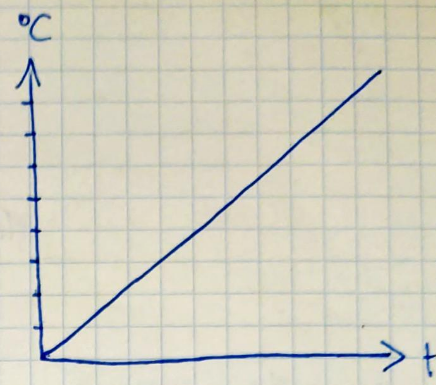
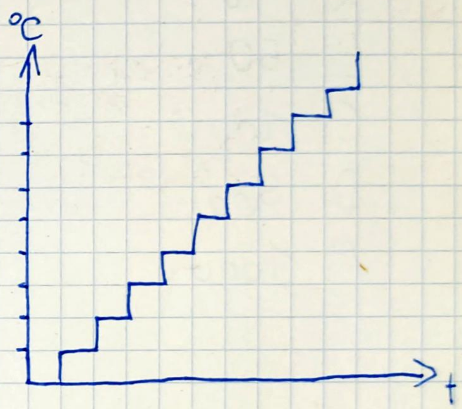
Analoges Signal:



Analoge Signale sind stetig und können jeden beliebigen Zwischenwert annehmen.

Bsp.: Analoge Uhr, Temperatursäule

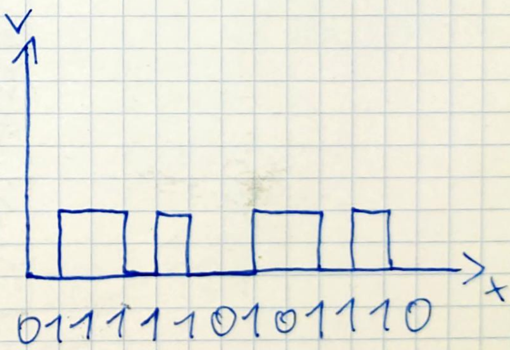
Digitales Signal:



Digitale Signale haben abzählbare Zustände.
Sind stufig

Bsp.: Dig. Multimeter

Binäres Signal:



Binäre Signale sind digitale Signale die nur zwei Zustände einnehmen können.

1 = H / High od. wahr od. true od. ja
0 = L / Low od. falsch od. false od. nein

1 8 9 5

10^3 10^2 10^1 10^0 — Basis 10

↓ ↓ ↓ ↓

1000 800 90 5 = 1895

I	1
V	5
X	10
L	50
C	100
D	500
M	1000

~~8421~~

8 4 2 1

↑ ↑ ↑ ↑

2^3 2^2 2^1 2^0

0+0+0+1	1
0+0+1+0	2
0+0+1+1	3
0+1+0+0	4
0+1+0+1	5
0+1+1+0	6
0+1+1+1	7
1+0+0+0	8
1+0+0+1	9
0+0+0+0	0

128 64 32 16 8 4 2 1
10111010 = 186

128 0 32 16 8 0 2 0

37 = 100101

235 = 11101011

Ubungen

4571 = 1000111011011
- 4096

475
- 256

219
- 128

91
- 64

27
- 16

11
- 8

3

2780 = 1010110111
- 2048

732
- 512

220
- 128

92
- 64

28
- 16

12
- 8

3421 = 110101110101
- 2048

1333
- 1024

309
- 256

53
- 32

21
- 16

1110010 = 114
64 32 16 0 0 2 0

1111100 = 124
64 32 16 8 4 0 0

1000011 = 67
64 0 0 0 0 2 1

1001	9
1111	15
11000 =	24

Zeichenvorrat

Binär: {0,1}
Dezimal: {0,1,2,3,4,5,6,7,8,9}
Oktal: {0,1,2,3,4,5,6,7}
Hexadezimal: {0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F}

Oktal:

:8

417	1
52	4
6	6
0	

641

641

8^2	8^1	8^0
64	8	1

384 + 32 + 1 = 417

Logische Verknüpfungen

Und

P

+

Q

$\wedge$

und

$P \wedge Q$

P	Q	$P \wedge Q$
W	W	W
W	F	F
F	W	F
F	F	F

W = Wahr
F = Falsch

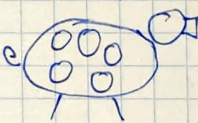
Oder

$\vee$

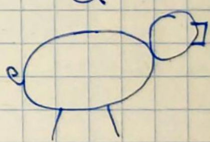
oder

P

R

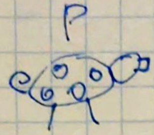


Q



P	Q	$P \vee Q$
W	W	W
W	F	W
F	W	W
F	F	F

P	Q	R	$P \vee Q \vee R$
W	W	W	W
W	W	F	W
W	F	W	W
F	W	W	W
F	F	W	W
W	F	F	W
F	W	F	W
F	F	F	F



Nicht

P	$\bar{P}$
F	W
W	F

Q	R	P	$\bar{P} \land \bar{Q} \lor Q \land R$	$\bar{P} \land \bar{Q}$	$Q \land R$
W	W	W	W	F	W
W	W	F	W	F	W
W	F	W	F	F	F
W	F	F	F	F	F
F	W	W	F	F	F
F	W	F	W	W	F
F	F	W	F	F	F
F	F	F	W	W	F

$\bar{P} \land \bar{Q} \lor \overline{Q \land R}$	$\overline{Q \land R}$

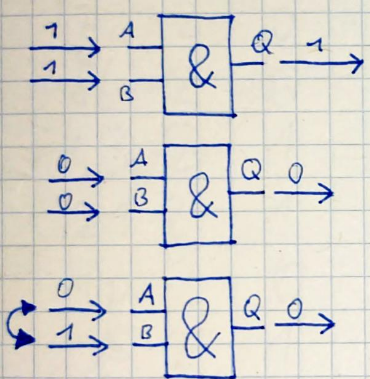
A	B	$\bar{A} \vee \bar{B}$
W	W	F
W	F	W
F	W	W
F	F	W

