Aide à la décision/ Decision aid

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Partie 2/Part 2
Gestion des incohérences/Inconsistency handling

What is a knowledge?

It is information we have about agents' beliefs or preferences.

Outline

- Classical logic
- Inference in classical logic
- Non-classical inference/Inconsistency handling methods: The flat case
- Non-classical inference/Inconsistency handling methods: The prioritized case
- Reasoning about prioritized knowledge

Brief background on classical propositional logic

- Let V be a set of propositional variables denoted by $a, b, c, \cdots$
- Propositional language PL_V is built on
  - V,
  - $\{\top, \bot\}$ (respectively tautology and contradiction),
  - usual connectors $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$.
- Formulas in PL_V are denoted by $\phi, \psi, \varphi, \cdots, \phi_1, \phi_2, \cdots, \psi_1, \psi_2, \cdots, \varphi_1, \varphi_2, \cdots$
- Inference is denoted by $\vdash$
- Let $\Sigma = \{\phi_1, \cdots, \phi_n\}$
  - $\bigwedge \Sigma = \phi_1 \wedge \cdots \wedge \phi_n$
  - $\bigvee \Sigma = \phi_1 \vee \cdots \vee \phi_n$

Inference

Given ϕ and $\phi \rightarrow \psi$ we deduce ψ.
We write $\phi, \phi \rightarrow \psi \vdash \psi$

Complementary definitions

- Let Σ be a multiset of formulas (the same formula may be present several times)
- A sub-base A of Σ ($A \subseteq \Sigma$) is consistent if and only if $\bigwedge A \nvdash \perp$
- We suppose that Σ is not deductively closed

$A \subseteq \Sigma$ is maximally consistent if and only if

- A is consistent, and
- either $A = \Sigma$ or adding any formula ϕ from $\Sigma \backslash A$ to A entails the inconsistency of $A \cup \{\phi\}$

$A \subseteq \Sigma$ is minimally inconsistent if and only if

- $A \vdash \perp$, and
- $\forall \phi \in A$, $A \backslash \{\phi\} \nvdash \perp$

Let $Inc(\Sigma) = \{\phi, \exists A \subseteq \Sigma$ such that $\phi \in A$, A is minimally inconsistent$\}$. $Inc(\Sigma)$ is the set of formulas belonging to at least one minimally inconsistent sub-base of Σ.

Where does inconsistency come from?

- The presence of exceptions

Example

- Penguins are birds
- Birds fly
- A penguin doesn't fly
- Titi is a penguin

- Knowledge is provided from multiple sources

Example

- Agent 1: We will have a course tomorrow
- Agent 2: If it rains then we don't have course
- Agent 3: It will rain tomorrow

How to solve conflicts?

- Revise the knowledge base and restore consistency
- Cope with inconsistency and define non-classical inference

Non-classical inference/Inconsistency handling methods

The flat case

An argumentative consequence relation (1)

A sub-base A of Σ is an argument for a formula ϕ iff

1. $A \nvdash \perp$ (A is consistent),
2. $A \vdash \phi$,
3. $\forall \psi \in A$, $A \backslash \{\psi\} \nvdash \phi$ (A is minimal)

We write $\langle A, \phi \rangle$ which stands for "A is an argument for ϕ".

- Why condition 1?
  From an inconsistent base we can deduce a formula and its contradiction.
- Why condition 2?
  (ϕ: it is raining), (ψ: I should use my umbrella).
  Is $\langle \{\phi\}, \psi \rangle$ an argument??
- Why condition 3?
  (ϕ: it is raining), ($\phi \rightarrow \psi$: if it is raining, then I should use my umbrella), (φ: I like paprika).
  Is $\langle \{\phi, \phi \rightarrow \psi, \varphi\}, \psi \rangle$ an argument??

An argumentative consequence relation (2)

ϕ is an argumentative consequence of Σ, denoted by $\Sigma \vdash_{\mathcal{A}} \phi$, iff

- there exists an argument for ϕ in Σ, and
- there is no argument for $\neg\phi$ in Σ

Example

Consider a discussion in a newspaper editorial office about whether or not to proceed with the publication of some indiscretion about a prominent politician. Suppose the key bits of information are captured by the following five statements.

- Simon Jones is a Member of Parliament
- If Simon Jones is a Member of Parliament then we need not keep quiet about details of his private life
- Simon Jones just resigned from the House of Commons
- If Simon Jones just resigned from the House of Commons then he is not a Member of Parliament
- If Simon Jones is not a Member of Parliament then we need to keep quiet about details of his private life

How good is argumentative consequence?

Example: Let $\Sigma = \{\psi, \psi \to \phi, \neg\psi\}$

- Is ϕ an argumentative consequence of Σ?

