Assignment 2:

ESS 318

Date 25/04/2025

Question 1;

Required to solve the ODE

$$\frac{d^\wedge 2v(t)}{dt^2} + \frac{1}{RC}\frac{dv(t)}{dt} + \frac{v(t)}{LC} = 0$$

This is the second order homogeneous differential equation with the standard form

$$\frac{ad^{2v}}{dt^2} + \frac{bdv}{dt} + cv = 0$$

Where a=1, b=1/RC and c=1/LC

Then forming the characteristic equation as below,

$$s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$$

And solving it using quadratic formula then

$$s = \frac{-1/RC \pm \sqrt{(\frac{1}{RC})^2 - 4/LC}}{2}$$

Depending on the discriminant

- If positive ;two real roots
- If zero; repeated real roots
- If negative; complex conjugate roots

Thus the general solution when there are complex roots is,

$v(t) = e^{-\alpha t}$ (Acos (βt) +Bsin (βt)

Question 02

Given the block diagram and required to find the closed loop transfer function

Having the blocks G1, G2, G3 and H1

Forward path transfer function is given as

G=G1G2G3

Feedback structure is a single feedback around G1G2 with H1 so

$$Greduced = \frac{G1G2}{1 + G1G2H1}$$

Now attaching G3 forward, then

$$T(s) = Greduced \times G3 = G1G2G3/(1 + G1G2H1)$$

Question 03

Required to design the value of k for the feedback system so that it responds with 10% overshoot

Overshoot formula for second order systems is given as

%OS=$e^{(-\pi\gamma/\sqrt{1-\gamma^2}}$ ×100

Where %OS=10

And γ stands for damping ratio and is obtained as $\gamma \approx 0.591$

Characteristic equation, given transfer function as

$$\frac{-K}{(s(s+5)+K)}$$

While the standard second order form is

$$s^2 + 2\gamma\omega ns + \omega^2$$

Where on comparing

- $2\gamma\omega n = 5$
- $\omega n^2 = k$

Thus, $\omega n = \frac{5}{2\gamma} = \frac{5}{2} \times 0.591 = 4.227$

Now $K = \omega^{2n} = (4.227)^2 = 17.86$

Thus the value of K is 17.86

Question 4

Required to determine the stability of a closed loop transfer function that is

$$T(s) = \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3}$$

Step 1: Write the characteristic equation:

S^5+2s^4+3s^3+6s^2+5s+3=0

Step 2: Build the Routh Array:

The first two rows are filled directly from the coefficients:

Power of s	coefficients
S^5	1 3 5
S^4	2 6 3

Now compute the next rows:

For s^3 row:

First element:

(2)(3)−(1)(6)/2=6−6/2=0

Second element:

(2)(5)−(1)(3)/2=10−3/2=7/2=3.5

Third element: since there are no more coefficients to pair, it's 0.

Thus:

| s^3 | 0 3.5 0 |

Notice: First element is zero in s^3 row.

This is a **problem**.
When the first element is zero, we **replace it with a small number epsilonϵ** and continue.

Thus, replace 0 with epsilonϵ.

New s^3 row:

| s^3 | ϵ 3.5 0 |

For s^2 row:

First element:

$(\epsilon)(6)-(2)(3.5)/\epsilon=6\epsilon-7/\epsilon$

Second element:

$(\epsilon)(3)-(2)(0)/\epsilon=3\epsilon/\epsilon=3$

Thus:

| s^2 | $6\epsilon-7/\epsilon$ |

For s^1 row:

First element:

$(6\epsilon-7/\epsilon)(3.5)-(\epsilon)(3)/(6\epsilon-7/\epsilon)$

Simplify numerator:

$(6\epsilon-7)(3.5)-\epsilon(3)=(21\epsilon-24.5)-3\epsilon=18\epsilon-24.$

Thus:

First element=$18\varepsilon - 24.5/(6\varepsilon - 7)$

Second element: no second element needed now.

Thus:

| s^1 | 18ϵ−24.5/6ϵ−7

For s^0 row:

Just take the last coefficient from previous row:

| s^0 | 3 |

Step 3: Analyze Sign Changes

- Now we examine the first column:

1,2,ϵ,6ϵ−7/ϵ,18ϵ−24.5/6ϵ−7,3

Let's check sign behavior as ϵ→0+\epsilon \to 0^+ϵ→0+:

- ϵ>0 small positive.
- 6ϵ−7/ϵ≈−7/ϵ→−∞ (very large negative value).
- 18ϵ−24.5/6ϵ−7≈−24.5/−7=3.5 - (positive value).

Changes:

- 1 (positive) to 2 (positive): no change
- 2 (positive) to ϵ (positive): no change
- ϵ (positive) to 6ϵ−7/ϵ: **one sign change**
- 6ϵ−7/ϵ (negative) to 18ϵ−24.56/ϵ−7 (positive): **another sign change**
- Then 3.5 (positive) to 3 (positive): no change.

Step 4: Conclusion

- **Total sign changes = 2**

Thus, the system has **2 right half-plane poles**.

The system is unstable

Question 5

Required to find the value of K so that there is 10% in a steady state control system

Step 1; find type of system

- Number of poles at s=0=1
- Type 1 system (good for step inputs)

Step 2; required to find the position error constant Kp

$$Kp = \lim_{s\to 0} \mathrm{G}(s) = \quad \lim_{s\to 0} \frac{K(0+5)}{(0+6)(0+7)(0+8)} = \infty$$

It is an open loop transfer function and we need to consider it in feedback system type 1

And if it is a ramp input velocity error constant Kv is given as

$$Kv = \lim_{s\to 0} sG(s)$$

Thus

$$Kv = \lim_{s\to 0} s \times K\,(s+5)/(s(s+6)(s+7)(s+8))$$

$$= 5K/336$$

But steady state error is given as

Ess = 1/Kv

For 10% error , Ess = 0.1 and thus Kv is 10 solving for the value of K

$$\frac{5K}{336} = 10$$

Thus, the value of K=672