Aide à la décision/ Decision aid

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Partie 2/Part 2
Gestion des incohérences/Inconsistency handling

What is a knowledge?

It is information we have about agents' beliefs or preferences.

Outline

- Classical logic
- Inference in classical logic
- Non-classical inference/Inconsistency handling methods: The flat case
- Non-classical inference/Inconsistency handling methods: The prioritized case
- Reasoning about prioritized knowledge

Brief background on classical propositional logic

- Let V be a set of propositional variables denoted by $a, b, c, \cdots$
- Propositional language PL_V is built on
  - V,
  - $\{\top, \bot\}$ (respectively tautology and contradiction),
  - usual connectors $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$.
- Formulas in PL_V are denoted by $\phi, \psi, \varphi, \cdots, \phi_1, \phi_2, \cdots, \psi_1, \psi_2, \cdots, \varphi_1, \varphi_2, \cdots$
- Inference is denoted by $\vdash$
- Let $\Sigma = \{\phi_1, \cdots, \phi_n\}$
  - $\bigwedge \Sigma = \phi_1 \wedge \cdots \wedge \phi_n$
  - $\bigvee \Sigma = \phi_1 \vee \cdots \vee \phi_n$

Inference

Given ϕ and $\phi \rightarrow \psi$ we deduce ψ.
We write $\phi, \phi \rightarrow \psi \vdash \psi$

Complementary definitions

- Let Σ be a multiset of formulas (the same formula may be present several times)
- A sub-base A of Σ $(A \subseteq \Sigma)$ is consistent if and only if $\bigwedge A \nvdash \perp$
- We suppose that Σ is not deductively closed

$A \subseteq \Sigma$ is maximally consistent if and only if

- A is consistent, and
- either $A = \Sigma$ or adding any formula ϕ from $\Sigma \backslash A$ to A entails the inconsistency of $A \cup \{\phi\}$

$A \subseteq \Sigma$ is minimally inconsistent if and only if

- $A \vdash \perp$, and
- $\forall \phi \in A,\ A \backslash \{\phi\} \nvdash \perp$ → Tout est nécessaire pour être incohérent

Let $Inc(\Sigma) = \{\phi, \exists A \subseteq \Sigma$ such that $\phi \in A$, A is minimally inconsistent$\}$. $Inc(\Sigma)$ is the set of formulas belonging to at least one minimally inconsistent sub-base of Σ. → au moins dans un conflit

Where does inconsistency come from?

- The presence of exceptions

Example

- Penguins are birds
- Birds fly
- A penguin doesn't fly
- Titi is a penguin

- Knowledge is provided from multiple sources

Example

- Agent 1: We will have a course tomorrow
- Agent 2: If it rains then we don't have course
- Agent 3: It will rain tomorrow

How to solve conflicts?

- Revise the knowledge base and restore consistency
- Cope with inconsistency and define non-classical inference

Non-classical inference/Inconsistency handling methods

The flat case

An argumentative consequence relation (1)

A sub-base A of Σ is an argument for a formula ϕ iff

1. $A \nvdash \bot$ (A is consistent),
2. $A \vdash \phi$,
3. $\forall \psi \in A$, $A \backslash \{\psi\} \nvdash \phi$ (A is minimal)

We write $\langle A, \phi \rangle$ which stands for "A is an argument for ϕ".

- Why condition 1?
  From an inconsistent base we can deduce a formula and its contradiction.
- Why condition 2?
  (ϕ: it is raining), (ψ: I should use my umbrella).
  Is $\langle \{\phi\}, \psi \rangle$ an argument??
- Why condition 3?
  (ϕ: it is raining), ($\phi \rightarrow \psi$: if it is raining, then I should use my umbrella), (φ: I like paprika).
  Is $\langle \{\phi, \phi \rightarrow \psi, \varphi\}, \psi \rangle$ an argument??

An argumentative consequence relation (2)

⚠ argument ≠ argumentative consequence

ϕ is an argumentative consequence of Σ, denoted by $\Sigma \vdash_{\mathcal{A}} \phi$, iff

- there exists an argument for ϕ in Σ, and
- there is no argument for $\neg\phi$ in Σ

An argumentative consequence relation
Example

$a \rightarrow b$ est l'implication matérielle $\equiv (a \vee \neg b)$

$a \Rightarrow b$ est la règle générale par défaut

Consider a discussion in a newspaper editorial office about whether or not to proceed with the publication of some indiscretion about a prominent politician. Suppose the key bits of information are captured by the following five statements.

