D – The Montagovian Generative Lexicon

D.1. Examples of Lexical Issues

Short roadmap:

- Restriction of selection, polysemy, felicity
- System-F and our framework
- Determiners and quantification
- Classical GL constructions
- Co-predication and constraints
- Deverbals
- Fictive motion
- Integrating plurals and their readings
- Specific issues

D.2. Restriction of Selection and Polysemy

Selection

- Predicates (syntactically) select arguments
- The lexical field of those arguments is restricted
- Other arguments can be forced to behave as expected

Differences in acceptability

- The dog barked.
- The chair barked.
- The drill sergeant barked.
- The hawker barked.

Contrastive ambiguity

- Different lexemes, homophobic / homographic
- Bank, Bar, Pen. . .

Relational polysemy

- A single word, different uses and related meanings
- The bank killed my account.
- The school is on strike.

D.3. Acceptability and Felicity: Semantics, Pragmatics or Both ?

Montague: everything is acceptable

- All syntactically valid items have the same semantic “meaning”
- We have to rely on pragmatics or interpretation

Too strong restriction from lexical semantics

- **e** is replaced by many sorts
- Barking dogs are licensed, everything else is blocked
- No language works that way

Creative uses and semantic licenses

- Fast runners, cars, computers... and phones
- A delicious game (Cooper)
- Expertly built (Adams)

Pragmatic licenses

- Variations from the lexicon in specific contexts external to the utterances
- The father becomes the son of the uncle on the left (Tree manipulation)
- The analyser cannot be expected to get a correct meaning without the external context

Our position on the integration of "Pragmatics"

- Creative use should be included
- Anything that can be comprehended in a self-contained text should be included
- We should not try to account for unknown contextual information
- World knowledge, background, social and universal contexts can be integrated
- Pragmatics should not be a pretext to give up on critical variations of meaning

D.4. System F

Types:

- $\mathbf{t}$ (prop)
- many entity types $\mathbf{e}_i$
- type variables $\alpha, \beta, \ldots$
- $\Pi\alpha.\ T$
- $T_1 \rightarrow T_2$

Terms

- Constants and variables for each type
- $(f^{T \rightarrow U} a^T) : U$
- $(\lambda x^T.\ u^U) : T \rightarrow U$
- $t^{(\Lambda\alpha.\ T)}\{U\} : T[U/\alpha]$
- $\Lambda\alpha.u^T : \Pi\alpha.T$ — no free α in a free variable of u.

The reduction is defined as follows:

- $(\Lambda\alpha.\tau)\{U\}$ reduces to $\tau[U/\alpha]$
  (remember that α and U are types).
- $(\lambda x.\tau)u$ reduces to $\tau[u/x]$ (usual reduction).

D.5. Unnecessary type operators

The following defined types have the same elimination and introduction behaviour.

(1) Product $A \wedge B$ can be defined as
$\Pi\alpha.\ (A \rightarrow B \rightarrow \alpha) \rightarrow \alpha$

(2) Sum $A \vee B$ can be defined as
$\Pi\alpha.\ (A \rightarrow \alpha) \rightarrow (B \rightarrow \alpha) \rightarrow \alpha$

(3) Existential quantification:
$\Sigma\beta.\ ((\Pi\alpha.\ (V[X] \rightarrow \beta)) \rightarrow \beta)$

(4) Inductive types (Church numerals, lists, trees, etc.)

A problem: encodings are unnatural. On going work: include predefined types (e.g. Gödel's integers of system T).

D.6. Inductive types (cf. ML, CaML, Haskell)

Integers
$\Pi\alpha.\ \alpha \to (\alpha \to \alpha) \to \alpha$

List of β objects. (the β can be quantified as well)
$\Pi\alpha.\ \alpha \to (\beta \to \alpha \to \alpha) \to \alpha$

Binary trees
$\Pi\alpha.\ \alpha \to (\alpha \to \alpha \to \alpha) \to \alpha$

Binary trees with leaves L and nodes N
$\Pi\alpha.\ (L \to \alpha) \to (N \to \alpha \to \alpha \to \alpha) \to \alpha$

D.7. What can be defined, computed

The function that can be programmed are the ones that can be proved total in second order Heyting arithmetic, that for such issues as the same power as second order Peano arithmetic.

All data types can be defined, and for such types their only normal terms are the expected ones.

More than polymorphic typed functional languages but every program terminates — there is no fixed point operator $Y : \Pi\alpha.\, (\alpha \to \alpha) \to \alpha$.

D.8. Basic facts on system F

Logicians / philosophers often ask whether system F is safe?

- We do not really need system F but any type system with quantification over types. F is syntactically the simplest. (Polynomial Soft Linear Logic of Lafont is enough)
- Confluence and strong normalisation — requires the comprehension axiom for all formulae of HA_2. (Girard 1971)
- A concrete categorical interpretation with coherence spaces that shows that there are distinct functions from A to B.
- Terms of type $\mathbf{t}$ with constants of mutisorted FOL (resp. HOL) correspond to multisorted formulae of FOL (resp. HOL)
- Possiblilty to have **coercive sub typing** for ontological inclusion (cats are animals etc.)

