Thermal Conductivity of Copper: Analysis under a One-Dimensional Heat Transfer Model

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This experiment investigates the heat transfer behavior of a copper rod subjected to transient heat conduction under a one-dimensional approximation. A 61.0 cm long, 1.0 cm diameter copper rod was heated at one end using a magnetic stirrer preheated to 150 ° C while temperature variations were measured at five equidistant points along the rod. The thermocouples, placed 10.1 cm apart and secured with thermal paste, recorded temperature values every 0.5 seconds via an automated Arduino-based system. The resulting temperature distribution as a function of position and time, $T(x,t)$, was analyzed using a polynomial surface fit to understand the thermal response of the system. The analysis neglected heat loss to the surroundings, justified by the academic nature of the experiment, and assumed a fixed temperature boundary condition at the heated end. The development of an experimental heat equation $Q(x,t)$ for the rod contributes to a quantitative understanding of the transient thermal conduction process in copper.

Keywords: Thermal conductivity, transient heat conduction, copper rod, polynomial surface fit, copper heat equation, heat transfer modeling.

I. INTRODUCTION

Thermal conductivity is a fundamental property that governs heat transfer in solid materials. It defines the ability of a material to conduct heat and is particularly relevant in engineering applications involving heat exchangers, electronic cooling systems, and industrial processing. Fourier's Law describes the relationship between heat flux and temperature gradient, forming the basis for analyzing thermal conduction in materials.

This experiment aims to study the heat transfer behavior of a copper rod under a one-dimensional approximation, through a function $T(x,t)$ that relates the temperature of the rod to a position within it and the time at which the temperature was reached at that position. The rod was subjected to transient heating at one end, while the temperature evolution was recorded at multiple points along its length. By plotting the temperature distribution over time and space, a polynomial surface fit function was obtained to characterize the conduction process. The study assumes negligible heat loss to the environment and focuses on determining the transient heat conduction through a heat equation $Q(x,t)$ for the rod of copper, which is presented as the final result of the study.

II. EXPERIMENTAL DETAILS

A. Design of Experimental Holder

A cylindrical copper rod with a length of 61.0 cm and an approximate diameter of 1.0 cm was selected for the experiment. The rod was vertically mounted using a support structure equipped with metal hooks. To minimize thermal interaction between the rod and its support, Kapton tape was applied at the points of contact.

A magnetic stirrer, preheated to 150°C, served as the heat source. The lower end of the rod was placed in direct contact with the heated surface, establishing a fixed-temperature boundary condition. Heat conduction along the rod was monitored using five K-type thermocouples, each spaced 10.1 cm apart. The thermocouples were affixed using thermal paste to ensure optimal thermal contact and minimize measurement errors. Fig. 1 shows a photograph of the described setup.

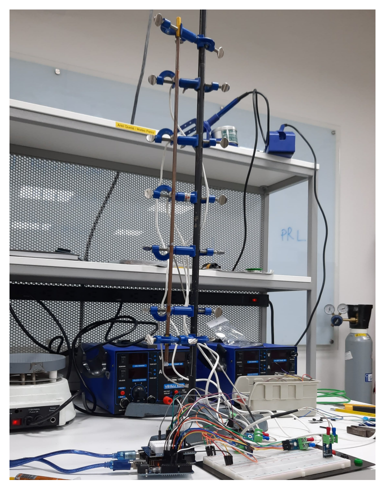

FIG. 1. Sample rod mounted on the support structure using metal hooks.

B. Experimental Procedure

The temperature measurements were automated using an Arduino-based data acquisition system, which recorded temperature values from all thermocouples at an interval of 0.5 seconds. To enhance measurement reliability, the thermocouples were pre-calibrated to ensure consistent ambient temperature readings. The overall measurement uncertainty was estimated to be ±0.25°C.

Given the academic nature of this experiment, heat losses to the surrounding air were assumed to be negligible, simplifying the analysis under a one-dimensional heat conduction model. The recorded temperature values were analyzed to determine a polynomial surface fit function that describes the transient heat conduction behavior in the copper rod.

C. Mathematical Model

To describe the heat conduction process in the copper rod, it is necessary to derive the heat equation governing the material. Thus, one starts with Fourier's law of heat conduction, which states:

$$\frac{1}{A}\frac{d\vec{Q}}{dt} = -k\vec{\nabla}T$$

where $\vec{Q}$ is the heat transferred over time, A is the cross-sectional area of the rod, k is the thermal conductivity of copper, and $\vec{\nabla}T$ is the temperature gradient.

