Cours Master 2

Contraintes

Organisation

- Intervenants :
  - Christian Bessiere (bessiere@lirmm.fr)
  - Clément Carbonnel
  - Michael Sioutis (responsable)

Contenu

- Bases (12h) par Bessiere
- Classes polynomiales (6h) par Carbonnel
- Raisonnement temporel et spatial (6h) par Sioutis

+ TDs par Carbonnel et Sioutis

Success stories

★ **Scientific fields**

- chemistry, biology, robotics, telecoms, software verification, production line, games

★ **Categories of problems**

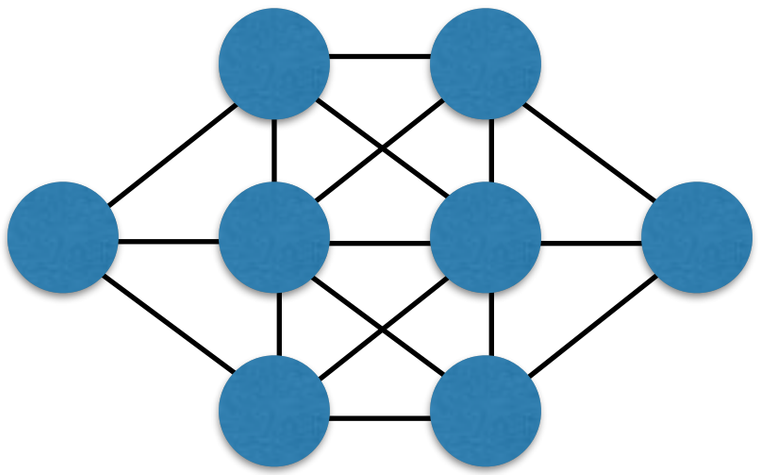
- resource allocation, time tabling, scheduling, configuration, planning

★ **Examples**

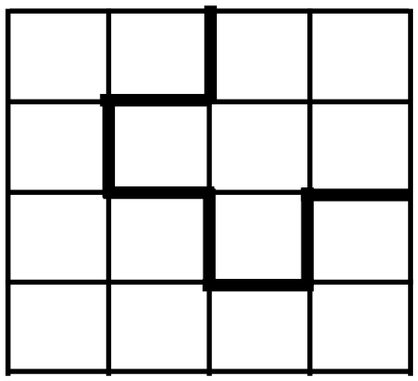
- sport league scheduling, nurse/hostess rostering, gates to planes/trains, excavators in mines, car assembly line, energy (data centers, power plants), port management …

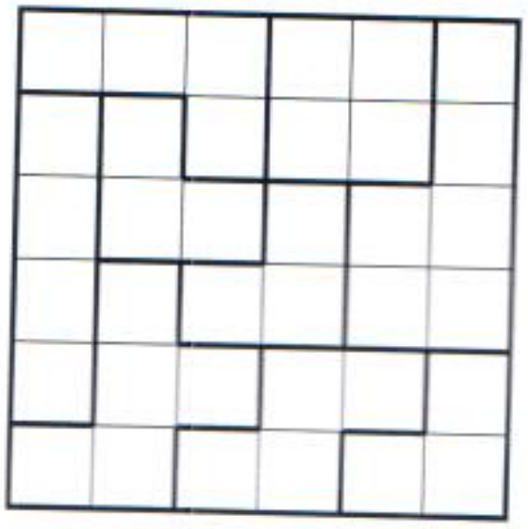
Games…

- Mettre les nombres de 1 à 8 sur les 8 sommets du graphe de sorte que deux nombres consécutifs ne sont pas voisins



- Mettre les nombres de 1 à *n* sur chaque ligne et chaque colonne de sorte que chaque région ait la même somme

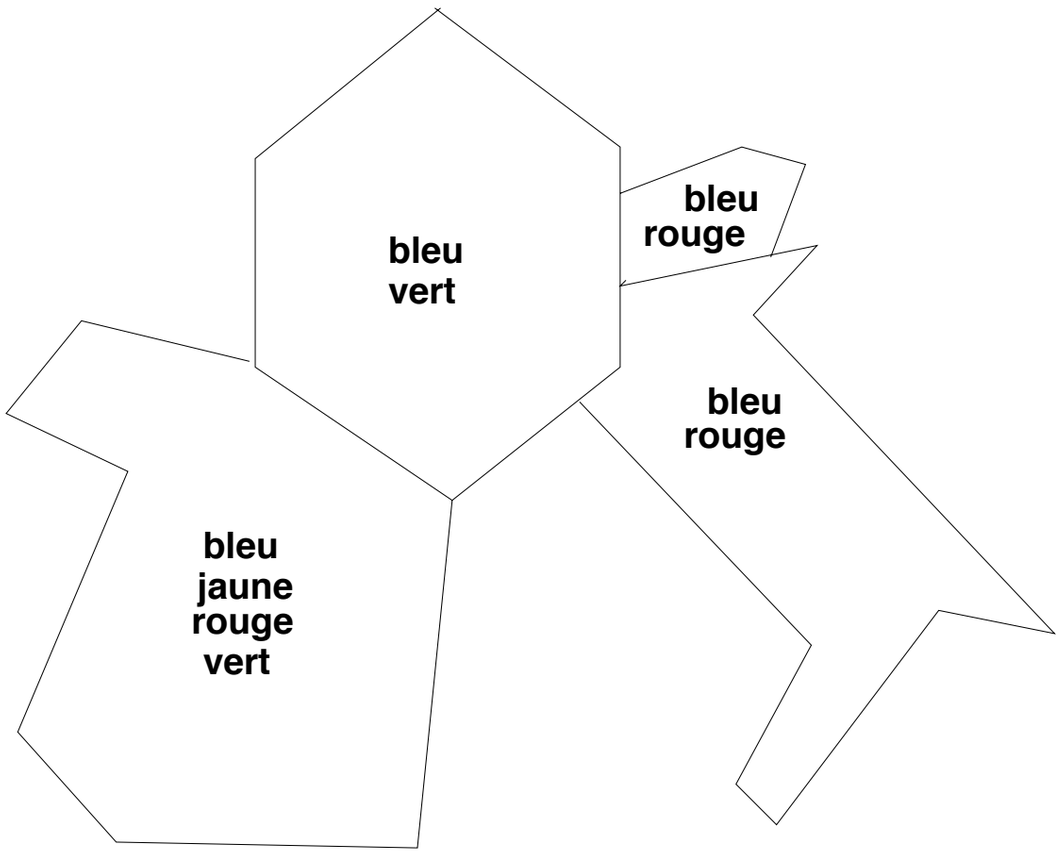




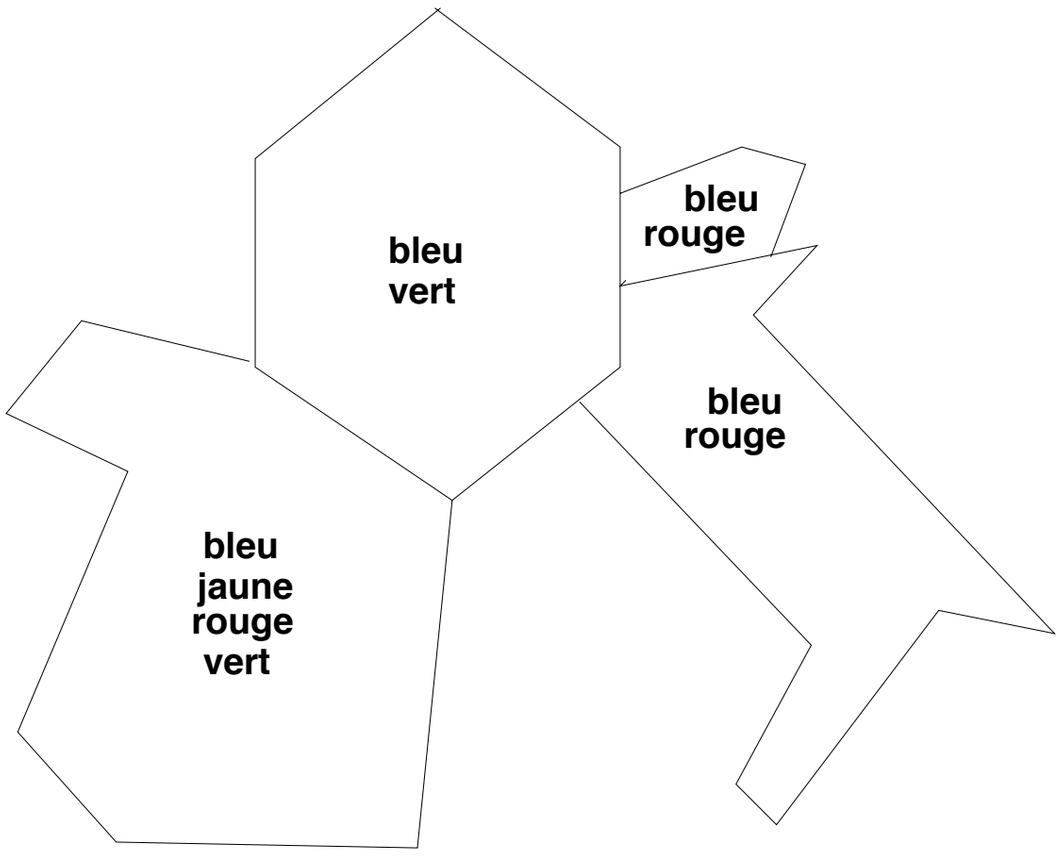
Running example

Running example

Graph coloring



Constraint network



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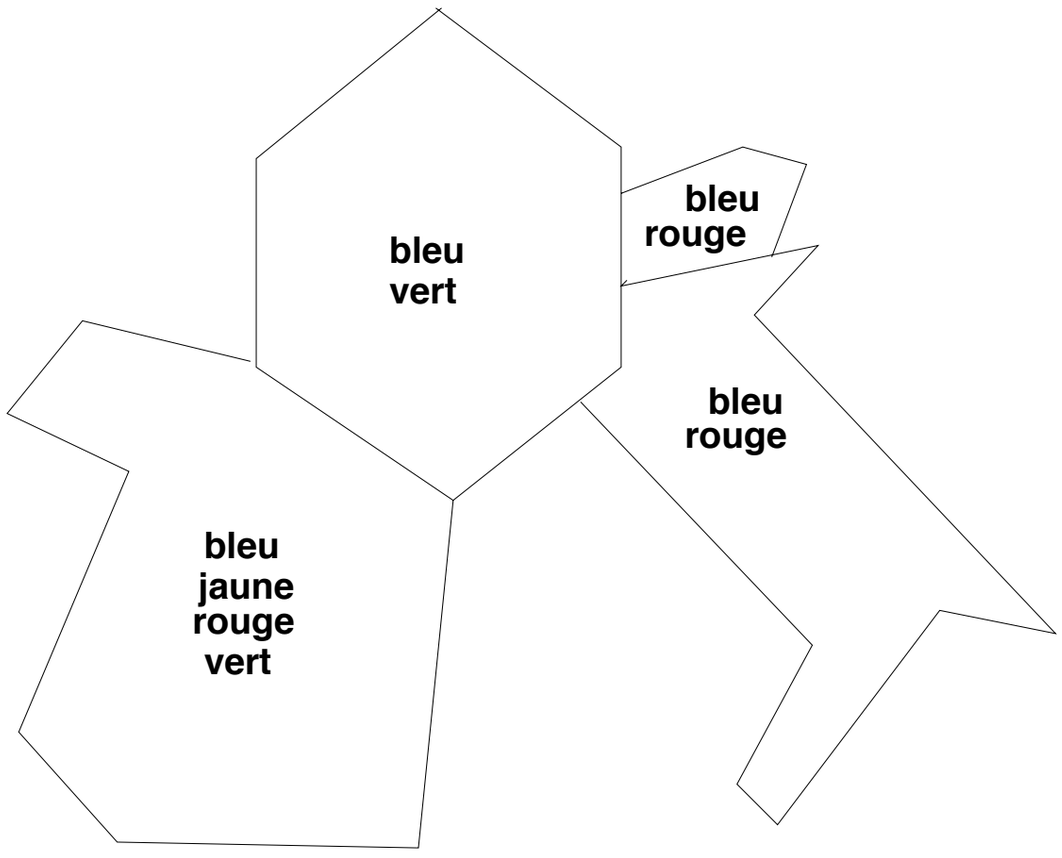
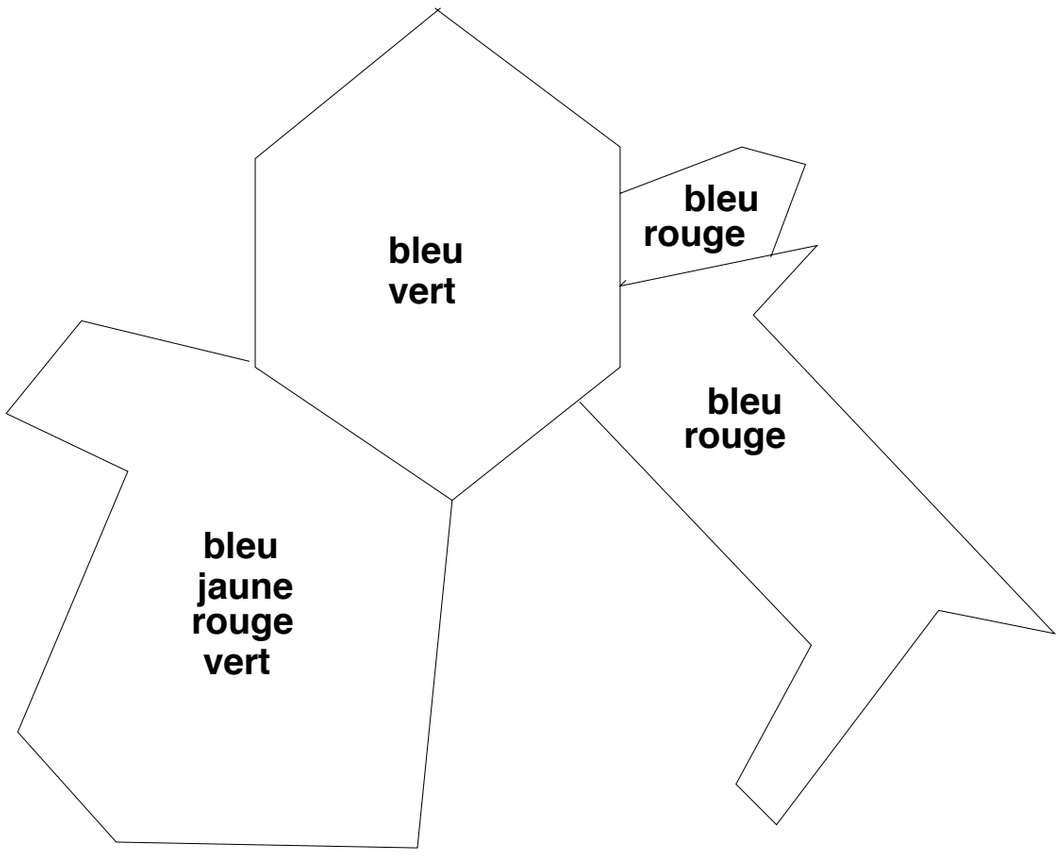


	components	on the example
for each variable	variables	France, Spain, etc
	domain of values	bleu, jaune, etc
on subsets of variables	constraints	not the same color

Constraint network

Constraint network



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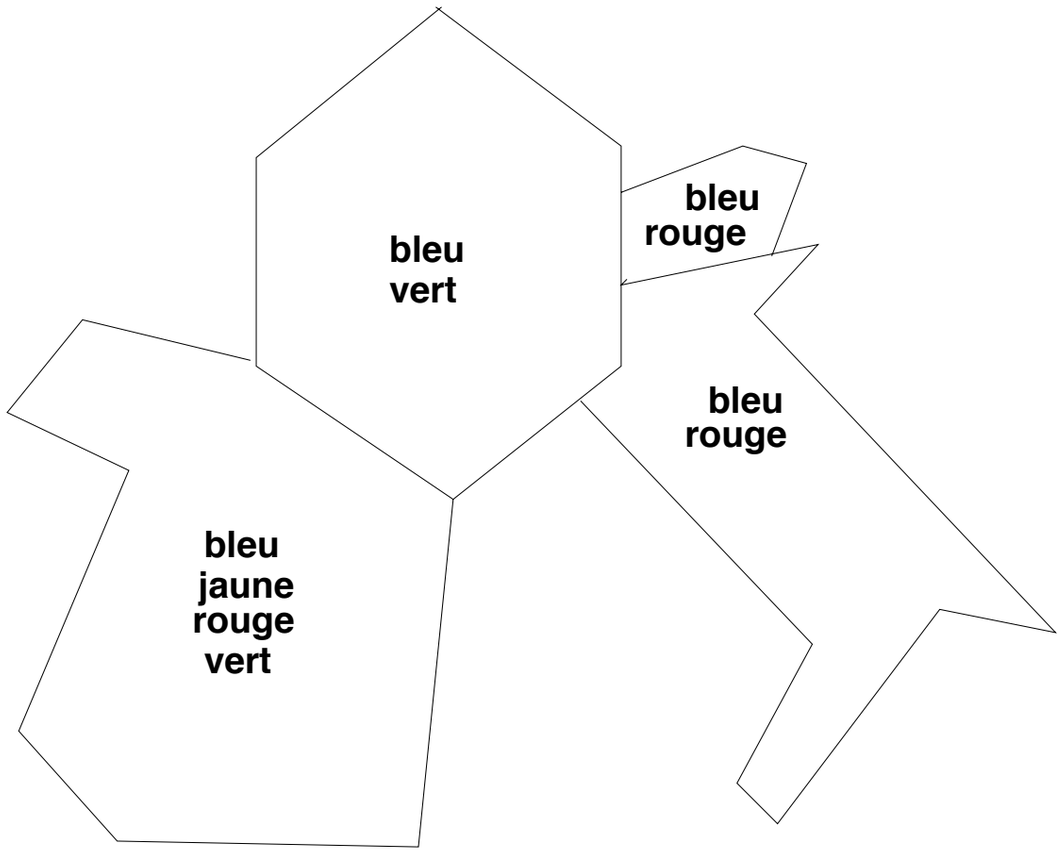

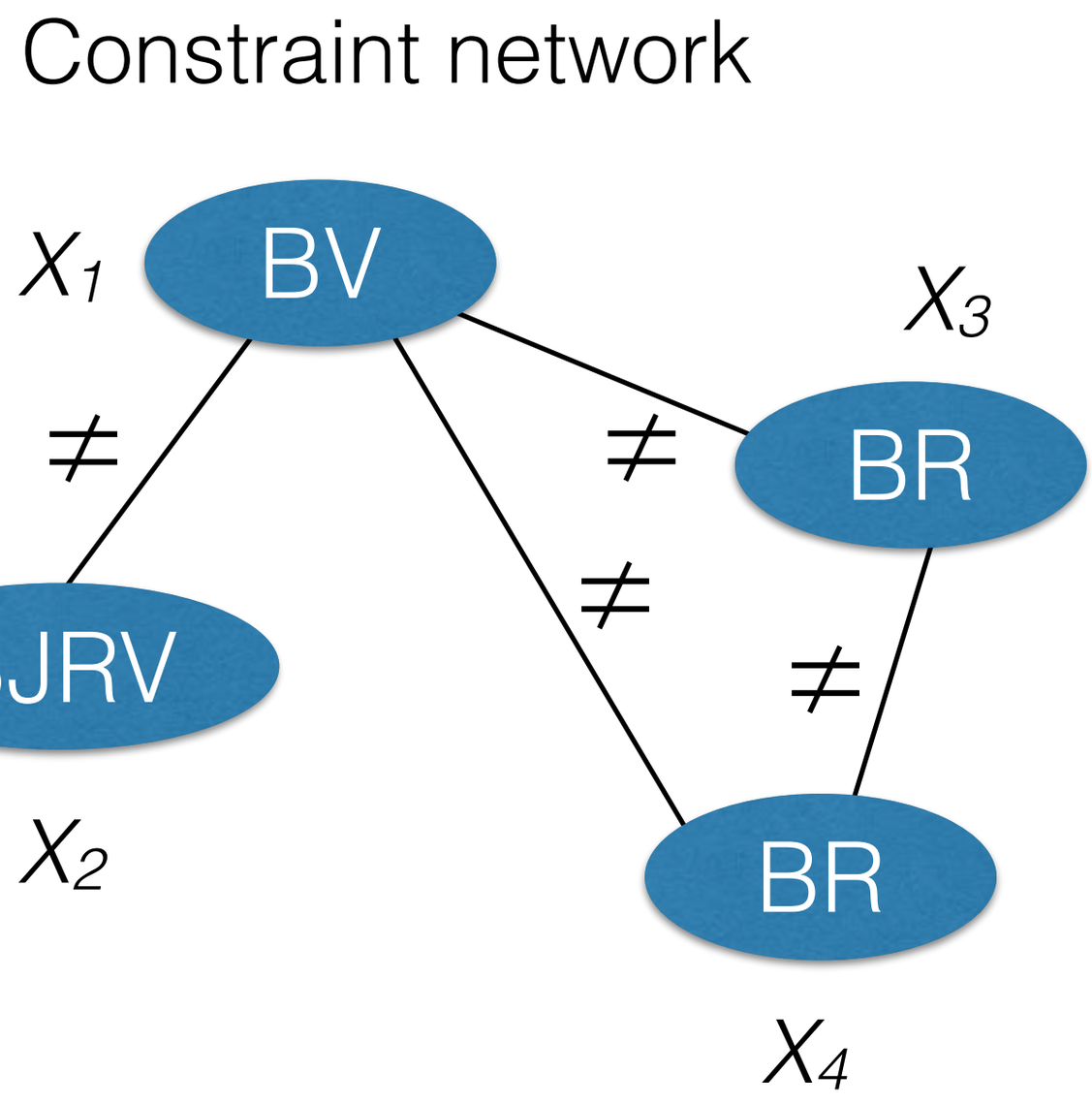

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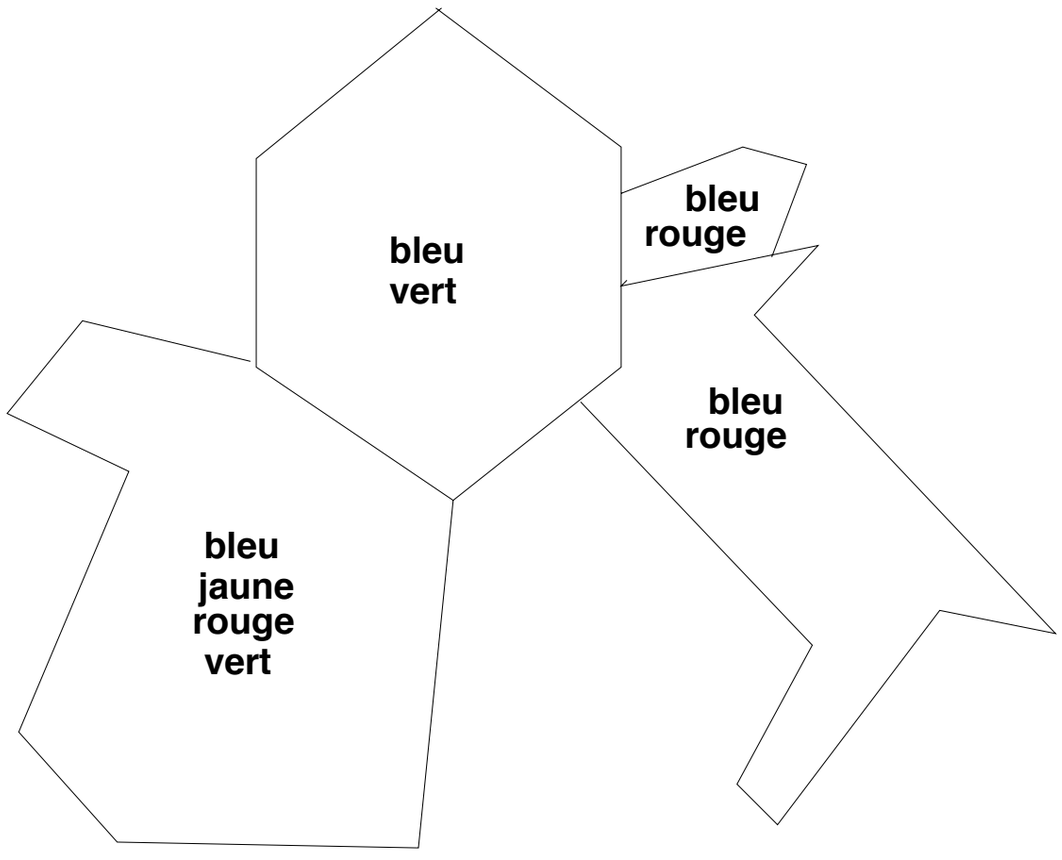
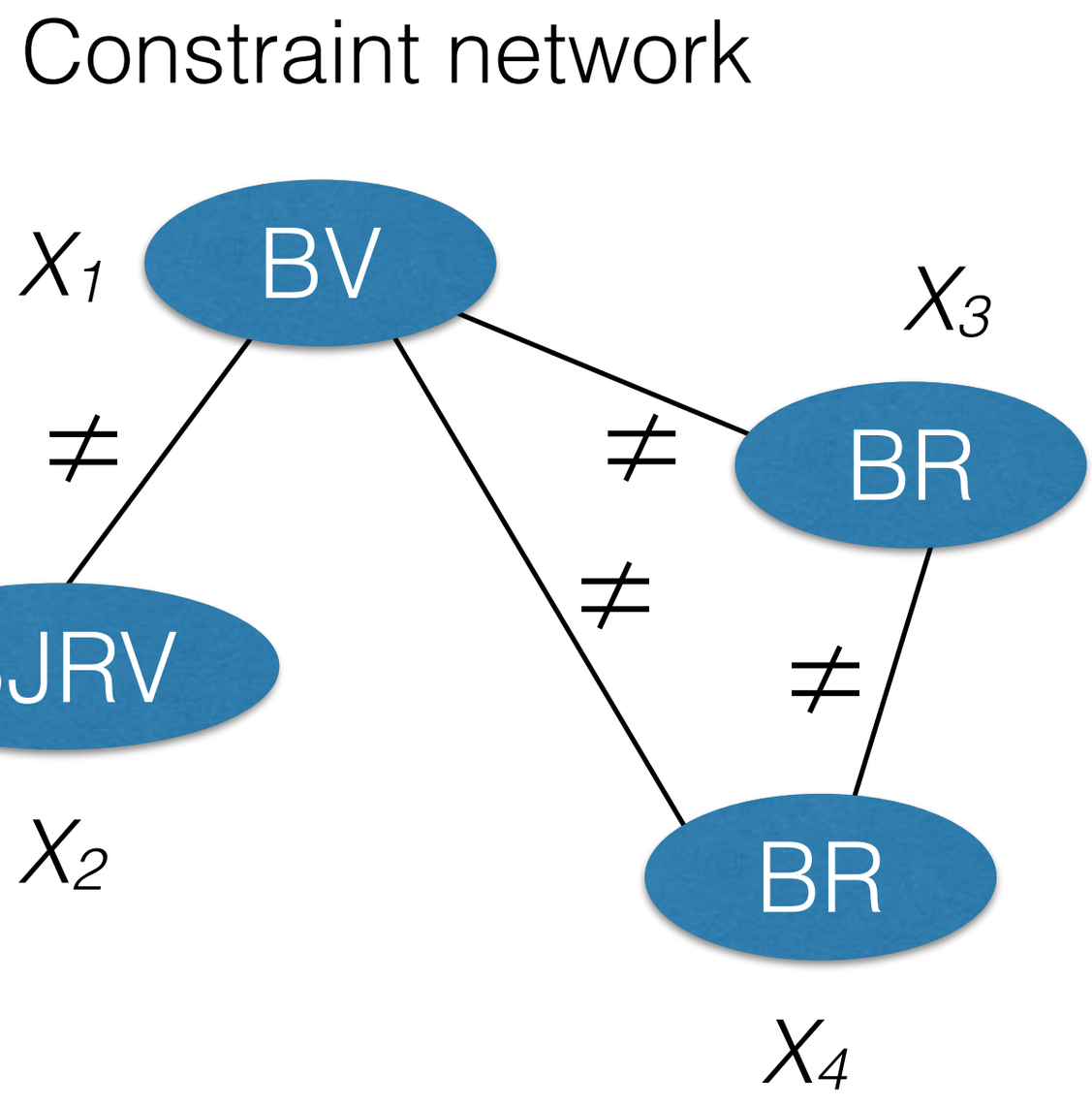
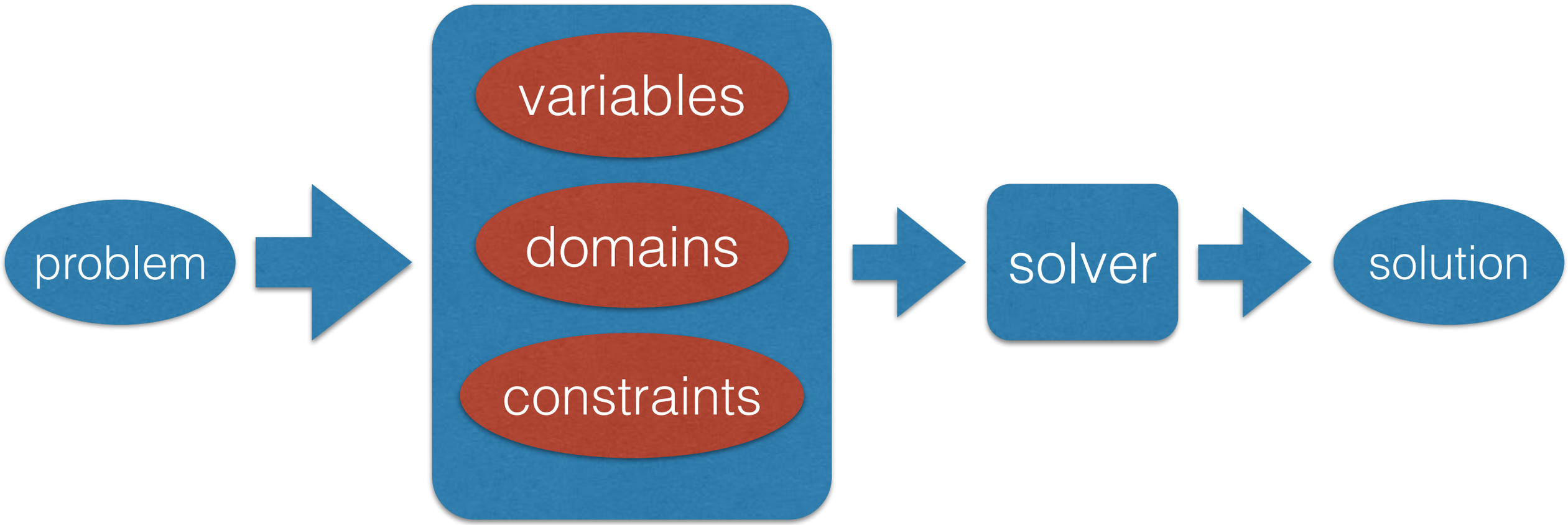




Solution : (X_1,V); (X_2,B); (X_3,B); (X_4,R)

France = vert, Espagne = bleu, Suisse = bleu, Italie = rouge

Formalisme de résolution de problèmes



constraint network

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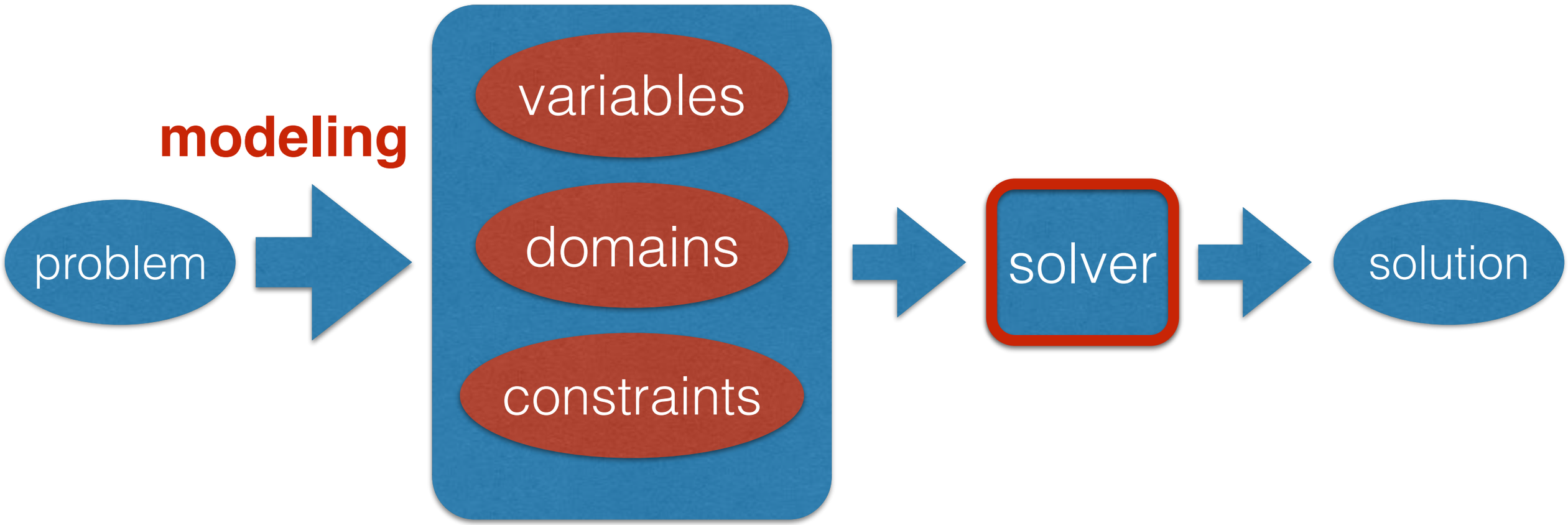

constraint network

Basic definitions

Constraint network

Definition 1 (Constraint network) *A* constraint network *(or* network*) is composed of :*

- *a set of variables* $X = \{X_1, \ldots, X_n\}$
- *a domain on* X*, that is, a set* $D = \{D(X_1), \ldots, D(X_n)\}$*, where* $D(X_i) \subset \mathbb{Z}$ *is the* finite *set of values that variable* X_i *can take (its* domain*), and*
- *a set of constraints* $C = \{c_1, \ldots, c_e\}$

Constraint

Definition 2 (Constraint) *A constraint c is a Boolean function involving a sequence of variables $X(c) = (X_{i_1}, \ldots, X_{i_q})$ called its* scheme. *The function is defined on $\mathbb{Z}^q$. A combination of values (or* tuple*) $\tau \in \mathbb{Z}^q$* satisfies *c if $c(\tau) = 1$ (also noted $\tau \in c$). If $c(\tau) = 0$ (or $\tau \notin c$), τ* violates *c.*

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- Notations:
  - c on $X(c)=(X_1,\ldots,X_k)$
  - $c(X_1,\ldots,X_k)$
  - $c(X_i,X_j) \longrightarrow c_{ij}$

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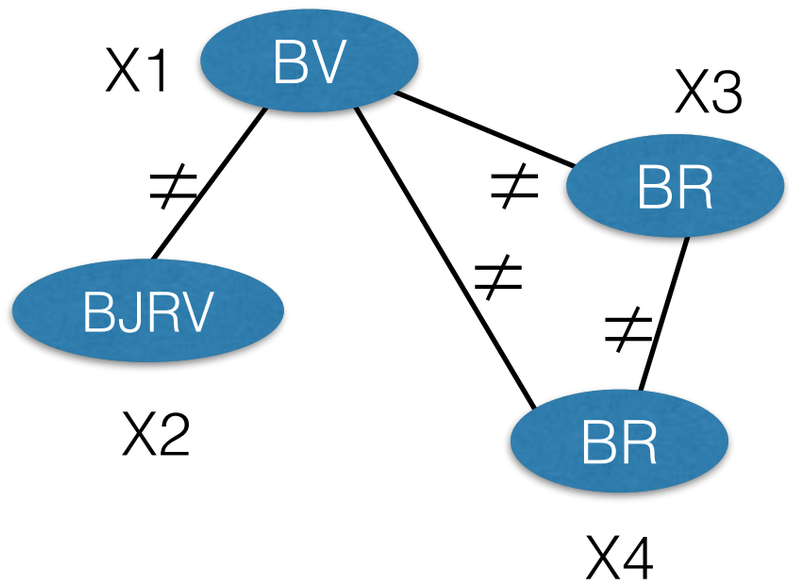
- Notations:
  - c on $X(c)=(X_1,\ldots,X_k)$
  - $c(X_1,\ldots,X_k)$
  - $c(X_i,X_j)$ —> c_{ij}
- Examples:
  - *alldifferent*$(X_1,\ldots,X_k)$
  - $X_1 + X_2 = 5$
  - $c(X_i,X_j) = \{(1,2),(2,1),(3,3),(4,4)\}$

Instantiation

Definition 3 (Instantiation) *Given a network* $N = (X, D, C)$,



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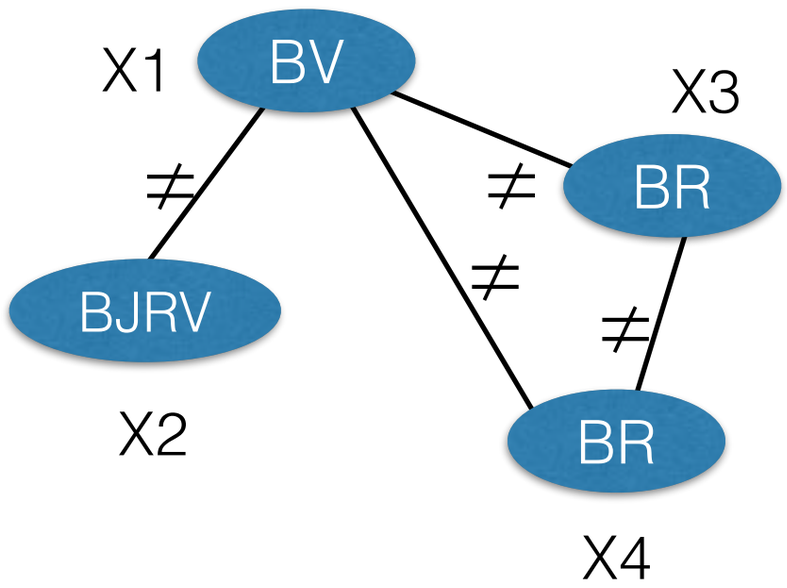
— *an* instantiation I *on a set* $Y = \{X_1, \ldots, X_k\} \subseteq X$ *of variables is an assignment of values* $v_1, \ldots v_k$ *to* $X_1, \ldots, X_k$

(X_1,R), (X_2,J), (X_3,B)



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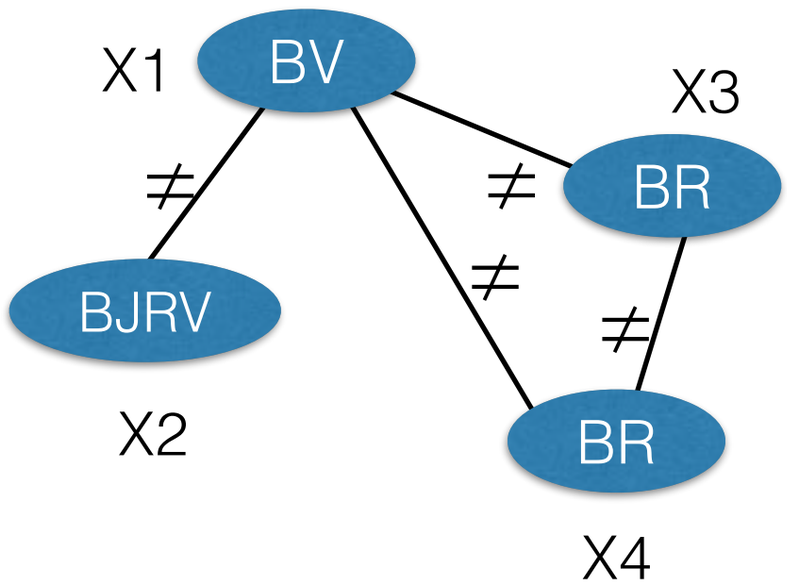

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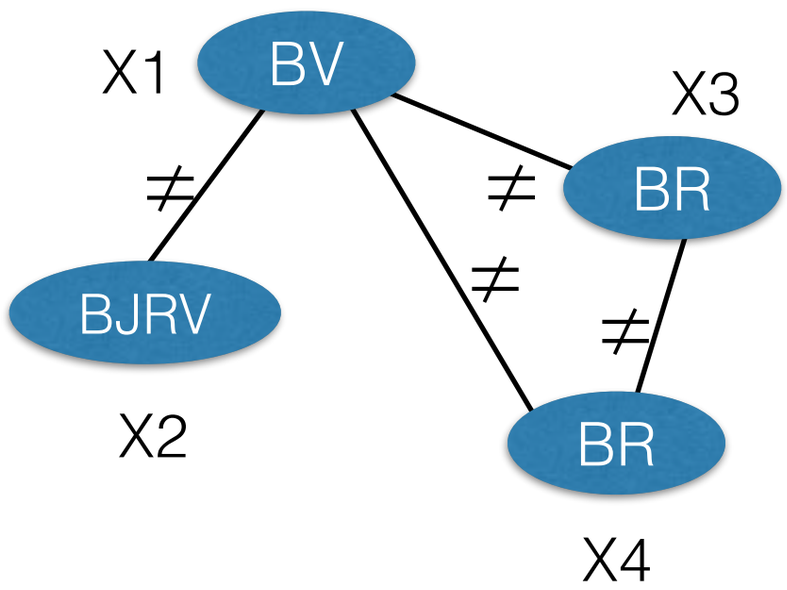

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- *an instantiation* I *on* Y *is* locally consistent *iff it is valid and does not violate any constraint*

(X_1,R), (X_2,J), (X_3,B)

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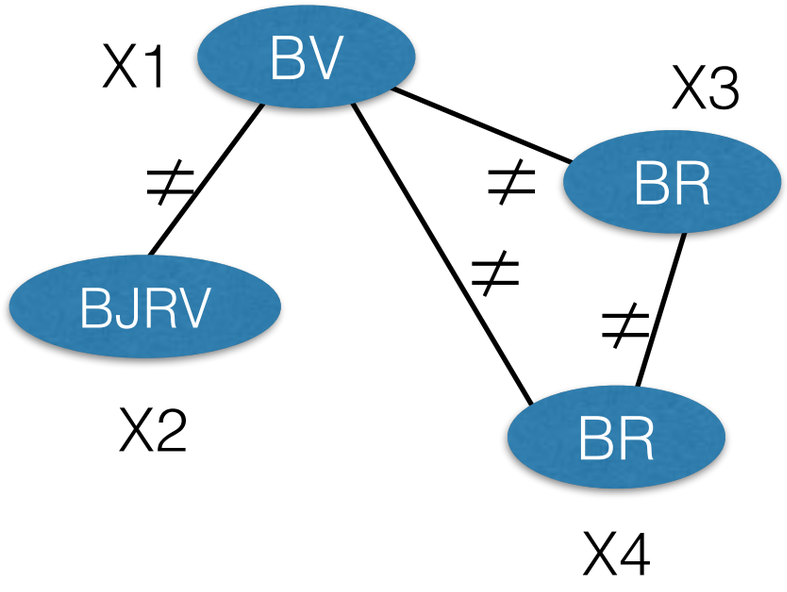

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— *an instantiation* I *on* Y *is* locally consistent *iff it is valid and does not violate any constraint*

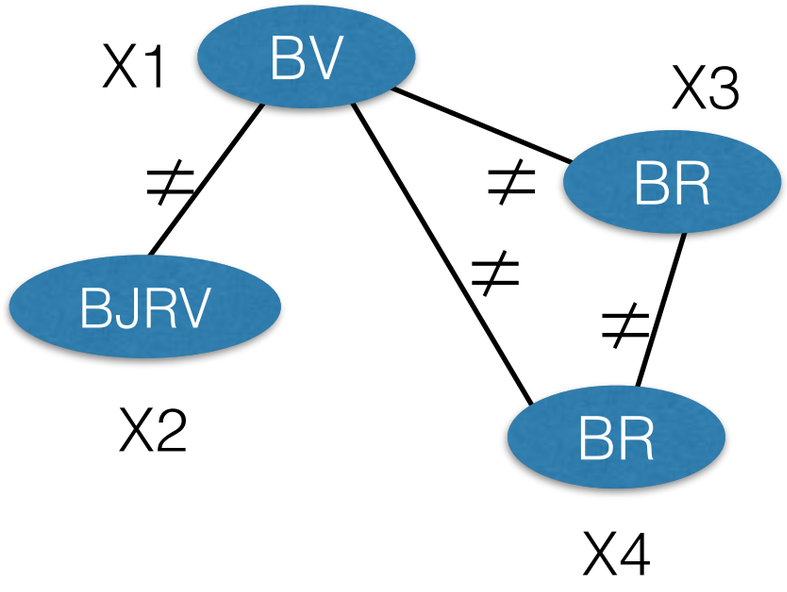
— *a* solution *is an instantiation* I *on* X *which is locally consistent.*

(X_1,*R*), (X_2,*J*), (X_3,B)

(X_1,*B*), (X_2,*J*), (X_3,B)

(X_1,*B*), (X_2,*J*), (X_3,R)

(X_1,*V*), (X_2,*J*), (X_3,B), (X_4,*R*)



Constraint Satisfaction Problem (CSP)

- Instance: *N = (X,D,C)*
- Question: Does there exist a solution to *N?*
- NP-complete

  ➔ search algorithm

Backtracking (BT)

function BT(N : problem ; I : instantiation) :**Boolean**
begin
  if $|I| = n$ **then** return **true**
  choose a variable X_i not in I
  for each $v_i \in D(X_i)$ **do**
    if $I \cup \{X_i, v_i\}$ locally consistent **then**
      if BT($N, I \cup \{X_i, v_i\}$) **then** return **true**
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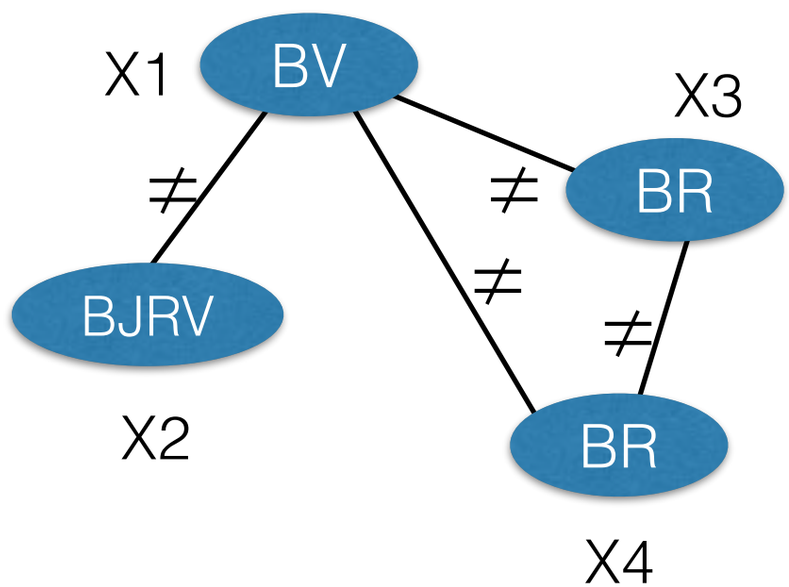
end

A call to BT(*N*,∅) decides CSP



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termination

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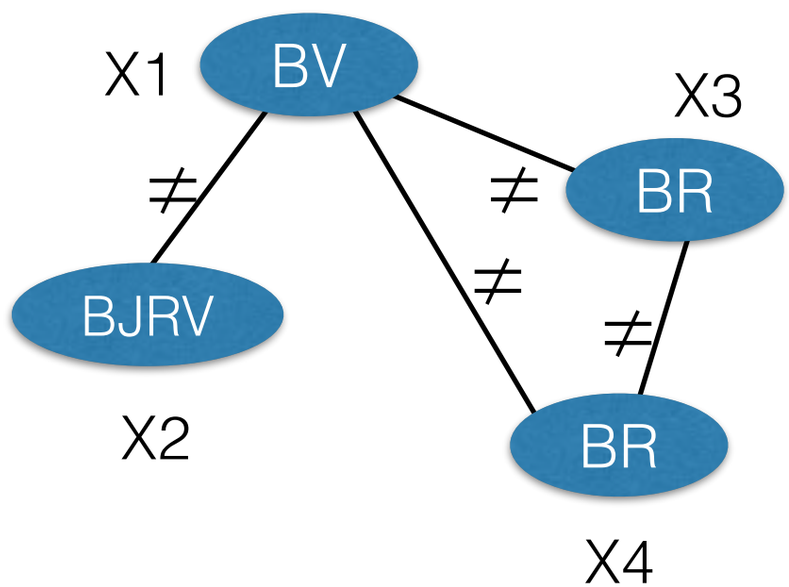

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branching

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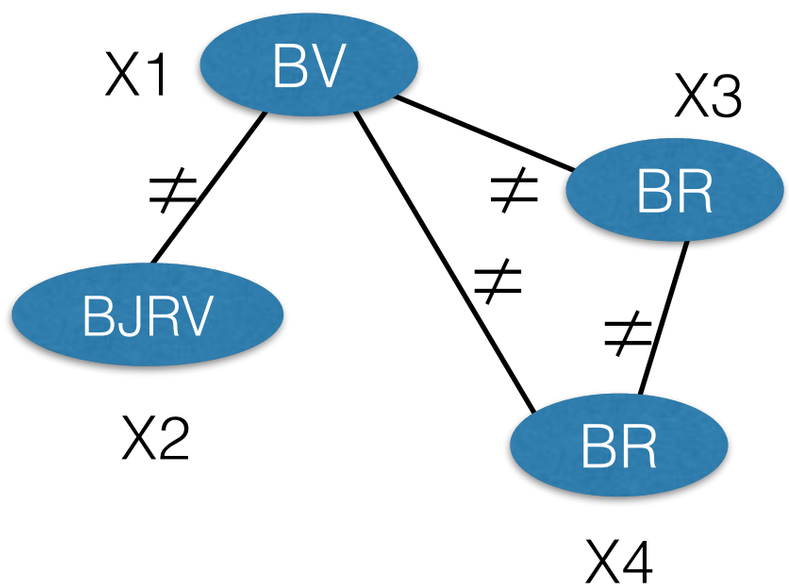

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branching

recursive call

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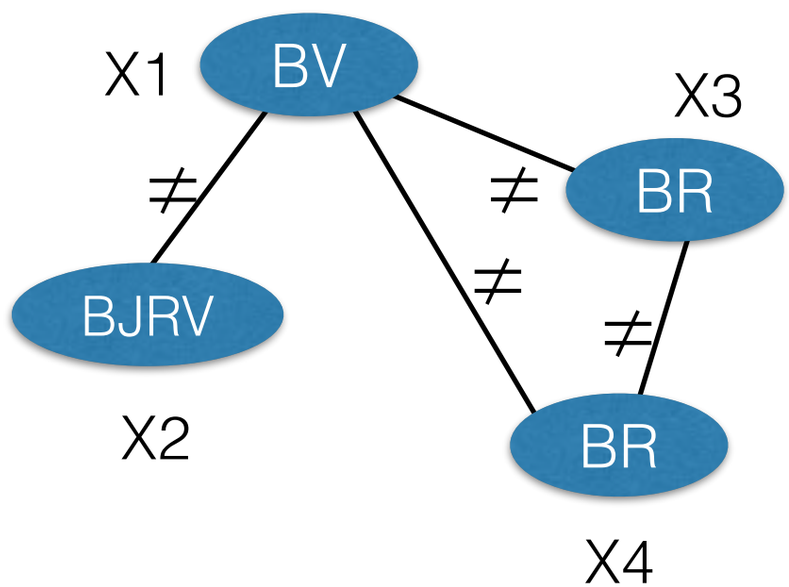

BT on running example

X1

X2

X3

X4

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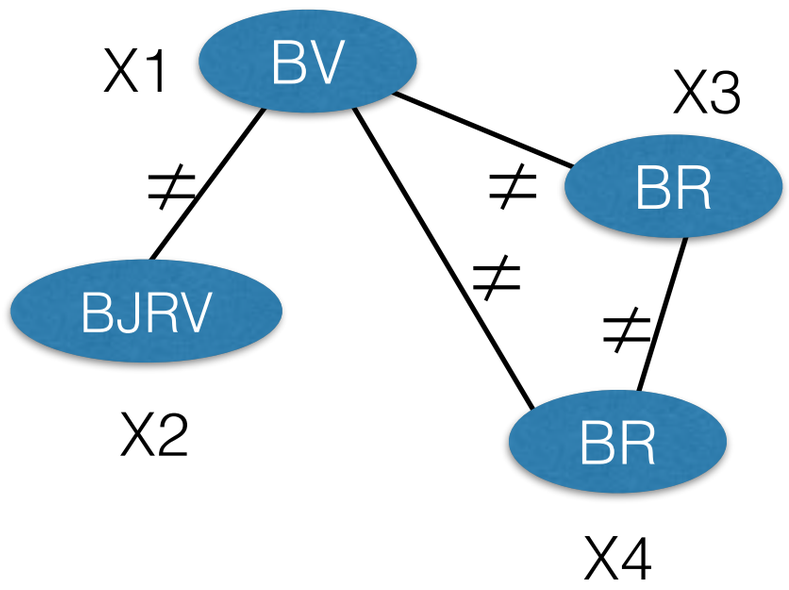

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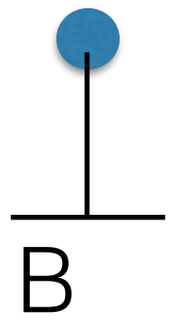


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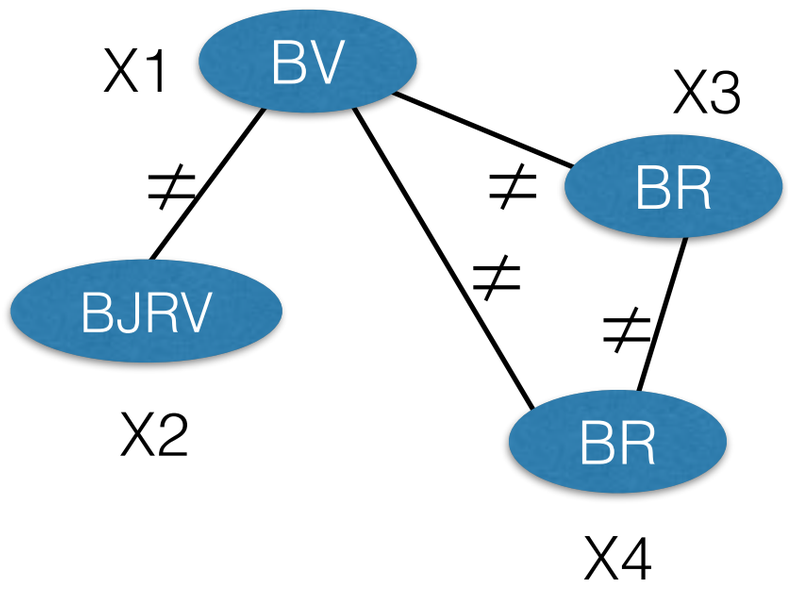
X2

X3

X4



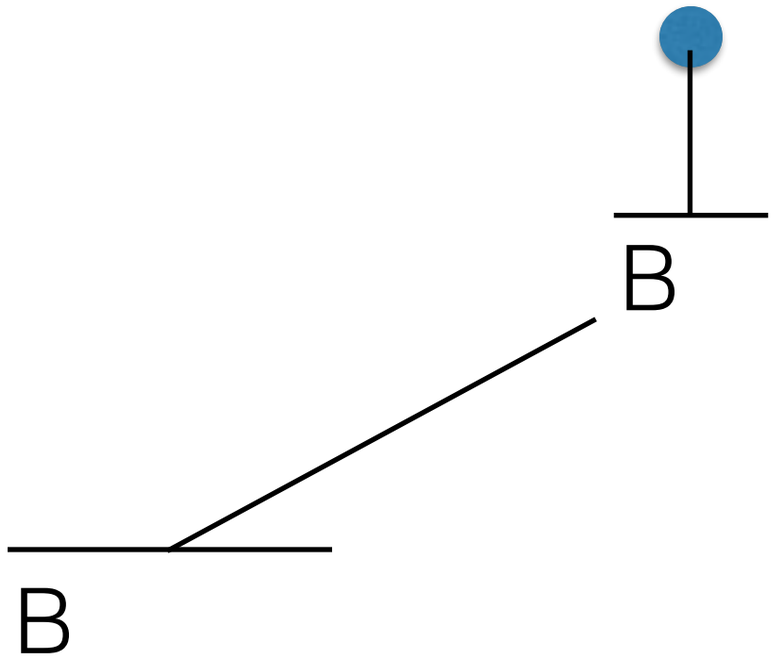
BT on running example

X1

X2

X3

X4



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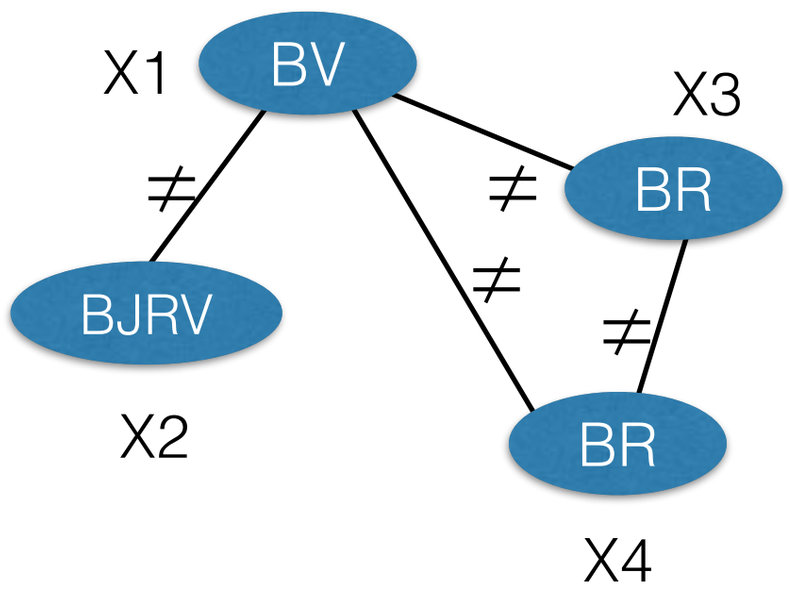

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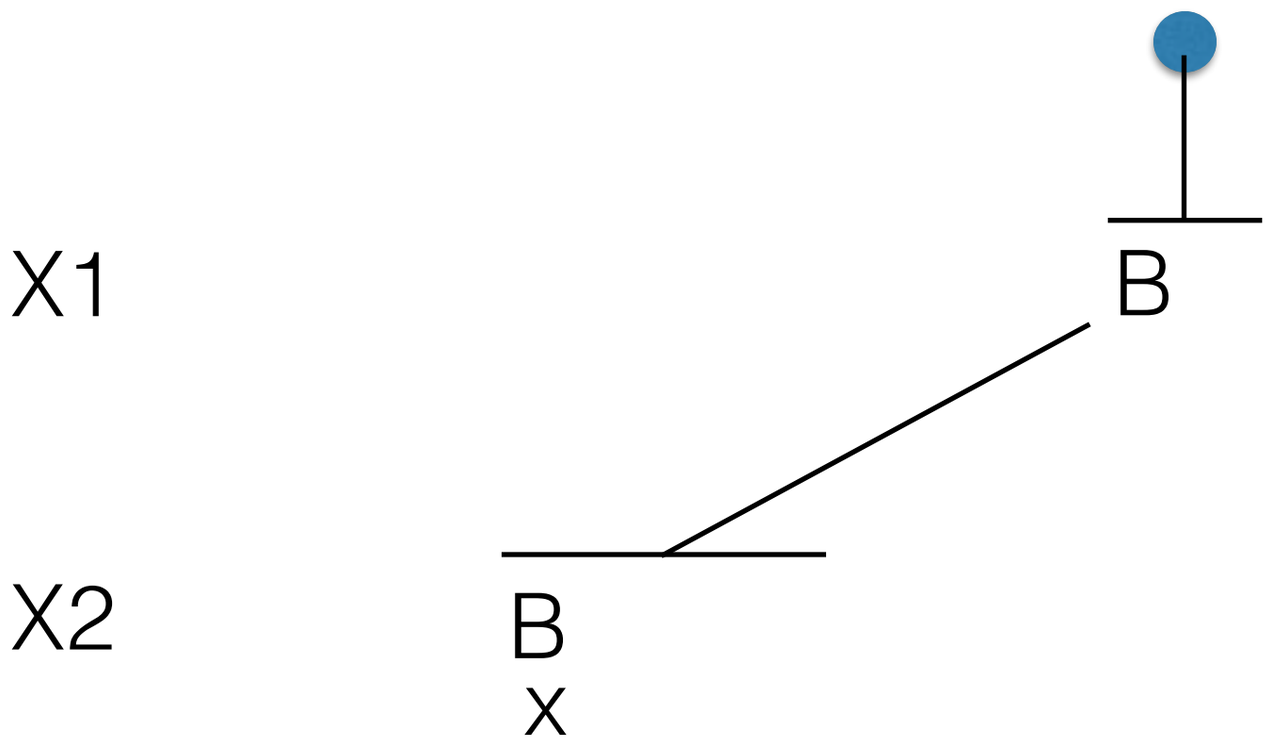

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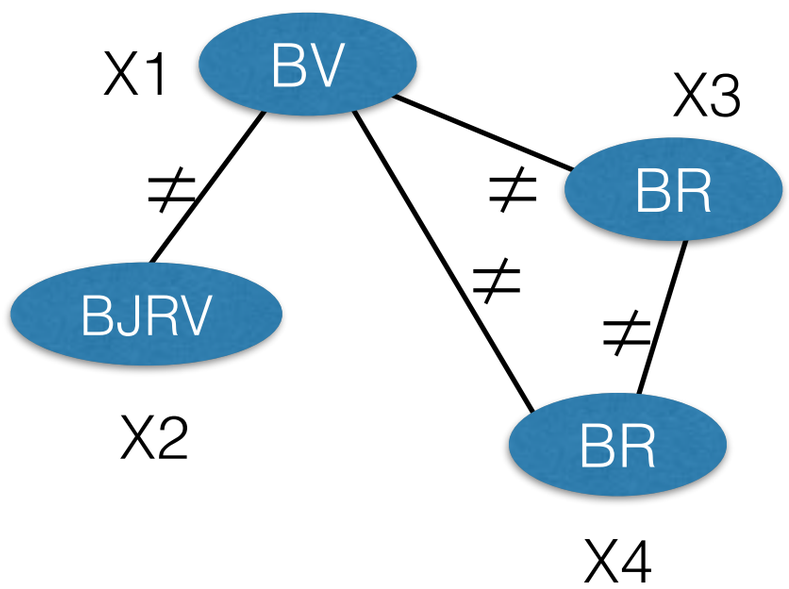
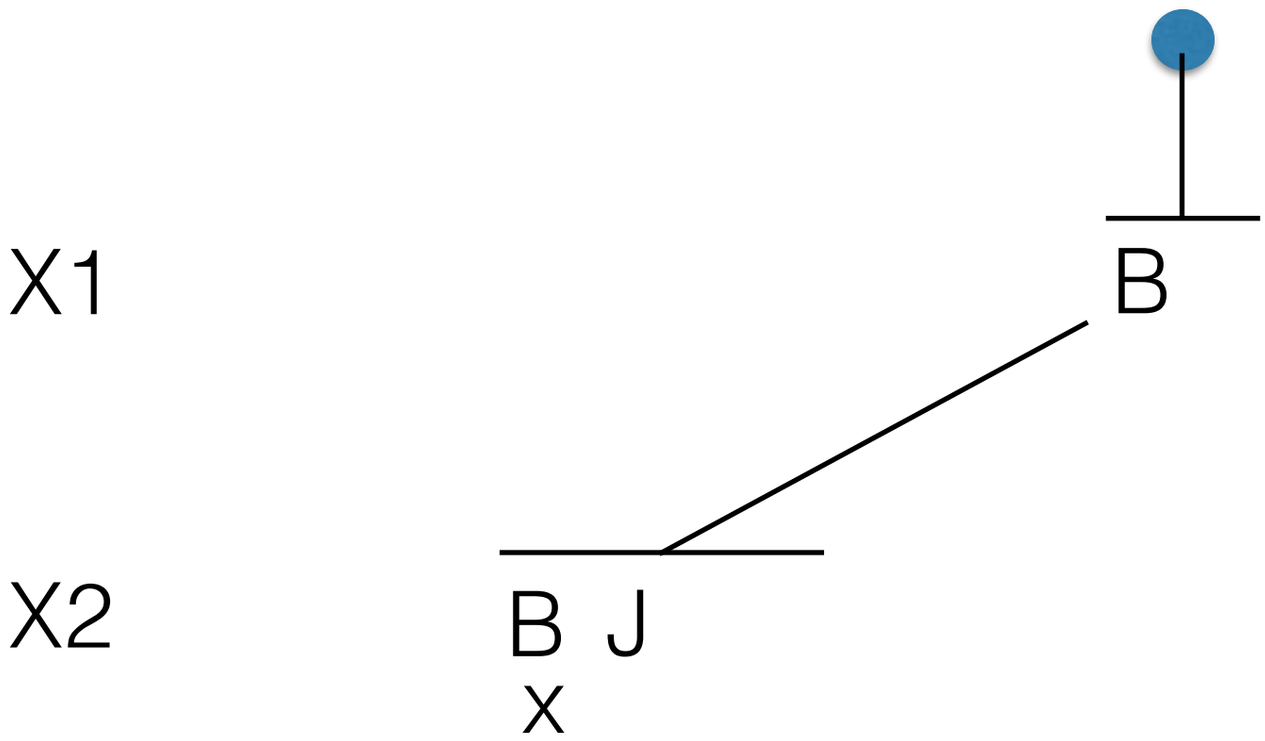


BT on running example



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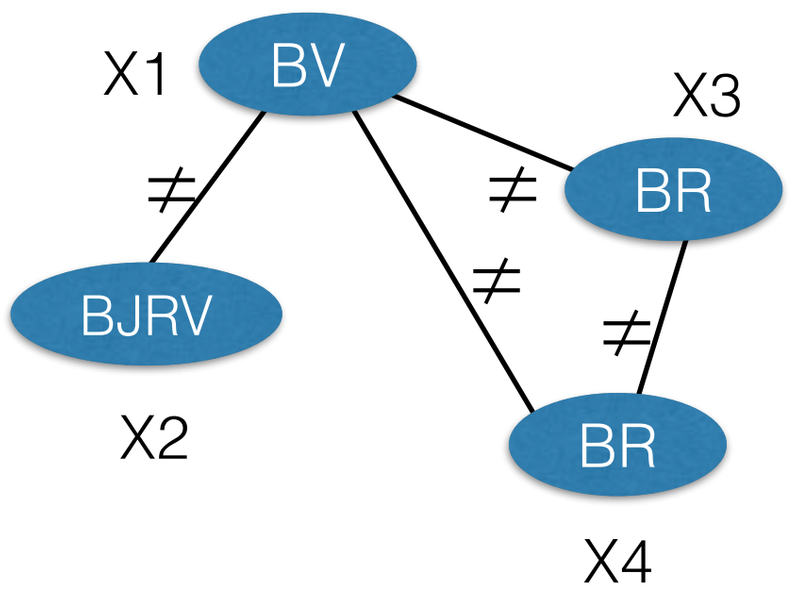

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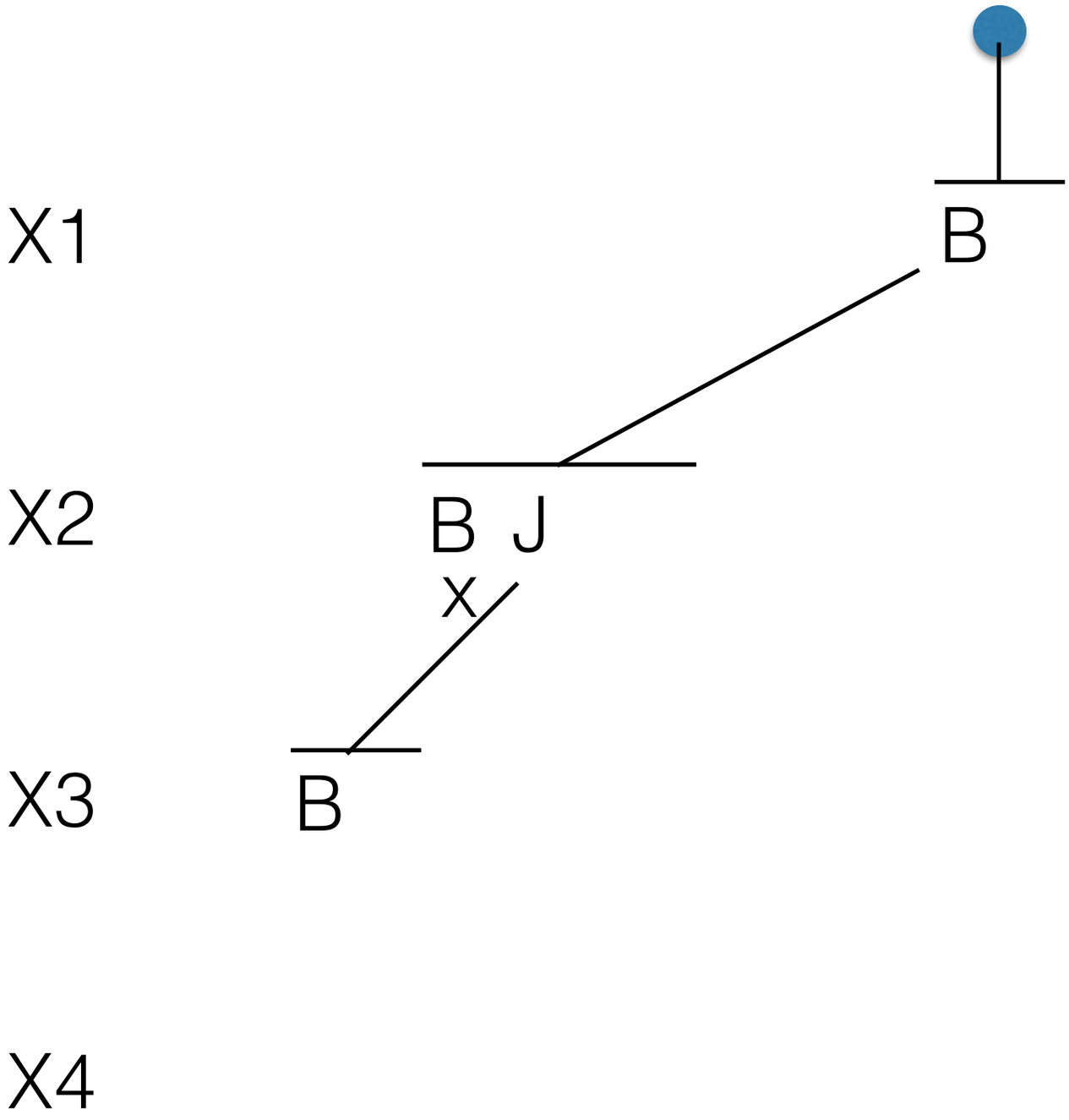

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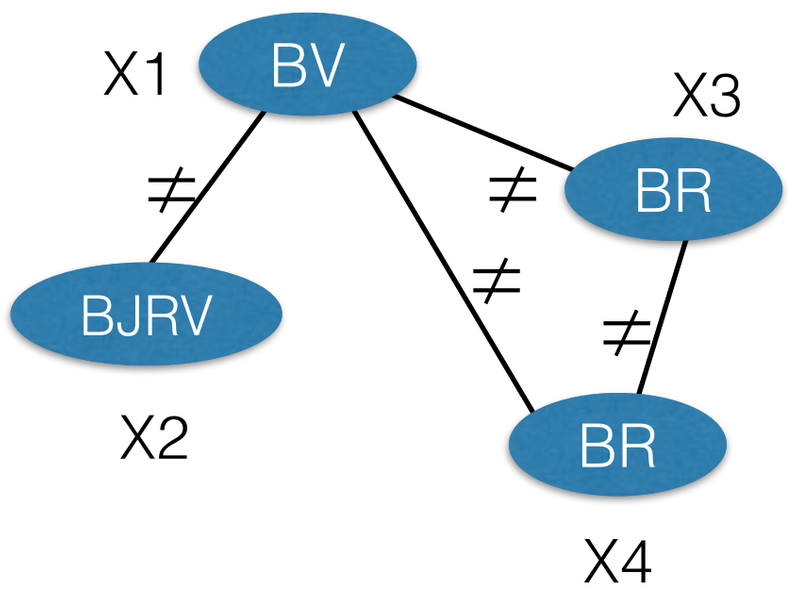

