D.8. Exemple de calcul sémantique : les enfants prendront une pizza

mot	***catégorie syntaxique*** u ***type sémantique*** u^* ***sémantique :*** λ***-term of type*** u^* x^v ***signifie*** x ***(variable, constante) de type*** v
les	$(S/(np\backslash S))/n$ (subject) $((S/np)\backslash S)/n$ (object) $(e \to t) \to ((e \to t) \to t)$ $\lambda P^{e\to t}\ \lambda Q^{e\to t}\ (\forall^{(e\to t)\to t}\ (\lambda x^e (\Rightarrow^{t\to(t\to t)}\ (P\ x)(Q\ x))))$
une	$((S/np)\backslash S)/n$ (object) $(S/(np\backslash S))/n$ (subject) $(e \to t) \to ((e \to t) \to t)$ $\lambda P^{e\to t}\ \lambda Q^{e\to t}\ (\exists^{(e\to t)\to t}\ (\lambda x^e (\wedge^{t\to(t\to t)} (P\ x)(Q\ x))))$
enfant(s)	n $e \to t$ $\lambda x^e (\mathtt{enfant}^{e\to t}\ x)$
pizza	n $e \to t$ $\lambda x^e (\mathtt{pizza}^{e\to t}\ x)$
prendront	$(np\backslash S)/np$ $e \to (e \to t)$ $\lambda y^e\ \lambda x^e\ ((\mathtt{prendront}^{e\to(e\to t)}\ x) y)$

D.9. Analyse syntaxique $\exists\forall$

Il y a deux analyse syntaxique possibles. Une :

$\exists\forall$

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{(S/(np\backslash S))/n \quad n}{(S/(np\backslash S))}/_e
\quad
\dfrac{(np\backslash S)/np \quad [np]^1}{(np\backslash S)}/_e
}{S}/_e
}{S/np}/_i(1)
\quad
\dfrac{((S/np)\backslash S)/n \quad n}{(S/np)\backslash S}\backslash_e
}{S}\backslash_e
$$

D.10. Syntaxe $\to$ λ-terme sémantique de la phrase

$\exists\forall$

$$\dfrac{\dfrac{\dfrac{\dfrac{\overset{\textit{les}}{(\mathbf{e}\to\mathbf{t})\to(\mathbf{e}\to\mathbf{t})\to\mathbf{t}} \quad \overset{\textit{enfants}}{(\mathbf{e}\to\mathbf{t})}}{(\mathbf{e}\to\mathbf{t})\to\mathbf{t}}\to_e \quad \dfrac{\overset{\textit{prendront}}{\mathbf{e}\to\mathbf{e}\to\mathbf{t}} \quad \overset{\textit{o}}{[\mathbf{e}]^1}}{\mathbf{e}\to\mathbf{t}}\to_e}{\mathbf{t}}\to_e}{\mathbf{e}\to\mathbf{t}}\to_i(1) \quad \dfrac{\overset{\textit{une}}{(\mathbf{e}\to\mathbf{t})\to(\mathbf{e}\to\mathbf{t})\to\mathbf{t}} \quad \overset{\textit{pizza}}{(\mathbf{e}\to\mathbf{t})}}{(\mathbf{e}\to\mathbf{t})\to\mathbf{t}}\to_e}{\mathbf{t}}\to_e$$

Le λ-terme correspondant est :

$\exists\forall = (\textit{une pizza})(\lambda o^{\mathbf{e}}(\textit{les enfants})(\textit{prendront } o))$

Il faut encore :

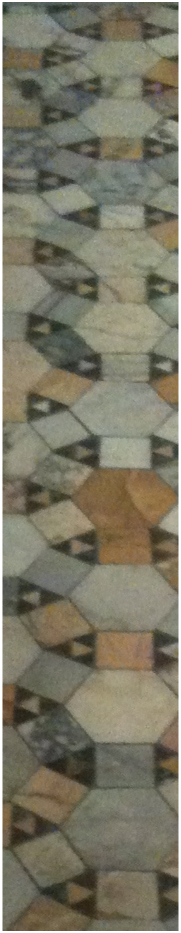
1. insérer les lambda terme lexicaux et
2. réduire/calculer