- Simon Jones is a Member of Parliament
- If Simon Jones is a Member of Parliament then we need not keep quiet about details of his private life
- Simon Jones just resigned from the House of Commons
- If Simon Jones just resigned from the House of Commons then he is not a Member of Parliament
- If Simon Jones is not a Member of Parliament then we need to keep quiet about details of his private life

How good is argumentative consequence?

Example: Let $\Sigma = \{\psi, \psi \to \phi, \neg\psi\}$

- Is ϕ an argumentative consequence of Σ?

↳ Oui, mais pas très fiable

Under which conditions we have

$$\Sigma \vdash \phi \text{ iff } \Sigma \vdash_{\mathcal{A}} \phi?$$

Si Σ est cohérente, car α et $\neg\alpha$ pas possible.

Ensemble des formules de Σ impliquées dans au moins un conflit

Definition: Free formula

$\varphi \in \Sigma$ is free iff $\varphi \notin Inc(\Sigma)$. $Free(\Sigma) = \Sigma \backslash Inc(\Sigma)$ → Formules jamais impliquées dans des conflits

Definition: Free consequence

ϕ is a free consequence of Σ, denoted by $\Sigma \vdash_{Free} \phi$, iff $Free(\Sigma) \vdash \phi$

Example: $\Sigma = \{\phi, \neg\phi \vee \neg\psi, \psi, \neg\phi \vee \varphi, \neg\psi \vee \varphi\}$

$$\begin{cases} Inc(\Sigma) = \{\phi, \neg\phi \vee \neg\psi, \psi\} \\ Free(\Sigma) = \{\neg\phi \vee \varphi, \neg\psi \vee \varphi\} \end{cases}$$

How do argumentative and free consequences relate?

- $\forall \phi$, si $\Sigma \vdash_{Free} \phi$ alors $\Sigma \vdash_{A} \phi$

- L'inverse n'est pas vrai, par exemple pour $\Sigma = \{a, a \rightarrow b, \neg a\}$, $Free(\Sigma) = \{a \rightarrow b\}$ et $\Sigma \vdash_{A} b$ mais $\Sigma \nvdash_{Free} b$.

Universal consequence (or MC-consequence)

$MC(\Sigma)$ is the set of maximally consistent sub-bases of Σ

Definition

ϕ is a MC-consequence of Σ, denoted by $\Sigma \vdash_{MC} \phi$, iff
$\forall A \in MC(\Sigma),\ A \vdash \phi$ $\longrightarrow \phi$ dans toutes les sous-bases maximalement cohérentes

Example: $\Sigma = \{\phi, \neg\phi \vee \neg\psi, \psi, \neg\phi \vee \varphi, \neg\psi \vee \varphi\}$

$MC(\Sigma) = \{\{\phi, \neg\phi \vee \neg\psi, \neg\phi \vee \varphi, \neg\psi \vee \varphi\},$
$\{\neg\phi \vee \neg\psi, \psi, \neg\phi \vee \varphi, \neg\psi \vee \varphi\}$
$\{\phi, \psi, \neg\phi \vee \varphi, \neg\psi \vee \varphi\}\}$

$\Sigma \vdash_{MC} \{\varphi, \neg\phi \vee \varphi, \ldots\}$

How do MC- and argumentative (resp. free) consequences relate?

- $\forall \phi$, si $\Sigma \vdash_{MC} \phi$ alors $\Sigma \vdash_{A} \phi$
- L'inverse n'est pas vrai, par exemple pour $\Sigma = \{a, a \to b, \neg a\}$
- $\forall \phi$, si $\Sigma \vdash_{free} \phi$ alors $\Sigma \vdash_{MC} \phi$
- L'inverse n'est pas vrai, par exemple pour $\Sigma = \{\neg a \vee c, \neg b \vee c\}$

One of the main difficulties for implementing the MC-consequence is the cardinality of $MC(\Sigma)$ which exponentially increases with the number of conflicts in Σ. The lexicographical consequence is based on a subset of $MC(\Sigma)$.

le ou les plus grands ensembles de MC

$$A \in L(\Sigma) \text{ iff } A \in MC(\Sigma) \text{ and } \forall B \in MC(\Sigma),\ |A| \geq |B|,$$

where $|\Gamma|$ is the cardinality (number of elements) of Γ.

