FIR Digital Filter Design By Using Windows Method With MATLAB

(1)Alia Ahmed Eleti and (2) Amer R. Zerek
(1)Lecture, EE Department, Higher Institute of industry technology, Tripoli- Libya
(2) Zawia University, Faculty, of Engineering/ EE Department
, Zawia, Libya,
(1)alia742002@yahoo.com , (2) anas_az94@yahoo.co.uk

Abstract- **The work reported in this paper deal with of Finite Impulse Response FIR digital filter design by using window method. The window method is easiest to design FIR, but lacks flexibility especially when the passband and stopband ripples are different.**

The digital filter is used to filter discrete time signals with the ability to modify the frequency response of the filter at any time and it used in many application such as data compression, biomedical signal processing, communication receivers, etc.

Using MATLAB package software programs are developed for designing FIR digital filter and good results are obtained .

Key words: FIR, Matlab, IIR and Filter

I. INTRODUCTION

. A filter is essentially a system or network that selectively changes the wave shape, amplitude–frequency and/or phase-frequency characteristics of a signal in desired manner. Common filtering objectives are to improve the quality of a signal, to extract information from signal or to separate two or more signals previously combined.

A digital filter is mathematical algorithm implemented in hardware and/or software that operates on a digital input signal to produce a digital output signal for the purpose of achieving a filtering objective. Digital filters often operate on digitized analog signals or just numbers, representing some variable, stored in a computer memory.

II. DIGITAL FILTERS TYPES

Digital filters are divided in two classes, infinite impulse response (IIR) and finite impulse response (FIR) filters. The input and the output signals are related by

$$y(n)=\sum_{k=0}^{\infty}h(k)x(n-k) \qquad (1)$$

for the IIR filter

and

$$y(n)=\sum_{k=0}^{N-1}h(k)x(n-k) \qquad (2)$$

for the FIR filter

Where h(k)the impulse response sequence h (k), k = 0,1,…….., from these equations we see that, for IIR filters, the impulse response is of infinite duration while for FIR filters, the impulse response is of finite duration, h (k) has only N values. In practical, we cannot compute the output of the IIR filter using equation (1) because the length of its impulse response is too long, instead, the IIR filtering equation is expressed in a recursive form

$$y(n)=\sum_{k=0}^{\infty}h(k)x(n-k)=\sum_{k=0}^{M}b_k x(n-k)-\sum_{k=1}^{N}a_k y(n-k) \qquad (3)$$

where the a_k and b_k are the coefficients of the filter. From equation (3) we note that, the current output sample, y (n) is a function of past values of output and the present and past input samples, that is the a feedback system of some sort. the equation (3) reduces to the FIR equation when the b_k are set to zero and we note that in the FIR filter current output sample, y(n) is a function only of past and present values of input sample.

The transfer functions of FIR and IIR filters are given in the equations (4) and (5) respectively which very useful equations in evaluating their frequency responses.

$$H(z) = \sum_{k=0}^{N-1} h(k) z^{-k} \tag{4}$$

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} \tag{5}$$

In the design of frequency selective filters, the desired filter characteristics are specified in the frequency domain in terms of the desired magnitude and phase response of the filter. In the filter design process, we determine the coefficients of a causal FIR, IIR filter that closely approximates the desired frequency responses specification. The issue of which type of filter to design, FIR or IIR, depends on the nature of the problem and on the specification of the desired frequency responses. [5].

III. FIR FILTER DESGINE BY WINDOWS METHODS

In this method we begin with the desired frequency response specification $H_D(\omega)$ and determine the corresponding impulse response $h_d(n)$. $h_d(n)$ is related to $H_D(\omega)$ by the inverse Fourier transform relation, this method usually called the Fourier series method.

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_D(\omega) e^{j\omega n} d\omega \tag{6}$$

We could start with the ideal lowpass response shown in figure (3a) where ω_c is the cutoff frequency and the frequency scale is normalized (T=1). The impulse response is given by.