A	B	$\overline{A \wedge B}$
W	W	F
W	F	F
F	W	F
F	F	W

A	B	$\bar{A} \wedge \bar{B}$
W	W	F
W	F	F
F	W	F
F	F	W

A	B	$\overline{A \vee B}$
W	W	F
W	F	W
F	W	W
F	F	W

Und



A	B	C	$(A \wedge B)$	$\vee$	$(B \vee C)$	$\wedge$	$(A \wedge \overline{B})$	— " —
W	W	W	W	W	W	F	F	W
W	W	F	W	W	W	F	F	W
W	F	W	F	W	W	W	W	W
W	F	F	F	F	F	F	W	F
F	W	W	F	W	W	F	F	W
F	W	F	F	W	W	F	F	W
F	F	W	F	W	W	F	F	W
F	F	F	F	F	F	F	F	F

$$\begin{array}{r} 10110 \\ -\ 01001 \\ \hline 01101 \end{array} \quad \begin{array}{r} 22 \\ -\ 9 \\ \hline =13 \end{array} \qquad\qquad \begin{array}{r} 10110 \\ -\ 01001 \\ \hline \end{array}$$

$$\begin{array}{r} 10110 \\ -\ 1001 \\ \hline \end{array} \qquad \begin{array}{r} 0110 \\ +1 \\ \hline 0111 \end{array} \begin{array}{l} \text{1. Komp.} \\ \\ \text{2. Komp.} \end{array}$$

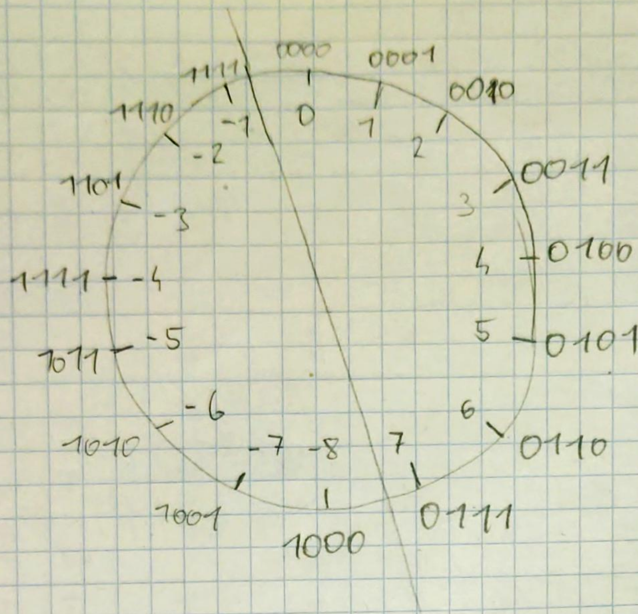
$$\begin{array}{r} 10110 \\ +\ 0111 \\ \hline 01101 \end{array}$$

53 : 2	1		24 : 2	0
26 : 2	0		12 : 2	0
13 : 2	1		6 : 2	0
6 : 2	0		3 : 2	1
3 : 2	1		1 : 2	1
1 : 2	1			

```
  110101        110101              110101
- 111000      -  11000    →       +  00111
  011011                            011100
```

```
  172  =  0101100                 0101100
-  69  =  000101                + 111011
              +1                  ~~1~~100111
```

172 : 2	0	69 : 2	1
86 : 2	0	34 : 2	0
43 : 2	1	17 : 2	1
21 : 2	1	8 : 2	0
10 : 2	0	4 : 2	0
5 : 2	1	2 : 2	0
2 : 2	0		




```
  0011
+ 1101
(1)0000
```

```
 1000
 0110
 ----
 1110
```

0000 0000

-128

0

+127

Multiplikation

57 · 23

57	1
28	0
14	0
7	1
3	1
1	1

23	1	
11	1	
5	1	
2	~~1~~	0
1	1	

```
111001 · 10111
111001
     00
  111001
   111001
    111001
-----------
10100011111
```

$$
\begin{array}{rcr}
10111 & & 21 \\
11011 & & 27 \\
11011 & & 27 \\
1111 & & 15 \\
\hline
1011100 & = & 92
\end{array}
$$

$64 + 16 + 8 + 4 =$

Teststoff

- Analoges, Digitales Signal, Binäres Signal (Unterschied, Aussehen)
- Zahlensysteme (Dezi, Binär, Hexa (Darstellung, Umrechnung)
- Logische Verknüpfung (Wahrheitstabelle, Und, oder, nicht)
- Binärsystem (+, -)
- BCD- Code
- Schaltung (in Wahrheitstabelle übertragen)

BCD - Code

$12_{10} \Rightarrow 1100_2$

DEZ	BCD 842
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

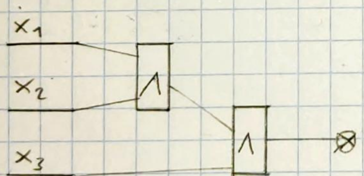
7 - Segment:

0110 0000 1001 0011
6 0 9 3

6093

73 = 0111 0011
64 32 16 2 1 = 115_2

0111 1111 = null / Fehler
↑
zu groß



x_1	x_2	x_3	$(x_1 \wedge x_2) \wedge x_3$	$\overline{(x_1 \wedge x_2) \wedge x_3}$
1	1	1	1	0
1	1	0	0	1
1	0	1	0	1
1	0	0	0	1
0	1	1	0	1
0	1	0	0	1
0	0	1	0	1
0	0	0	0	1

Gray- Code

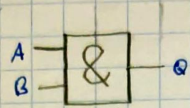
	BCD	Distanz
0	0000	
1	0001	1
2	0010	2
3	0011	1
4	0100	3
5	0101	1
6	0110	2
7	0111	1
8	1000	4
9	1001	1