D.11. Calculs, par étapes 1/2

$$
\begin{aligned}
&(\textit{une pizza})\\
&= (\lambda P^{e\to t}\ \lambda Q^{e\to t}\ (\exists^{(e\to t)\to t}\ (\lambda x^{e}(\wedge^{t\to(t\to t)}(P\ x)(Q\ x))))) (\lambda z^{e}(\mathtt{pizza}^{e\to t}\ z))\\
&= (\lambda Q^{e\to t}\ (\exists^{(e\to t)\to t}\ (\lambda x^{e}(\wedge^{t\to(t\to t)}((\lambda z^{e}(\mathtt{pizza}^{e\to t}\ z))\ x)(Q\ x)))))\\
&= (\lambda Q^{e\to t}\ (\exists^{(e\to t)\to t}\ (\lambda x^{e}(\wedge^{t\to(t\to t)}((\mathtt{pizza}^{e\to t}\ x)))(Q\ x))))
\end{aligned}
$$

$$
\begin{aligned}
&(\textit{les enfants})\\
&= (\lambda P^{e\to t}\ \lambda Q^{e\to t}\ (\forall^{(e\to t)\to t}\ (\lambda x^{e}(\Rightarrow^{t\to(t\to t)}\ (P\ x)(Q\ x))))) (\lambda u^{e}(\mathtt{enfant}^{e\to t}\ u))\\
&= (\lambda Q^{e\to t}\ (\forall^{(e\to t)\to t}\ (\lambda x^{e}(\Rightarrow^{t\to(t\to t)}\ ((\lambda u^{e}(\mathtt{enfant}^{e\to t}\ u))\ x)(Q\ x)))))\\
&= (\lambda Q^{e\to t}\ (\forall^{(e\to t)\to t}\ (\lambda x^{e}(\Rightarrow^{t\to(t\to t)}\ (\mathtt{enfant}^{e\to t}\ x)(Q\ x)))))
\end{aligned}
$$

$$
\begin{aligned}
&(\textit{les enfants})(\textit{prendront } o) =\\
&(\lambda Q^{e\to t}\ (\forall^{(e\to t)\to t}\ (\lambda w^{e}(\Rightarrow^{t\to(t\to t)}(\mathtt{enfant}^{e\to t}\ w)(Q\ w)))))((\lambda y^{e}\ \lambda x^{e}\ ((\mathtt{prendront}^{e\to(e\to t)}\ x)\ y))\ o)\\
&=(\lambda Q^{e\to t}\ (\forall^{(e\to t)\to t}\ (\lambda w^{e}(\Rightarrow^{t\to(t\to t)}(\mathtt{enfant}^{e\to t}\ w)(Q\ w)))))(\lambda x^{e}\ ((\mathtt{prendront}^{e\to(e\to t)}\ x)\ o))\\
&=\forall^{(e\to t)\to t}\ (\lambda w^{e}(\Rightarrow^{t\to(t\to t)}(\mathtt{enfant}^{e\to t}\ w)((\lambda x^{e}\ ((\mathtt{prendront}^{e\to(e\to t)}\ x)\ o))\ w)))\\
&=\forall^{(e\to t)\to t}\ (\lambda w^{e}(\Rightarrow^{t\to(t\to t)}(\mathtt{enfant}^{e\to t}\ w)(((\mathtt{prendront}^{e\to(e\to t)}\ w)\ o))))
\end{aligned}
$$