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} 1 \times e^{j\omega n} d\omega = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega n} d\omega \tag{7}$$

$$= \frac{2f_c \sin(n\omega_c)}{n\omega_c}, \quad \text{for all n except n = 0}$$

$$= 2f_c, \quad \text{for n = 0}$$

The impulse response of the ideal highpass and lowpass filters are obtained from equation (7) and are summarized in TABLE I. In general, the impulse response $h_d(n)$ obtained from equation (6) is symmetrical about n=0, so that the filter will have a linear phase response, and is infinite in duration (ca not be realized) as shown in the fig.1a then must be truncated at some point, say at n = N-1 / 2, to a length N is equivalent to multiplying $h_d(n)$ by a rectangular window defined by:

$$w(n) = \begin{cases} 1, & |n| = 0,1,....,\frac{N-1}{2} \\ 0, & otherwise \end{cases} \tag{8}$$

Thus the impulse response of the FIR filter becomes

$h(n) = h_d(n)\, w(n)$

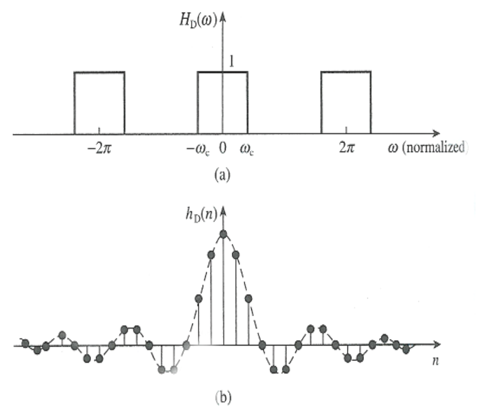

Fig. 1. (a) Ideal frequency response of a lowpass filter. (b) Impulse response of the ideal lowpass filter.

$$h(n) = \begin{cases} h_d, & |n| = 0,1,....,\frac{N-1}{2} \\ 0, & otherwise \end{cases} \tag{9}$$

This truncation introduces undesirable ripples and overshoots as shown in the fig.1. Fig.1. illustrates the effects of discarding coefficients on the filter response the more coefficients that are retained, the closer the filter spectrum is to the ideal response.

In the frequency domain this is equivalent to convolving $H_D(\omega)$ and $W(\omega)$. Where $W(\omega)$ is the Fourier transform of w (n). As $W(\omega)$ has the classic (sin x /x) shape, truncation of h_d (n) leads to the over shoots and ripples in the frequency response.

TABLE I

Impulse responses for standard frequency selective filters.

Filter type	Hd(n),n≠0	Hd(n)
Low pass	2fcsin(nωc)⁄nωc	2fc
high pass	2fcsin(nωc)⁄nωc	1-fc

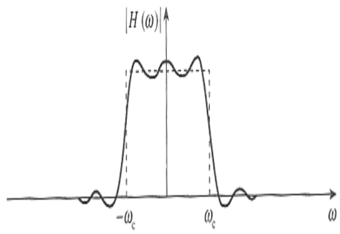

Fig. 2. Effects of truncating the ideal impulse response on the frequency response.

A practical approach is to multiply the ideal impulse response, h_d (n), by a suitable window function, w (n), whose duration is finite. This way the resulting impulse response decays smoothly towards zero the process is illustrated in fig.3. fig.3a. shows the ideal frequency response and the corresponding ideal impulse response. fig.3b. shows a finite duration window function and its spectrum. fig.3c. shown h (n) which is obtained by multiplying h_d (n) by w (n). The corresponding frequency response shows that ripples and overshoots, characteristic of direct truncation, are much reduced. However, the transition width is wider than for the rectangular case. The transition of the filter is determined by the width of the main lobe of the window. The side lobes produce ripples in both pass band and stopband. [2].

We also note that a filter designed by the window method has equal passband and stopband ripples. That is $\delta_p = \delta_s$.

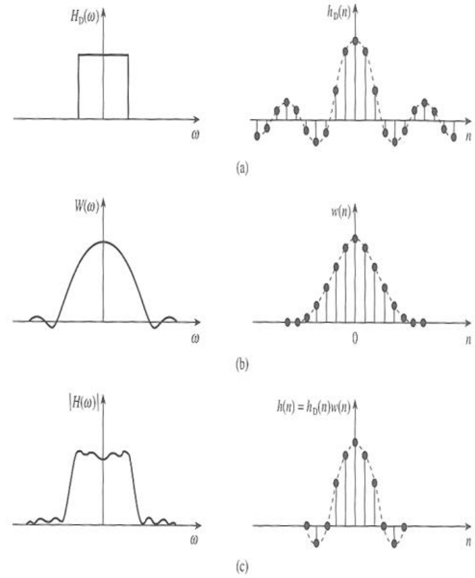

Fig.3. How the filter coefficients, h(n), are determined by the window method.

