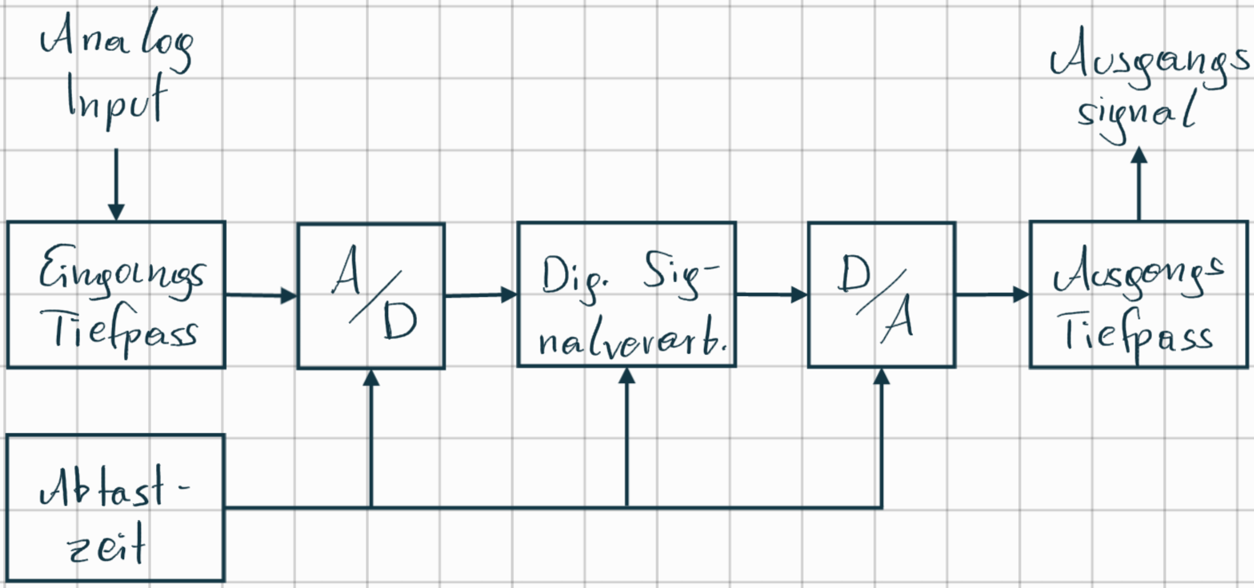
Signaltheorie

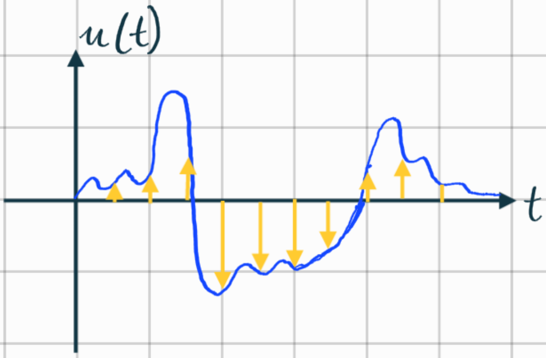
1. Einführung



2. Signalabtastung & Rekonstruktion

2 Problembereiche:

- Zeitbereich:



Shanon-Theorem (Abtasttheorem)

$f_A > 2 \cdot f_{max}$

t_A $\left(f_A = \frac{1}{t_A}\right)$: muss exakt konst. sein

- Quantitativ:
  - bestimmter Bereich
  - definierte Auflösung

2.1 Quantisierungsfehler d. Amplitude

12-Bit A/D-Wandler $2^{12} = 4096$

5V: $\Delta U = \frac{5V}{4096} = 1{,}2\,mV$

Quantisierungsrauschen

$$Q_{SNR} = 10 \cdot \log\left(1{,}5 \cdot 2^{2 \cdot n}\right) dB = (n \cdot 6{,}02 + 1{,}76) dB$$

8-Bit: $Q_{SNR}(8) = 49{,}92 \; dB$
16-Bit: $Q_{SNR}(16) = 98{,}08 \; dB$
24-Bit: $Q_{SNR}(24) = 146{,}24 \; dB$

2.2 Zeitliche Diskretierung

Überlegung: Mit welcher Frequenz muss ein Signal abgetastet werden?
-> Abtasttheorem: $f_A > 2 \cdot f_{max}$

Beweis des Abtasttheorems: T_A ... Abtastzeit
Signal: $x(t) = \sin(2\pi \cdot f_0 \cdot t)$ k ... $-\infty$ ... $+\infty$
Abtastung mit $t = k \cdot T_A$

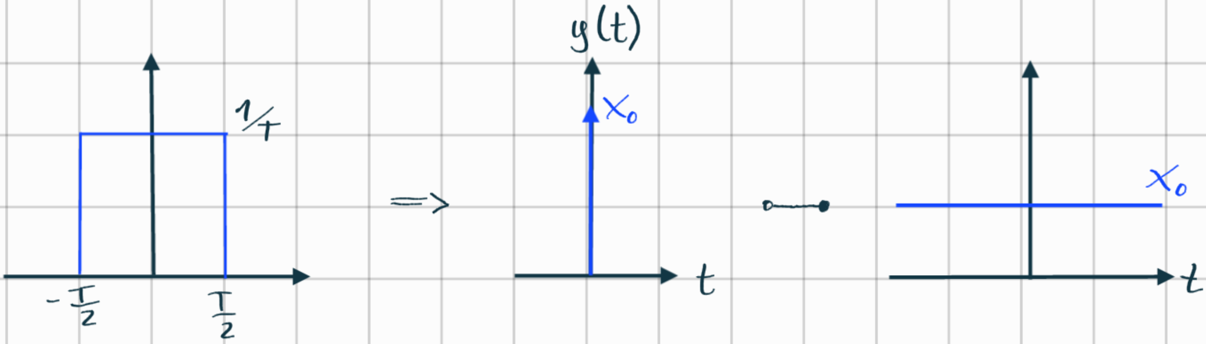
$$\begin{aligned}
x(k) = x(k \cdot T_A) &= \sin(2\pi \cdot f_0 \cdot k \cdot T_A) \\
&\ \sin(2\pi (f_0 + n \cdot f_A) \cdot k \cdot T_A) \\
&= \sin\left(2\pi\left(f_0 + n \cdot \frac{1}{T_A}\right) \cdot k \cdot T_A\right) \\
&= \sin\left(2\pi \cdot f_0 \cdot k \cdot T_A + 2\pi \cdot n \cdot \frac{k \cdot T_A}{T_A}\right) \\
&= \sin(2\pi \cdot f_0 \cdot k \cdot T_A + 2\pi \cdot n \cdot k) \\
&= \sin(2\pi \cdot f_0 \cdot k \cdot T_A)
\end{aligned}$$