Definition: L-consequence

ϕ is a L-consequence of Σ, denoted by $\Sigma \vdash_L \phi$, iff $\forall A \in L(\Sigma)$, $A \vdash \phi$ → ϕ dans toutes les sous-bases de L

Example: $\Sigma = \{\psi \to \phi, \psi, \neg\psi, \neg\phi \wedge \neg\psi, \neg\phi\}$

$MC(\Sigma) = \{\{\psi \to \phi, \psi\},$ A

$\{\neg\psi, \neg\phi \wedge \neg\psi, \neg\phi, \psi \to \phi\},$ B

$\{\psi, \neg\phi\}\}$ C

$L(\Sigma) = B$

How do lexicographical and MC- (resp. argumentative-) consequences relate?

- $\forall \phi$, si $\Sigma \vdash_{MC} \phi$ alors $\Sigma \vdash_{L} \phi$
- L'inverse n'est pas vrai
- Les conséquences Arg et L sont incomparables

Existential consequence

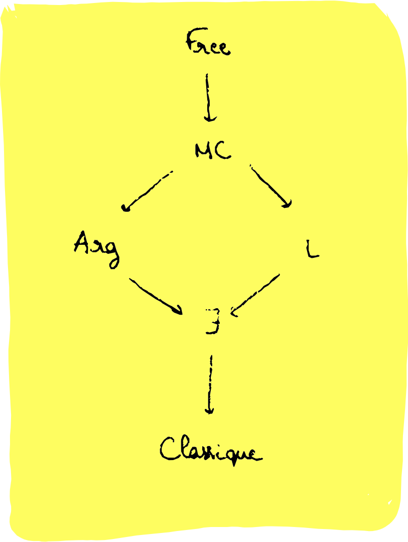
Definition

ϕ is an existential consequence of Σ, denoted by $\Sigma \vdash_{\exists} \phi$, iff $\exists A \in MC(\Sigma),\ A \vdash \phi$ $\rightarrow$ ϕ dans toutes au moins une sous-bases de L

Example: $\Sigma = \{\psi \rightarrow \phi, \psi, \neg\psi, \neg\phi \wedge \neg\psi, \neg\phi\}$

How do existential and lexicographical (resp. argumentative) consequences relate?

Link between the consequence relations



Non-classical inference/Inconsistency handling methods

The prioritized case

What are priorities and where do they come from?

- Priority is very important in the study of knowledge-based systems
- A priority represents uncertainty or preferences associated with knowledge
- Priorities may represent the reliability/preference of sources/knowledge
- The task of coping with inconsistency is greatly simplified since conflicts have better chance to be solved

Prioritized knowledge bases

- Prioritized knowledge bases are layered, that is, of the form $\Sigma = S_1 \cup \cdots \cup S_n$, such that formulas in S_i have the same → Strates priority and are more prioritized than formulas in S_j with $j > i$. Sometimes we write $\Sigma = \{S_1, \cdots, S_n\}$
- Each S_i is a multiset: copies of the same formula may appear in the same layer or in different layers. They are considered as distinct
- $\phi \in \Sigma$ means that there is a copy of ϕ in Σ
- We sometimes use the notation $\phi \in_i \Sigma$ for $\phi \in S_i$
- $\phi \notin_i \Sigma$ means that $\phi \notin S_i$ (however it may belong to other layers of Σ)
- Adding a formula ϕ to the ith layer S_i of Σ is denoted $\Sigma \cup \{\phi\}_i$
- Sub-bases are denoted by capital letters $A, B, C, \ldots$. They are also represented in a stratified form $A = A_1 \cup \cdots \cup A_n$, with $A_i \subseteq S_i$ (A_j may be empty)

Possibilistic consequence (1)

Definition: i-consequence

Let $\Sigma = S_1 \cup \cdots \cup S_n$ be a layered belief base. A formula ϕ is an i-consequence of Σ, denoted by $\Sigma \vdash_i \phi$, iff:

1. $S_1 \cup \cdots \cup S_i$ is consistent, → la strate n'est pas incohérente avec les strates plus prioritaires
2. $S_1 \cup \cdots \cup S_i \vdash \phi$, and → la strate et les plus prioritaires montrent ϕ
3. $\forall j < i$, $S_1 \cup \cdots \cup S_j \nvdash \phi$. → Aucune strate plus prioritaire ne montre ϕ

Definition: π-consequence

ϕ is a π-consequence of Σ if ϕ is an i-consequence of Σ for some i, and denote it by $\Sigma \vdash_\pi \phi$.