	Gray
0	0000
1	0001
2	0011
3	0010
4	0110
5	0111
6	0101
7	0100
8	1100
9	1101

1 Stellenunterschied (immer)
bei 16 Stellen zyklisch

Logikgatter

Und - Gatter



$A \wedge B = Q$
$A \cdot B = Q$
$AB = Q$
$A \& B = Q$

A	B	Q
F	F	F
F	W	F
W	F	F
W	W	W

Oder - Gatter

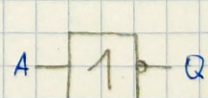
$A \vee B = Q$
$A + B = Q$

A	B	Q
F	F	F
F	W	W
W	F	W
W	W	W

Nicht - Gatter

$\overline{A} = Q$
$\neg A = Q$

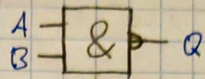
A	Q
F	W
W	F



Nicht - Und - Gatter

NAND

$$Q = \overline{A \wedge B}$$
$$Q = \overline{AB}$$

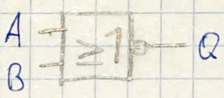


A	B	Q
F	F	W
F	W	F
W	F	F
W	W	F

Nicht - Oder - Gatter

NOR

$$Q = \overline{A \vee B}$$
$$Q = \overline{A + B}$$

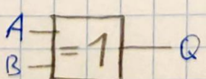


A	B	Q
F	F	W
F	W	W
W	F	W
W	W	F

Exklusives - Oder - Gatter

XOR

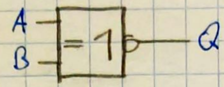
$$Q = A \underline{\vee} B$$
$$Q = A \oplus B$$



A	B	Q
F	F	F
F	W	W
W	F	W
W	W	F

Exklusives-Nicht-Oder-Gatter XNOR

$$Q = \overline{A \underline{\vee} B}$$
$$Q = \overline{A \oplus B}$$



A	B	Q
F	W	F
F	F	W
W	W	W
W	F	F

Boolsche Algebra

Und-Verknüpfung

$A \wedge 0 = 0$
$A \wedge 1 = A$
$A \wedge A = A$
$A \wedge \overline{A} = 0$

Oder-Verknüpfung

$A \vee 0 = A$
$A \vee 1 = 1$
$A \vee A = A$
$A \vee \overline{A} = 1$

Nicht-Verknüpfung

$$A = \neg\neg A = \overline{\overline{A}}$$

Gesetze der Schaltalgebra

1. Kommutativgesetz:

$$A \wedge B = B \wedge A$$
$$A \vee B = B \vee A$$

2. Assozativgesetz

$$(A \wedge B) \wedge C = A \wedge (B \wedge C)$$
$$(A \vee B) \vee C = A \vee (B \vee C)$$

3. Distributivgesetz

$$A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$$
$$[\, a \cdot (b + c) = (a \cdot b) + (a c) \,]$$

$$A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$$
$$[\, a + b \cdot c = (a + b) \cdot (a + c)$$

4. Neutrales Element

$$A \wedge 1 = A$$
$$A \vee 0 = A$$

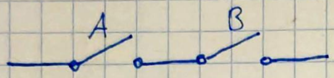
5. Komplementäres Element

$$A \wedge \overline{A} = 0$$
$$A \vee \overline{A} = 1$$

6. Demorgan Gesetze

$$\bar{A} \land \bar{B} = \overline{(A \lor B)} = \neg\overline{(A \lor B)}$$
$$\bar{A} \lor \bar{B} = \overline{(A \land B)} = \neg(A \land B)$$

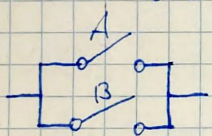
Reihenschaltung



$f(A,B) \Rightarrow A \land B$

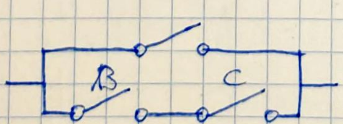
A	B	$A \land B$
1	1	1
1	0	0
0	1	0
0	0	0

Parallelschaltung



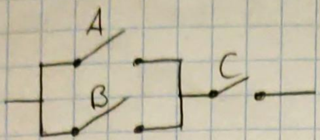
$f(A,B) \Rightarrow A \lor B$

A	B	$A \lor B$
1	1	1
1	0	1
0	1	1
0	0	0

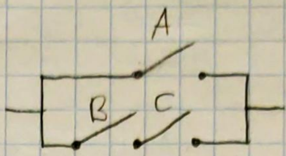


$A \lor (B \land C)$

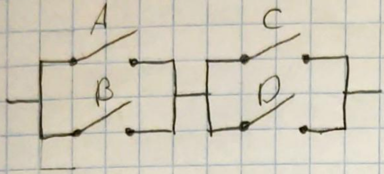
1 $(A\lor B)\land C$



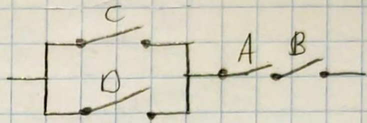
2 $A\lor(B\land C)$



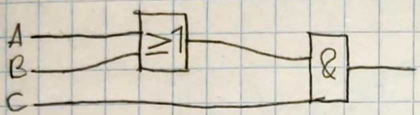
3 $(A\lor B)\land(C\lor D)$



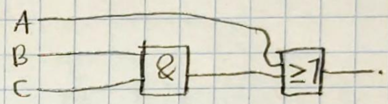
4 $(A\land B)\land(C\lor D)$



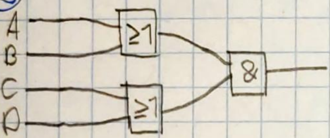
1



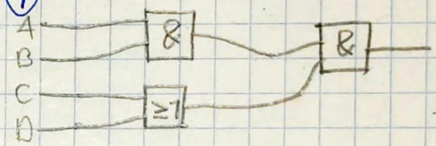
2



3



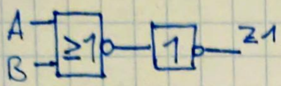
4



$(A\land B\land C)\lor(A\land B\lor C)\lor(A\land\bar{B}\land C)\lor(\bar{A}\land B\land C)$