2.3 Ideale Abtastung & Rekonstruktion

Mathematische Beschreibung d. idealen Abtastung

$$\delta(t) = \begin{cases} 0 & \text{für } t \neq 0 \\ \infty & \text{für } t = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t) = 1$$



Abtastfunktion: $a(t) = \sum_{k=-\infty}^{\infty} \delta(t - k \cdot T_A)$

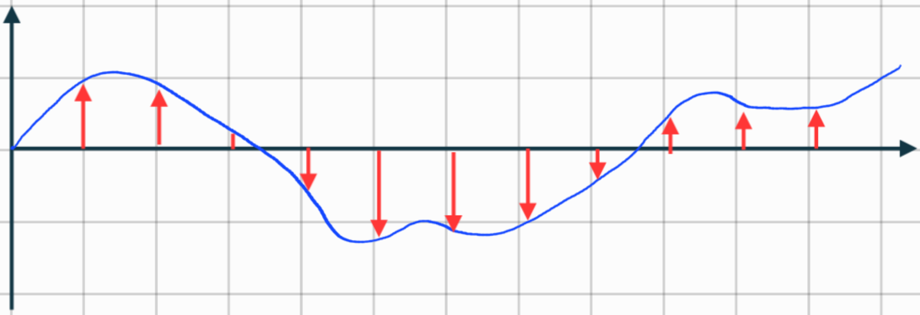
Abtastung von $x(t)$:

$$x_A(t) = x(t) \cdot a(t) = x(t) \cdot \sum_{k=-\infty}^{\infty} \delta(t - k \cdot T_A)$$

$$= \ldots\, x(t) \cdot \delta(t + 2k \cdot T_A) + x(t) \cdot \delta(t + k \cdot T_A) + \ldots$$

$$= \ldots\, x(-2kT_A)\,\delta(t + 2kT_A) + x(-kT_A)\,\delta(t + kT_A) + \ldots$$

$$= \sum_{k=-\infty}^{\infty} x(kT_A) \cdot \delta(t - kT_A)$$



Die Höhe des Impulses entspricht der Gewichtung des Signals.

2.4 Faltung

Faltungsintegral

$$y(t) = \int_{-\infty}^{\infty} u(\tau) \cdot g(t - \tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} u(t - \tau) \cdot g(\tau)\, d\tau$$

$u(t)$ ... Eingangssignal
$g(t)$ ... Impulsantwort

Das Faltungsintegral wird zum Zeitpunkt t ausgewertet. Es ist die Fläche unter einer Funktion die sich aus dem Produkt zweier Teilfunk. ergibt.

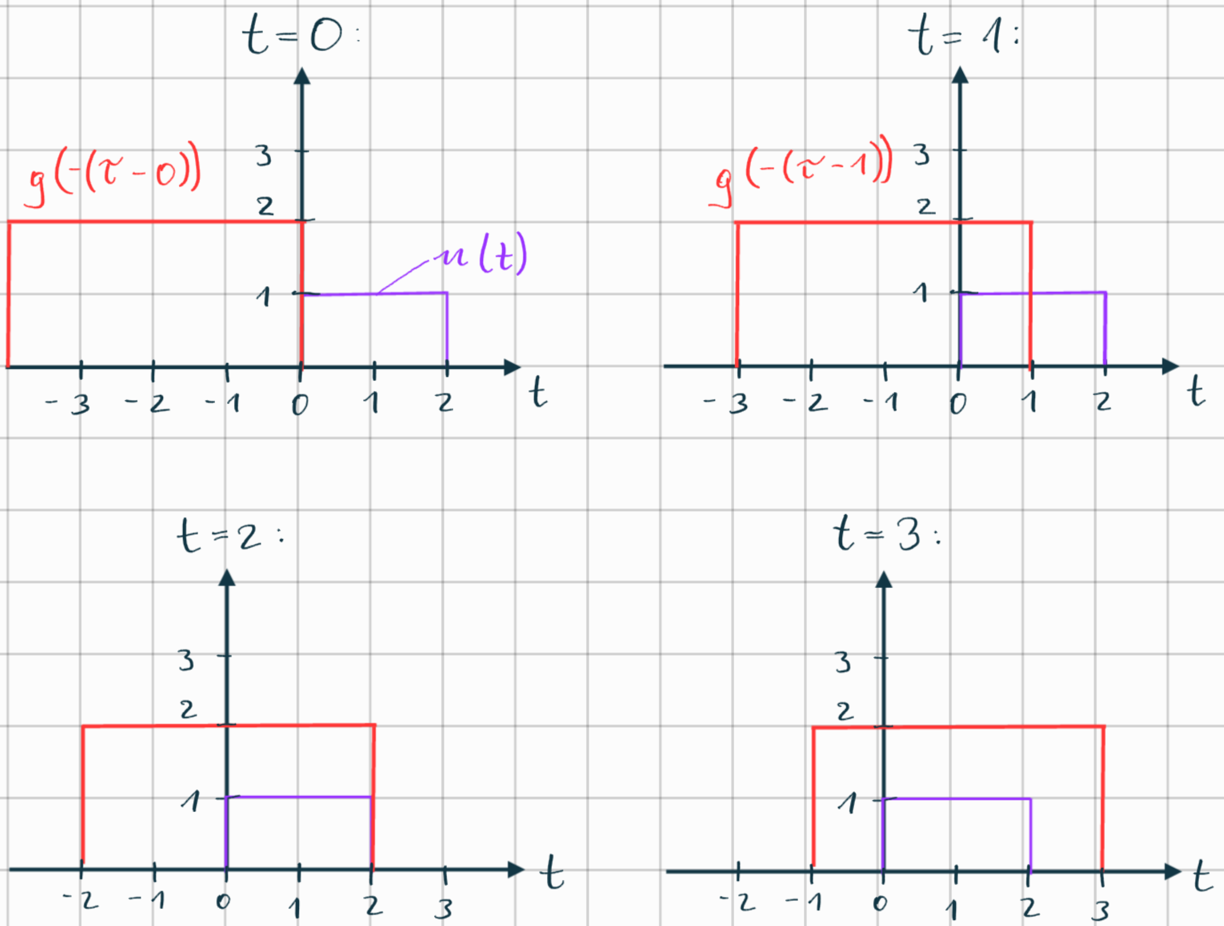
Grafische Auswertung

- Skizzieren der Funktion $u(\tau)$
- Skizzieren von $g(-(\tau - t))$ durch Spiegeln der Funktion $g(\tau)$ und Verschieben nach rechts
- Berechnen des Produkts der beiden Funkt. $u(\tau)\, g(-(\tau - t))$
- Auswertung d. Fläche unter der Kurve $u(\tau)\, g(-(\tau - t))$

