Database Theory and Knowledge Representation

1st Lecture

David Carral

University of Montpellier

September 28, 2023

Personal Information

- Name: David Carral
- Job Title: CRCN Researcher at Inria
- Research Team: Boreal
- Email: david.carral@inria.fr
- Webpage: `https://www-sop.inria.fr/members/David.Carral/`
- Photo:

Contact, Questions, and Course Materials

- I will only teach 3 lectures: 28/9, 5/10, and 12/10
- You can ask me questions after each lecture or via email. If this is not enough, we can arrange a meeting at LIRMM:

  Salle 131 - Batîment 5
  Campus Saint Priest - Université Montpellier
  860 Rue St - Priest, 34090 Montpellier

- I will upload all slides in Moodle and provide you with additional study materials (see the following slides).

Goals of (this Part of) the Lecture

Obtain an understanding of database theory with a focus on different query languages:

- Relational data model
- Query languages: first-order, conjunctive, and tree-like
- Expressive power of query languages
- Complexity of query answering

Remark

We will not only discuss theoretical results but will also do some exercises to understand how to use the above languages.

Prerequisites

1. Turing machines
2. First-order logic: syntax and semantics
3. Computational complexity

Remark

I will reintroduce most of the concepts from computational complexity that are applied in this lecture (if necessary).

Additional Materials

1. Database Theory:
   - *Foundations of Databases*
     Serge Abiteboul, Richard Hull, Victor Vianu
     Available at `http://webdam.inria.fr/Alice/`
   - *Database Theory* Course at TU Dresden
     Available at `https://iccl.inf.tu-dresden.de/web/Database_Theory_(SS2020)/en`
2. Computational Complexity:
   - *Foundations of Complexity Theory* Course at TU Dresden
     Available at `https://iccl.inf.tu-dresden.de/web/Complexity_Theory_(WS2020)/en`
   - *Introduction to the Theory of Computation*
     Michael Sipser
   - *Computational Complexity: A Modern Approach*
     Boaz Barak and Sanjeev Arora

Acknowledgements

This lecture is heavily inspired by the *Database Theory* lectures at TU Dresden, which were created by Prof. Markus Krötzsch:

Additional links to lectures from previous years:

- 2016: https://iccl.inf.tu-dresden.de/web/Database_Theory_(SS2016)/en
- 2018: https://iccl.inf.tu-dresden.de/web/Database_Theory_(SS2018)/en
- 2019: https://iccl.inf.tu-dresden.de/web/Database_Theory_(SS2019)/en

(Tentative) Outline

1. What is a database?
2. The relational data model
3. Relational algebra queries
4. First-order queries
5. Conjunctive queries
6. Tree-like queries

What is a database?

A Database Management System (DBMS) is a software to manage collections of data.

$\rightsquigarrow$ highly important class of software systems
$\rightsquigarrow$ major role in industry and in research
$\rightsquigarrow$ extremely wide variety of concepts and implementations

General three-level architecture of DBMS:

- External Level: application-specific user views
- Logical Level: abstract data model, independent of implementation, conceptual view
- Physical Level: data structures and algorithms, platform-specific

In this lecture: focus on logical view for relational data model

What is a database? (2)

Basic functionality of DBMS:

- Schema definition: specify how do we logically organise data
- Update: insert/delete/update stored data
- Query: retrieve stored data or information derived from it
- Administration: user rights management, configuration, recovery, data export, etc.

Many related concerns:

- Persistence: data retained when DBMS is shut down
- Optimisation: ensure maximal efficiency
- Scalability: cope with increasing loads by adding resources
- Concurrency: support update/query operations in parallel
- Distribution: combine data from several locations
- Interfaces: APIs, query languages, update languages, etc.

In this lecture: schema and query languages

Database = Collection of Tables

Lines:

Line	Type
85	bus
3	tram
F1	ferry
...	...

