Interferometric Determination of the Refractive Index Variation of Air with Temperature Using a Modified Mach-Zehnder Setup

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This study presents an interferometric method to measure the variation of the refractive index of air with temperature using a modified Mach-Zehnder interferometer. In this configuration, planar mirrors replace the second beam splitter, allowing retroreflection and interference at the original beam splitter. A collimated red laser beam ($\lambda = 640$ nm) was split into two coherent paths, one of which passed through a region subjected to controlled heating. The resulting interference pattern was recorded and analyzed to extract phase shifts corresponding to temperature-induced optical path differences. A cosine-based model was used to estimate the number of displaced fringes and infer changes in the refractive index. The results confirmed the expected inverse relationship between temperature and refractive index, with a measured variation on the order of 10^{-6}.
Keywords: Mach-Zehnder interferometer, refractive index, temperature, path difference, phase shift, fringe analysis

I. INTRODUCTION

Accurate knowledge of the refractive index of air and its dependence on temperature is essential in a wide range of high-precision optical and metrological applications. From interferometric length measurements to the calibration of laser based sensors and the correction of atmospheric effects in astronomical observations, understanding how air responds optically to thermal variations is of fundamental importance. Since air is a dispersive and thermally sensitive medium, even slight temperature changes induce measurable variations in its refractive index, which can accumulate over macroscopic distances and significantly impact phase sensitive measurements.

Interferometry provides a robust and highly sensitive method to detect such small refractive index changes through phase shifts arising from variations in optical path length. Among various interferometric techniques, the Mach-Zehnder interferometer (MZI) is particularly suited for this task due to its versatility in separating and recombining beams, its compatibility with collimated light sources, and its capacity to isolate environmental perturbations along a single arm.

In this study, we employ a modified Mach-Zehnder interferometer to investigate how temperature variations affect the refractive index of air. By introducing a controlled thermal perturbation along one arm of the interferometer, and monitoring the resulting interference pattern, we extract the temperature dependent optical phase shift and use it to infer changes in the refractive index. This approach not only validates the theoretical relationship between temperature and refractive index but also demonstrates the capability of low cost interferometric setups for precision optical measurements under laboratory conditions.

II. EXPERIMENTAL DETAILS

A. Experimental Procedure

The setup shown in Figure 1 was designed to measure the variation of the refractive index of air with temperature using an interferometric setup based on a modified Mach-Zehnder interferometer. In this configuration, the second beam splitter of the classical Mach-Zehnder design was replaced with planar mirrors, allowing the beams to be retroreflected back to the original beam splitter, where they interfered.

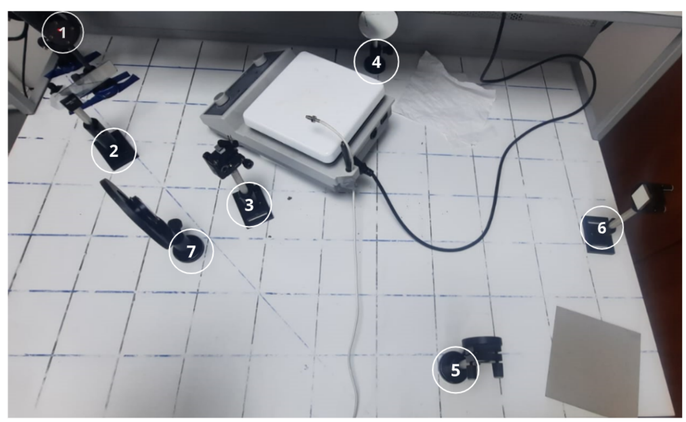

FIG. 1. The photograph of the sample holder for the emissivity measurement. ① Laser; ② Dichroic filter; ③ Cube beam splitter; ④ Mirror 1; ⑤ Mirror 2; ⑥ Mirror 3; ⑦ Concave lens.

A collimated red laser beam ($\lambda = 640$ nm) was first directed through a dichroic filter, which extracted a small portion of the beam's power for auxiliary use. The remaining beam then passed through a non-polarizing cube beam splitter that divided it into two coherent arms. One beam propagated through ambient air, while the other passed through a region subjected to a controlled temperature increase via a resistive heating element. Both beams were reflected at the end of their respective paths using planar mirrors, sending them back to the original beam splitter. Upon recombination, the beams interfered, and the resulting fringes were projected onto a screen. A digital camera recorded the intensity of a selected fringe at a rate of 1 frame per second.

The heating element was positioned adjacent to one arm, producing a localized and gradual thermal gradient in the air. A temperature sensor was placed in the heated region to monitor the air temperature as a function of time throughout the experiment. The region where heating occurred spanned a total length of approximately $L = 19$ cm along the beam path.

Interference fringes were monitored in real time. As the temperature increased, changes in the optical path length due to the refractive index variation caused observable fringe shifts. A single interference maximum-to-minimum to maximum transition was associated with a phase shift of 2π. Therefore, the number of fringe oscillations directly corresponded to the change in optical path difference between the two arms.

B. Mathemathical Model

The phase difference $\Delta\phi$ accumulated in a region of length L with a change of refractive index Δn is expressed as

$$\Delta\phi = \frac{2\pi}{\lambda} \cdot \Delta n \cdot L \tag{1}$$

where, λ is the wavelength of the laser beam, L the length of the path in which the thermal change occurs, and $\Delta n = n_{cold} - n_{hot}$, the difference in the refractive index given by the temperature change.