Argumentative vs classical inference

Under which conditions we have

$$\Sigma \vdash \phi \text{ iff } \Sigma \vdash_{\mathcal{A}} \phi?$$

Free consequence

Definition: Free formula

$\varphi \in \Sigma$ is free iff $\varphi \notin \mathit{Inc}(\Sigma)$. $\mathit{Free}(\Sigma) = \Sigma \backslash \mathit{Inc}(\Sigma)$

Definition: Free consequence

ϕ is a free consequence of Σ, denoted by $\Sigma \vdash_{\mathit{Free}} \phi$, iff $\mathit{Free}(\Sigma) \vdash \phi$

Example: $\Sigma = \{\phi, \neg\phi \vee \neg\psi, \psi, \neg\phi \vee \varphi, \neg\psi \vee \varphi\}$

How do argumentative and free consequences relate?

Universal consequence (or MC-consequence)

$MC(\Sigma)$ is the set of maximally consistent sub-bases of Σ

Definition

ϕ is a MC-consequence of Σ, denoted by $\Sigma \vdash_{MC} \phi$, iff $\forall A \in MC(\Sigma)$, $A \vdash \phi$

Example: $\Sigma = \{\phi, \neg\phi \vee \neg\psi, \psi, \neg\phi \vee \varphi, \neg\psi \vee \varphi\}$

How do MC- and argumentative (resp. free) consequences relate?

Lexicographical (or cardinality-based) consequence (1)

One of the main difficulties for implementing the MC-consequence is the cardinality of $MC(\Sigma)$ which exponentially increases with the number of conflicts in Σ. The lexicographical consequence is based on a subset of $MC(\Sigma)$.

$$A \in L(\Sigma) \text{ iff } A \in MC(\Sigma) \text{ and } \forall B \in MC(\Sigma),\ |A| \geq |B|,$$

where $|\Gamma|$ is the cardinality (number of elements) of Γ.

Definition: L-consequence

ϕ is a L-consequence of Σ, denoted by $\Sigma \vdash_L \phi$, iff $\forall A \in L(\Sigma)$, $A \vdash \phi$

Lexicographical (or cardinality-based) consequence (2)

Example: $\Sigma = \{\psi \rightarrow \phi, \psi, \neg\psi, \neg\phi \wedge \neg\psi, \neg\phi\}$

How do lexicographical and MC- (resp. argumentative-) consequences relate?

Existential consequence

Definition

ϕ is an existential consequence of Σ, denoted by $\Sigma \vdash_\exists \phi$, iff $\exists A \in MC(\Sigma)$, $A \vdash \phi$

Example: $\Sigma = \{\psi \rightarrow \phi, \psi, \neg\psi, \neg\phi \wedge \neg\psi, \neg\phi\}$

How do existential and lexicographical (resp. argumentative) consequences relate?

Link between the consequence relations

Non-classical inference/Inconsistency handling methods

The prioritized case

What are priorities and where do they come from?

- Priority is very important in the study of knowledge-based systems
- A priority represents uncertainty or preferences associated with knowledge
- Priorities may represent the reliability/preference of sources/knowledge
- The task of coping with inconsistency is greatly simplified since conflicts have better chance to be solved

Prioritized knowledge bases

- Prioritized knowledge bases are layered, that is, of the form $\Sigma = S_1 \cup \cdots \cup S_n$, such that formulas in S_i have the same priority and are more prioritized than formulas in S_j with $j > i$. Sometimes we write $\Sigma = \{S_1, \cdots, S_n\}$
- Each S_i is a multiset: copies of the same formula may appear in the same layer or in different layers. They are considered as distinct
- $\phi \in \Sigma$ means that there is a copy of ϕ in Σ
- We sometimes use the notation $\phi \in_i \Sigma$ for $\phi \in S_i$
- $\phi \notin_i \Sigma$ means that $\phi \notin S_i$ (however it may belong to other layers of Σ)
- Adding a formula ϕ to the ith layer S_i of Σ is denoted $\Sigma \cup \{\phi\}_i$
- Sub-bases are denoted by capital letters $A, B, C, \ldots$. They are also represented in a stratified form $A = A_1 \cup \cdots \cup A_n$, with $A_i \subseteq S_i$ (A_j may be empty)

Possibilistic consequence (1)

Definition: i-consequence

Let $\Sigma = S_1 \cup \cdots \cup S_n$ be a layered belief base. A formula ϕ is an i-consequence of Σ, denoted by $\Sigma \vdash_i \phi$, iff:

1. $S_1 \cup \cdots \cup S_i$ is consistent,
2. $S_1 \cup \cdots \cup S_i \vdash \phi$, and
3. $\forall j < i$, $S_1 \cup \cdots \cup S_j \not\vdash \phi$.

Definition: π-consequence

ϕ is a π-consequence of Σ if ϕ is an i-consequence of Σ for some i, and denote it by $\Sigma \vdash_\pi \phi$.

Possibilistic consequence (2)

It is possible to characterize the set of π-consequences of Σ.

- We compute the set $\pi(\Sigma)$ defined as: $\pi(\Sigma) = S_1 \cup \cdots \cup S_i$, such that $S_1 \cup \cdots \cup S_i$ is consistent and $S_1 \cup \cdots \cup S_{i+1}$ is inconsistent.
- If Σ is consistent then $\pi(\Sigma) = \Sigma$.
- $i + 1$ is called the *inconsistency degree* of Σ. If Σ is consistent then $Inc(\Sigma) = 0$.
- $\Sigma \backslash \pi(\Sigma)$ is simply inhibited.