D.9. Examples of second order usefulness

Arbitrary modifiers: $\Lambda\alpha\lambda x^{A} y^{\alpha} f^{\alpha\to R}.((\mathrm{read}^{A\to R\to t}\ x)\ (f\ y))$

Polymorphic conjunction:

> Given predicates $P^{\alpha\to\mathbf{t}}$, $Q^{\beta\to\mathbf{t}}$ over respective types α, β,
>
> given any type ξ with two morphisms from ξ to α and to β
>
> we can coordinate the properties P, Q
> of (the two images of) an entity of type ξ:

The polymorphic conjunction $\&^{\Pi}$ is defined as the term

$$
\begin{aligned}
\&^{\Pi} = \Lambda\alpha\Lambda\beta\lambda P^{\alpha\to\mathbf{t}}\lambda Q^{\beta\to\mathbf{t}} \\
\Lambda\xi\lambda x^{\xi}\lambda f^{\xi\to\alpha}\lambda g^{\xi\to\beta}. \\
(\mathrm{and}^{\mathbf{t}\to\mathbf{t}\to\mathbf{t}}\ (P\ (f\ x))(Q\ (g\ x)))
\end{aligned}
$$

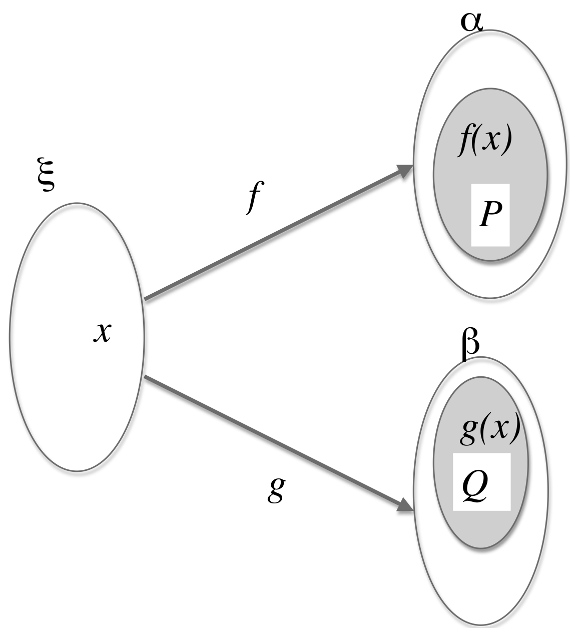

Figure 1: Polymorphic conjunction: $P(f(x))\&Q(g(x))$ with $x : \xi$, $f : \xi \to \alpha$, $g : \xi \to \beta$.

D.10. Coercive subtyping for F (Luo, Soloviev, Retoré)

Key property: at most one coercion between any two types.

Given coercions between base types.

Propagates through type hierarchy (unique possible restoration).

coercive application $\dfrac{f : A \to B \quad u : A_0 \quad A_0 < A}{(f\ a) : B}$

. . . .

$$\frac{A < B \quad C < D}{B \to A < C \to D} \qquad \frac{A < B}{X \to A < X \to B} \qquad \frac{A < B}{B \to X < A \to X}$$

. . . .

$$\frac{U < T[X]}{U < \Pi X.\ T[X]}\ X \textit{ not free in } U \qquad \frac{U < \Pi X.\ T[X]}{U < T[W]}$$

D.11. Coercive subtyping

Theorem: [hierarchical coherence] Whenever the above system derives $a < b$ where a and b are base types the coercion $a < b$ was a given coercion.

Coercive sub typing seems adequate to model ontological inclusions in particular between base types A car is a vehicle

These morphisms / coecions are identity on the object, hence only one morphism may exist between two given base types.

Possibly there are more coercions than ontological inclusions.

D.12. In a Nutshell

The Generative Lexicon

- Pustejovsky, 1995 (and precursors)
- Discussed and refined by Asher, Cooper, Luo...
- Idea: the lexicon provides enough data to generate word meanings in context

Framework sketch

- Types (and terms) from System-F
- Lexical entries are typed with many sorts
- Each word has a single main λ-term
- Each word can have any number of optional λ-terms
- Those terms are transformations, and are word-based
- Normal application is the same
- Transformations are used when types clash
- Types guide the selection of transformations

D.13. Lexicon v. Type

Why do we think transformations are lexical ?

(Rather than type-driven)

In a word: idiosyncrasy.

Consider:

(5) La classe est finie. (Event)

(6) La classe est fermée. (Location)

(7) La classe est de bon niveau. (People)

(8) La promotion est de bon niveau. (univoque: People)

In French, the two words do not have the same possible uses, but represent exactly the same group of people.

(This also seems to be the case in American English.)

D.14. Strong Idiosyncrasy

Linguistic constructs are not independent of the language

(Pleonasm ?)
Idioms and specific constructs are illustrations of this.

Differences of language
I have punctured

Differences of dialect
Un demi-fraise

Differences of jargon
Redresse la #16

D.15. Toy Example

Named towns are examples of highly polysemous words that can be referred to for their location, population, and many other aspects.

- Types : T (town), Pl (place), P (people)
- Usual predications:
  1. Birmingham is spread out
  2. Birmingham voted labour
  3. 1 & 2

Lexical item	Main λ-term	Modifiers
Birmingham	$birmingham^T$	$Id_T : T \to T$ $t_2 : T \to P$ $t_3 : T \to Pl$
is_spread_out	$spread\_out : Pl \to \mathbf{t}$	
voted	$voted : P \to \mathbf{t}$	

1. Type mismatch in $\mathit{spread\_out}^{Pl \to \mathbf{t}}(\mathit{Birmingham}^{T})$), resolved using t_3:

$$\mathit{spread\_out}^{Pl \to \mathbf{t}}({t_3}^{T \to Pl}\mathit{Birmingham}^{T}))$$

2. The same, using t_2:

$$\mathit{voted} P \to \mathbf{t}({t_2}^{T \to Pl}\ \mathit{Birmingham}^{T})$$

3. We use a polymorphic conjunction operator, $\&^{\Pi}$.

   –as seen.

   $$\Lambda\xi\lambda x^{\xi}\ \lambda f^{\xi \to \alpha}\lambda g^{\xi \to \beta}(\mathrm{and}^{(\mathbf{t} \to \mathbf{t}) \to \mathbf{t}}\ (\mathit{spread\_out}\ (f\ x))(\mathit{voted}\ (g\ x)))$$

   After application, we have:

   $$(\mathrm{and}\ (\mathit{spread\_out}^{Pl \to \mathbf{t}}\ ({t_3}^{T \to Pl}\ \mathit{Birmingham}^{T}))$$

   $$(\mathit{voted}^{Pl \to \mathbf{t}}\ ({t_2}^{T \to P}\ \mathit{Birmingham}^{T})))$$

D.16. Determiners and Quantifiers

The following is a discussion on the modelling determiners and quantifiers in our system.