A	B	C	
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

1



$\overline{\overline{A \vee B}} = A \vee B$

A	B	$A \vee B$
W	F	W
W	W	W
F	F	F
F	W	W

2

A	B	C	D	$A \vee B \wedge C \wedge D$
W	W	W	W	W
W	W	W	F	W
W	W	F	W	W
W	W	F	F	W
W	F	W	W	W
W	F	W	F	W
W	F	F	W	W
W	F	F	F	W
F	W	W	W	W
F	W	W	F	F
F	W	F	W	F
F	W	F	F	F
F	F	W	W	F
F	F	W	F	F
F	F	F	W	F
F	F	F	F	F

$$z_2 = a \wedge (a \vee b) = a$$

$$z_1 = a \vee (a \wedge b) = a$$

$$z_3 = a \vee (\bar{a} \wedge b) = a \vee b$$

$$z_4 = a \wedge (\bar{a} \vee b) = a \wedge b$$

$$Z_5 = (a \wedge b) \vee (a \wedge \bar{b}) = a$$

$$Z_6 = (a \vee b) \wedge (a \vee \bar{b}) = a$$

$$Z_7 = (a \wedge b \wedge c) \vee (\bar{a} \wedge b \wedge c) = (a \wedge (b \wedge c)) \vee (\bar{a} \wedge (b \wedge c)) =$$
$$(a \vee \bar{a}) \wedge (b \wedge c) = 1 \wedge (b \wedge c) = b \wedge c$$

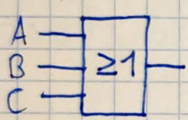
$$Z_8 = (\bar{a} \wedge \bar{b} \wedge \bar{c} \wedge \bar{d}) \vee (\bar{a} \wedge b \wedge \bar{c} \wedge \bar{d}) \vee (\bar{a} \wedge \bar{b} \wedge \bar{c} \wedge d)$$
$$\vee (\bar{a} \wedge b \wedge \bar{c} \wedge d) =$$
$$\bar{a} \wedge \bar{c} \wedge (\bar{b} \wedge \bar{d} \vee b \wedge \bar{d} \vee \bar{b} \wedge c \vee b \wedge d)$$
$$\bar{a} \wedge \bar{c} \wedge 1 = \bar{a} \wedge \bar{c}$$

$$Z_9 = \overline{\bar{a} \vee \bar{b}} \wedge \overline{\bar{a} \wedge \bar{b}} = (a \wedge b) \wedge (a \vee b) =$$
$$a \wedge a \wedge b \vee a \wedge b \wedge b = a \wedge b \vee a \wedge b = a \wedge b$$

$$Z_{10} = (a \wedge \bar{b} \wedge c) \vee (\bar{a} \wedge \bar{b} \wedge \bar{c}) \vee (a \wedge b \wedge c) \vee (\bar{a} \wedge b \wedge \bar{c}) =$$
$$(a \wedge c) \wedge (b \vee b) \vee (\bar{a} \wedge \bar{c}) \wedge (b \vee \bar{b}) =$$
$$((a \wedge c) \vee (\bar{a} \wedge \bar{c})) \wedge (b \wedge b) =$$
$$a \wedge c \vee \bar{a} \wedge \bar{c}$$

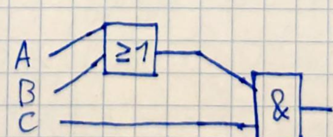
2. Test - Stoff:

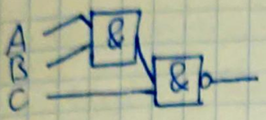
- Schaltungsvereinfachung (ab S. 38-47)
  - Schaltalgebra
  - Rechengesetze
- Schaltungen aufstellen
  - Textaufgabe
  - aus Gattersymbole



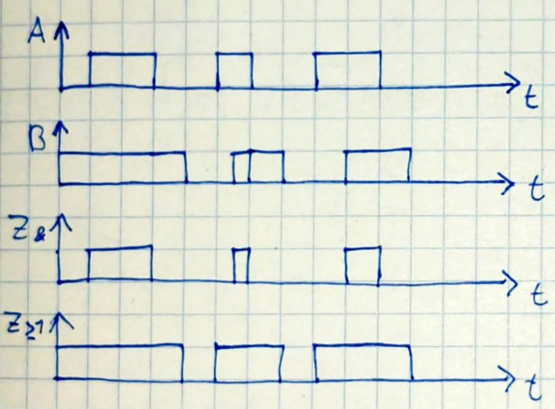
A	B	C	A∨B∨C
W	W	W	W
W	W	F	W
W	F	W	W
W	F	F	W
F	W	W	W
F	W	F	W
F	F	W	W
F	F	F	F

A	B	C	(A∨B)∧C
W	W	W	W
W	W	F	F
W	F	W	W
W	F	F	F
F	W	W	W
F	W	F	F
F	F	W	F
F	F	F	F





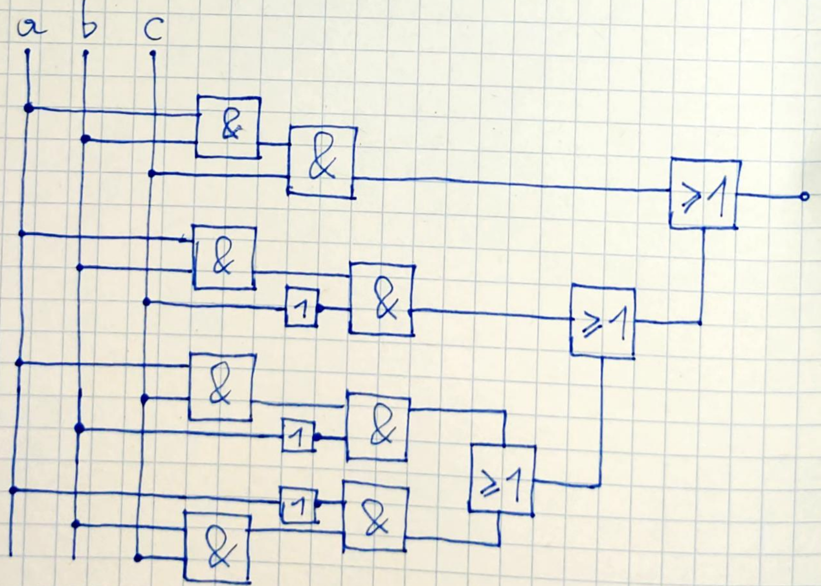
A	B	C	$\overline{(A \wedge B \wedge C)}$
W	W	W	F
W	W	F	F
W	F	W	F
W	F	F	F
F	W	W	F
F	W	F	F
F	F	W	F
F	F	F	W