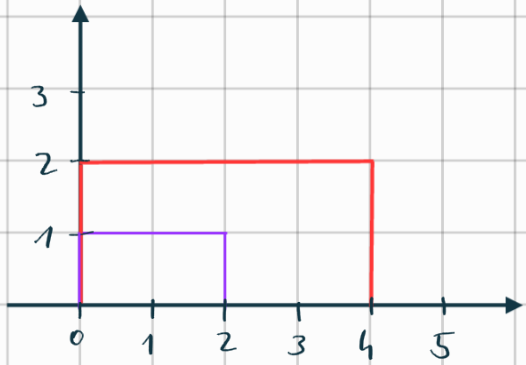
Beispiel:

$u(t) = \delta(t) - \delta(t-2)$ $\quad\quad \delta(t) \ldots$ Sprungfunktion

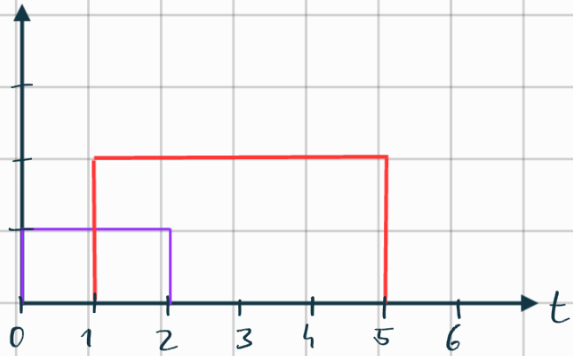
$g(t) = 2(\delta(t) - \delta(t-4))$



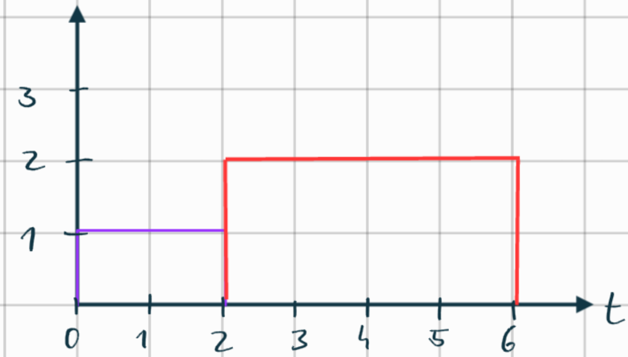
$t = 4:$

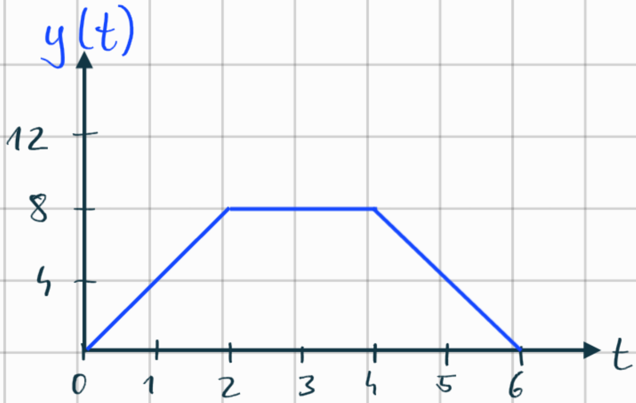


$t = 5:$

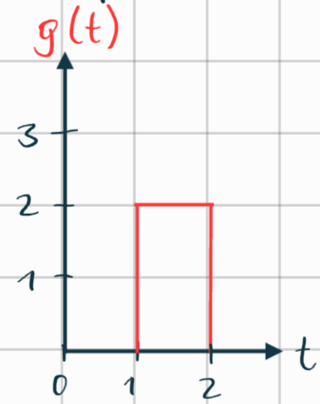


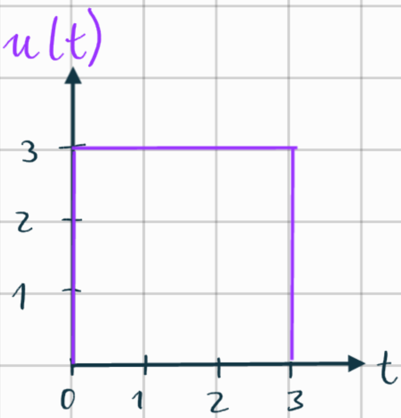
$t = 6:$

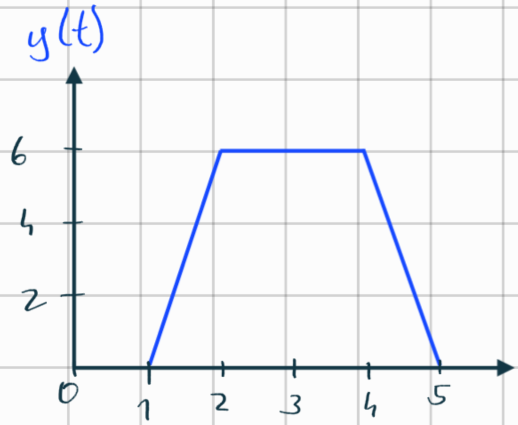




Beispiel:







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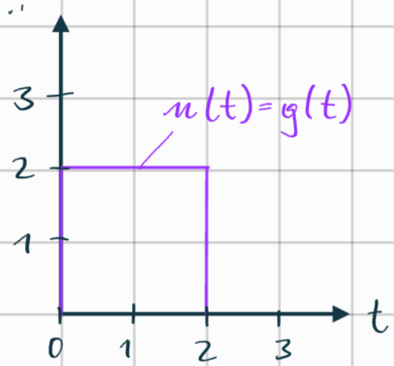
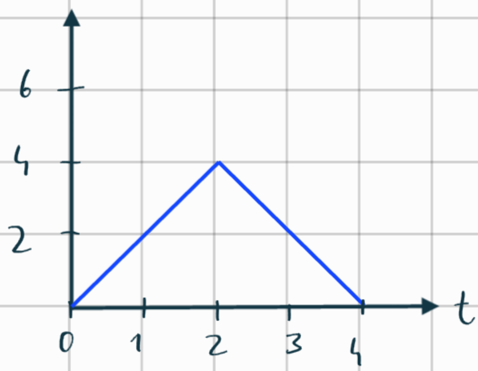