D.12. Calculs, par étapes 2/2

$$
\begin{aligned}
&(\mathit{une\ pizza})(\lambda o\ (\mathit{les\ enfants})(\mathit{prendront}\ o))\\
&= (\lambda Q^{e\to t}\ (\exists^{(e\to t)\to t}\ (\lambda x^{e}(\wedge^{t\to(t\to t)}((\texttt{pizza}^{e\to t}\ x)))(Q\ x))))\\
&\qquad (\lambda o \forall^{(e\to t)\to t}\ (\lambda w^{e}(\Rightarrow^{t\to(t\to t)}\ (\texttt{enfant}^{e\to t}\ w)(((\texttt{prendront}^{e\to(e\to t)}\ w)\ o)))))\\
&= (\exists^{(e\to t)\to t}\ (\lambda x^{e}(\wedge^{t\to(t\to t)}((\texttt{pizza}^{e\to t}\ x)))\\
&\qquad ((\lambda o \forall^{(e\to t)\to t}\ (\lambda w^{e}(\Rightarrow^{t\to(t\to t)}\ (\texttt{enfant}^{e\to t}\ w)(((\texttt{prendront}^{e\to(e\to t)}\ w)\ o)))))\ x)))\\
&= (\exists^{(e\to t)\to t}\ (\lambda x^{e}(\wedge^{t\to(t\to t)}((\texttt{pizza}^{e\to t}\ x)))\\
&\qquad (\forall^{(e\to t)\to t}\ (\lambda w^{e}(\Rightarrow^{t\to(t\to t)}\ (\texttt{enfant}^{e\to t}\ w)((\texttt{prendront}^{e\to(e\to t)}\ w)\ x))))))
\end{aligned}
$$

ce qui s'écrit communément :

$$\exists x.\ \mathit{pizza}(x) \wedge \forall w.\ (\mathit{enfant}(w) \Rightarrow \mathit{prendront}(w,x))$$

D.13. Avec l’autre analyse syntaxique...

$\forall\exists$

$$
\dfrac{
  \dfrac{(S/(np\backslash S))/n \quad n}{(S/(np\backslash S))}\;/_e
  \qquad
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{[np]^1 \quad \dfrac{(np\backslash S)/np \quad [np]^2}{(np\backslash S)}\;/_e}{S}\;\backslash_e
      }{S/np}\;/_i(2)
      \quad
      \dfrac{((S/np)\backslash S)/n \quad n}{(S/np)\backslash S}\;\backslash_e
    }{S}\;\backslash_e
  }{np\backslash S}\;\backslash_i(1)
}{S}\;/_e
$$

Qui correspond à l’analyse :

$\forall\exists$

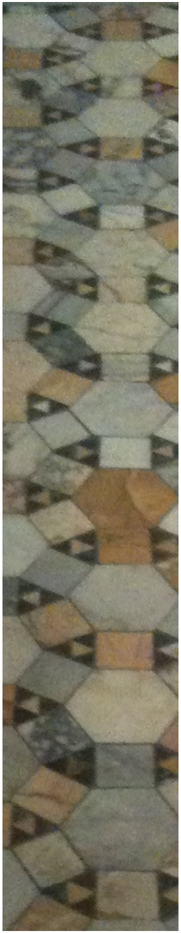
$$
\begin{prooftree}
\AxiomC{les}
\noLine
\UnaryInfC{$(\mathbf{e}\to\mathbf{t})\to(\mathbf{e}\to\mathbf{t})\to\mathbf{t}$}
\AxiomC{$\mathit{enfants}$}
\noLine
\UnaryInfC{$(\mathbf{e}\to\mathbf{t})$}
\RightLabel{$\to_e$}
\BinaryInfC{$(\mathbf{e}\to\mathbf{t})\to\mathbf{t}$}
\AxiomC{s}
\noLine
\UnaryInfC{$[\ \mathbf{e}]^1$}
\AxiomC{$\mathit{prendront}$}
\noLine
\UnaryInfC{$\mathbf{e}\to\mathbf{e}\to\mathbf{t}$}
\AxiomC{o}
\noLine
\UnaryInfC{$[\mathbf{e}]^2$}
\RightLabel{$\to_e$}
\BinaryInfC{$\mathbf{e}\to\mathbf{t}$}
\RightLabel{$\to_e$}
\BinaryInfC{$\mathbf{t}$}
\RightLabel{$\to_i(2)$}
\UnaryInfC{$\mathbf{e}\to\mathbf{t}$}
\AxiomC{une}
\noLine
\UnaryInfC{$(\mathbf{e}\to\mathbf{t})\to(\mathbf{e}\to\mathbf{t})\to\mathbf{t}$}
\AxiomC{pizza}
\noLine
\UnaryInfC{$(\mathbf{e}\to\mathbf{t})$}
\RightLabel{$\to_e$}
\BinaryInfC{$(\mathbf{e}\to\mathbf{t})\to\mathbf{t}$}
\RightLabel{$\to_e$}
\BinaryInfC{$\mathbf{t}$}
\RightLabel{$\to_i(1)$}
\UnaryInfC{$\mathbf{e}\to\mathbf{t}$}
\RightLabel{$\to_e$}
\BinaryInfC{$\mathbf{t}$}
\end{prooftree}
$$

λ-terme de la phrase :

$$\forall\exists = (\mathit{les\ enfants})(\lambda s.\ (\mathit{une\ pizza})(\lambda o\ ((\mathit{prendront}\ o)\ s)))$$

on insère les λ-termes lexicaux et on calcule...