Some common window function

TABLE I lists several most widely used window function that possess desirable

frequency response Characteristics. All of these window function have significantly lower side lobes compared with the rectangular window. However, for the same value of N, the width of the main lobe is also wider for these windows compared to the rectangular window. Then, these windows functions provide more smoothing through the convolution operation in the frequency domain, but the transition region in the FIR filter response is wider. To reduce the width of this transition region, we can increase the length of the window that results in a larger filter. The frequency – domain of the important various window functions are summarizes in TABLE III.

TABLE II

Summary of important features of common window function.

Window. Function. Window. Function.	Trans. width Trans. width (hz)	P. B. Ripple P. B. Ripple (db)	S. B. (db) S. B. (db) Max.
rectangular	0.9/N	0.7416	21
hanning	3.1/N	0.0546	44
hamming	3.3/N	0.0194	53
Blackman	5.5/N	0.0017	75

TABLE III

Frequency-domain characteristics of some window function.

Types of window	Approximate Transition width of main lobe	Peak Side lobe (dB)
rectangular	4Π/N	-13
hanning	8Π/N	-32
hamming	8Π/N	-43
Blackman	12Π/N	-58

From TABLE II above we note that the first four window functions (rectangular, hanning, hamming and Blackman) have fixed characteristics such as transition width and stop band attenuation.

The Kaiser window is given by:

$$w(n)=\frac{I_0\left\{\beta\left[1-\left(\frac{2n}{N-1}\right)^2\right]^{1/2}\right\}}{I_0(\beta)},\quad \frac{-(N-1)}{2}\leq n\leq\frac{(N-1)}{2} \qquad (10)$$

where β controls the way the window function tapers at the edges in the time- domain, and I_0 (x) is the zero–order modified Bessel function of the first kind.

$$I_0(x)=1+\sum_{k=1}^{L}\left[\frac{(x/2)^k}{k\,!}\right]^2 \qquad (11)$$

where typically L < 25. When β = 0, the Kaiser window becomes the rectangular window, and when β = 5.44, the Kaiser window corresponds to the hamming window. The value of β is often determined by the stopband attenuation requirements and may be obtained from one of the following relationships;

β=0 if A ≤ 21 dB

β = 0.5 842 (A - 21)$^{0.4}$ + 0.078 86 (A - 21) if 21dB < A < 50dB

β = 0.1102 (A – 8.7) if A ≥ 50 dB

where A = -20 $\log_{10}$ (δ) is the stopband attenuation, δ = min (δ_p , δ_s). The number, of filter coefficients, N is given by:

$$N\geq\frac{A-7.95}{14.36\Delta f} \qquad (12)$$

where Δf is the normalized transition width.

IV. FIR FILTER DESGINE WITH MATLAB

In this section we will illustrate how to use some of the MATLAB functions and

I_0 (x) is evaluated using the following power series expansion.

programs to design linear phase FIR filters. In particular, we will illustrate how to calculate the coefficients and plot the magnitude and phase frequency response of linear phase FIR filters using window method.

The MATLAB program (B1) in appendix using for designing FIR filters using window method. To illustrate the use of the program we will give example.

Example

By using an above MTLAB programs determine the coefficients and plot the frequency response of lowpass FIR filter to meet the specifications given below using the window method.

passband edge frequency	1.5 KHz
transition width	0.5 KHz
stopband attenuation	> 60 dB
sampling frequency	8KHZ

Result

Filter length: N = 89

Coefficients of FIR filter:

hn =[0 0.0000 0.0000 -0.0000 -0.0001 -
0.0000 0.0001 0.0001 -0.0002
-0.0003 0.0002 0.0006 -0.0000 -0.0010 -
0.0005 0.0013 0.0013 -0.0012 -0.0024
0.0006 0.0036 0.0008 -0.0045 -0.0032
0.0047 0.0063 -0.0033
-0.0098 0.0000 0.0128 0.0057 -0.0142 -
0.0138 0.0124 0.0238 -0.0058
-0.0348 -0.0080 0.0454 0.0336 -0.0544 -
0.0866 0.0604 0.3115 0.4375 0.3115 0.0604
-0.0866 -0.0544 0.0336 0.0454 -0.0080 -

