Ejercicio 1

Equipo

2024-03-26

1. Regresión a través del origen.

$$y_i = \beta x_i + \xi_i \quad i : 1, \ldots, n$$

donde $\xi_1, \ldots, \varepsilon_n$ son v.a.i talque $\xi_i \sim N\left(0, \frac{\sigma^2}{w_i}\right) \forall i = 1, \ldots, n$ Suponiendo σ^2 conocida y $w_i = \frac{1}{x_i^2} \quad i = 1, \ldots, n$

I)

$$\sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} (y_i - \beta x_i)^2$$

$$\frac{d}{d\beta} = 2\sum_{i=1}^{n} (y_i - \beta x_i)(-x_i) = 0$$

$$\rightarrow \sum_{i=1}^{n} (y_i - \beta x_i)(-x_i) = 0 \rightarrow \sum_{i=1}^{n} y_i (-x_i) + \sum_{i=1}^{n} \beta x_i^2 = 0$$

$$\therefore \hat{\beta} = \frac{\sum_{i=1}^{n} y_i x_i}{\sum_{i=1}^{n} x_i^2}$$

II) $\text{Var}(\hat{\beta})$

$$\text{Var}(\hat{\beta}) = \text{Var}\left(\frac{\sum Y_i X_i}{\sum X_i^2}\right)$$

$$= \text{Var}\left(\frac{\sum X_i (\beta X_i + \xi_i)}{\sum X_i^2}\right)$$

Continúa. . . .

5+5

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