It is not hard to check that

$\Sigma \vdash_\pi \phi$ iff $\pi(\Sigma) \vdash \phi$

Example

- Let $\Sigma_1 = \{\{p\}, \{\neg p \vee b, \neg p \vee \neg f\}, \{\neg b \vee f\}\}$
- Let $\Sigma_2 = \{\{p\}, \{\neg p \vee b, \neg p \vee \neg f\}, \{\neg b \vee w\}\}$

Possibilistic consequence (3)

Dealing with inconsistency using possibilistic consequence is not entirely satisfactory. The latter suffers from the "drowning problem".

Example

Let $\Sigma = \{\{\neg\phi \vee \neg\psi\}, \{\phi\}, \{\psi\}, \{\xi\}\}$.

A particular case of the drowning effect is called "blocking of property inheritance".

$\Sigma = \{\{p\}, \{\neg p \vee b, \neg p \vee \neg f\}, \{\neg b \vee f, \neg b \vee w\}\}$,

p, b, f, w respectively mean penguin, bird, fly, wings.
Based on Σ, it is not possible for a penguin to inherit properties of birds (in our example, to inherit property of having wings), while the only undesirable property for a penguin in our example is "flying".

Free consequence (1)

Definition: Dominant sub-base

The dominant sub-base of Σ is
$\Sigma^* = Free(S_1) \cup Free(S_1 \cup S_2) \cup \cdots \cup Free(S_1 \cup \cdots \cup S_n)$.

Properties of the dominant sub-base

- For a given $i \geq 1$, if a formula ϕ in $S_1 \cup \cdots \cup S_i$ does not belong to $Free(S_1 \cup \cdots \cup S_i)$ then it will not belong to $Free(S_1 \cup \cdots \cup S_k)$ for $k \geq i$.
- In general, there is no inclusion relation between $Free(S_1 \cup \cdots \cup S_i)$ and $Free(S_1 \cup \cdots \cup S_k)$ with $k > i$.

Definition: Free consequence

A formula ϕ is a free consequence of Σ, denoted by $\Sigma \vdash_{ND}$, iff $\Sigma^* \vdash \phi$.

Free consequence (2)

Example

Let $\Sigma = \{\{p\}, \{\neg p \vee b, \neg p \vee \neg f\}, \{\neg b \vee f, \neg b \vee w\}\}$

Linear consequence (1)

The sub-base $l(\Sigma)$ is obtained by dropping layers S_i when they are inconsistent with the previous ones. Namely, $l(\Sigma)$ is constructed in the following way:

$$l(S_1) = \begin{cases} S_1 & \text{if } S_1 \text{ is consistent} \\ \emptyset & \text{otherwise.} \end{cases}$$

for $i > 1$:

$$l(S_1 \cup \cdots \cup S_i) = \begin{cases} l(S_1 \cup \cdots \cup S_{i-1}) \cup S_i & \text{if } \textit{consistency} \\ l(S_1 \cup \cdots \cup S_{i-1}) & \text{otherwise.} \end{cases}$$

Definition: Linear consequence

ϕ is a linear consequence of Σ, denoted by $\Sigma \vdash_l \phi$, iff $l(\Sigma) \vdash \phi$.

Linear consequence (2)

Example

$\Sigma = \{\{p\}, \{\neg p \vee b, \neg p \vee \neg f\}, \{\neg b \vee f, \neg b \vee w\}\}$

Remark

- Linear consequence is more productive than possibilistic consequence. However it does not solve the problem of the drowning effect.
- Linear consequence and free consequence are incomparable.

Argumentative consequence (1)

Definition: A reason/argument of rank i

A sub-base A of Σ is a reason of rank i for a formula ϕ if it satisfies the following conditions:

1. $A \nvdash \bot$
2. $A \vdash \phi$
3. $\forall \psi \in A,\ A \backslash \{\psi\} \nvdash \phi$
4. $R(A) = \max\{j | A \cap S_j \neq \emptyset\} = i$

Definition: Argumentative consequence

ϕ is an argumentative consequence of Σ, denoted by $\Sigma \vdash_{\mathcal{A}} \phi$, iff:

- there exists an argument of rank i for ϕ in Σ, and
- arguments for $\neg\phi$ (if any) are of rank $j > i$.

Argumentative consequence (2)

Example

Let $\Sigma = \{\{\neg c\}, \{\neg a \vee \neg b \vee c, \neg d \vee c, \neg e \vee c\}, \{d, e, f, \neg f \vee \neg g \vee c\}, \{a, b, g, h\}\}$

Application: Access Control in Medical Problem

1. If Mary plays patient role then she plays non staff member role.
2. If Mary plays patient role then she is permitted to read her medical record.
3. If Mary plays a non staff member role then she is not permitted to read her medical record.
   If Jean plays doctor role then he is permitted to read Mary's record.

Question

Is Mary permitted to read her medical record?