Disjunktive Normal form

a	b	c	f(a, b, c)
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

$$f(a, b, c) = (a \wedge b \wedge c) \vee (a \wedge b \wedge \bar{c}) \vee (a \wedge \bar{b} \wedge c) \vee (\bar{a} \wedge b \wedge c)$$



1. Markiere alle Zeilen, in denen das Ergebnis gleich 1 ist

2. Verknüpfe die Eingangsvariablen jeder Zeile mit Und ($\land$)

3. Verknüpfe die Und-($\land$)-Glieder mit Oder ($\lor$)

HÜ 1

a	b	c	d	f(a, b, c, d)
1	1	1	1	1
1	1	1	0	1
1	1	0	1	0
1	1	0	0	0
1	0	1	1	1
1	0	1	0	1
1	0	0	1	1
1	0	0	0	1
0	1	1	1	1
0	1	1	0	0
0	1	0	1	1
0	1	0	0	0
0	0	1	1	1
0	0	1	0	1
0	0	0	1	1
0	0	0	0	1

$f(a,b,c,d) = (a \land b \land c \land d) \lor (a \land b \land c \land \bar{d}) \lor (a \land b \land \bar{c} \land d) \lor$
$(a \land b \land \bar{c} \land \bar{d}) \lor (a \land \bar{b} \land c \land d) \lor (a \land \bar{b} \land c \land \bar{d}) \lor (a \land \bar{b}$
$\land \bar{c} \land d) \lor (a \land \bar{b} \land \bar{c} \land \bar{d}) \lor (\bar{a} \land b \land c \land d) \lor (\bar{a} \land b \land c \land \bar{d}) \lor (\bar{a} \land b \land \bar{c} \land d)$
$\lor (\bar{a} \land b \land \bar{c} \land \bar{d}) \lor (\bar{a} \land \bar{b} \land c \land d) \lor (\bar{a} \land \bar{b} \land c \land \bar{d}) \lor (\bar{a} \land \bar{b} \land \bar{c} \land d) \lor (\bar{a} \land \bar{b} \land \bar{c} \land \bar{d}) = *$

a	b	c	f(a,b,c)
1	1	1	1
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	0

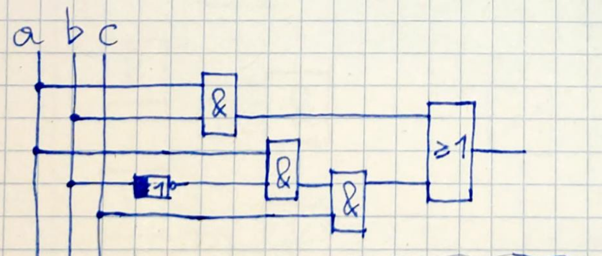
⊗ $f(a,b,c) = (a \land b \land c) \lor (a \land b \land \bar{c}) = (a \land b) \land (c \lor \bar{c}) = a \land b$

✓ $f(a,b,c) = (a \land b) \lor (a \land b \land c) = (a \land b \land a) \lor (a \land b \land c) = (a \land b) \land (a \lor c)$

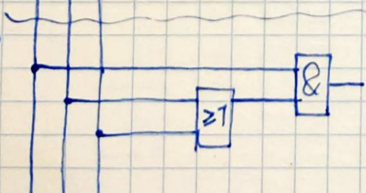
⊗ $(a \land b) \lor (a \land \bar{b} \land c) = (a \land a \land c) \land (b \lor \bar{b}) = (a \land c) \land 1 = a \land c$

✓ $= a \land (b \lor (\bar{b} \land c)) = a \land ((b \lor \bar{b}) \land (b \lor c)) = a \land (b \lor c)$

①



②

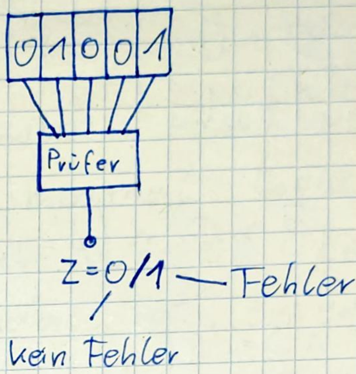


Fehlererkennende Codes

BCD Code

	1	2	3	4	E
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	0
4	0	1	0	0	1
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	0

← Das letzte Bit der 5-Bit Einheit wird auch Prüfbit genannt.



Redundanz

Redundanz: Liegt immer dann vor, wenn außer der eigentlichen Information noch zusätzliche Informationen übertragen werden, die eine Fehlererkennung oder eine Fehlerkorrektur ermöglichen.
Für die Fehlerkorrektur benötigt man eine größere Redundanz als für d. Fehlererkennung.

#	A	B	C	f
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

ges.: Formel im DNF für die Funktion A, B, C die genau dann wahr / 1 ist wenn # = Primzahl

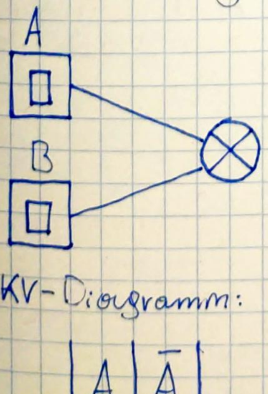
$$DNF = (\bar{A} \land \bar{B} \land C) \lor (\bar{A} \land B \land \bar{C}) \lor (\bar{A} \land B \land C) \lor (A \land \bar{B} \land C) \lor (A \land B \land C)$$

$$KNF = (\bar{A} \lor \bar{B} \lor \bar{C}) \land (\bar{A} \lor \bar{B} \lor C) \land (A \lor B \lor \bar{C})$$

Teststoff:
Boolsche Algebra vereinfachen
Fehlerkennende Codes (BCD)
Schaltungsaufstellung (DNF/KNF)
KV-Diagramm m. 3 Variablen
Gatter

KV-Diagramme

Mittels eines KV-Diagramms lässt sich jede beliebige DNF in einen minimal logischen Ausdruck umwandeln. Der Vorteil gegenüber anderer Verfahren ist, dass d. erzeug? Term meist minimal ist. Sollte d. Term noch nicht minimal sein, ist eine weitere Vereinfachung durch anwenden d. Bool'schen Algebra möglich. Das Umwandeln beginnt mit d. Erstellen einer Wahrheitstabelle, aus der dann die DNF abgeleitet wird, die dann wiederrum direkt in ein KV-Diagramm umgewandelt wird.