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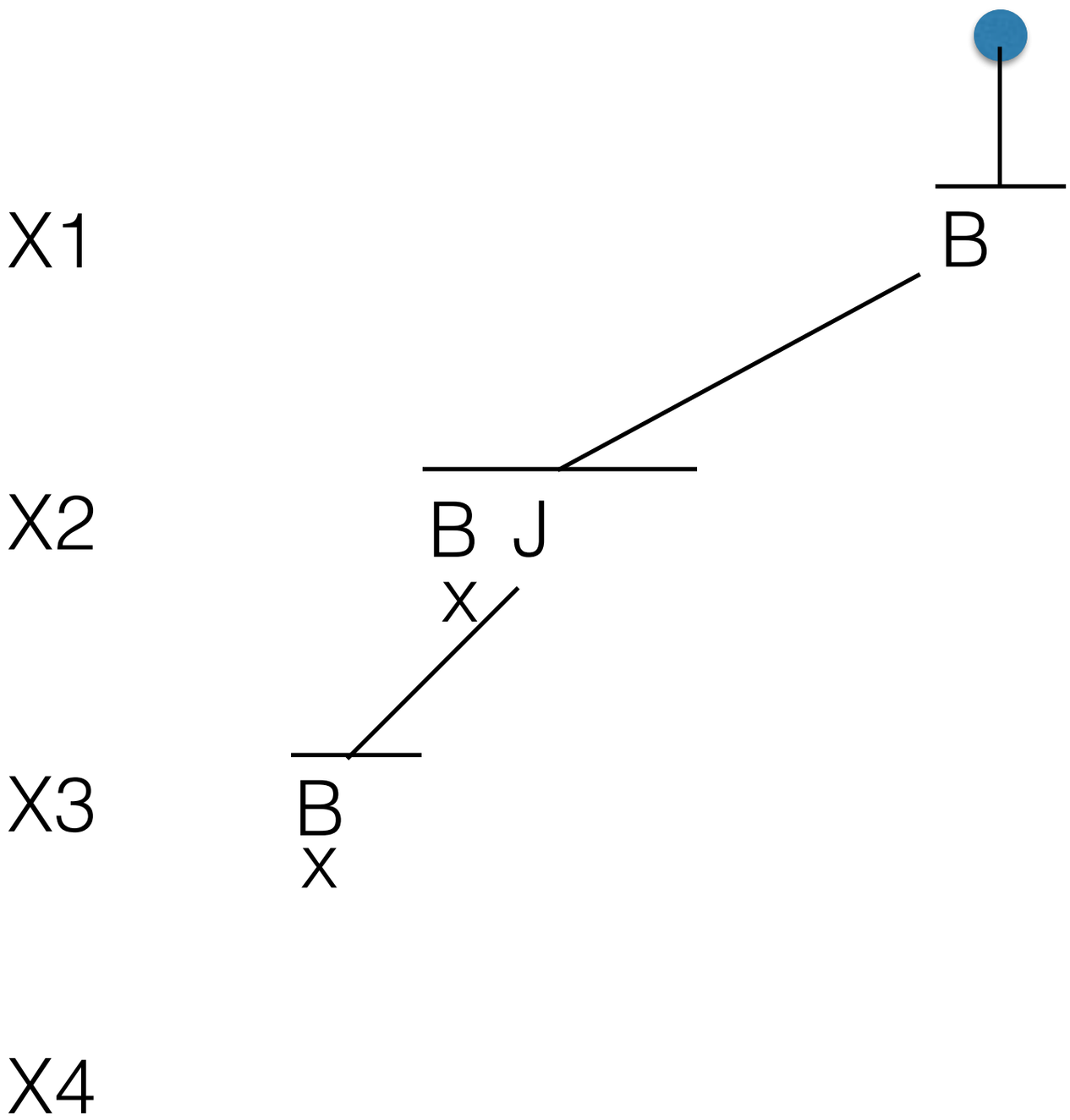


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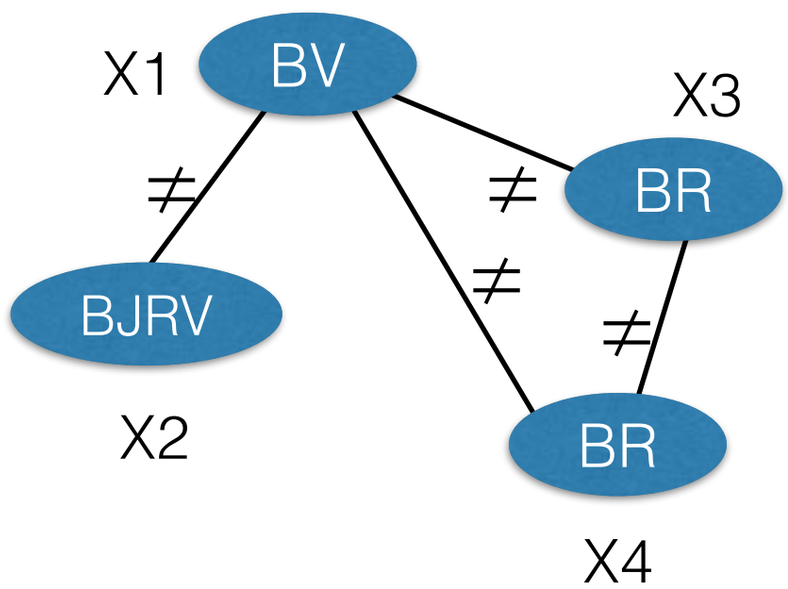

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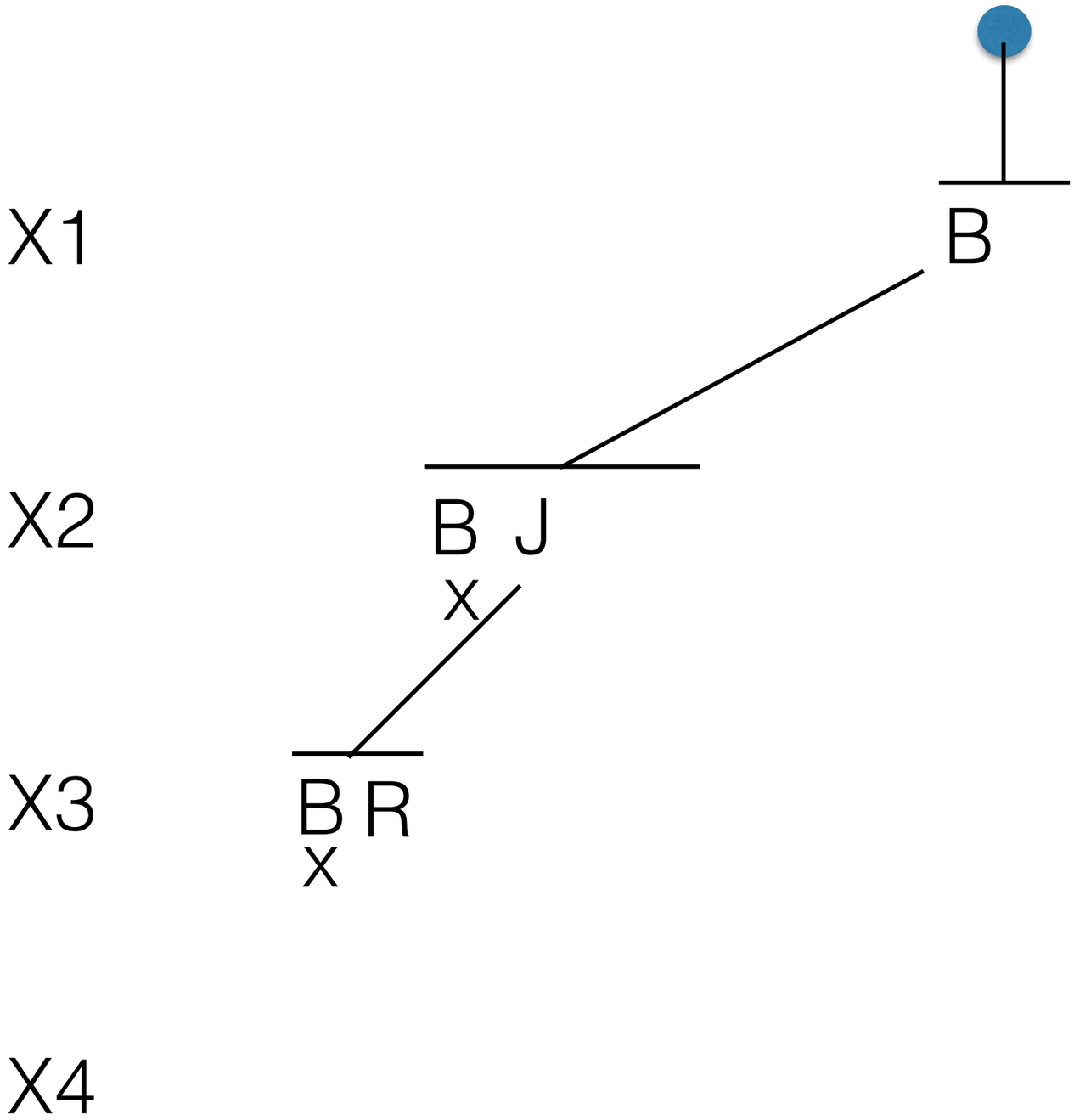


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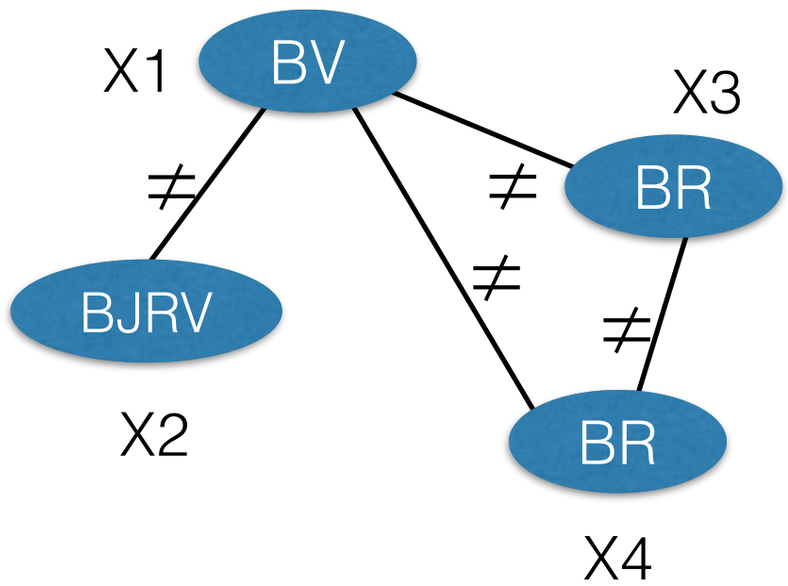

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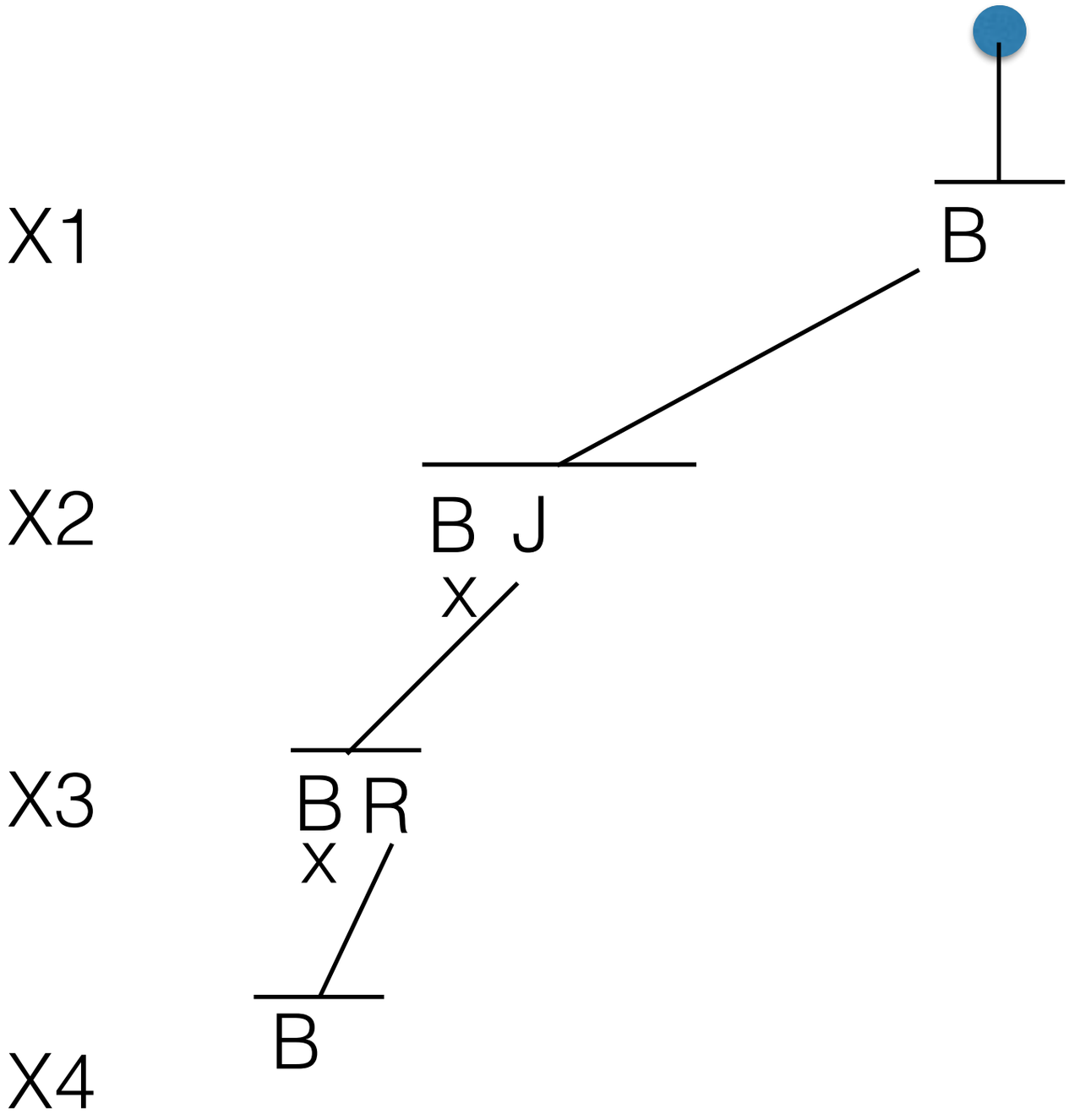


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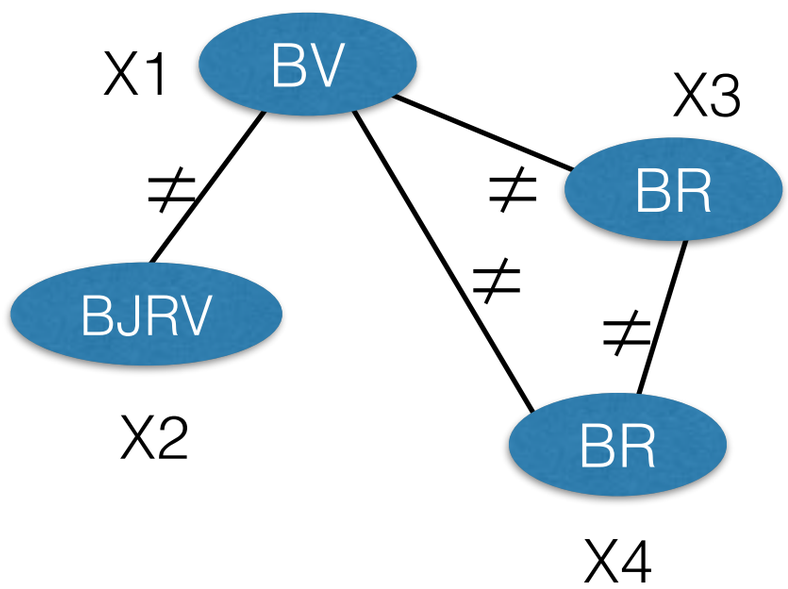

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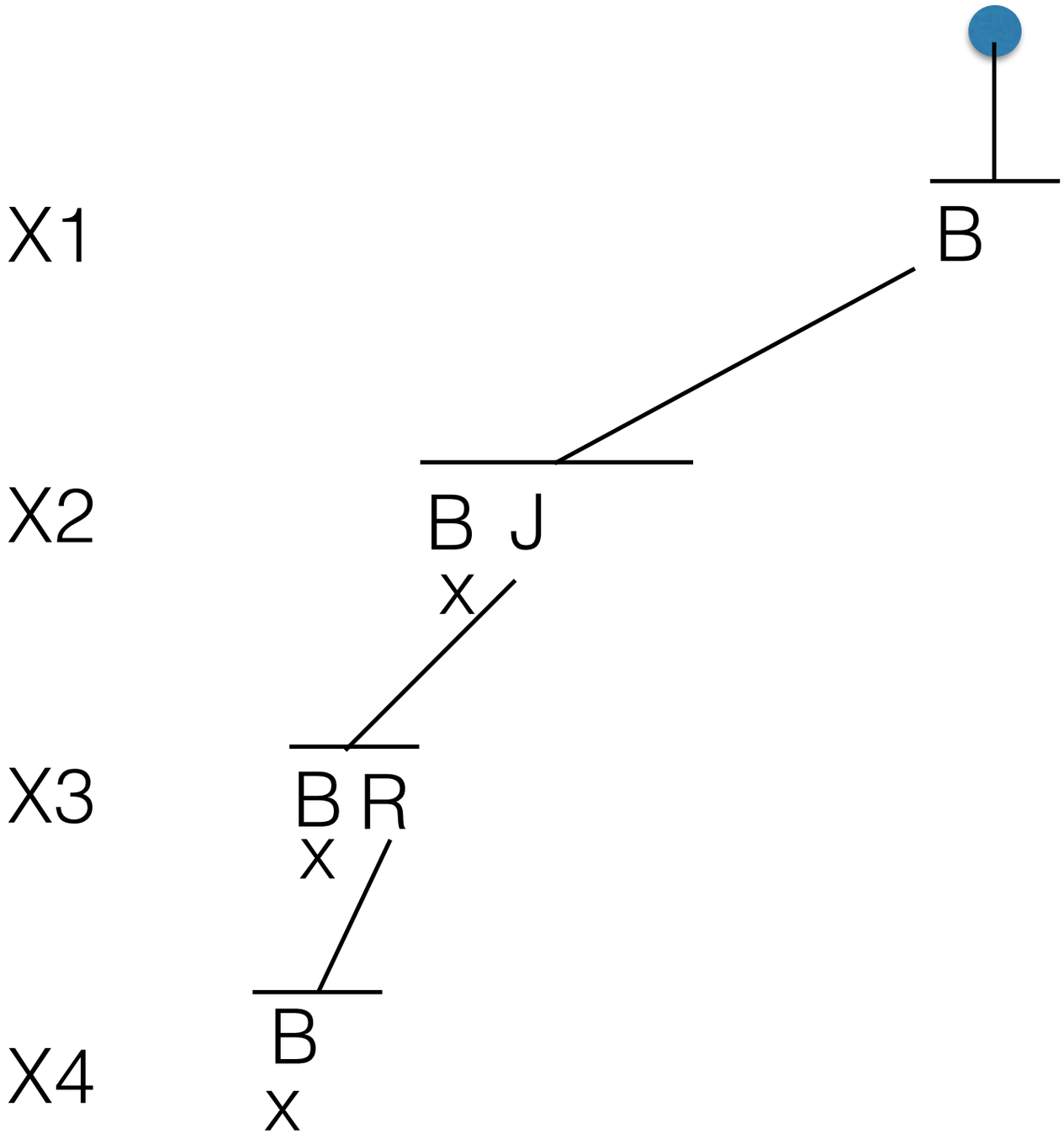

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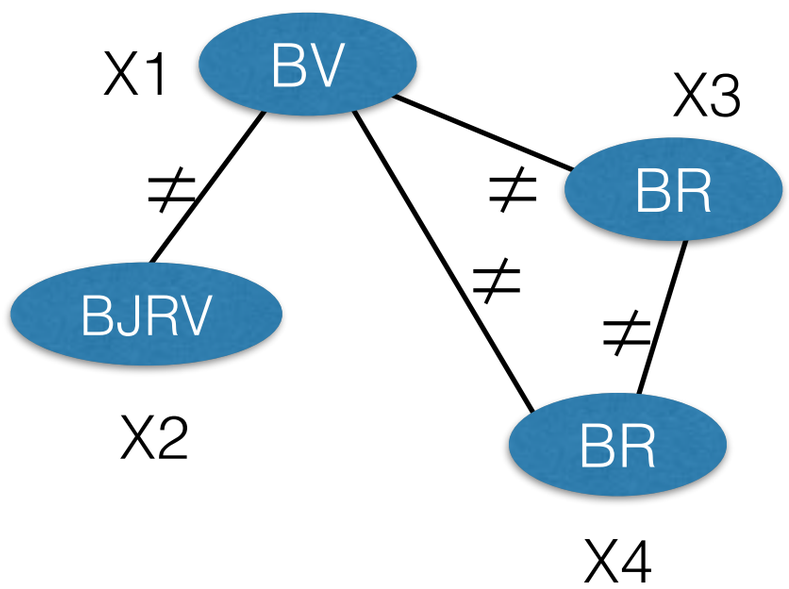

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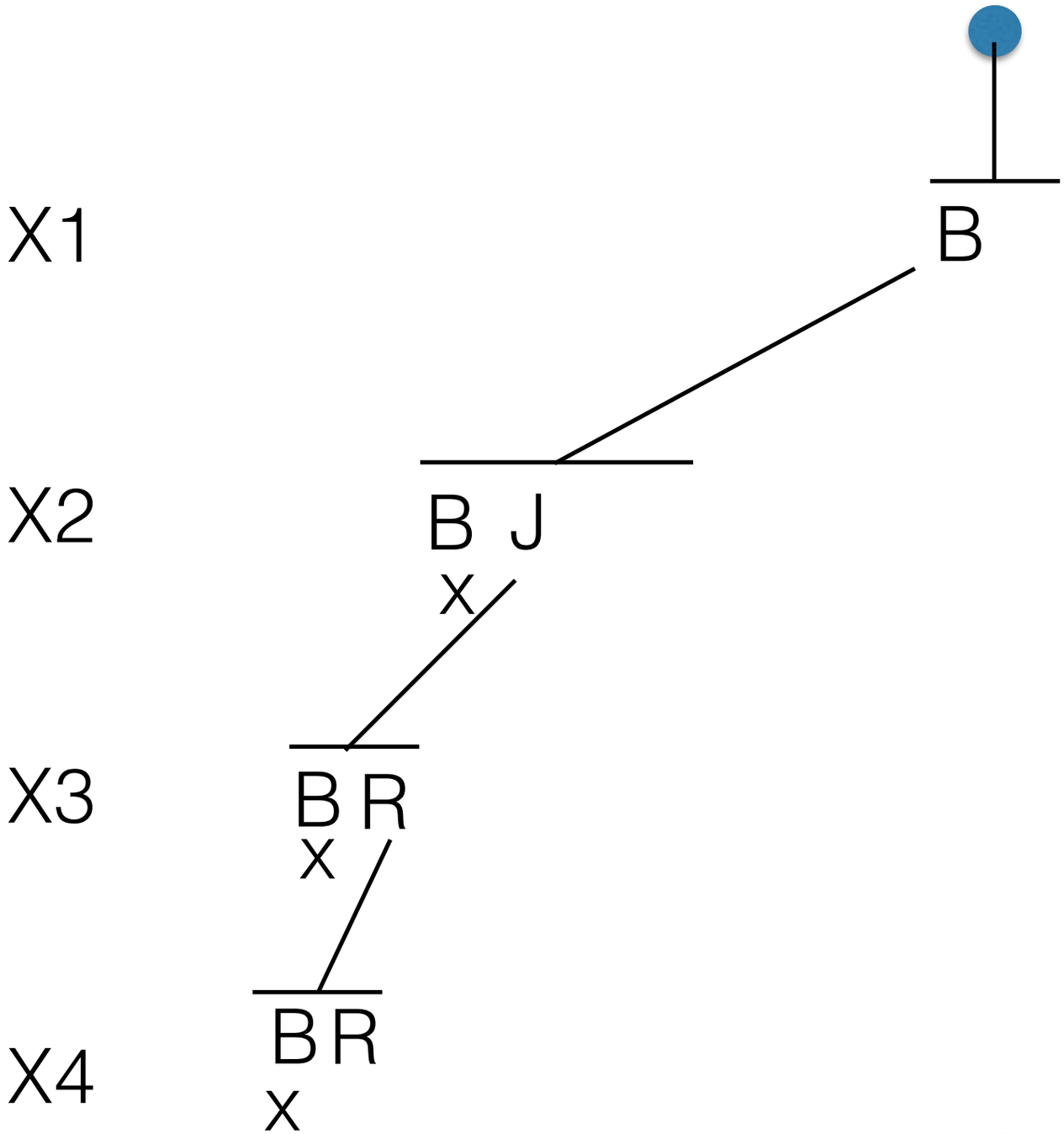

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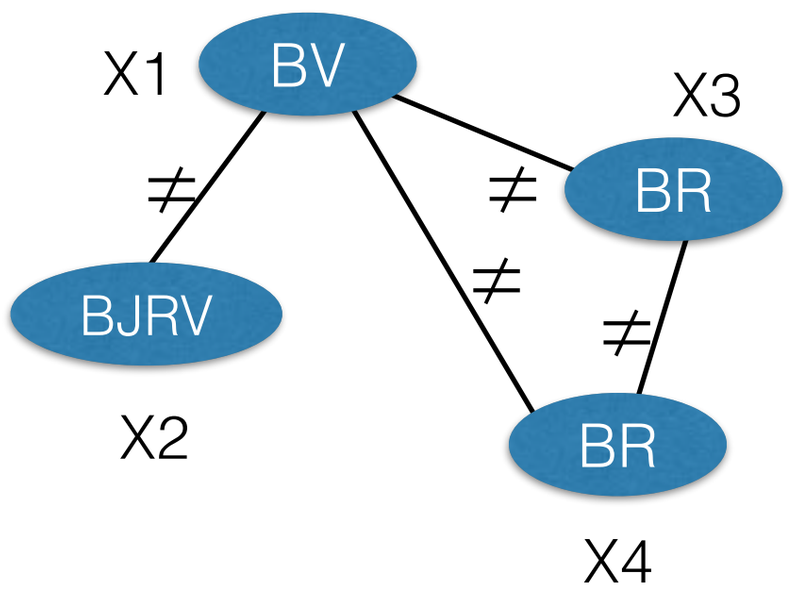

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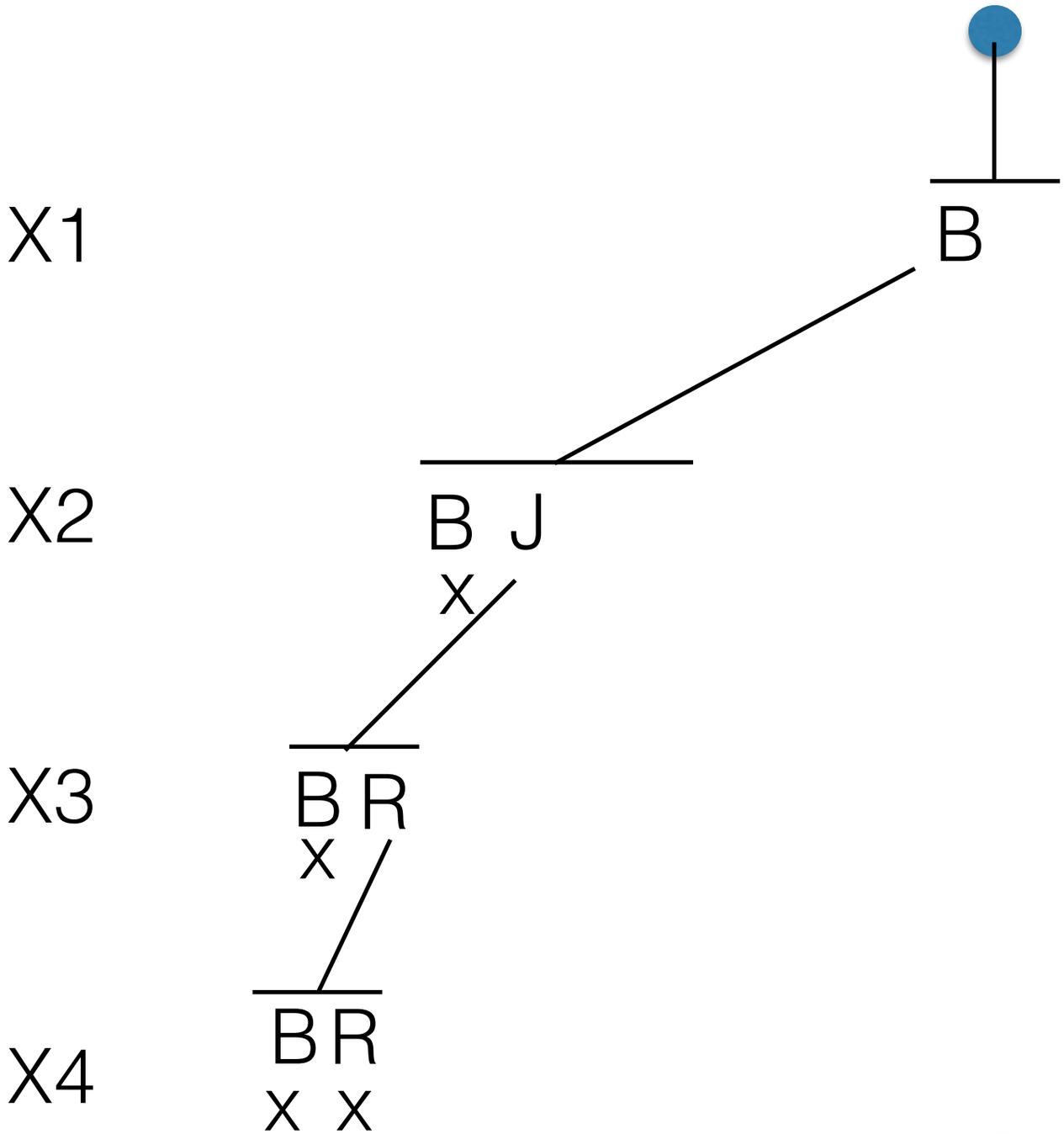

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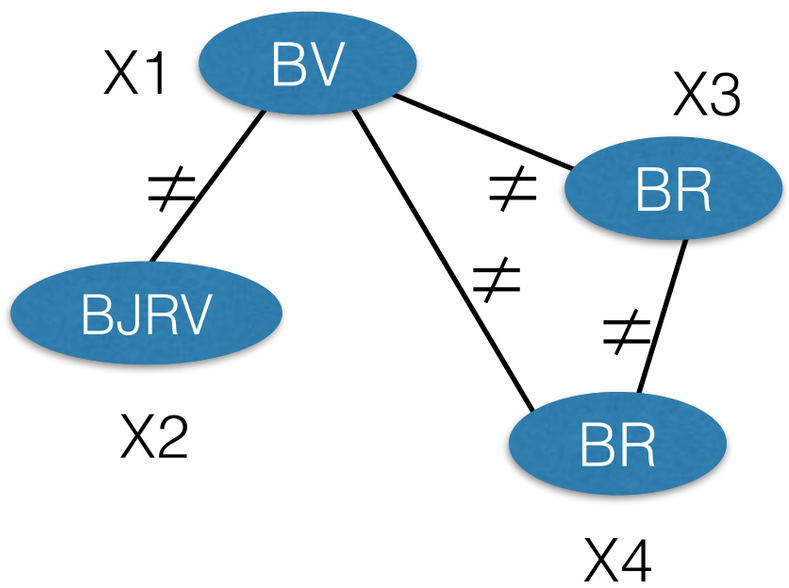

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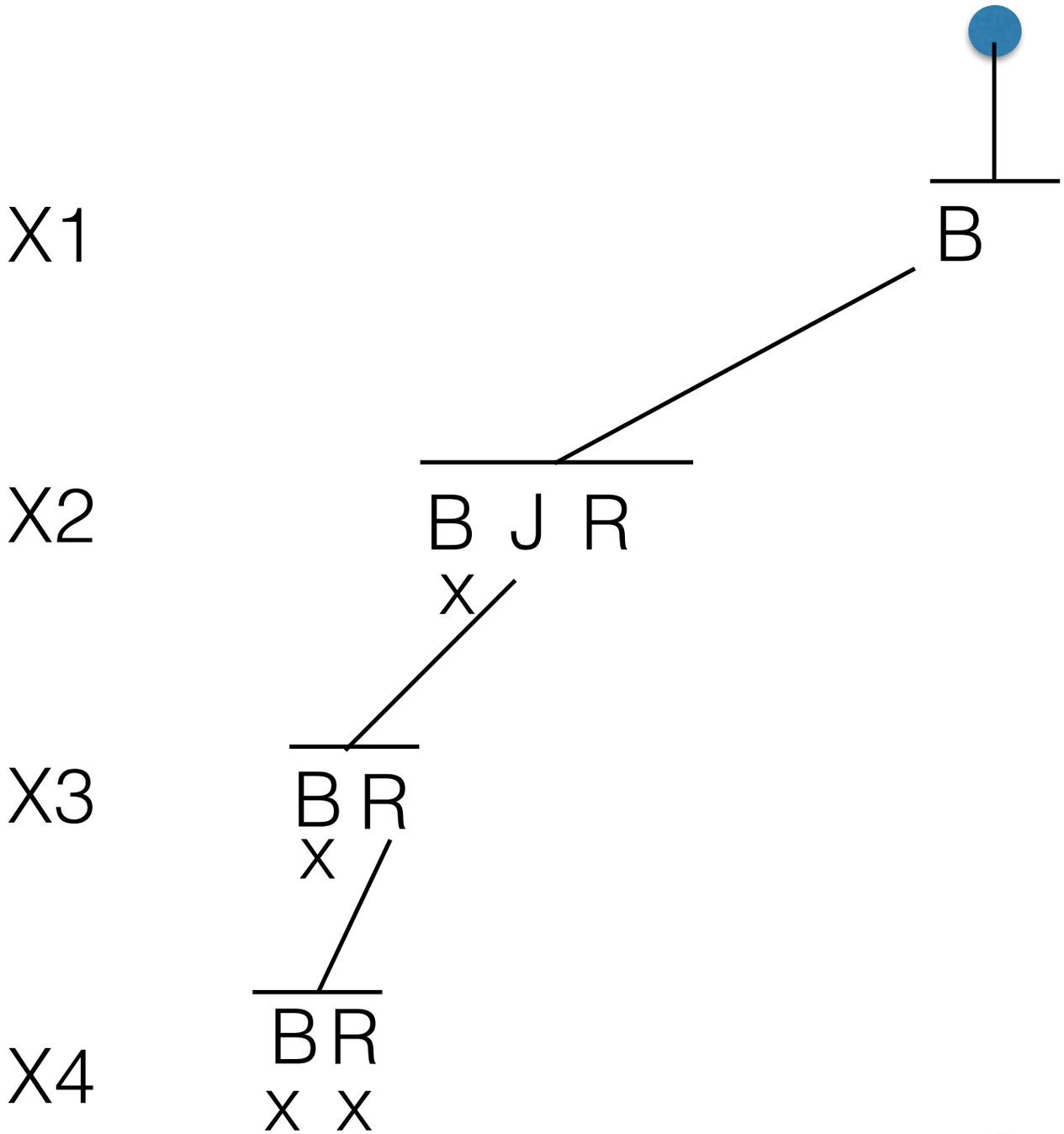

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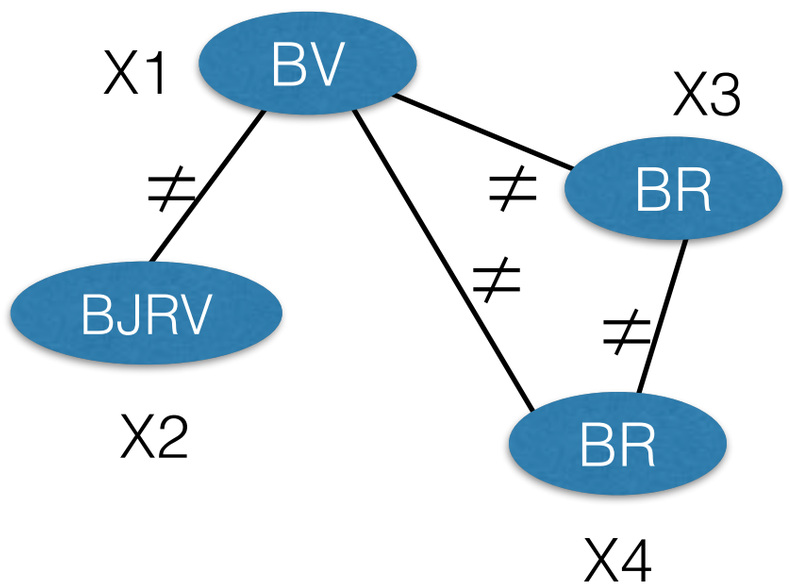

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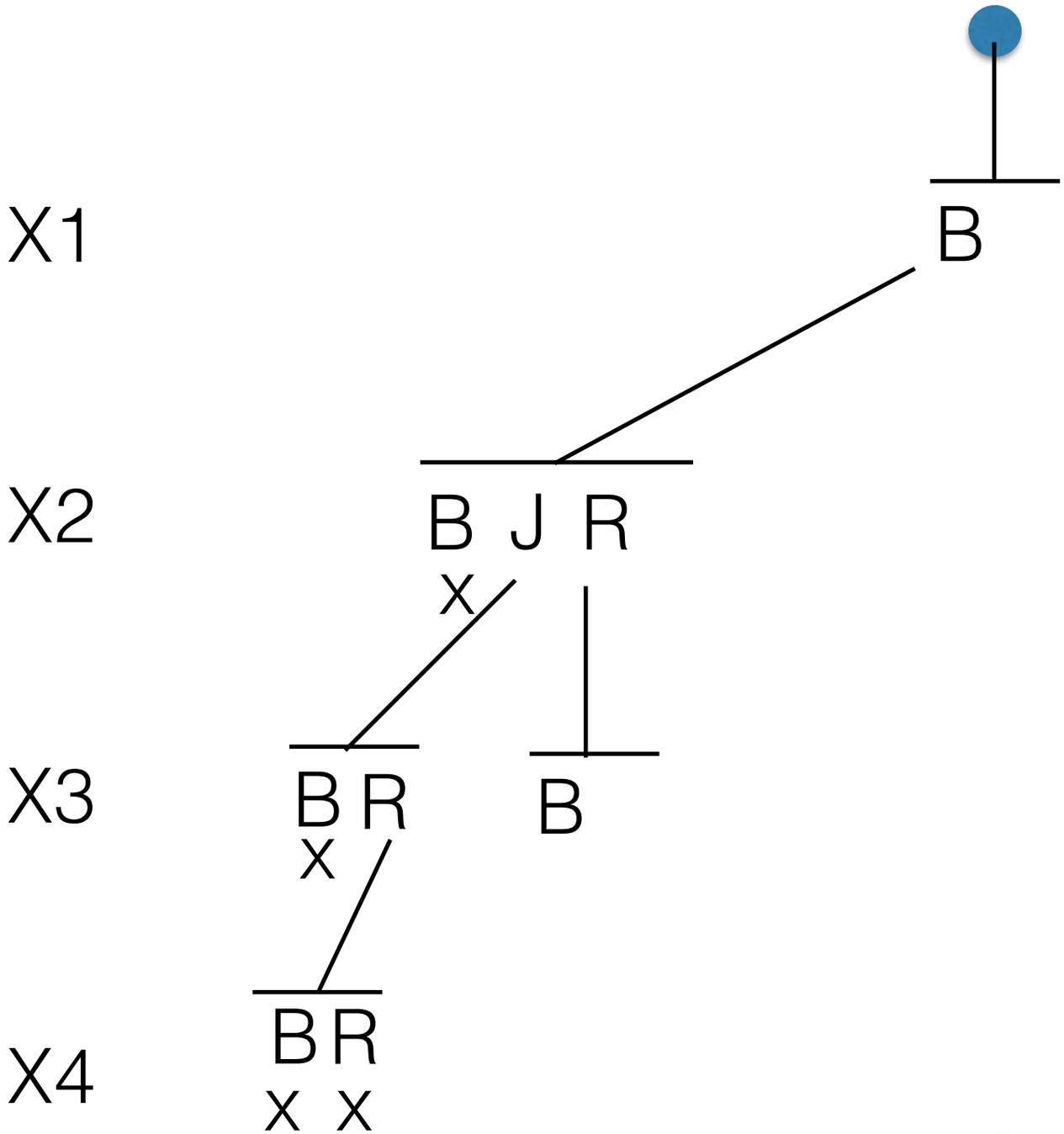

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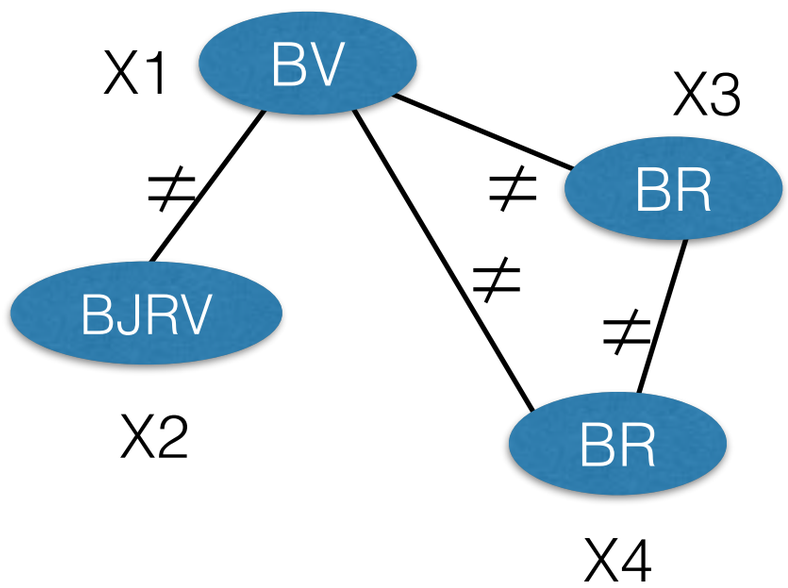

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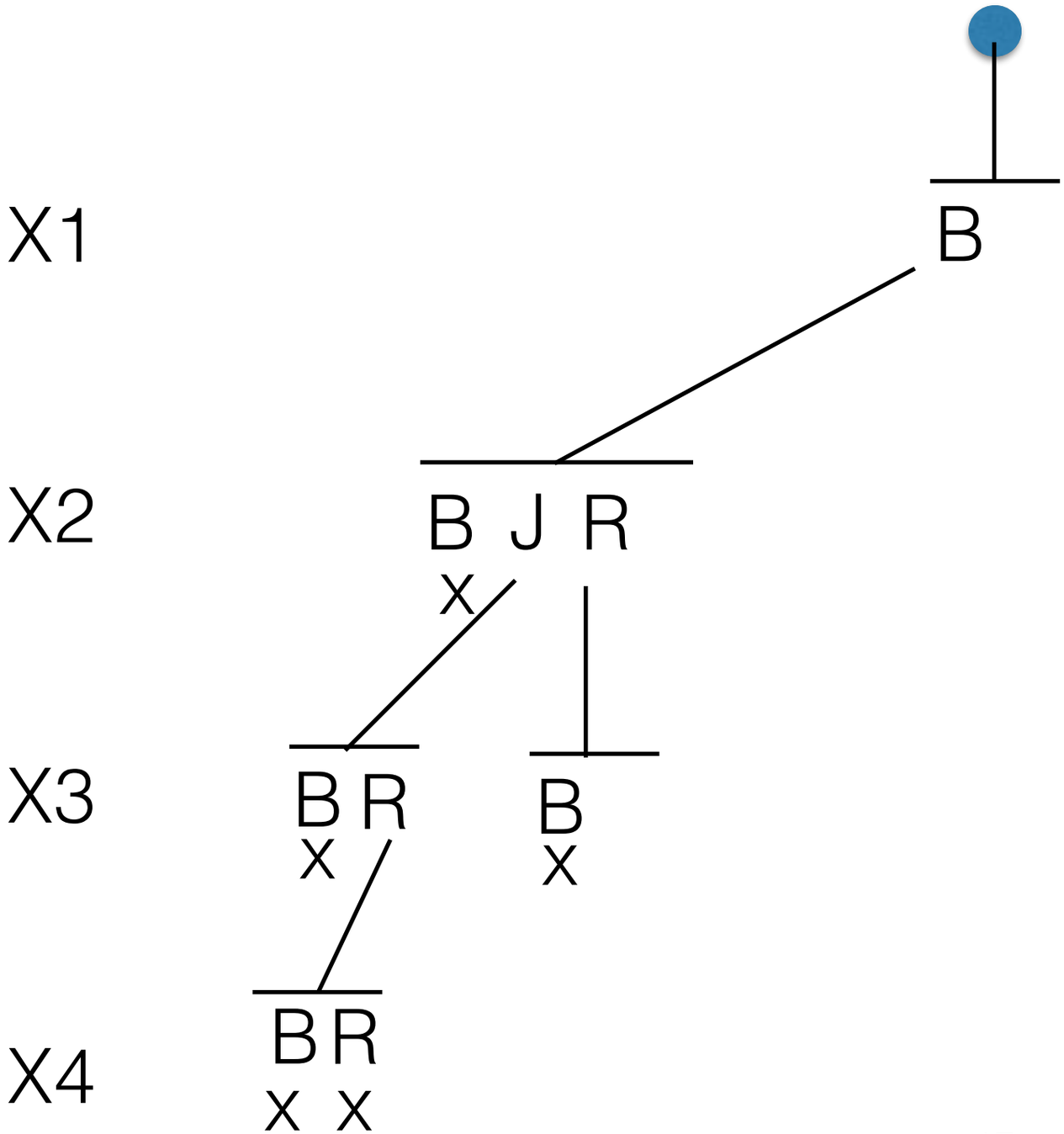

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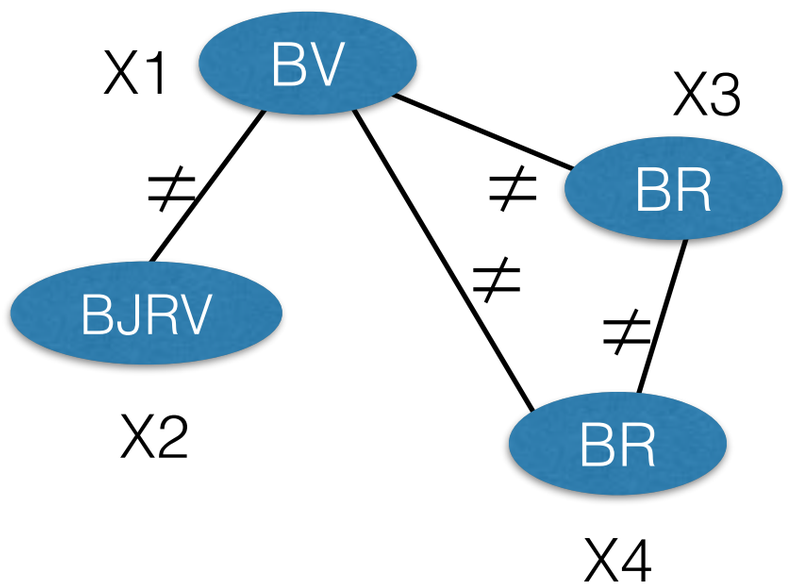

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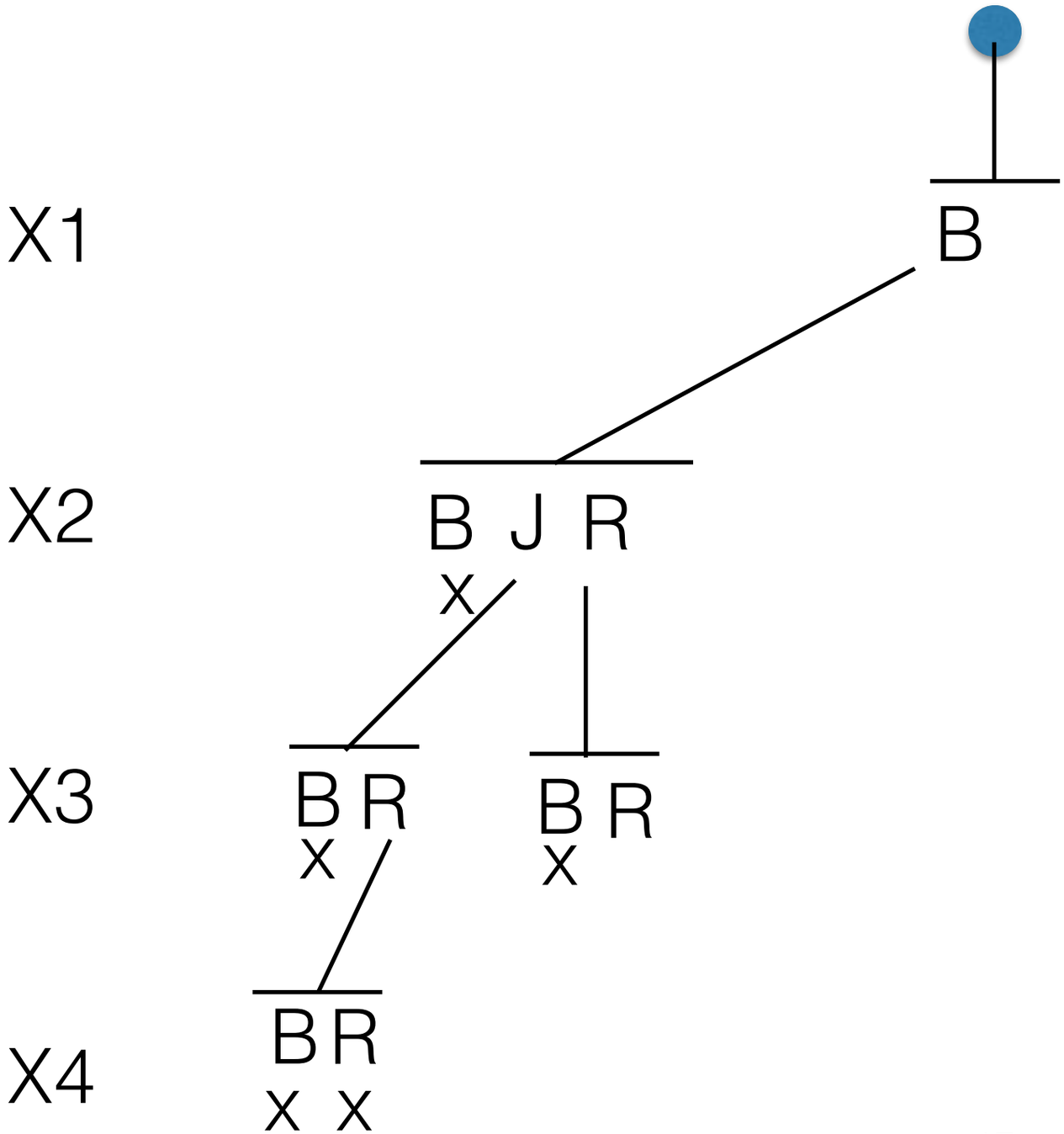


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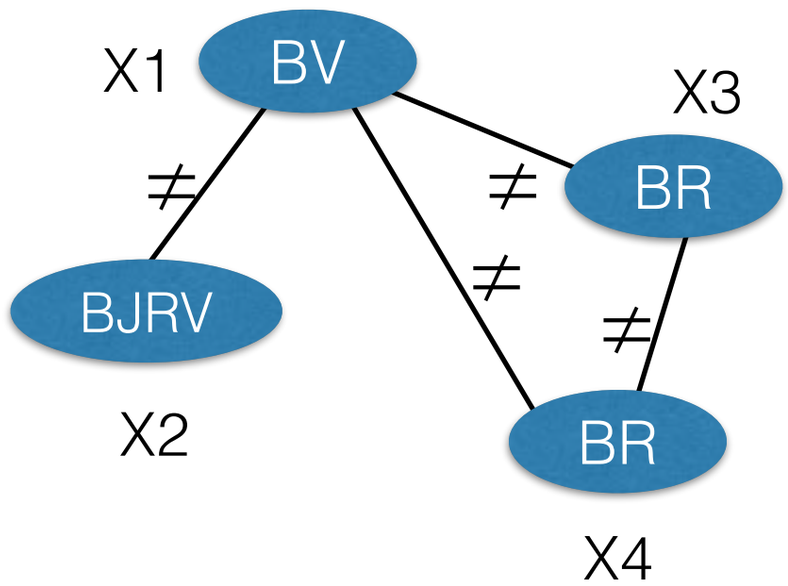

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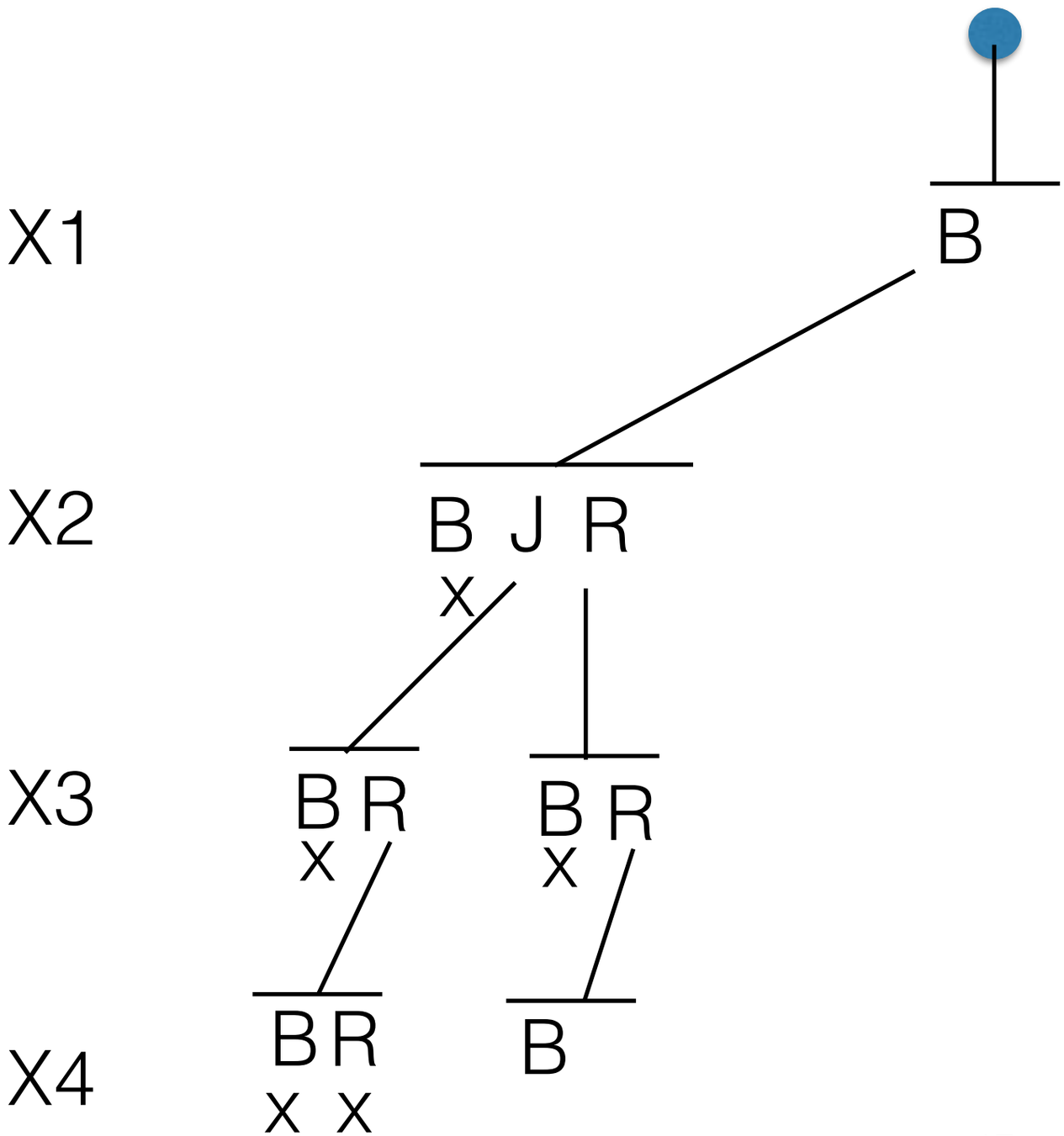

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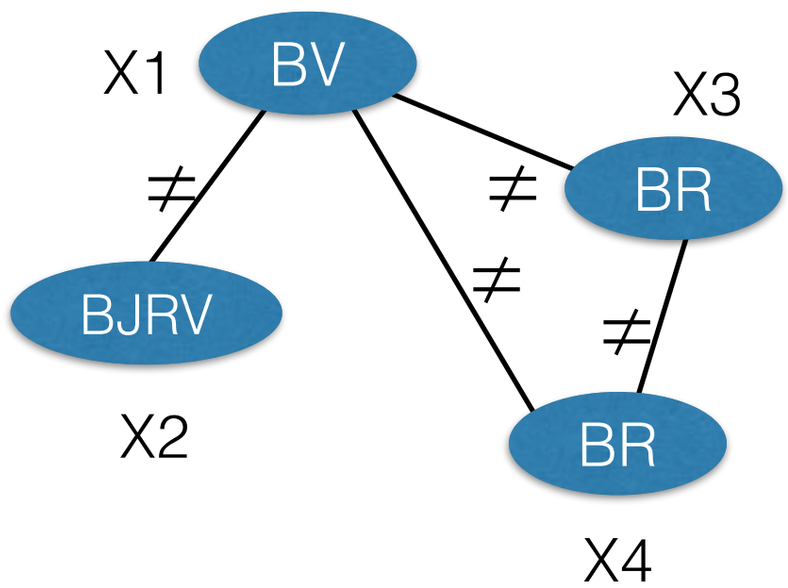

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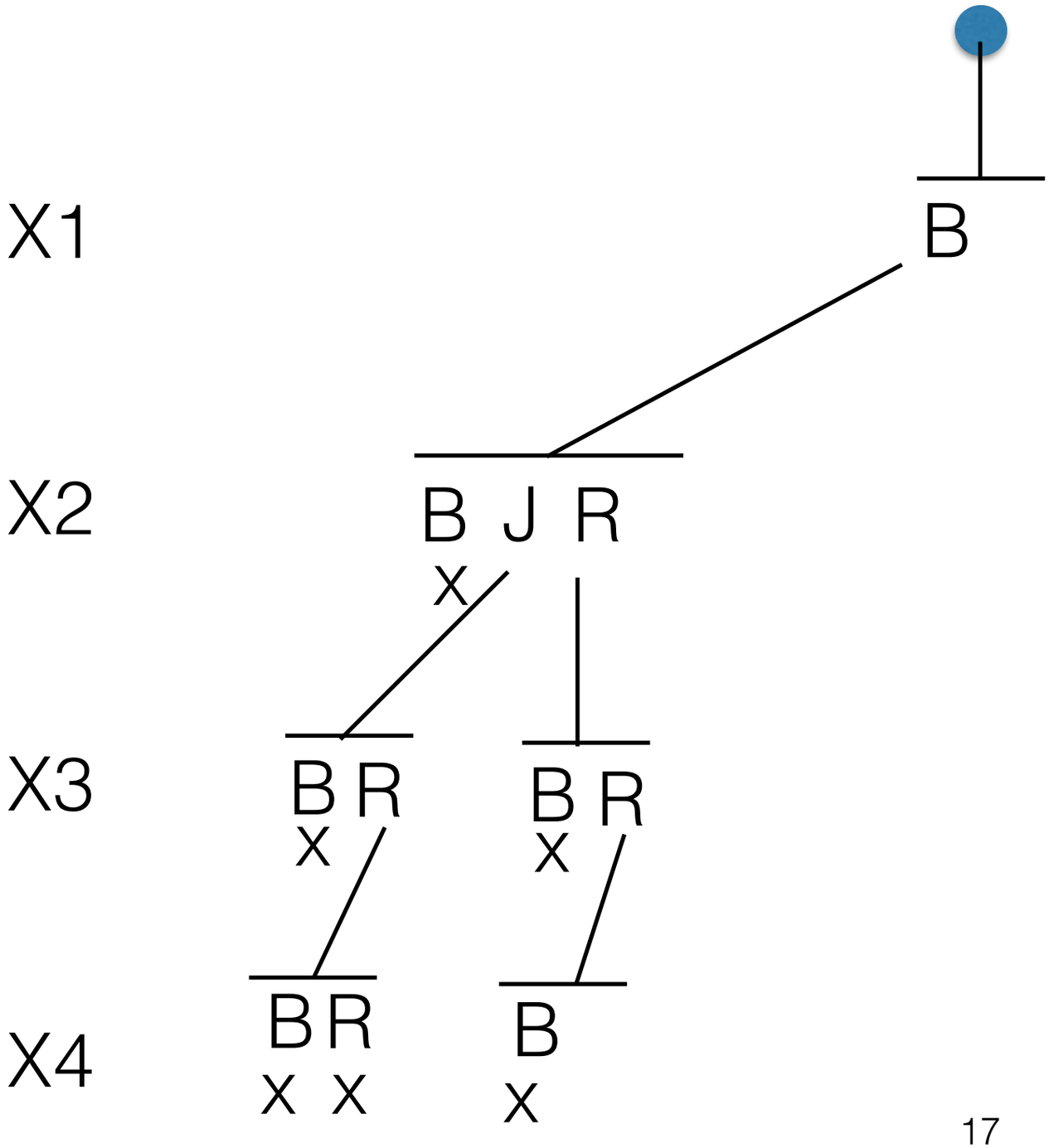

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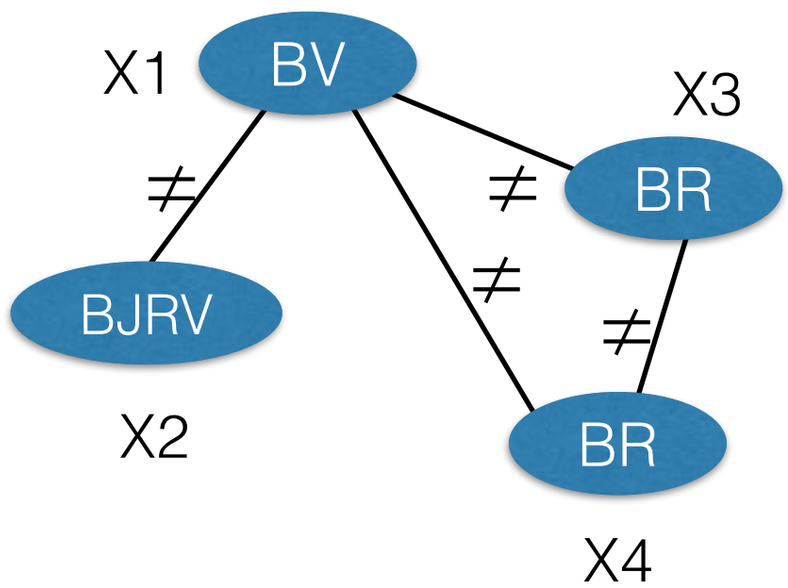

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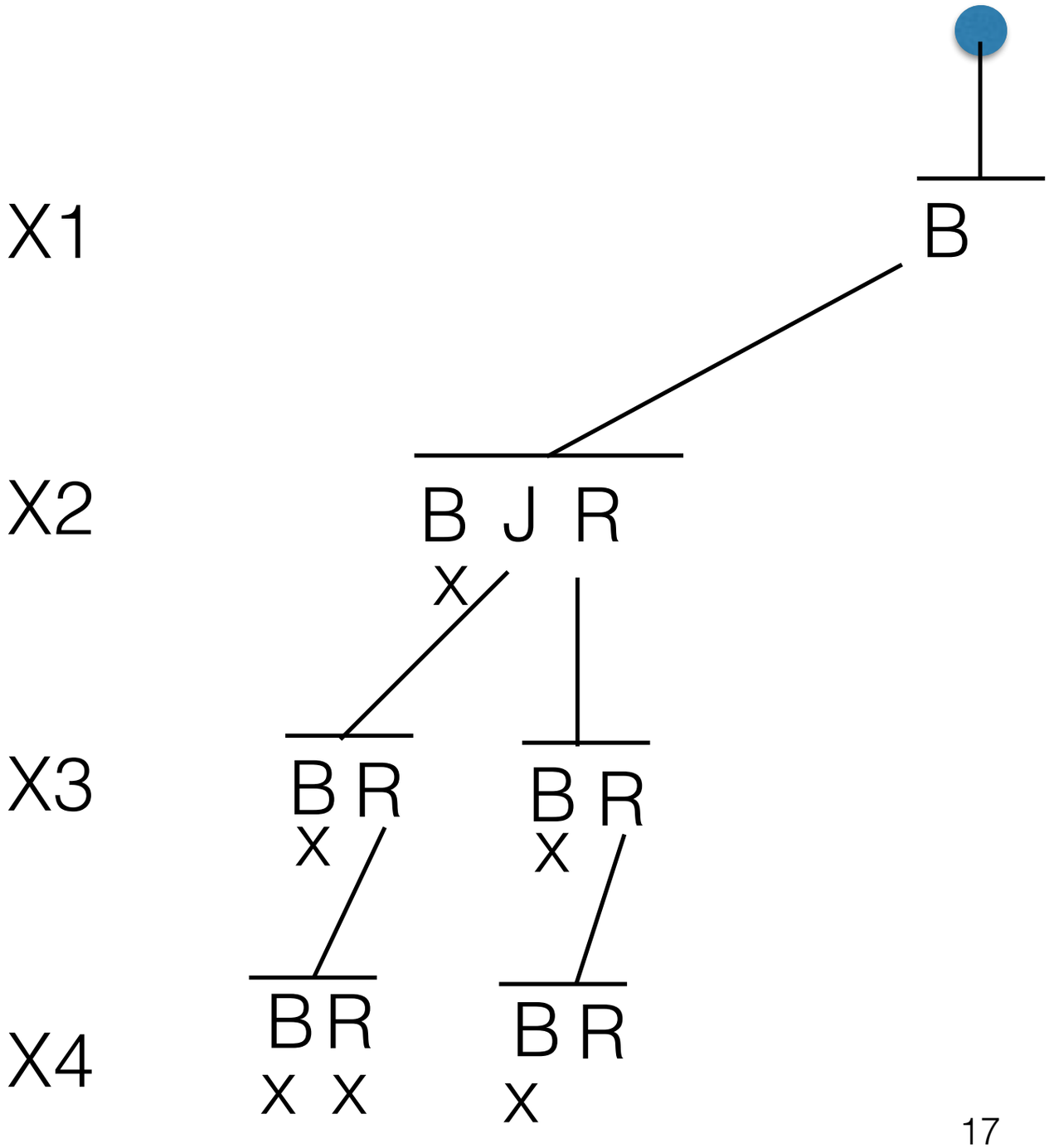

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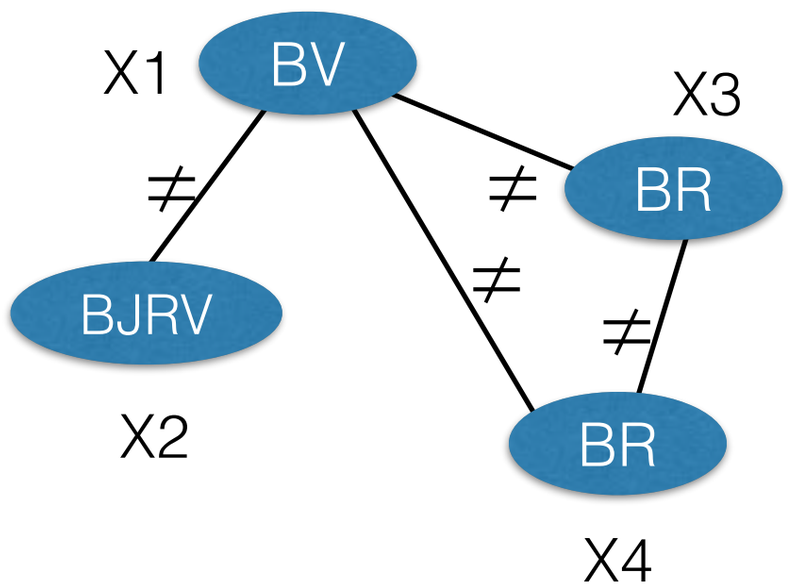

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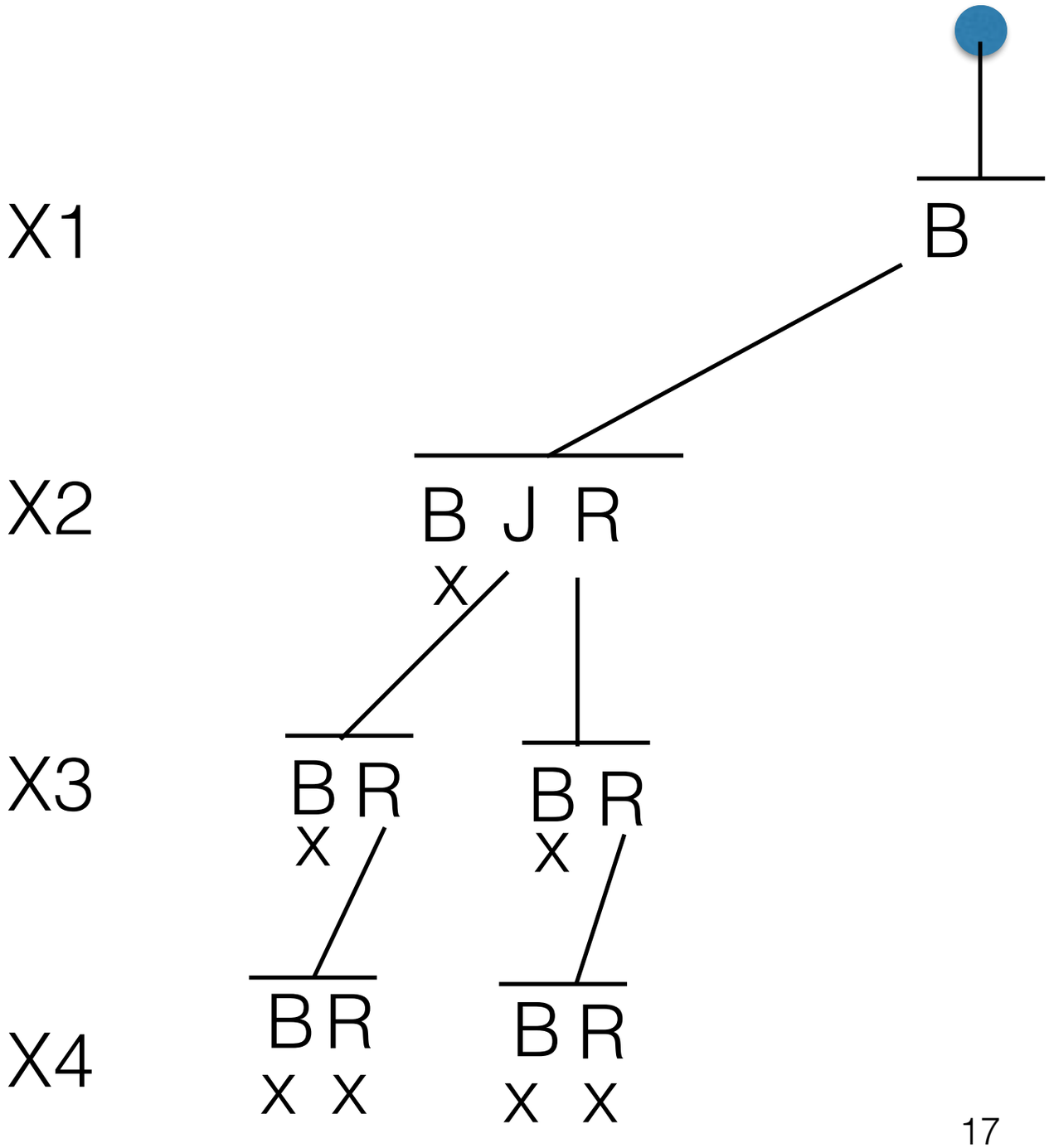