Stops:

SID	Stop	Accessible
17	St-Guilhem	true
42	Foch	true
57	Comédie	true
123	Av. Liberté	false
...	...	...

Connect:

From	To	Line
57	42	85
17	789	3
...	...	...

Every table has a schema:

- Lines[Line:string, Type:string]
- Stops[SID:int, Stop:string, Acc:bool]
- Connect[From:int, To:int, Line:string]

Remark

Answering queries requires integrating info from different tables.

Towards a Formal Definition of "Table"

A table row has one value for each column. Hence, a row can be represented by a function from attributes to values.

Example: The row

SID	**Stop**	**Accessible**
...	...	...
42	Comédie	true
...	...	...

can be represented by the function:

$$f : \{\text{SID} \mapsto 42, \text{Stop} \mapsto \texttt{"Comédie"}, \text{Accessible} \mapsto \text{true}\}$$

Remark

The above is an "abstract data model" and not a "data structure". We focus in the logical level of the DBMS architecture.

The Domain of a Database

In order to give a definition for a database instance, we need to introduce the universe of elements that appear in the tables.

Definition: Domain

Let **dom** ("domain") be the (possibly infinite) set of conceivable values in tables.

Remark

For simplicity, we drop the datatypes of database columns and assume that each column uses the same datatype that supports all values in **dom**.

Database = Set of Tables

Definition: Named Perspective

- A relation schema $R[U]$ consists of a relation name R and a finite set U of attributes ($|U|$ is the arity of $R[U]$)
- A table for $R[U]$ is a finite set of functions from U to **dom**
- A database instance $\mathcal{I}$ is a finite set of tables

Remark

Note that we disregard the order and multiplicity of rows.

Under the "set of tables" perspective, tables are also called *relation instances*. The table with relation schema $R[U]$ in the database instance $\mathcal{I}$ is written $R^{\mathcal{I}}$.

Database = Set of Tables

Example 2.5

Consider the database instance $\mathcal{I}$ that contains the tables:

Lines:

Line	**Type**
85	bus
33	tram

Stops:

SID	**Stop**	**Accessible**
17	St-Guilhem	true

Under the "Set of Tables" perspective, the database instance $\mathcal{I}$ is the set $\{\text{Lines}^{\mathcal{I}}, \text{Stops}^{\mathcal{I}}\}$ where:

$$\begin{aligned}\text{Lines}^{\mathcal{I}} &= \{\{\textbf{Line} \mapsto 85, \textbf{Type} \mapsto \text{bus}\},\\ &\qquad \{\textbf{Line} \mapsto 33, \textbf{Type} \mapsto \text{tram}\}\}\\ \text{Stops}^{\mathcal{I}} &= \{\{\textbf{SID} \mapsto 17, \textbf{Stop} \mapsto \text{St-Guilhem}, \textbf{Accessible} \mapsto \text{true}\}\}\end{aligned}$$

Database = Set of Facts

Another way to encode the rows in a database is using facts:

$$\mathrm{Lines}(85, \texttt{"bus"}) \qquad \mathrm{Stops}(42, \texttt{"Comédie"}, \mathrm{true})$$

$$\mathrm{Lines}(\mathrm{F1}, \texttt{"ferry"})$$

Definition: Unnamed Perspective

A fact is an expression $p(t_1, \ldots, t_n)$ where

- p is an n-ary predicate symbol
- t_1, …, t_n are constant symbols

A database instance is a finite set of facts.

Remark

Constant symbols in facts are elements of **dom**.