0.0348 -0.0058 0.0238 0.0124 -0.0138 -0.0142 0.0057 0.0128 0.0000 -0.0098 -0.0033 0.0063 0.0047 -0.0032 -0.0045 0.0008 0.0036 0.0006 -0.0024 -0.0012 0.0013 0.0013 0.0005 -0.0010 -0.000 0.0006 0.0002 -0.0003 -0.0002

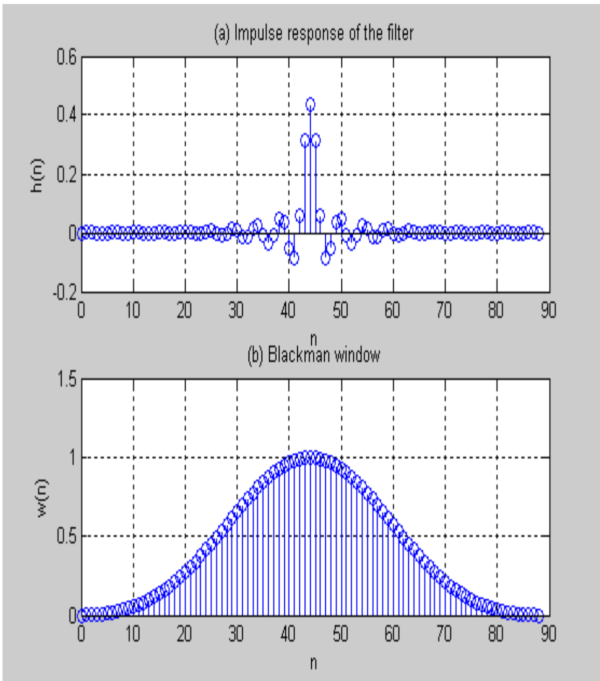

Fig.4. Frequency response of the filter using Blackman window

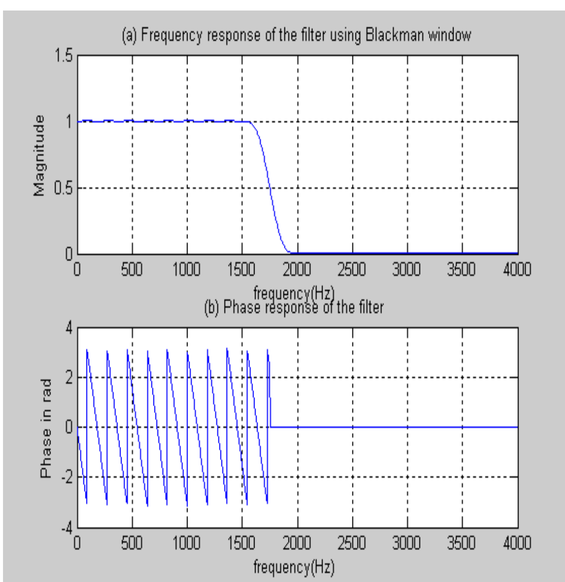

Fig. 5. (a) Impulse response of the filter

(b) Blackman window

CONCLUSION

0.0001 0.0001 -0.0000 -0.0001 -0.0000 0.0000 0.0000 0]

The achieved simulation results are represented in figures 4 to 7.