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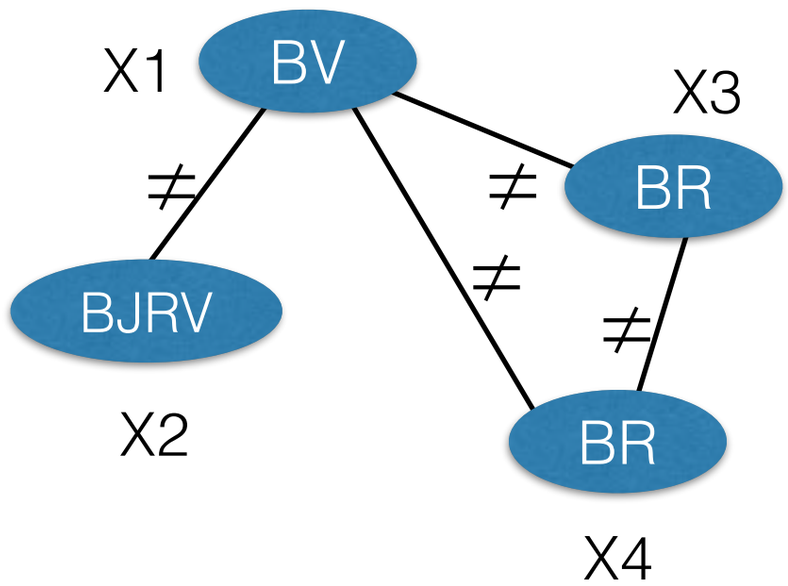

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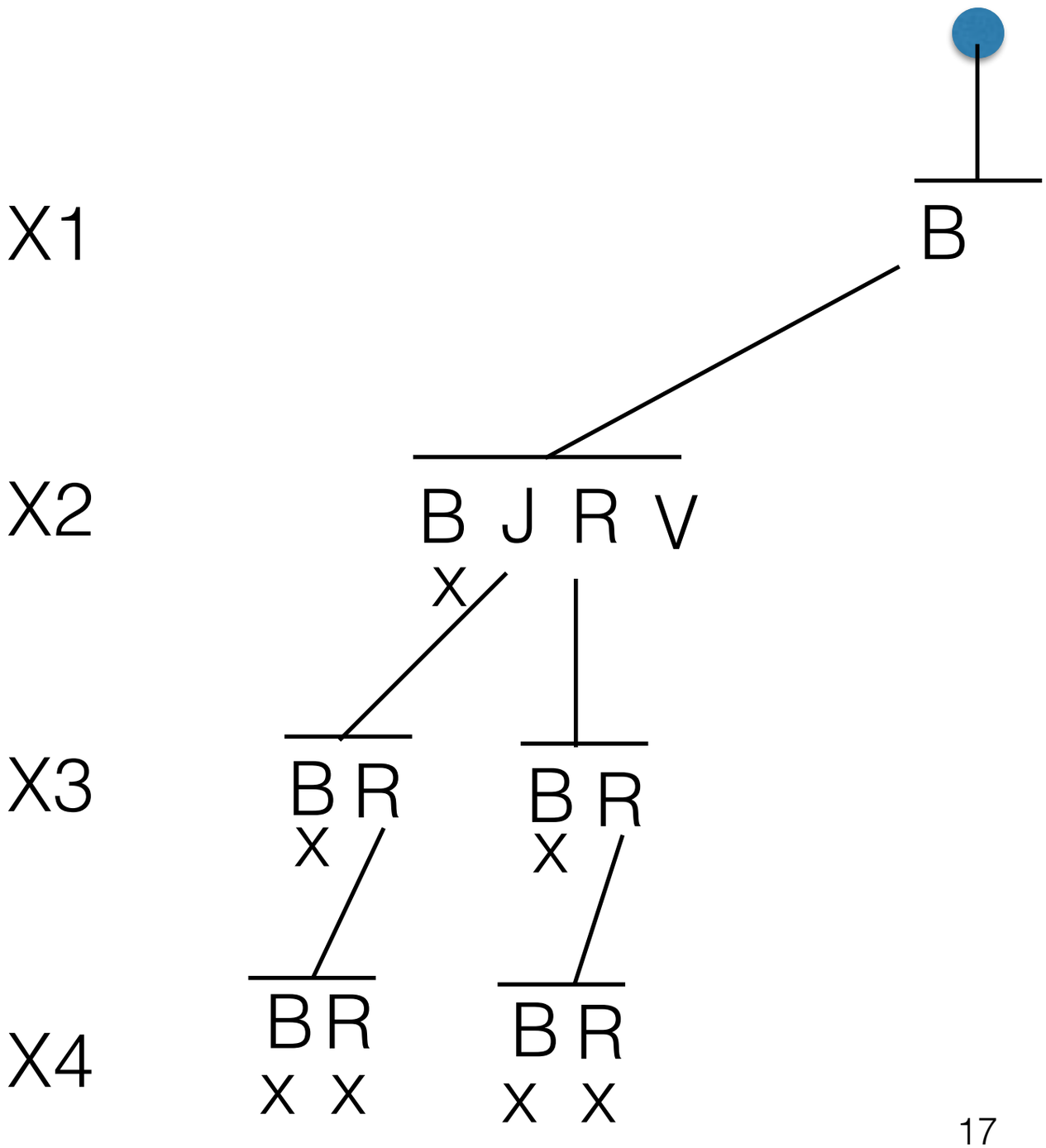

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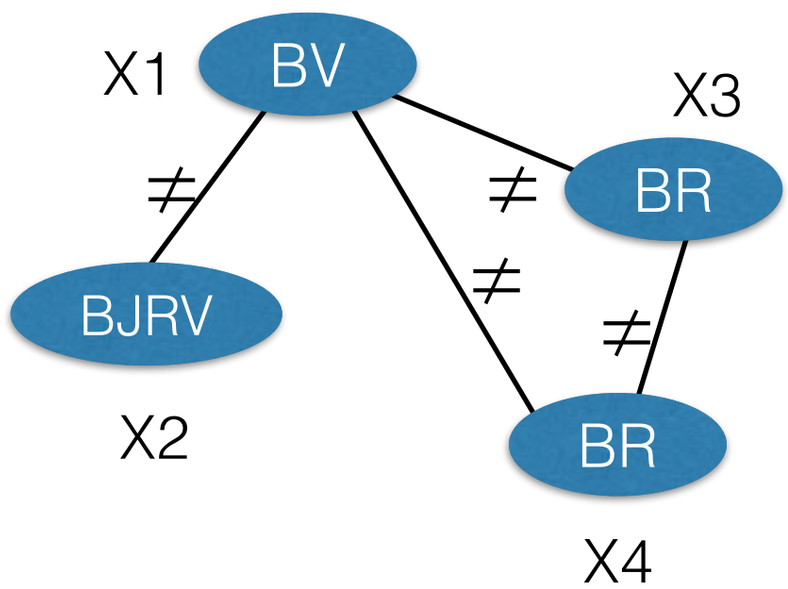

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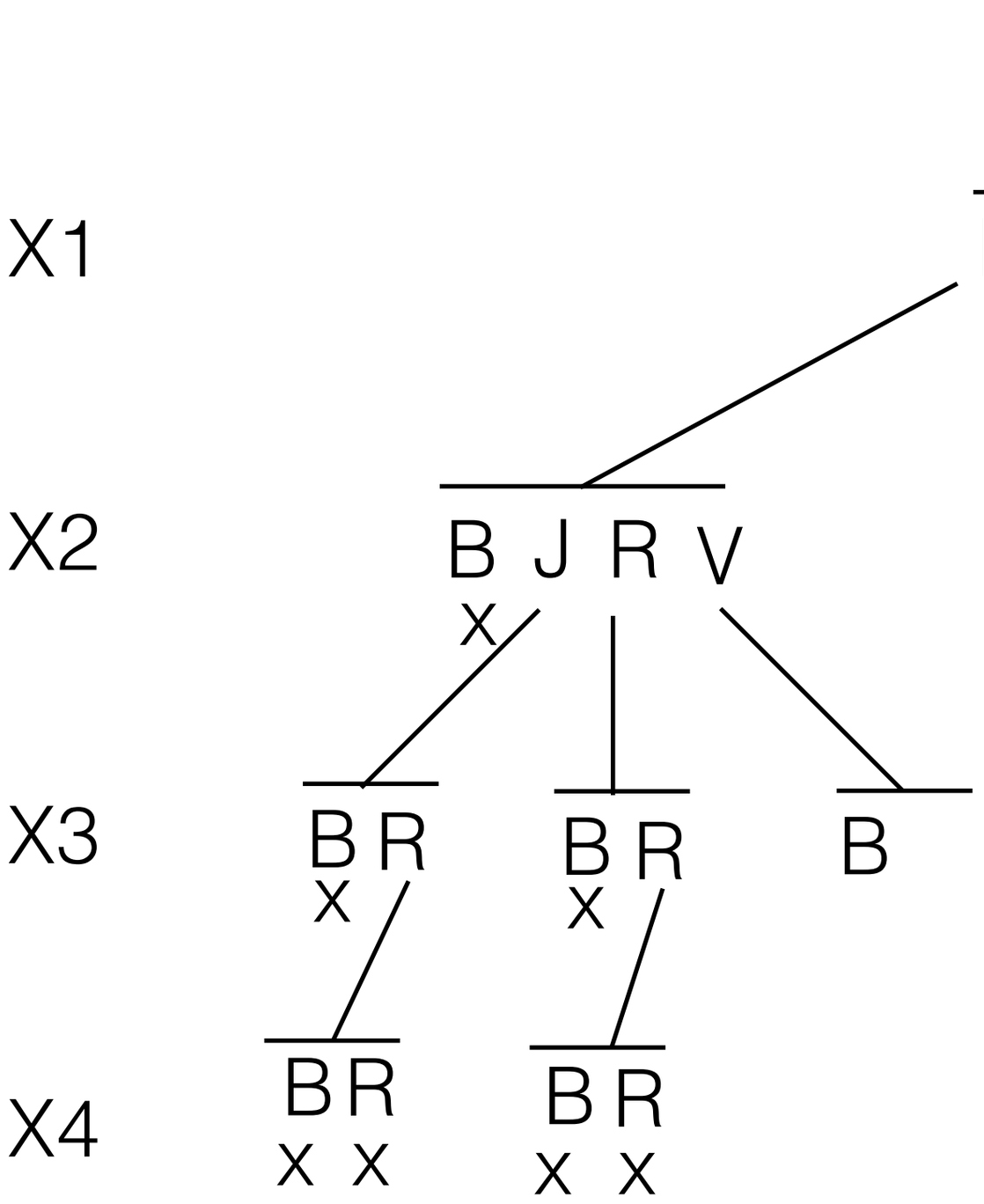

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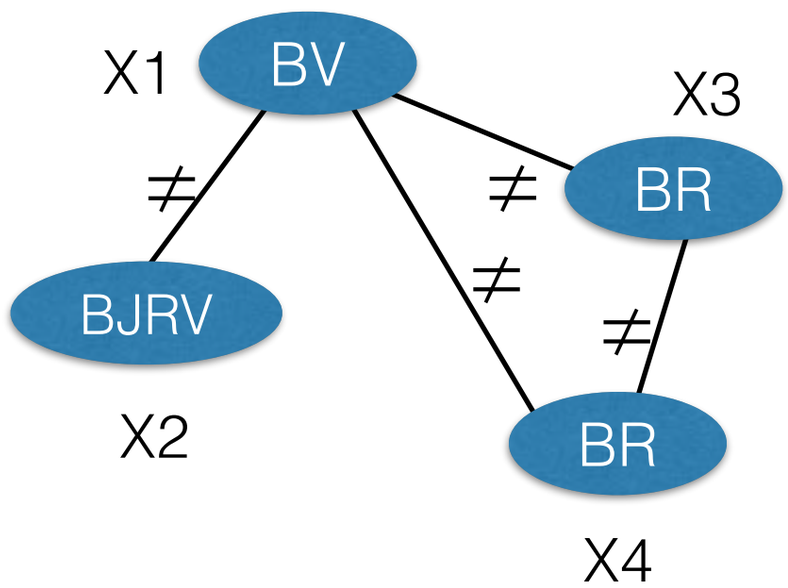

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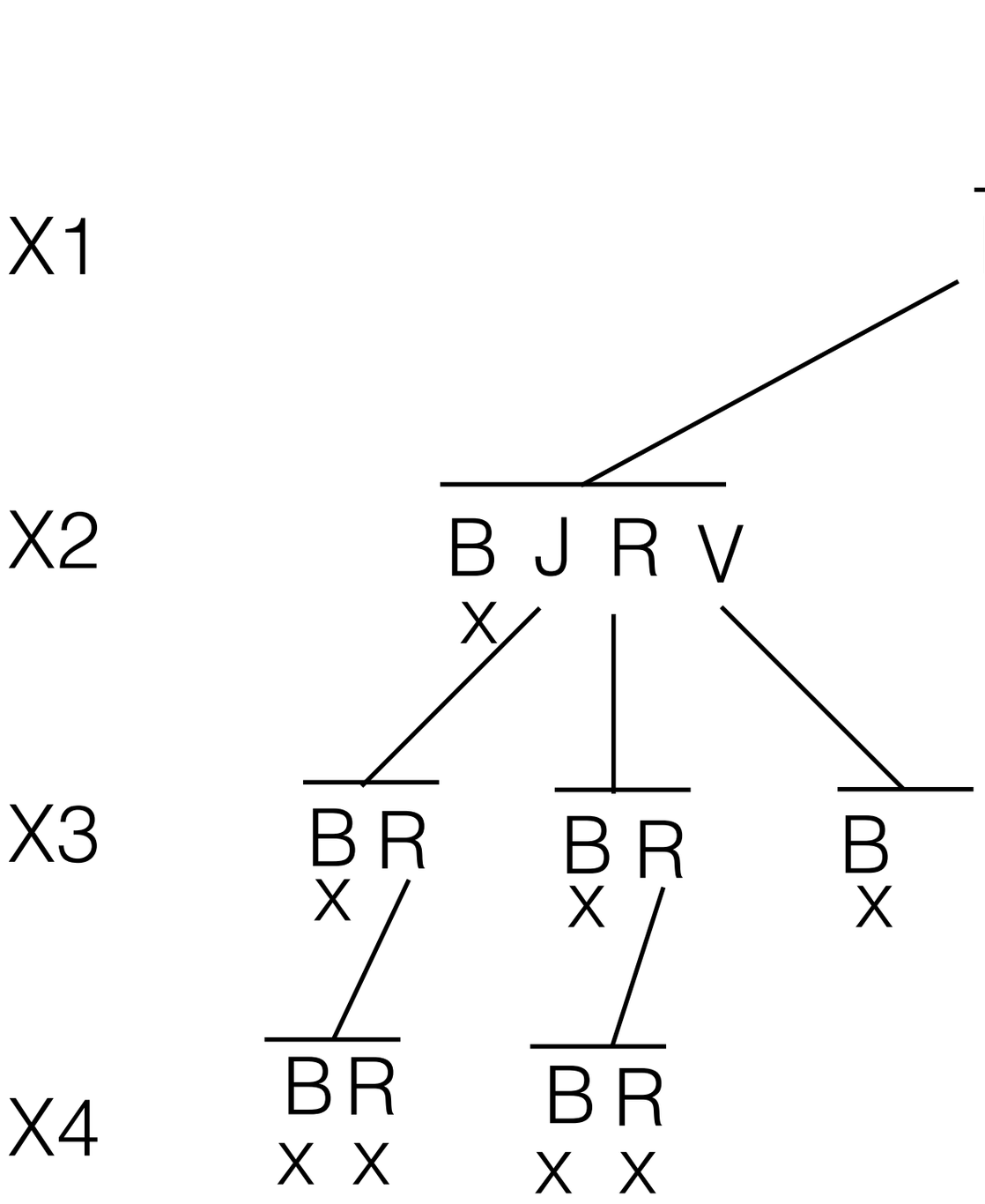

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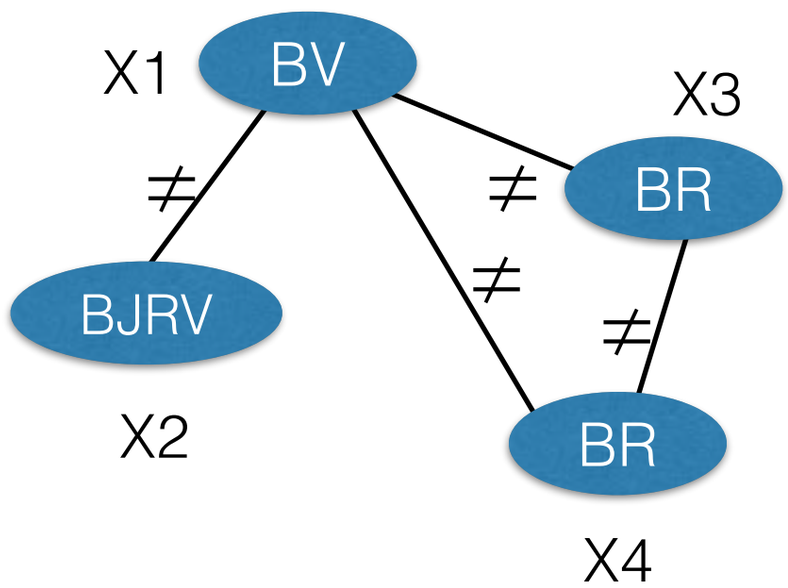
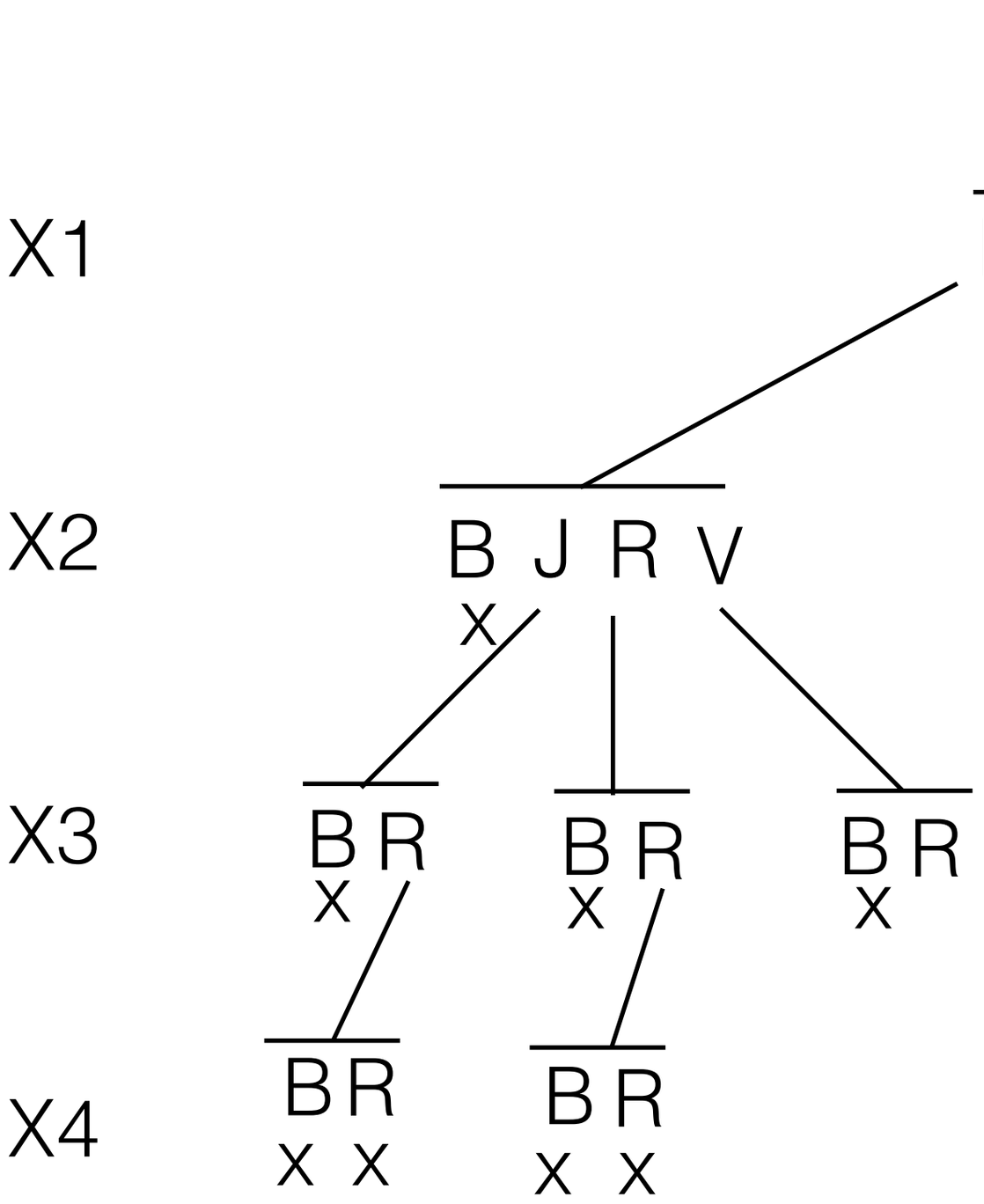


BT on running example



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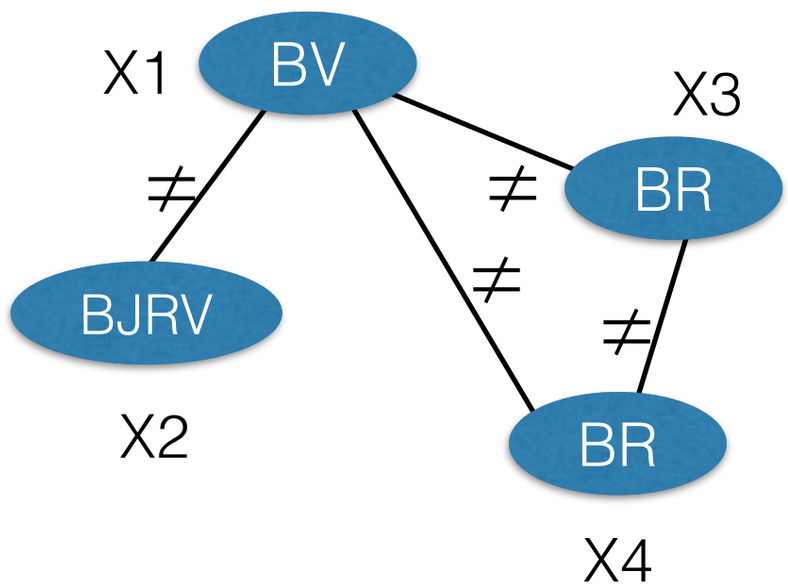

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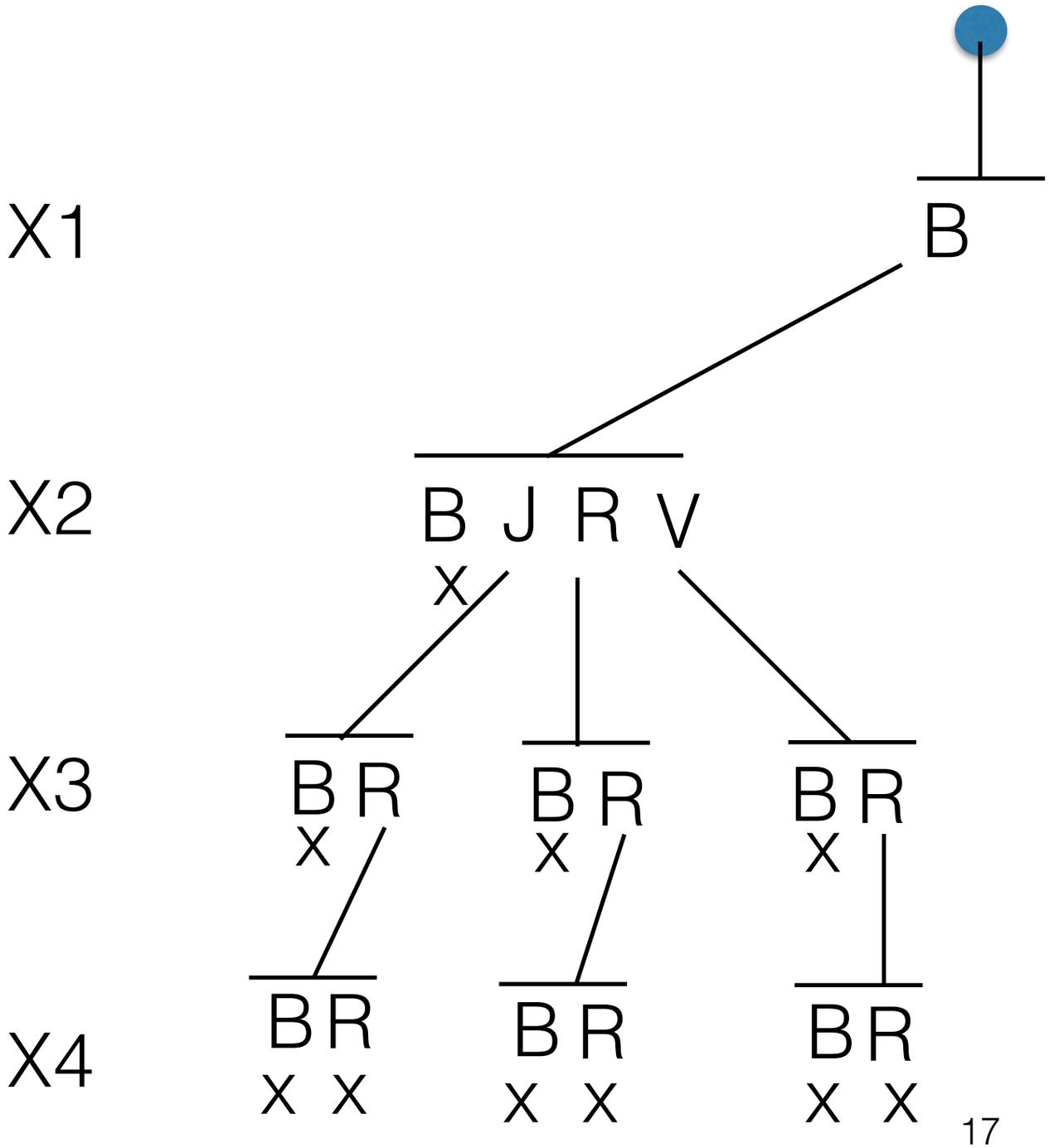


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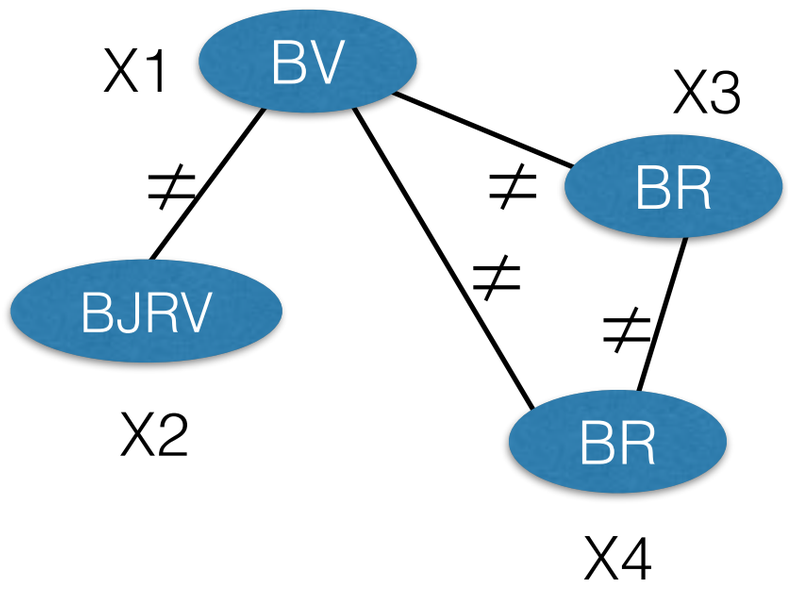

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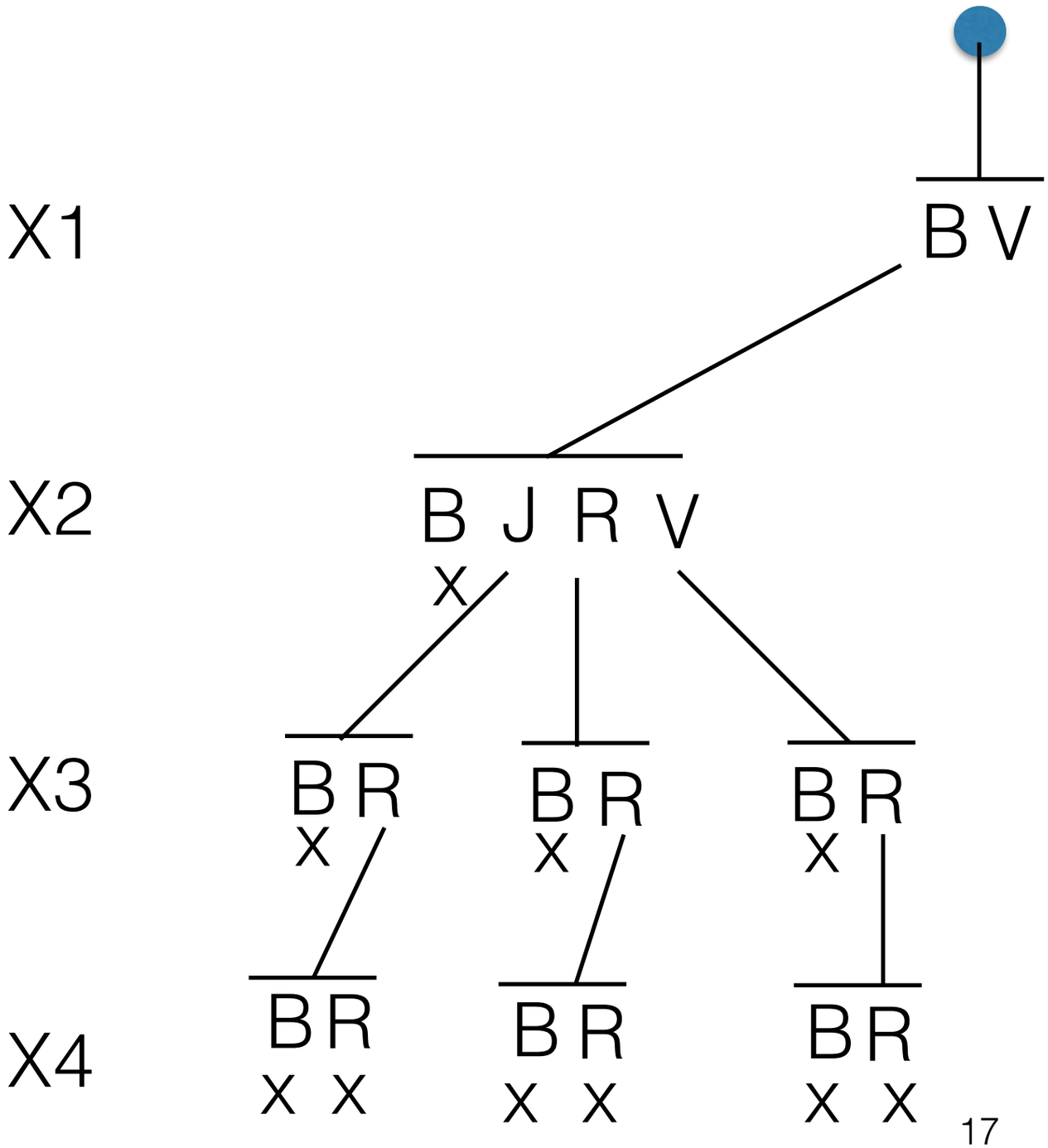

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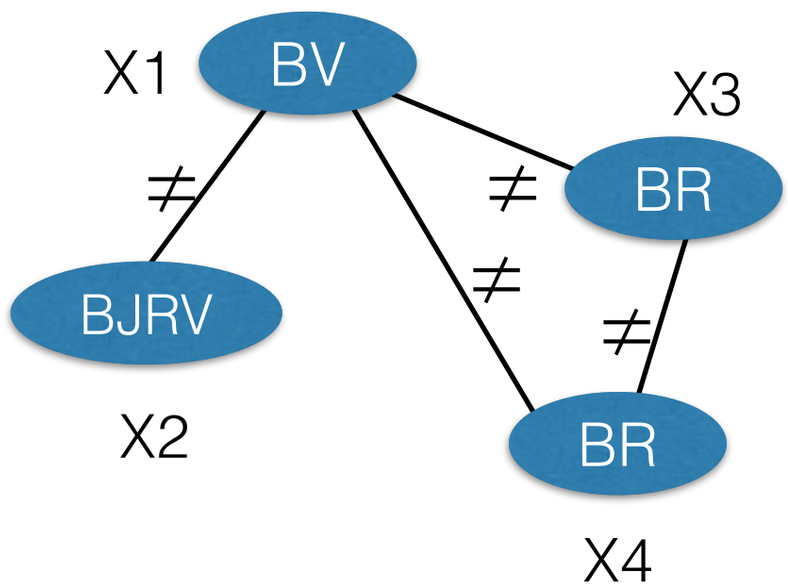

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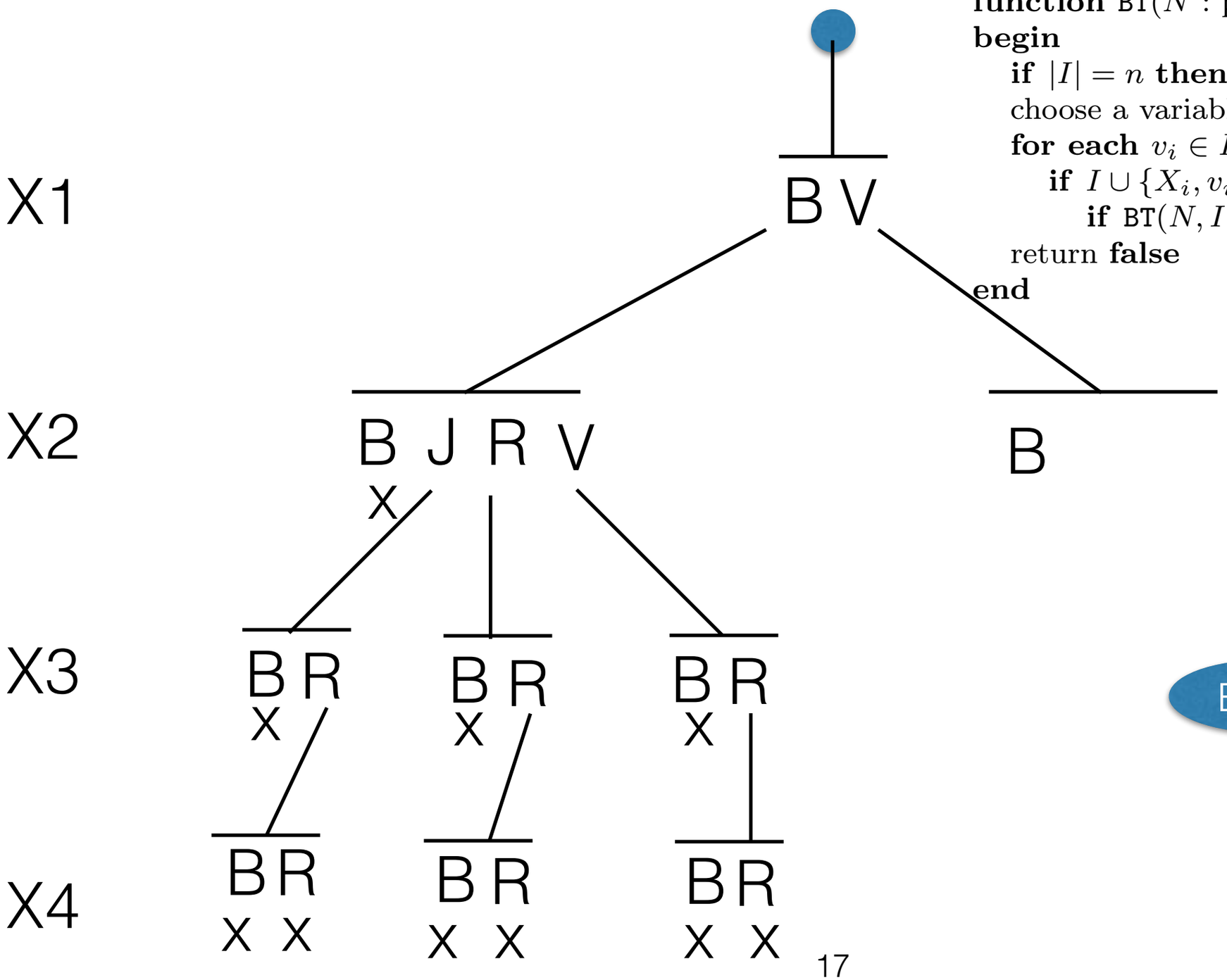

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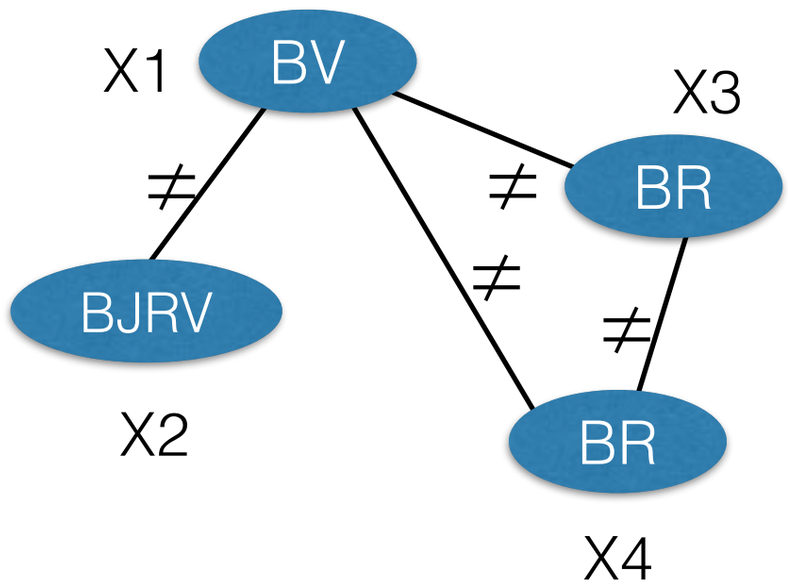
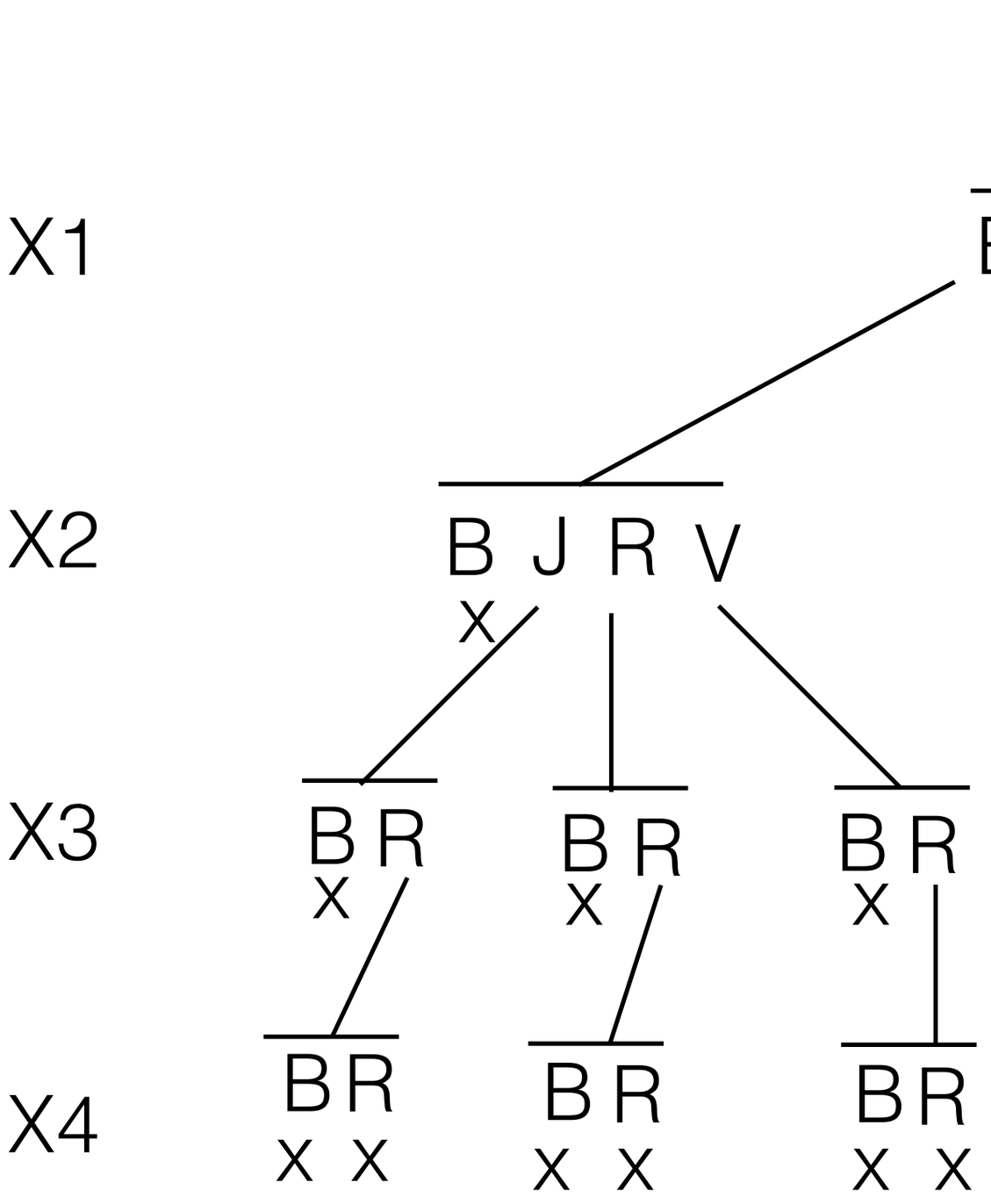


BT on running example



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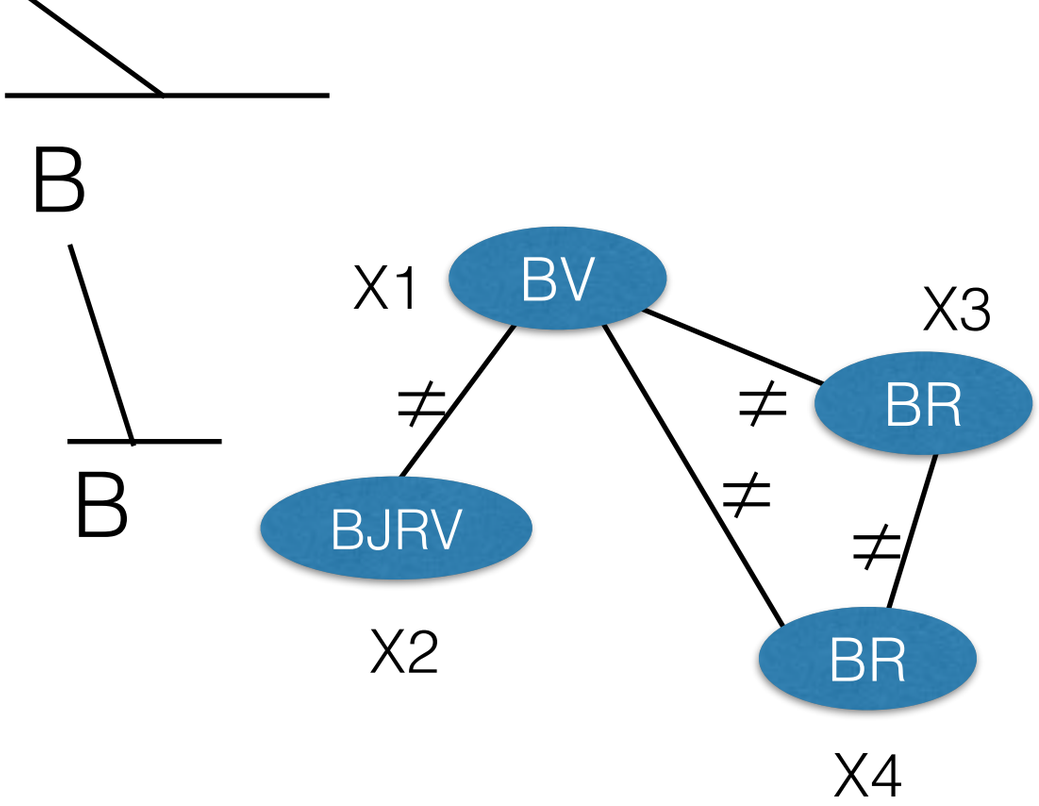
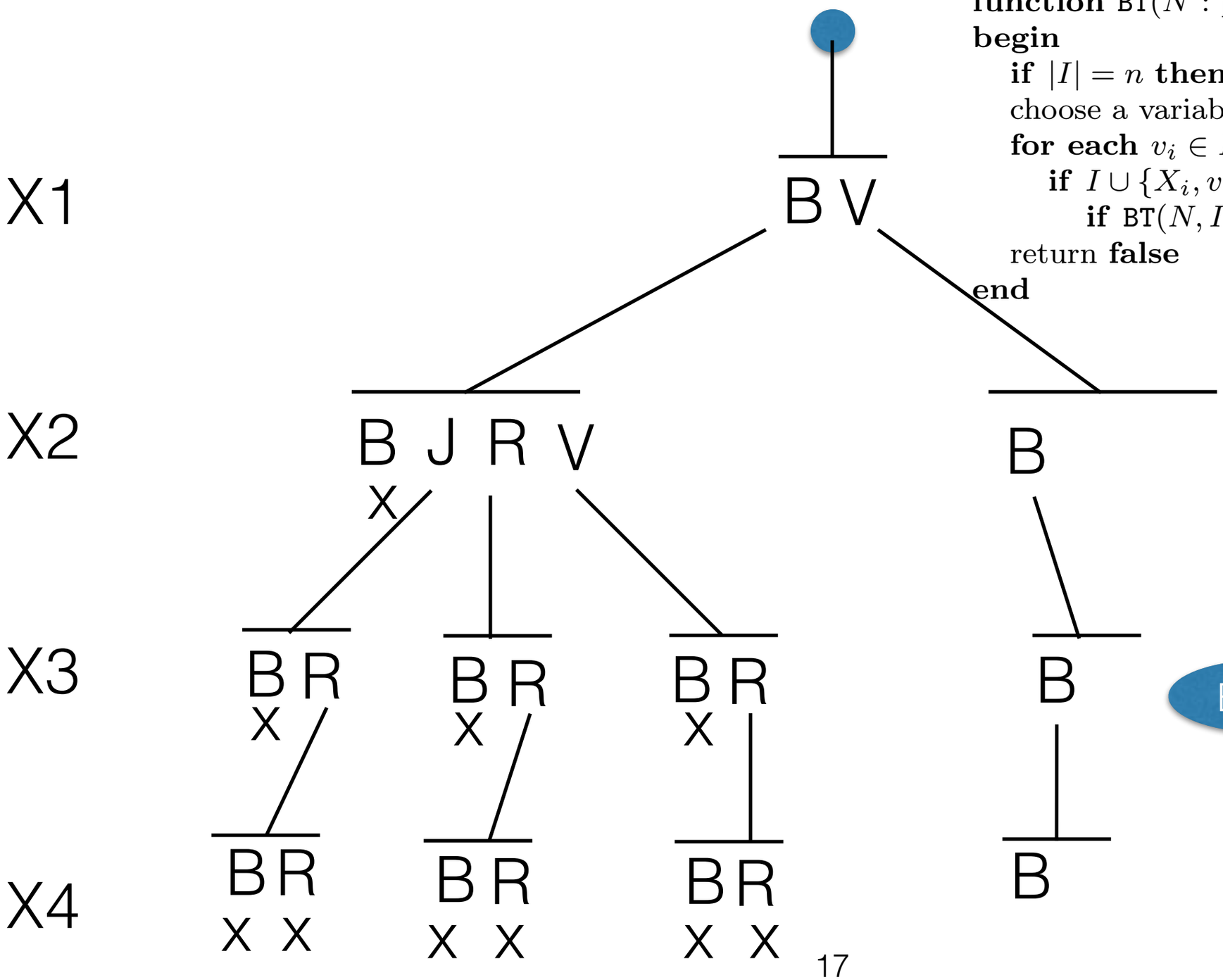


BT on running example



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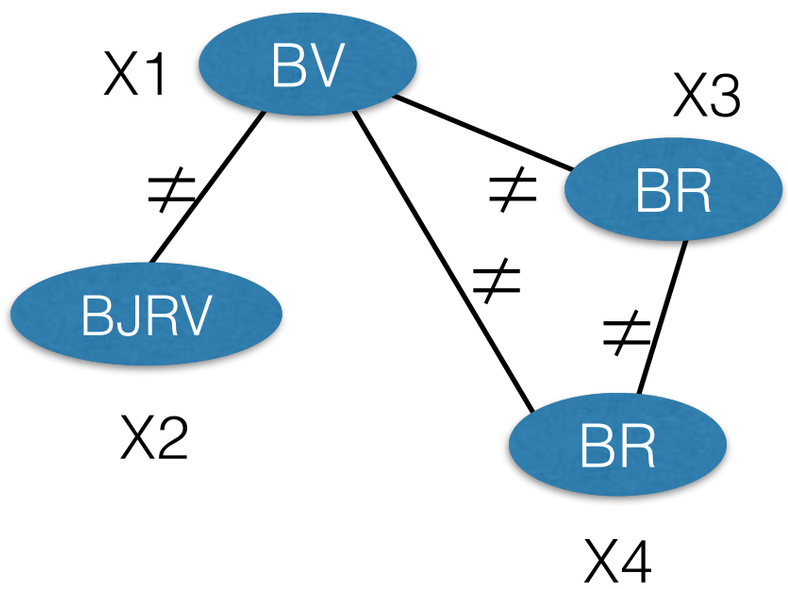

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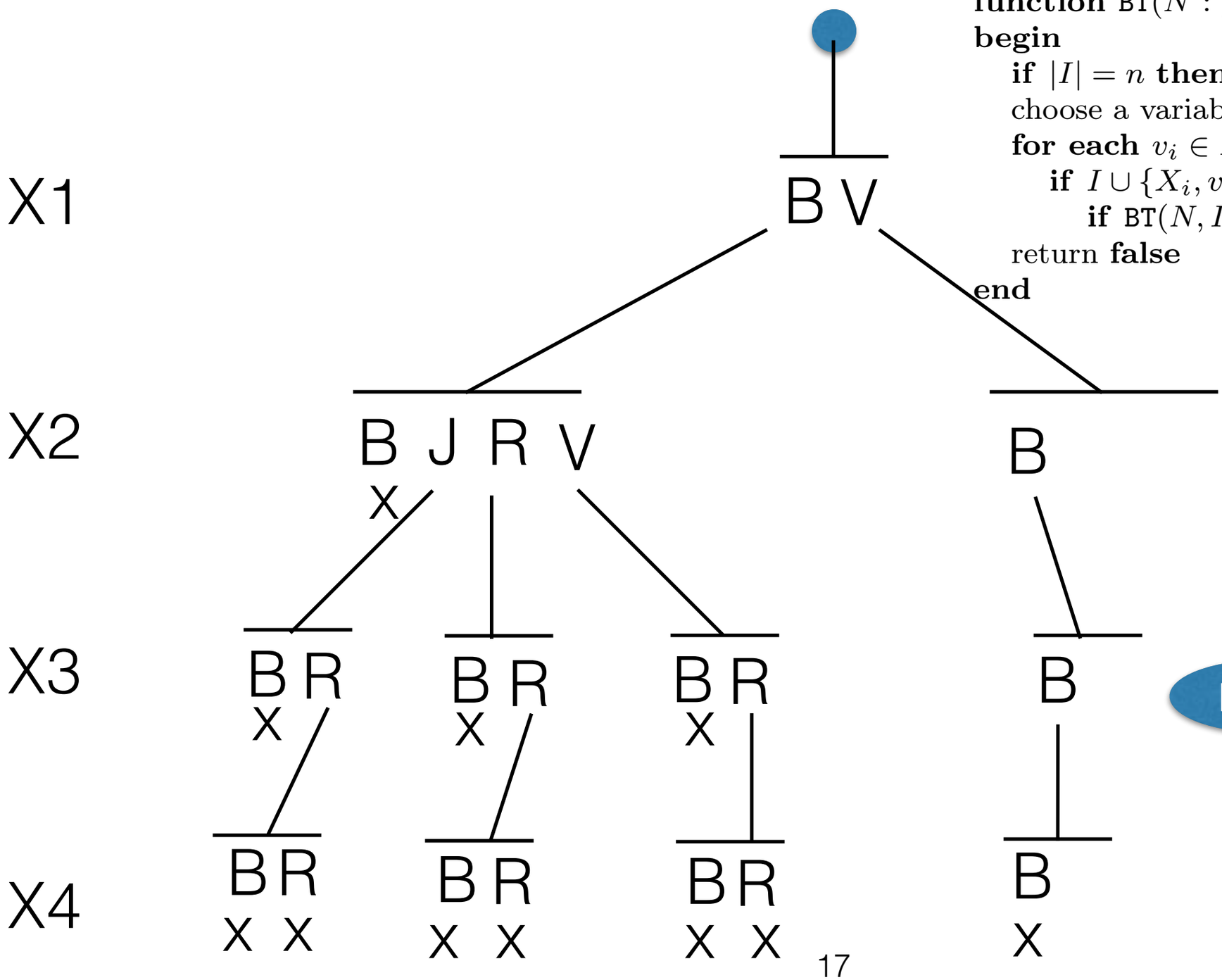

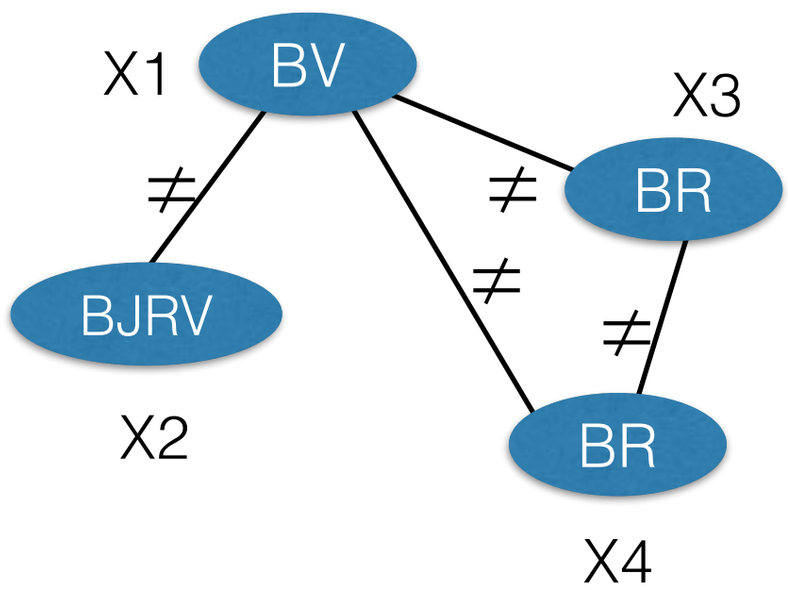

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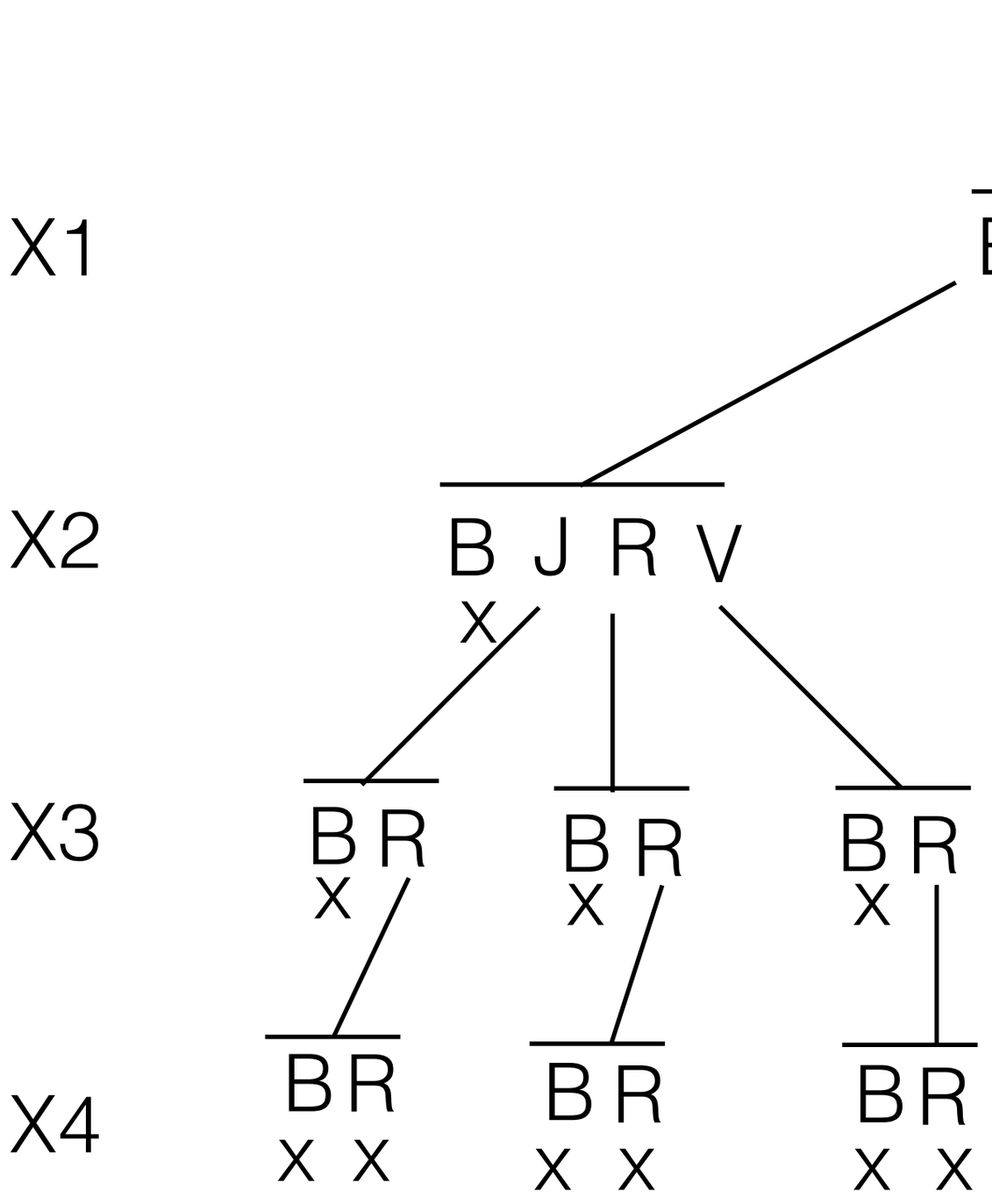
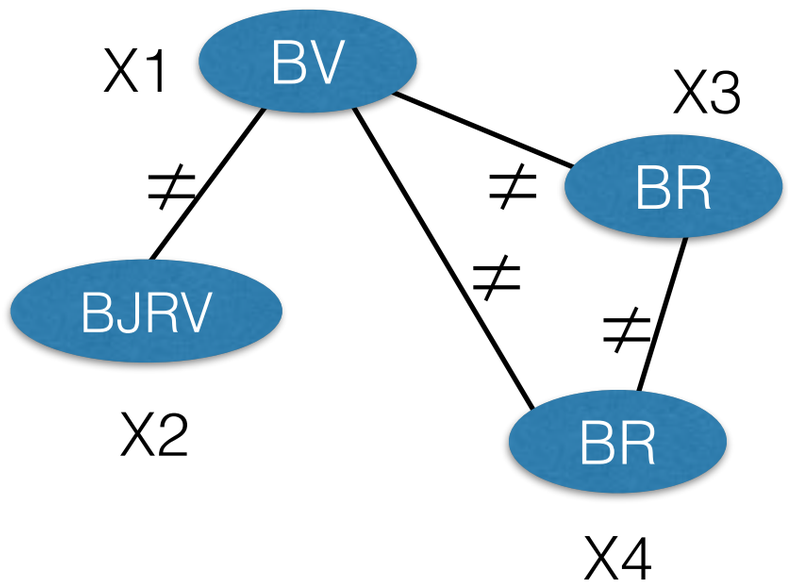
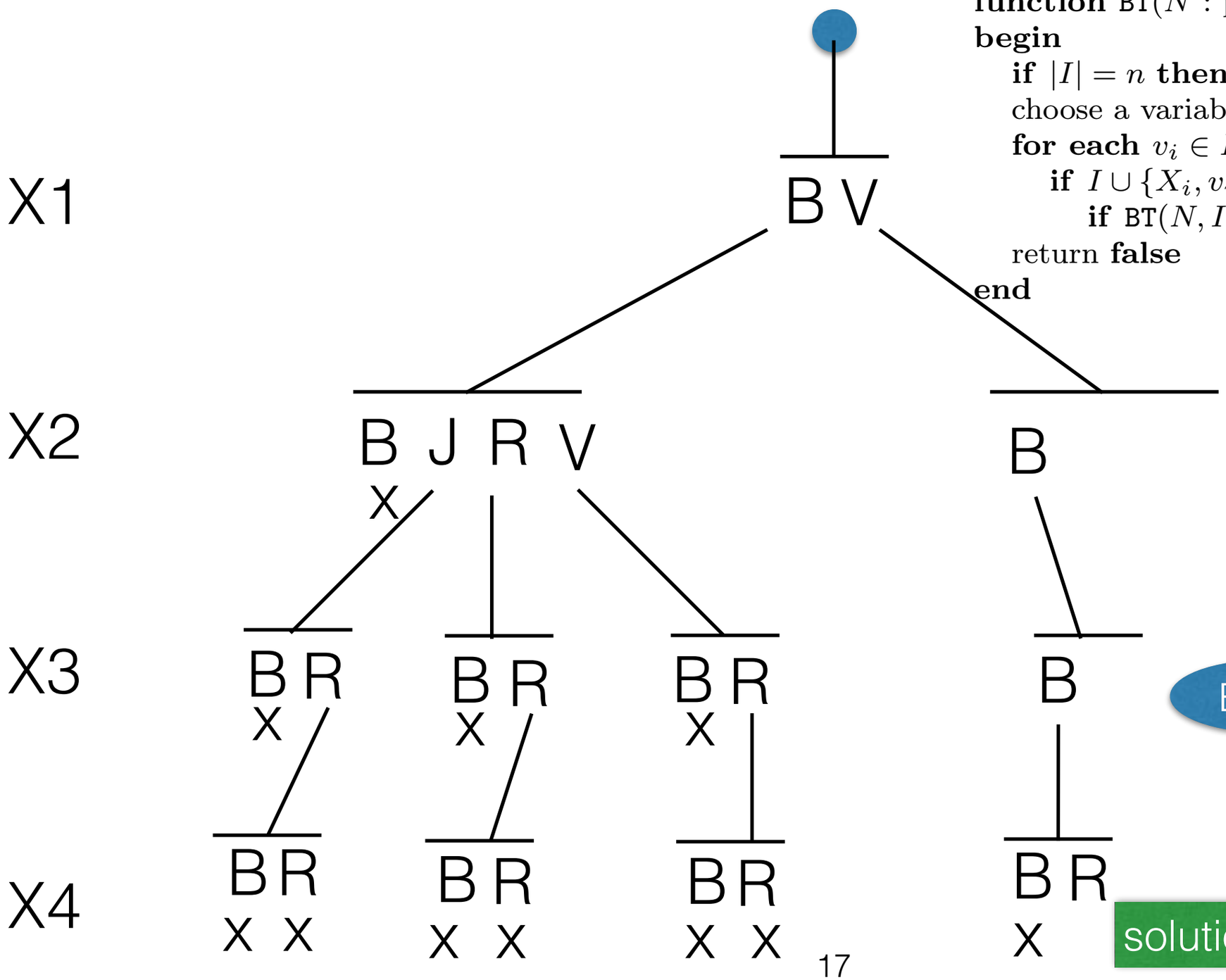




BT on running example



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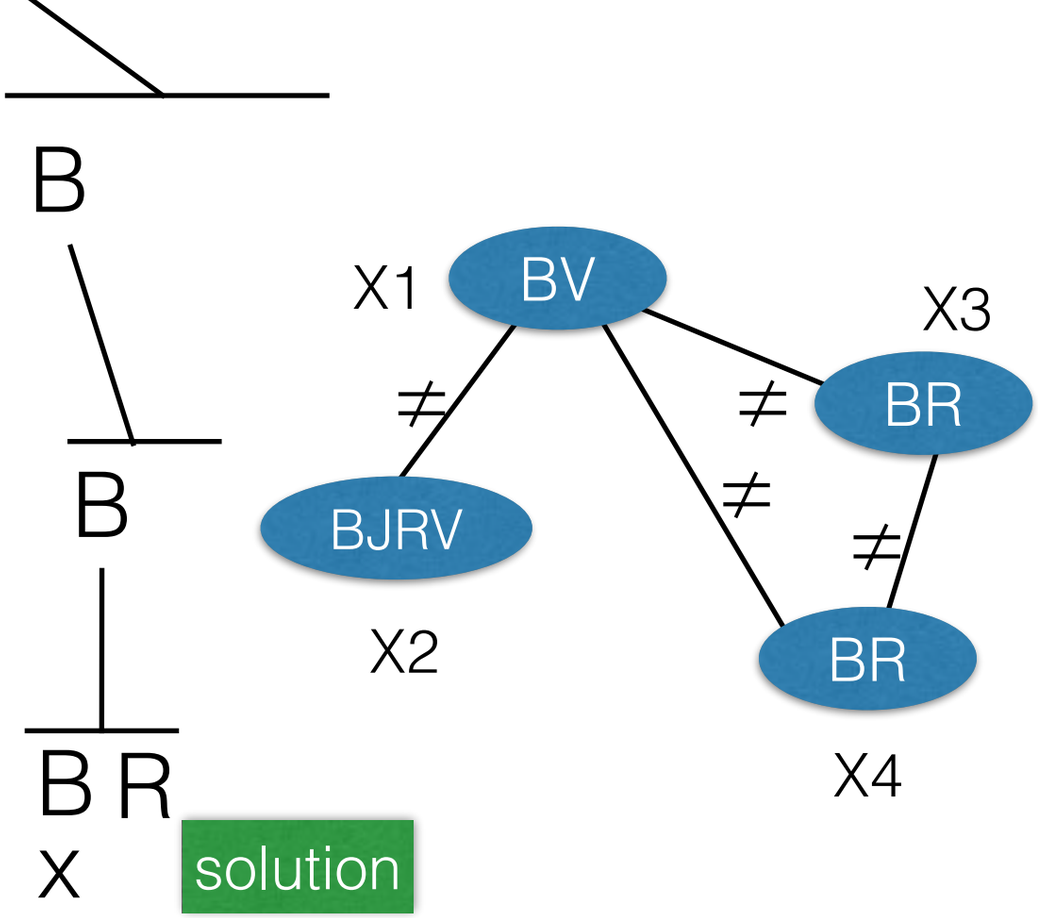


Complexity of BT

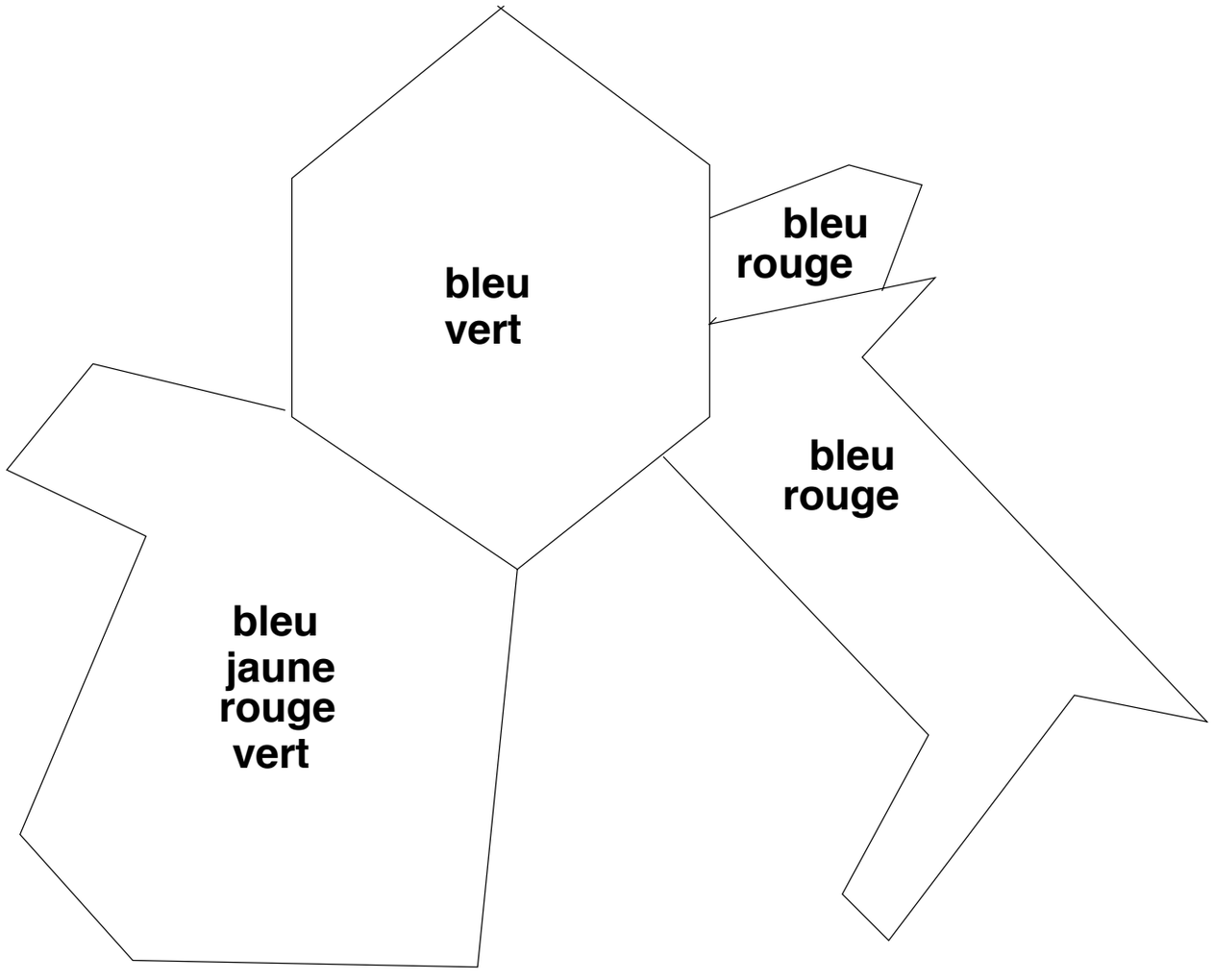
- Space complexity: linear
- Time complexity:

  $O(ed^n)$, where $n = |X|$, $e = |C|$, and $d = max_{1..n}(D(X_i))$

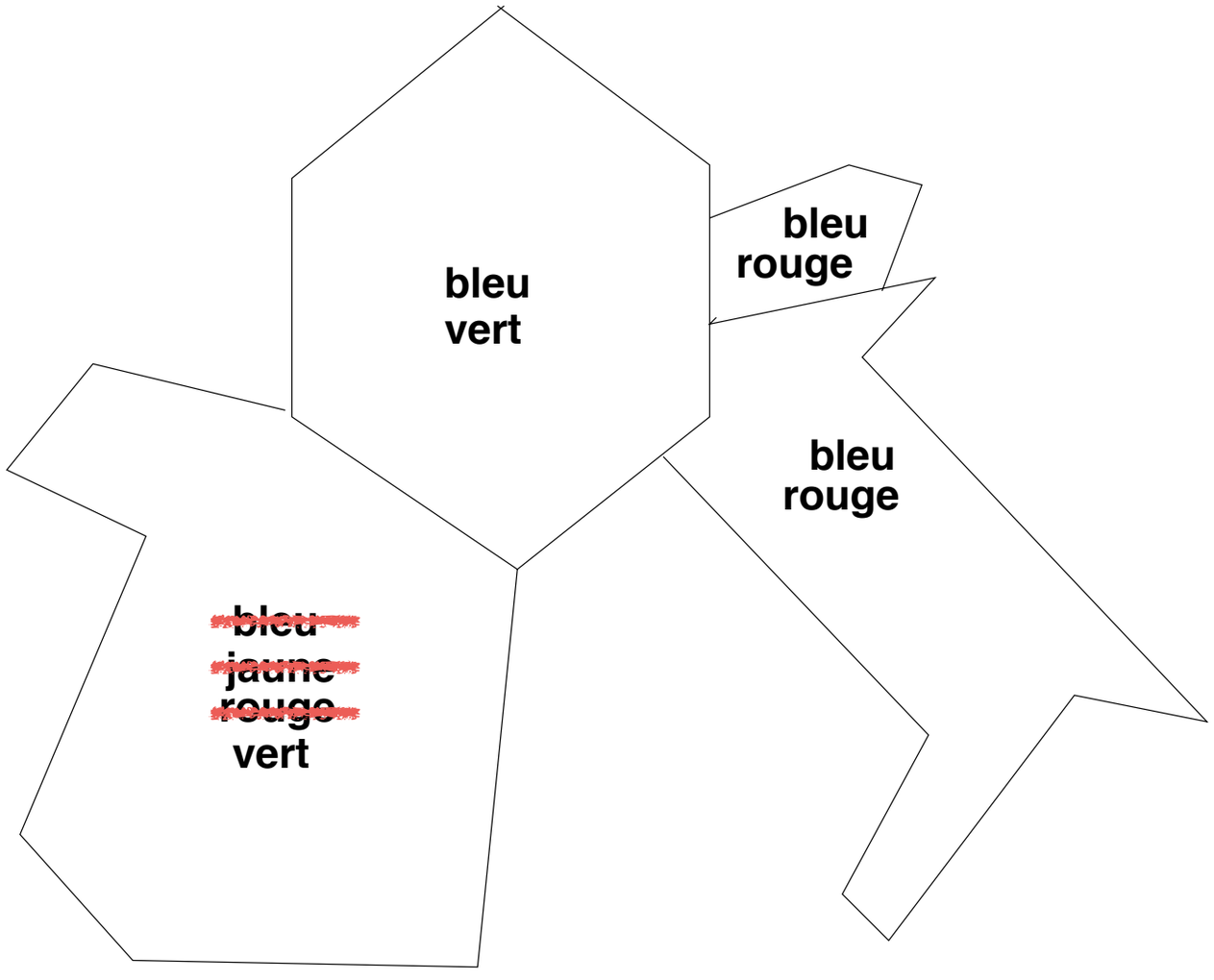
- Repeatedly discovers the same inconsistencies
- Limited to very small problems

Local consistencies (constraint propagation)

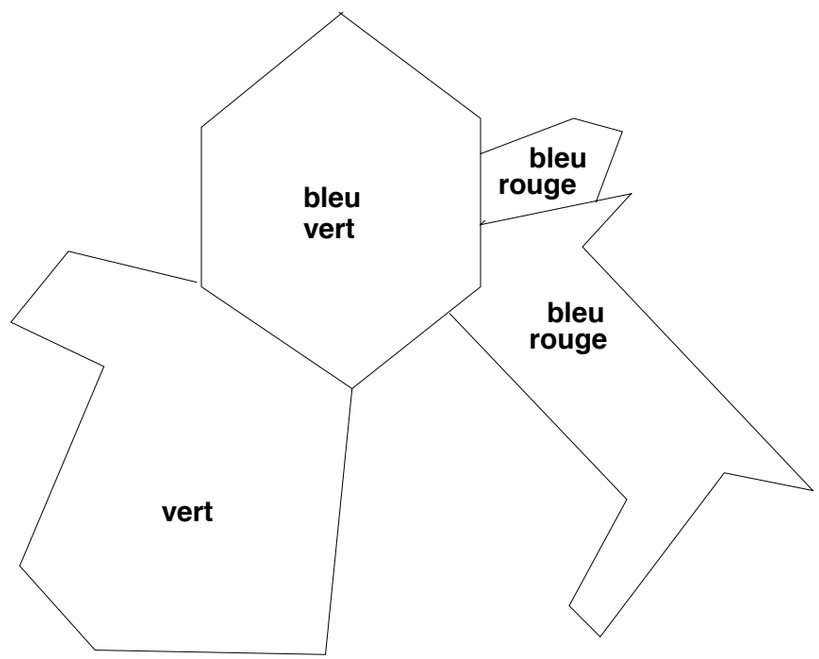
Modified running example



Modified running example



BT



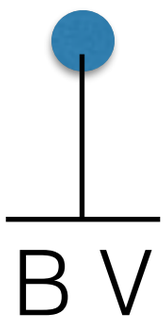
X1

X2

X3

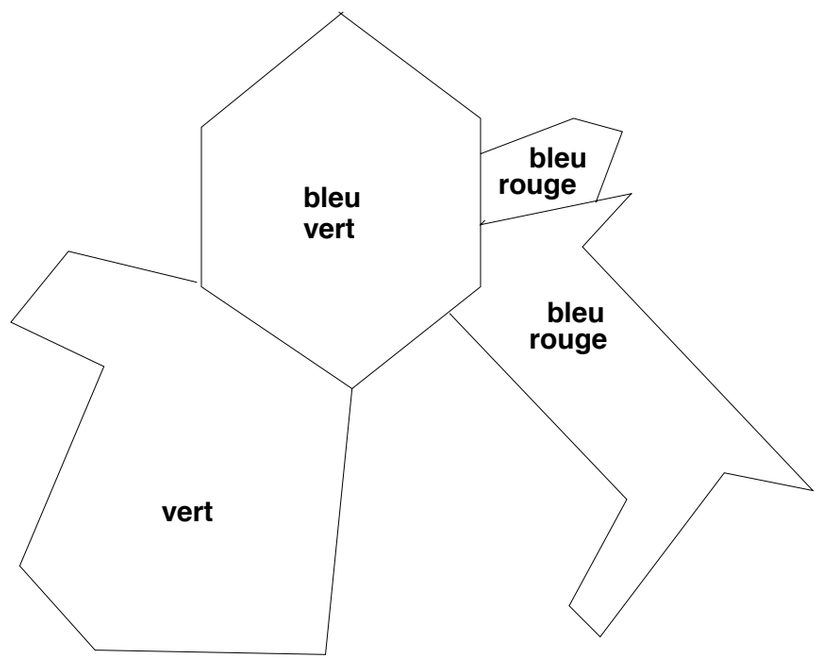
X4

BT



B V

X1

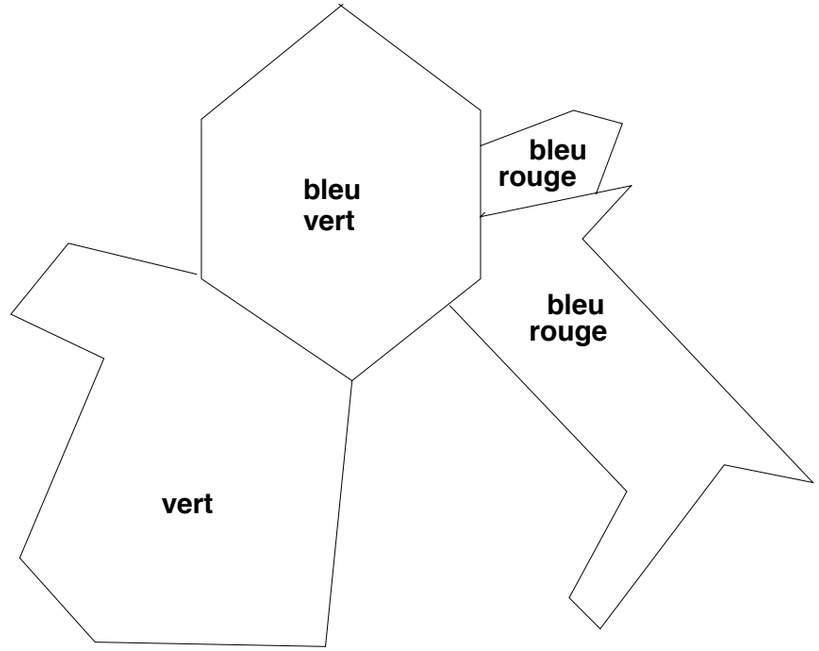


X2

X3

X4

BT



X1

B V

X2

V

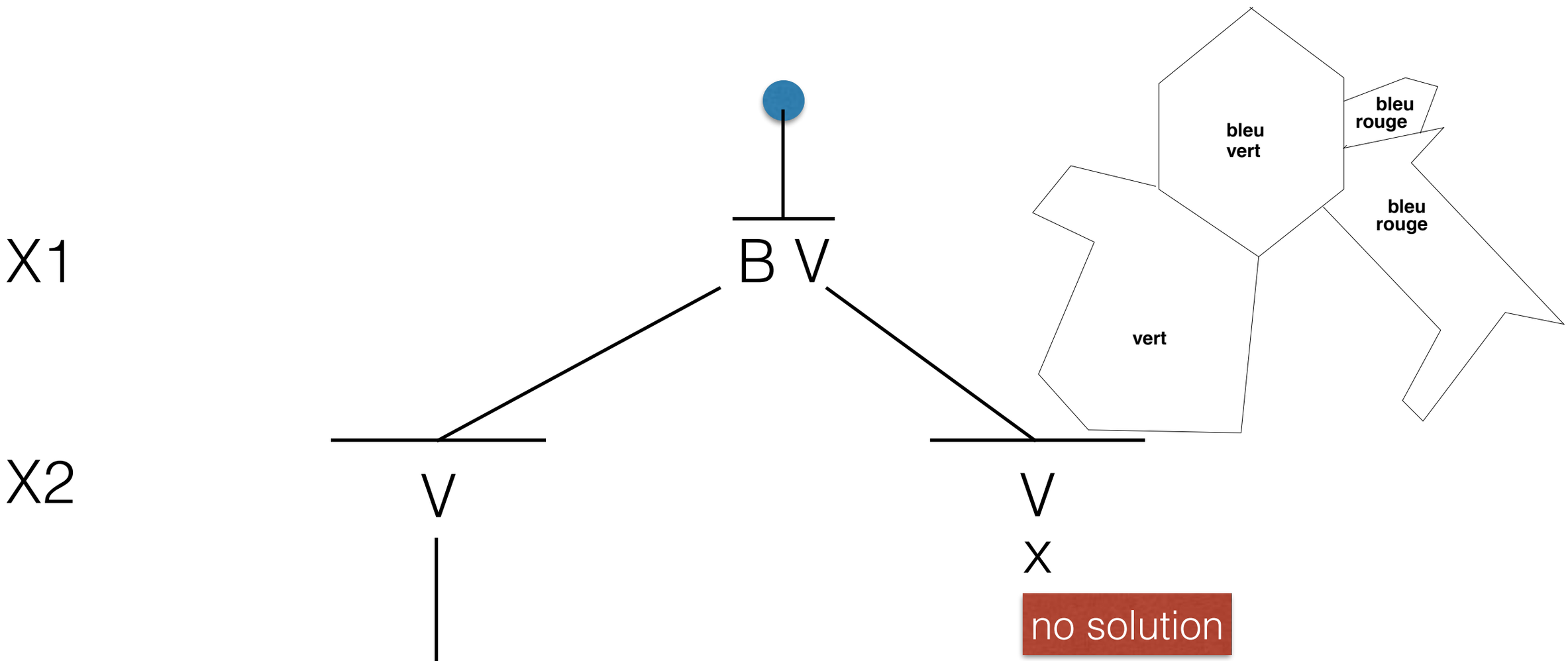
X3

B R
X

X4

B R
X X

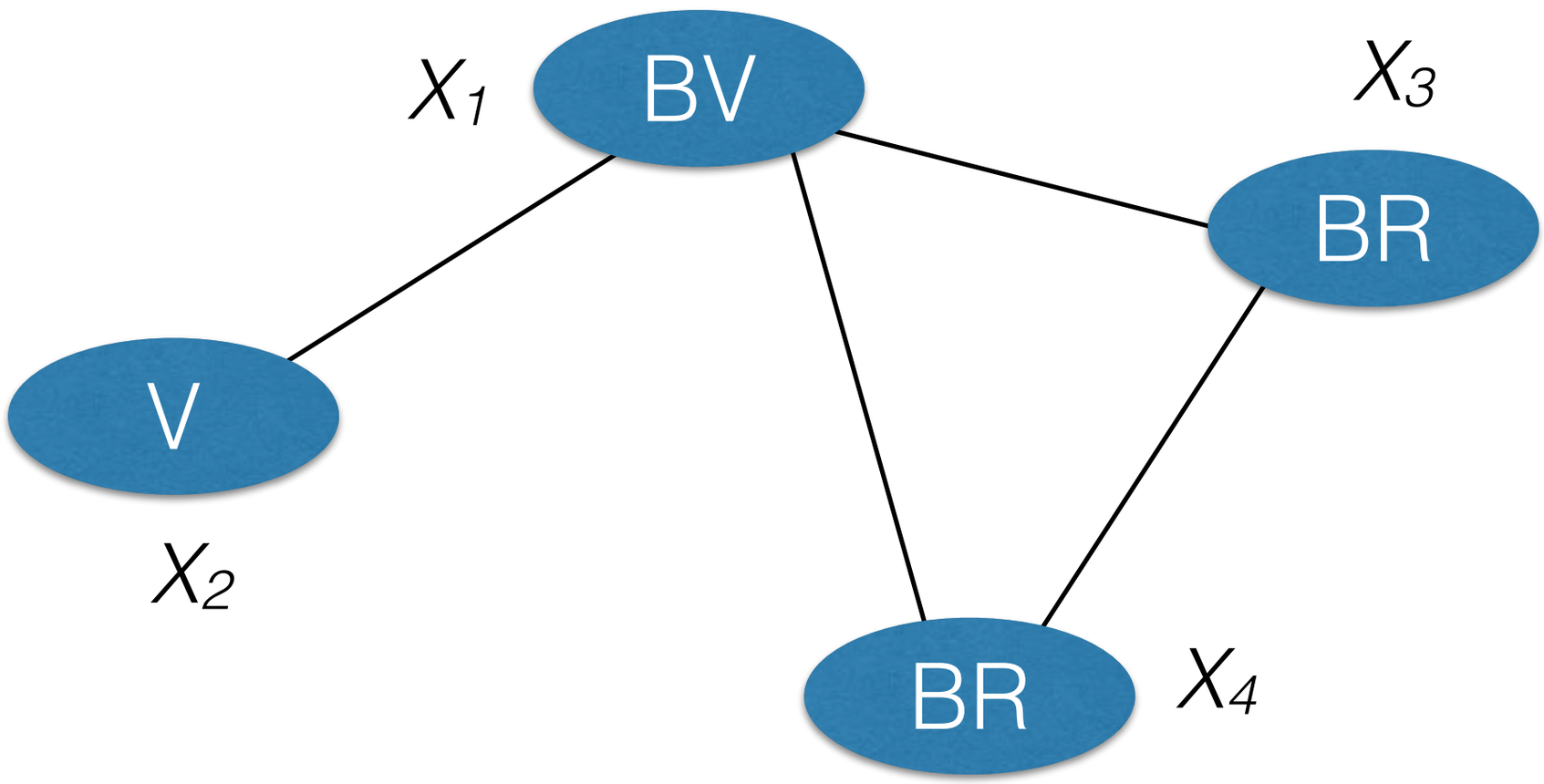
BT


X1
B V
bleu
vert
bleu
rouge
bleu
rouge
vert
X2
V
V
x
no solution
X3
B R
x
X4
B R
x x

Could we detect this inconsistency with a polynomial procedure?

Constraint Propagation

- Eliminate once and for all inconsistencies that BT could reach many times otherwise



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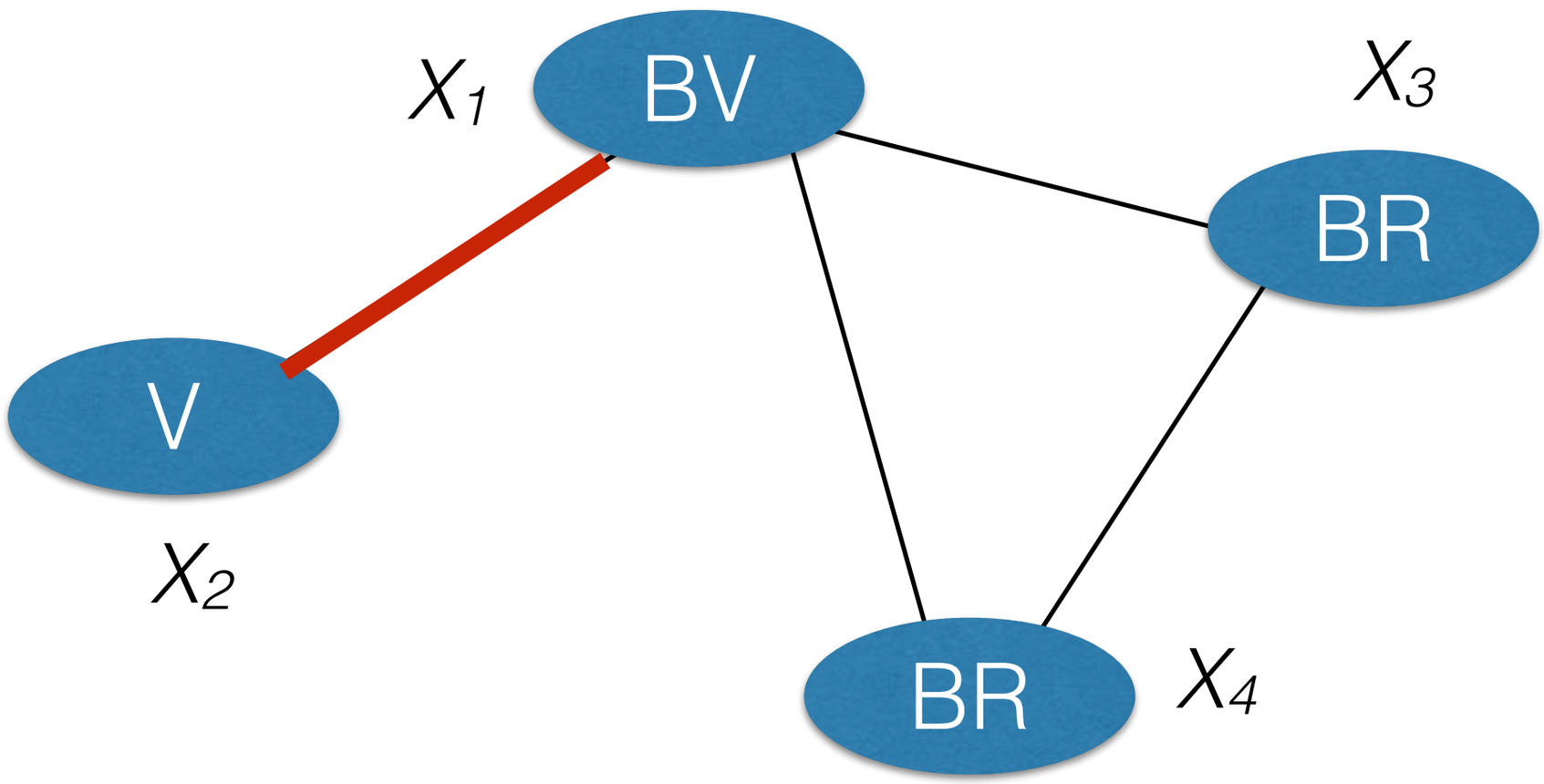

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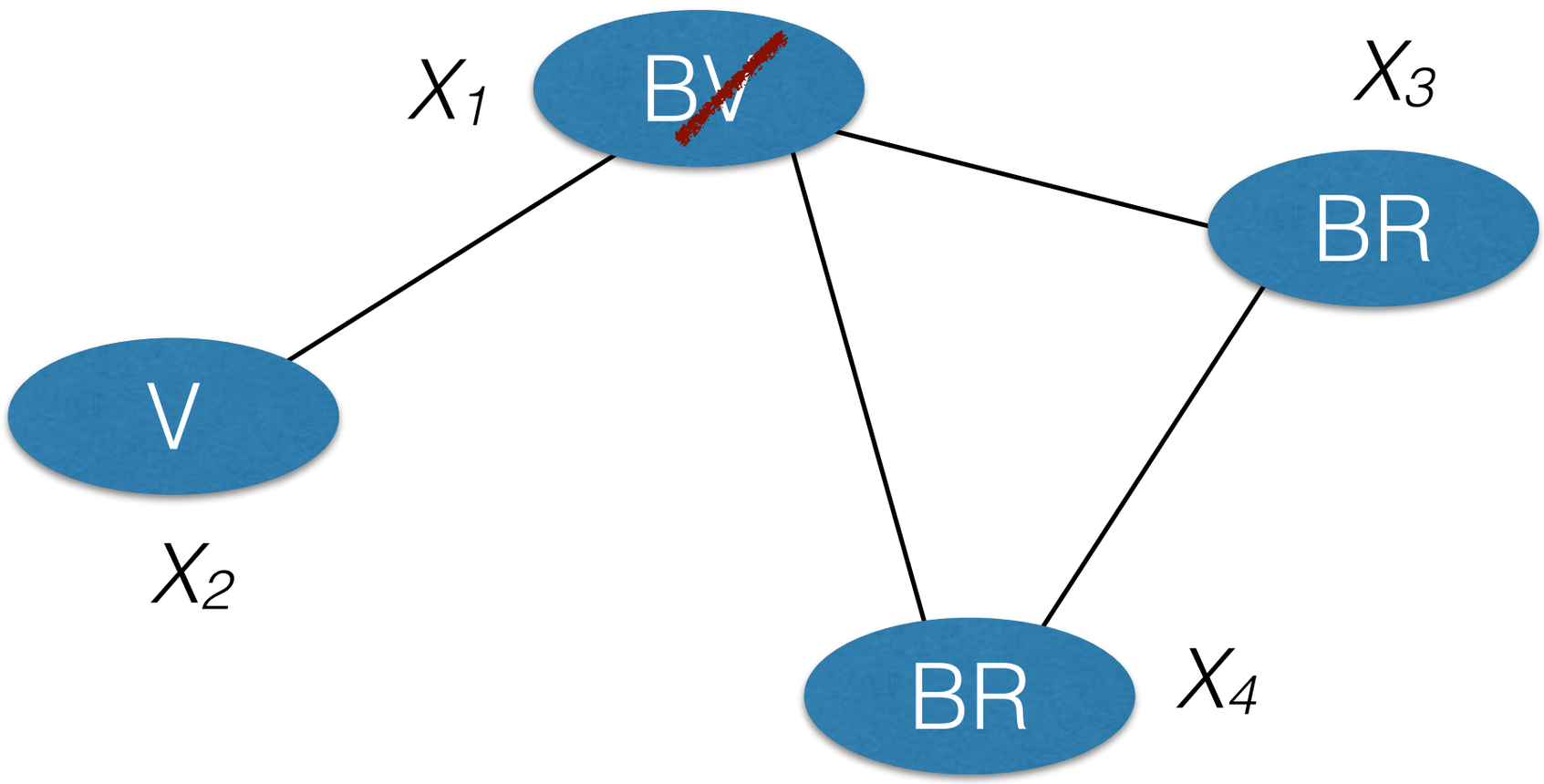

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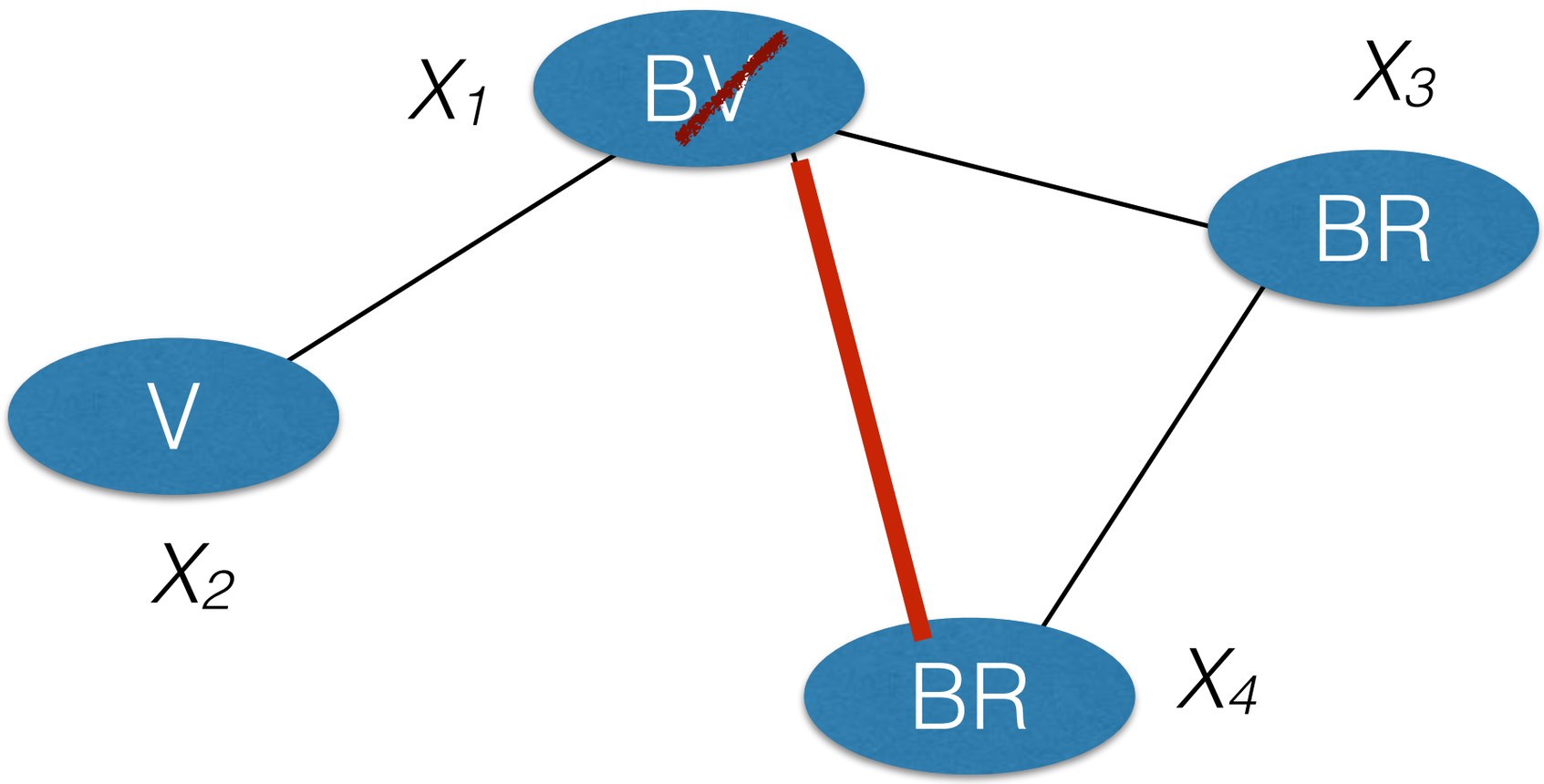

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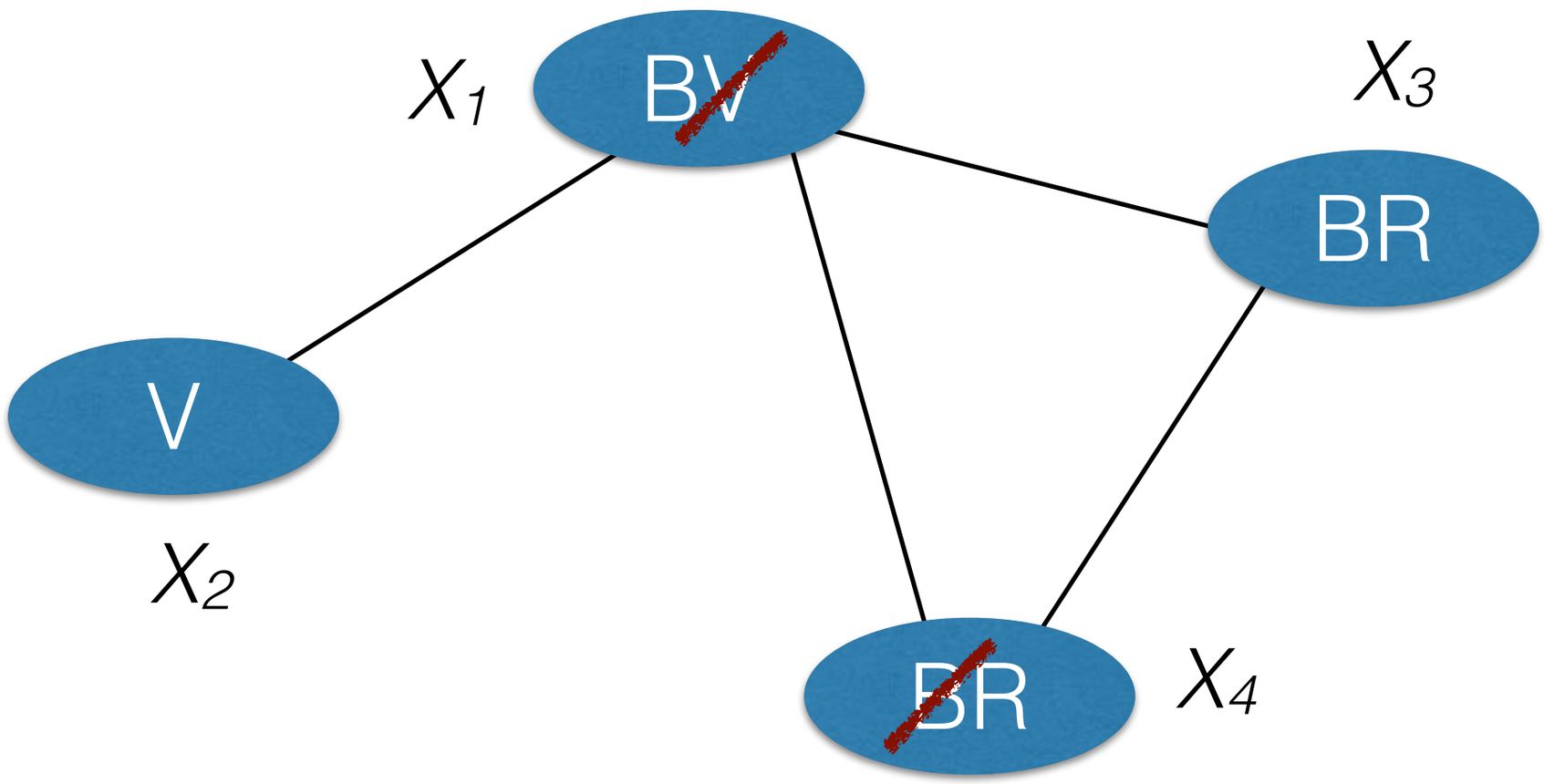

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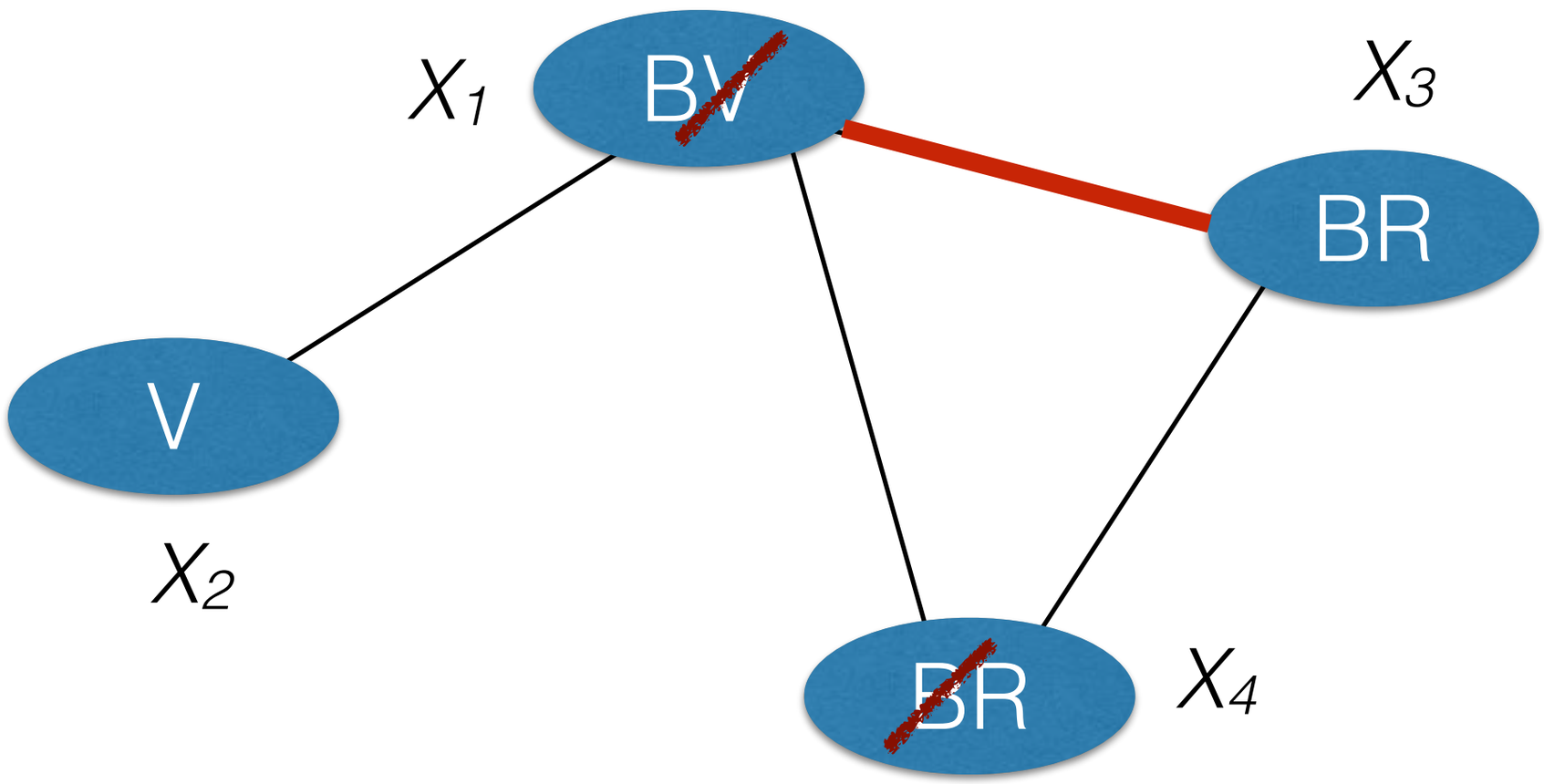

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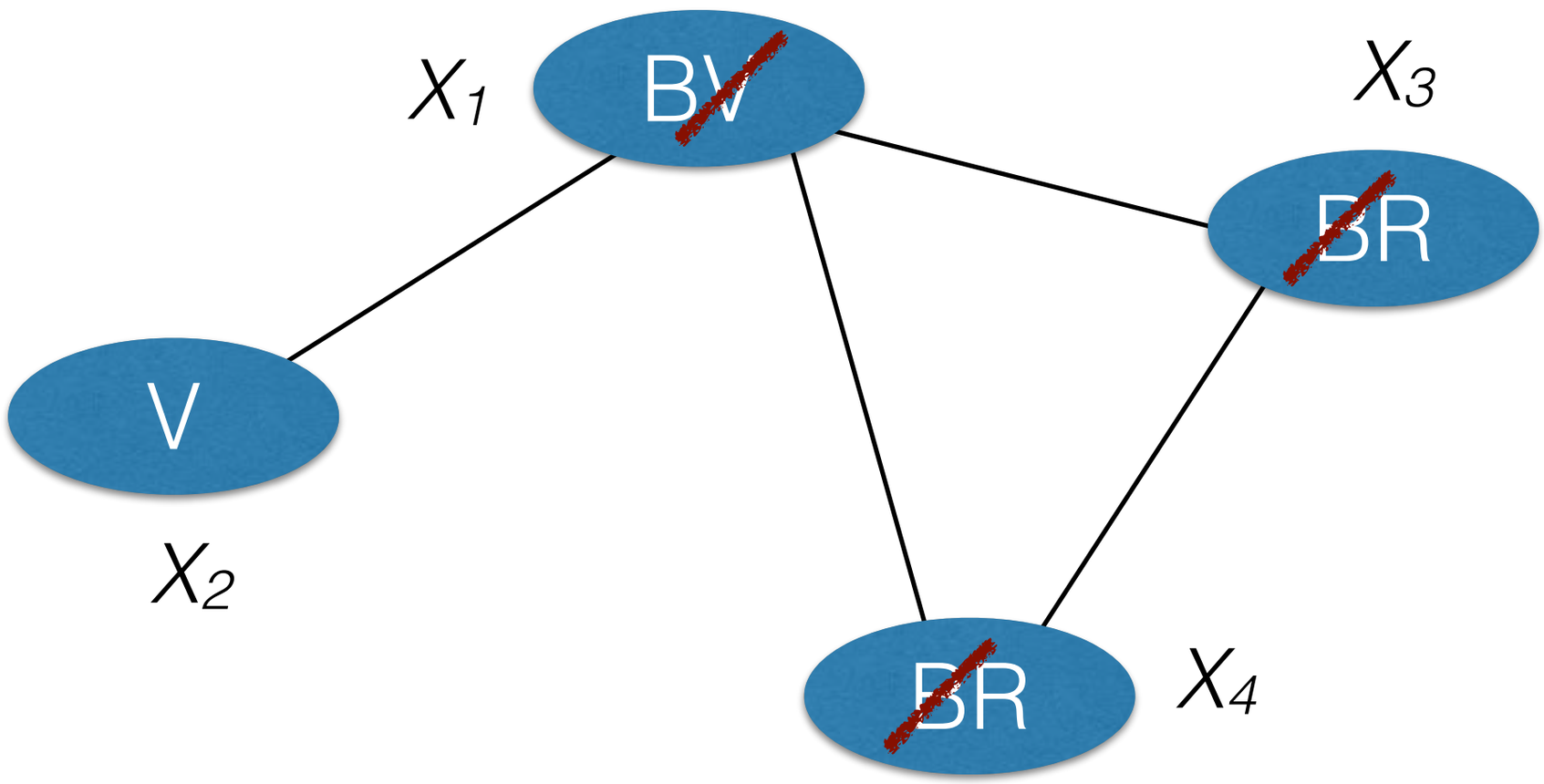

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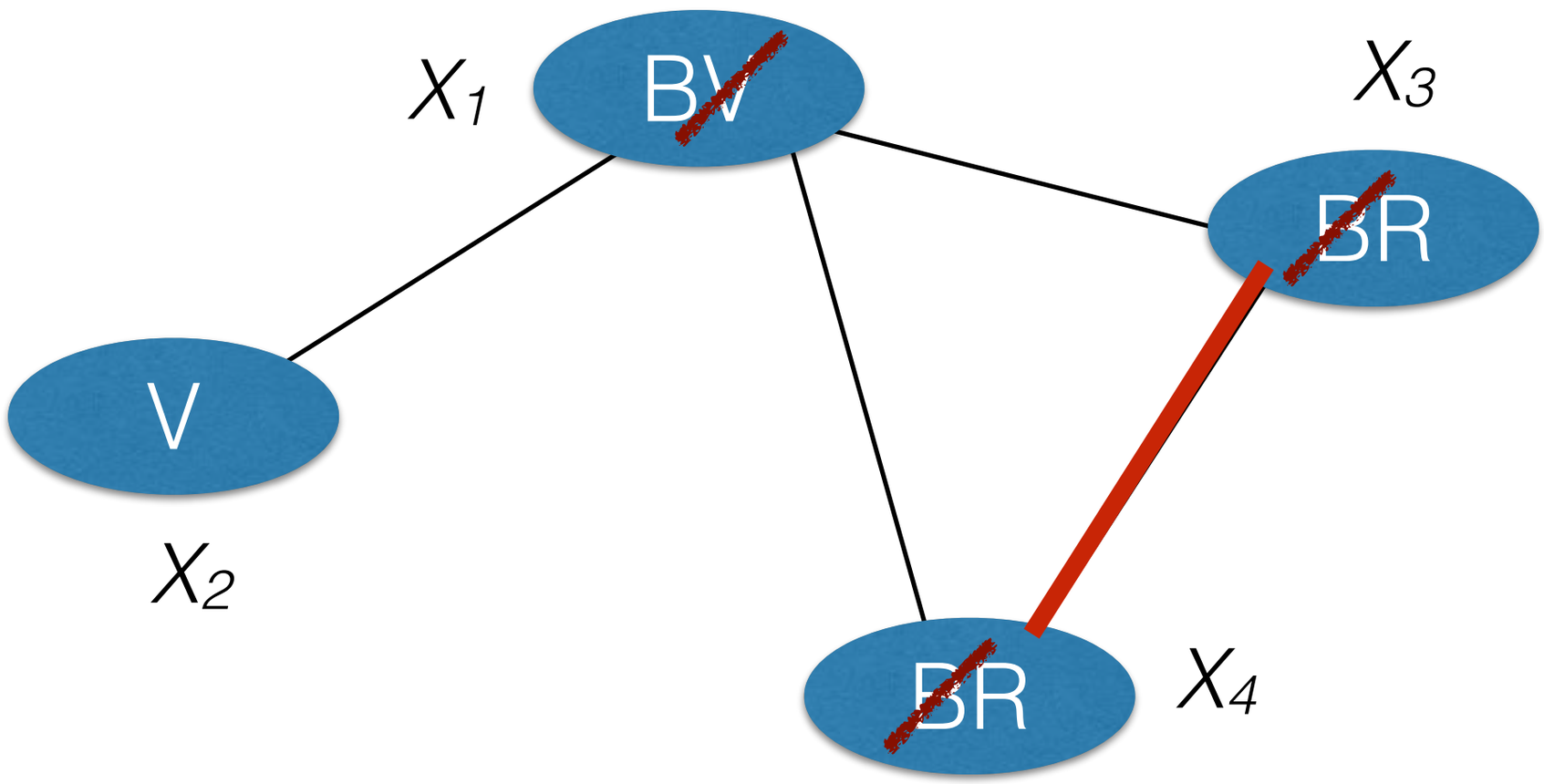

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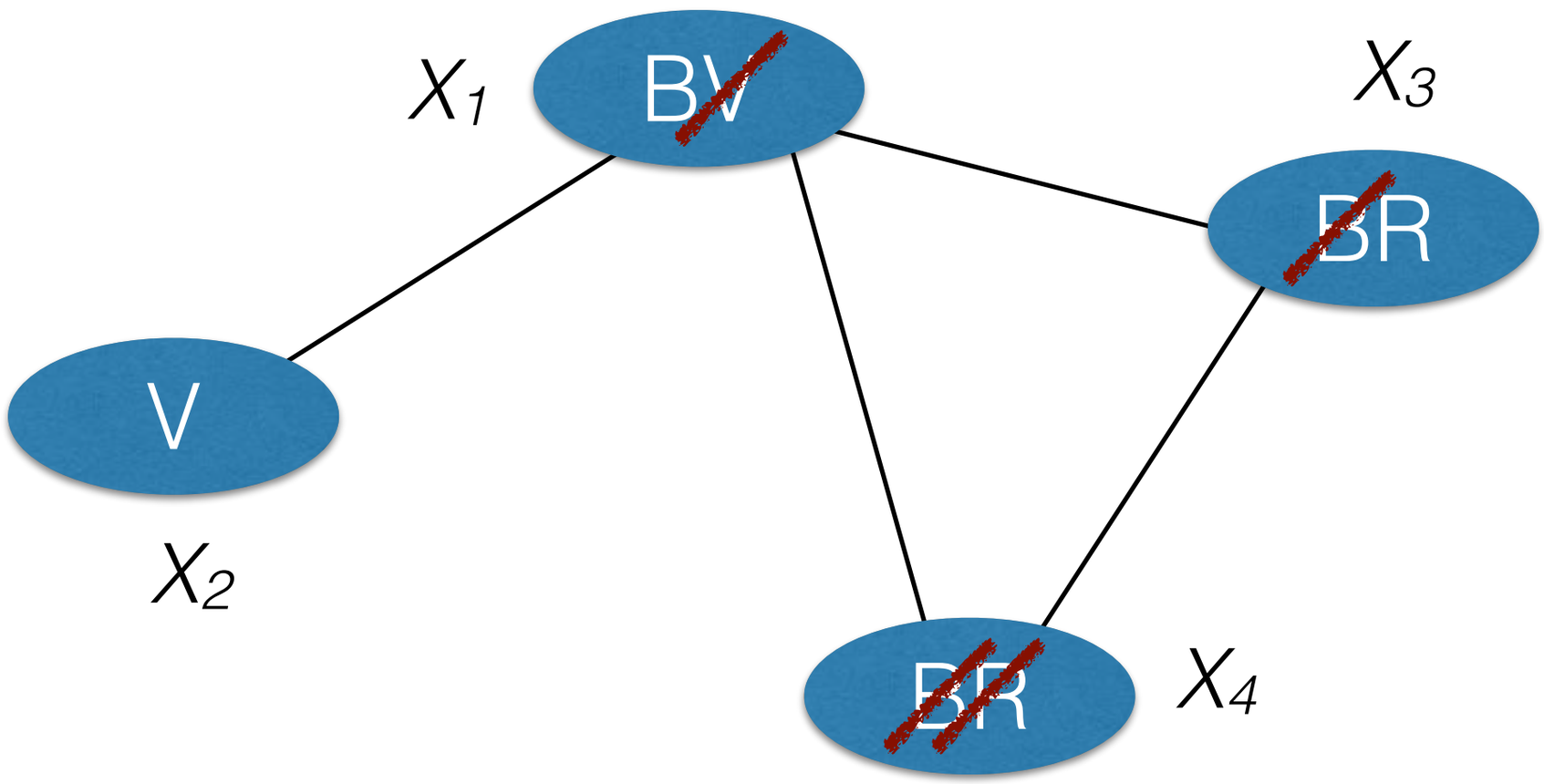

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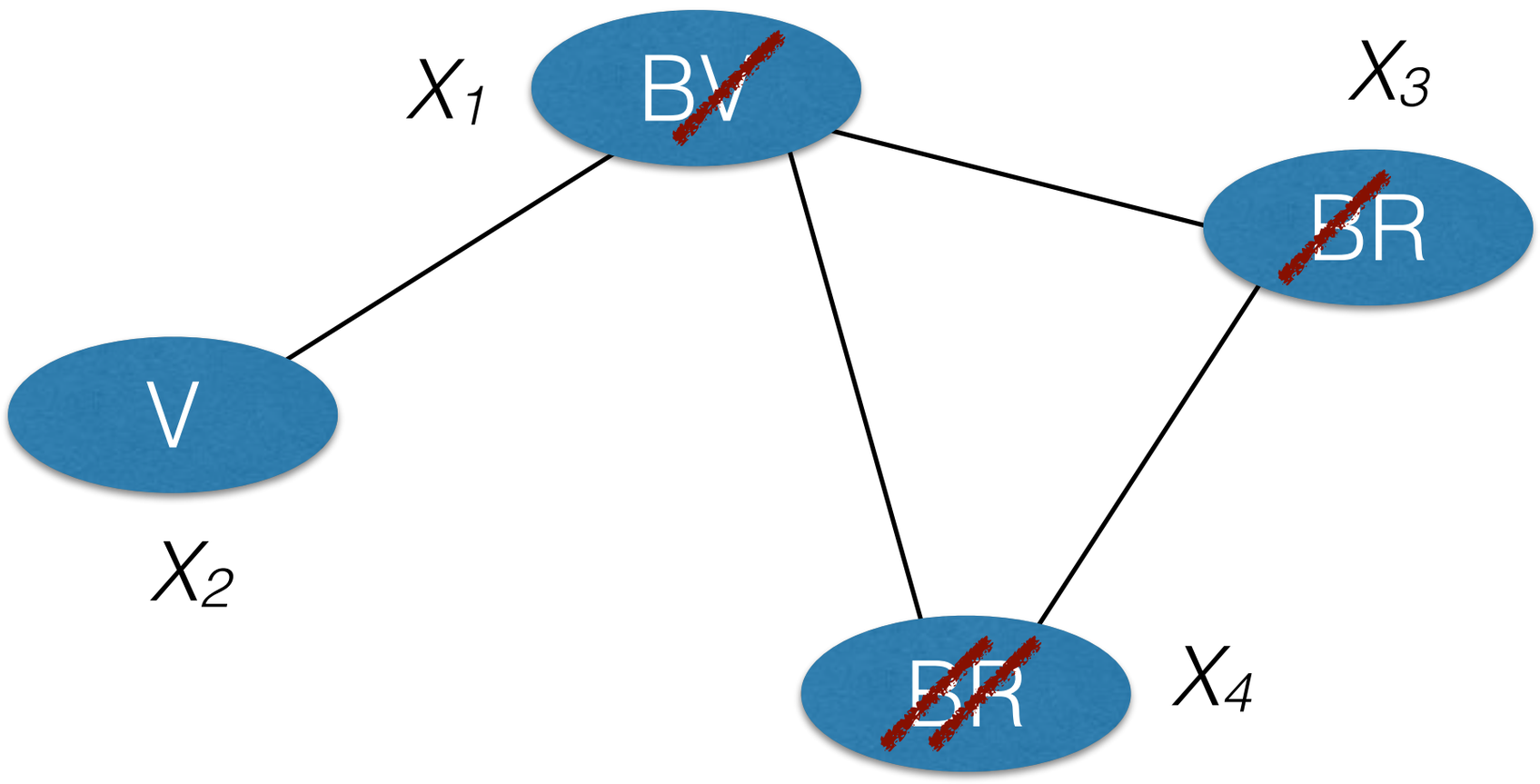

empty domain!

Arc consistency

Definition 4 (Arc consistency (AC)) *Let* $N = (X, D, C)$ *a network and* c *a constraint in* C*.*

D: X_1 X_2 X_3

1	1	1
2	2	2
3	3	3
7	7	7
8	8	8

$X_1+X_2=X_3$

1	1	2
1	2	3
2	1	3
1	7	8
7	1	8

Arc consistency

Definition 4 (Arc consistency (AC)) *Let* $N = (X, D, C)$ *a network and* c *a constraint in* C*.*

— *a tuple* $\tau \in c \cap D^{X(c)}$ *is called a* support *on* c

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- *a tuple* $\tau \in c \cap D^{X(c)}$ *is called a* support *on* c
- *a value* $v_i \in D(X_i)$ *is* viable *iff* $\forall c \in C, X_i \in X(c), \exists$ *a support* τ *on* c *such that* $\tau[X_i] = v_i$

D: X_1 X_2 X_3

1	1	1
2	2	2
3	3	3
7	7	7
8	8	8

$X_1 + X_2 = X_3$

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1	2	3
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1	7	8
7	1	8

Arc consistency

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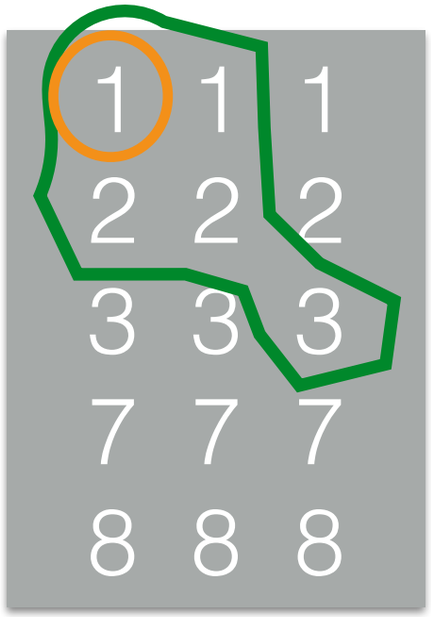
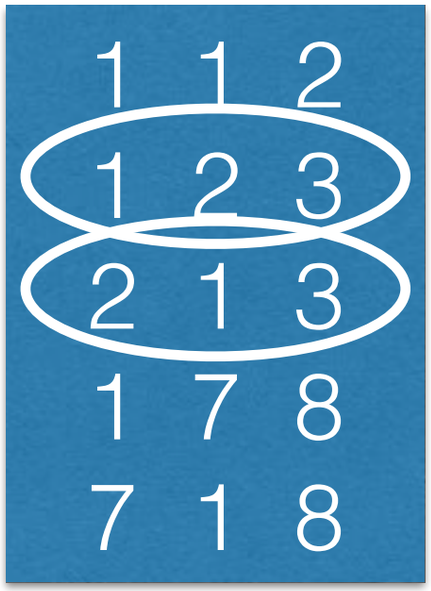
— *the domain* D *is* arc consistent *iff* $\forall X_i \in X, \forall v_i \in D(X_i)$, v_i *is viable*

D: X_1 X_2 X_3



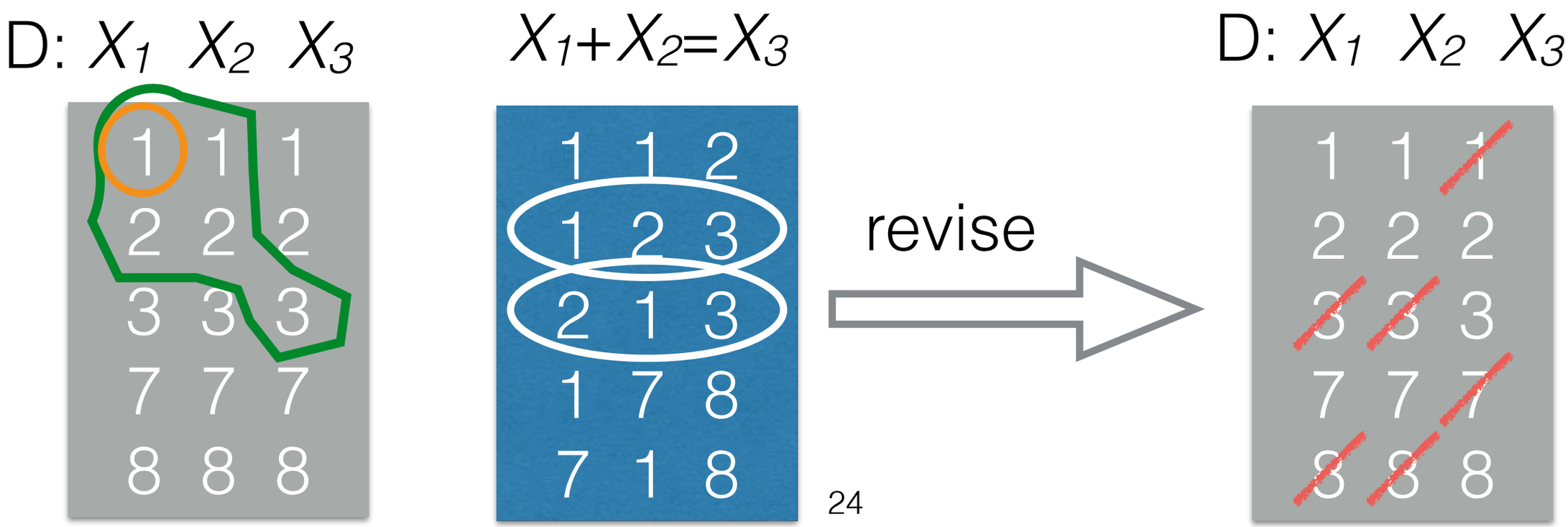
$X_1+X_2=X_3$



Arc consistency

Definition 4 (Arc consistency (AC)) *Let $N = (X, D, C)$ a network and c a constraint in C.*

- *a tuple $\tau \in c \cap D^{X(c)}$ is called a* support *on c*
- *a value $v_i \in D(X_i)$ is* viable *iff $\forall c \in C, X_i \in X(c), \exists$ a support τ on c such that $\tau[X_i] = v_i$*
- *the domain D is* arc consistent *iff $\forall X_i \in X, \forall v_i \in D(X_i)$, v_i is viable*
- *the* AC-closure *of D is the domain D_{AC} which is obtained by iteratively removing non viable values from D until no more exists*



Arc consistency

★ **Important observations**

- D_{AC} is arc consistent
- $sol(X,D,C) = sol(X,D_{AC},C)$
- D_{AC} is the union of all subdomains of D that arc consistent (this is the non-operational definition of AC)

Long history of AC algorithms since 1970:

- AC1,…, AC8, AC2001, AC3.2, AC3.3, AC3d…, STR1,…, STR3, etc.