Database = Set of Facts

Example

Consider the database instance $\mathcal{I}$ that contains the tables:

Lines:

Line	**Type**
85	bus
33	tram

Stops:

SID	**Stop**	**Accessible**
17	St-Guilhem	true

Under the "Set of Facts" perspective, the database instance $\mathcal{I}$ is the set of facts:

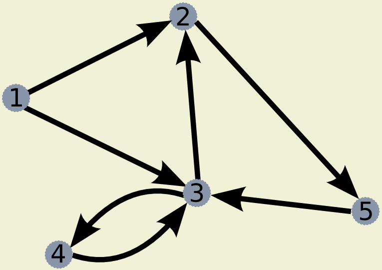
$$\mathcal{I} = \{\mathsf{Lines}(85, \mathsf{bus}), \mathsf{Lines}(33, \mathsf{tram}), \\ \mathsf{Stops}(17, \mathsf{St\text{-}Guilhem}, \mathsf{true})\}$$

Visualising relations

Example

Binary relations (sets of pairs) can be viewed as directed graphs.

Source	**Target**
1	2
1	3
2	5
3	2
3	4
4	3
5	3



Many binary tables in one graph? Use table name to label edges!

Database = Hypergraph

What to do with tables of arity $\neq 2$?
$\rightsquigarrow$ Generalise graphs to hypergraphs!

Definition 2.10: Hypergraph Perspective

A hypergraph is a triple $\langle V, E, \rho \rangle$, where

- V is a set of vertices
- E is a set of edge names
- ρ maps each edge name $e \in E$ to an n-ary relation $\rho(e) \subseteq V^n$

In other words: finite hypergraphs are databases!

Definitions of a Database

We consider three different definitions for a database:

- A *set of tables*, which are sets of functions from the corresponding set of attributes to **dom**.
- A *set of facts* wherein the predicates are the table names and the constants are elements of **dom**.
- A *hypergraph* where hyperedges are labeled with table names and vertices are elements of **dom**.

Relational Algebra Queries

Query language based on a set of operations on databases.

Each operation takes one or more tables as input and produces another table as output

(we often simplify notation and write a table name rather than a table instance)

Main operations:

- Selection σ
- Projection π
- Join $\bowtie$
- Renaming δ
- Difference $-$
- Union $\cup$
- Intersection $\cap$

Remark

The RA is the theoretical language underpinning SQL.

Selection

"Find all bus lines"

$$\sigma_{\mathsf{Type}=\texttt{"bus"}}\mathsf{Lines}$$

"Find all connections that begin and end in the same stop"

$$\sigma_{\mathsf{From}=\mathsf{To}}\mathsf{Connect}$$

Definition 3.1: Selection Operator

The selection operator has the form $\sigma_{n=m}$

- n is an attribute name
- m is an attribute name or a constant value

Consider a table $R^{\mathcal{I}}$ for $R[U]$.

- For m constant value: $\sigma_{n=m}(R^{\mathcal{I}}) = \{f \in R^{\mathcal{I}} \mid f(n) = m\}$
- For m attribute name: $\sigma_{n=m}(R^{\mathcal{I}}) = \{f \in R^{\mathcal{I}} \mid f(n) = f(m)\}$

This is only defined if U contains the required attribute names.

Projection

"Find all possible types of lines"

$$\pi_{\mathsf{Type}}\mathsf{Lines}$$

"Find all pairs of adjacent stops on line 85"

$$\pi_{\mathsf{From},\mathsf{To}}(\sigma_{\mathsf{Line}=\texttt{"85"}}\mathsf{Connect})$$

Definition 3.2: Projection Operator

The projection operator has the form $\pi_{a_1,\ldots,a_n}$ where each a_i is an attribute name.

Consider a table $R^{\mathcal{I}}$ for $R[U]$.

$$\pi_{a_1,\ldots,a_n}(R^{\mathcal{I}}) = \left\{ f_{\{a_1,\ldots,a_n\}} \mid f \in R^{\mathcal{I}} \right\}$$

where $f_{\{a_1,\ldots,a_n\}}$ is the restriction of f to the domain $\{a_1,\ldots,a_n\}$; that is, the function $\{a_1 \mapsto f(a_1),\ldots,a_n \mapsto f(a_n)\}$. Of course, this projection is only defined if $a_i \in U$ for each a_i.

Natural join

"Find all connections and their type of line":

Connect:

Fr	To	Line
57	42	85
17	789	3
...	...	...