D.17. Usual Montagovian treatment

(9) A tramp died on the pavement.

(10) Something happened to me yesterday.

Usual view (e.g Montague)

Quantifiers apply to the main predicate,

$[\mathit{something}] = \exists : (\mathbf{e} \to \mathbf{t}) \to \mathbf{t}$

and when there is a restriction to a class (e.g. $[\mathit{some}]$) the quantifier applies to two predicates:

$\lambda P^{\mathbf{e}\to\mathbf{t}} \lambda Q^{\mathbf{e}\to\mathbf{t}} (\exists \lambda x^{\mathbf{t}}. \&(P\ x)(Q\ x)) : (\mathbf{e} \to \mathbf{t}) \to (\mathbf{e} \to \mathbf{t}) \to \mathbf{t}$

D.18. Quantifier: critics of the standard solution 1/3

Syntactical structure of the sentence $\neq$ logical form.

(11) Orlando di Lasso composed some motets.

(12) syntax (Orlando di Lasso (composed (some (motets))))

(13) semantics: (some (motets)) $\underline{(\lambda x.\ \text{OdL composed } x)}$

The underlined predicate is not a proper phrase.

D.19. Quantifier: critics of the standard solution 2/3

Asymmetry class / predicate

(14) a. Some politicians are crooks.

b. ?? Some crooks are politicians.

(15) a. Some students are employees.

b. Some employees are students.

The different focus makes a big difference.

D.20. Quantifier: critics of the standard solution 3/3

There can be a reference before the utterance of the main predicate (if any):

(16) Cars, cars, cars,... (Blog)

(17) Premier voyage, New-York. (B. Cendrars)

(18) What a thrill — My thumb instead of an onion. (S. Plath)

(19) Lundi, mercredi et vendredi, une machine de couleurs, mardi et jeudi, une machine de blanc, le samedi, les draps, le dimanche, les serviettes. (Blog)

Even when there is a main predicate, I do think that we interpret the quantified NP as soon as we hear it.

(20) Most students go out on Thursday night.

D.21. Sorts, classes,...

Intuitively, there are several ways to quantify. For instance universal quantification can be viewed:

- as a conjunction over the domain (model theoretical view)
- as a property of the generic member of its class (proof theoretical view)

Completeness makes sure that they both agree.

Nertheless the generic view requires a class, a type.

It is very rare to quantify over all possible entities.

D.22. Sorts and classes for generalised quantifiers

Frege's single sorted logic:

(21) a. $\forall x \in M\ P(x) \quad \equiv \forall x\ (M(x) \Rightarrow P(x))$
b. $\exists x \in M\ P(x) \quad \equiv \exists x\ (M(x) \& P(x))$

This treatment does not apply to other quantifiers:

(22) a. for $\frac{1}{3}$ of the $x \in M\ P(x) \not\equiv$ for $\frac{1}{3}$ of the $x\ (M(x) \Rightarrow P(x))$
b. for few $x \in M\ P(x) \not\equiv$ for few $x\ (M(x) \& P(x))$

Sorts and classes with specific quantifiers may be a good direction.

D.23. Russell's iota

The first operator like ε was Russell's ι for definite descriptions: the unique individual satisfying P: a term $\iota_x P(x)$.

A technical problem is that the negation of there exists a unique individual such that P is that there are no such individual or at least two.

As observed by von Heusinger, it should be observed that there is little difference between the logical form of definite descriptions and indefinite noun phrase...

The uniqueness is not always observed,

(23) Recueilli très jeune par les moines de l'abbaye de Reichenau, sur **l'ile du lac de Constance**, en Allemagne, qui le prennent en charge totalement; Hermann étudie et devient l'un des savants les plus érudits du XIème siècle.

D.24. Hilbert's epsilon

$$F(\varepsilon_x F) \equiv \exists x.\ F(x)$$

A term (of type individual) $\varepsilon_x F$ associated with F: as soon as an entity enjoys F the term $\varepsilon_x F$ enjoys F.

The operator ε binds the free occurrences of x in F.

D.25. Rules for ε

Hilbert's work: fine! (Grundlagen der Mathematik, with P. Bernays)

Introduction of the universal quantifier

Rule 1: From $P(x)$ with x generic infer:
$P(\varepsilon_x.\neg P(x)) \equiv P(\tau_x.P(x)) \equiv \forall x\ P(x)$

Introduction of the existential quantifier:

Rule 2: From $P(t)$ infer $P(\varepsilon_x P(x)) \equiv \exists x\ P(x)$

D.26. “Loose” use of ε

Some A are B. (E sentences of Aristotle)

$B(\varepsilon x.\ A(x))$

Not equivalent to an ordinary formula, in particular not equivalent to the standard: $\exists x.\ A \& B(x)$ but

$B(\varepsilon x.\ A(x)) \wedge A(\varepsilon x.\ A(x)) \vdash \exists x.\ B \& A(x)$

Indeed:

$$
\begin{aligned}
& B(\varepsilon x.\ A(x)) \wedge A(\varepsilon x.\ A(x)) \\
\vdash\ & B(\varepsilon x.\ B \& A(x)) \wedge A(\varepsilon x.\ B \& A(x)) \\
\vdash\ & B \& A(\varepsilon x.\ (B \& A(x))
\end{aligned}
$$

On the other hand, one has:

$\exists x.A(x) \& \forall y (A(y) \Rightarrow B(y)) \vdash B(\varepsilon x.A(x))$

because ε-terms are usual terms.

D.27. Intuitive interpretation

Kind of Henkin witnesses but actually no good interpretation that would entail completeness.

Here is a pleasant intituitive interpretation rule due to von Heusinger: both "*a*" and "*the*" are interpreted by the an epsilon term, but the "*a*" always refers to a **new** individual in the class, while "*the*" refers to the most **salient** one.

(24) A student entered the lecture hall. He sat down. A student left the lecture hall.