AC3 [Mackworth1977]

```
function AC3(in X : set) : Boolean
    begin
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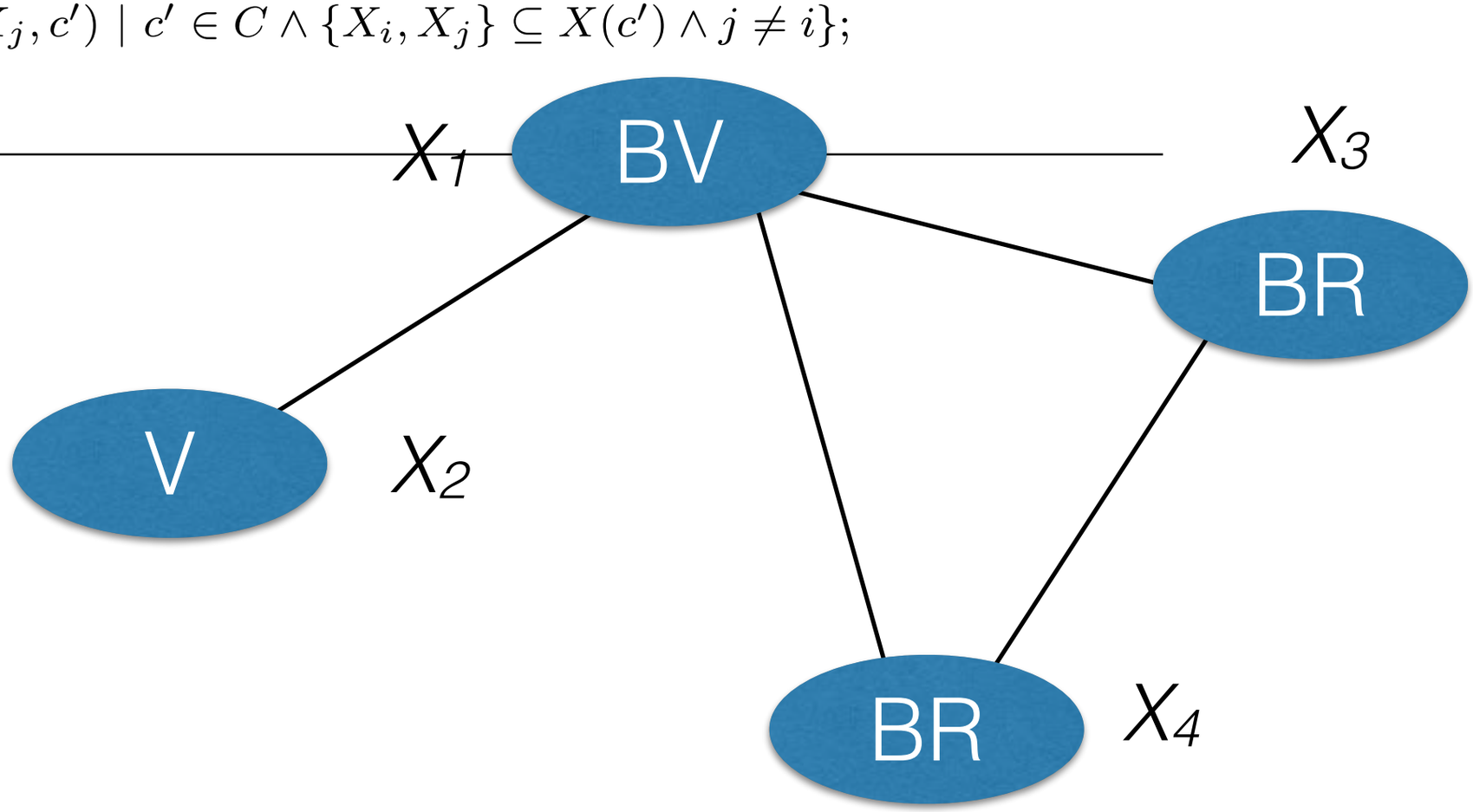

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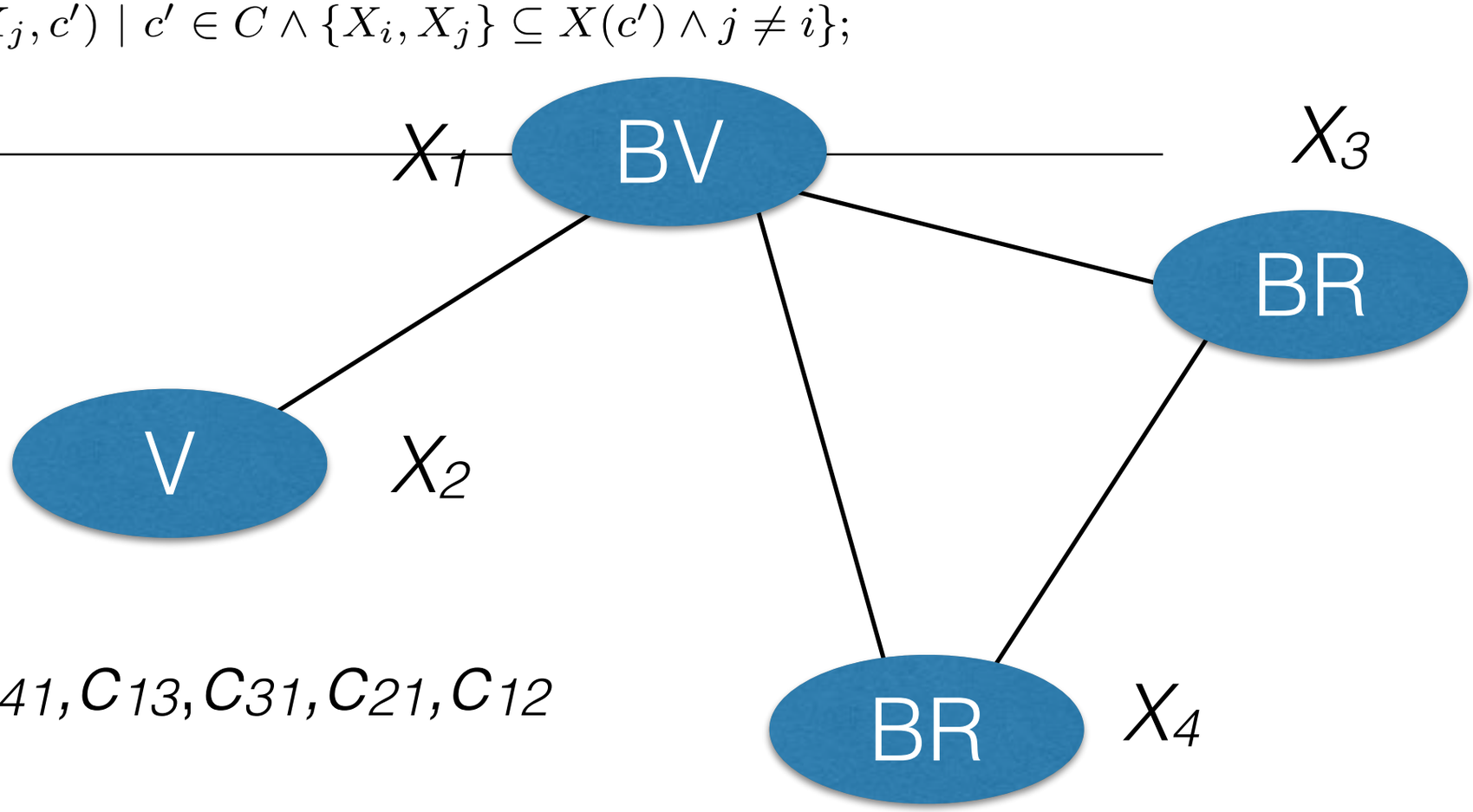

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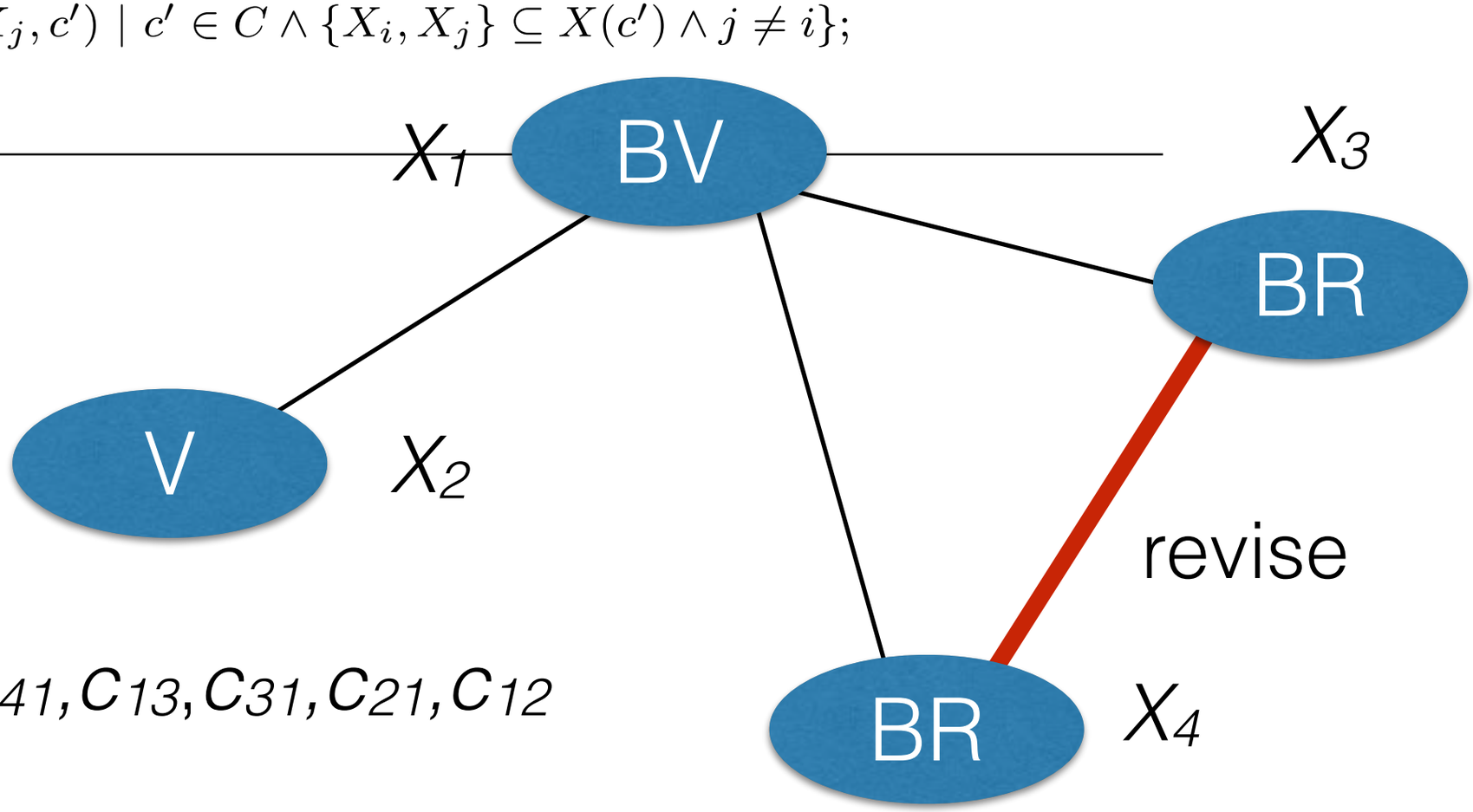

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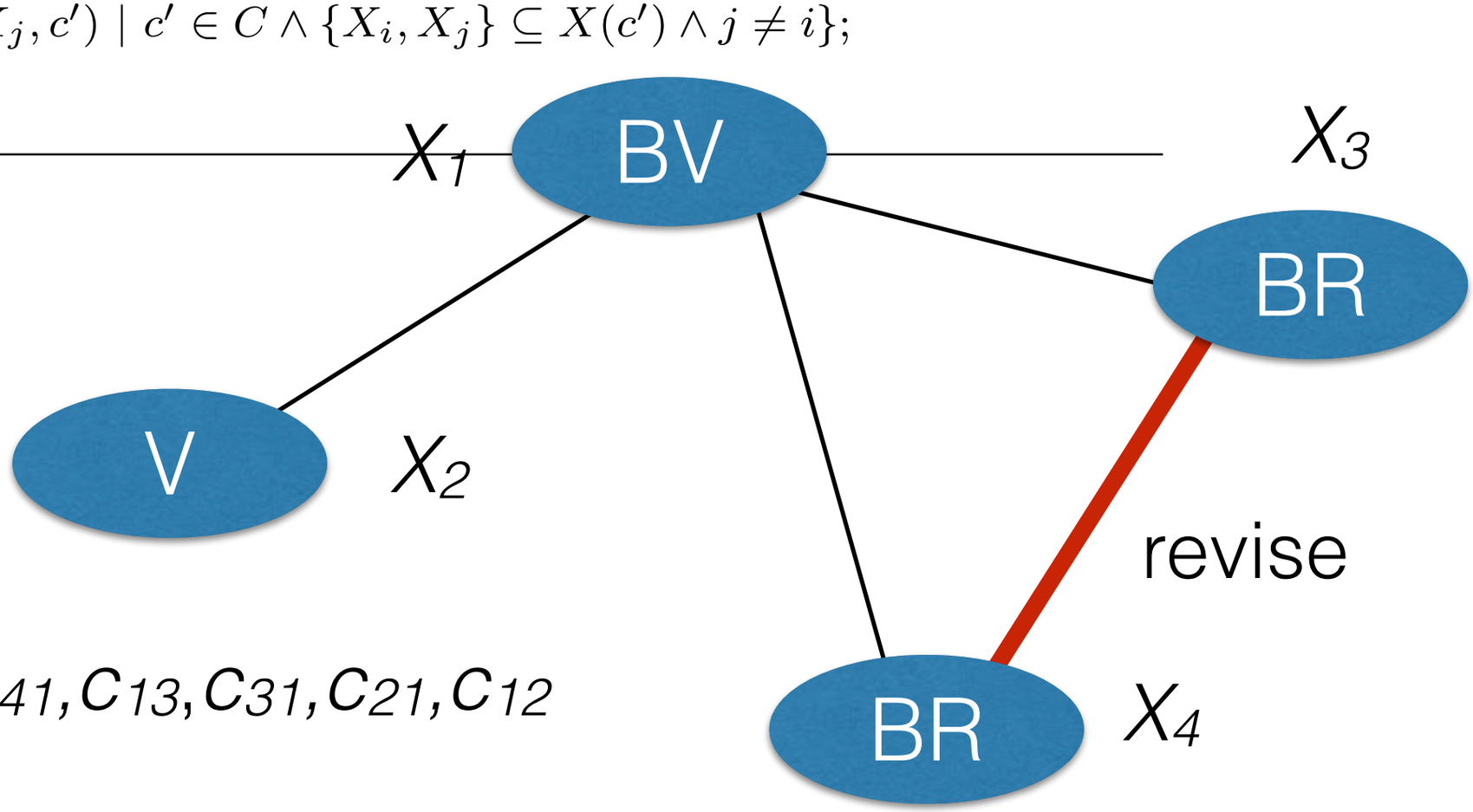

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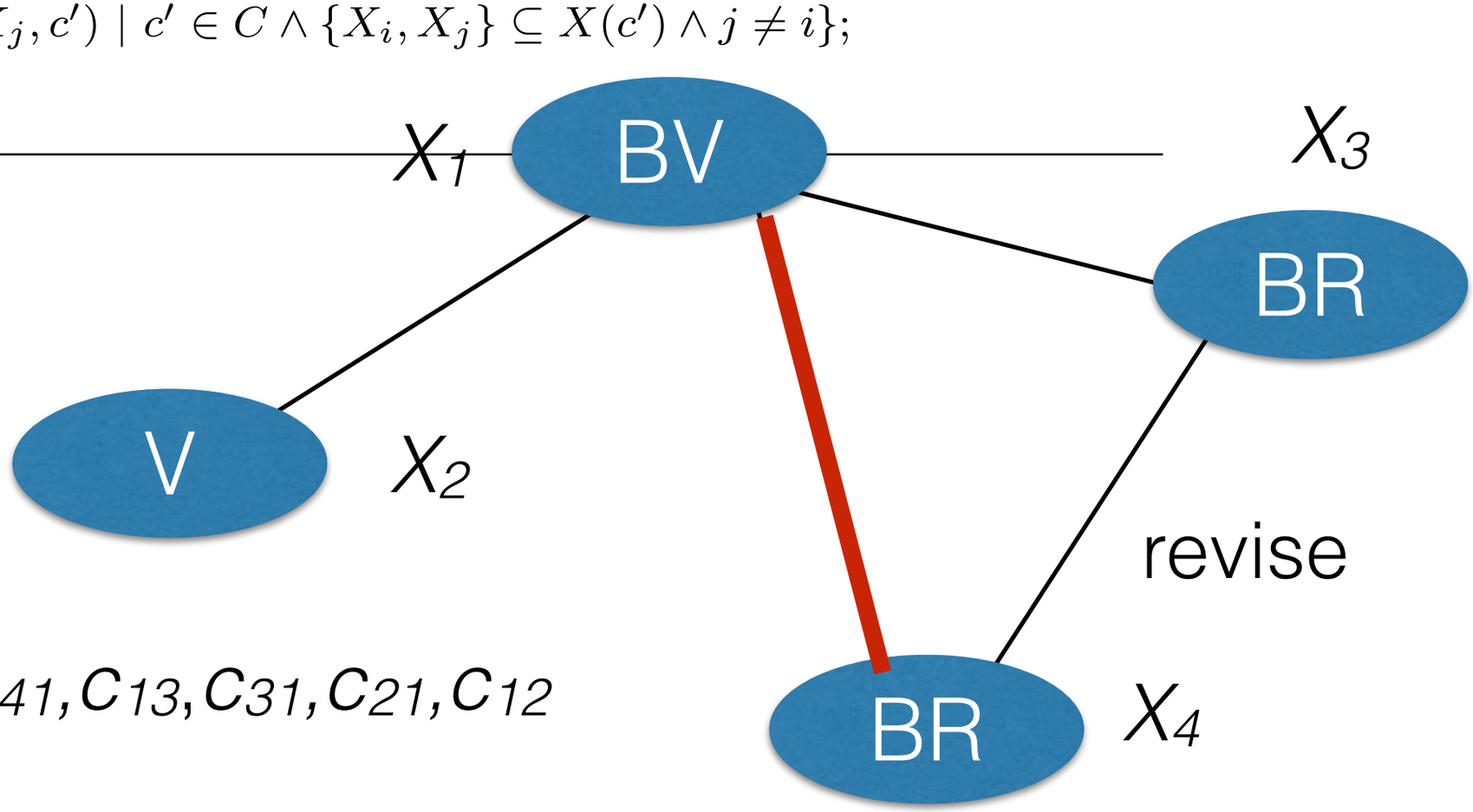

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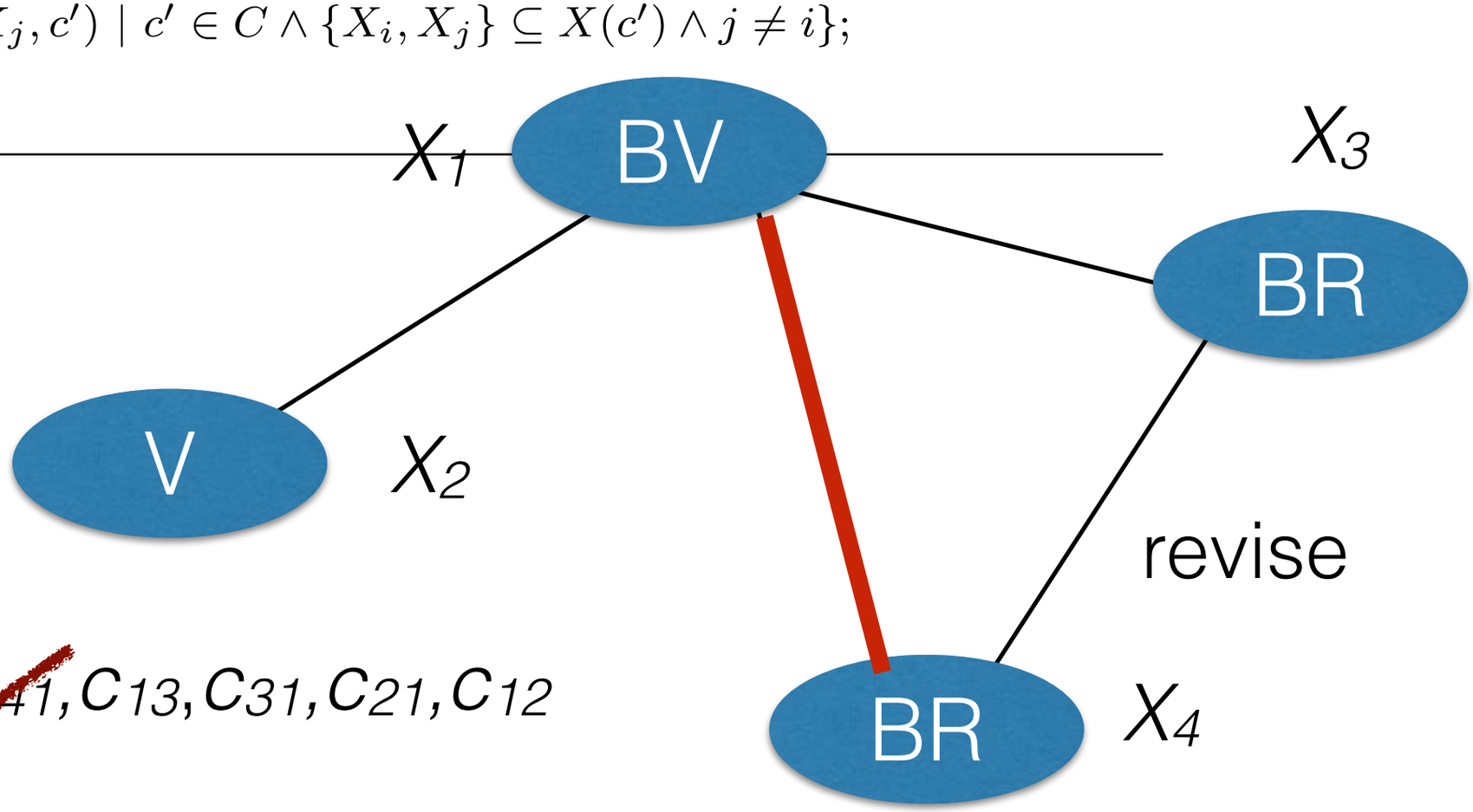

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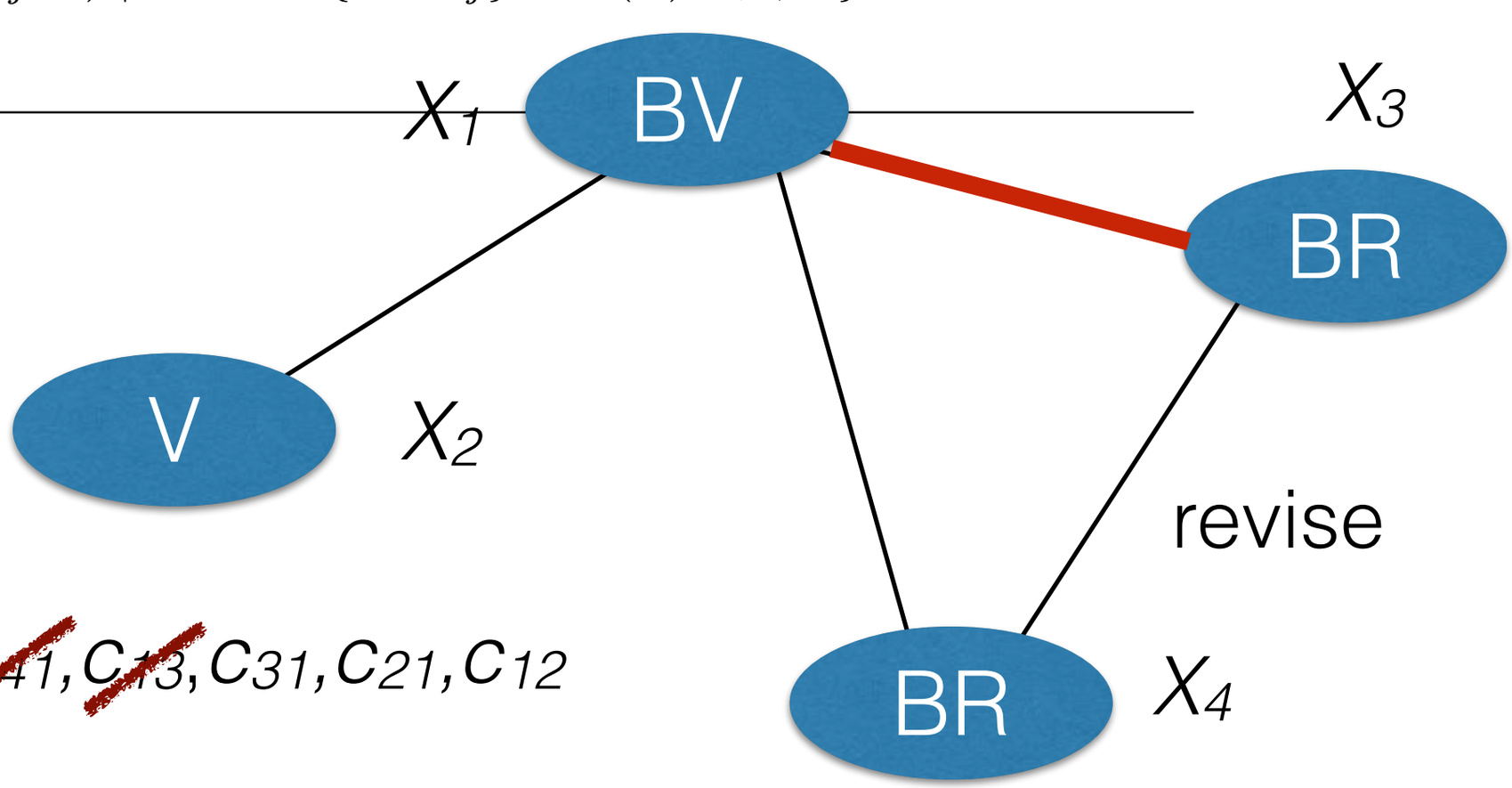

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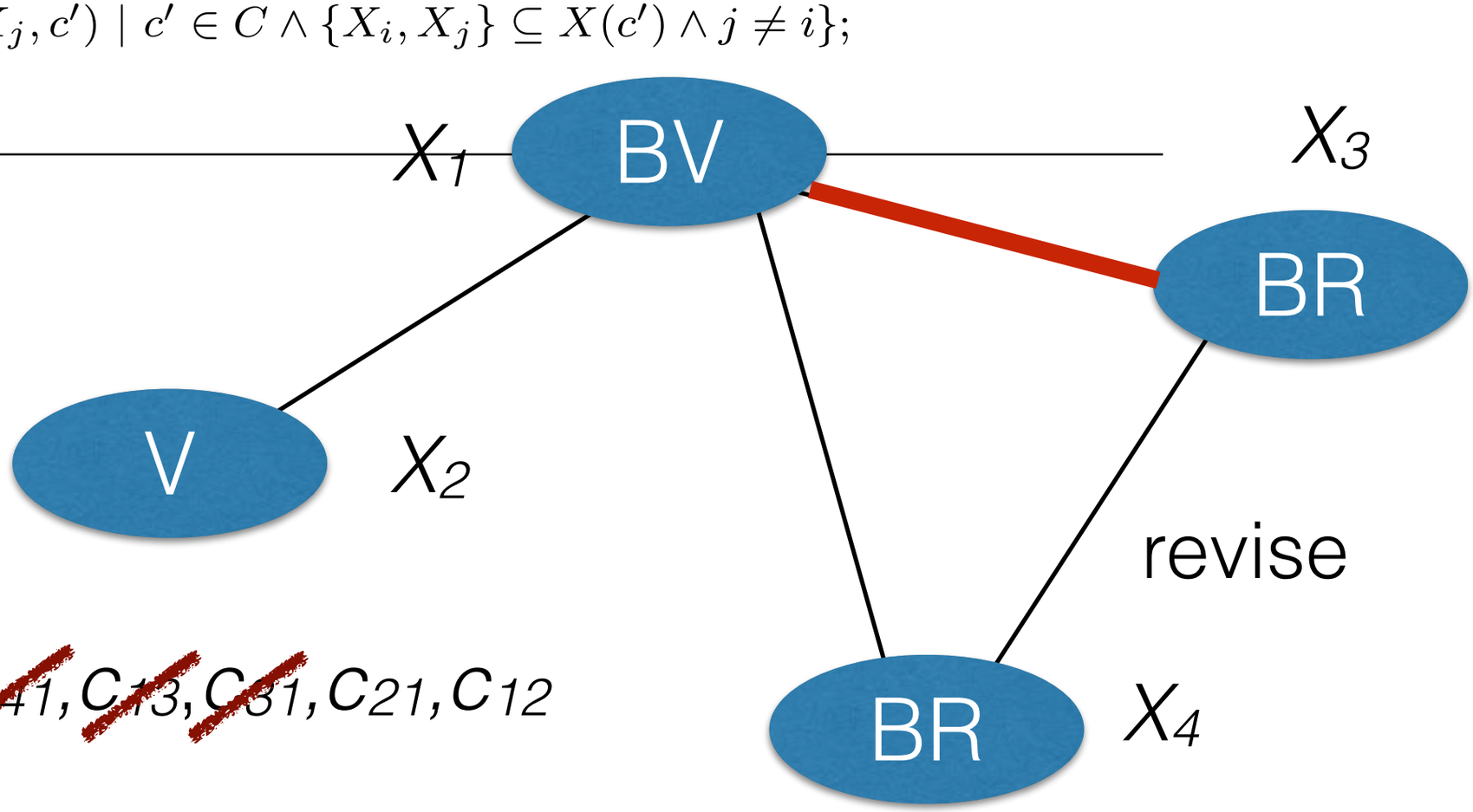

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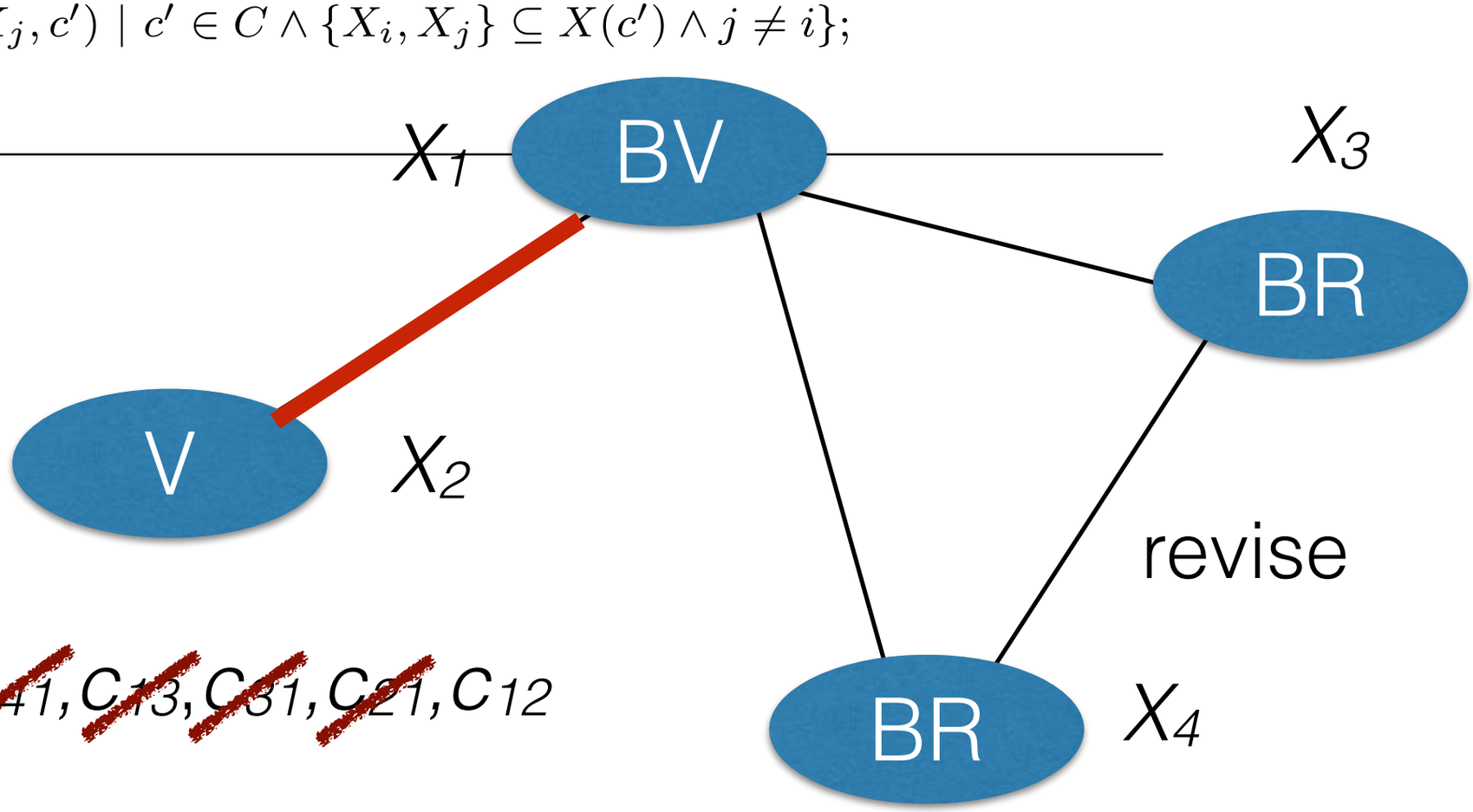

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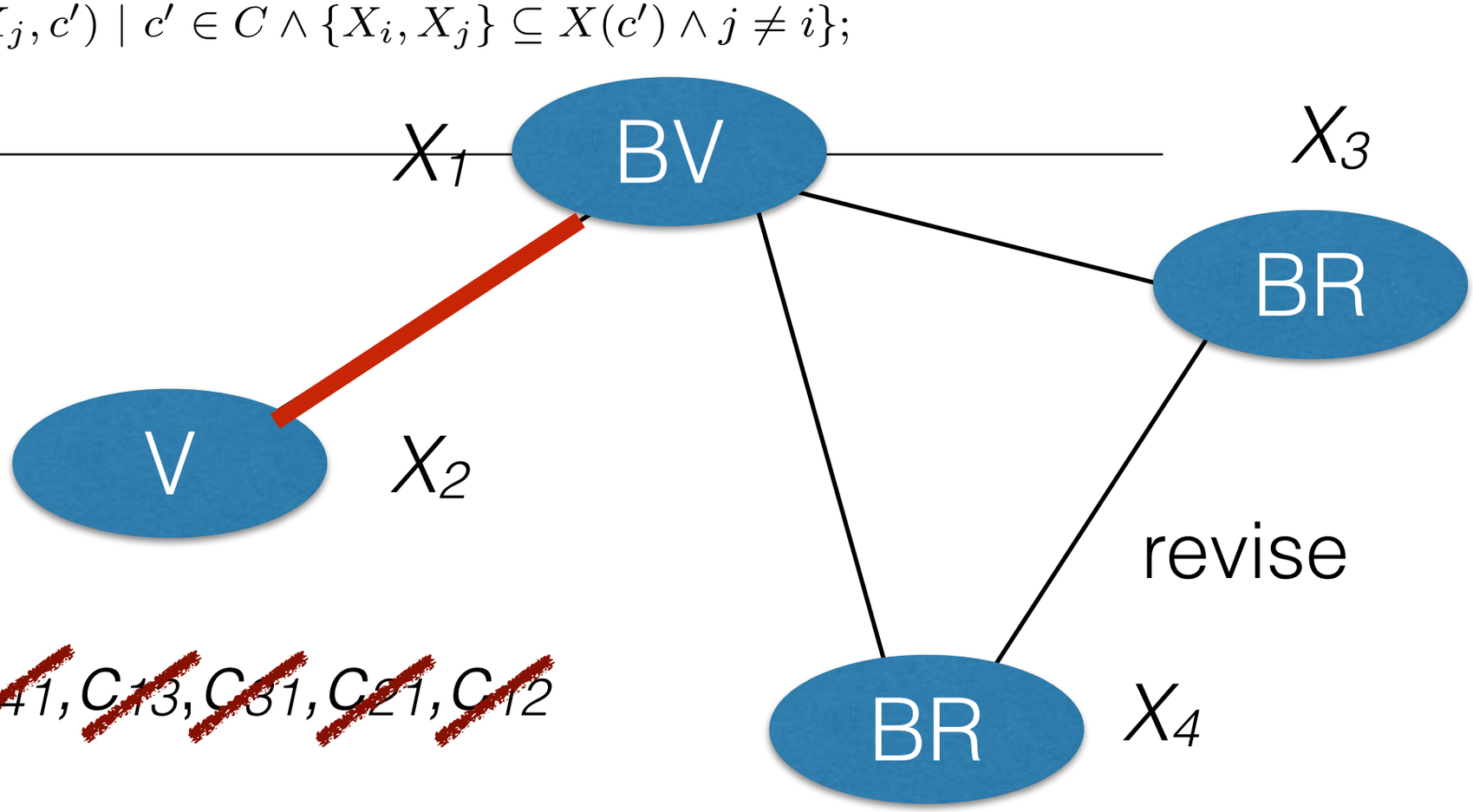

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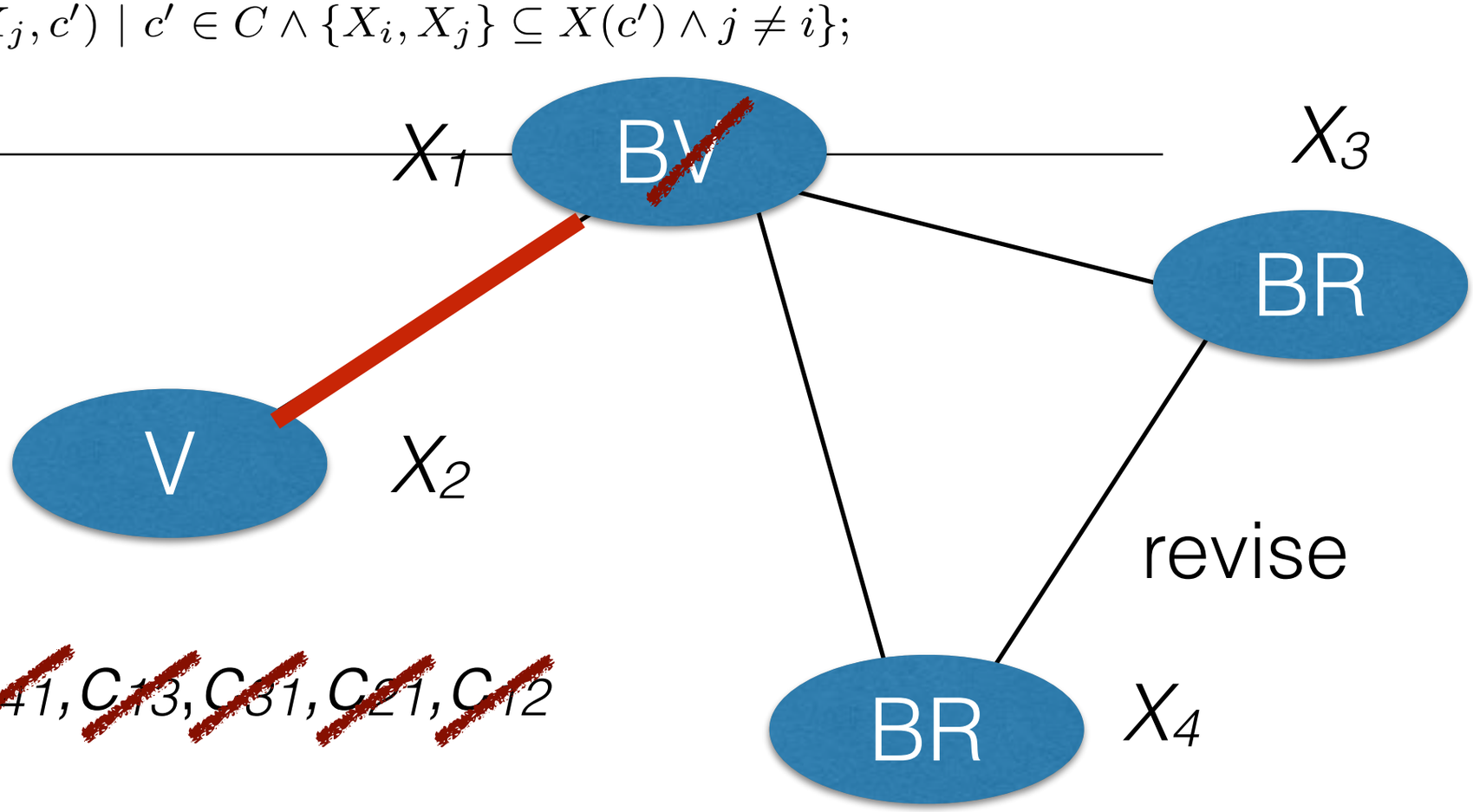

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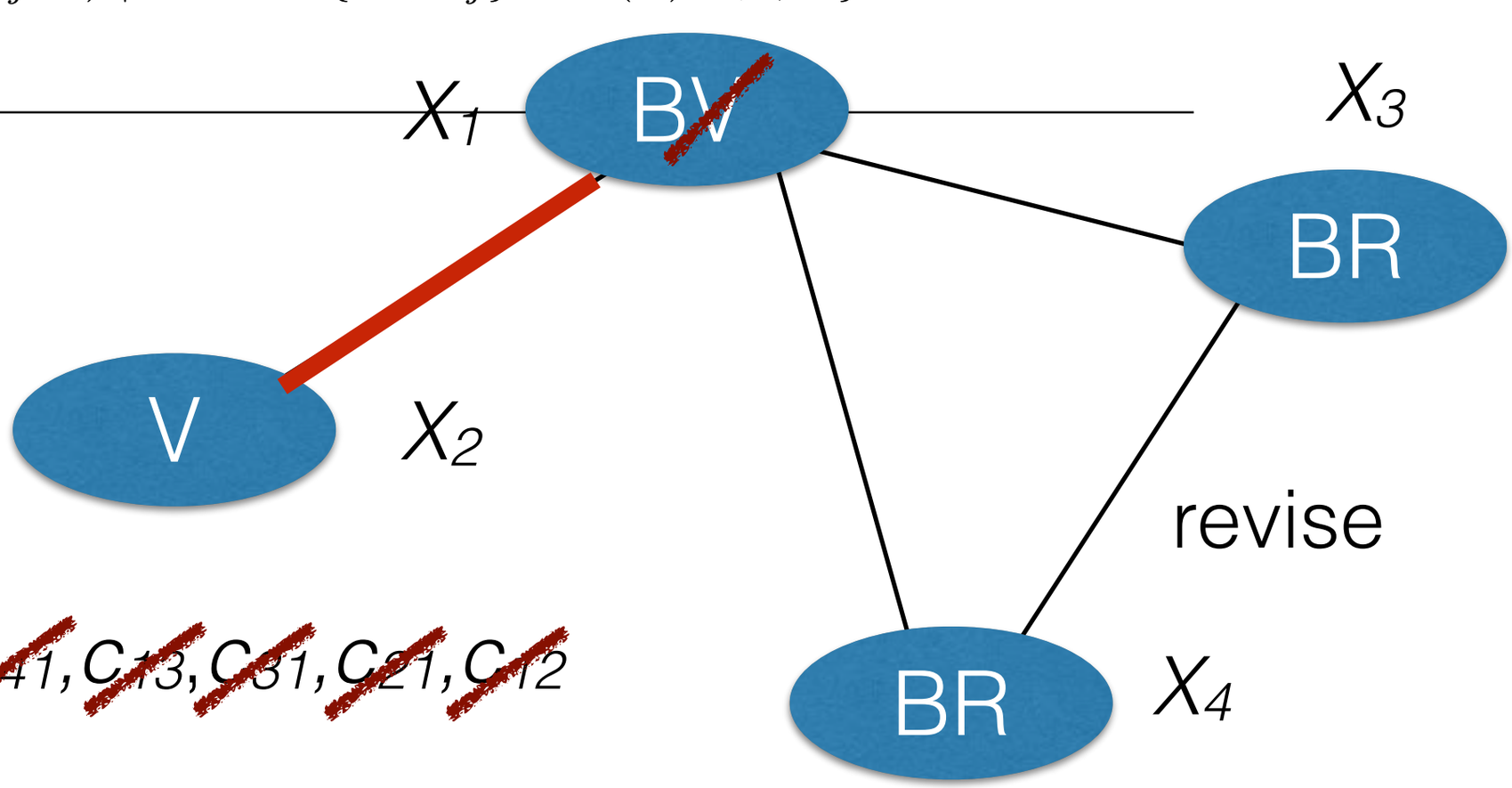

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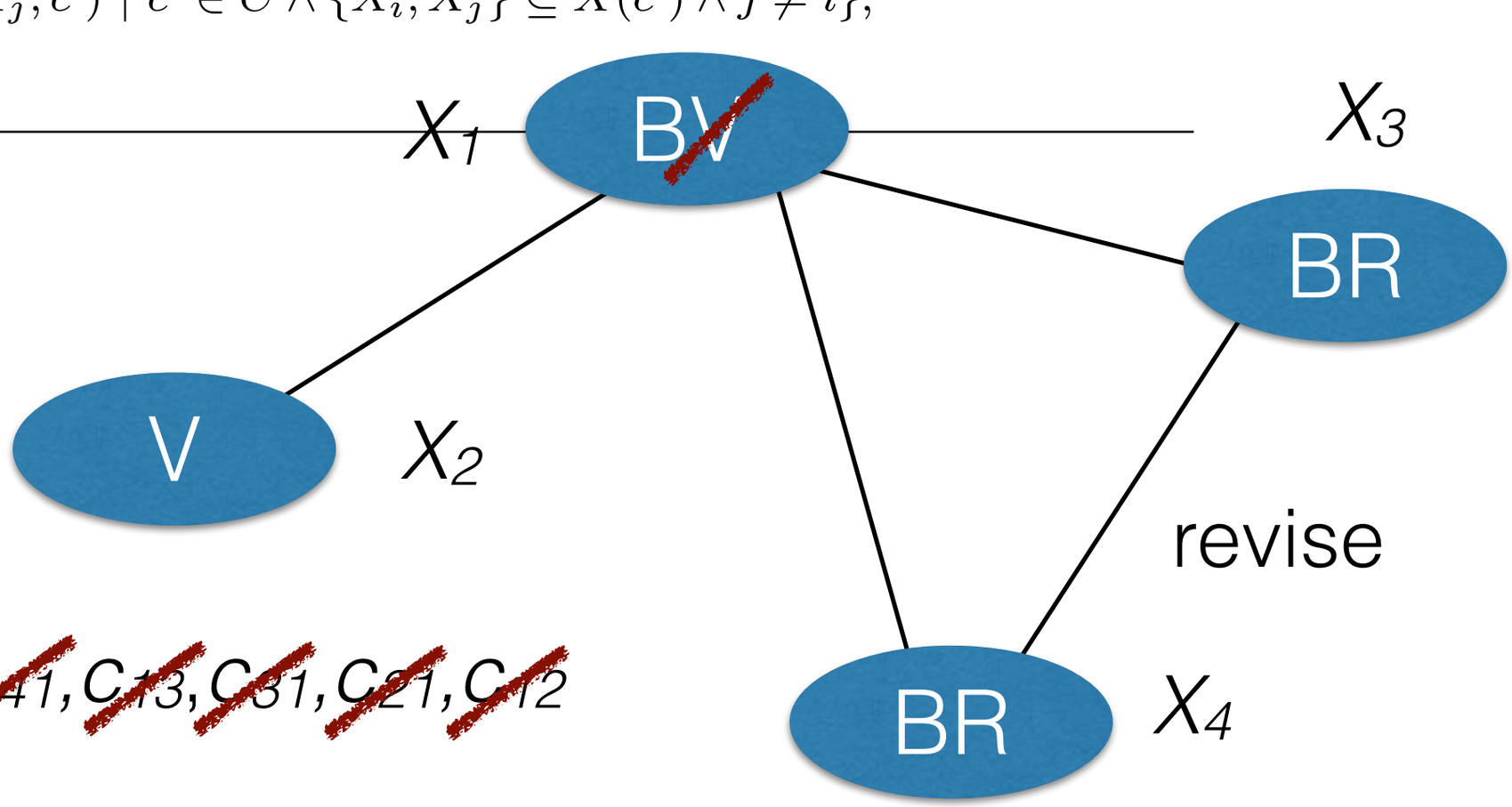

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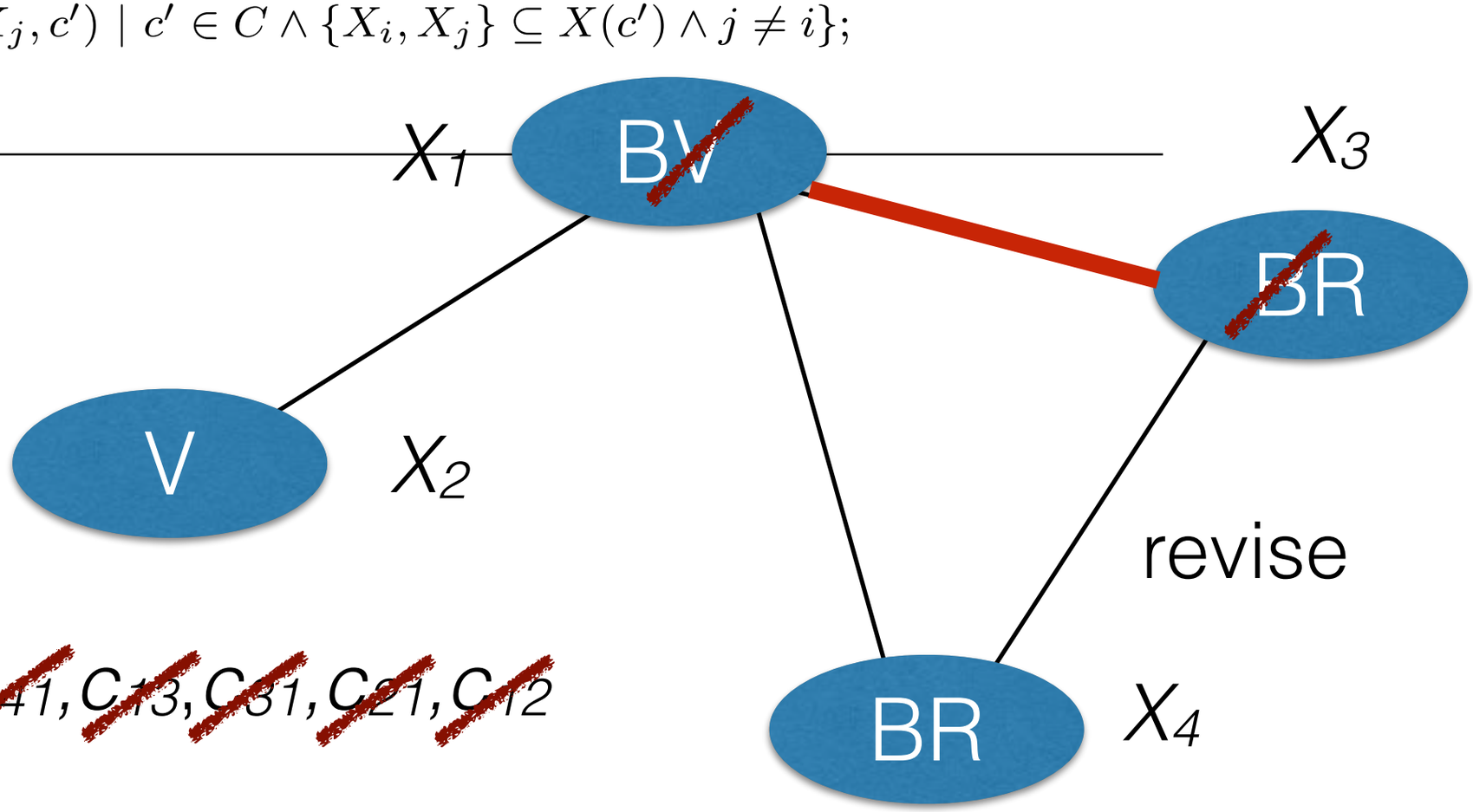

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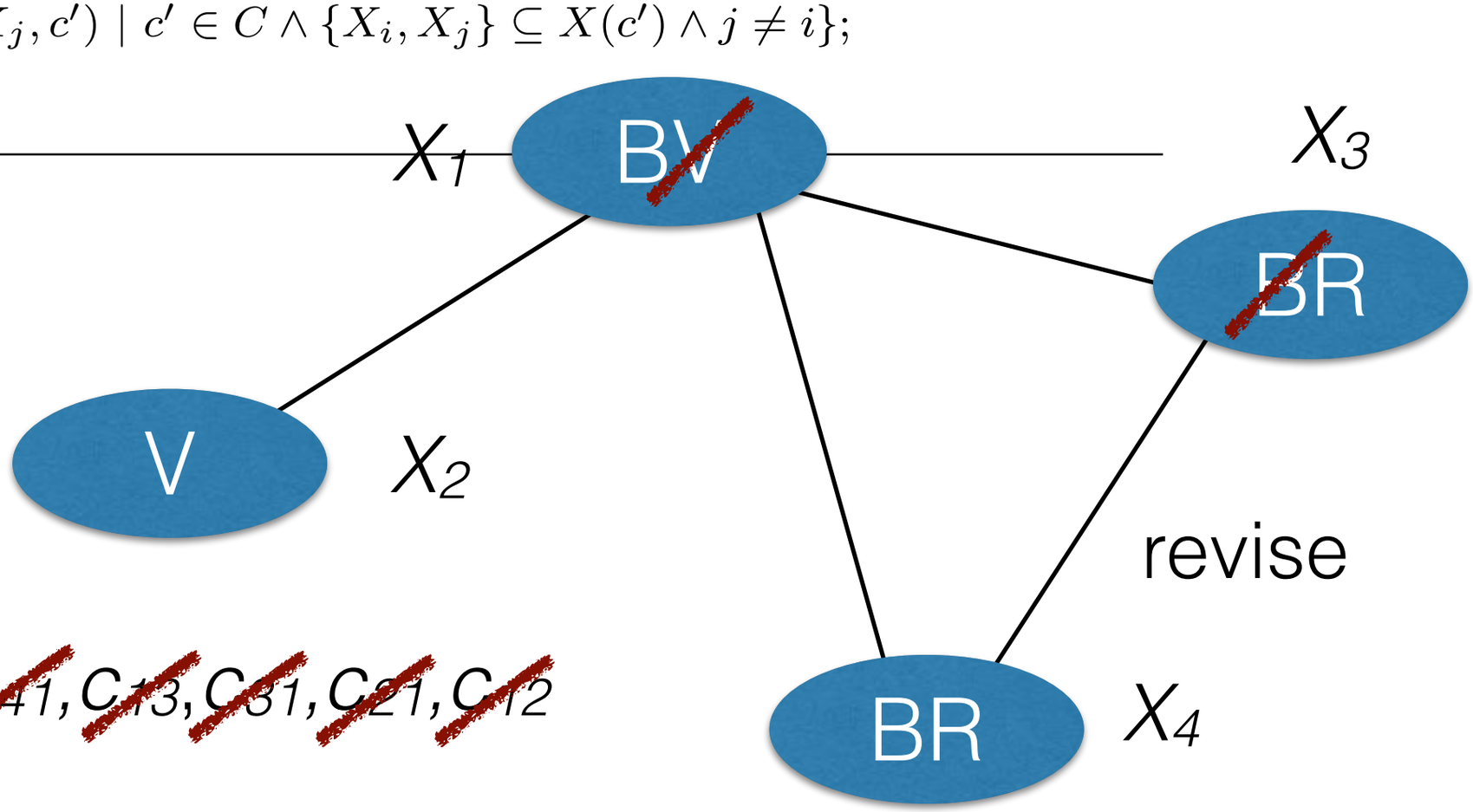

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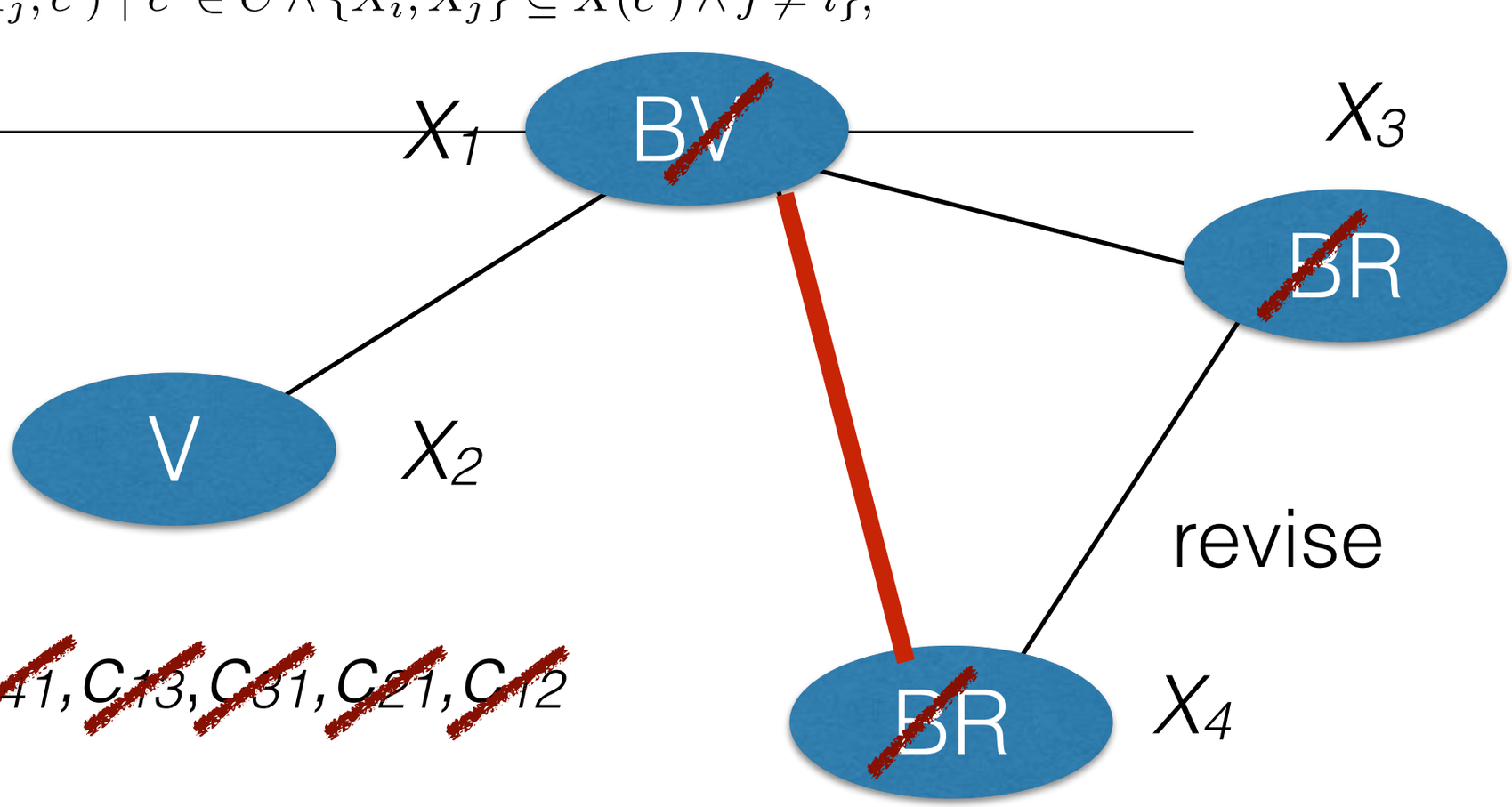

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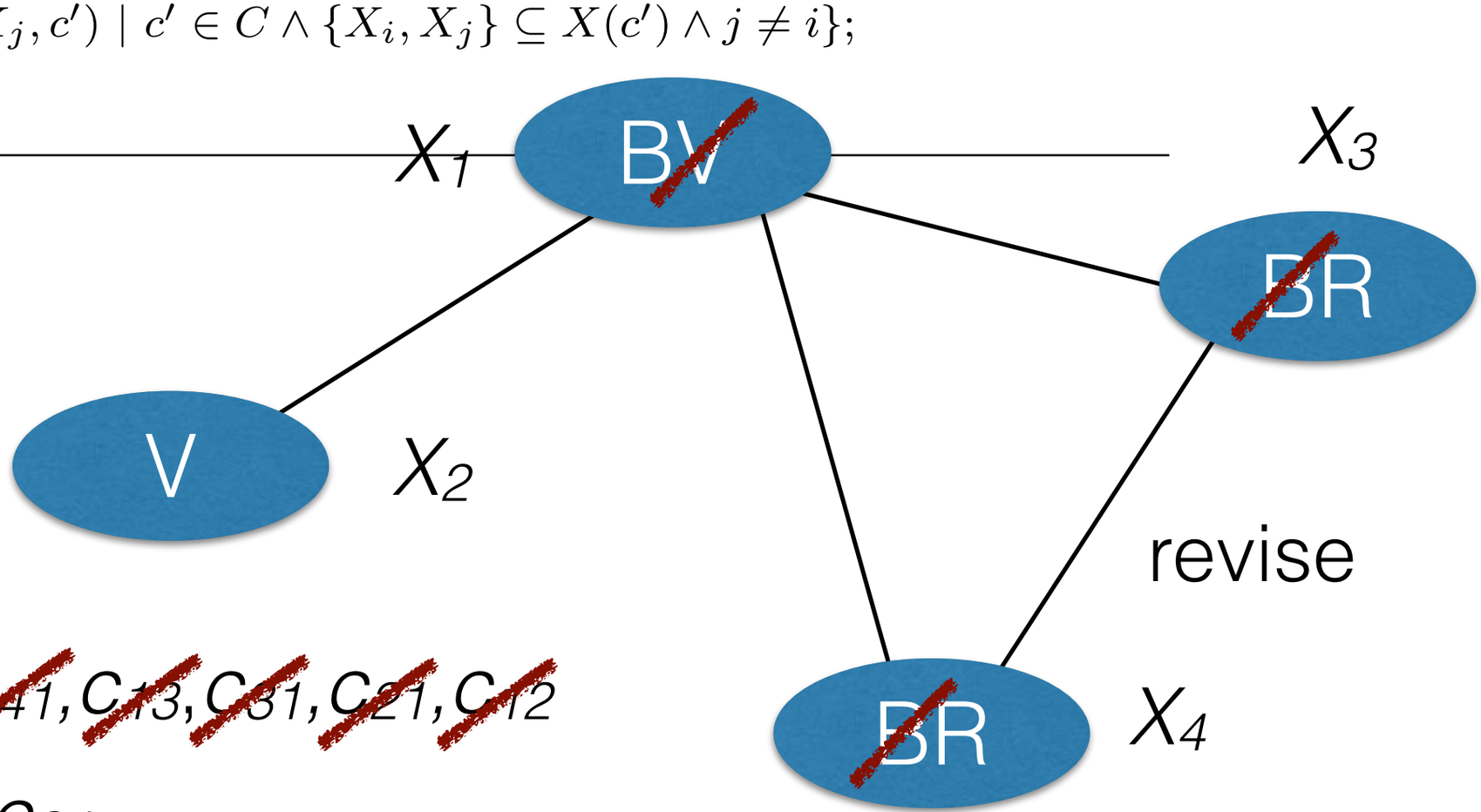

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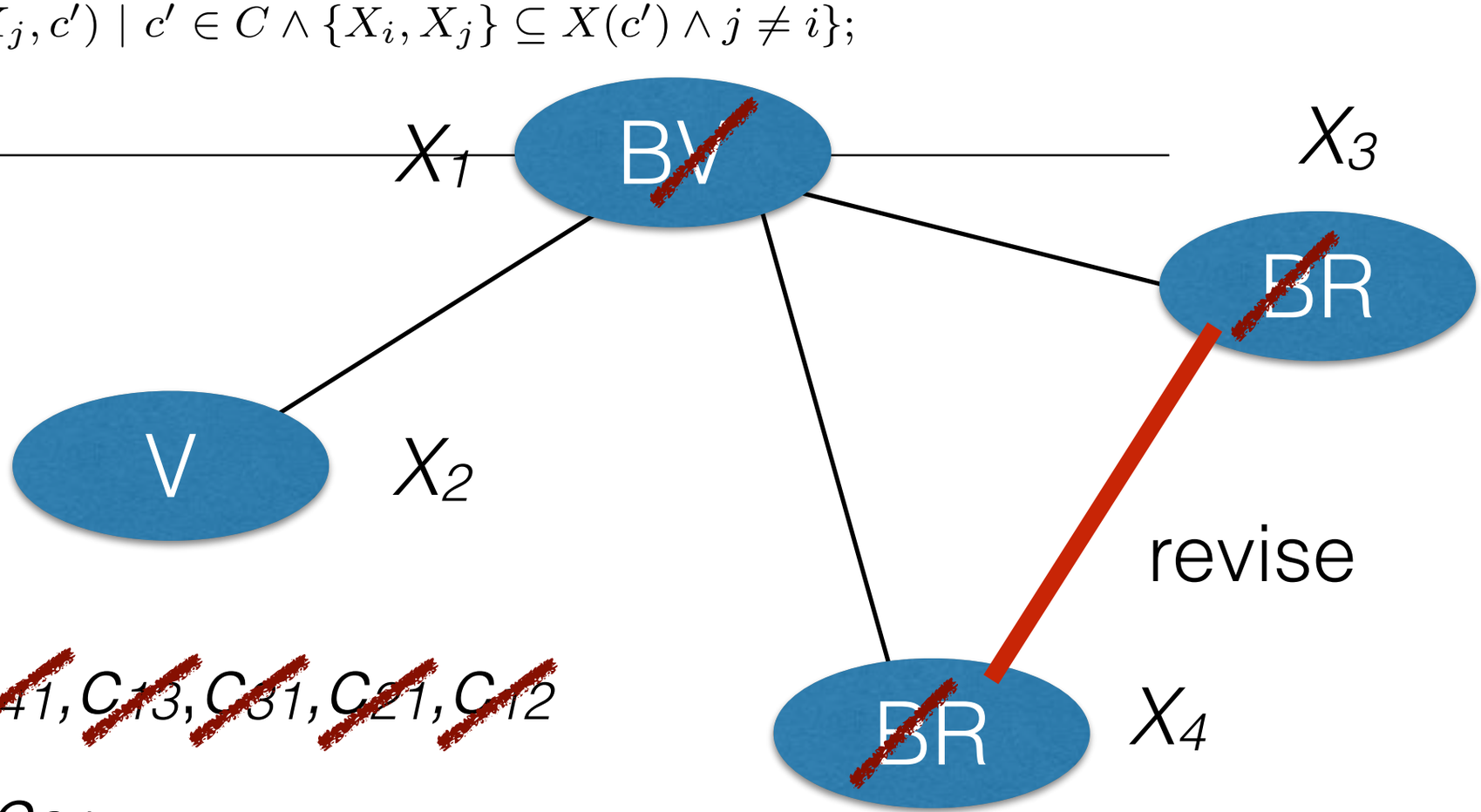

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c31,c41 c43 c34

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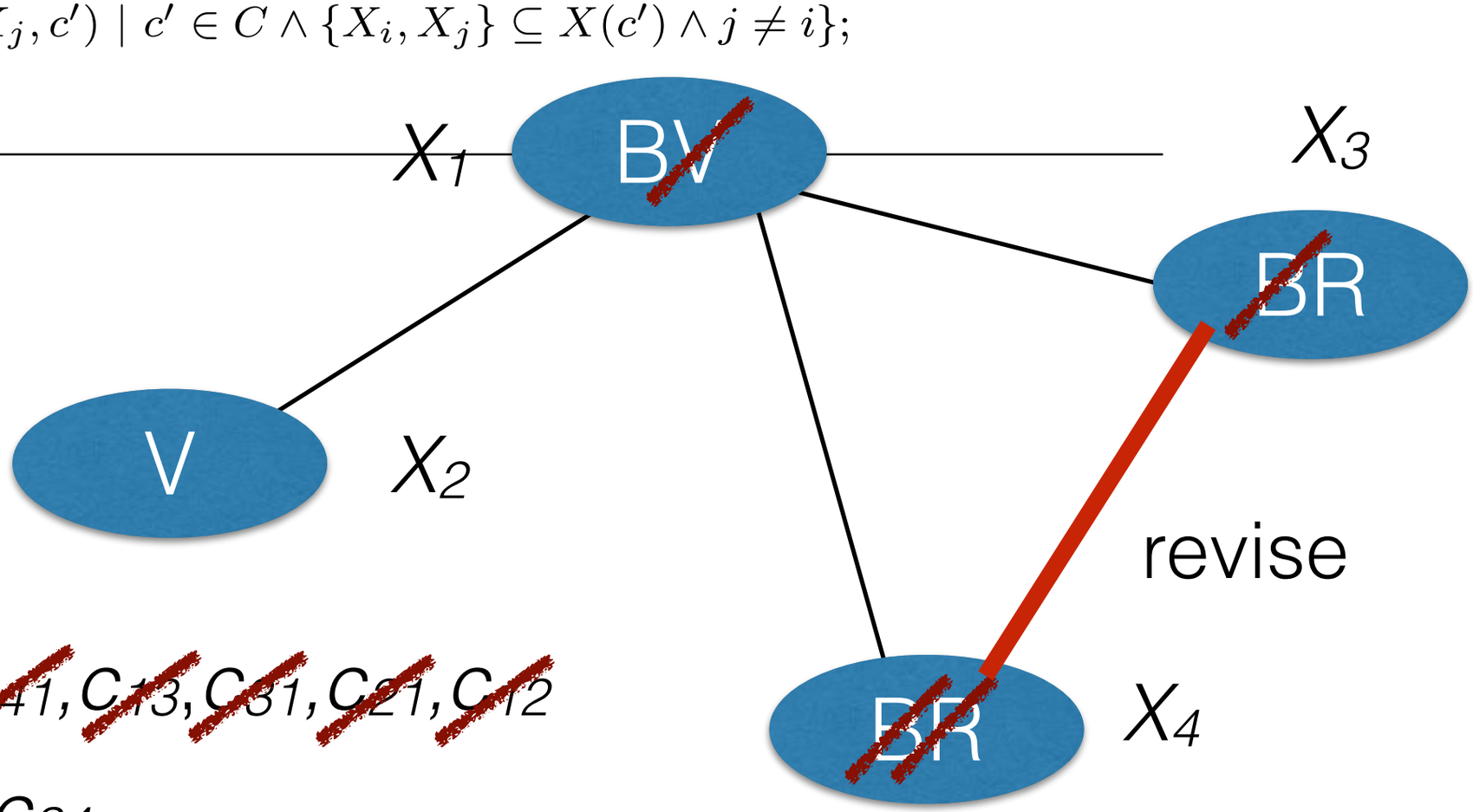


Q= {c34, c43, c14, c41, c13, c31, c21, c12

c31, c41 c43 c34

empty domain

Revise of AC3

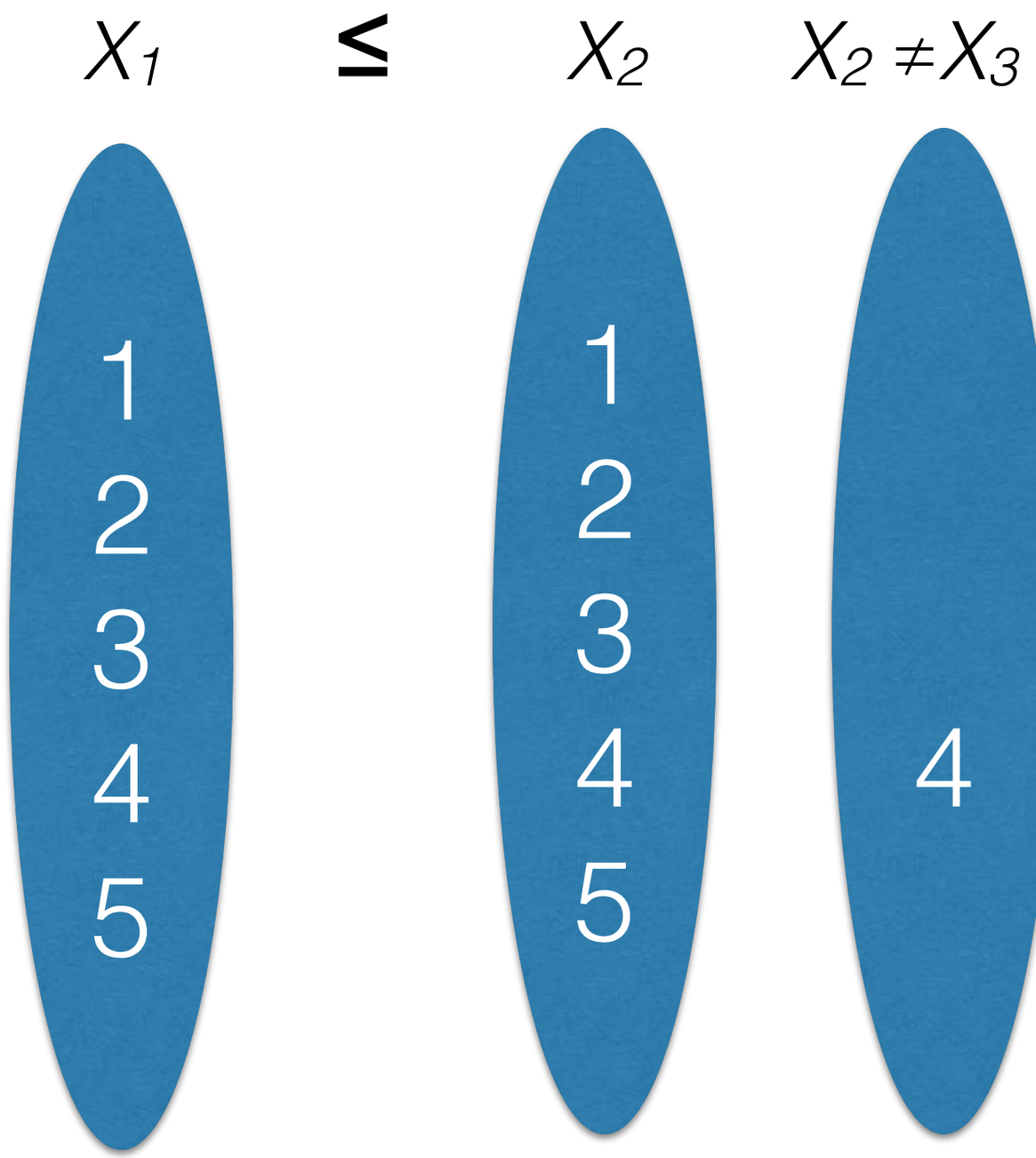
Revise of AC3