Lines:

Line	Type
85	bus
3	tram
F1	ferry
...	...

Connect $\bowtie$ Lines:

Fr	To	Line	Type
57	42	85	bus
17	789	3	tram
...	...	...	...

Definition 3.3: Natural Join Operator

The natural join operator has the form $\bowtie$.

Consider tables $R^{\mathcal{I}}$ for $R[U]$ and $S^{\mathcal{I}}$ for $S[V]$.

$$R^{\mathcal{I}} \bowtie S^{\mathcal{I}} = \{f : U \cup V \to \mathbf{dom} \mid f_U \in R^{\mathcal{I}} \text{ and } f_V \in S^{\mathcal{I}}\}$$

where f_U (f_V) is the restriction of f to elements in U (V) as before

Renaming

"Find all lines that depart from an accessible stop"

Stops:

SID	Stop	Accessible
57	Comédie	true
123	St-Guilhem	false
...	...	...

Connect:

From	To	Line
57	42	85
17	789	3
...	...	...

We need to join Stops.SID with Connect.From $\rightsquigarrow$ use renaming

$$\pi_{Line}\big(\sigma_{\text{Accessible}=\texttt{"true"}}(\text{Stops} \bowtie \delta_{\text{From,To,Line}\to\text{SID,To,Line}}(\text{Connect}))\big)$$

Definition 3.3: Renaming Operator

The renaming operator has the form $\delta_{a_1,\ldots,a_n\to b_1,\ldots,b_n}$ with all a_i mutually distinct attribute names, and likewise for all b_i.

Consider a table $R^{\mathcal{I}}$ for $R[\{a_1,\ldots,a_n\}]$.

$$\delta_{a_1,\ldots,a_n\to b_1,\ldots,b_n}(R^{\mathcal{I}}) = \{f \circ g \mid f \in R^{\mathcal{I}} \text{ and } g : \{b_i \mapsto a_i\}_{1\leq i\leq n}\}$$

where $f \circ g$ is function composition: $(f \circ g)(x) = f(g(x))$

Difference, Union, Intersection

Binary operators on tables of the same relational schema, defined like the usual set operations.

"Find all stops where line 3 departs, but line 8 does not depart."

"Find all stops where either line 3 or line 8 departs."

"Find all stops where both line 3 and line 8 depart."

Table constants in queries

It is sometimes convenient to define constant tables in queries.

"Find all stops near Helmholtzstr. (SID 42), including Helmholtzstr."

$$\delta_{\mathsf{To} \to \mathsf{StopId}}(\pi_{\mathsf{To}}(\sigma_{\mathsf{From}=\texttt{"42"}} \mathsf{Connect})) \cup \left\{\{\mathsf{StopId} \mapsto 42\}\right\}$$

One can generalise this to constant tables with more than one column or more than one table.

Reachability

Generalising the previous example:

"Stops that are Helmholtzstr."

$$R_0 = \{\{\mathsf{From} \mapsto 42\}\}$$

"Stops that are next to Helmholtzstr."

$$R_1 = \delta_{\mathsf{To}\to\mathsf{From}}(\pi_{\mathsf{To}}(\mathsf{Connect} \bowtie R_0))$$

"Stops at distance 2 from Helmholtzstr."

$$R_2 = \delta_{\mathsf{To}\to\mathsf{From}}(\pi_{\mathsf{To}}(\mathsf{Connect} \bowtie R_1))$$

Stops reachable from Helmholtzstr. with a short-distance ticket:

$$R_0 \cup R_1 \cup R_2 \cup R_3 \cup R_4$$

What about all stops reachable from Helmholtzstr.?
$\rightsquigarrow$ see upcoming lectures . . .

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
Internet's Own Boy	Knappenberger	Swartz
Internet's Own Boy	Knappenberger	Lessig
Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck

Venues

Cinema	Address	Phone
UFA	St. Peter St. 24	4825825
Diagon	King St. 55	8032185
...	...	...