(25) A student arrived lately. The professor looked upset. The student left.

D.28. Typed Hilbert operators

Single sorted logic, Frege / Montague style: $\varepsilon : (\mathbf{e} \to \mathbf{t}) \to \mathbf{e}$

Many sorted:

$\varepsilon^* : \Lambda\alpha.\ \alpha$

or

$\varepsilon : \Lambda\alpha.\ (\alpha \to \mathbf{t}) \to \alpha$

???

either type/formula entails the other:

$\varepsilon^* = \varepsilon\{\Lambda\alpha.\alpha\}(\lambda x^{\Pi\alpha.\alpha}.\ x\{\mathbf{t}\}) : \Lambda\alpha.\ \alpha$
$\varepsilon = \varepsilon^*\{\Lambda\alpha.\ (\alpha \to \mathbf{t}) \to \alpha\}$

ε is more general because type can be mirrored as predicates, but not the converse.

There is no problem of consistency with such constants whose type in unprovable (like fix point Y).

D.29. Intuitive interpretation and logic: some perspectives

Cohabitation of types and formulae of first/higher order logic:

Typing ($\sim$ presupposition) is irrefutable $sleeps(x : cat)$

Type to Formula:

type cat mirrored as a predicate $\widehat{cat} : \mathbf{e} \to \mathbf{t}$

Formula to Type?

Formula with a single free variable $\sim$ type?

$cat(x) \wedge belong(x, john) \wedge sleeps(x) \sim$ type?

At least it is not a natural class.

D.30. Computing the proper semantics reading

A cat. $\quad \mathrm{cat}^{animal \to \mathbf{t}} \quad (\varepsilon\{animal\} cat^{animal \to \mathbf{t}}) : animal$

Presupposition $F(\varepsilon_x F)$ is added: $cat(\varepsilon\{animal\} cat^{animal \to \mathbf{t}})$

For applying ε to a type say cat,
any type has a predicative counterpart cat (type) $\mathrm{pred}cat : \mathbf{e} \to \mathbf{t}$.
(domains can be restrained / extended)

D.31. Avoiding the infelicities of standard Montague semantics

$\varepsilon_x F$: individual.

1. Can be interpreted as an individual without the main predicate:
   it is a term.
2. Follows syntactical structure:
   it is a term, the semantics of an NP.
3. Asymmetry subject/predicate:
   $P(\varepsilon Q) \not\equiv Q(\varepsilon P)$.

D.32. E-type pronouns

ε solves the so-called E-type pronouns interpretation (Gareth Evans) where the semantic of the pronoun is the copy of the semantic of its antecedent:

(26) A man came in. He sat dow.

(27) $''He'' = ''A\ man'' = (\varepsilon_x\ M(x))$.

D.33. Categorical model (F. Pasquali)

[Sorry for giving little details, this construction is ”heavy” category theory]

Very recently Fabio Pasquali proposed a categorical model: an epsilon logic/language can be interpreted in a Boolean hyper doctrine with a specific property (corresponding to the Axiom of Choice).

Formulae, terms and proofs are all interpreted by arrows. The $\exists x F$ arrow correspond to the ε-term arrow.

Such an hyper doctrine can be constructed from any elementary Topos enjoying the Axiom of choice.

It is important to have a many sorted logic defined with types for his construction.

D.34. Qualia Exploitation

Classic GL: The ”actual” meaning is conveyed by qualia

Formal An heavy sword (physical property)

Constitutive A well-tempered honed sword (blade, edge)

Agentive A master’s sword (smith)

Telic An effective sword (for battle, practice, fruit-cutting. . .)

For instance, books are written (agentive) and read (telic) We have (on top of other things) transformations

$$f_{Agentive}{}^{Book \to \mathbf{v}}, f_{Telic}{}^{Book \to \mathbf{v}}$$

So I finished the book is ambiguous between

$$(\text{finished}^{A \to \mathbf{v} \to \mathbf{t}}\ I^{A}\ (f_{Agentive}\ \text{the\_book}))$$

and

$$(\text{finished}^{A \to \mathbf{v} \to \mathbf{t}}\ I^{A}\ (f_{Telic}\ \text{the\_book}))$$

(This is expected behaviour in GL-esque theories.)

D.35. Dot Objects

"Dots" are very special objects in GL. Examples:

- The book was heavy and interesting.
- The lunch was delicious but took forever.
- The pressure is 120 psi and rising.
- The fair city of Perth, by the river Tay, is a bustling shopping and trade centre that nevertheless retains a tranquil atmosphere...

We do not differentiate between qualia, dot "facets" and provide transformations for everything, such as

$$f_{Phys}{}^{Book \to \varphi}, f_{Info}{}^{Book \to I}$$

D.36. Constraints

Possible/Hazardous co-predicative constructions

- Important point: this is not (only) about toy examples.
- *The salmon was fast and delicious.
- The salmon was lightning fast. It is delicious.
- Birmingham is a large city and voted labour.
- *Birmingham is a large city and won the cup.

Constraints on flexibility: FLEXIBLE v. RIGID

- Are lexically fixed on modifiers, computed for terms
- FLEXIBLE: anything goes.
- RIGID: nothing else can go (even the original typing).

Complex example

Lexicon

word	principal λ-term	optional λ-terms	rigid/flexible
Birmingham	$Birmingham^T$	$Id_T : T \to T$	(F)
		$t_1 : T \to F$	(R)
		$t_2 : T \to P$	(F)
		$t_3 : T \to Pl$	(F)
is_spread_out	$spread\_out : Pl \to \mathbf{t}$		
voted	$voted : P \to \mathbf{t}$		
won	$won : F \to \mathbf{t}$		

where the base types are defined as follows:

T	town
F	football club
P	people
Pl	place

Birmingham is spread out and won

Polymorphic AND yields:

$(\&^{\Pi}(\mathit{spread\_out})^{Pl\to\mathbf{t}}(\mathit{won})^{P\to\mathbf{t}})$

Forces $\alpha := Pl$ and $\beta := P$, the properly typed term is

$$\&^{\Pi}\{Pl\}\{P\}(\mathit{spread\_out})^{Pl\to\mathbf{t}}(\mathit{won})^{P\to\mathbf{t}}$$

It reduces to:

$\Lambda\xi\lambda x^{\xi}\ \lambda f^{\xi\to\alpha}\lambda g^{\xi\to\beta}(\mathrm{and}^{\mathbf{t}\to\mathbf{t})\to\mathbf{t}}\ (\mathit{spread\_out}\ (f\ x))(\mathit{won}\ (g\ x)))$

Should apply to t_1 and t_3 – but t_1 is RIGID.