```
function Revise3(in X_i : variable; c_ij : constraint) : Boolean
    begin
1       CHANGE ← false;
2       foreach v_i ∈ D(X_i) do
3           v_j ← first(D(X_j));
4           while (v_j ≠ nil) and ¬c_ij(v_i, v_j) do v_j ← next(v_j, D(X_j));
5           if v_j = nil then
6               remove v_i from D(X_i);
7               CHANGE ← true;
8       return CHANGE ;
    end
```

AC3

- Uses the queue Q as we did before: Q={c_{12},c_{21},c_{23},c_{32}}

- Function Revise traverses *$D(X_1)$ x $D(X_2)$*



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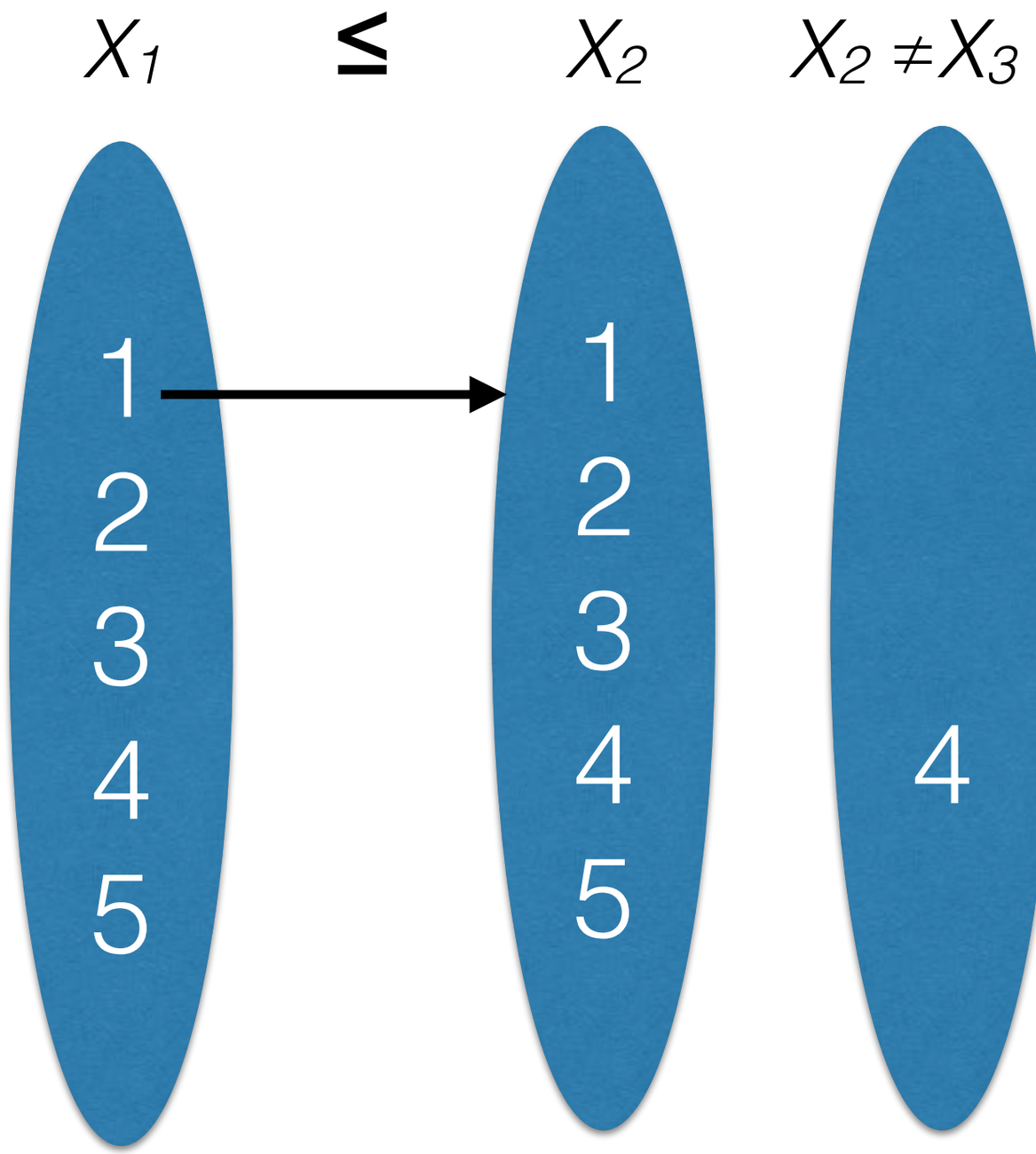

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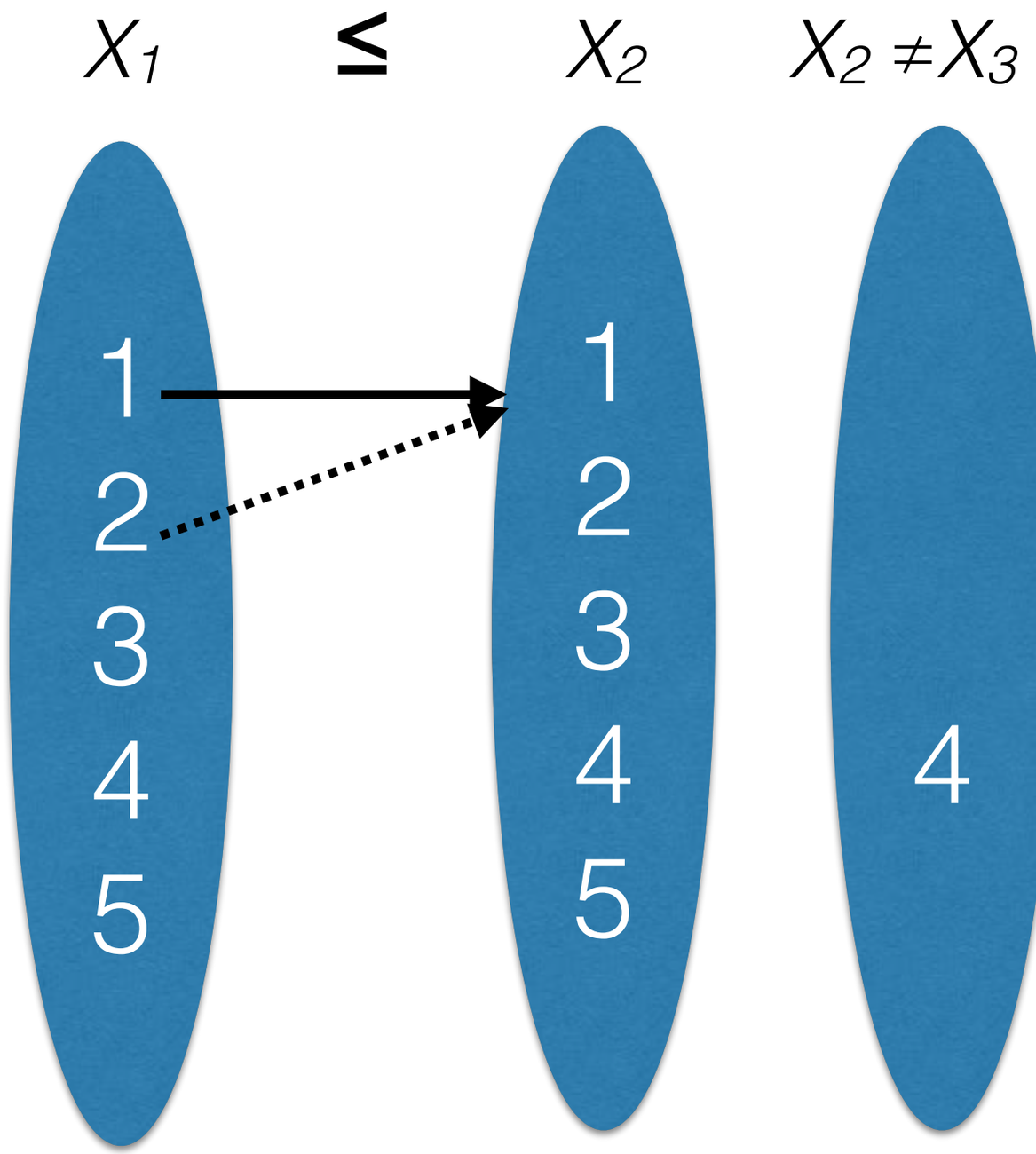

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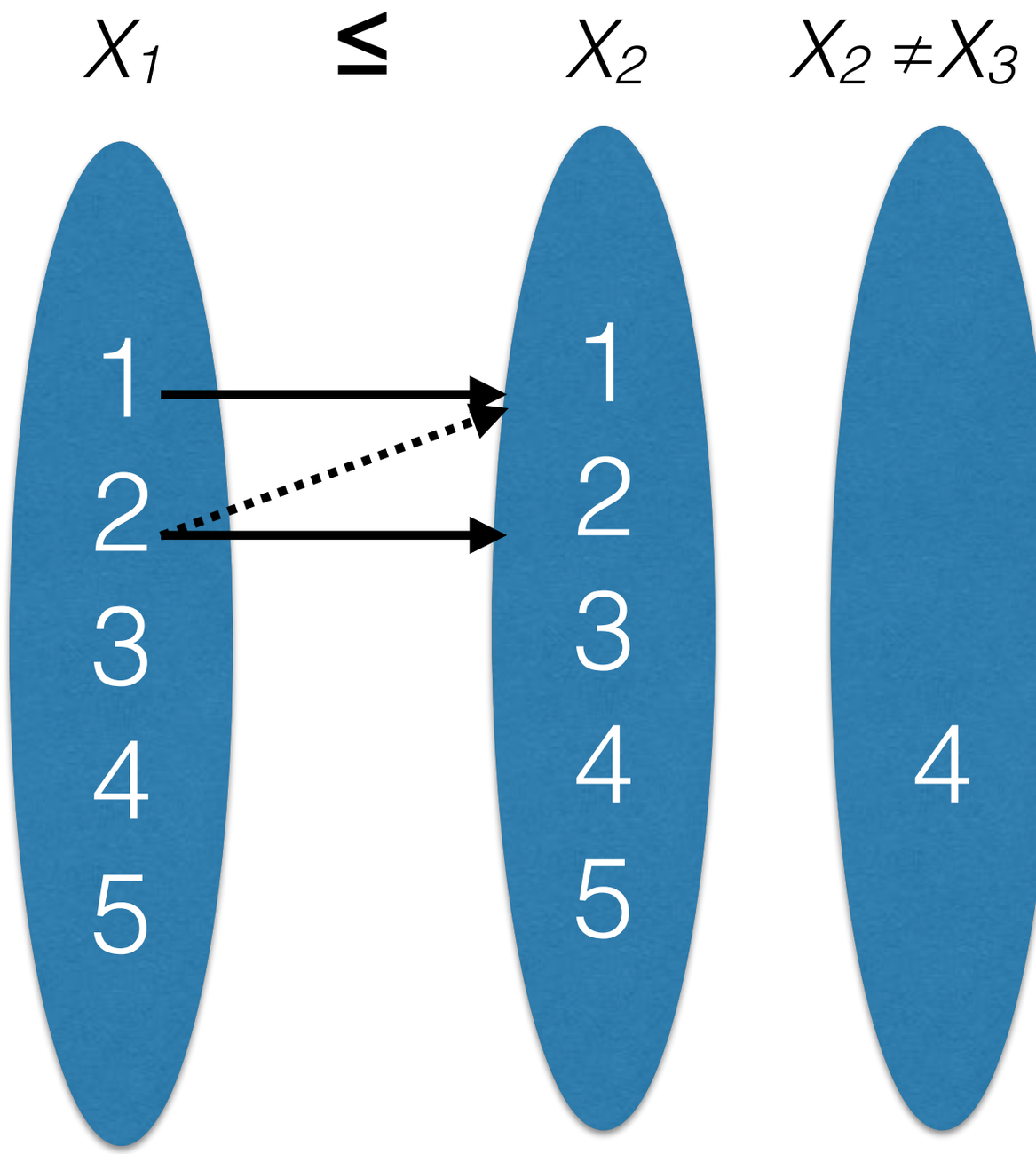

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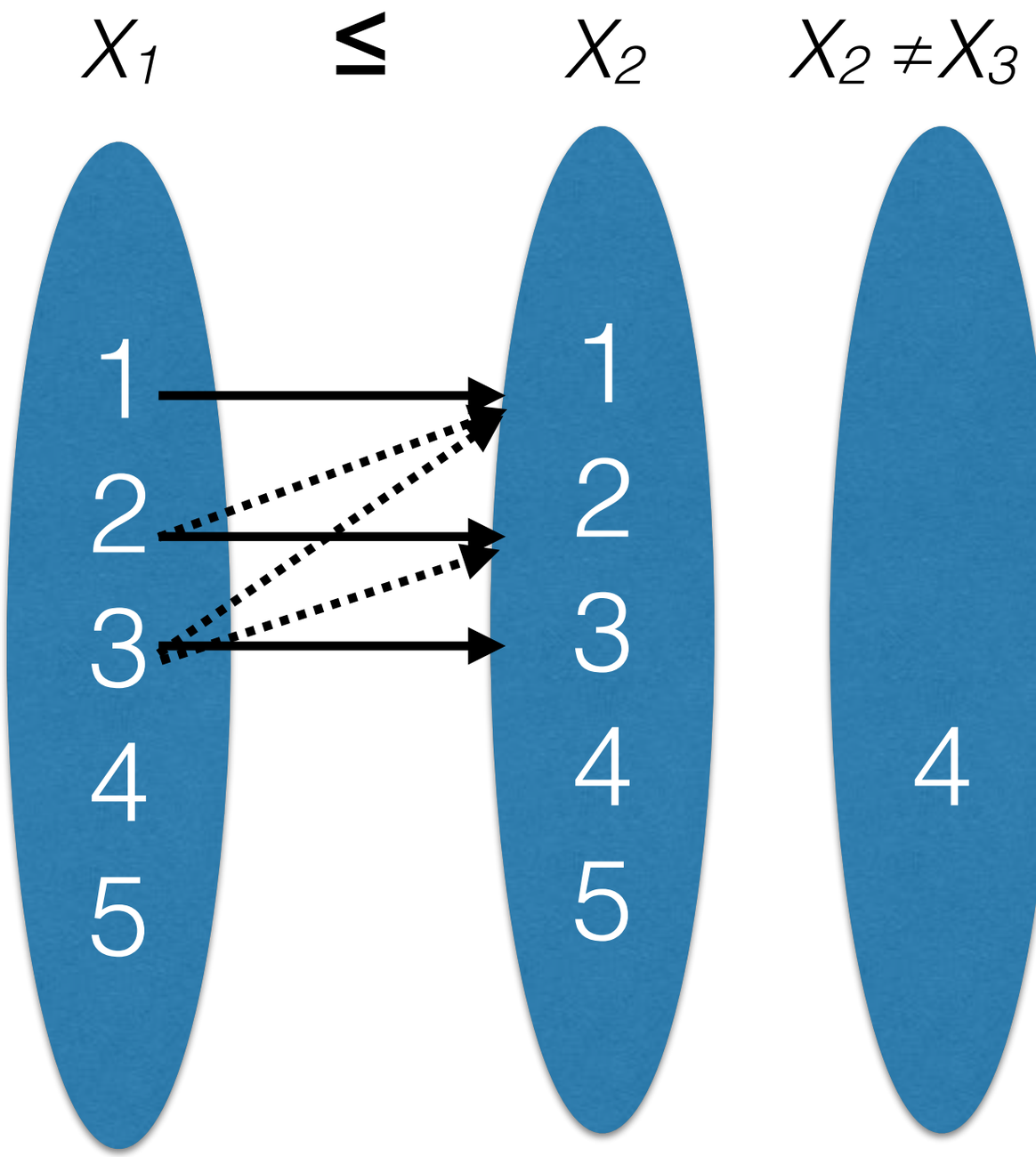

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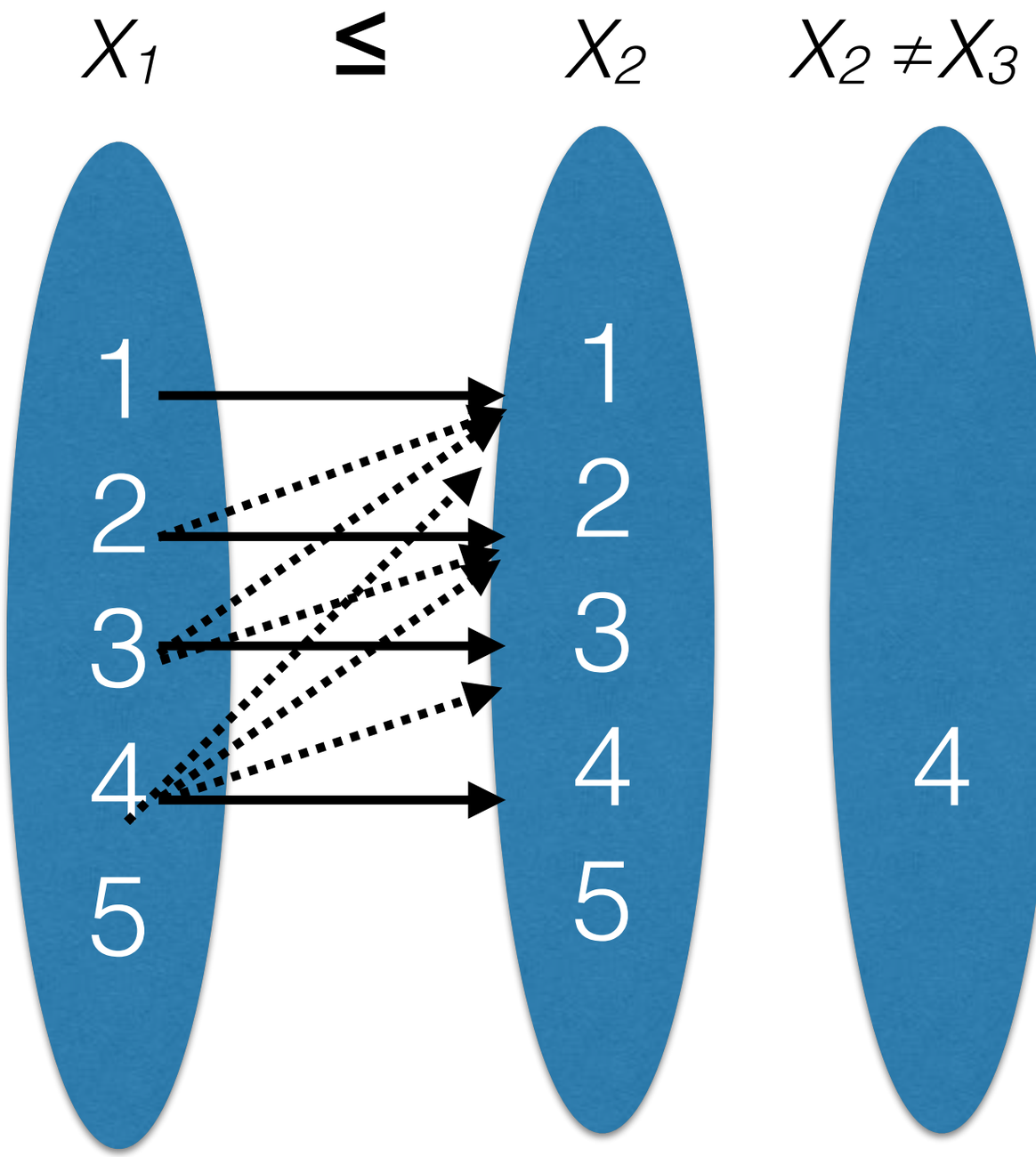

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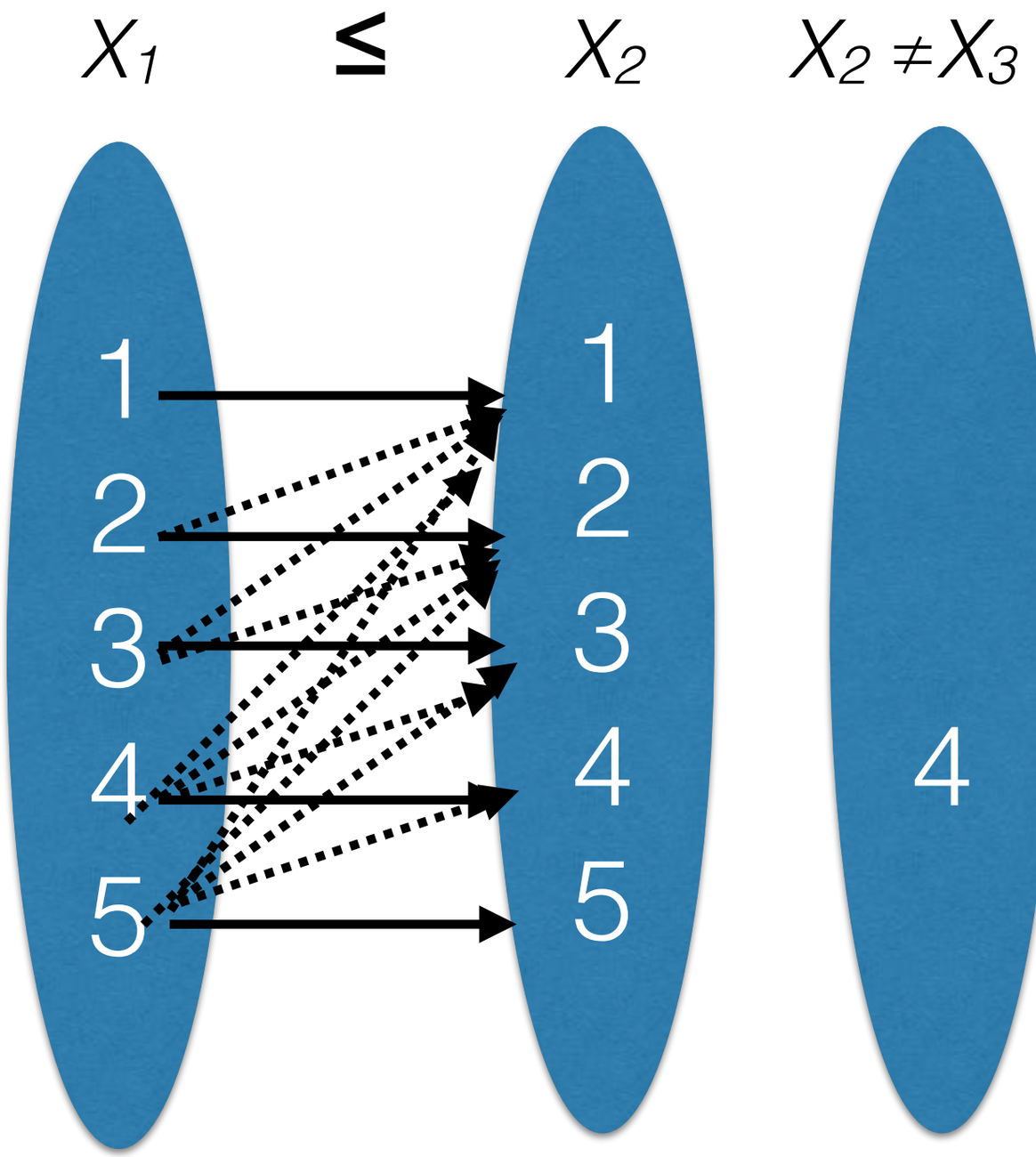

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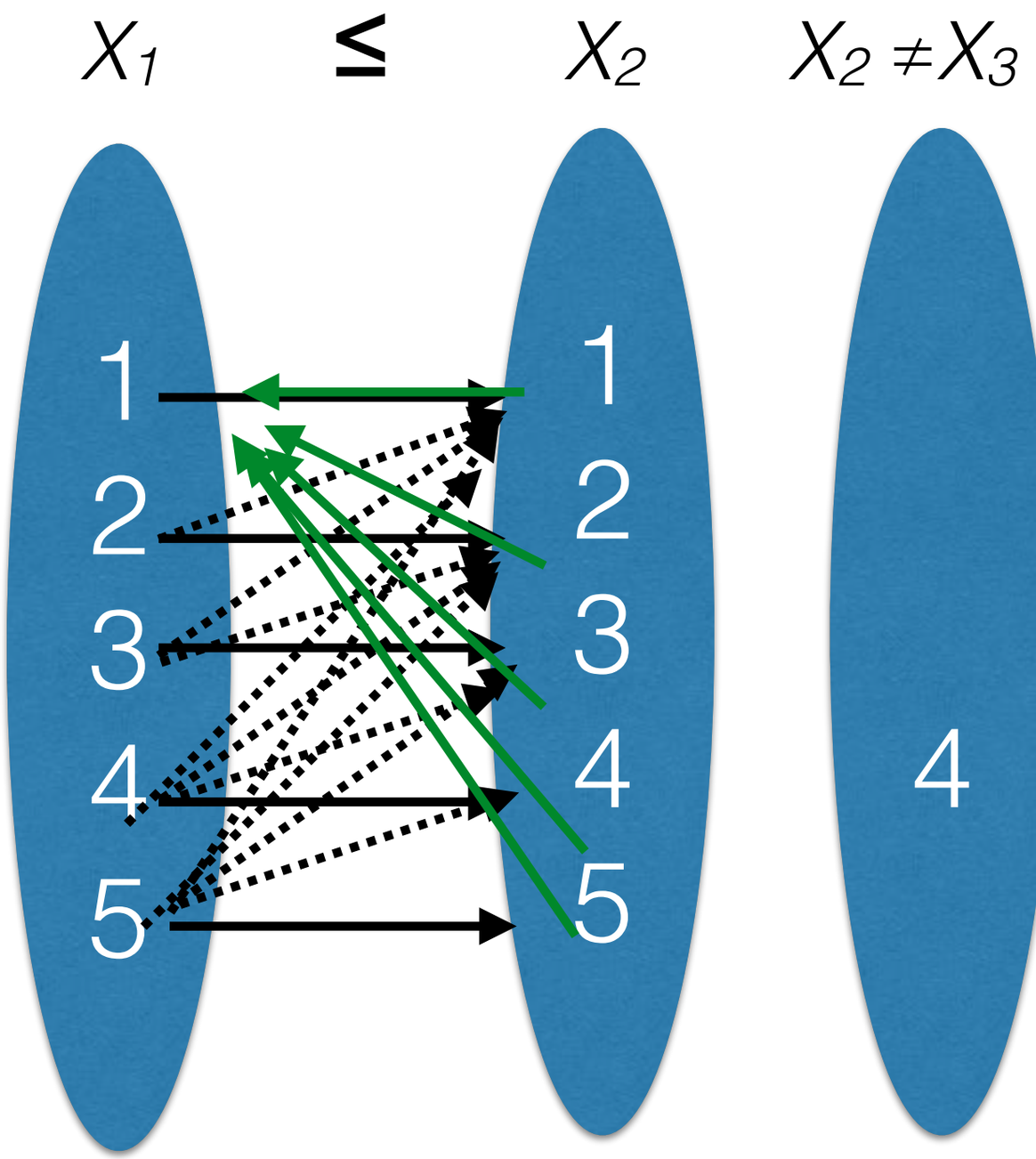

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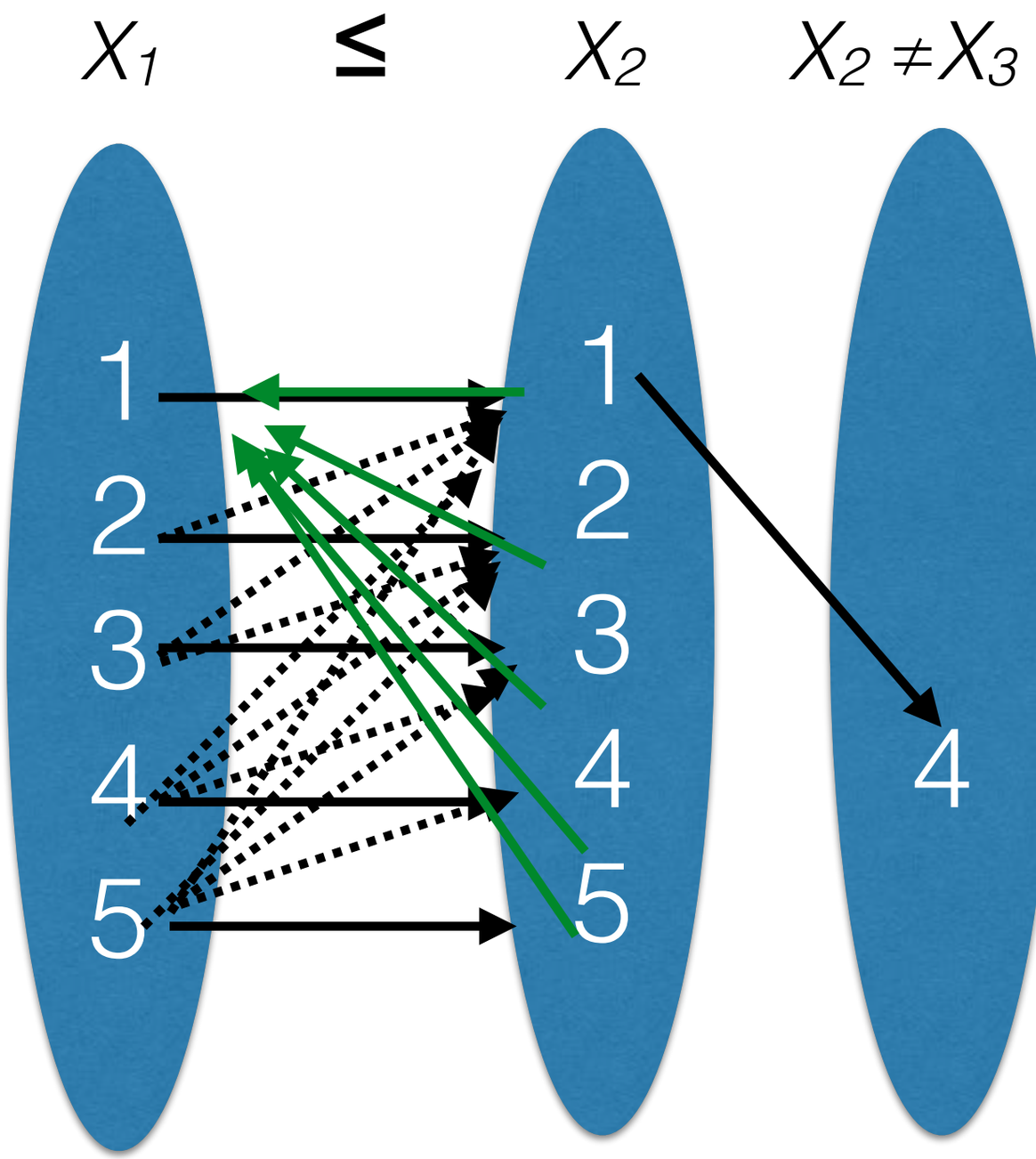

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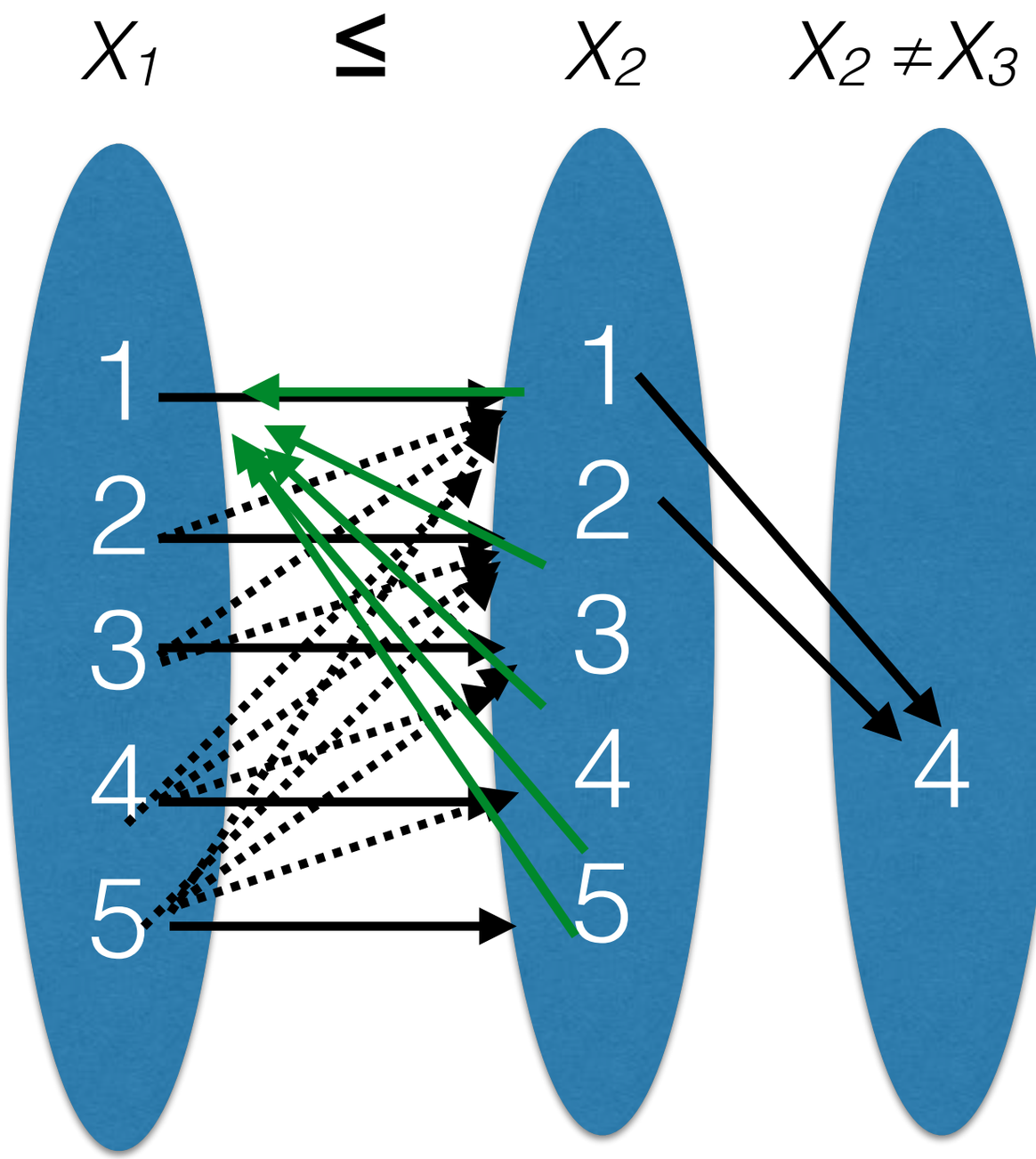

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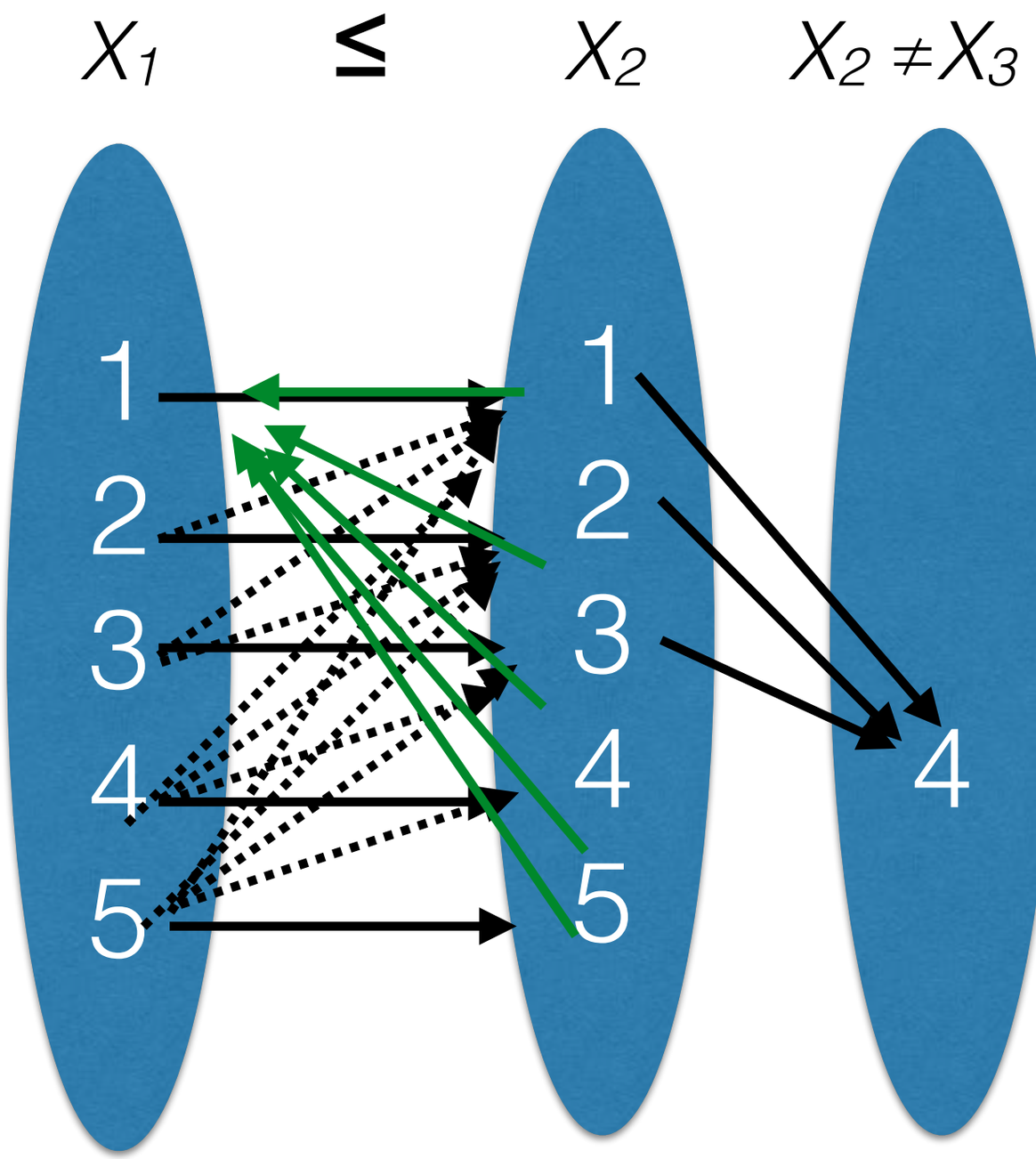

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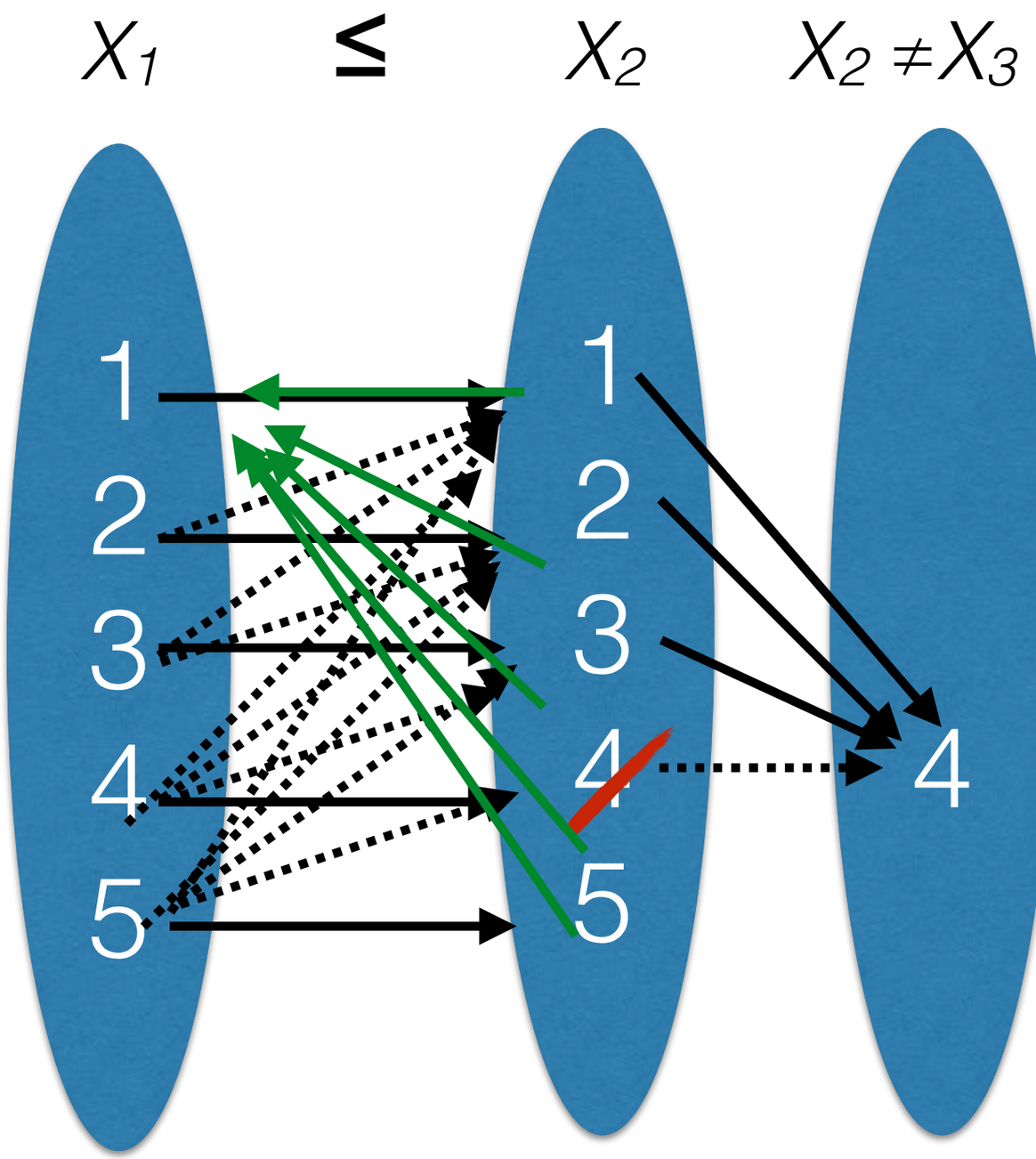

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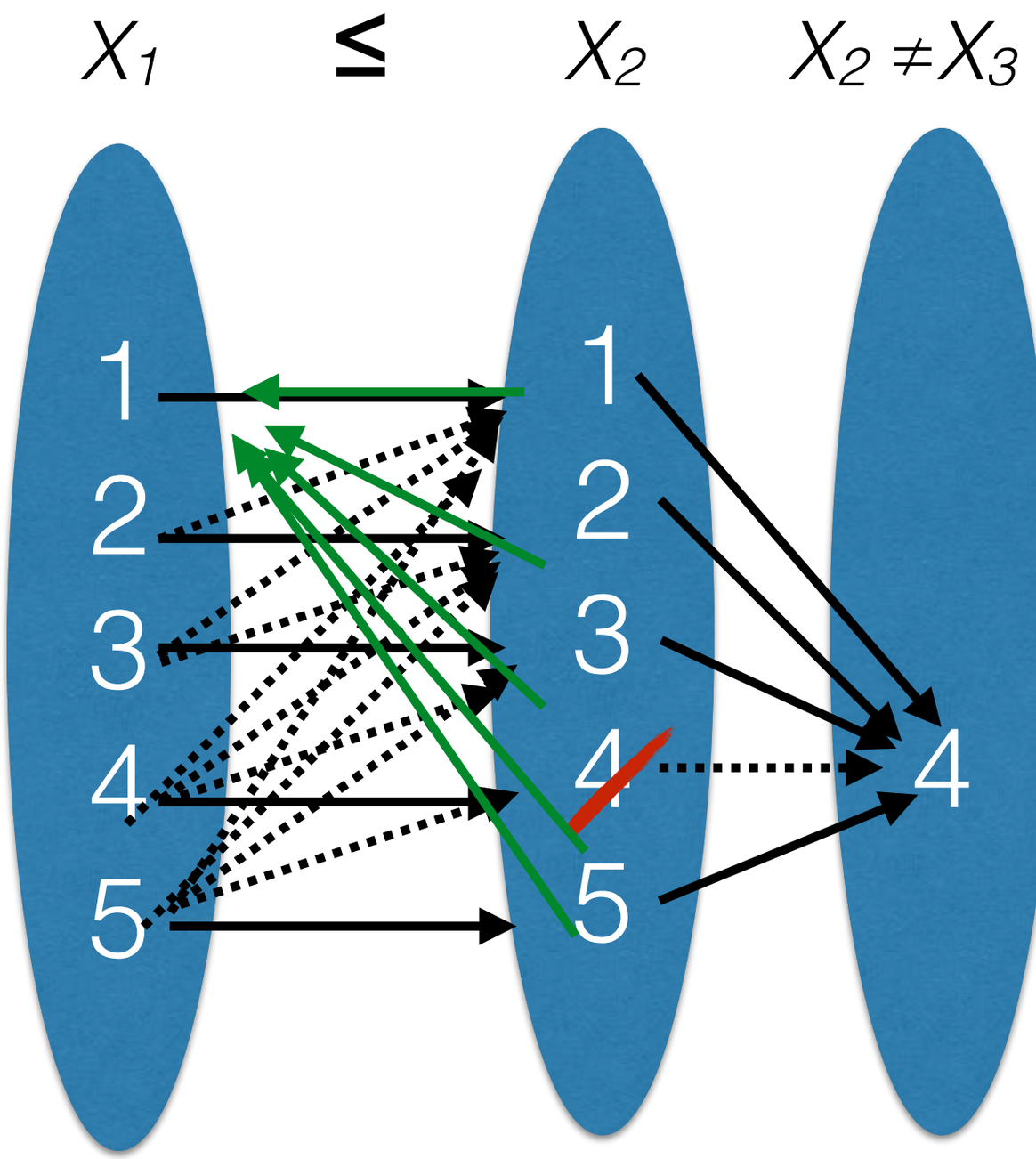

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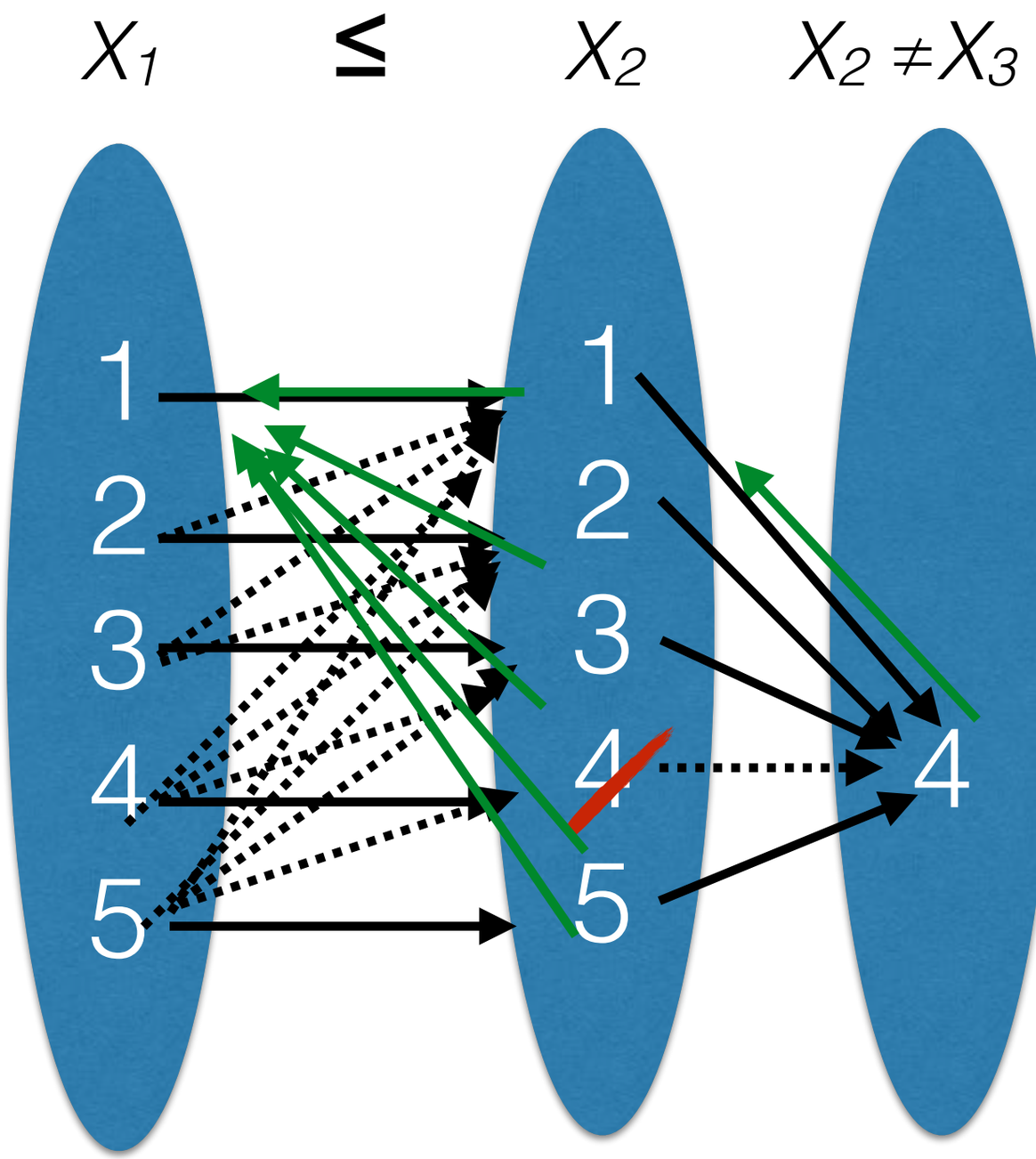
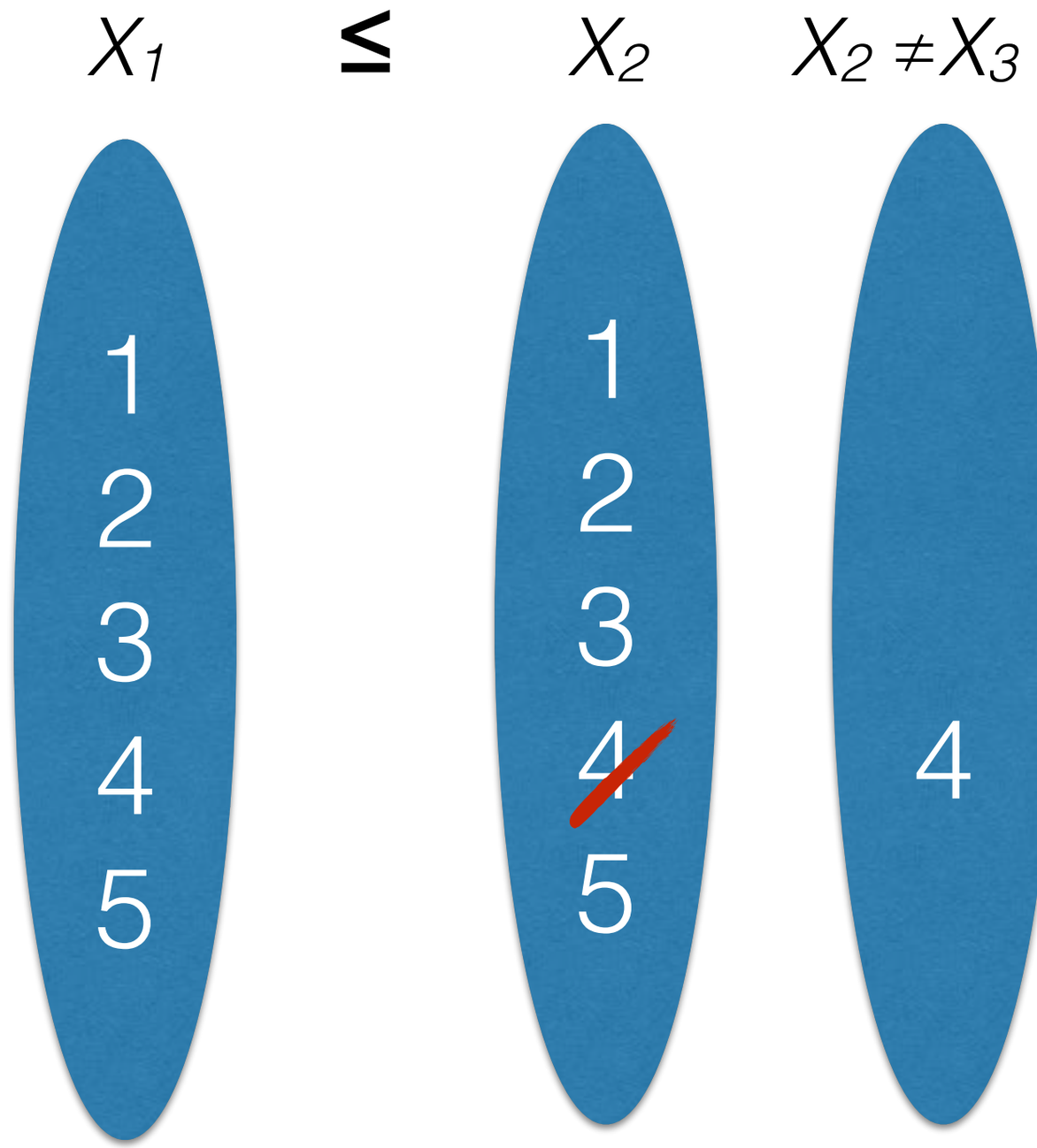
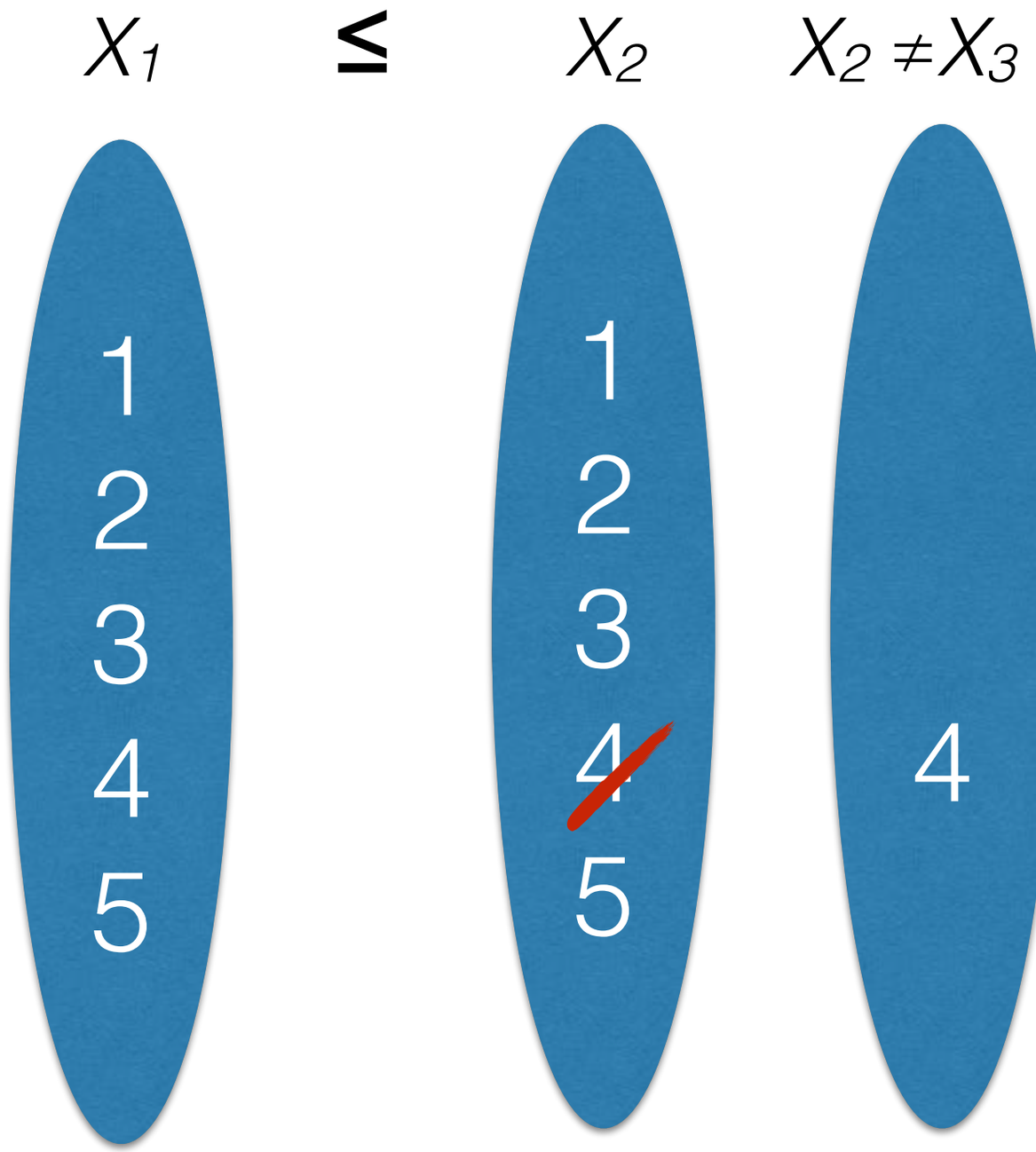


AC3



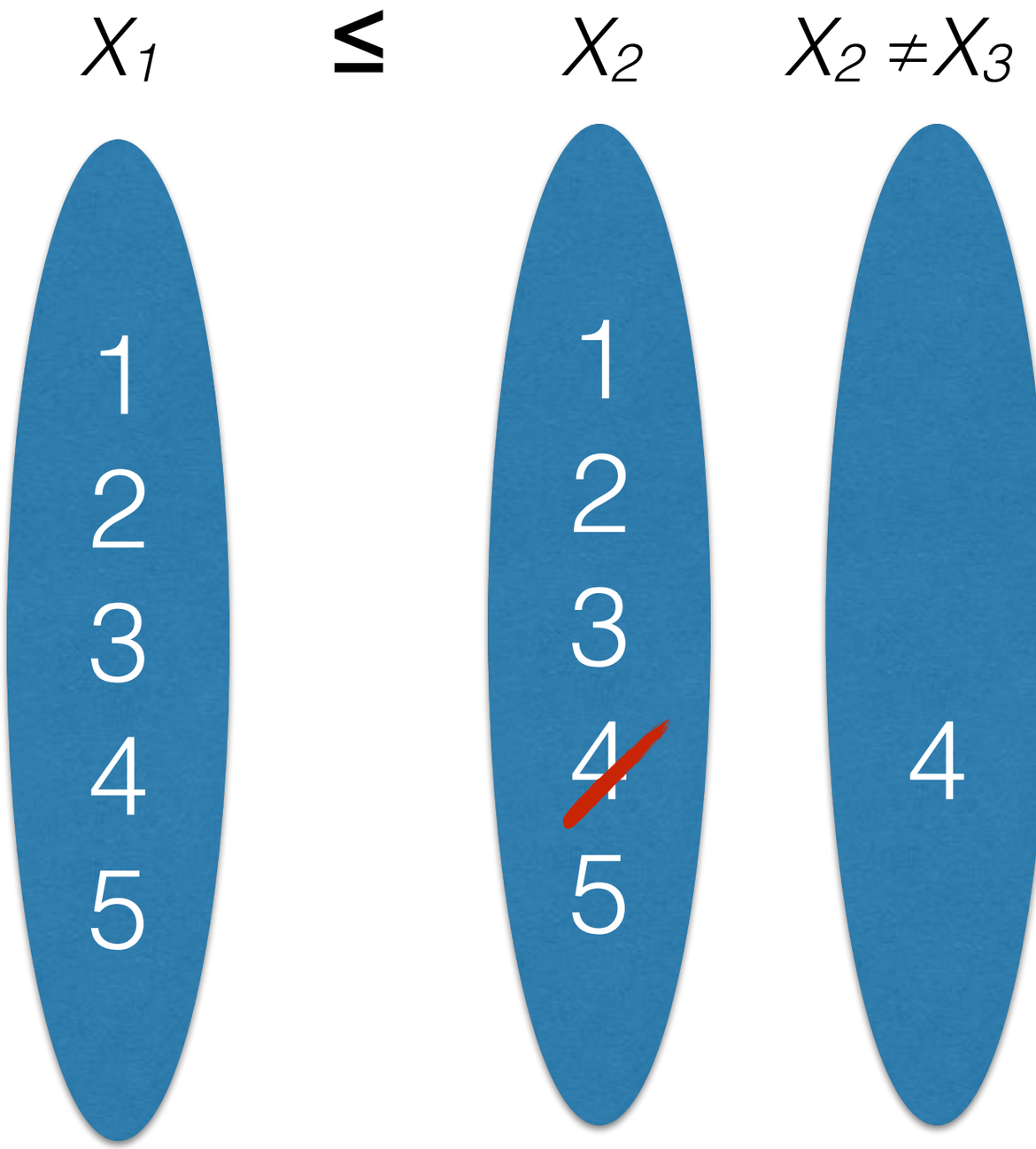
AC3

- We have to put X1 ≤X_2 in Q because of the removal of (X_2,4)



AC3

- We have to put X1 ≤X_2 in Q because of the removal of (X_2,4)
- AC3 forgot what it did before



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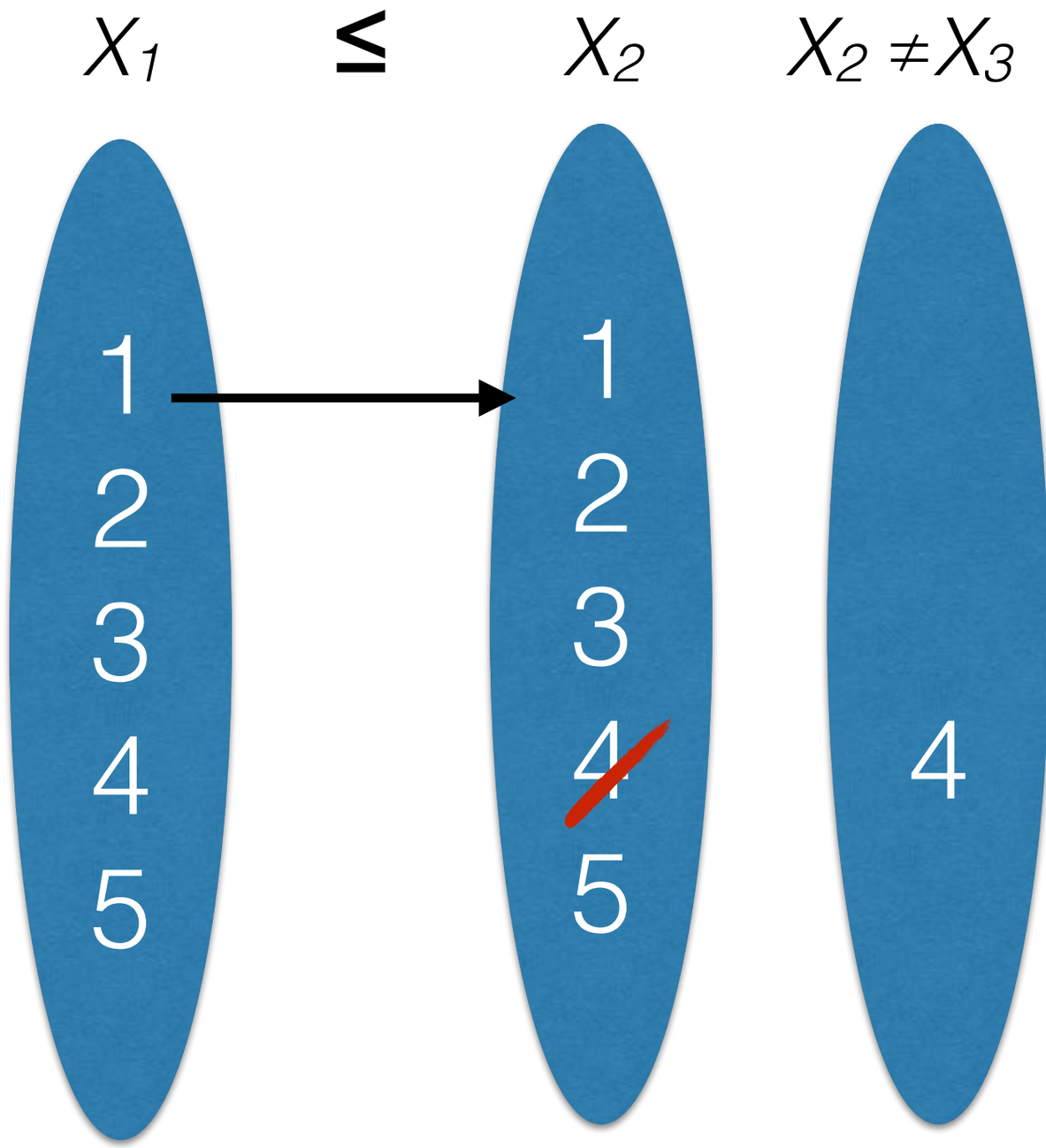

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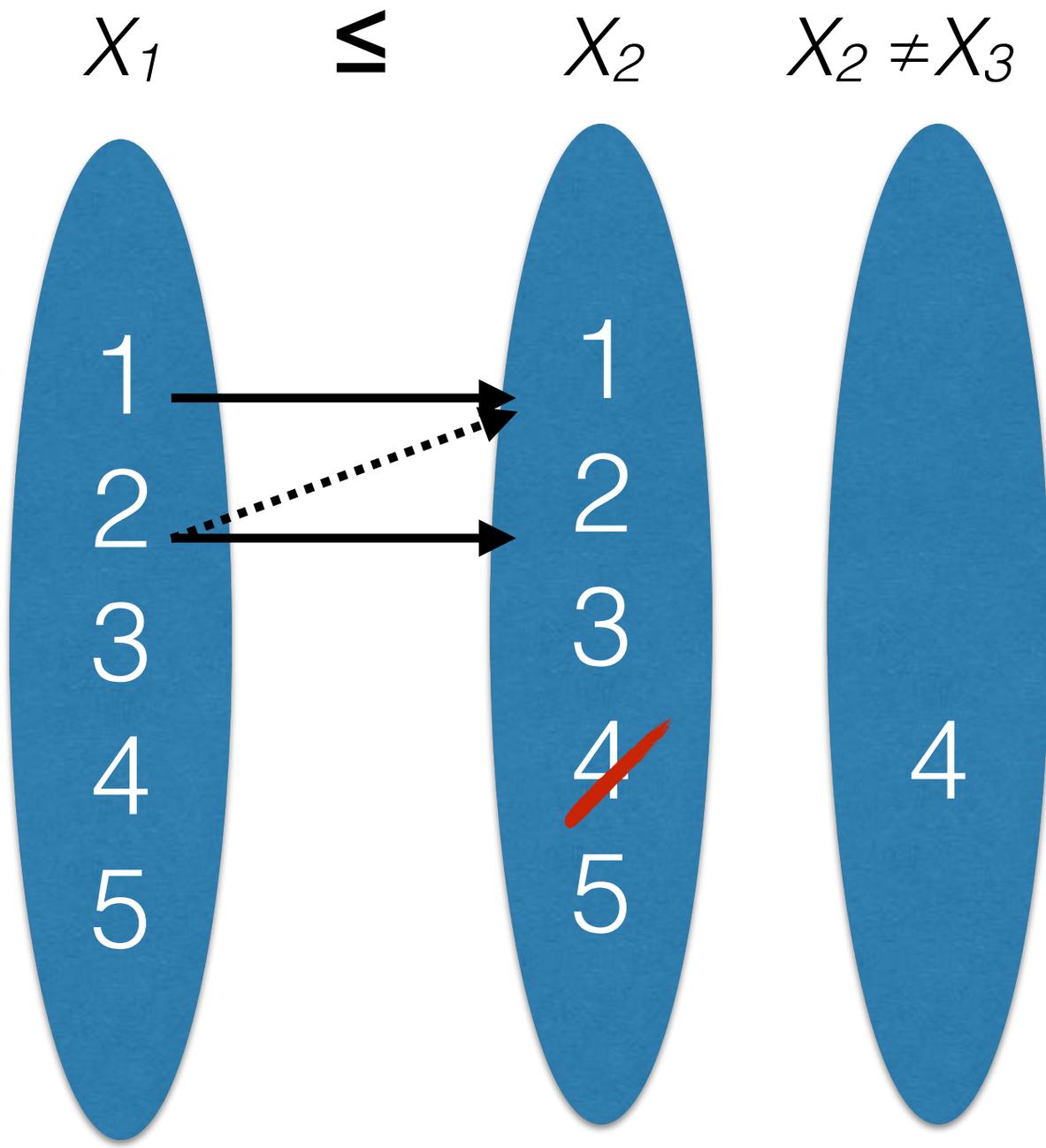

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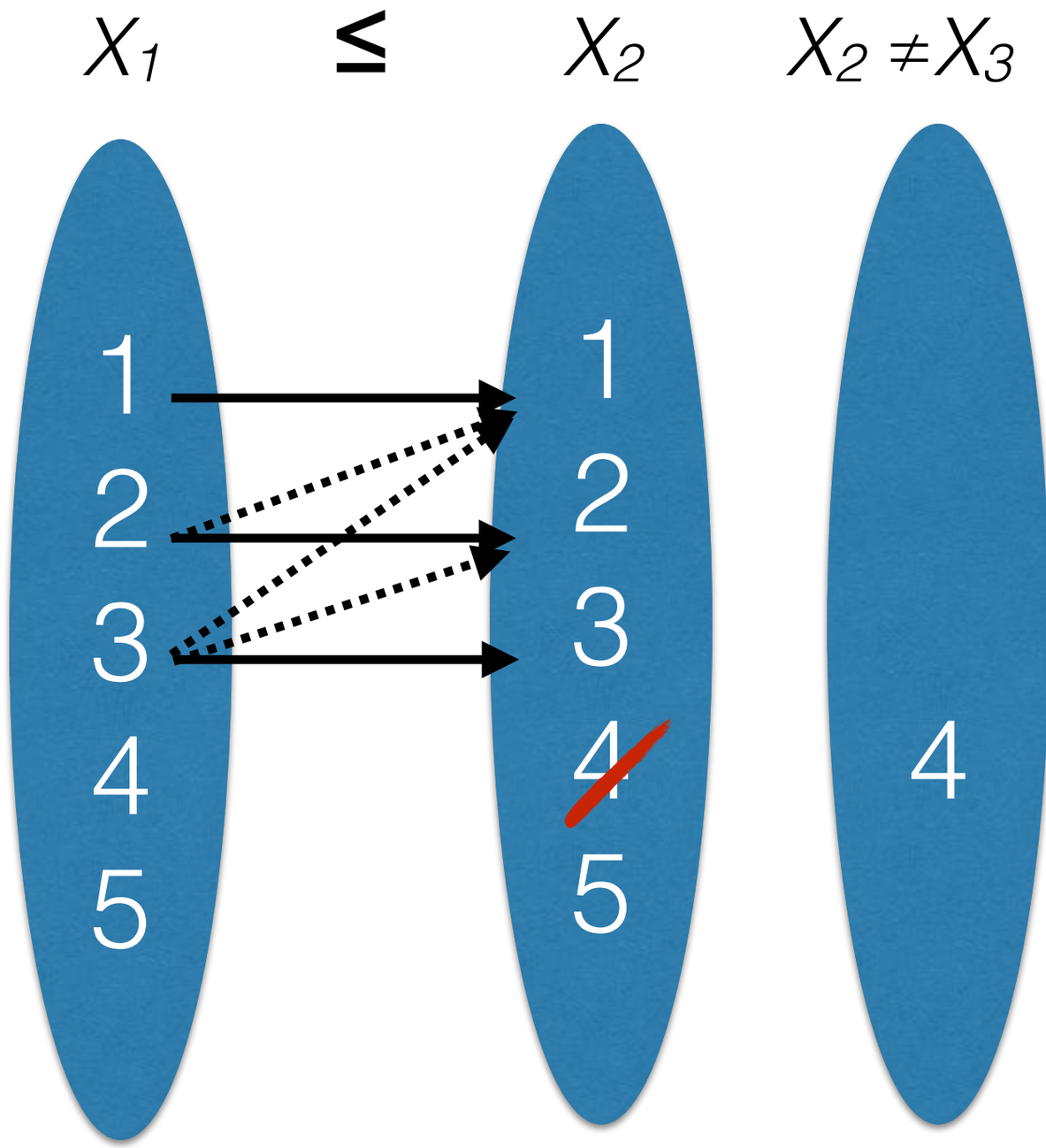