Program

Cinema	Title	Time
Diagon	The Imitation Game	19:30
Diagon	Dogma	20:45
UFA	The Imitation Game	22:45

1. Who is the director of "The Imitation Game"?

$$\pi_{Director}(\sigma_{Title=\text{“The Imitation Game”}}(Films))$$

2. Which cinemas feature "The Imitation Game"?

$$\pi_{Cinema}(\sigma_{Title=\text{“The Imitation Game”}}(Program))$$

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
Internet's Own Boy	Knappenberger	Swartz
Internet's Own Boy	Knappenberger	Lessig
Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck

Venues

Cinema	Address	Phone
UFA	St. Peter St. 24	4825825
Diagon	King St. 55	8032185
...	...	...

Program

Cinema	Title	Time
Diagon	The Imitation Game	19:30
Diagon	Dogma	20:45
UFA	The Imitation Game	22:45

3. What are the address and phone number of "Schauburg"?

$$\pi_{Address,Phone}(\sigma_{Cinema=\text{“Schauburg”}}(Venues))$$

4. *Boolean query:* Is a film directed by "Smith" playing?

$$\pi_{\emptyset}(\sigma_{Director=\text{“Smith”}}(Films) \bowtie Program)$$

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
Internet's Own Boy	Knappenberger	Swartz
Internet's Own Boy	Knappenberger	Lessig
Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck

Venues

Cinema	Address	Phone
UFA	St. Peter St. 24	4825825
Diagon	King St. 55	8032185
...	...	...

Program

Cinema	Title	Time
Diagon	The Imitation Game	19:30
Diagon	Dogma	20:45
UFA	The Imitation Game	22:45

5. List the pairs of persons such that the first directed the second in a film, and vice versa.

$$\pi_{Director,D}(\sigma_{Director=A}(\sigma_{Actor=D}(\delta_{Title,Director,Actor \rightarrow T,D,A}(Films) \bowtie Films)))$$

6. List the names of directors who have acted in a film they directed.

$$\pi_{Director}(\sigma_{Actor=Director}(Films))$$

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
Internet's Own Boy	Knappenberger	Swartz
Internet's Own Boy	Knappenberger	Lessig
Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck

Venues

Cinema	Address	Phone
UFA	St. Peter St. 24	4825825
Diagon	King St. 55	8032185
...	...	...

Program

Cinema	Title	Time
Diagon	The Imitation Game	19:30
Diagon	Dogma	20:45
UFA	The Imitation Game	22:45

7. Return $\{\text{Title} \mapsto \texttt{"Ap. Now"}, \text{Director} \mapsto \texttt{"Coppola"}\}$ as the answer.

$$\{\{\mathit{Title} \mapsto \text{"Ap. Now"}\}\} \bowtie \{\{\mathit{Director} \mapsto \text{"Coppola"}\}\}$$

8. Find the actors cast in at least one film by "Smith".

$$\pi_{\mathit{Actor}}(\sigma_{\mathit{Director}=\text{"Smith"}}(\mathit{Films}))$$

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
Internet's Own Boy	Knappenberger	Swartz
Internet's Own Boy	Knappenberger	Lessig
Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck

Venues

Cinema	Address	Phone
UFA	St. Peter St. 24	4825825
Diagon	King St. 55	8032185
...	...	...

Program

Cinema	Title	Time
Diagon	The Imitation Game	19:30
Diagon	Dogma	20:45
UFA	The Imitation Game	22:45

9. Find the actors that are NOT cast in a movie by "Smith."

$$\pi_{Actor}(Films) - \pi_{Actor}(\sigma_{Director=\text{"Smith"}}(Films))$$

10. Find all pairs of actors who act together in at least one film.

$$\begin{aligned} q = \ & \pi_{Actor',Actor}[\delta_{Actor \to Actor'}(Films) \bowtie Films] \\ & q - \sigma_{Actor=\text{Actor}'}(q) \end{aligned}$$

Exercise 2: DIY

Consider the following identities and decide for each whether it is true or false. If true, prove your answer using the definitions from the lecture; if false, give a counterexample.