Syntactical relaxation of semi-flexible constraints

- The salmon was lightning fast. It is delicious.
- Semi-flexible: acts as Rigid, but is reset by reference.

D.37. Deverbal Nouns

(With Livy Real)

* The construction was delayed and is around the corner.

Even with the same suffix, deverbal polysemy is rich and largely idiosyncratic with diverse facets, incompatible most of the time

Specialised events, e. g. $\mathbf{v}_\varphi$. Example of signature:

- The signature took a long time. $\mathbf{v}$
- The signature was an angry jab. $\mathbf{v}_\varphi$
- The signature doesn't stand out on the page. φ

Our analysis: primary type $\mathbf{v}$, transformations $f^{\mathbf{v}\to\mathbf{v}_\varphi}$, $f^{\mathbf{v}\to\varphi}$

However, compare:

- * The signature took three months and is in black ink.
- * The signature was hard to obtain and was neatly executed.
- The signature, that nearly pierced the page, is preserved in the National Library.

We get around this by having constraints: both transformations are flexible, but the identity is rigid

$$f^{\mathbf{v}\to\mathbf{v}_\varphi} : F$$

$$f^{\mathbf{v}\to\varphi} : F$$

$$id^{\mathbf{v}\to\mathbf{v}} : R$$

. . . So resultatives can have various facets X-or the original event

D.38. Fictive motion (R. Moot & L. Prévot & Ch. Retoré

Example from an Historical and Regional corpus (travel stories in the Pyrenees, XIXth century):

(28) (...) cette route monte jusqu'à Lux où l'on arrive par une jolie avenue de peupliers.

(29) (...) cette route qui monte sans cesse pendant deux heures

(30) The road descends (...) A simpler variant

In 28: possibly static

In 29 one needs to introduce a traveller because of pendant deux heures. this traveller does not necessarily exists.

This is a case of meaning transfer:

$$\left(P^{human \to \mathbf{t}}\left(u^{path}\right)\right) \qquad human \neq path$$

The u^{path} introduces a x^{human}, but if u remained an argument of P turned into a *human*, two problems can arise:

1. Firstly, the quantifier corresponding the virtual traveller could not get get the proper scope over the predicate.
2. Secondly, properties of the path, like asphalted, could become properties of the virtual traveller! [1] Observe that the pleasant path is easy to handle, because we have an event variable.

---

[1] This may only happen in the FarWest!

To avoid those problems, the coercion applies not to path but to its type raised version $(path \to \mathbf{t}) \to \mathbf{t}$, which is turned into a "type raised" human $(human \to \mathbf{t}) \to \mathbf{t}$.
Actually the transformation is performed with $\mathtt{t}' := \mathtt{v} \to \mathbf{t}$ instead of $\mathbf{t}$
the: $\Lambda\alpha\lambda P^{\alpha\to\mathbf{t}}(\tau^{(\alpha\to\mathbf{t})\to\alpha}\ P)$
— that's the ι of Russel or a variant of the ε of Hilbert.
When applied to a type α, the choses one object that satisfies P.

path: $\lambda x^{path} path(x)$

(the road): $((\Lambda\alpha\lambda P^{\alpha\to\mathbf{t}}(\tau^{(\alpha\to\mathbf{t})\to\alpha}\ P))\{path\}\ \lambda x^{path} road(x)))$
$=_\beta (\lambda P^{road\to\mathbf{t}}(\tau^{(road\to\mathbf{t})\to road}\ P))\lambda x^{path} road(x)))\quad =_\beta (\tau\ \lambda x^{path} road(x)) : path$
$\Rightarrow \lambda P^{path\to\mathtt{v}\to\mathbf{t}}\lambda e^{\mathtt{v}}(P\ (\tau\ \lambda x^{path} road(x))\ e)$ (*type raising*)

h: $\lambda Q^{(path\to\mathtt{v}\to\mathbf{t})\to\mathtt{v}\to\mathbf{t}}\lambda P^{human\to\mathtt{v}\to\mathbf{t}}$
$(Q\ (\lambda c^{path}\lambda e^{\mathtt{v}}$
$\forall(\lambda v^{human}\ follow(e, v, c) \Rightarrow ((P\ v)\ e))))$ (*type transformation*)

(h (the road)):

$$((\lambda Q^{(path\to \mathrm{v}\to \mathbf{t})\to \mathrm{v}\to \mathbf{t}}\lambda P^{human\to \mathrm{v}\to \mathbf{t}}$$
$$(Q\ (\lambda c^{path}\lambda e^{\mathrm{v}}$$
$$\forall(\lambda v^{human}\ follow(e,v,c)\Rightarrow((P\ v)\ e)))))$$
$$(\lambda P^{path\to \mathrm{v}\to \mathbf{t}}\lambda e^{\mathrm{v}}(P\ (\tau\ \lambda x^{path} road(x))\ e)))$$

$$=_{\beta}\lambda P^{human\to \mathrm{v}\to \mathbf{t}}\lambda e^{\mathrm{v}}$$
$$\forall(\lambda y^{human} follow(e,y,(\tau\ \lambda x^{path} road(x)))\Rightarrow((P\ x)\ e))$$

descends: $\lambda x^{human}\lambda e^{\mathrm{v}} descends(e,x)$

((h (the road)) descends):

$$((\lambda P^{human\to \mathrm{v}\to \mathbf{t}}\lambda e^{\mathrm{v}}$$
$$\forall(\lambda y^{human} follow(e, y, (\tau\ \lambda x^{path} road(x)))\Rightarrow((P\ x)\ e)))$$
$$(\lambda x^{human}\lambda e^{\mathrm{v}} descends(e, x)))$$

$$=_{\beta}\lambda e^{\mathrm{v}}\forall(\lambda y^{human}$$
$$follow(e, y, (\tau\ \lambda x^{path} road(x)))\Rightarrow descends(e, y))$$

Modifiers like during two hours or abruptly are defined as usual: either higher order predicate, or first order that applies to the the event/reification variable $e : \mathrm{v}$.