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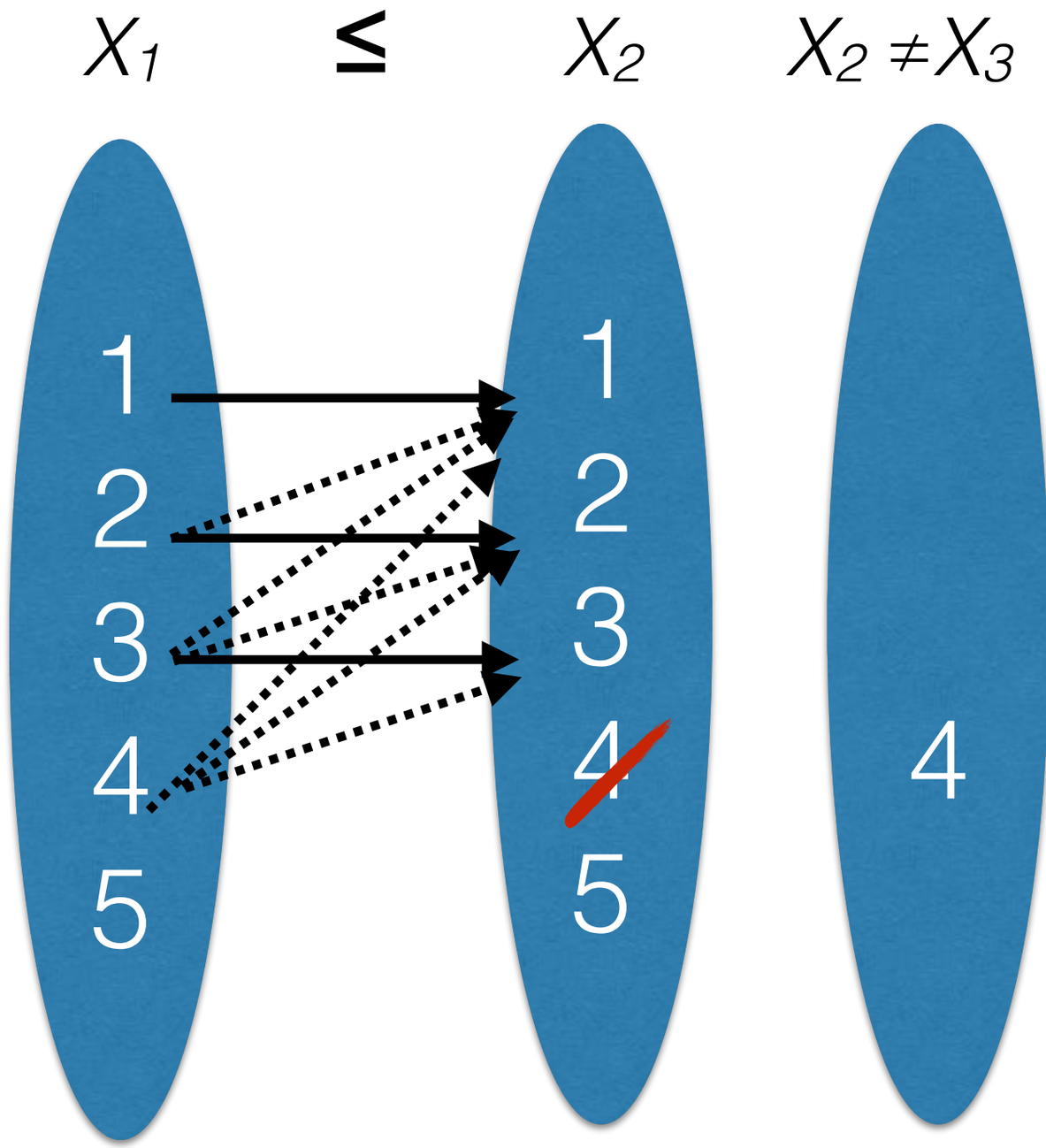

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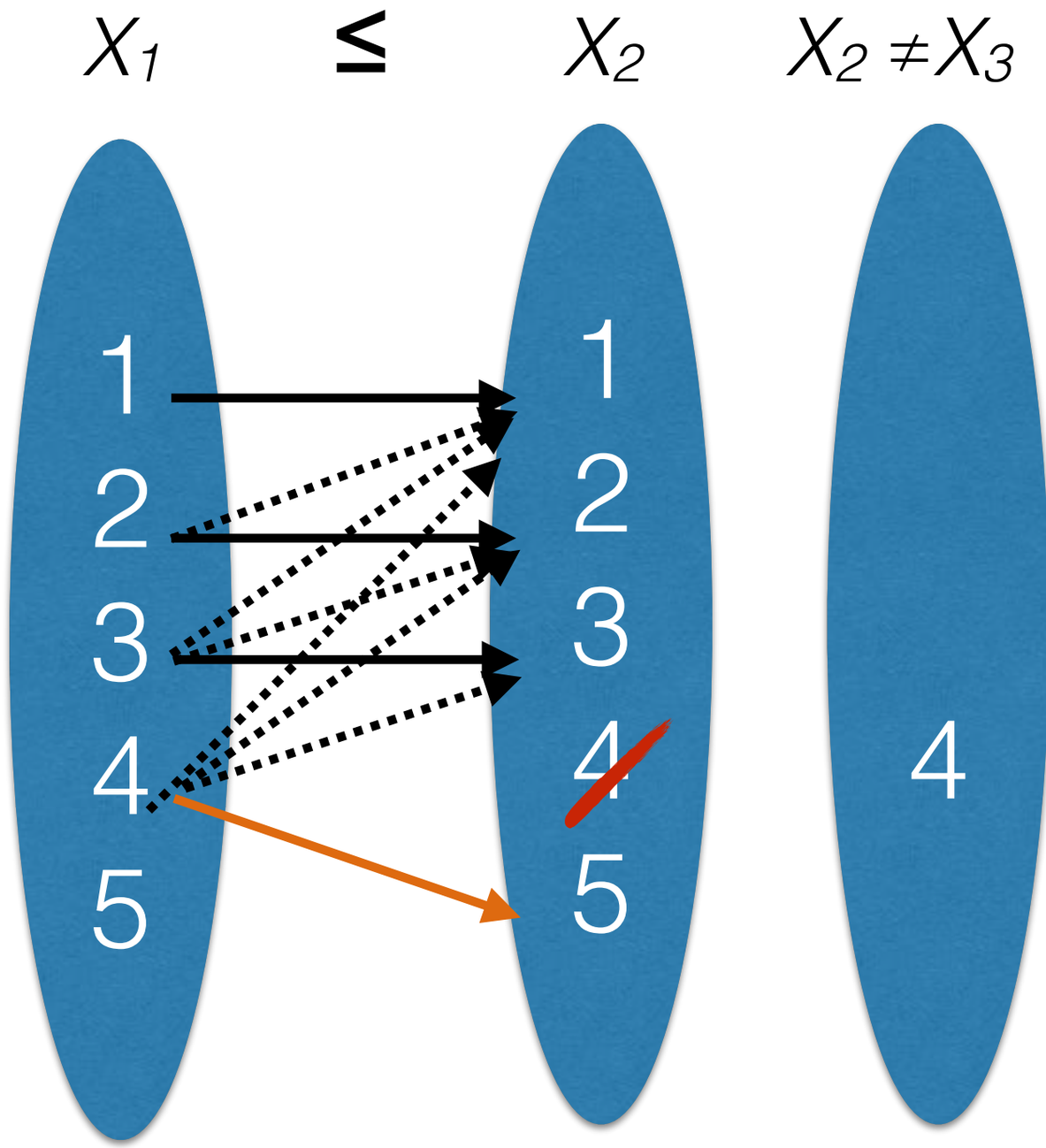

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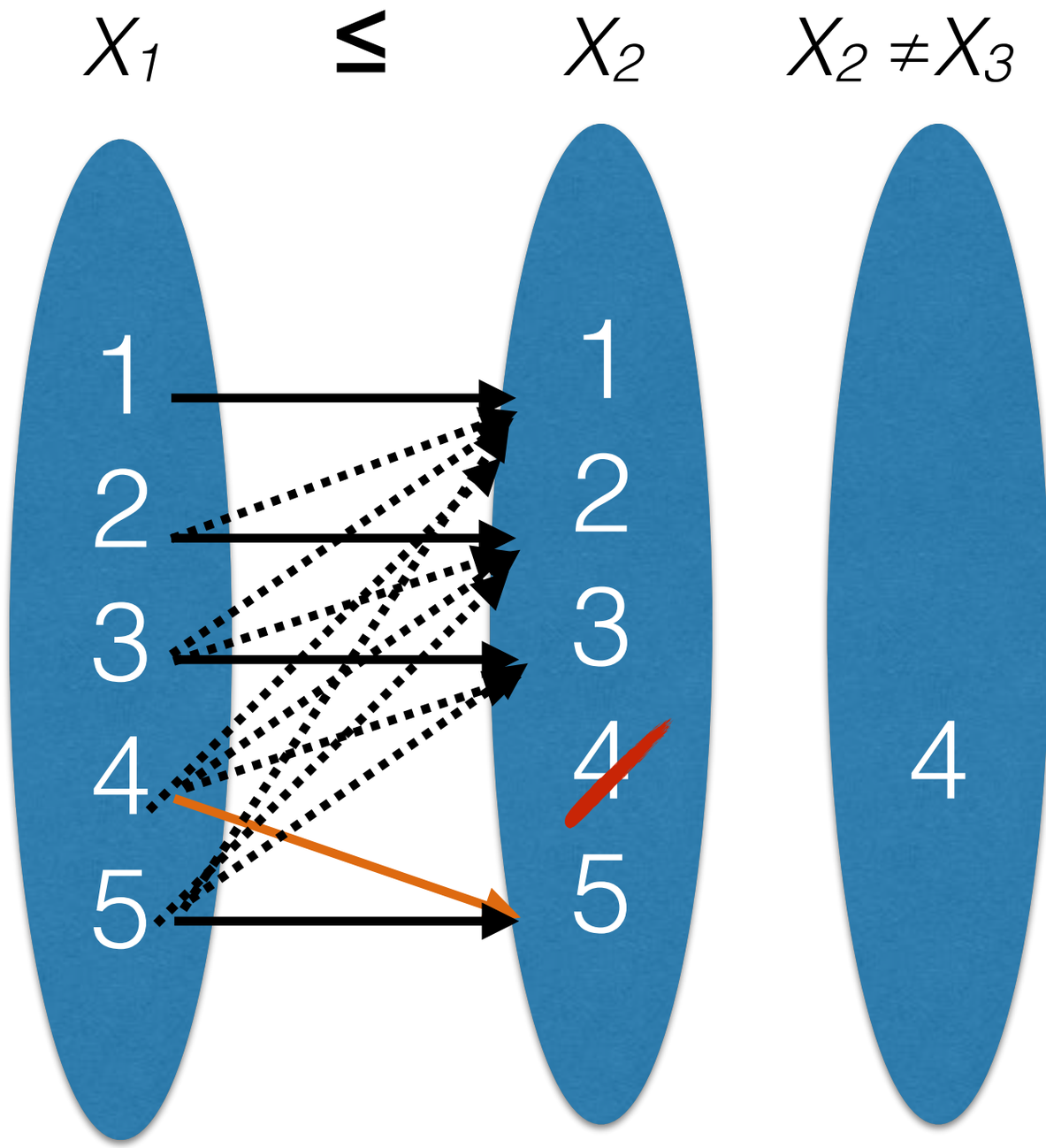

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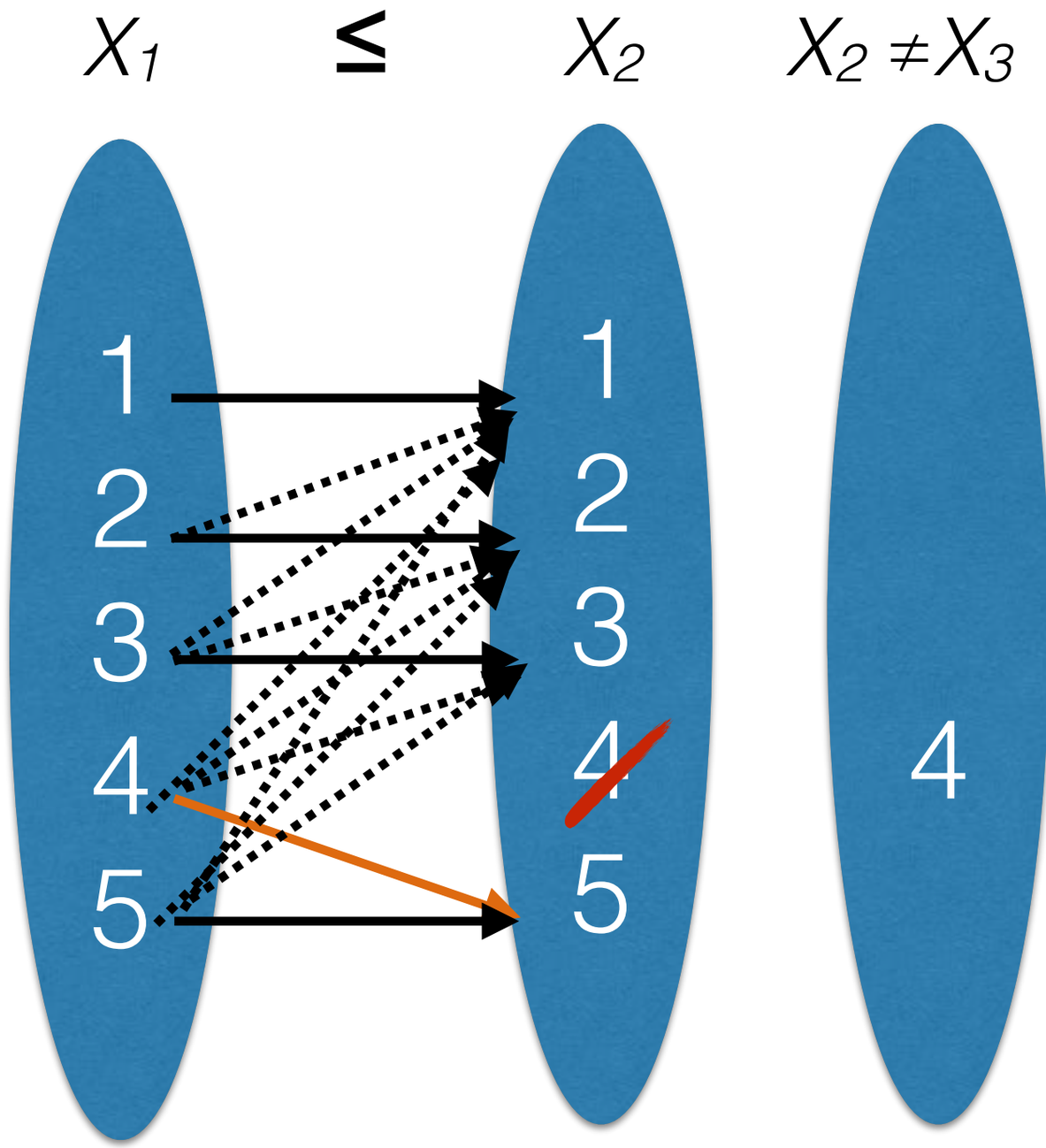
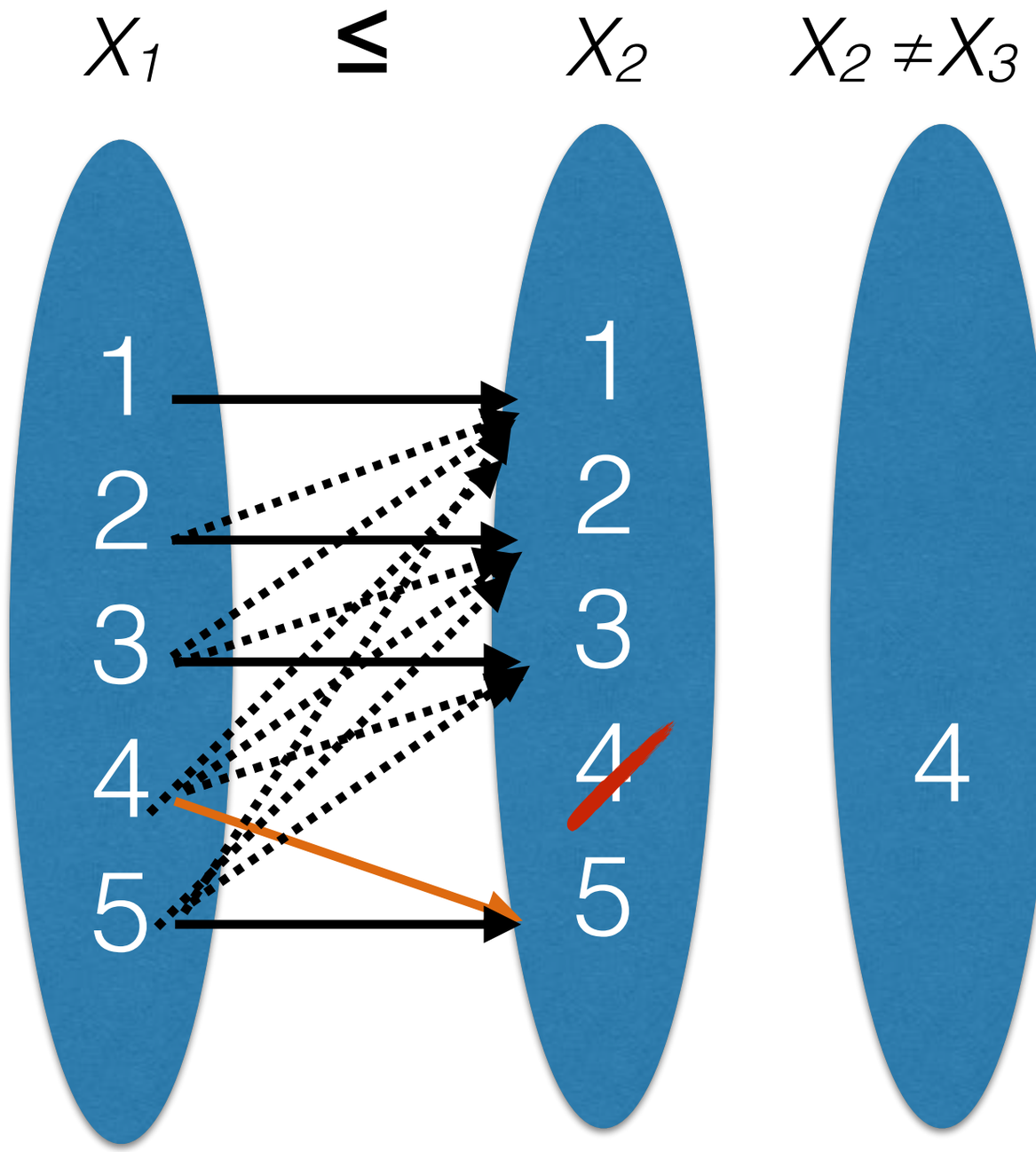
➡ **AC3 redoes what it already did**



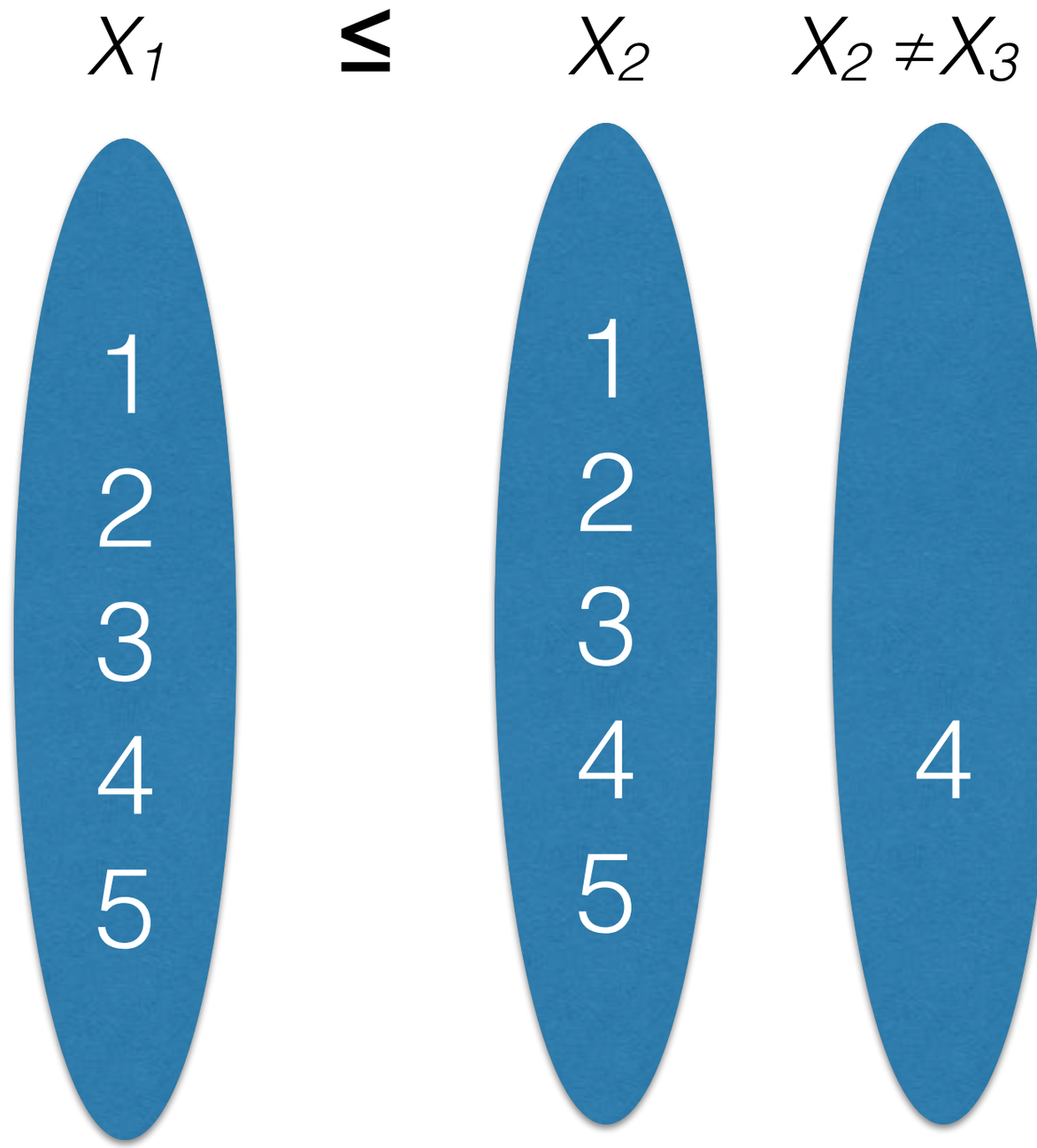
AC3

- We have to put X1 ≤X_2 in Q because of the removal of (X_2,4)
- AC3 forgot what it did before
- ➡ **AC3 redoes what it already did**
- AC3 is not optimal



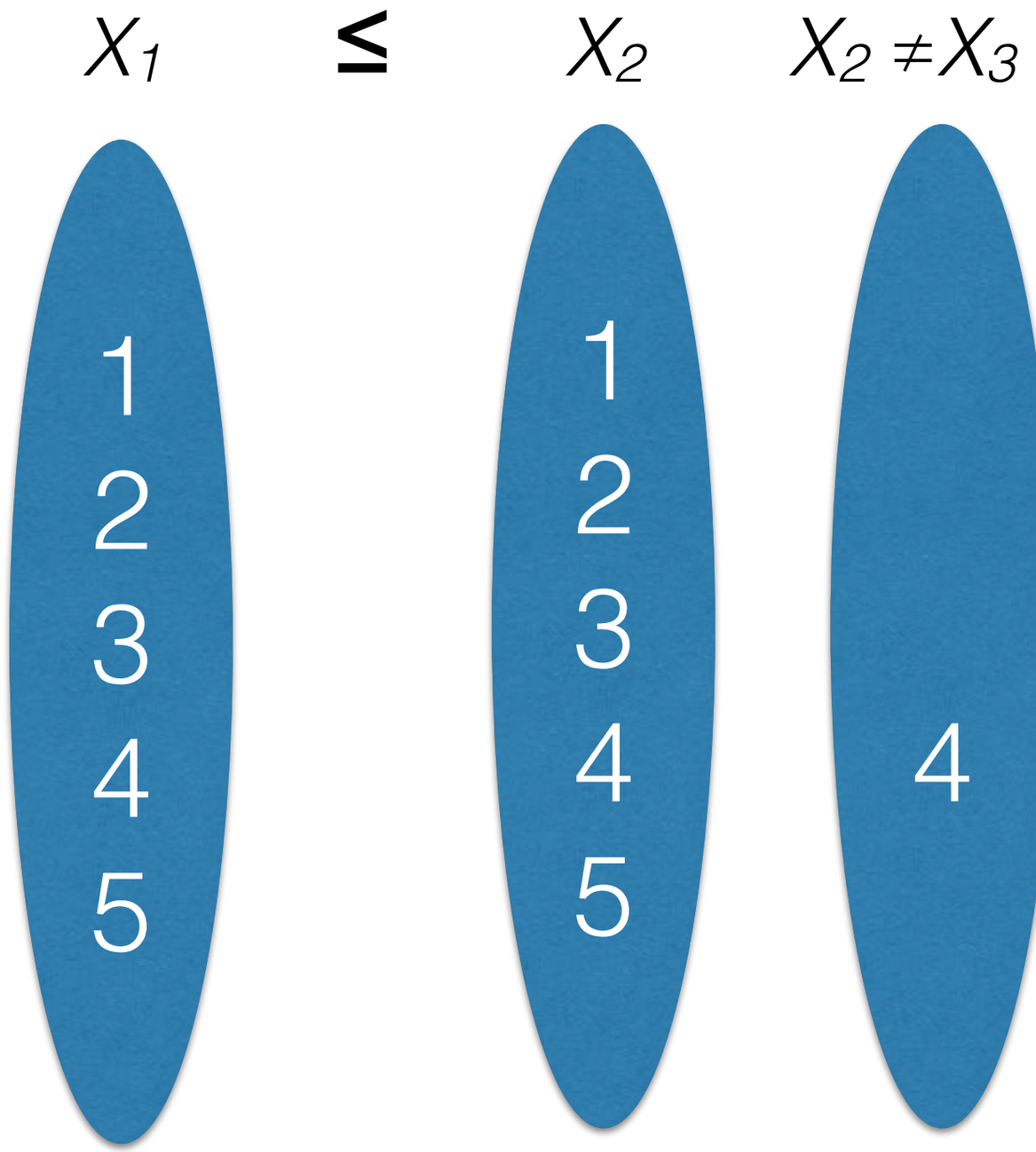
AC2001: an (easy) optimal algorithm

AC2001



AC2001

- At the first call, AC2001 does like AC3



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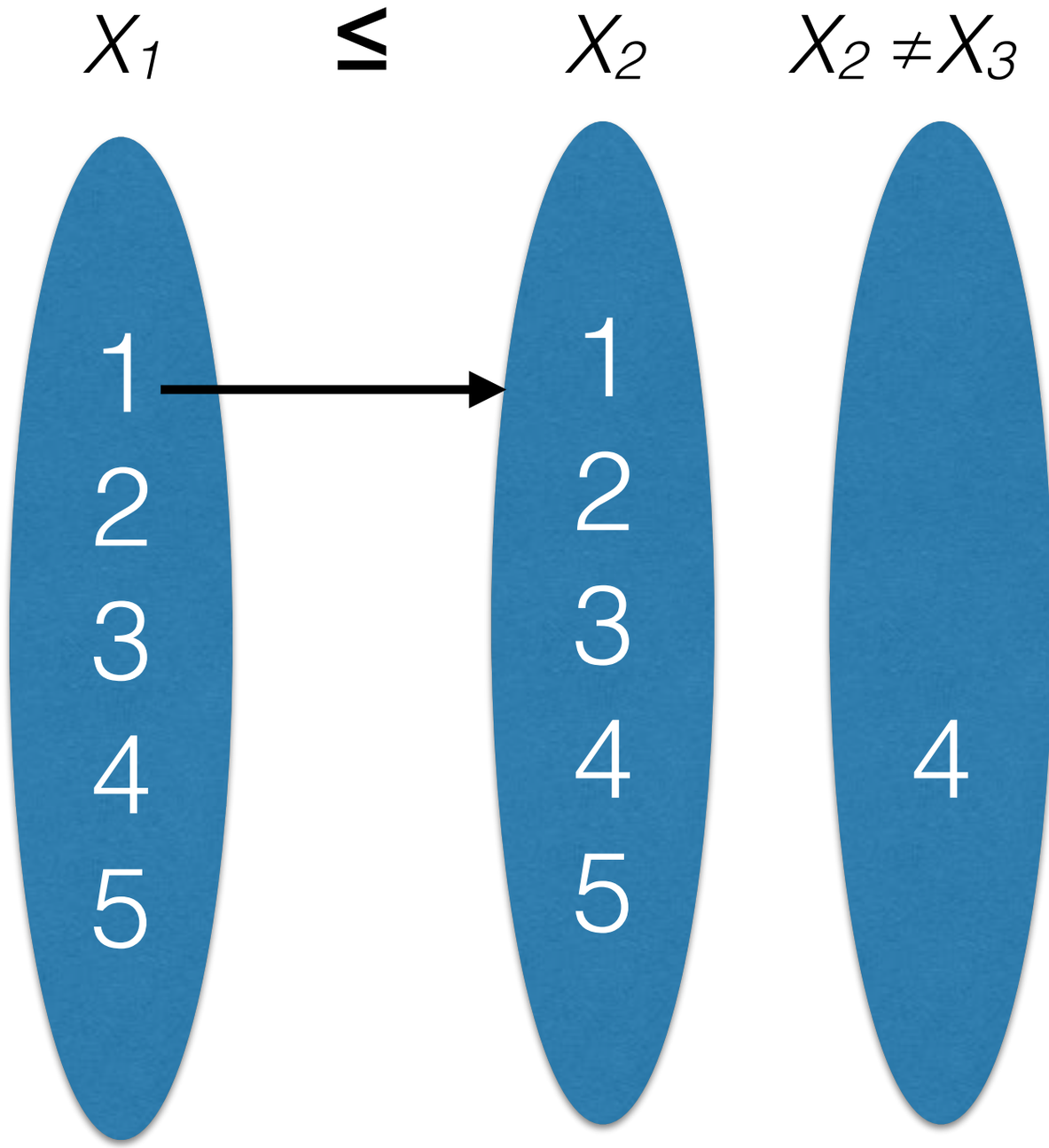
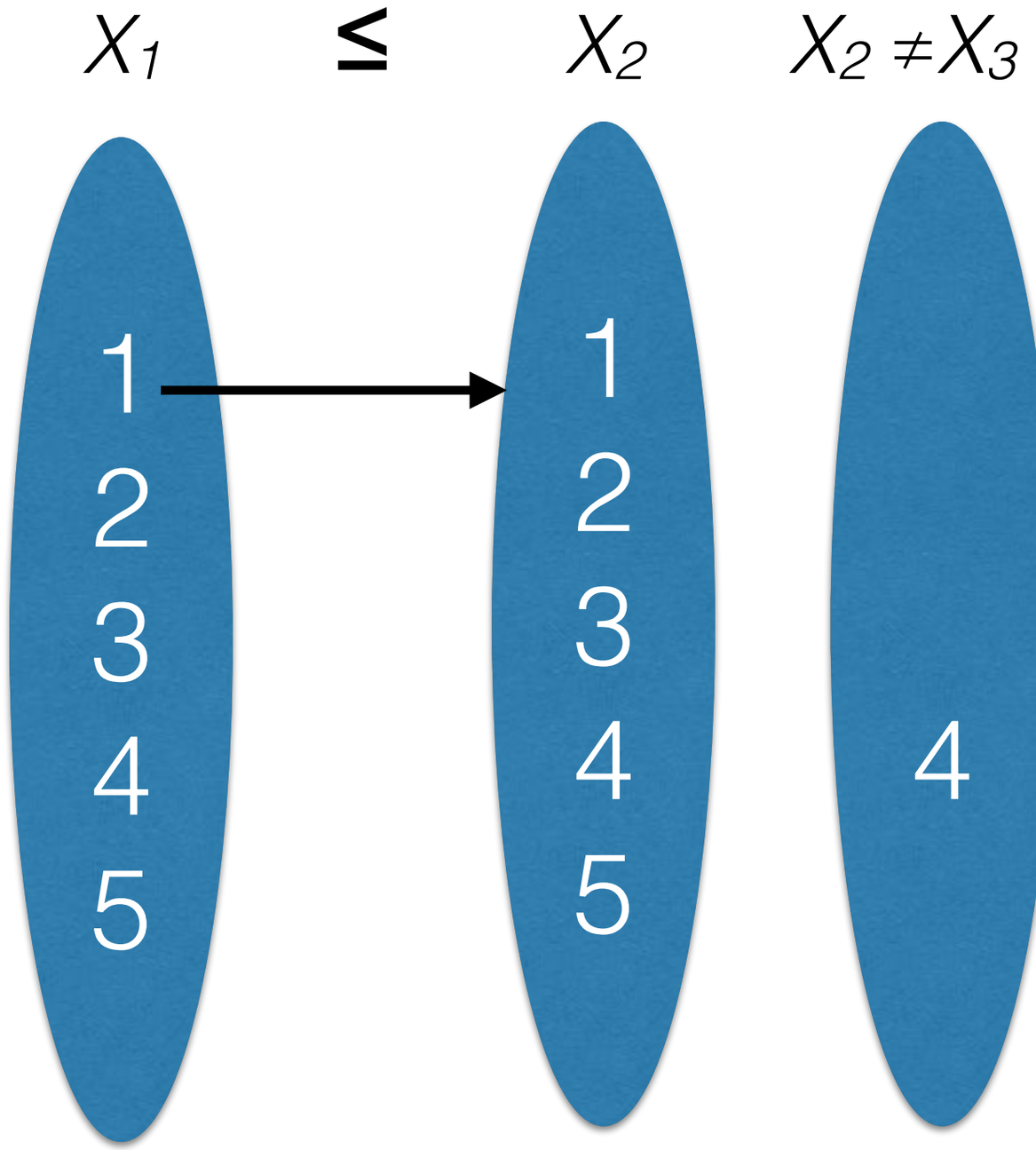


AC2001

- At the first call, AC2001 does like AC3
- **But** it stores the *last* support checked



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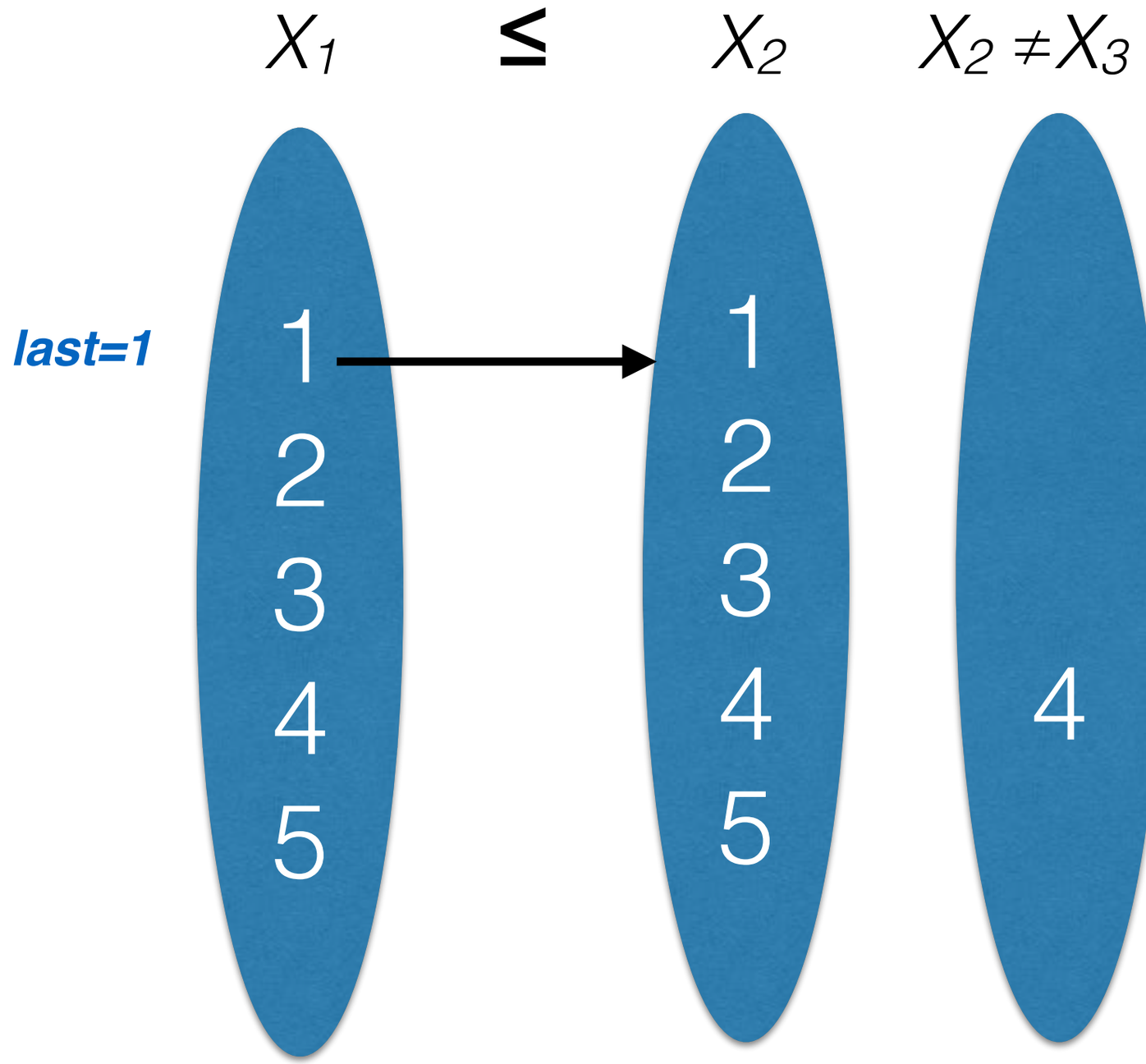

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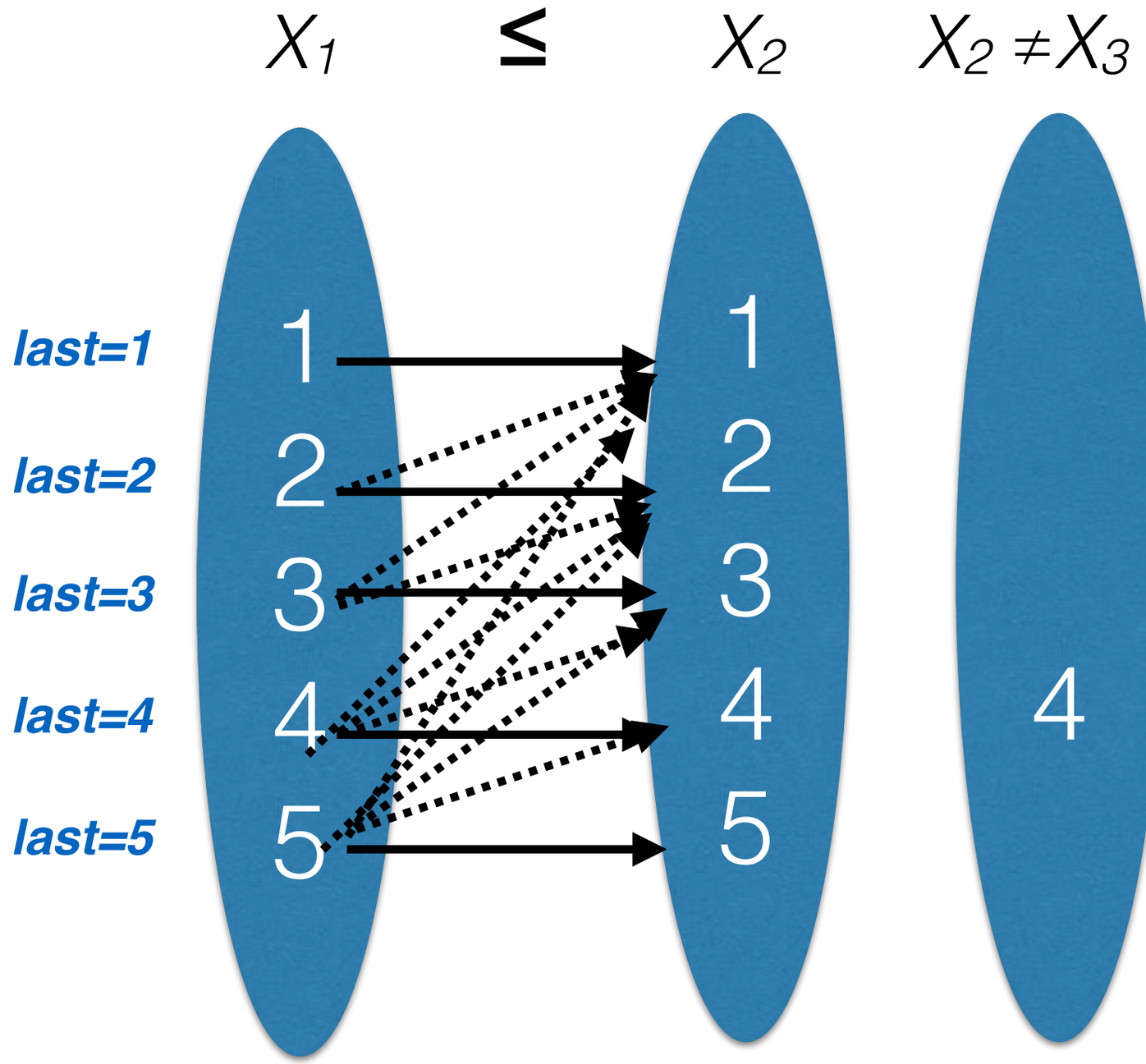

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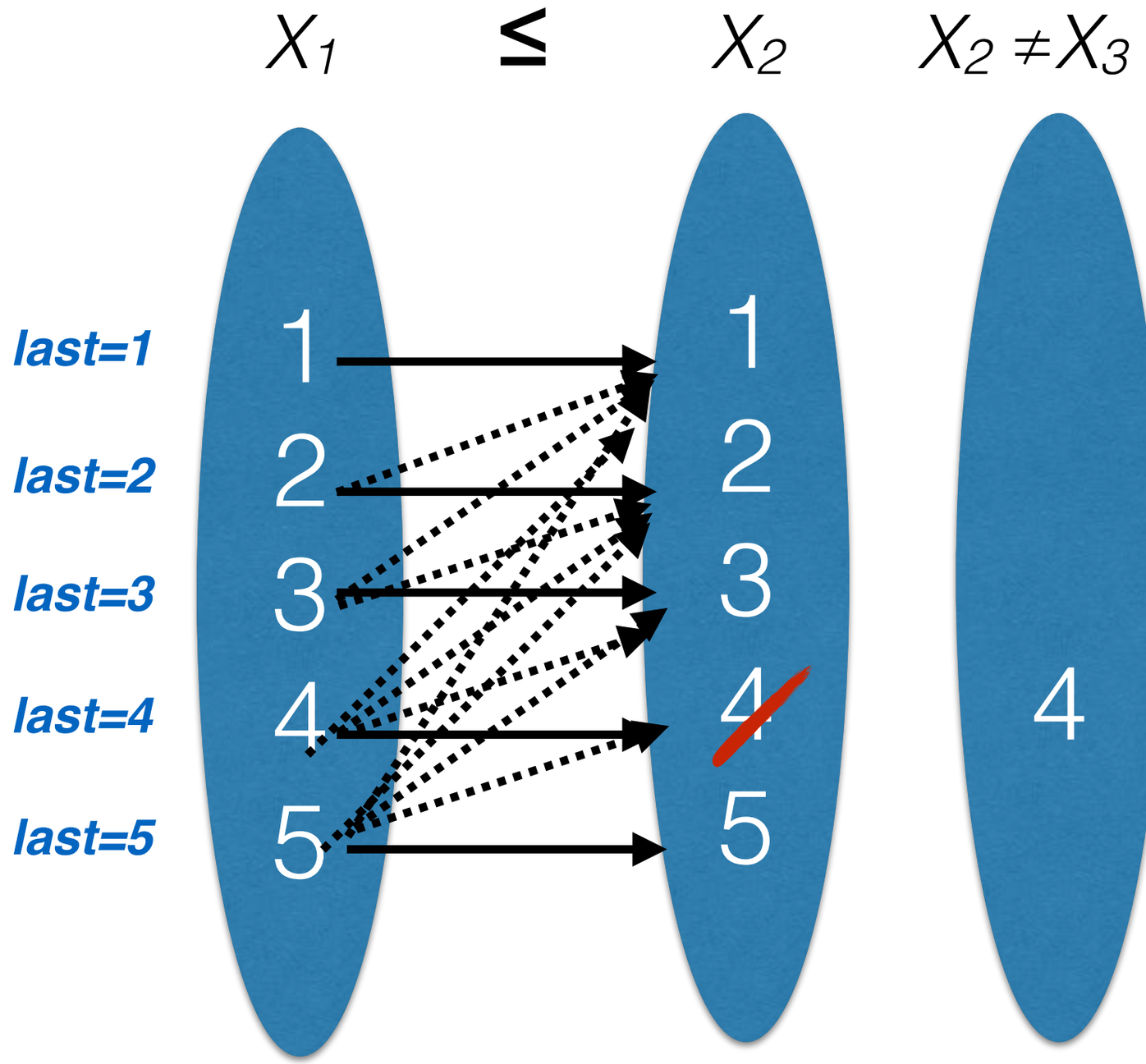

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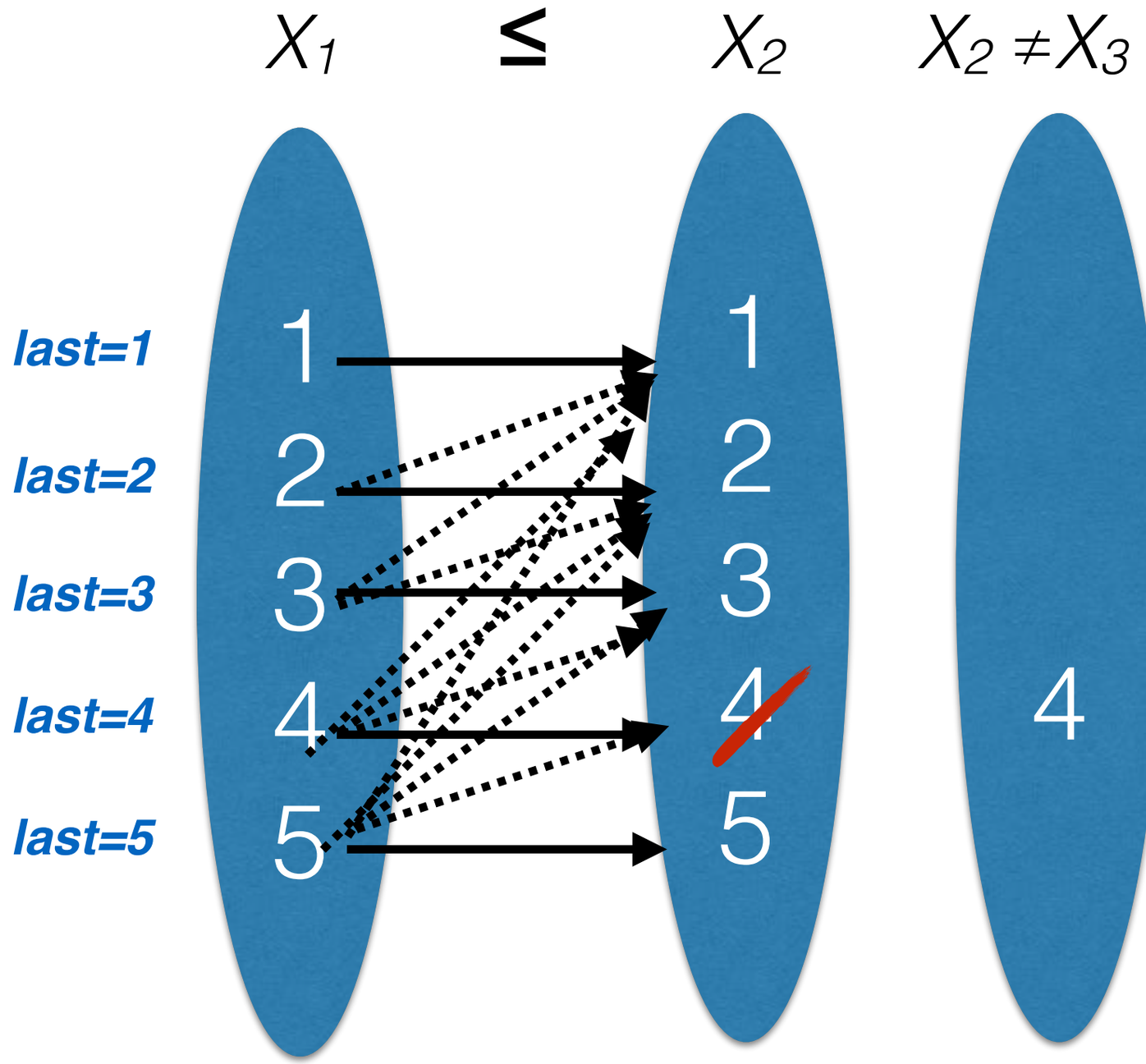
- When coming back on this constraint, it uses the Last



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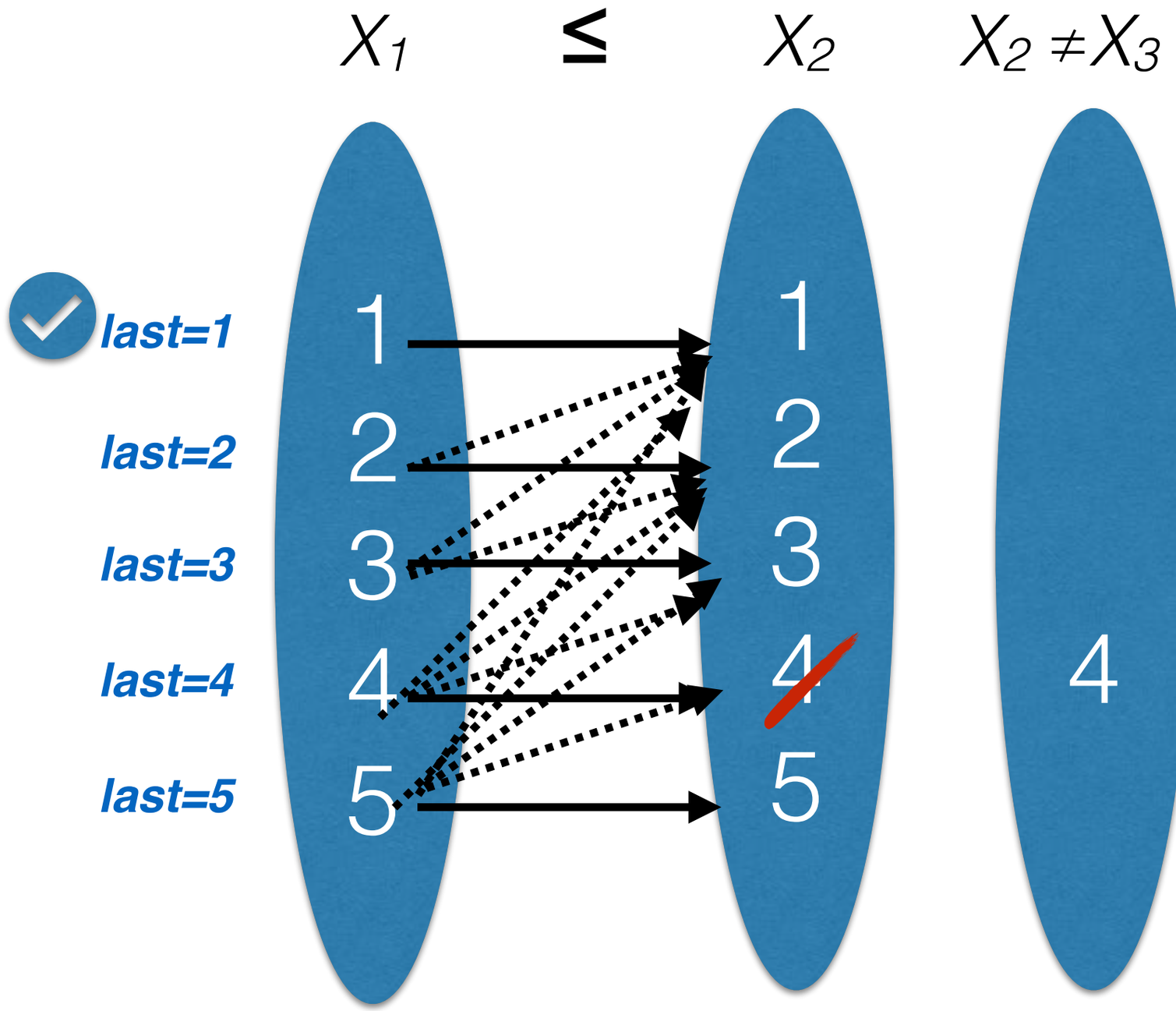

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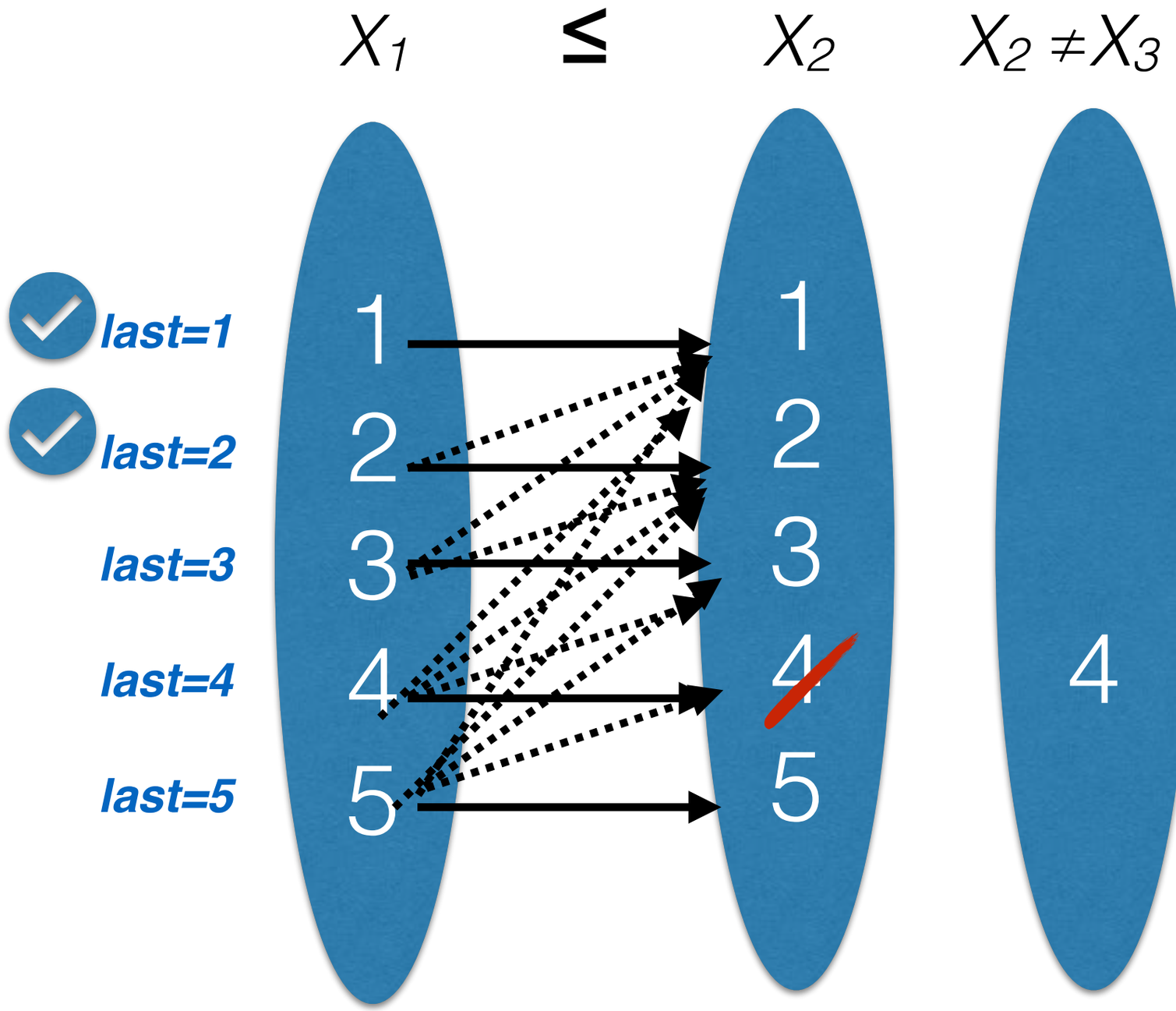

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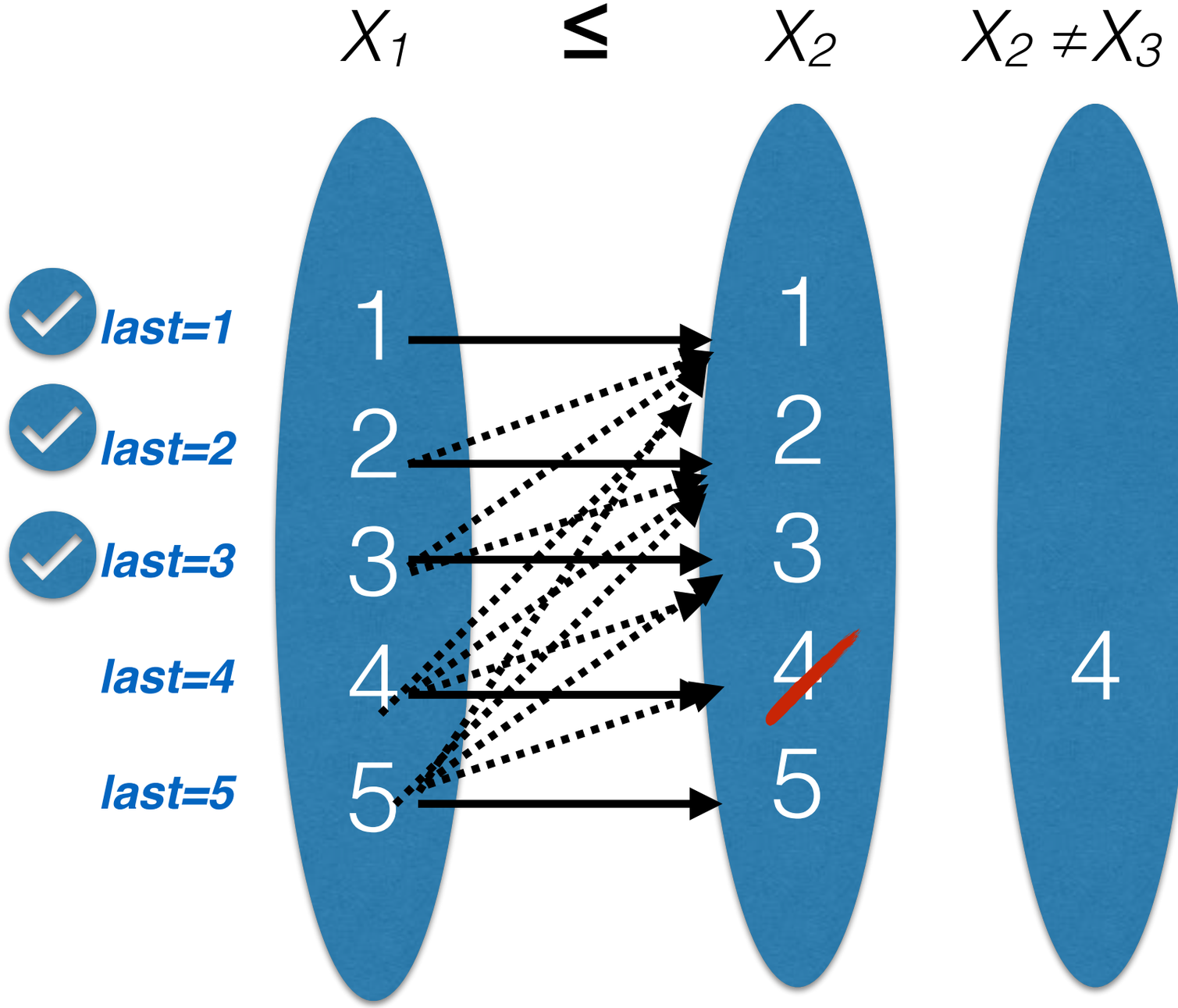

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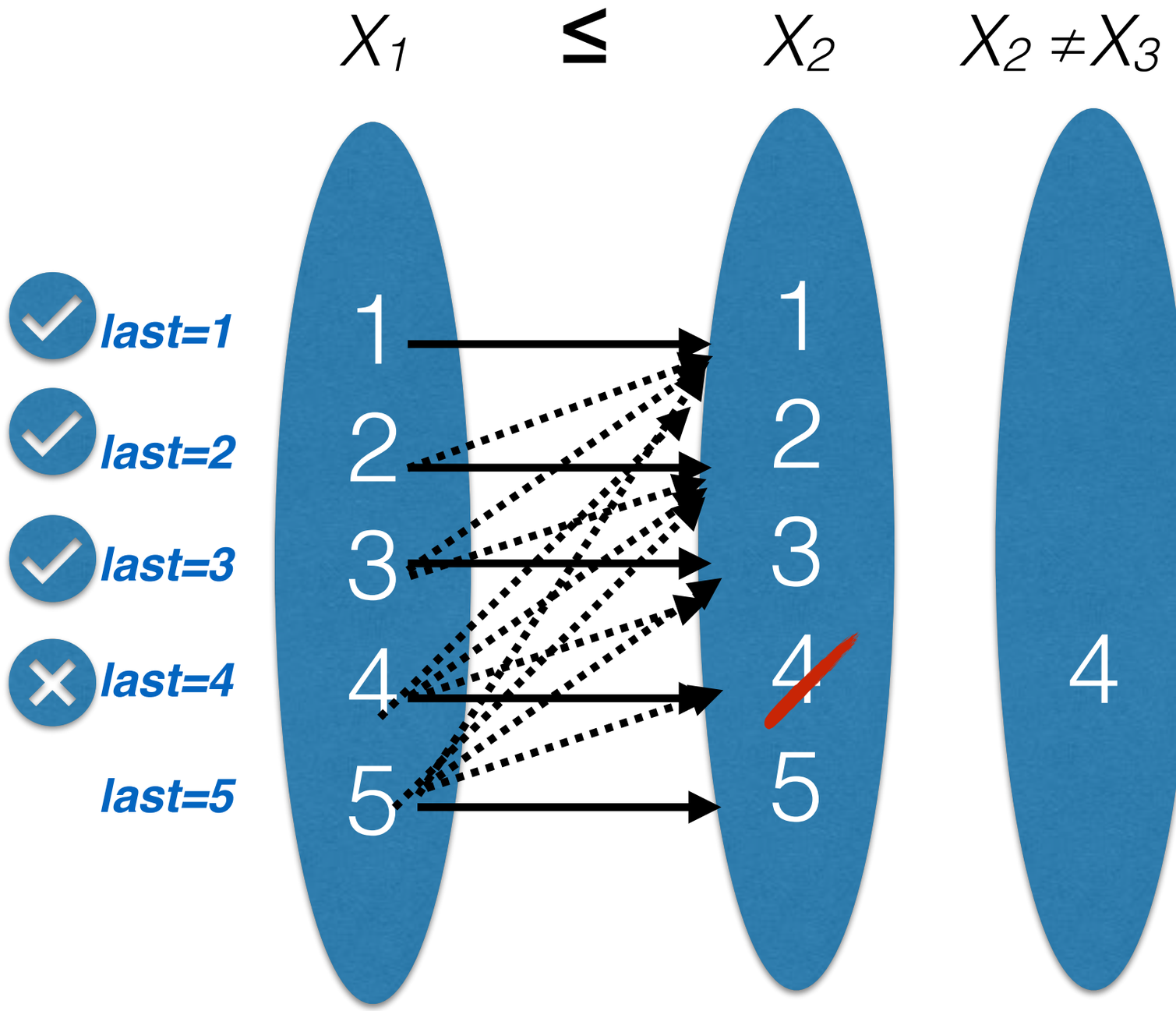

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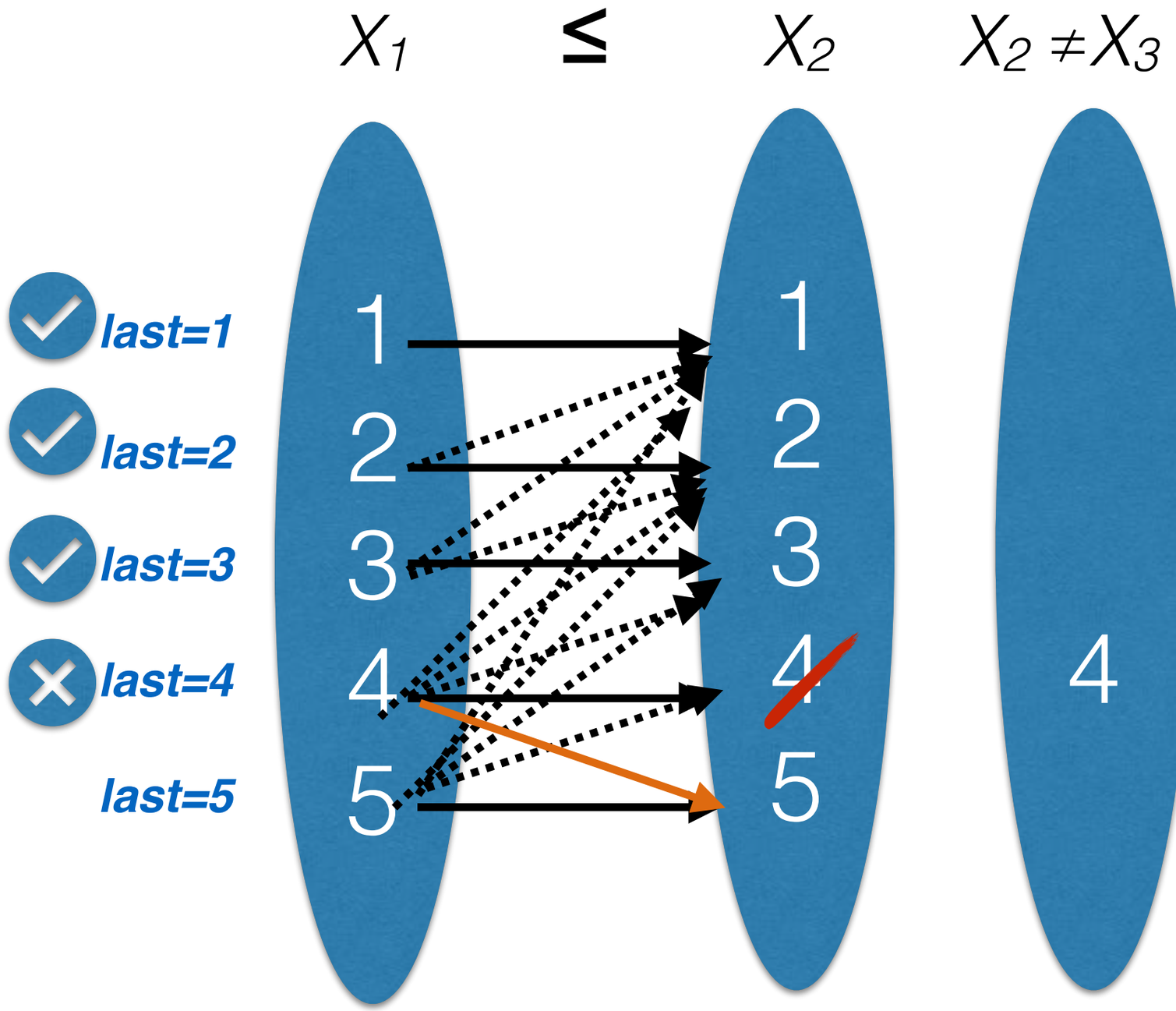

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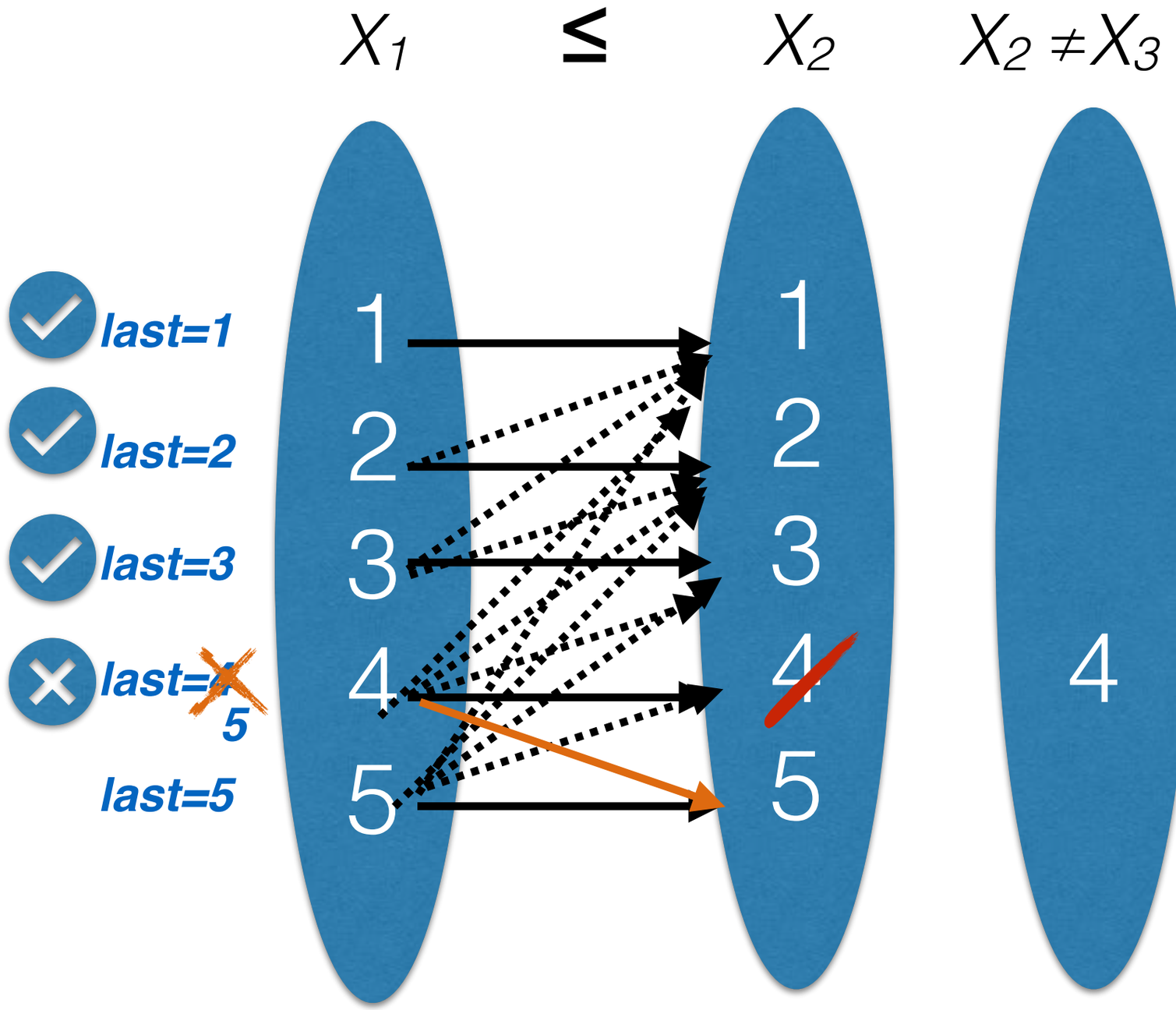

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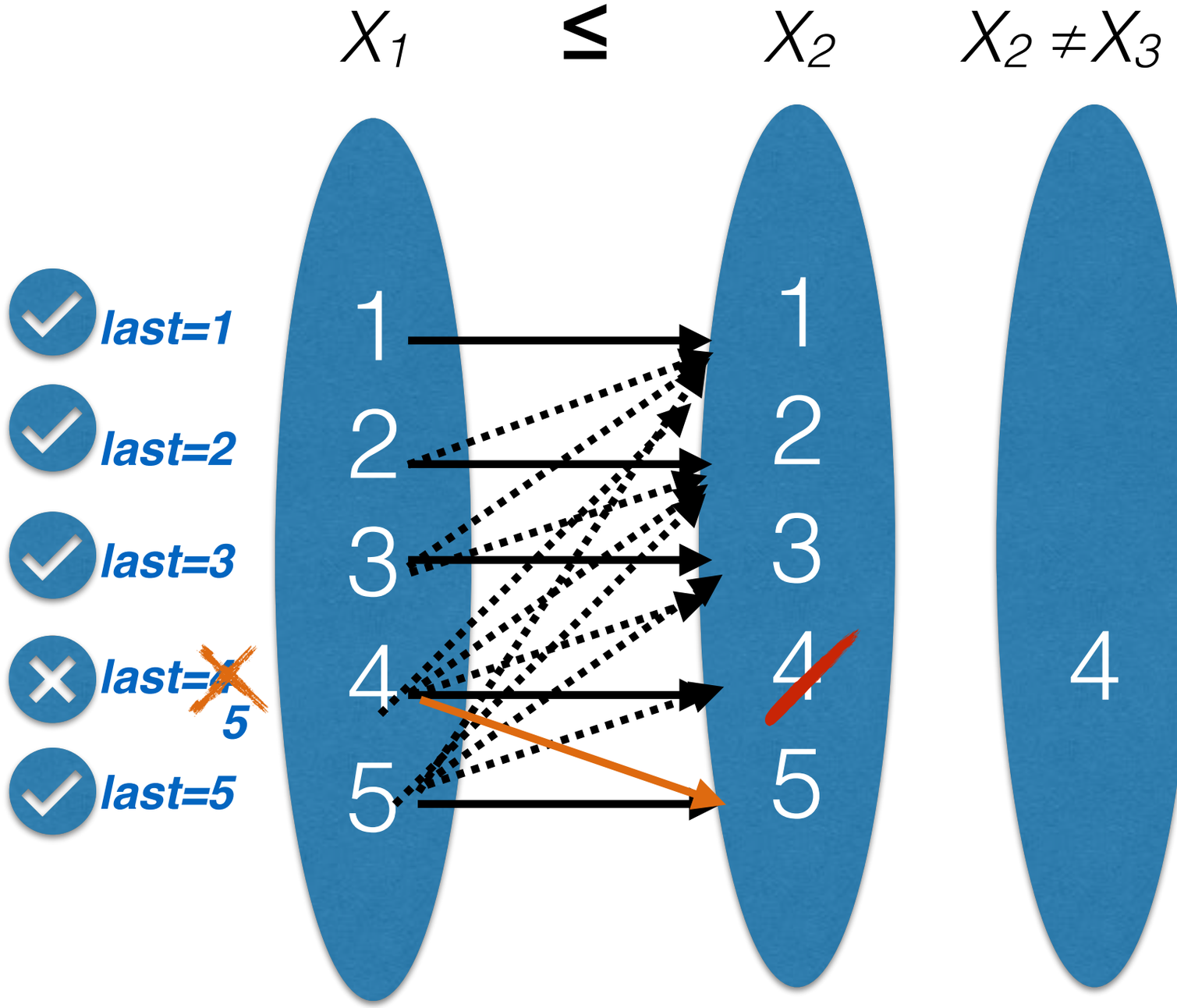

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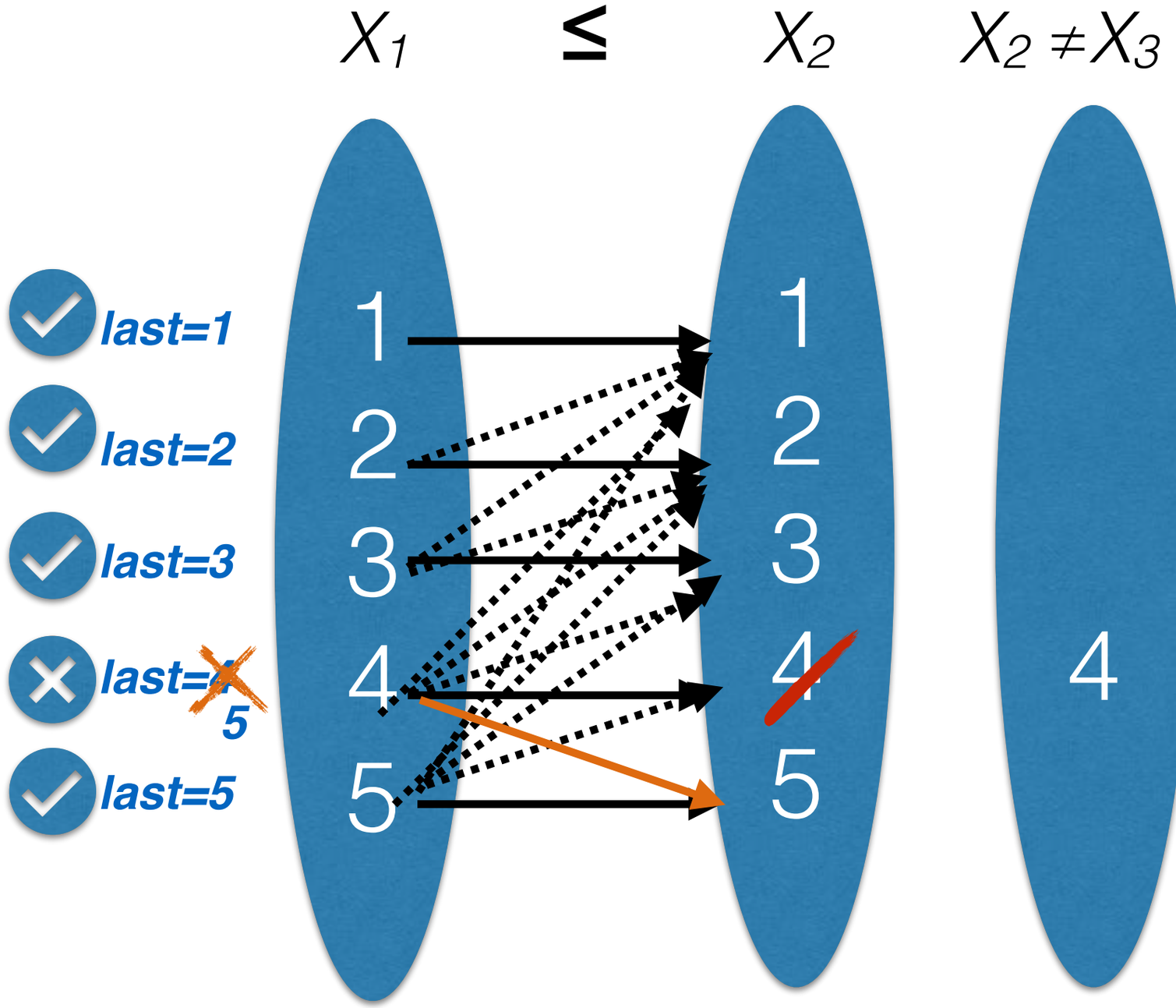
➡ **AC2001 only checks unknown pairs**



Revise of AC2001

Revise of AC2001