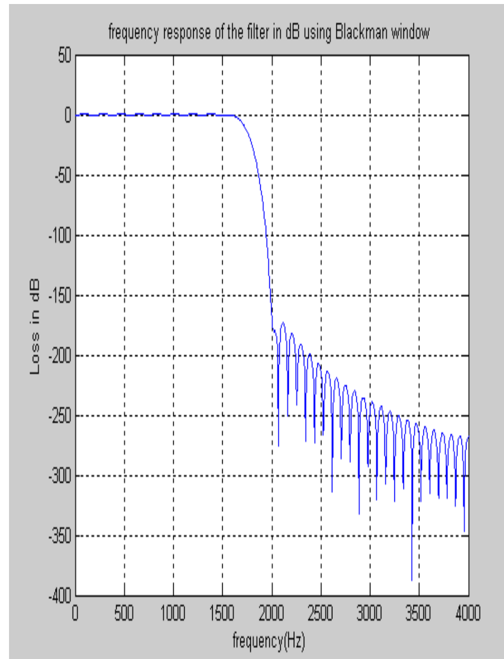

Fig.6. Frequency response of the filter in dB

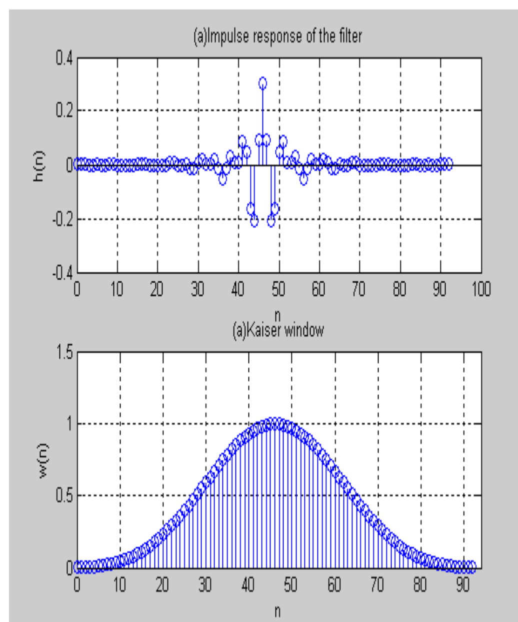

Fig.7. Frequency response of the filter in dB

The main advantage of the window method is simple to apply and simple to understand.

-An important disadvantages is lack of flexibility because both the peak passband δ_p and stop band δ_s ripples are approximately equal, so that the designer cannot make a passband ripple very small or a stopband attenuation very large.

From the effect of convolution of the spectrum of the desired response and the window function, the passband and stopband edge frequencies cannot be precisely specified.

In some application, the expression of the desired filter response, $H_D(\omega)$ is very complicated then we cannot obtained $h_d(n)$ from equation (7). In these cases $h_d(n)$ may be obtained from frequency sampling method before the window function is applied.

REFERENCES

[1] Sanjit K. Mitra, "Digital signal processing A computer-Based Approach", Department of Electrical and Computer Engineering University of California, McGraw-Hill, Second Edition 2002

[2] Emmanuel C. Ifeachor & Barrie W. Jervis, "Digital Signal Processing A Practical Approach", Prentic Hall, Second Edition 2002

[3] Paulo S. R. Diniz, Eduardo A. B. da Silva, and Sergio L. Netto, "Digital Signal Processing System Analysis and Design", Combridge University Press, 2002

[4] Alan V. Oppenheim, and Ronald W. Schafer, "Discrete-Time Signal Processing"

[5] John G. Proakis & Dimitis G. Manoakis, "Digital Signal Processing Principles, Algorithms, and Applications", PRENTICE-HALL INTERNATINAL, INC., Third Edition 1996

[6] Lawrence R. Rabiner, Bernard Gold, "Theory and Applications of Digital Signal processing", Prentice-Hall, 1975

[7] C. Britton Rorabangh, "DSP PRIMER", McGraw-Hill, 1999

[8] Andreas Antoniou, "filters Analysis, Design, and Application", McGraw-Hill, Second Edition 1993

[9] Johnny R. Johnson, "Introduction to Digital Signal Processing", Prentice-Hall, INC., 1989

[10] http://www.duptutor.freeuk.com/, "Digital signal processing", November 2005

[11] http://www.duptutor.freeuk.com/ dfilt1.htm, "Digital filters", November 2005

[12] http://www.freqdev.com/digital.html ,"Digital signal processing – Filters", November 2005

[13] http://www.bores.com/courses/intro/basics ,Introduction to DSP", August 2005 [14] http://en.wikipedia.org/wiki/Digital-signal-processing ,"Digital signal processing" , August 2005

[15] Rahman Jamal, Mike Cerna, Koun Hanks, http://www..sss_mag.com/pdf/sdigf/tr.pdf,"Designing filters using the digital filter design toolkit", , August 2005

.