D.39. Details on Plural Readings

The following illustrates the difficult issue of plurals:

(31) * Jimi met.

(32) Jimi and Dusty met.

(33) The committee met.

(34) Jimi and Dusty walked.

(35) Jimi and Dusty lifted a piano.

D.40. Plurality

Generalities

- Plurals appear in many difficult cases of language
- The original Montague Grammar is not sufficient
- Logic deals with individuals
- Language has some people, the guys, a bit of. . .
- Further complicated with generalised quantification (most) and mass nouns
- Different syntax and semantics among languages

Cases

- Elementary cases (intransitive verbs, individuals, groups)
- Collective predicates: to meet
  - *The student met v. The students met
  - The committee met v. The committees met
- Distributive predicates: to walk, to sneeze. . .
- Ambiguous (covering) predicates: to discuss, to lift a piano. . .

D.41. An Elaboration on Link and Partee

(1983)

- Foundation for most semantic accounts of plurality
- Individuals and Groups are on the same semantic level
- Groups can be formed by joining a number of individuals or groups
- $\oplus$ as a lattice-theoretical joint operation
- $\leq_i$ as individual-part relation
- atomic predicate or atomic/group distinction
- Unary predicates P can be distributive-ified as P^*

D.42. Distinguishing Types and Sets

Types are not sets

- Type/token and Set/element analogy
- However, in intuitionistic logics such as TY_n...
- Negation is not complementation
- Disjunction of types is difficult
- There is no intersection between types

Accounting for sets

- Predicates define sets easily
- Sets of type $\mathbf{e}_i$ can be defined with the property x is of type $\mathbf{e}_i$: $\widehat{\mathbf{e}_i}(x) : \mathbf{e} \to \mathbf{t}$
- Plurals: individuals, groups, and some set operations

D.43. Operators for Sets

Basic elements

- Sets are predicates of type $\alpha \to \mathbf{t}$ (mostly $\mathbf{e} \to \mathbf{t}$)
- Integers can be defined, let $|P| = |\lambda x.P(x)|$ be the cardinality of P (the number of x satisfying $P(x)$)
- Inclusion: $P \subseteq Q$ is defined as

$$\Lambda\alpha\lambda P^{\alpha\to\mathbf{t}}\lambda Q^{\alpha\to\mathbf{t}}\lambda x^{\alpha}.P(x) \Rightarrow Q(x)$$

- $P \cup Q$ is $\lambda x.P(x) \vee Q(x)$, $P \cap Q$ is $\lambda x.P(x) \wedge Q(x)$

Groups

- Used for nouns such as committee, team...
- Groups are a specific type g
- Members are retrieved (as a set) from a group using the *member* function:

$$member : g \to \mathbf{e} \to \mathbf{t}$$

- Group union is noted $\oplus$
- We do not use group complement and intersection

Alternatives

- Groups can be defined for any sort...
- ...or a group type for every sort i, g_i
- We present the single-sort version

D.44. Individual Nouns

We illustrate the constructions using a single sort **e** and a basic categorial grammar.

Simple individuals - predicates over simple type **e** individuals:

$$\text{student} \;\Big|\; n \;\Big|\; \lambda x^{e}.student(x)$$

Group individuals - predicates over type g

$$\text{committee} \;\Big|\; n \;\Big|\; \lambda x^{g}.committee(x)$$

Member Of - in addition to *member*:

$$\begin{array}{r|c|l} \text{member\_of} & n/np & \lambda y^{g} x^{e}.member\_of(x, y) \\ \text{committee member} & n/n & \lambda y^{g} x^{e}.member\_of(x, y) \end{array}$$

Proper nouns (names) - individuals with Quine characteristic

$$\begin{array}{r|c|l} John & np & j^{e} \\ q & np/np & \Lambda\alpha\lambda x^{\alpha}\lambda y^{\alpha}.y = x \\ John^{q} & np & \lambda y^{e}.y = j \end{array}$$

D.45. Basic Operations

Plural suffix type-shifting individuals to sets (card. > 1)

$$\textit{-s} \;\Big|\; n\backslash n \;\Big|\; \Lambda\alpha\lambda P^{\alpha\to\mathbf{t}}\lambda Q^{\alpha\to\mathbf{t}}.|Q| > 1 \wedge \forall x^{\alpha}.Q(x) \Rightarrow P(x)$$

Plural forms : suffix with type **e** or g (or appropriate sorts)

$$\begin{array}{r|c|l} \textit{students} & n & \lambda Q^{\mathbf{e}\to\mathbf{t}}.|Q| > 1 \wedge \forall x^{\mathbf{e}}.Q(x) \Rightarrow \textit{student}(x) \\ \textit{committee} & n & \lambda Q^{g\to\mathbf{t}}.|Q| > 1 \wedge \forall x^{g}.Q(x) \Rightarrow \textit{committee}(x) \end{array}$$

Conjunction - set-theoretic union for predicates

$$\begin{array}{r|c|l} \textit{and} & (np\backslash np)/np & \Lambda\alpha\lambda P^{\alpha\to\mathbf{t}}\lambda Q^{\alpha\to\mathbf{t}}\lambda x^{\alpha}.P(x) \vee Q(x) \\ \textit{John and Mary} & np & \lambda y^{\mathbf{e}}.(y = j) \vee (y = m) \end{array}$$