Possibilistic consequence (2)

It is possible to characterize the set of π-consequences of Σ.

- We compute the set $\pi(\Sigma)$ defined as: $\pi(\Sigma) = S_1 \cup \cdots \cup S_i$, such that $S_1 \cup \cdots \cup S_i$ is consistent and $S_1 \cup \cdots \cup S_{i+1}$ is inconsistent. → Maximally consistent set of strates
- If Σ is consistent then $\pi(\Sigma) = \Sigma$.
- $i + 1$ is called the *inconsistency degree* of Σ. If Σ is consistent then $Inc(\Sigma) = 0$. → ⚠ Ici c'est le degré d'incohérence, pas l'ensemble des conflits
- $\Sigma \backslash \pi(\Sigma)$ is simply inhibited.

It is not hard to check that

$\Sigma \vdash_\pi \phi$ iff $\pi(\Sigma) \vdash \phi$

Example

- Let $\Sigma_1 = \{\{p\}, \{\neg p \vee b, \neg p \vee \neg f\}, \{\neg b \vee f\}\}$
- Let $\Sigma_2 = \{\{p\}, \{\neg p \vee b, \neg p \vee \neg f\}, \{\neg b \vee w\}\}$

Dealing with inconsistency using possibilistic consequence is not entirely satisfactory. The latter suffers from the "drowning problem":

on perd toutes les infos après le conflit, même si elles n'ont aucun rapport avec le conflit

Example

Let $\Sigma = \{\{\neg\phi \vee \neg\psi\}, \{\phi\}, \{\psi\}, \{\xi\}\}$. → info perdue pour rien

↳ stop car incohérence

A particular case of the drowning effect is called "blocking of property inheritance".

p p → b, p → ¬f ; b → f (pas pris car incohérence) b → w

$\Sigma = \{\{p\}, \{\neg p \vee b, \neg p \vee \neg f\}, \{\neg b \vee f, \neg b \vee w\}\},$

p, b, f, w respectively mean penguin, bird, fly, wings.
Based on Σ, it is not possible for a penguin to inherit properties of birds (in our example, to inherit property of having wings), while the only undesirable property for a penguin in our example is "flying".

Free consequence (1)

Definition: Dominant sub-base

The dominant sub-base of Σ is
$\Sigma^* = Free(S_1) \cup Free(S_1 \cup S_2) \cup \cdots \cup Free(S_1 \cup \cdots \cup S_n)$.

Properties of the dominant sub-base

- For a given $i \geq 1$, if a formula ϕ in $S_1 \cup \cdots \cup S_i$ does not belong to $Free(S_1 \cup \cdots \cup S_i)$ then it will not belong to $Free(S_1 \cup \cdots \cup S_k)$ for $k \geq i$.
- In general, there is no inclusion relation between $Free(S_1 \cup \cdots \cup S_i)$ and $Free(S_1 \cup \cdots \cup S_k)$ with $k > i$.

Definition: Free consequence

A formula ϕ is a free consequence of Σ, denoted by $\Sigma \vdash_{ND}$, iff $\Sigma^* \vdash \phi$.

Example

Let $\Sigma = \{\{p\}, \{\neg p \vee b, \neg p \vee \neg f\}, \{\neg b \vee f, \neg b \vee w\}\}$

$\Sigma^* = \text{Free}(S_1) \cup \text{Free}(S_1 \cup S_2) \cup \text{Free}(S_1 \cup S_2 \cup S_3)$

$= \{p\} \cup \{p, \neg p \vee b, \neg p \vee \neg f\} \cup \{\neg b \vee w\}$

$= \{p, \neg p \vee b, \neg p \vee \neg f, \neg b \vee w\}$

$\Sigma \vdash_{NO} \{p, \neg p \vee b, \neg p \vee \neg f, \neg b \vee w\}$

Linear consequence (1)