As mentioned, the observed interference pattern is a consequence of the recombined optic waves. The intensity registered by a photodetector (camera) varies according to the phase difference between the two branches

$$I(t) = I_o[1 + V \cdot \cos(\Delta\phi(t))] \tag{2}$$

with I_o the mean intensity of the signal and V the visibility of the interference pattern (varying from 0 to 1). In practice, as V is not known with precision, it's common to determine the phase shift by counting the oscillations of the interference pattern in time. Every oscillation (maximum to minimum to maximum) represents a phase shift of 2π, which is appreciated as a displaced interferometric fringe.

Guided by eq.(2), for the estimation of the number of displaced fringes N, it is assumed that the measured intensity follows pattern $I(t) = \bar{I} + A \cdot \cos(\Delta\phi)$. Where $\bar{I}$ is the mean intensity of the measured interference pattern and A the maximum value of the measurements, i.e. its amplitude. This assumption naturally leads to a centering and normalization of the measured intensities over time, $(I(t) - \bar{I})/A$.

To obtain $I(t)$, the original intensities were smoothed using a moving average technique in order to diminish the noise produced by the electronic devices, mechanic vibrations and random fluctuations of the laser or the air. Hence, the phase shift is given by

$$\Delta\phi(t) = \arccos\left(\frac{I(t) - \bar{I}}{A}\right) \tag{3}$$

However, given that $\arccos(x)$ has a bounded domain, the python function `np.unwrap()` was used to obtain a continuous phase $\Delta\phi_{total}$.

The number of fringes follows the relation $N = \Delta\phi_{total}/2\pi$. Therefore, the total change of the refractive index of air can be determined by

$$\Delta n(t) = \frac{N(t) \cdot \lambda}{L} \Rightarrow n(t) = n_o + \Delta n(t) \tag{4}$$

And the dependence on the temperature T follows from the measurements of Temperature vs. Time.

III. RESULTS AND DISCUSSION

As explained in Section II A, the experiment was carried out using a red laser ($\lambda = 640$ nm), a heated path of $L = 19$ cm and an adapted interferometer to detect changes in the refractive index of air due to temperature. The temporal response of the system can be described with two complementary dependencies. First, the temperature in the heated branch rises monotonically and smoothly over the heating interval, reflecting the controlled input power and the thermal inertia of the cell. Second, the photodetector signal, collected at a fringe initially exhibiting constructive interference, then oscillates about its mean value as the heated arm's optical path length changes. These oscillations correspond to successive fringe shifts induced by the gradually increasing temperature,.

To quantify the variation of the refractive index with temperature, the number of shifted fringes was estimated from the unwrapped phase of the normalized intensity signal. The total number of fringes was:

$$N_1 = 0.372 \quad \Rightarrow \quad \Delta n_1 = (1.25 \pm 0.05) \times 10^{-6}$$

These values are consistent with the expected order of magnitude for the refractive index variation of air with temperature in the visible spectrum.

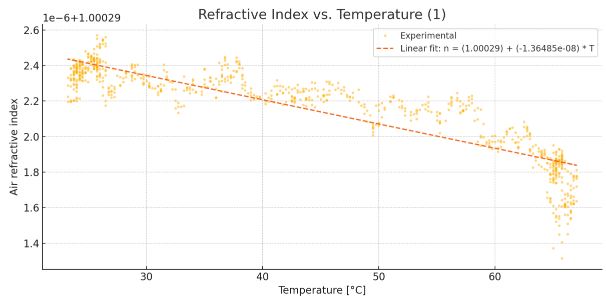

FIG. 2. Experimental refractive index of air as a function of temperature (experiment 1), together with linear fitting.

Figure 2 shows the experimental values of the refractive index $n(T)$ obtained from the experiment, along with a linear fit. The linear adjustment used the form:

$$n(T) = (1.00029) + (-1.36485 \times 10^{-8}) \cdot T$$

which results in a linear correlation of $R^2 = 0.779$, reflecting the limitations imposed by the significant noise in the measured intensity signal.

In both cases, the trend confirms the expected decrease of refractive index with temperature. The visibility of the interference pattern and the smoothness of the temperature increase allowed the extraction of phase information and estimation of the optical path difference.

Uncertainties in the values of Δn are estimated from the standard deviation in the phase extraction, and from the thermal and temporal resolution of the setup.

IV. CONCLUSION

This study investigated the variation of the refractive index of air with temperature using an interferometer adapted to the objectives of the experiment and limitations of the laboratory. By analyzing the temporal evolution of the interference pattern and applying a cosine-based optical model, we were able to extract phase shifts induced by thermal gradients and estimate the corresponding changes in refractive index.

The experimental results confirmed the expected inverse relationship between temperature and refractive index, in agreement with theoretical predictions and the Edlén model. The measured values of Δn were on the order of 10^{-6}, demonstrating the system's sensitivity to subtle optical path changes. Linear regression applied to the experimental data yielded trendlines that reasonably captured the temperature dependence, despite the presence of measurement noise.

The successful use of a fringe-counting technique and phase unwrapping to quantify optical path differences validates the underlying interferometric approach for studying thermally induced refractive index changes. However, the relatively low values of R^2 in the fits reflect the limitations imposed by mechanical instability and the low frame rate of the acquisition system.

Future improvements could include the use of a stabilized optical table to minimize vibrational noise, and a higher-resolution photodetector capable of operating at significantly higher sampling rates than the 1 frame per second used in this experiment. Additionally, enhanced environmental control and finer temperature resolution would contribute to more precise measurements and better agreement with theoretical models.

Overall, the results highlight the viability of interferometric techniques for detecting thermal effects on optical properties in gaseous media, and provide a foundation for future studies with improved instrumentation.

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