Faltung mit der Impulsfunktion $\delta(t)$

$$u(t) * \delta(t-t_0) = \int_{-\infty}^{\infty} u(t-\tau)\cdot\delta(\tau-t_0)\,d\tau = u(t-t_0)$$

$$\tau - t_0 = 0 \to \tau = t_0$$

Die Faltung eines Signals $u(t)$ mit einem Impuls an der Stelle t_0 verschieb das Signal an die Stelle des Impulses

$$H(f) = \mathcal{F}(h(t))$$

$$H(f) = \int_{-\infty}^{\infty} h(t)\cdot e^{-2\pi f t}\,dt$$

$$G(f) = H(f)\cdot X(f) \quad \text{- Filter}$$

$H(f)$ .. Übertragungsfunktion des Filters
$X(f)$ .. Eingangssignal - Spektrum

$h(t) = \mathcal{F}^{-1}\cdot H(f)$
$h(t)$ = Impulsantwort

$$g(t) = \mathcal{F}^{-1}(G(f)) = \int_{-\infty}^{\infty} h(t-\tau)\cdot x(\tau)\,d\tau$$

$$h(t) * x(t) \bullet\!\!-\!\!\circ\; H(f)\cdot X(f)$$

$*$ .. Faltung

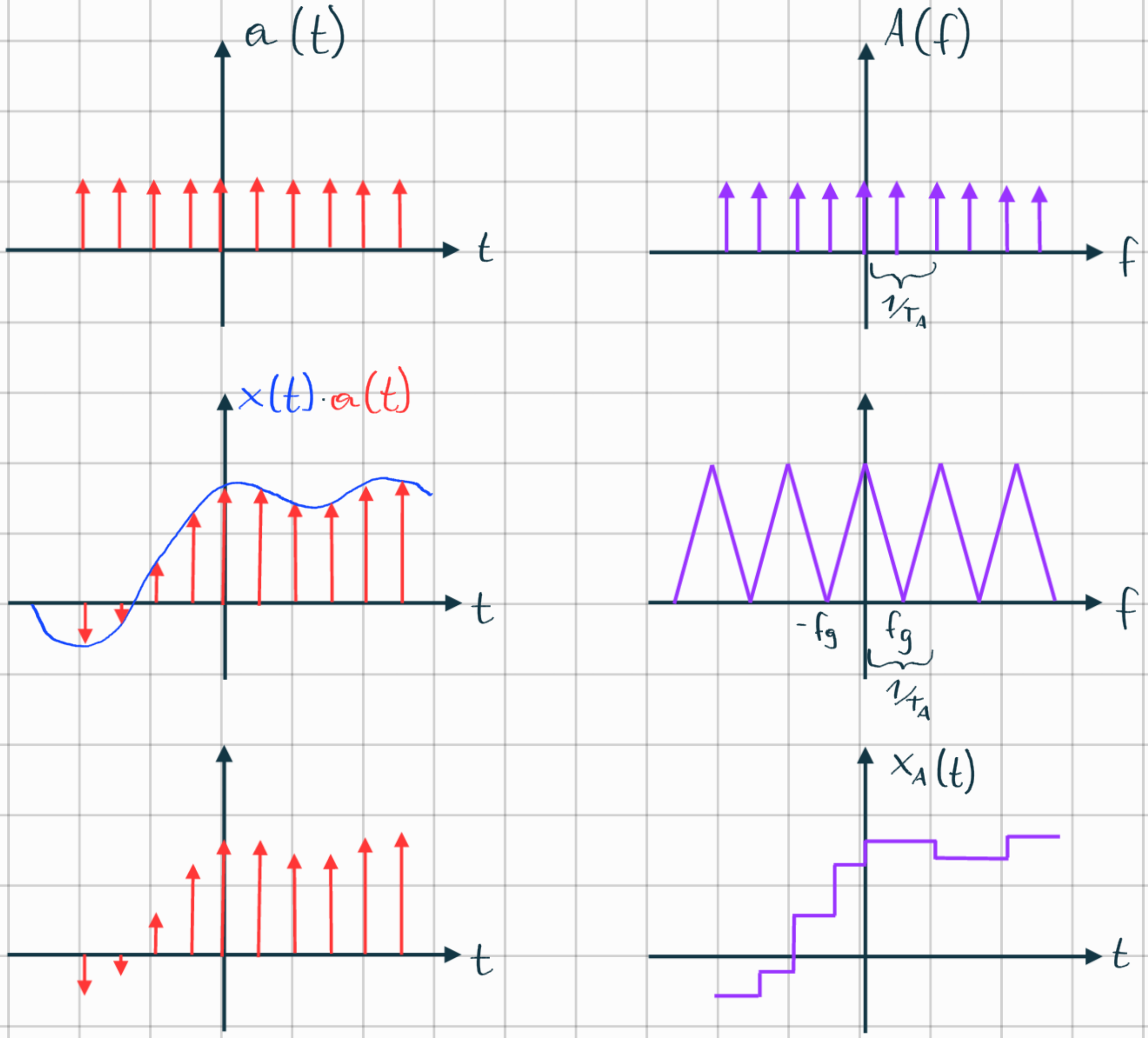
$$h(t)\cdot x(t) \circ\!\!-\!\!\bullet\; H(f) * X(f)$$

$$\mathcal{F}(\delta(t)) = \int_{-\infty}^{\infty} \delta(t)\cdot e^{-j\omega t}\,dt = 1$$

$$\mathcal{F}^{-1}(\delta(\omega)) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega)\cdot e^{j\omega t}\,d\omega = \frac{1}{2\pi}\delta(t)$$

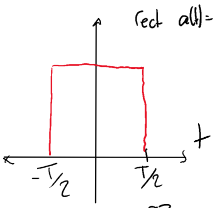
$$\mathcal{F}^{-1}(1) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 1\cdot e^{j\omega t}\,d\omega = \frac{1}{2\pi}\delta(t)$$

2.5 Reale Abtastung & Rekonstruktion



$$\mathrm{rect}\ a(t) = \begin{cases} 1 & \text{wenn } t < \frac{1}{2} \\ 0 & \text{wenn } t > \frac{1}{2} \end{cases}$$