It is reasonable to assume that the copper rod is homogeneous and isotropic, which indicates that its thermal conductivity is consistent throughout the material. Additionally, due to the rod's small diameter, any temperature gradients in the radial and angular directions can be considered negligible. This allows us to conclude that heat transfer primarily occurs along the longitudinal (z-axis) direction. Under these conditions, the heat equation can be simplified to a one-dimensional form:

$$\frac{1}{A}\frac{dQ}{dt} = -k\frac{\partial T}{\partial z}$$

Thus, the heat transfer equation is in terms of spatial and temporal dependence:

$$Q(z,t) = -Ak\int \frac{\partial T}{\partial z}\,dt \tag{1}$$

Since, the thermal conductivity of cooper is dependent of the temperature, $k = k(T(z,t))$, it is necessary to consider the one dimensional thermal diffusion equation

$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial z^2}$$

where $D = \frac{k}{\rho \cdot c_p}$ is the diffusivity, ρ the density of the rod and c_p the specific heat capacity at constant pressure. Rearranging this equation, the thermal conductivity k can be expressed as:

$$k = \rho c_p \frac{\frac{\partial T}{\partial t}}{\frac{\partial^2 T}{\partial z^2}} \tag{2}$$

For the experimental conditions and material properties of copper, the following values were considered $A = 4\pi$ cm^2, $\rho = 8.96\ \frac{g}{cm^3}$ and $c_p \approx 0.385\frac{J}{g\cdot K}$ at 75 kPa.

III. RESULTS AND DISCUSSION

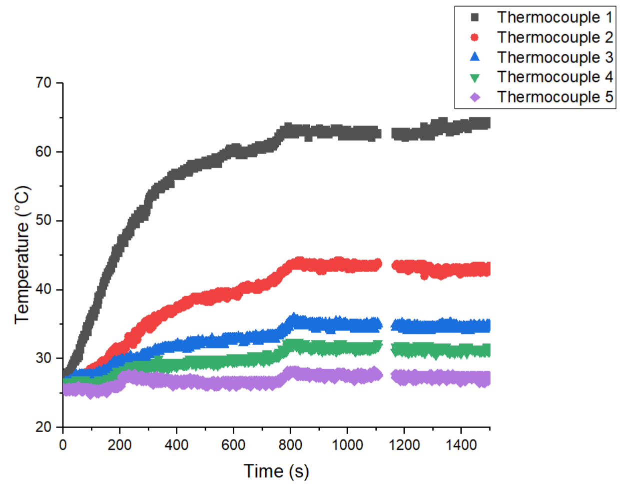

FIG. 2. Temperature variation over time for each thermocouple, arranged in increasing distance from the heat source

The temperature evolution along the copper rod was recorded at five equidistant points as mentioned in section II B. The collected data, presented in Fig. 2, shows the temporal progression of temperature at different distances from the heat source. As expected, the thermocouple closest to the heat source recorded the highest temperatures, with a gradual decrease observed as the measurement points moved further away. This trend is consistent with Fourier's law of heat conduction, which predicts a temperature gradient along the length of the material.

To further analyze the heat transfer process, the recorded temperature values were fitted using a polynomial surface fit with temperature as a function of both time and position along the rod. The resulting fit is depicted in Fig.3 and provides an empirical model for the temperature distribution:

$$\begin{aligned} T(x,t) = {} & 58.8789 - 3.4544z + 0.08367t + 0.1035z^2 \\ & - 0.0019zt - 5.0095t^2 - 9.5734\times 10^{-4}z^3 \\ & + 9.1051\times 10^{-6}z^2t + 6.3340\times 10^{-7}zt^2 \\ & + 8.7485\times 10^{-9}t^3 \end{aligned} \tag{3}$$

The selection of a third-order polynomial fit was based on an adjusted R-squared value of 0.98747, which demonstrated superior agreement compared to a second-order fit (R-squared = 0.90870). The accuracy of this model suggests that it effectively captures the dominant trends in heat transfer within the experimental setup.

From the polynomial fit, key characteristics of the heat conduction process can be inferred such that a dominant linear temperature decay along the rod's length, primarily dictated by the z term or that a strong time-dependent variation in temperature, with higher-order t-terms reflects transient conduction effects.