```
function Revise3(in X_i : variable ; c_ij : constraint) : Boolean
   begin
1      CHANGE ← false;
2      foreach v_i ∈ D(X_i) do
3          v_j ← first(D(X_j));
4          while (v_j ≠ nil) and ¬c_ij(v_i, v_j) do v_j ← next(v_j, D(X_j));
5          if v_j = nil then
6              remove v_i from D(X_i);
7              CHANGE ← true;
8      return CHANGE ;
   end
```

```
function Revise2001(in X_i : variable ; c_ij : constraint) : Boolean
   begin
1      CHANGE ← false;
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3          v_j ← next(Last(X_i, v_i, X_j), D(X_j));
4          while (v_j ≠ nil) and c(v_i, v_j) ∉ c_ij do v_j ← next(v_j, D(X_j));
5          if v_j ≠ nil then Last(X_i, v_i, X_j) ← v_j;
6          else
7              remove v_i from D(X_i);
8              CHANGE ← true;
9      return CHANGE ;
   end
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Revise of AC2001

```
function Revise3(in X_i : variable ; c_ij : constraint) : Boolean
   begin
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2      foreach v_i ∈ D(X_i) do
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Complexity

- AC3
  - time $O(ed^3)$ on binary, space $O(e)$
- AC2001
  - time $O(ed^2)$ on binary ($O(ed^k)$ for k-ary), **optimal** space $O(ed)$
- STR
  - time $O(T)$, space $O(T)$ (T = taille de la table)

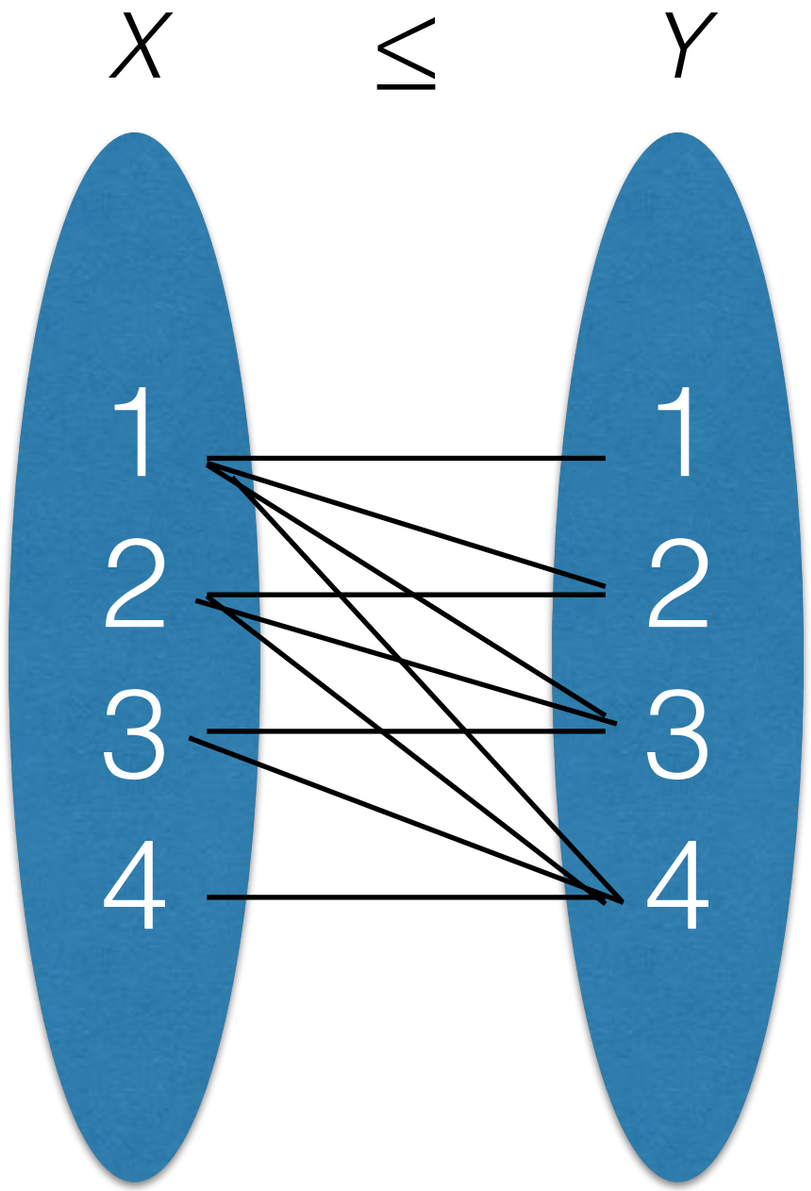
Arc consistency in solvers

- People who designed the first generation of solvers were not CP researchers

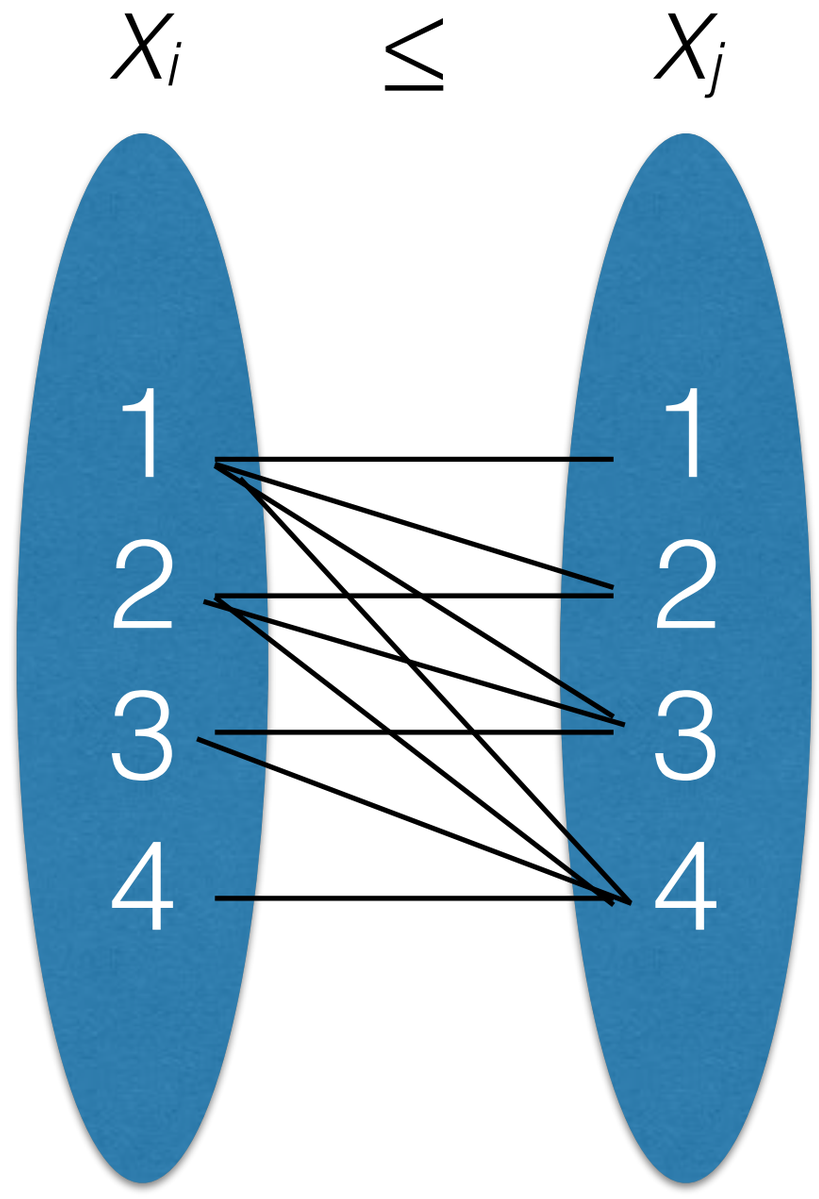
- They wanted efficient propagators for the constraints occurring often —> arithmetic

Removal events

- Each domain reduction is labeled with an ‘event’
  - ***remValue***(X_i): a value is removed from D(X_i)
  - ***incMin***(X_i): the smallest value is removed from D(X_i)
  - ***decMax***(X_i): the greatest value is removed from D(X_i)
  - ***instantiate***(X_i): D(X_i) is reduced to a singleton



Removal events



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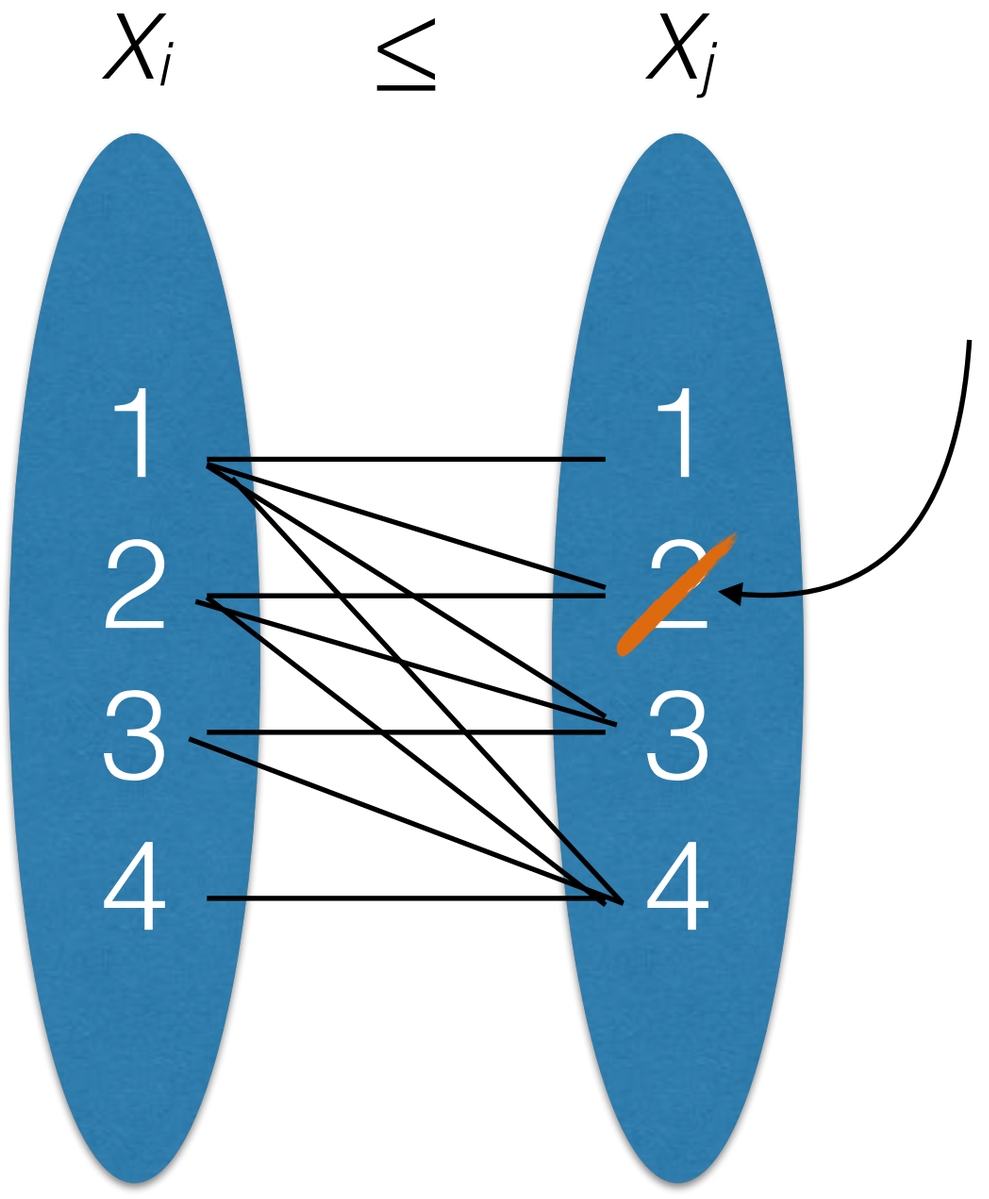
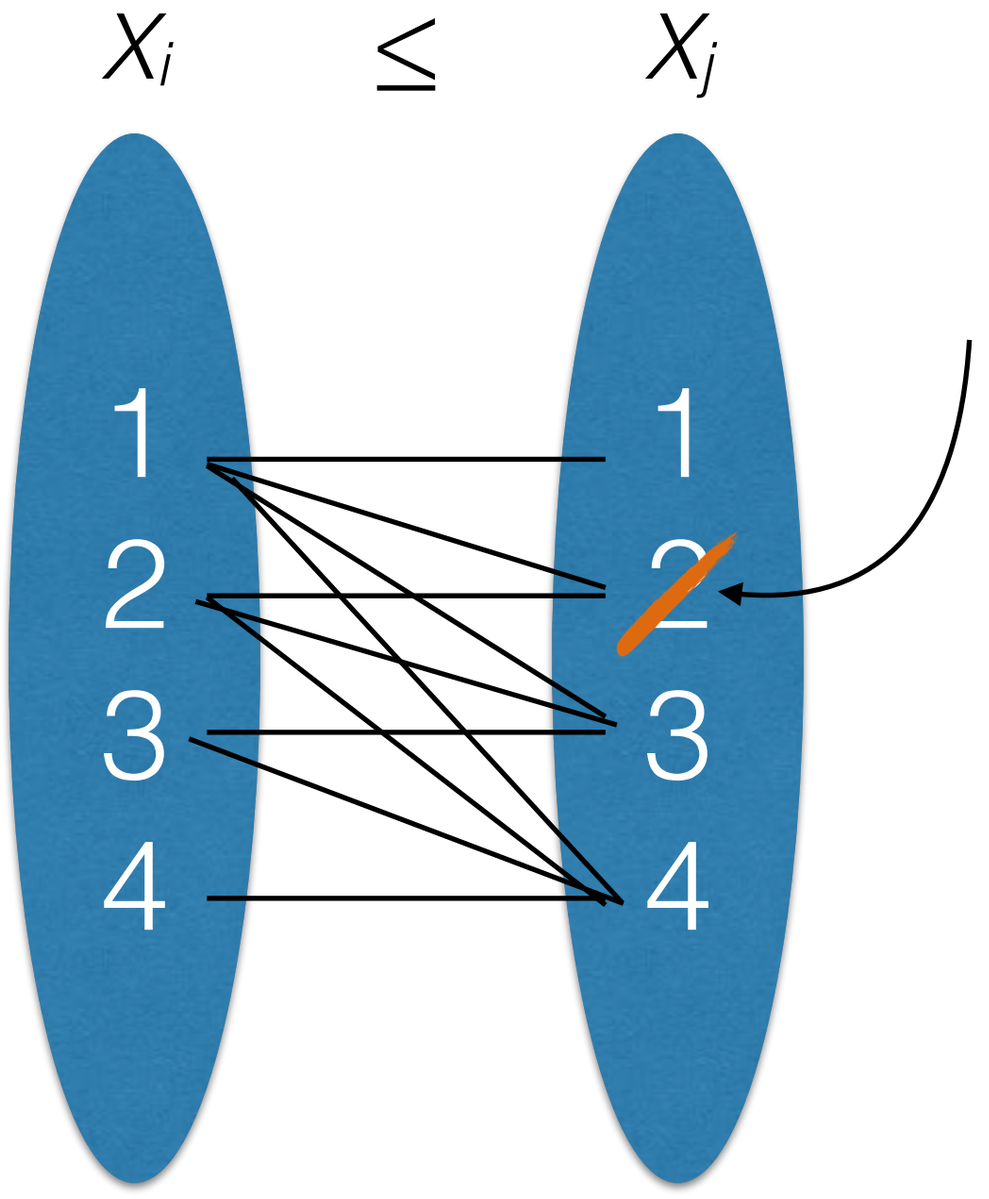


Removal events

- Removing (X_j,*2*) has no impact on X_i



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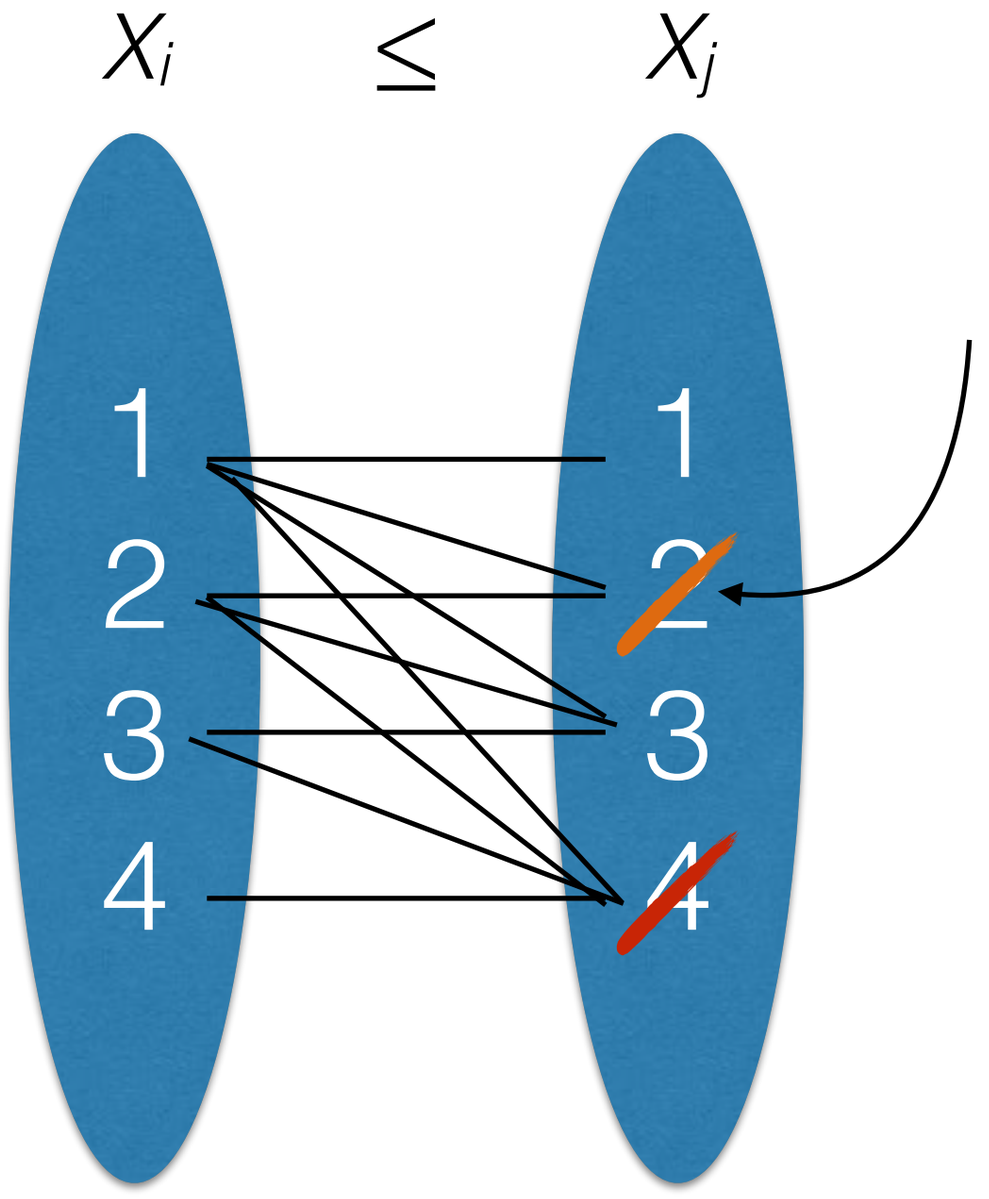

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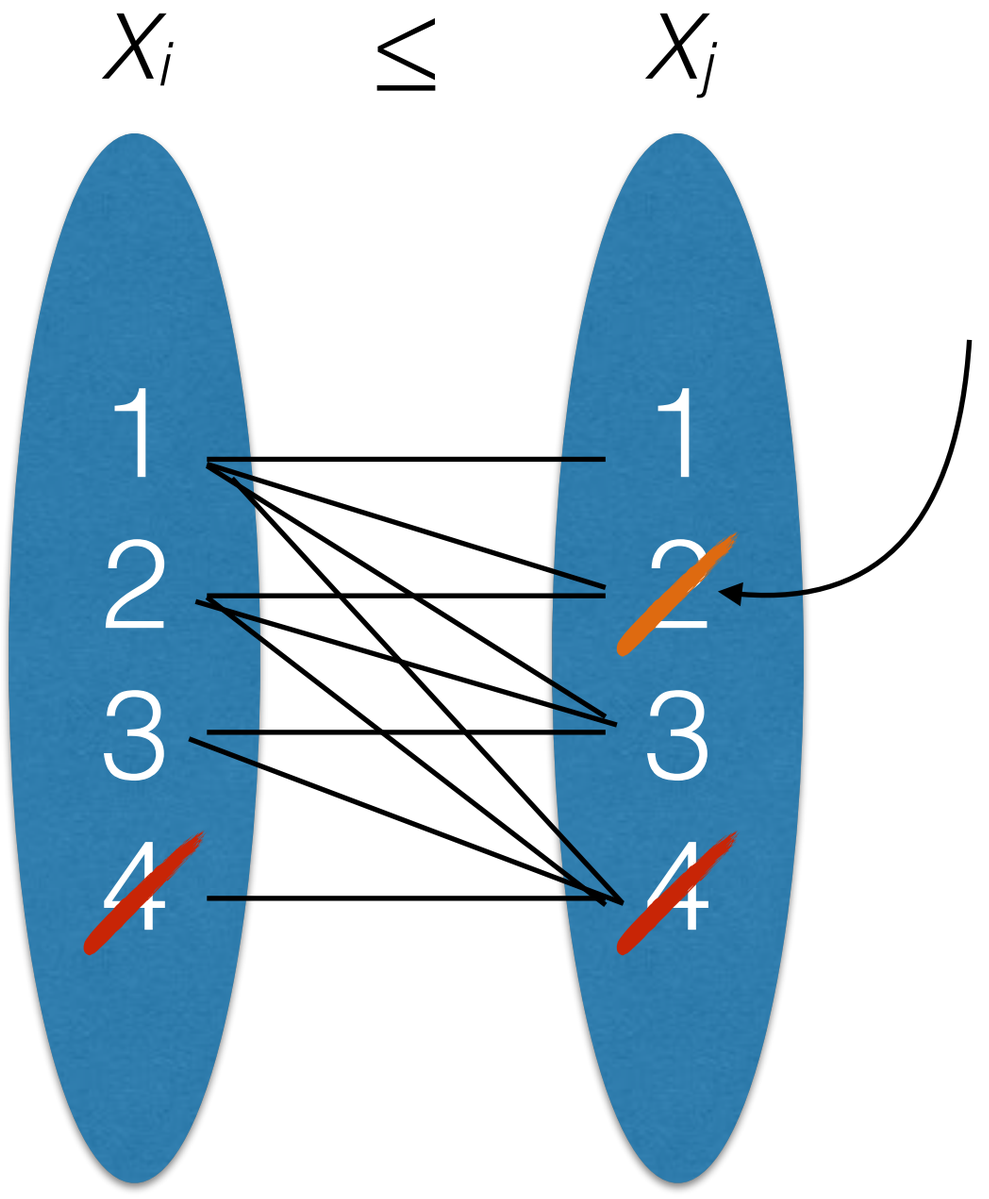

Removal events

- Removing (X_j,*2*) has no impact on X_i
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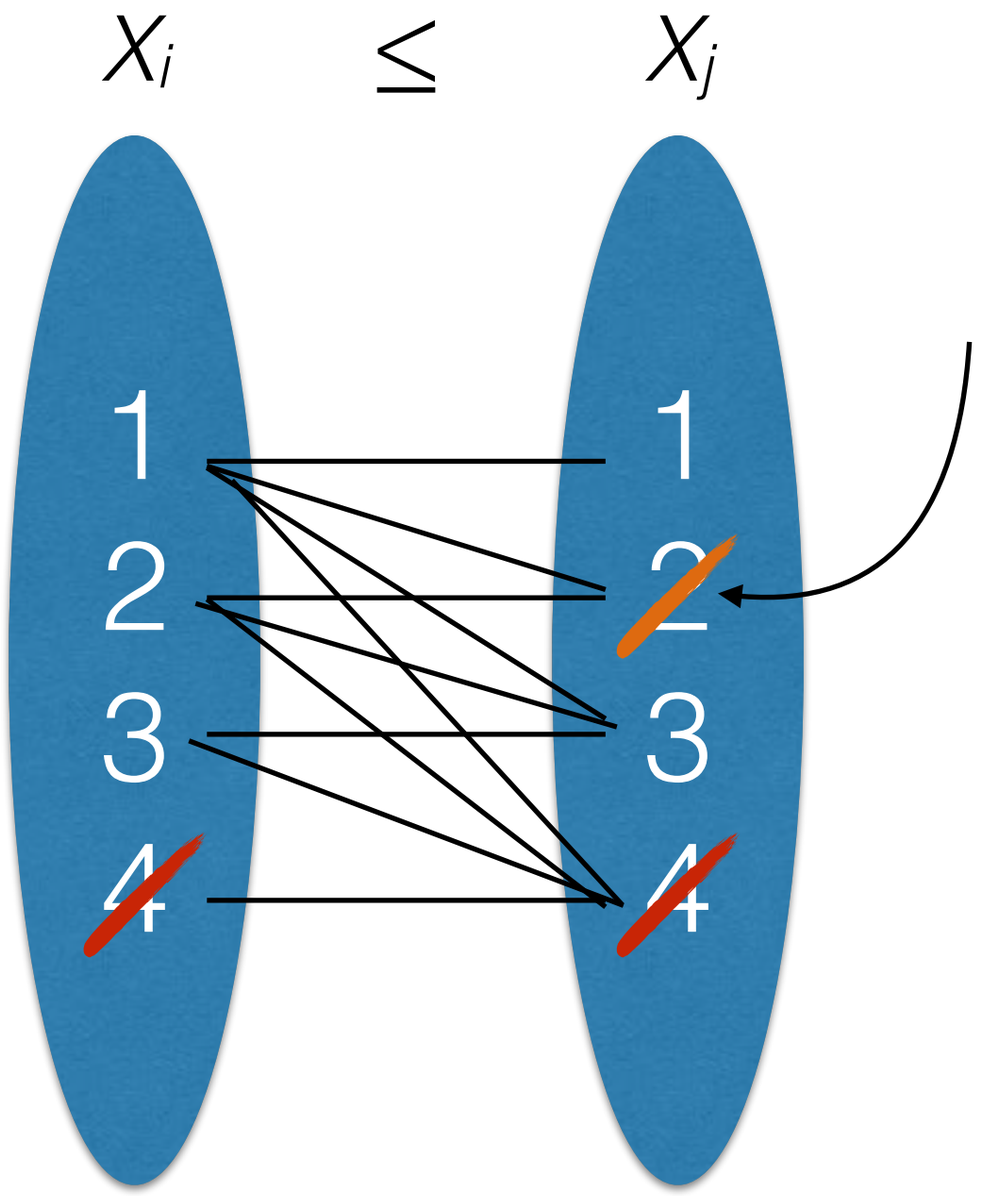

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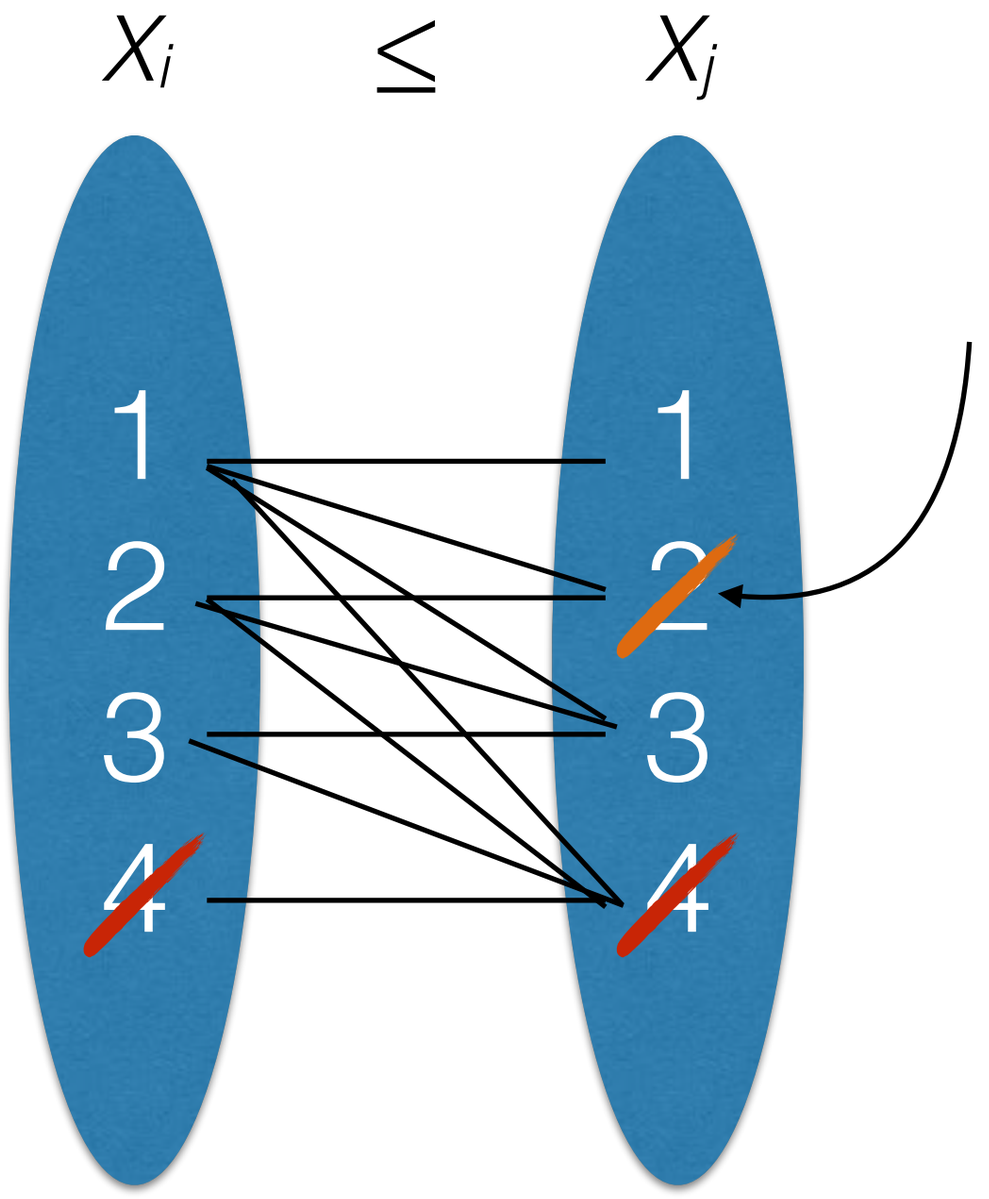

function `Revise`$(X_i, X_i \leq X_j)$:**Boolean**
  begin
    if $decmax(X_j)$ **then**
      remove $v \in D(X_i) \mid v > max(X_j)$
  end

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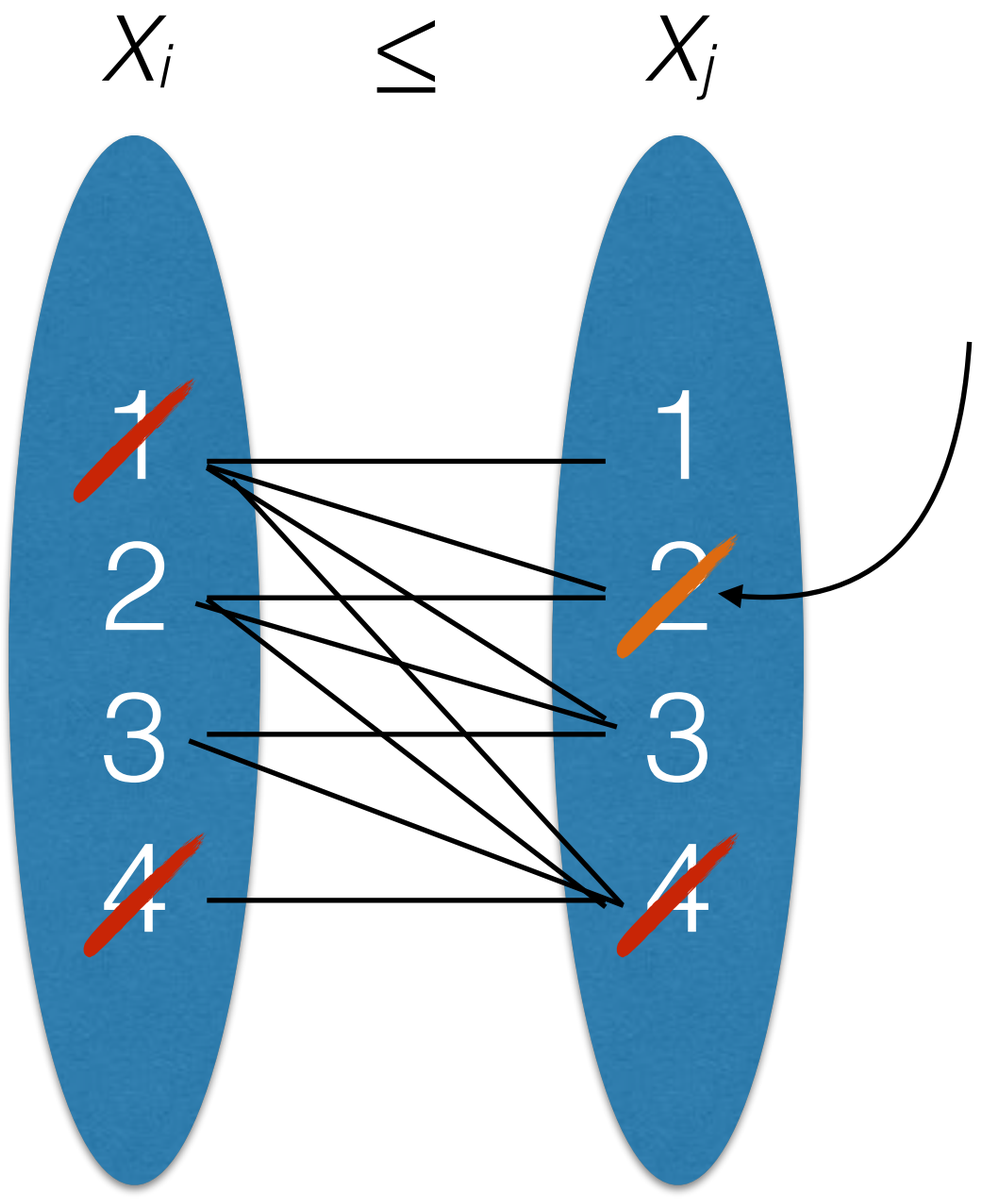

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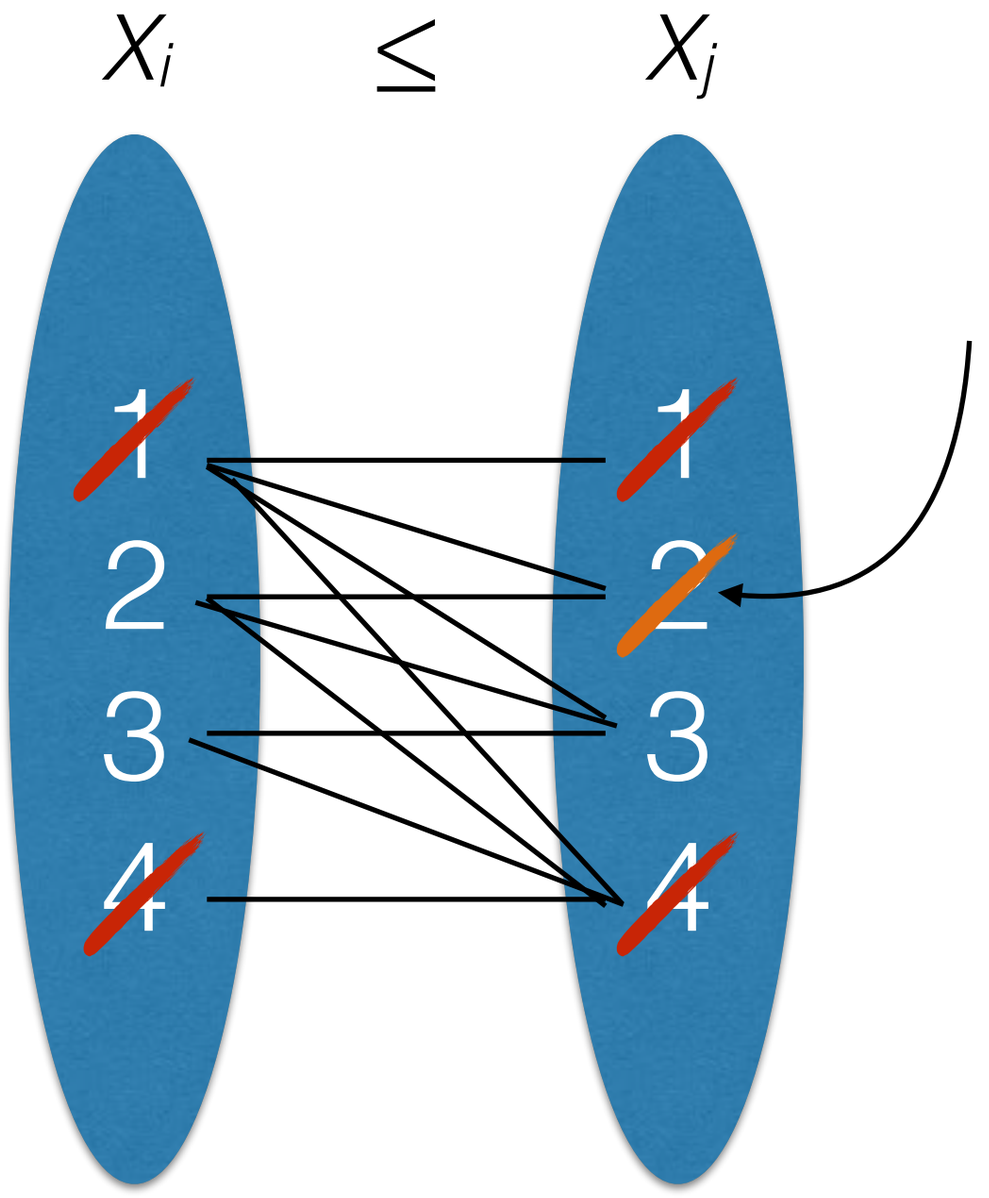

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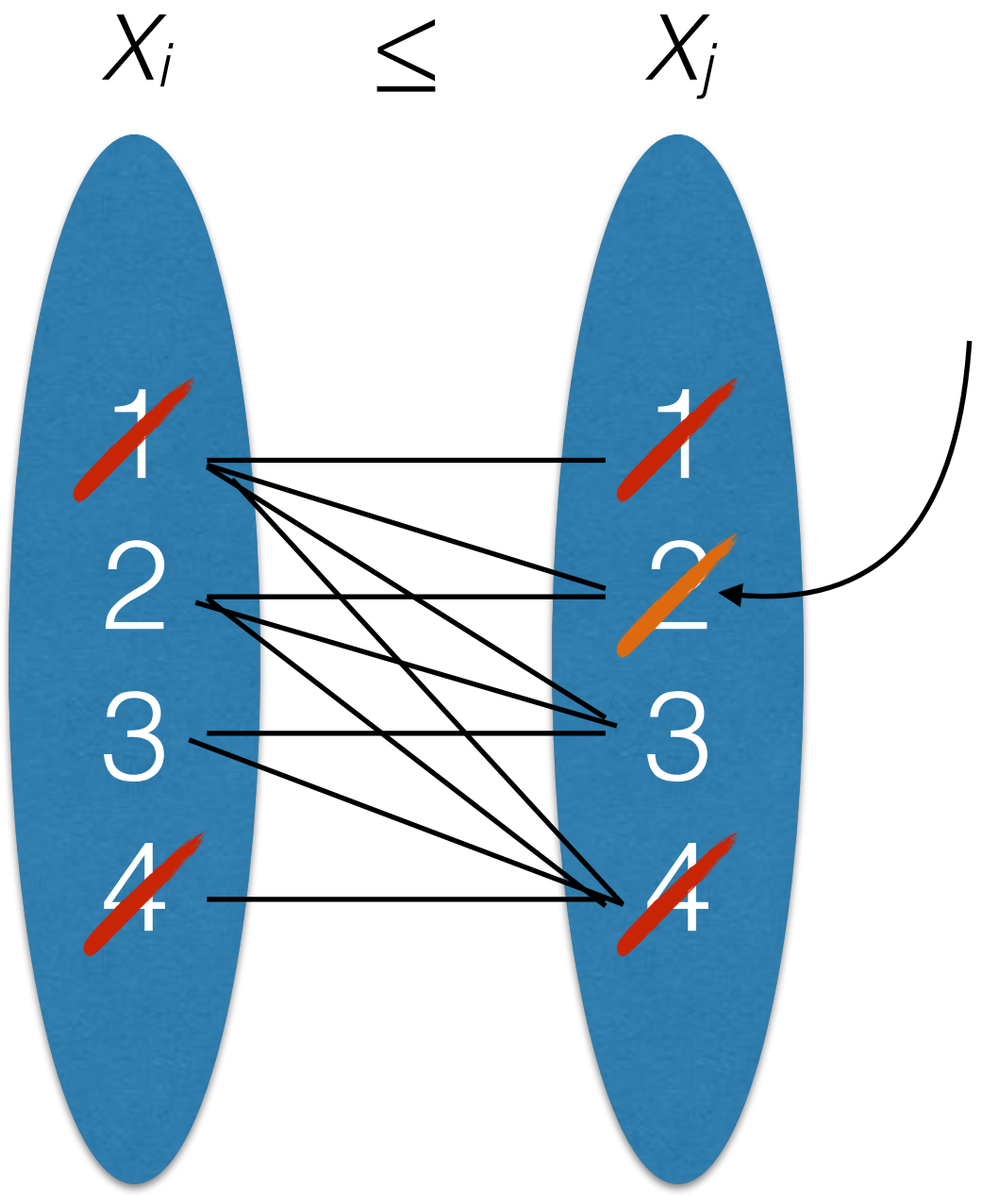

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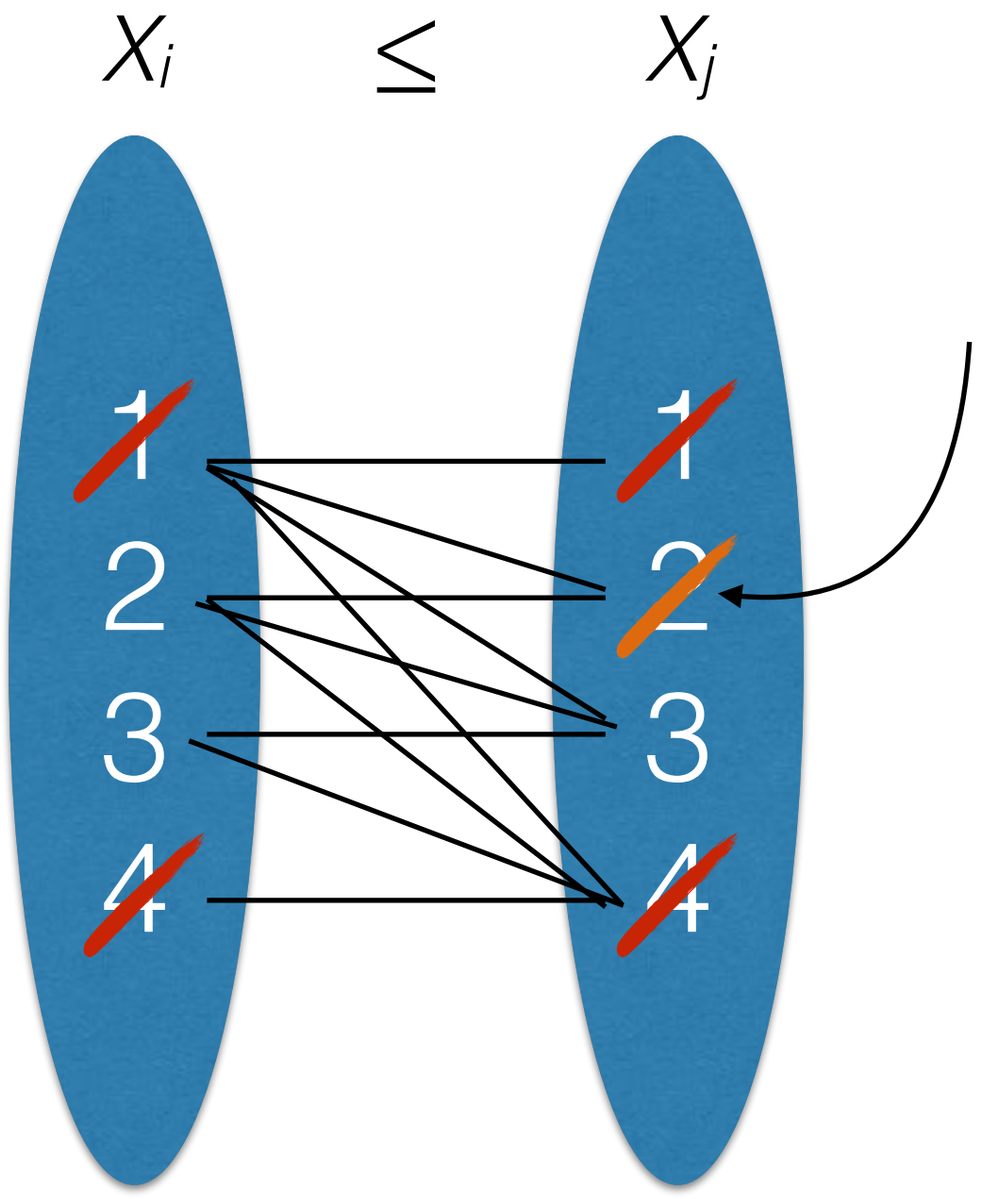

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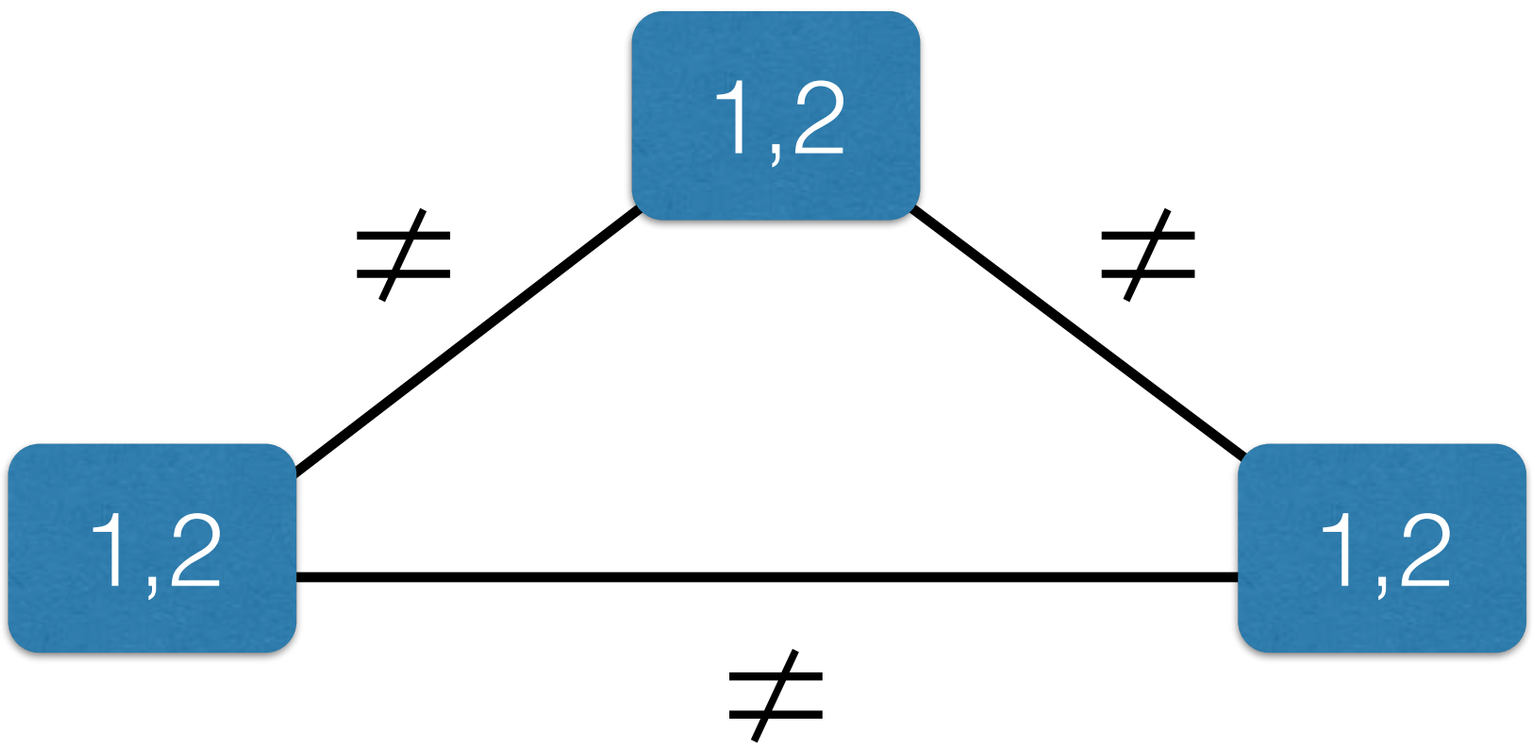
function `Revise`$(X_j, X_i \leq X_j)$:**Boolean**
  begin
    if $incmin(X_i)$ **then**
      remove $v \in D(X_j) \mid v < min(X_i)$
  end

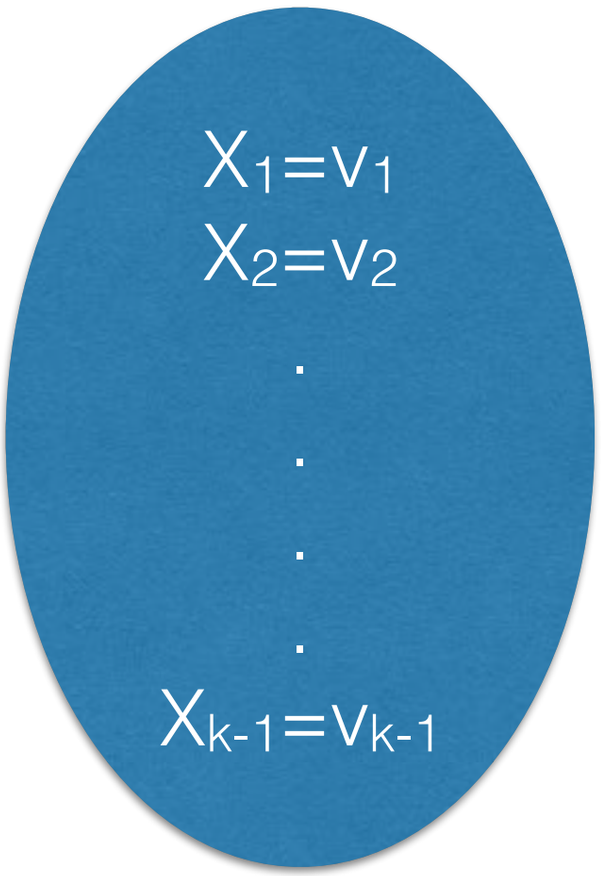
Warning on AC

- A network can be arc consistent while being inconsistent (no solution)

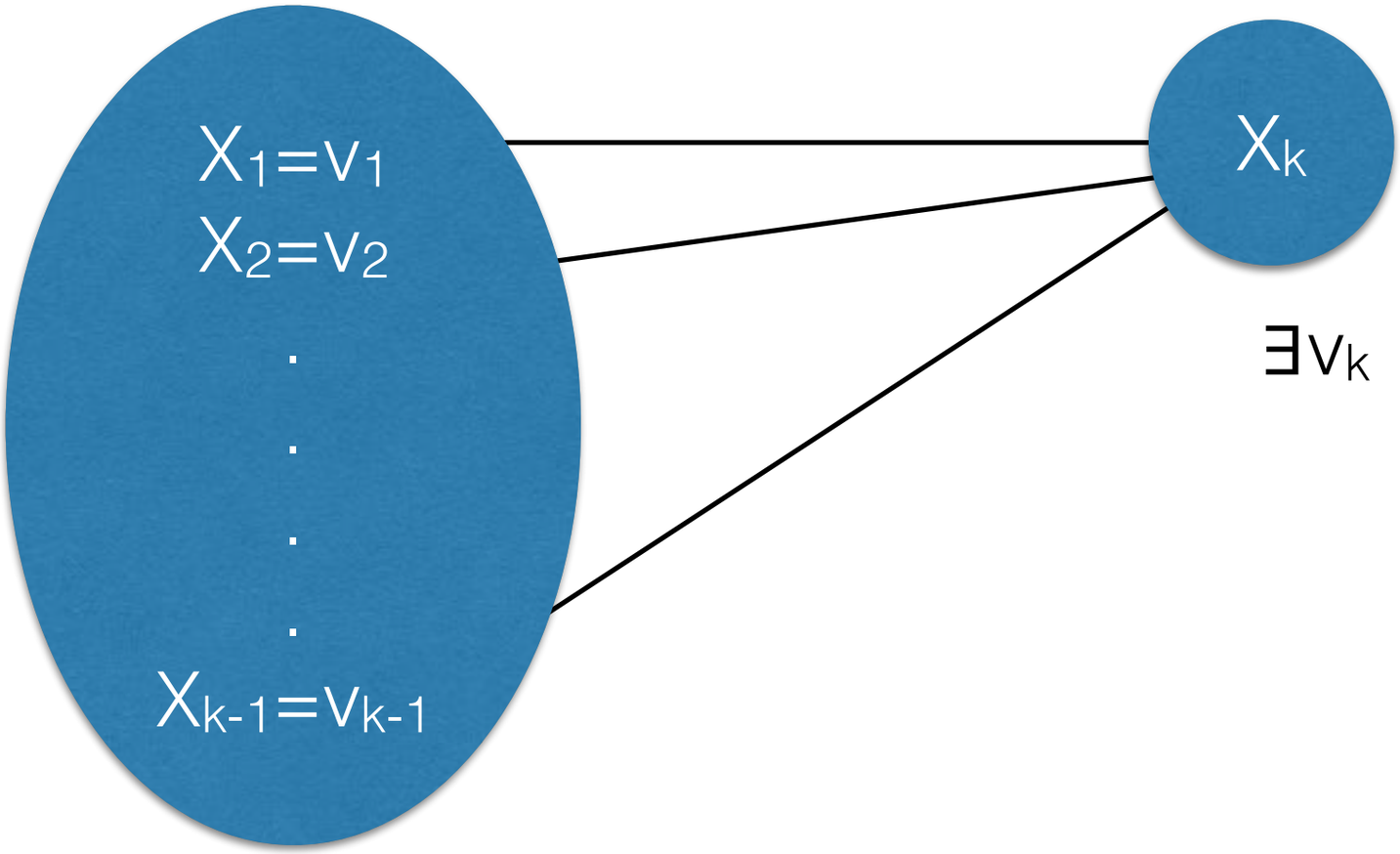


Stronger levels of consistency

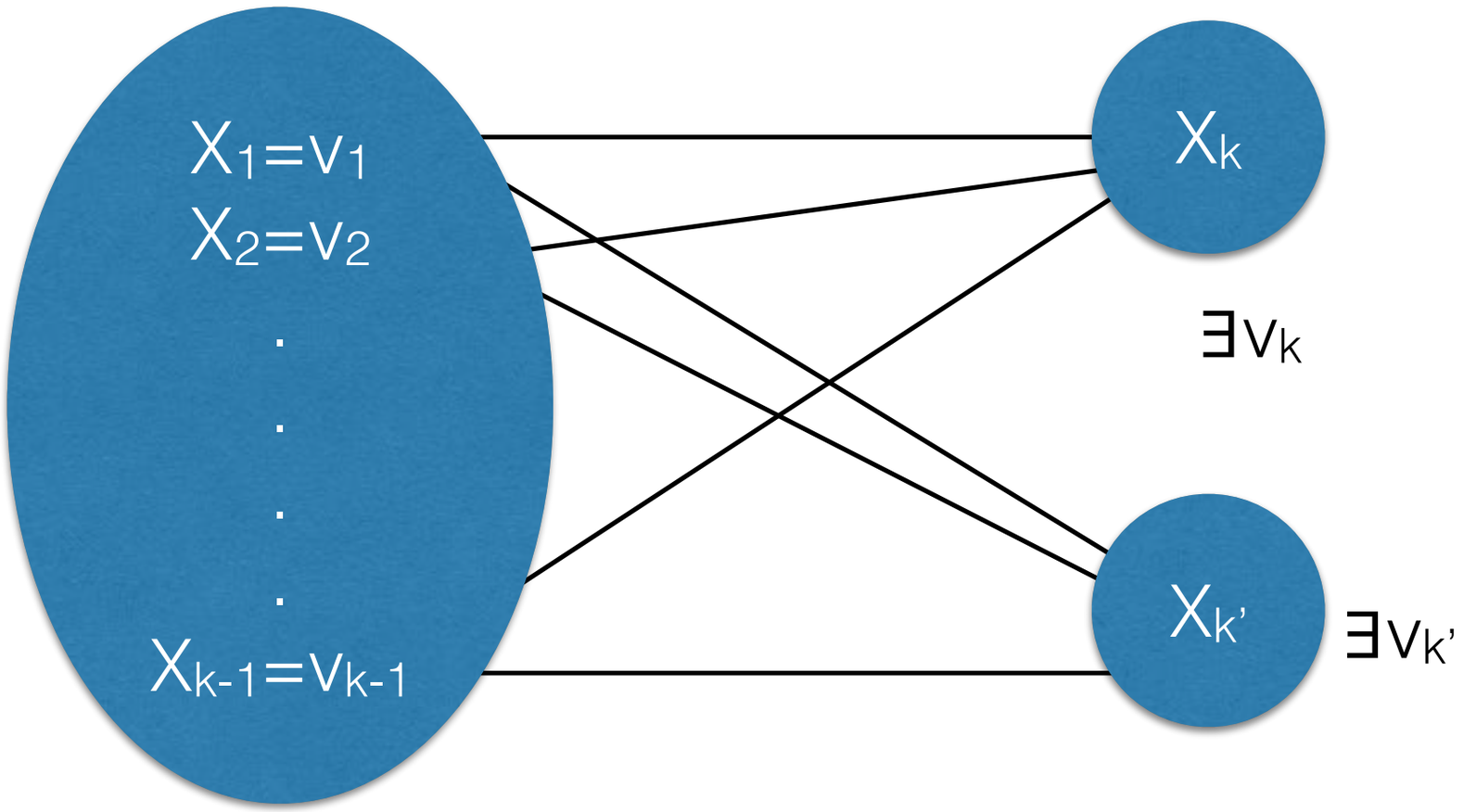
k-consistency



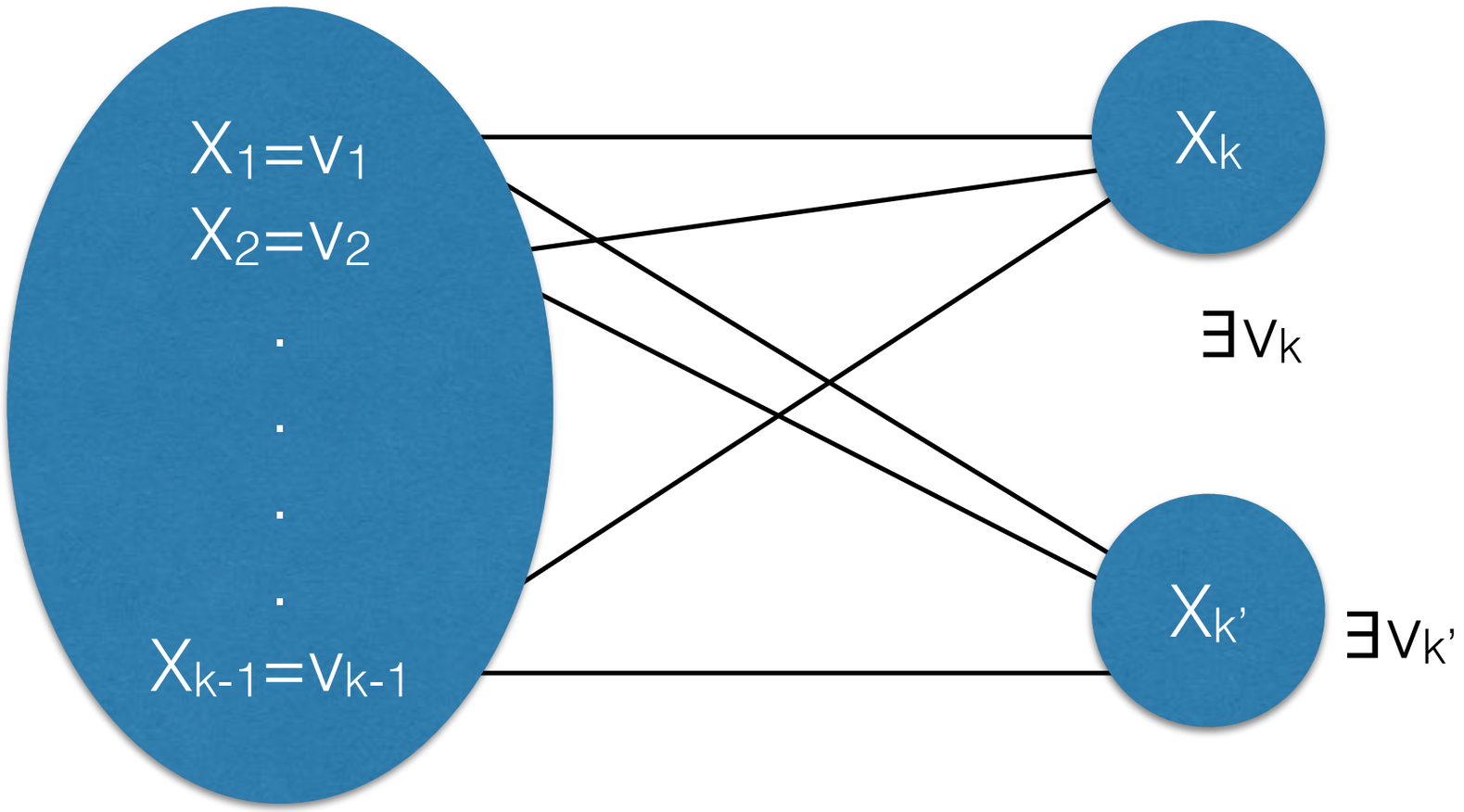
k-consistency



k-consistency

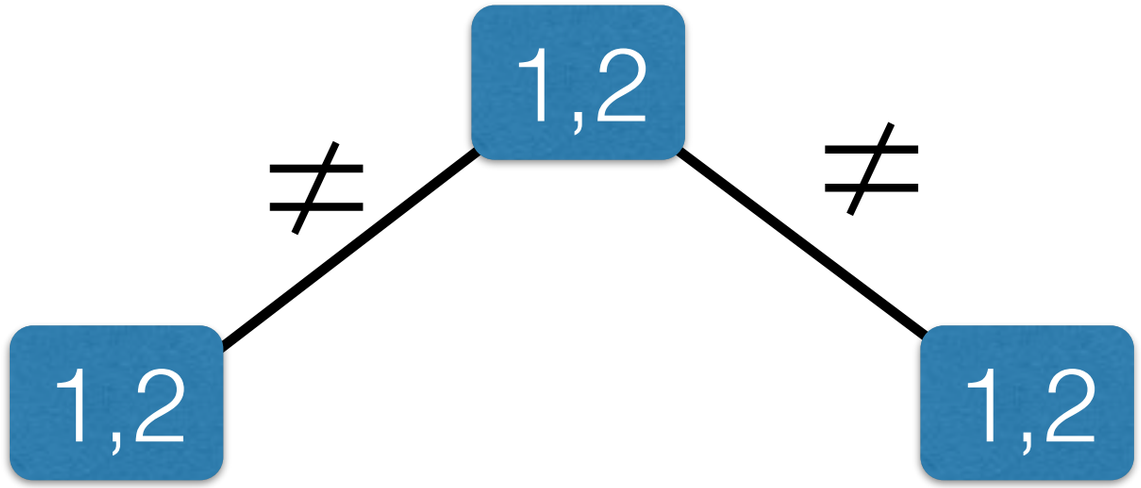


k-consistency



Definition 5 (*k*-consistency) *A network N is* k-consistent *iff* $\forall Y \subseteq X, |Y| = k-1$, $\forall I$ *locally consistent instantiation on* Y, $\forall X_{i_k} \in X \setminus Y$, $\exists v_{i_k} \in D(X_{i_k})$ *such that* $I \cup (X_{i_k}, v_{i_k})$ *is locally consistent.*

Example: 3-consistency



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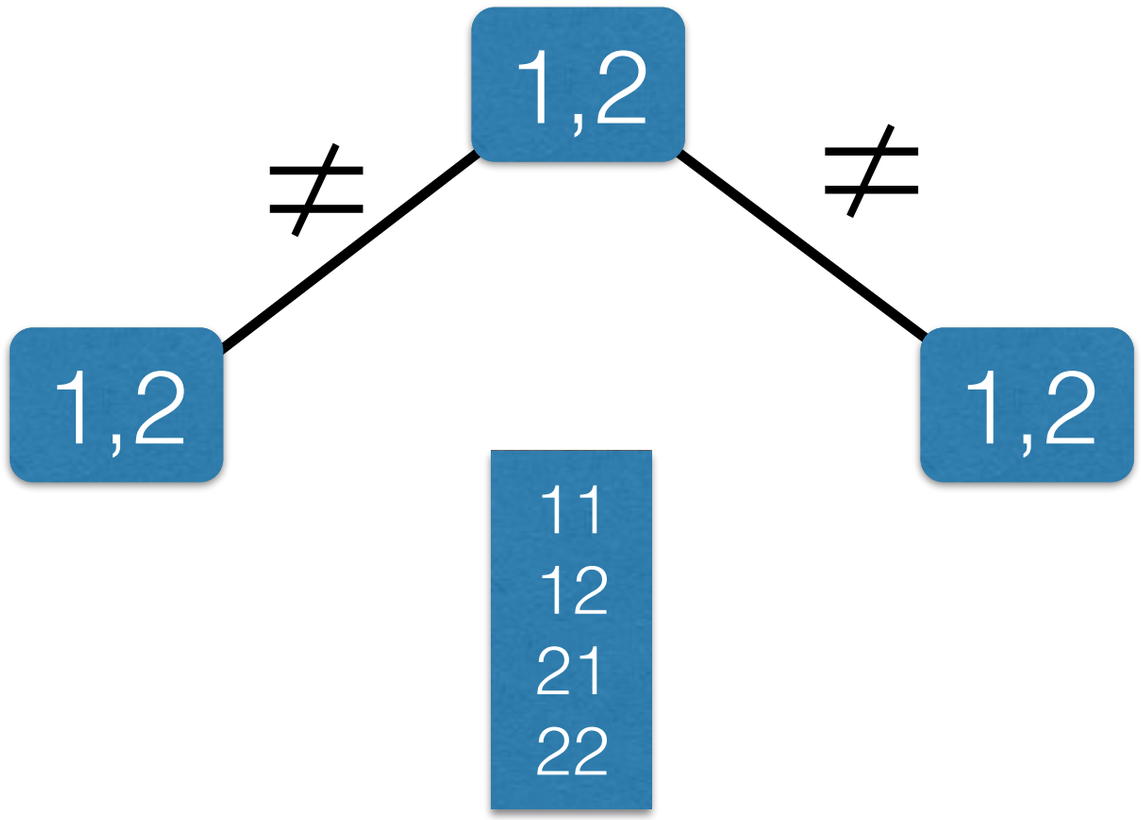

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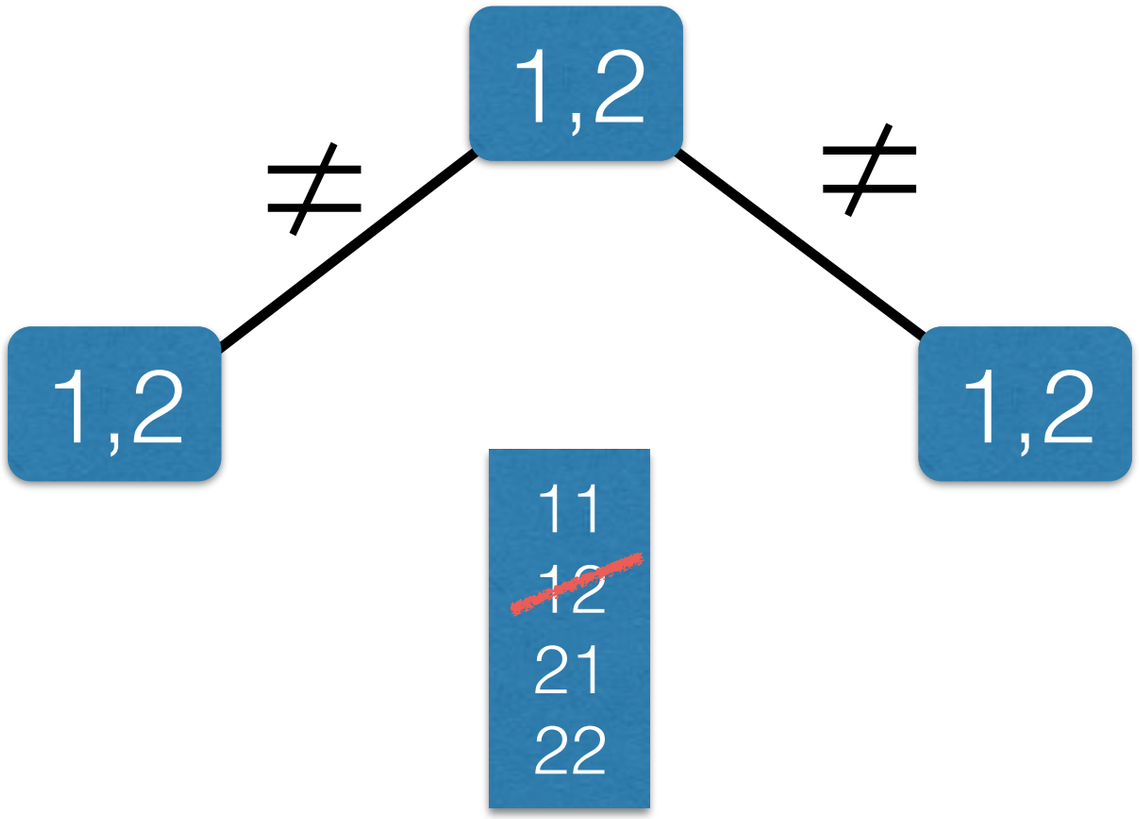

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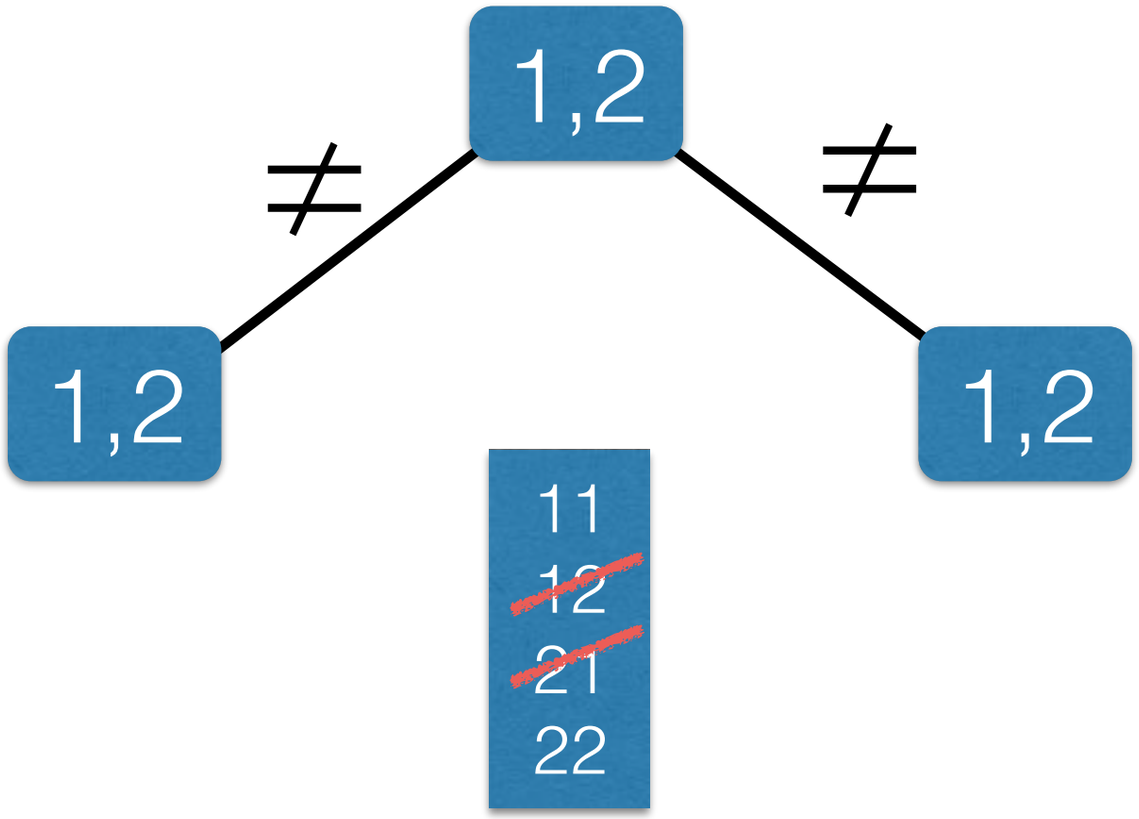
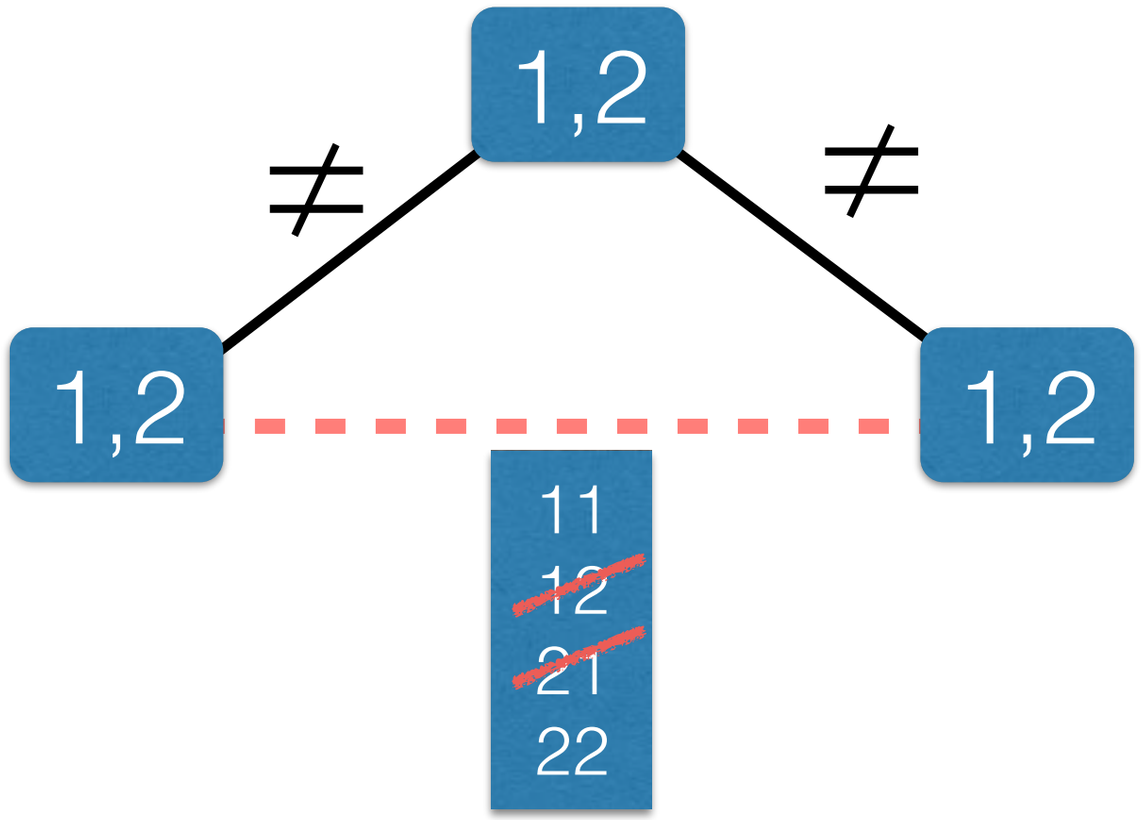


Example: 3-consistency

- Enforcing *k*-consistency creates new constraints!

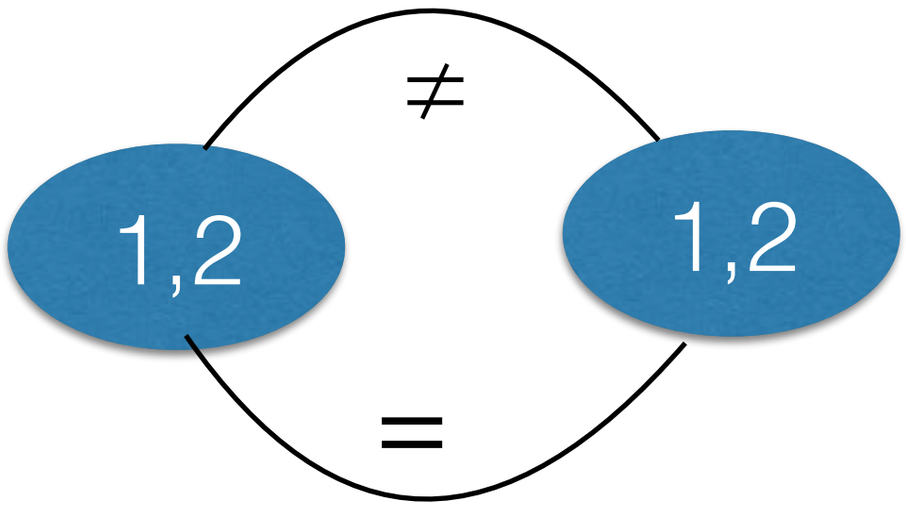


2-consistency

	normalized network	non normalized
binary network	2-c = AC	2-c > AC
non binary	2-c < AC	2-c ≠ AC

Normalized = not two constraints with the same scope

2-c > AC:



2-c < AC:

X+Y=Z

1	1	1
2	2	2
3	3	3

Strong *k*-consistency

Strong k-consistency

Definition 6 (Strong k-consistency) *A network is* strongly k-consistent *iff it is j-consistent for any $j, 1 \leq j \leq k$.*

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- time complexity is in $O(n^{k+1}d^{k+1})$ if space in $O(n^{k-1}d^{k-1})$
- time complexity is in $O(n^k d^k)$ if space in $O(n^k d^k)$

Global consistency

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Definition 7 *An instantiation is said* globally consistent *iff it can be extended to a solution. A network is said globally consistent iff every locally consistent instantiation is also globally consistent.*

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Proposition 4 *A network is globally consistent iff it is strongly n-consistent.*