D.46. Verbs

Collective readings apply to plural sets

$$\textit{met} \mid np\backslash s \mid \lambda P^{\mathbf{e}\to\mathbf{t}}.|P| > 1 \wedge \textit{meet}(P)$$

Distributive readings apply to individuals

$$\textit{sneezed} \mid np\backslash s \mid \lambda x^{\mathbf{e}}.\textit{sneeze}(x)$$

Ambiguous verbs can have either reading

$$\textit{wrote a paper} \mid np\backslash s \mid \lambda P^{\mathbf{e}\to\mathbf{t}}.\textit{write\_a\_paper}(P)$$

Coercions - different for each reading

- Collective: $\#$ enables collective verbs to have sets of sets as arguments

$$\# \mid \quad \mid \Lambda\alpha\lambda R^{(\alpha\to\mathbf{t})\to\mathbf{t}}\lambda S^{(\alpha\to\mathbf{t})\to\mathbf{t}}\forall P^{\alpha\to\mathbf{t}}.S(P) \Rightarrow R(P)$$

- Distributive: $*$ distributes over individuals in of a set

$$\begin{array}{c|c|l} * & & \Lambda\alpha\lambda P^{\alpha\to\mathbf{t}}\lambda Q^{\alpha\to\mathbf{t}}\forall x^{\alpha}.Q(x) \Rightarrow P(x) \end{array}$$

- Ambiguous: c (covering), any possible combination (including collective and distributive as special cases)

$$\begin{array}{c|c|l} c & & \Lambda\alpha\lambda R^{(\alpha\to\mathbf{t})\to\mathbf{t}}\lambda P^{\alpha\to\mathbf{t}}\forall x^{\alpha}.P(x) \Rightarrow \\ & & \quad \exists Q^{\alpha\to\mathbf{t}}.Q(x) \wedge Q \subseteq P \wedge R(S) \end{array}$$

Coerced forms for the previous cases:

$$\begin{array}{r|c|l} \mathit{met}^{\#} & np\backslash s & \lambda R^{(\mathbf{e}\to\mathbf{t})\to\mathbf{t}}\forall P^{\mathbf{e}\to\mathbf{t}}.R(P) \Rightarrow |P| > 1 \wedge \mathit{meet}(P) \\ \mathit{sneezed}^{*} & np\backslash s & \lambda P^{\mathbf{e}\to\mathbf{t}}.\forall x^{\mathbf{e}}.P(x) \Rightarrow \mathit{sneeze}(x) \\ \mathit{wrote}\ \ldots^{c} & np\backslash s & \lambda P^{\mathbf{e}\to\mathbf{t}}.\forall x^{\mathbf{e}}.P(x) \Rightarrow \\ & & \quad \exists Q^{\mathbf{e}\to\mathbf{t}}.Q(x) \wedge Q \subseteq P \wedge \mathit{write\_a\_paper}(Q) \end{array}$$

Quantification different for each determiner

- The: selection (ι)

$$\textit{the} \mid np/n \mid \Lambda\alpha.\iota^{(\alpha\to\mathbf{t})\to\alpha}$$

- Each: force distributivity

$$\textit{each} \mid (s/(np\backslash s))/n \mid \Lambda\alpha\lambda P^{\alpha\to}\lambda Q^{\to\mathbf{t}}\forall x^{\alpha}.P(x) \Rightarrow Q(x)$$

D.47. Plural Examples

- Jimi and Dusty met

  Straightforward collective reading for a conjunction of individuals

Jimi	j^e
Dusty	d^e
Jimi and Dusty	$\lambda y^e.(y = j) \vee (y = d)$
To meet	$\lambda P^{e \to t}.\lvert P \rvert > 1 \wedge meet(P)$
Jimi and Dusty met	$\lvert \lambda y^e.(y = j) \vee (y = d) \rvert > 1 \wedge$ $meet(\lambda y^e.(y = j) \vee (y = d))$
Jimi and Dusty met	$meet(\lambda y^e.(y = j) \vee (y = d))$

- Jimi and Dusty lifted a piano

  Two individuals, two possible readings

To lift a piano	$\lambda P^{e\to t}.\mathit{piano}(P)$
c Coercion for lift	$\Lambda\alpha\lambda R^{(\alpha\to t)\to t}\lambda P^{\alpha\to t} \quad \forall x^{\alpha}.P(x)$
	$\Rightarrow \exists Q^{\alpha\to t} Q(x) \wedge Q \subseteq P \wedge R(Q)$
R1 Jimi and Dusty ...	$\mathit{piano}(\lambda y^{e}.(y=j)\vee(y=d))$

This is the collective reading (a single lifting event)

R2 Jimi and Dusty ...	$\forall x^{e}.((x=j)\vee(x=d)) \Rightarrow$
	$\exists Q^{e\to t} Q(x)\wedge$
	$Q \subseteq (\lambda y^{e}.(y=j)\vee(y=d))\wedge \mathit{piano}(Q))$

All subgroups are possible, including distributive (an event for each individual).

- Jimi and Dusty were walking

  Straightforward distributive reading

	To be walking	$\lambda x^{e}.walk(x)$
*	Coercion for walk	$\Lambda\alpha\lambda P^{\alpha\to t}\lambda Q^{\alpha\to t}\forall x^{\alpha}.\ Q(x) \Rightarrow P(x)$
	…were walking	$\forall x^{e}.((x = j) \vee (x = d)) \Rightarrow walk(x)$

- The committee met

  Use of group nouns with collective readings

the	$\Lambda\alpha.\iota^{(\alpha\to t)\to\alpha}$
committee	$\lambda x^{g}.committee(x)$
the committee	tc^{g}
to meet	$\lambda P^{e\to t}.\|P\| > 1 \wedge meet(P)$
the committee met	$\|\lambda x^{e}.member\_of(x, tc)\| > 1$ $\wedge\ meet(\lambda x^{e}.member\_of(x, tc))$