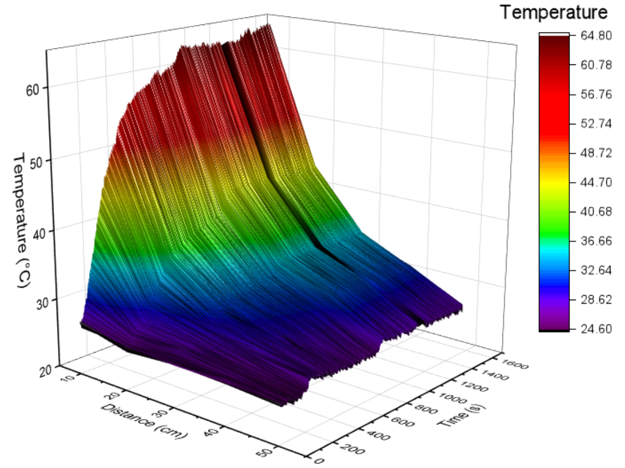

FIG. 3. Polynomial surface fit modeling temperature as a function of position and time

Furthermore, using Eq. 1, the polynomial fit allows direct computation of the heat transfer along the rod.

$$
\begin{aligned}
Q(z,t) = &-4\pi k(z,t)(-3.4544t + 0.2070zt - 0.0001t^2 \\
&- 0.002872z^2t + 9.1052\times 10^{-6}zt^2 \\
&+ 2.1113\times 10^{-7}t^3) + C(z)
\end{aligned}
\tag{4}
$$

The integration constant $C(z)$ can be determined using the boundary conditions of the system, while $k(z,t)$ is determined by Eq.2:

$$
k = 3.4496(\frac{0.08367 - 0.00191z}{0.2070} \frac{-1.001768\times 10^{-4}t + 9.1051\times 10^{-6}z^2}{-0.005744z} \frac{1.2668\times 10^{-6}zt + 2.6246\times 10^{-8}t^2}{1.8210\times 10^{-5}t}) \tag{5}
$$

To simplify the analysis, we impose the condition $Q(z,t=0) = 0$ which leads to $C(z) = 0$. This assumption facilitates the study of the system by ensuring a well-defined initial state. The behavior depicted in Fig. 4 aligns with physical expectations: as time progresses, the heater continuously transfers energy to the metal bar, leading to an increase in temperature until thermal equilibrium is reached at sufficiently large t. Thus, the heat equation seems to accurately describe the thermal evolution of this system.

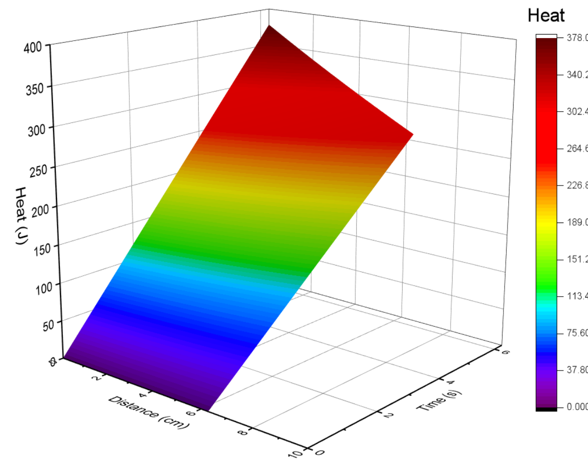

FIG. 4. Heat transfer as a function of position and time

IV. CONCLUSION

This study analyzed the transient heat conduction in a copper rod using a one-dimensional heat transfer model. By implementing an automated temperature measurement system, we obtained high-resolution data that allowed for an accurate characterization of the thermal response of the system. The experimental results demonstrated a clear temperature gradient along the rod, consistent with Fourier's Law of heat conduction, and validated the assumption that heat transfer occurs primarily along the longitudinal axis.

The polynomial surface fit model, derived from the collected data, provided a reliable representation of the temperature evolution over time and space. The high adjusted R-squared value (0.98747) of the third-order polynomial fit confirmed the robustness of this model in capturing the dominant trends in heat conduction. Furthermore, the application of the thermal diffusion equation enabled an estimation of the temperature-dependent thermal conductivity of copper, reinforcing the theoretical framework underlying this experiment.

Although the assumption of negligible heat losses to the surroundings facilitated a simplified analysis, future work could incorporate convective and radiative losses to refine the model further. Additionally, exploring different boundary conditions, such as varying the heat source temperature or introducing thermal insulation, would provide deeper insights into the dynamics of transient heat conduction.

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