The sub-base $l(\Sigma)$ is obtained by dropping layers S_i when they are inconsistent with the previous ones. Namely, $l(\Sigma)$ is constructed in the following way:

$$l(S_1) = \begin{cases} S_1 & \text{if } S_1 \text{ is consistent} \\ \emptyset & \text{otherwise.} \end{cases}$$

for $i > 1$:

$$l(S_1 \cup \cdots \cup S_i) = \begin{cases} l(S_1 \cup \cdots \cup S_{i-1}) \cup S_i & \text{if } \textit{consistency} \\ l(S_1 \cup \cdots \cup S_{i-1}) & \text{otherwise.} \end{cases}$$

Definition: Linear consequence

ϕ is a linear consequence of Σ, denoted by $\Sigma \vdash_l \phi$, iff $l(\Sigma) \vdash \phi$.

$\ell(S_1) = \{p\}$

$\ell(S_1 \cup S_2) = \ell(S_1) \cup S_2 = \{p, \neg p \vee b, \neg p \vee \neg f\}$

$\ell(S_1 \cup S_2 \cup S_3) = \ell(S_1 \cup S_2) = \{p, \neg p \vee b, \neg p \vee \neg f\}$

Example

$\Sigma = \{\{p\}, \{\neg p \vee b, \neg p \vee \neg f\}, \{\neg b \vee f, \neg b \vee w\}\}$

Remark

- Linear consequence is more productive than possibilistic consequence. However it does not solve the problem of the drowning effect.
- Linear consequence and free consequence are incomparable.

Argumentative consequence (1)

Definition: A reason/argument of rank i → $A_i \subseteq S_i$

A sub-base A of Σ is a reason of rank i for a formula ϕ if it satisfies the following conditions:

1. $A \nvdash \bot$ → pas incohérent
2. $A \vdash \phi$ → prouve ϕ
3. $\forall \psi \in A,\ A \backslash \{\psi\} \nvdash \phi$ → est minimale pour prouver ϕ
4. $R(A) = \max\{j | A \cap S_j \neq \emptyset\} = i$ → raison = strate la moins prioritaire avec une intersection non vide

Definition: Argumentative consequence

ϕ is an argumentative consequence of Σ, denoted by $\Sigma \vdash_{\mathcal{A}} \phi$, iff:

- there exists an argument of rank i for ϕ in Σ, and
  ↳ argument pour ϕ
- arguments for $\neg\phi$ (if any) are of rank $j > i$.
  → pas d'argument pour $\neg\phi$ plus prioritaire

S'il y a plusieurs arguments $A_1, \ldots, A_n$ pour Φ, on calcule $R(A_1), \ldots, R(A_n)$ et on garde A_i tel que $R(A_i)$ est minimale.

Example

Let $\Sigma = \{\{\neg c\}, \{\neg a \vee \neg b \vee c, \neg d \vee c, \neg e \vee c\}, \{d, e, f, \neg f \vee \neg g \vee c\}, \{a, b, g, h\}\}$

- $A_1: \langle \{d\}, d \rangle$, $R(A_1) = 3$
- $A_2 = \langle \{\neg c, \neg d \vee c\}, \neg d \rangle$, $R(A_2) = 2$
- $A_3 = \langle \{e\}, e \rangle$, $R(A_3) = 3$
- $A_4 = \langle \{\neg c, \neg e \vee c\}, \neg e \rangle$, $R(A_4) = 2$
- $A_5 : \langle \{a, b\}, a \wedge b \rangle$, $R(A_5) = 4$
- $A_6 = \langle \{\neg c, \neg a \vee \neg b \vee c\}, \neg a \vee \neg b \rangle$, $R(A_6) = 2$
- $A_7 = \langle \{g\}, g \rangle$, $R(A_7) = 4$
- $A_8 = \langle \{\neg c, f, \neg f \vee \neg g \vee c\}, \neg g \rangle$, $R(A_8) = 3$

$\Sigma_R \vdash \{\neg d, \neg e, \neg a \vee \neg b, \neg g\}$

Application: Access Control in Medical Problem

1. If Mary plays patient role then she plays non staff member role.
2. If Mary plays patient role then she is permitted to read her medical record.
3. If Mary plays a non staff member role then she is not permitted to read her medical record.
   If Jean plays doctor role then he is permitted to read Mary's record.

Question

Is Mary permitted to read her medical record?