A	B	L
0	0	0
0	1	1
1	0	1
1	1	0

DNF:

$$(A \wedge \bar{B}) \vee (\bar{A} \wedge B)$$

KV-Diagramm:

	A	$\bar{A}$
B	0	1
$\bar{B}$	1	0

A	B	X
1	1	1
1	0	0
0	1	1
0	0	0

Identität v. B

$(A \land B) \lor (\bar{A} \land B)$

	A	$\bar{A}$
B	1	1
$\bar{B}$		

$A \lor \bar{A} = 1$

$B \land 1 = B$

$(A \land B) \lor (\bar{A} \land B) =$

$B \land (A \lor \bar{A}) =$

$B \land 1 = B$

Negation v. A

A	B	X
0	0	1
0	1	1
1	0	0
1	1	0

DNF: $(\bar{A} \land \bar{B}) \lor (\bar{A} \land B)$

	A	$\bar{A}$
B		1
$\bar{B}$		1

$\Rightarrow \bar{A}$

$(\bar{A} \land \bar{B}) \lor (\bar{A} \land B) = \bar{A} \land (\bar{B} \lor B) = \bar{A} \land 1 = \bar{A}$

Implikation v. A auf B

A	B	X
1	1	1
1	0	0
0	1	1
0	0	1

DNF: $(A \wedge B) \vee (\bar{A} \wedge B) \vee (\bar{A} \wedge \bar{B}) =$

$B \wedge (A \vee \bar{A}) \vee (\bar{A} \wedge \bar{B}) =$

$B \vee (\bar{A} \wedge \bar{B}) =$

$(B \vee \bar{A}) \wedge (B \vee \bar{B}) =$

$(B \vee \bar{A}) \wedge 1 = B \vee \bar{A}$

	A	$\bar{A}$
B	1	1
$\bar{B}$		1

$\Rightarrow B \vee \bar{A}$

Tautologie

A	B	X
1	1	1
1	0	1
0	1	1
0	0	1

DNF: $(A \wedge B) \vee (A \wedge \bar{B}) \vee (\bar{A} \wedge B) \vee (\bar{A} \wedge \bar{B}) =$

$A \wedge (B \vee \bar{B}) \vee \bar{A} \wedge (B \vee \bar{B}) =$

$A \wedge 1 \vee \bar{A} \wedge 1 = A \vee \bar{A} = 1$

	A	$\bar{A}$
B	1	1
$\bar{B}$	1	1

$\Rightarrow 1$

A	B	C	X
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

DNF: $(\bar{A} \land \bar{B} \land \bar{C}) \lor (\bar{A} \land \bar{B} \land C) \lor$
$(\bar{A} \land B \land \bar{C}) \lor (\bar{A} \land B \land C) \lor$
$(A \land B \land \bar{C}) \lor (A \land B \land C)$

	A	A	$\bar{A}$	$\bar{A}$
B	1	1	1	1
$\bar{B}$			1	1
	C	$\bar{C}$	$\bar{C}$	C

$B \lor \bar{A}$

Test-Verbesserung

1

429,71 =

0100 0010 1001 , 0111 0001

4	0100	1
2	0010	1
9	1001	0
7	0111	1
1	0001	1

2

$(x \land y) \lor \bar{y} = (x \lor \bar{y}) \land (y \lor \bar{y}) =$

$(x \lor \bar{y}) \land 1 = x \lor \bar{y}$

x	y	$x \lor \bar{y}$	$(x \land y) \lor \bar{y}$
1	1	1	
1	0	1	
0	1	0	
0	0	1	

(3)

$\boxed{=1}\circ$ XNOR

$\boxed{\&}$ AND

$\boxed{\geq 1}$ OR

$\boxed{1}\circ$ NOT

$\boxed{=1}$ XOR

(4)

a	b	c	x
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	0

DNF: $(a \wedge b \wedge c) \vee (a \wedge b \wedge \bar{c}) \vee$
$(a \wedge \bar{b} \wedge c) =$
$a \wedge ((b \wedge c) \vee (b \wedge \bar{c}) \vee (\bar{b} \wedge c)) =$
$a \wedge (b \wedge (c \vee \bar{c}) \vee (\bar{b} \wedge c)) =$
$a \wedge (b \wedge 1 \vee (\bar{b} \wedge c)) =$
$a \wedge (b \vee (\bar{b} \wedge c)) =$
$a \wedge ((b \vee \bar{b}) \wedge (b \vee c)) =$
$a \wedge (b \vee c)$

Arbeitsaufgabe für 24. 4.

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Seite 64 Aufgabe 1-6

KV- Diagramm

A	B	X	Zeile
0	0	0	1
0	1	1	2
1	0	0	3
1	1	1	4

	B	$\bar{B}$
A	1_4	0_3
$\bar{A}$	1_2	0_1

DNF: $(\bar{A} \wedge B) \vee (A \wedge B) =$

$B \wedge (A \vee \bar{A}) = B \wedge 1 = B$

A	B	C	X	Zeile
0	0	0	1	1
0	0	1	0	2
0	1	0	1	3
0	1	1	1	4
1	0	0	1	5
1	0	1	0	6
1	1	0	1	7
1	1	1	0	8

	B	B	$\bar{B}$	$\bar{B}$
A	1_7	0_8	0_6	1_5
$\bar{A}$	1_3	1_4	0_2	1_1
	$\bar{C}$	C	C	$\bar{C}$