Mit ~~abeglied~~ Halteglied rekonstruier- Signal



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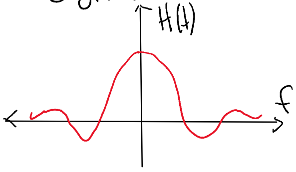
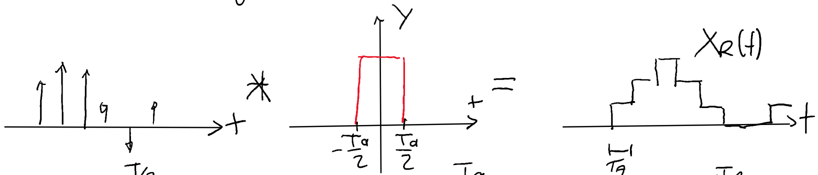


$$H(f) = \int_{-\infty}^{\infty} \mathrm{rect}(t) \cdot e^{-j2\pi f \cdot t}\, dt$$

$$H(f) = \int_{-T/2}^{T/2} e^{-j2\pi f \cdot t}\, dt = \left.\frac{e^{-j2\pi f \cdot t}}{-j2\pi f}\right|_{-\frac{T}{2}}^{+\frac{T}{2}} = \frac{e^{-j2\pi f \cdot \frac{T}{2}}}{-j2\pi f}$$

$$H(f) = -\frac{e^{j2\pi f \frac{T}{2}}}{-j2\pi f}$$



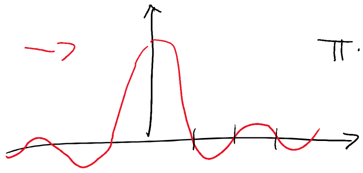
$$H(f) = \int_{-\frac{T}{2}}^{T/2} e^{-j2\pi f t}\, dt = \left.\frac{e^{-j2\pi f t}}{-j2\pi f}\right|_{-\frac{T_a}{2}}^{\frac{T_a}{2}} = \frac{1}{j2\pi f t}\left(-e^{-j2\pi f \frac{T_A}{2}} + e^{+j2\pi f \frac{T_A}{2}}\right)$$

$$\int e^{ax} = \frac{e^{ax}}{a}$$

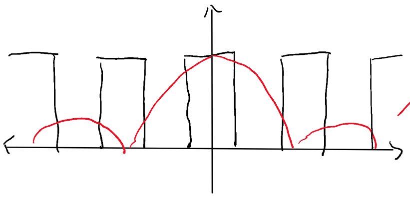
$$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

$$H(f) = \frac{1}{j2\pi f t} \cdot \sin \pi f T_A = k \cdot \frac{\sin x}{x}$$

$\frac{\sin x}{x}$ →



$\pi \cdot f \cdot T_A = \frac{\pi}{2} \rightarrow \sin() = 0$

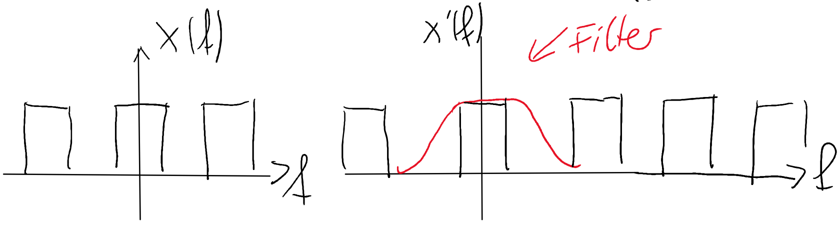


$\left|\frac{\sin \pi \cdot f \cdot T_A}{f}\right|$

Durch den Betrag wird die Ampl. verzerrt

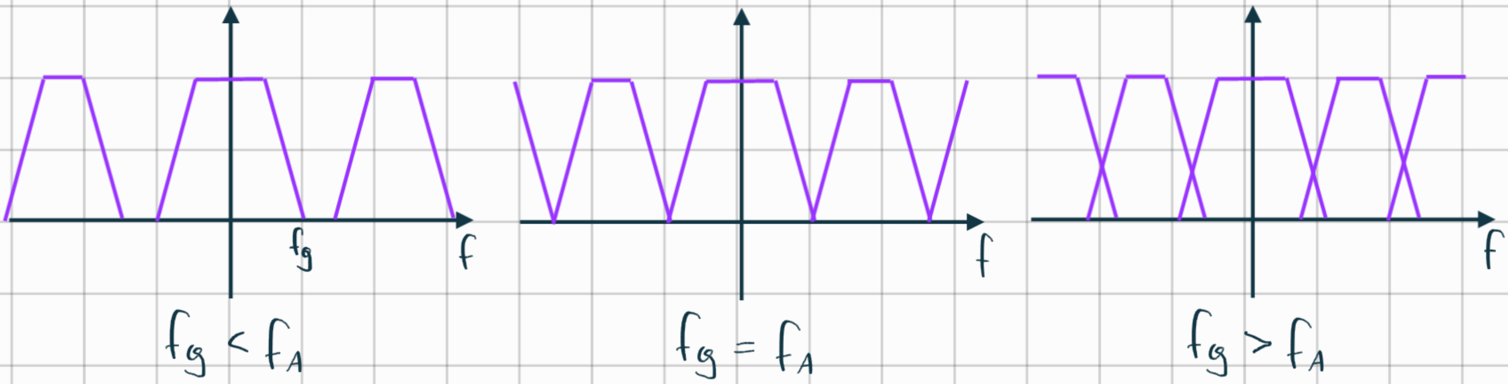
Abhilfe - Vorverzerrung

- Erhöhung Abtastfrequenz
- Multiplikation der Abtastpunkte vor DA Wandler + digitaler Filter



2.6 Abtasttheorem nach Shannon

Tiefpass: $f_{g,TP} < f_A - f_g$



2.7 Übungsaufgaben

2.7.1 Abtastzeit und Signalfolgen

Signal $x(t)$ wird mit T_A abgetastet

$$x(t) = \sin(20\pi t) + \cos(40\pi t)$$

$\hookrightarrow k \cdot T_A$

Abtastung des Signals ergibt:

$$x[k] = \sin\frac{\pi}{5}k + \cos\frac{2\pi}{5}k$$

ges.: a) T_A

b) Lösung eindeutig

a) $20\cancel{\pi} \cdot \cancel{k} \cdot T_A = \frac{\cancel{\pi}}{5} \cdot \cancel{k}$

$$T_A = \frac{1}{5 \cdot 20} = \frac{1}{100}$$

$$40 \cancel{\pi} \cdot \cancel{k} \cdot T_A = \frac{2\cancel{\pi}}{5} \cancel{k}$$
$$T_A = \frac{2}{200} = \frac{1}{100}$$

b) $f = 110\ Hz,\ 210\ Hz,\ 310\ Hz$

$$x'(t) = \sin\left(2\pi(10+100)\,t\right) + \cos\left(2\pi(10+100)t\right)$$
$$x'(k) = x(k) \quad \text{für} \quad T_A = \frac{1}{100}$$