1. $R \bowtie S = S \bowtie R$
2. $R \bowtie (S \bowtie T) = (R \bowtie S) \bowtie T$
3. $\pi_X(R \circ S) = \pi_X(R) \circ \pi_X(S)$ for all $\circ \in \{\cup, \cap, -, \bowtie\}$
4. $\sigma_{n=m}(R \circ S) = \sigma_{n=m}(R) \circ \sigma_{n=m}(S)$ for all $\circ \in \{\cup, \cap, -\}$.
5. $\sigma_{n=m}(R \bowtie S) = \sigma_{n=m}(R) \bowtie S$, for n and m attributes of R only.

Discussion

Why are these identities of interest?

Bonus Exercise: DIY

The set of operations $\{\sigma, \pi, \cup, -, \bowtie, \delta\}$ can express all queries of relational algebra. (Note that $\cap$ is syntactic sugar.) Show that it is not possible to reduce this set any further.

Tip

For each operator, define a query that can be expressed using that operator. Then, show that this query cannot be expressed using the remaining operators.

Remark

Video solutions available at: `https://iccl.inf.tu-dresden.de/web/Database_Theory_(SS2020)/en`

What is a query?

The relational queries considered so far produced a result table from a database.

Other query languages can be completely different, but they usually agree on this:

Definition

- Syntax: a *query expression* q is a word from a query language (algebra expression, logical expression, etc.)
- Semantics: a *query mapping* $M[q]$ is a function that maps a database instance $\mathcal{I}$ to a database table $M[q](\mathcal{I})$

Review: Example from Section

Lines:

Line	Type
85	bus
3	tram
F1	ferry
...	...

Stops:

SID	Stop	Accessible
17	St-Guilhem	true
42	Foch	true
57	Comédie	true
123	Av. Liberté	false
...	...	...

Connect:

From	To	Line
57	42	85
17	789	3
...	...	...

Every table has a *schema*:

- Lines[Line:string, Type:string]
- Stops[SID:int, Stop:string, Accessible:bool]
- Connect[From:int, To:int, Line:string]

First-order Logic as a Query Language

Idea: database instances are finite first-order interpretations
$\rightsquigarrow$ use first-order formulae as query language
$\rightsquigarrow$ use the fact perspective (more natural here)

Examples (using schema as in previous lecture):

- Find all bus lines: $\mathit{Lines}(x, \text{bus})$
- Find all possible types of lines: $\exists y.\text{Lines}(y, x)$
- Find all lines that depart from an accessible stop:

$$\exists y_{\text{SID}}, y_{\text{Stop}}, y_{\text{To}}.\big(\text{Stops}(y_{\text{SID}}, y_{\text{Stop}}, \text{true}) \land \text{Connect}(y_{\text{SID}}, y_{\text{To}}, x_{\text{Line}})\big)$$

First-order Logic with Equality: Syntax

Basic building blocks:

- Predicate names with an arity ≥ 0: p, q, Lines, Stops
- Variables: x, y, z
- Constants: a, b, c
- Terms are variables or constants: s, t

Formulae of first-order logic are defined as usual:

$$\varphi ::= p(t_1, \ldots, t_n) \mid t_1 \approx t_2 \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \exists x.\varphi \mid \forall x.\varphi$$

where p is an n-ary predicate, t_i are terms, and x is a variable.