Zweierblock: $\bar{A} \wedge B$

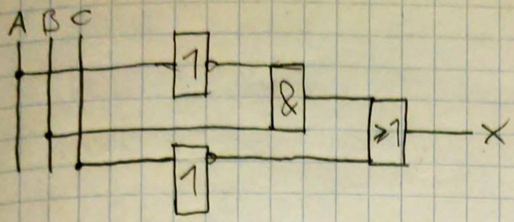
Viererblock: $\bar{C}$

$X = \bar{C} \vee \bar{A} \wedge B$

DNF: $(\bar{A} \wedge \bar{B} \wedge \bar{C}) \vee (\bar{A} \wedge B \wedge \bar{C}) \vee (\bar{A} \wedge B \wedge C) \vee$
$(A \wedge \bar{B} \wedge \bar{C}) \vee (A \wedge B \wedge \bar{C})$

$\vdots$

$= \bar{C} \vee \bar{A} \wedge B$



	B	B	$\bar{B}$	$\bar{B}$
A	1_7	0_8	0_6	1_5
$\bar{A}$	1_3	1_4	0_2	1_1
	$\bar{C}$	C	C	$\bar{C}$

$(A \wedge C) \vee (\bar{B} \wedge C) =$
$C \wedge (A \vee \bar{B})$

A	B	C	D	X	Zeile
0	0	0	0	1	1
0	0	0	1	0	2
0	0	1	0	0	3
0	0	1	1	1	4
0	1	0	0	1	5
0	1	0	1	0	6
0	1	1	0	1	7
0	1	1	1	0	8
1	0	0	0	1	9
1	0	0	1	0	10
1	0	1	0	1	11
1	0	1	1	0	12
1	1	0	0	1	13
1	1	0	1	0	14
1	1	1	0	1	15
1	1	1	1	1	16

	A	A	$\bar{A}$	$\bar{A}$	
B	1_{13}	1_{15}	1_7	1_5	$\bar{D}$
B	0_{14}	1_{16}	0_8	0_6	D
$\bar{B}$	0_{10}	0_{12}	1_4	0_2	D
$\bar{B}$	1_9	1_{11}	0_3	1_1	$\bar{D}$
	$\bar{C}$	C	C	$\bar{C}$	

DNF: $(A \wedge \bar{D}) \vee (B \wedge \bar{D}) \vee (\bar{C} \wedge \bar{D})$
$\vee (A \wedge B \wedge C) \vee (\bar{A} \wedge \bar{B} \wedge C \wedge D)$

KNF: $(\bar{C} \wedge D) \vee (\bar{A} \wedge B \wedge D) \vee (A \wedge \bar{B} \wedge D) \vee (\bar{A} \wedge C \wedge \bar{D} \wedge \bar{B})$

4. Test-Stoff: (Untis)

- Kapitel 4 Schaltsynthese
  -> KNF / DNF / KV-Diagr. (bis 4 Var.)
  bis Punkt 4.2.2.3 (inkl.) - S. 48 - 57

- Binäres Rechnen
  -> Add. / Subtr. v. Binärz. / Umrechnen Dez. - Okt. - Hex - Bin

KV-Diagramm -Regeln

* Ein KV-Diagramm hat immer so viele Felder, wie die zugehörige Tabelle Zeilen
  -> 2 Variablen = $2^2 = 4$ Felder
  -> 3 Variablen = $2^3 = 8$ Felder
  -> 4 Variablen = $2^4 = 16$ Felder
* Die Felder sind immer rechteckförmig angeordnet
* An jeder Kante des KV-Diagramms darf immer nur <u>eine</u> Variable stehen, zur Hälfte negiert, die andere nicht.
  -> Bei 2 Var. 2 Kanten beschriften
  -> Bei 3 Var. 3 Kanten beschriften
  -> Bei 4 Var. 4 Kanten beschriften
* Gegenüber liegende Kanten <u>müssen</u> unterschiedlich aufgeteilt werden
* Die Blöcke sollten so groß wie möglich gebildet werden. Ein Viererblock ist z.B. besser als zwei Zweierblöcke
* Es können nur 2er, 4er, 8er, ... Blöcke gebildet werden (2er-Potenzen)
  Die Felder eines Blockes müssen rechteckförmig oder quadratisch sein.
* Kanten die sich gegenüber liegen, gelten als benachbart, sodass die vier Eckfelder einen 4er-Block bilden können (Bsp. zuvor)

* sind wesentlich weniger 1en als 0en im KV-Diagramm, kann es sinnvoll sein Blöcke mit 0en anstatt 1en zu bilden.

* Den zum jeweiligen Block gehörenden Gleichungsterm findet man, indem die Variable die in allen Reihen des Blocks vorkommen mit $\wedge$ (und) verknüpft.

	B	B	$\bar{B}$	$\bar{B}$	
A	1	0	0	1	$\bar{D}$
A	1	1	0	1	D
$\bar{A}$	1	1	1	0	D
$\bar{A}$	1	0	1	1	$\bar{D}$
	$\bar{C}$	C	C	$\bar{C}$	

DNF: $(\bar{C} \wedge \bar{D}) \vee (A \wedge \bar{C}) \vee (B \wedge D) \vee (C \wedge \bar{A} \wedge \bar{B})$

KNF: $(C \wedge B \wedge \bar{D}) \vee (A \wedge C \wedge \bar{D}) \vee (A \wedge \bar{B} \wedge C) \vee (\bar{A} \wedge \bar{B} \wedge \bar{C} \wedge D)$

Test - Verbesserung A:

1)

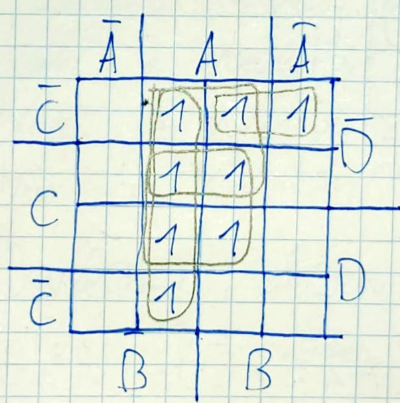
DNF:

$$\bar{A} B \bar{C} \bar{D} \vee A \bar{B} \bar{C} \bar{D} \vee A \bar{B} \bar{C} D \vee A \bar{B} C \bar{D} \vee A \bar{B} C D \vee A B \bar{C} \bar{D} \vee$$
$$A B C \bar{D} \vee A B C D$$