2.7.2 Abtasttheorem Aliasing

$x_1(t)$ & $x_2(t)$ werden mit $T_{A1} = \frac{1}{400}$ und $T_{A2} = \frac{1}{1500}$ abgetastet

$$x_1(t) = \sin(2\pi \cdot 100\,t)$$
$$x_2(t) = \cos(4000\,\pi \cdot t)$$

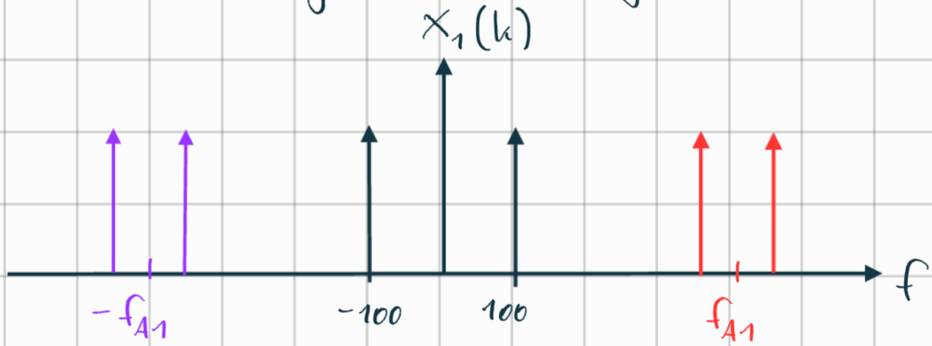
a) Abtasttheorem eingehalten?

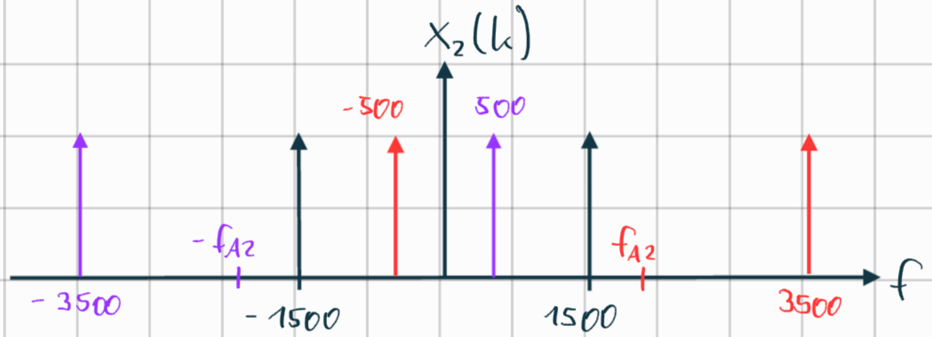
$f_1 = 100\ Hz$	$f_2 = 2000\ Hz$
$f_{A1} = 400\ Hz$	$f_{A2} = 1500\ Hz$
→ Eingehalten	→ Nicht eingehalten

b) Stellen Sie das abgetastete Signal mathematisch dar.

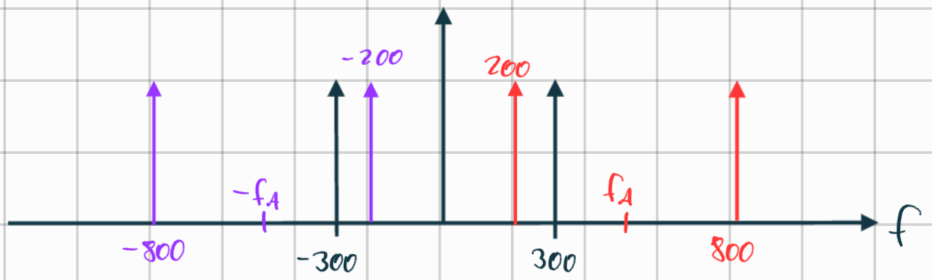
$$x_1(k) = \sum_{k=-\infty}^{\infty} \sin(2\pi \cdot 100 \cdot k \cdot T_{A1}) = \sum_{k=-\infty}^{\infty} \sin\left(\frac{\pi}{2} k\right)$$
$$x_2(k) = \sum_{k=-\infty}^{\infty} \cos(4000\,\pi \cdot k \cdot T_{A2}) = \sum_{k=-\infty}^{\infty} \cos\left(\frac{40\pi}{15} k\right)$$

c) Stellen Sie die Spektren von $X_{A1}(\omega)$ & $X_{A2}(\omega)$ der abgetasteten Signale dar.





$f_S = 300$ Hz, $f_A = 500$ Hz: Spektrum?



3 Zeitdiskrete Signale

3.1 Eigenschaften von Signalfolgen

3.1.1 Zeitlich begrenzte & kausale Signalfolgen

Kausal: Wir betrachten ein Signal ab dem Zeitpunkt 0

Zeitlich begrenzt: Signal wird als endlich betrachtet

3.1.2 Symmetrieeigenschaften von Signalfolgen

gerade Signale: $x(k) = x(-k)$ Bsp.: $x(k) = \cos(\dots)$

ungerade Signale: $x(k) = -x(-k)$ Bsp.: $x(k) = \sin(\dots)$

Jedes Signal kann in einen geraden & ungeraden Anteil zerlegen. $x(k) = x_g(k) + x_{ug}(k)$

Berechnung von $x_g(k)$ & $x_{ug}(k)$

(1) $x(k) = x_g(k) + x_{ug}(k)$

$x(-k) = x_g(-k) + x_{ug}(-k)$

(2) $x(-k) = x_g(k) - x_{ug}(k)$

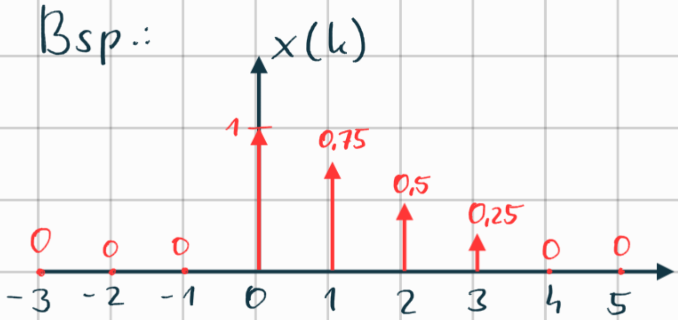
(1) + (2):