On remarque que $(\mathit{une\ pizza})$ et $(\mathit{les\ enfants})$ déjà faits.



D.14. Calculs (bis repetita placent)

$$
\begin{aligned}
&(\mathit{une\ pizza})(\lambda o\ ((\mathit{prendront}\ o)\ s))\\
&= (\lambda Q^{e\to t}\ (\exists^{(e\to t)\to t}\ (\lambda x^e(\wedge^{t\to(t\to t)}((\mathtt{pizza}^{e\to t}\ x)))(Q\ x))))\\
&(\lambda o\ (((\lambda y^e\ \lambda x^e\ ((\mathtt{prendront}^{e\to(e\to t)}\ x)\ y))\ o)\ s)))\\
&= (\lambda Q^{e\to t}\ (\exists^{(e\to t)\to t}\ (\lambda x^e(\wedge^{t\to(t\to t)}((\mathtt{pizza}^{e\to t}\ x)))(Q\ x))))\\
&(\lambda o\ ((\mathtt{prendront}^{e\to(e\to t)}\ s)\ o))\\
&= (\exists^{(e\to t)\to t}\ (\lambda x^e(\wedge^{t\to(t\to t)}((\mathtt{pizza}^{e\to t}\ x)))((\lambda o\ ((\mathtt{prendront}^{e\to(e\to t)}\ s)\ o))\ x)))\\
&= (\exists^{(e\to t)\to t}\ (\lambda x^e(\wedge^{t\to(t\to t)}((\mathtt{pizza}^{e\to t}\ x)))((\mathtt{prendront}^{e\to(e\to t)}\ s)\ x)))\\
&\forall\exists = (\mathit{les\ enfants})(\lambda s.\ (\mathit{une\ pizza})(\lambda o\ ((\mathit{prendront}\ o)\ s)))\\
&= (\lambda Q^{e\to t}\ (\forall^{(e\to t)\to t}\ (\lambda u^e(\Rightarrow^{t\to(t\to t)}\ (\mathtt{enfants}^{e\to t}\ u)(Q\ u)))))\\
&(\lambda s.\ (\exists^{(e\to t)\to t}\ (\lambda x^e(\wedge^{t\to(t\to t)}((\mathtt{pizza}^{e\to t}\ x)))((\mathtt{prendront}^{e\to(e\to t)}\ s)\ x))))\\
&= (\forall^{(e\to t)\to t}\ (\lambda u^e(\Rightarrow^{t\to(t\to t)}\ (\mathtt{enfants}^{e\to t}\ u)\\
&((\lambda s.\ (\exists^{(e\to t)\to t}\ (\lambda x^e(\wedge^{t\to(t\to t)}((\mathtt{pizza}^{e\to t}\ x)))((\mathtt{prendront}^{e\to(e\to t)}\ s)\ x))))\ u)))))
\end{aligned}
$$

$$
\begin{aligned}
&= (\forall^{(e\to t)\to t}\ (\lambda u^e(\Rightarrow^{t\to(t\to t)}\ (\mathtt{enfants}^{e\to t}\ u)\\
&(\exists^{(e\to t)\to t}\ (\lambda x^e.\ (\wedge^{t\to(t\to t)}((\mathtt{pizza}^{e\to t}\ x)))((\mathtt{prendront}^{e\to(e\to t)}\ u)\ x))))))
\end{aligned}
$$

ce qui s'écrit communément :

$$\forall u.\ \mathit{enfants}(u) \Rightarrow \exists .x\ \mathit{pizza}(x) \wedge \mathit{prendront}(u,x)$$