- The committees met

  A plural form for group nouns: two readings. . .

	committee	$\lambda x^{g}.committee(x)$
	committees	$\lambda Q^{g\to t}.\lvert Q\rvert > 1 \wedge \forall x^{g}\, Q(x) \Rightarrow committee(x)$
	the	$\Lambda\alpha.\iota^{(\alpha\to t)\to\alpha}$
	the committees	$tcs^{g\to t}$
	to meet	$\lambda P^{e\to t}.\lvert P\rvert > 1 \wedge meet(P)$
#	Coercion for meet	$\Lambda\alpha\lambda R^{(\alpha\to t)\to t}\lambda S^{(\alpha\to t)\to t}.\ S(P) \Rightarrow R(P)$

Optional coercion that enables set combination

$$\text{R1} \;\left|\; \begin{array}{l} |\lambda x^e.(\forall y^g.tcs(y) \Rightarrow member\_of(x, y))| > 1 \wedge \\ meet(\lambda x^e.(\forall y^g.tcs(y) \Rightarrow member\_of(x, y))) \end{array}\right.$$

Collective-collective reading. No coercion, a single meeting event.

$$\text{R2} \;\left|\; \begin{array}{l} \forall P^{e \to t}(\forall y^g x^e.P(x) \wedge tcs(y) \Rightarrow \\ member\_of(x, y)) \Rightarrow |P| > 1 \wedge meet(P) \end{array}\right.$$

Collective-distributive reading with coercion. An independent meeting per committee.

- JIMI tachi ha saikai shita (Jimi and his group had a reunion)
  Language-specific construction $\rightarrow$ collective reading

JIMI	j^e
tachi	$\lambda x^e.entourage^{e \rightarrow g}(x)$
JIMI tachi	jt^g
saikai suru	$\lambda P^{e \rightarrow t}.\lvert P \rvert > 1 \wedge saikai(P)$
...saikai shita	$\lvert \lambda x^e.member\_of(x, jt) \rvert > 1$ $\wedge saikai(\lambda x^e.member\_of(x, jt))$

Tachi: plural with specific selection

D.48. Remaining Issues

Coming up are a few more mostly informal points:

- Framework comparison
- The type system
- Better constraints
- Variations of lexica
- Implementation

D.49. The Differences between our Proposal and Related Formulations

Ontological Types

- Most other approaches
- Asher, Bekki, Pustejovski. . .
- Ontology (concept) provides types
- Types provide all adaptations (co-compositions, shifts, accommodations. . .)

Lexical Sorts

- Basis for our approach
- Types provide a mechanism for recognising clashes
- Transformations come from the lexicon
- Idiosyncrasies in languages and dialects are possible
- Closer to the linguistic data

Our approach

- Based on the latter
- General type-based transformations are still used
- The lexicon can overload type transformations
- (Simply for reasons of scale and practicality)

Other possibilities

- Nunberg & Sag, transfers of meaning
- Cooper: type records
- Luo: words as types

D.50. Issues of Granularity

- How should we build a (comprehensive) type system ?
- What would the sorts be ?
- How big shall we make it ?

Many different approaches:

1. Do not. (Keep **e** from Montague, not acceptable.)
2. Assume it exists. (Actually, most publications.)
3. Keep it restricted to a specific domain. (It works.)
4. Choose a dozen sorts or two. (animated, physical, abstract, edible. . .)
5. Every noun is a sort. (Luo.)
6. Every single-free-variable-formula can be a sort.

D.51. The Classifier Approach

Features of the classifier systems

- Common to several language families
- Asian languages, Sign languages. . .
- Used for counting and measuring
- Apply to most entities in the lexicon

Granularity

- Many classifiers, but a lot less than the lexicon
- Semi-organized: generic, common, specialized
- Variations of use and lexical licenses
- Naturally occuring

Implicit sorts in English and French In progress

D.52. Linear Constraints

Current constraints

- Flexible (compatible with everything)
- Rigid (not compatible with anything)
- Plus syntax-based relaxation

Possible configurations

- Set A modifiers compatible with each others, but not with set B
- Using a modifier f_1 imply we may not use a modifier f_2, except if another g is used first
- . . . and even more complicated constraints
- This is not a priority, as we have no examples

Linear Fromulation

- Idea: associate to lexemes a linear formula describing the available modifiers (rather than a list)
- If possible sets are $\{k\}, \{f, g\}, \{g, h\}, \{h, f\}$, excluding any other configurations...
- That formula is: $!k \& (!f \otimes !g) \& (!g \otimes !h) \& (!h \otimes !f)$
- (The actual results are somewhat simpler than this, to appear.)

D.53. Contexts and Variations of Lexica

Variable Lexica and the need to be adaptable

Idioms from language, dialect, and jargon As seen

Literary settings

- Some narratives assume background knowledge from a specific period / location
- Tales, Fables, Fantasy, SF... Routinely use an expanded lexicon
- Sometimes types are overloaded (Animals as Agents)

Example with implicit introduction (Lewis Caroll)

“Beware the Jabberwock, my son!
The jaws that bite, the claws that catch!
Beware the Jubjub bird, and shun
The frumious Bandersnatch!”

Example with explicit introduction (George Martin)

Their driver awaited them beside his *hathay*.
In Westeros, it might have been called an oxcart, though it [...] lacked an ox.
The *hathay* was pulled by a dwarf elephant[...]

Example with in-character introduction (J. K. Rowling)

‘I’d like ter see a great Muggle like you stop him,’ he said.
‘A what?’ said Harry, interested.
‘A Muggle,’ said Hagrid.
‘It’s what we call non-magic folk like them.’

D.54. Per Aspera

This is ongoing work (and ongoing discussion)

However:

- We keep Montague's compositionality
- The same processing chain is applied
- We can integrate some issues from lexical semantics
- Iterative process towards better analysis
- Keeping things simple
- ”Easy” to implement with logical OR functional programming
- Offers a change of perspective on formal semantics