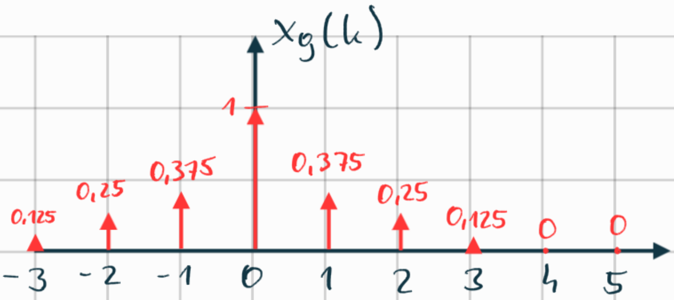
$$x_g(k) = \frac{x(k) + x(-k)}{2}$$

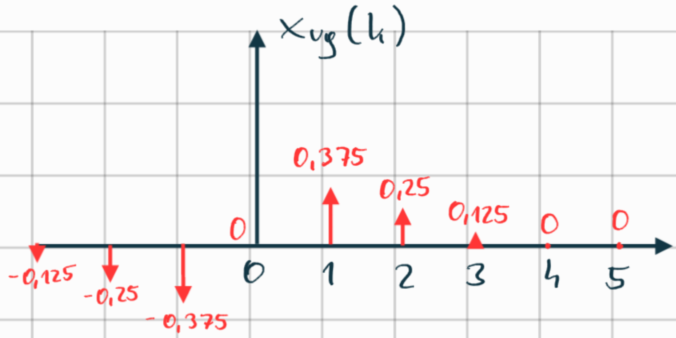
(1) - (2):

$$x_{ug}(k) = \frac{x(k) - x(-k)}{2}$$

Bsp.:







3.2 Sprung- & Impulsfolgen

Impulsfolge:

$$\delta(k) = \begin{cases} 1 & \text{für } k = 0 \\ 0 & \text{für } k \neq 0 \end{cases}$$

Ausblendeigenschaft d. Impulsfolge:

$$\sum_{k=-\infty}^{\infty} x(k) \cdot \delta(k - k_0) = x(k_0)$$

Periodische Impulsfolge:

$$x(k) = \sum_{k=-\infty}^{\infty} \delta(k - n \cdot C)$$

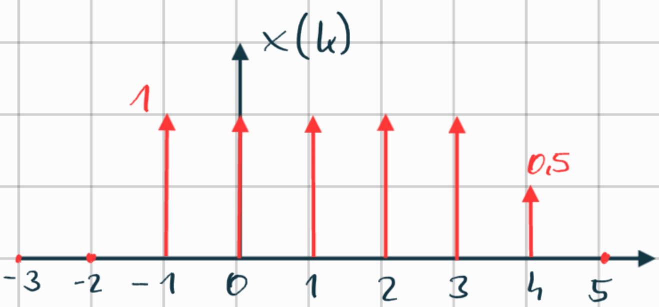
Sprungfolge:

$$\sigma(k) = \begin{cases} 1 & \text{für } k \geq 0 \\ 0 & \text{für } k < 0 \end{cases}$$

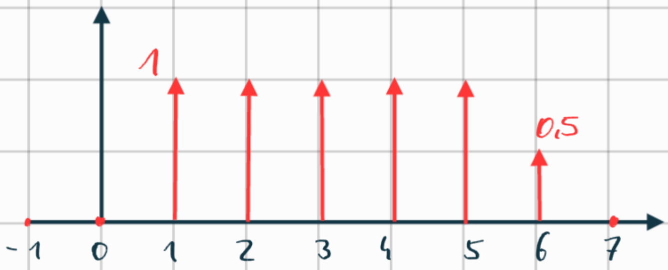
$$\sigma(k) = \sum_{i=0}^{\infty} \delta(k - i)$$

$$\delta(k) = \sigma(k) - \sigma(k - 1)$$

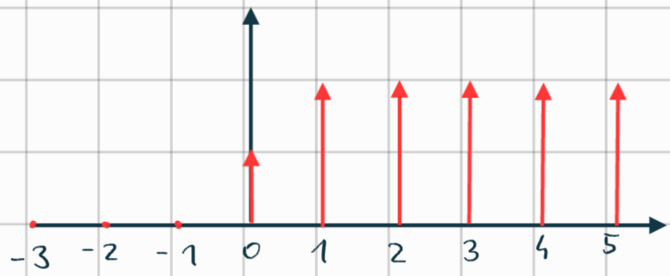
Bsp.:



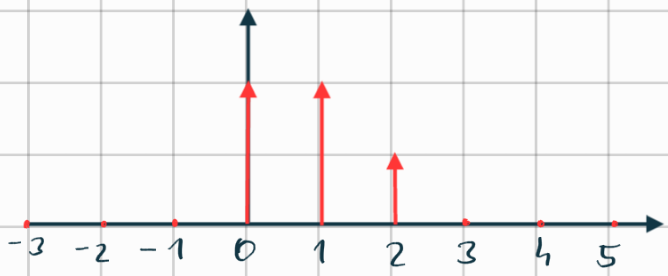
a) $y_1(k) = x(k-2)$



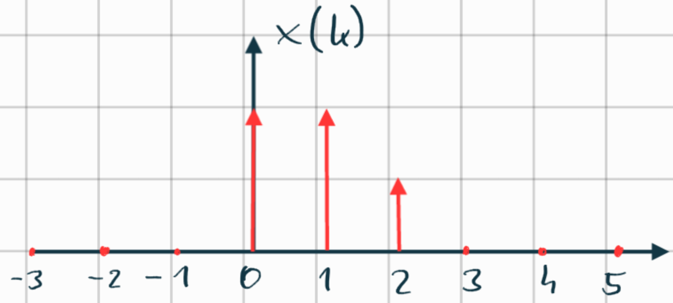
b) $y_2(k) = x(4-k)$



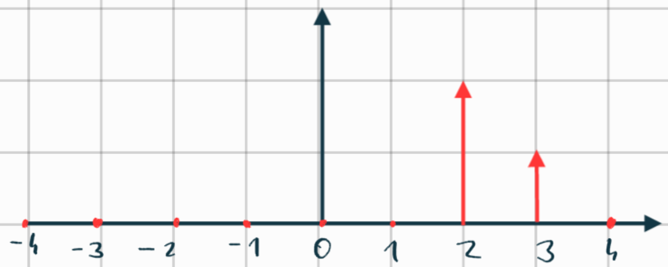
c) $y_3(k) = x(2k)$



d) $y_4(k) = x(2k) \cdot \delta(2-k)$



e) $y_5(k) = x(k-1) \cdot \delta(k-3)$



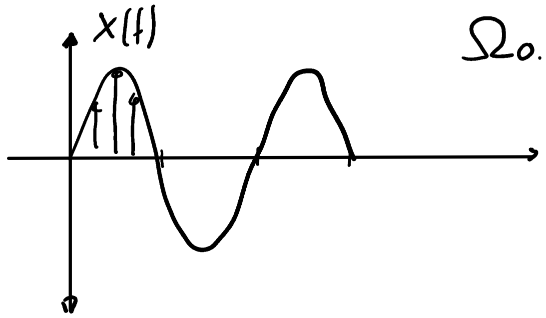
$$X[k] = A\cos(\omega_0 T_A \cdot k + \varphi) = A\cos[\Omega_0 k + \varphi]$$