- An atom is a formula of the form $p(t_1, \ldots, t_n)$
- A literal is an atom or a negated atom
- Occurrences of variables in the scope of a quantifier are bound; other occurrences of variables are free

First-order Logic Syntax: Simplifications

We use the usual shortcuts and simplifications:

- flat conjunctions ($\varphi_1 \wedge \varphi_2 \wedge \varphi_3$ instead of $(\varphi_1 \wedge (\varphi_2 \wedge \varphi_3))$)
- flat disjunctions (similar)
- flat quantifiers ($\exists x, y, z.\varphi$ instead of $\exists x.\exists y.\exists z.\varphi$)
- $\varphi \rightarrow \psi$ as shortcut for $\neg\varphi \vee \psi$
- $\varphi \leftrightarrow \psi$ as shortcut for $(\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)$
- $t_1 \not\approx t_2$ as shortcut for $\neg(t_1 \approx t_2)$

But we always use parentheses to clarify nesting of $\wedge$ and $\vee$.

For instance, "$\varphi_1 \wedge \varphi_2 \vee \varphi_3$" is not allowed!

First-order Logic with Equality: Semantics

First-order formulas are evaluated over sets of facts $\mathcal{I}$.[1]

To interpret formulas with free vars, we need a variable assignment $\mathcal{Z} : \mathrm{Var} \to \mathbf{dom}$. For an atom α, let $\mathcal{Z}(\alpha)$ be the atom that results from replacing every variable x in α with $\mathcal{Z}(x)$.

A formula φ is satisfied by $\mathcal{I}$ and $\mathcal{Z}$, written $\mathcal{I}, \mathcal{Z} \models \varphi$, if:

- $\mathcal{I}, \mathcal{Z} \models p(t_1, \ldots, t_n)$ if $\mathcal{Z}(p(t_1, \ldots, t_n)) \in \mathcal{I}$
- $\mathcal{I}, \mathcal{Z} \models t_1 \approx t_2$ if $\mathcal{Z}(t_1 = t_2)$
- $\mathcal{I}, \mathcal{Z} \models \neg\varphi$ if $\mathcal{I}, \mathcal{Z} \not\models \varphi$
- $\mathcal{I}, \mathcal{Z} \models \varphi \land \psi$ if $\mathcal{I}, \mathcal{Z} \models \varphi$ and $\mathcal{I}, \mathcal{Z} \models \psi$
- $\mathcal{I}, \mathcal{Z} \models \varphi \lor \psi$ if $\mathcal{I}, \mathcal{Z} \models \varphi$ or $\mathcal{I}, \mathcal{Z} \models \psi$
- $\mathcal{I}, \mathcal{Z} \models \exists x.\varphi$ if there is $c \in \mathbf{dom}$ with $\mathcal{I}, \mathcal{Z} \cup \{x \mapsto c\} \models \varphi$[2]
- $\mathcal{I}, \mathcal{Z} \models \forall x.\varphi$ if for all $c \in \mathbf{dom}$ we have $\mathcal{I}, \mathcal{Z} \cup \{x \mapsto c\} \models \varphi$

[1] Reminder: a set of facts is also a database.

[2] W.l.o.g., assume that variables are quantified by at most one quantifier.

First-order Logic Queries

Definition: FO Queries

An n-ary first-order query q is an expression $\varphi[x_1, \ldots, x_n]$ where $x_1, \ldots, x_n$ are exactly the free variables of φ (in a specific order).

Definition: FO Query Answering

An answer to $q = \varphi[x_1, \ldots, x_n]$ over an interpretation $\mathcal{I}$ is a tuple $\langle a_1, \ldots, a_n \rangle$ of constants such that

$$\mathcal{I} \models \varphi[x_1/a_1, \ldots, x_n/a_n]^3$$

where $\varphi[x_1/a_1, \ldots, x_n/a_n]$ is φ with each free x_i replaced by a_i.

The result of q over $\mathcal{I}$ is the set of all answers of q over $\mathcal{I}$.

[3] Note that $\varphi[x_1/a_1, \ldots, x_n/a_n]$ does not feature answer variables.

Summary and Outlook

We have covered the following topics:

- The relational data model
- Relational queries
- First-order queries

Future Content:

- FO-Queries: exercises
- Complexity of query answering
- Query expressivity: comparing RA and FO queries