^2}{n}
\end{aligned}
$$

III) Como $\hat{\beta} = \sum_{i=1}^{n} \frac{y_i}{x_i n}$, Sea $c_i = \frac{1}{x_i n}$ entonces

$$\hat{\beta} = \sum_{i=1}^{n} c_i y_i \quad \therefore \text{ es estimador lineal}$$

Ademas

$$
\begin{aligned}
\mathbb{E}(\hat{\beta}) = \mathbb{E}\left(\sum_{i=1}^{n} \frac{y_i}{x_i n}\right) &= \frac{1}{n} \sum_{i=1}^{n} \mathbb{E}\left(\frac{y_i}{x_i}\right) \\
&= \frac{1}{n} \sum_{i=1}^{n} \frac{1}{x_i} \mathbb{E}\left(y_i\right) = \frac{1}{n} \sum_{i=1}^{n} \beta = \beta
\end{aligned}
$$

De esta forma $\hat{\beta}$ es estimador lineal y ademas es insesgado, por el teorema Gauss - Markoy $\hat{\beta}$ es el UMVUE