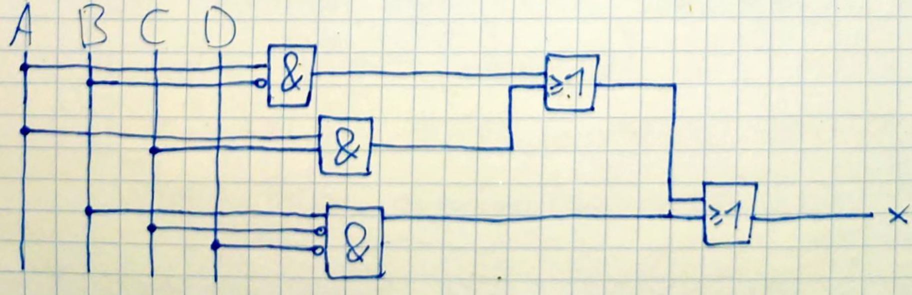
KNF:

$$(\bar{A} \vee \bar{B} \vee \bar{C} \vee \bar{D}) \wedge (\bar{A} \vee \bar{B} \vee \bar{C} \vee D) \wedge (\bar{A} \vee \bar{B} \vee C \vee \bar{D}) \wedge$$
$$(\bar{A} \vee \bar{B} \vee C \vee D) \wedge (\bar{A} \vee B \vee \bar{C} \vee D) \wedge (\bar{A} \vee B \vee C \vee \bar{D}) \wedge$$
$$(\bar{A} \vee B \vee C \vee D) \wedge (A \vee B \vee \bar{C} \vee D)$$

A	B	C	D	x
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1



$$x = A\bar{B} \vee AC \vee B\bar{C}\bar{D}$$



2)

312D

3	1	2	13
16^3	16^2	16^1	16^0

$12288 + 256 + 32 + 13 = 12589$

A1D

10	1	13
16^2	16^1	16^0

$2560 + 16 + 13 = 2589$

Halbaddierer

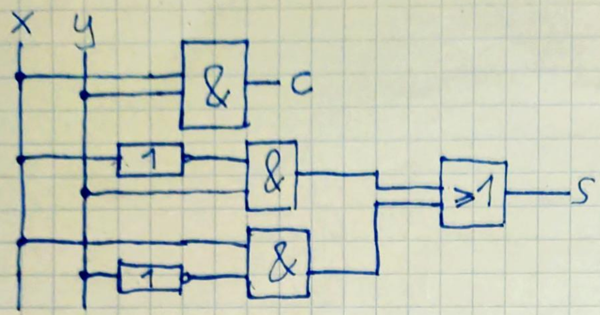
Mit einem Halbaddierer kann man 2 einstellige Binärzahlen addieren

c = „Carry" / Übertrag
s = „Summs" / Summe

x	y	s	c
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

DNF (c): $c = x \wedge y$

DNF (s): $s = (\bar{x} \wedge y) \vee (x \wedge \bar{y})$



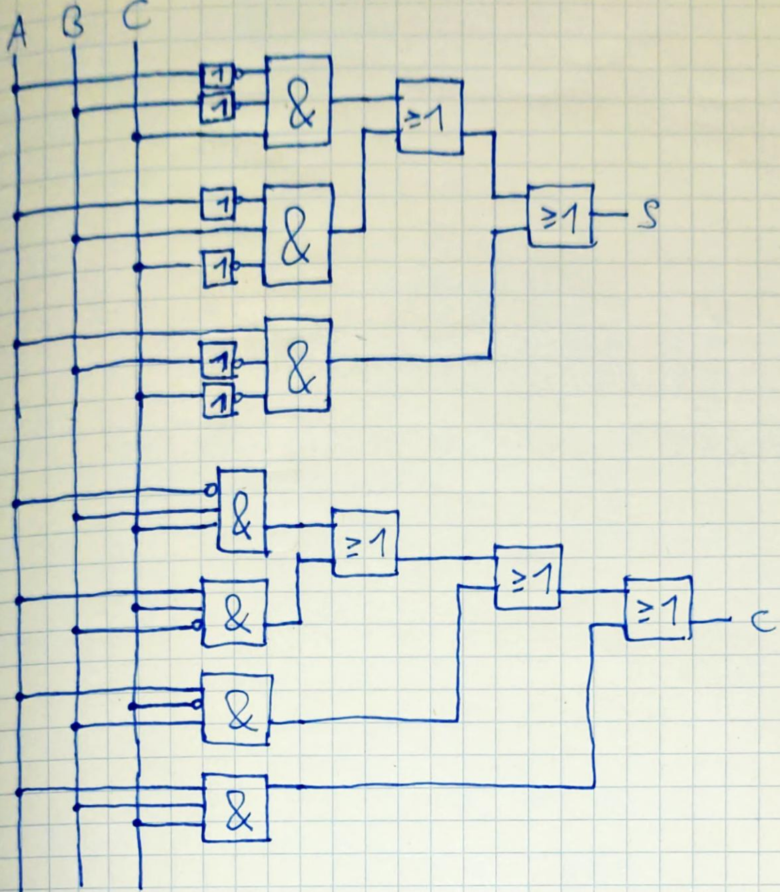
Addition einstelliger Binärzahlen

0 + 0 + 0 = 0
0 + 0 + 1 = 1
0 + 1 + 0 = 1
0 + 1 + 1 = 10
1 + 0 + 0 = 1
1 + 0 + 1 = 10
1 + 1 + 0 = 10
1 + 1 + 1 = 11

Input			Output	
A	B	C	S	C
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

DNF (S): $(\bar{A}\bar{B}C) \vee (\bar{A}B\bar{C}) \vee (A\bar{B}\bar{C}) \vee (ABC)$

DNF (C): $(\bar{A}BC) \vee (A\bar{B}C) \vee (AB\bar{C}) \vee (ABC)$



A
B
C
1
&
≥1
S
c