Normierte Frequenz Ω_0

$\boxed{\Omega_0 = \omega_0 \cdot T_A}$ T_A... Abtastfrequenz

Ω_0... Phasenwinkel zw 2 Abtastpunkten

Bereich Ω_0: $0 \ldots \pi$



Beispiel $\Omega_0 = 0,2$ $f_A = 44,1kHz$; $\omega_0 = ?$

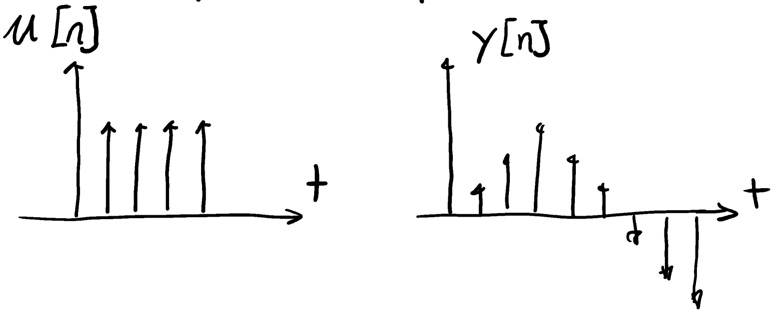
$$\Omega_0 = 2\pi f_0 \cdot \frac{1}{f_A}$$

$$f_0 = \frac{\Omega_0 \cdot f_A}{2\pi} = \frac{0,2 \cdot 44,1kHz}{2\pi} = 1403,74\ Hz$$

4.) Lineare Zeitinvariante Systeme (LTI)

Linearität

$$u[k] = v_1 \cdot u_1[k] + v_2 \cdot u_2[k]$$
$$y[k] = v_1 \cdot y_1[k] + v_2 \cdot y_2[k]$$



LTI: Lineare Differenzen gleichung
Konstante Koeffizienten

$$y(k) = (1-GF)\cdot u(k) + GF\cdot y(k-1)$$

GF ... Gewichtsfaktor

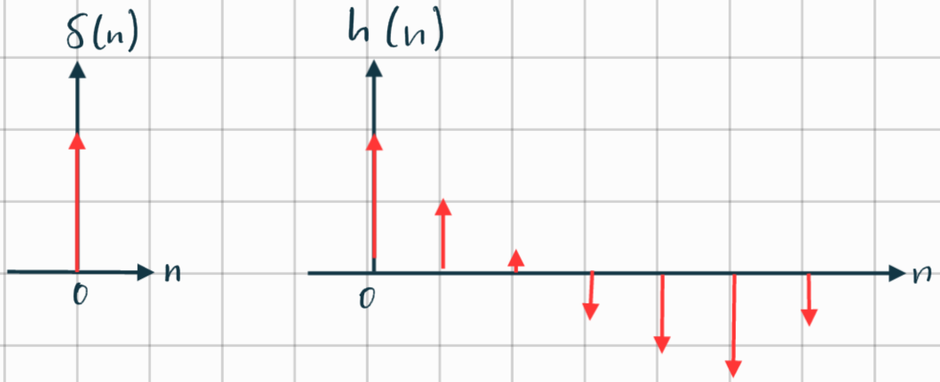
$u(k)$ ... Eingangssignal

$y(k)$ ... Ausgangssignal

Bsp.: Gleitender Mittelwert

$$y(k) = \frac{1}{5}\cdot \sum_{i=0}^{4} u(k-i)$$

4.1 Impulsantwort



Kausalität:

$$h(n) = 0 \quad \text{für } k < 0$$

Stabilität:

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

daraus folgt:

$$\lim_{n\to\infty} |h(n)| = 0$$

4.2 Diskrete Faltung

$$\delta(n) \to h(n)$$
$$\delta(n-i) \to h(n-i)$$

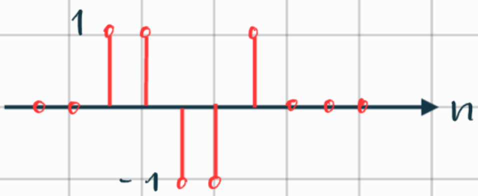
$$x(i) \cdot \delta(n-i) \to x(i) \cdot h(n-i)$$

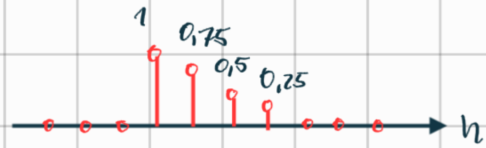
$$\sum_{i=-\infty}^{\infty} x(i) \cdot \delta(n-i) \to \sum_{i=-\infty}^{\infty} x(i) \cdot h(n-i) \quad \ldots \text{ Faltungssumme}$$

Diskrete Faltung

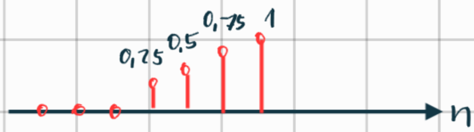
$$\boxed{y(n) = \sum_{i=-\infty}^{\infty} x(i) \cdot h(n-i)}$$

Bsp.: Diskrete Faltung





$$y(n) = x(n) * h(n) = \sum_{i=-\infty}^{\infty} x(i) \cdot h(n-i)$$



5 Z-Transformation

Laplace - zeitkontinuierlich

Z-Transf. - zeitdiskret

5.1 Grundlagen d. Z-Transformation

5.1.1 Definitionsgleichung

$$x_A(t) = \sum_{k=0}^{\infty} x(k \cdot T_A) \cdot \delta(t - k \cdot T_A) \quad \ldots \text{ abget. Signal}$$

$$X_A(s) = \mathcal{L}\left\{\sum_{k=0}^{\infty} x(k \cdot T_A) \cdot \delta(t - k \cdot T_A)\right\}$$

$$X_A(s) = \sum_{k=0}^{\infty} x(k \cdot T_A) \cdot \mathcal{L}\left\{\delta(t - k \cdot T_A)\right\}$$

$$= \sum_{k=0}^{\infty} x(k \cdot T_A) \cdot \int_0^{\infty} \delta(t - k \cdot T_A)\, dt$$

$$= \sum_{k=0}^{\infty} x(k) \cdot e^{-kT_A \cdot s} = \sum_{k=0}^{\infty} x(k) \cdot \left(e^{T_A \cdot s}\right)^{-k}$$

Def.: $z = e^{T_A \cdot s}$

$$\boxed{X(z) = \sum_{k=0}^{\infty} x(k) \cdot z^{-k}}$$

5.1.2 Z-Transf. grundlegender Signale

Diskrete Impulsfolge

$$x(k) = \delta(k) = \begin{cases} 0 & \text{für } k \neq 0 \\ 1 & \text{für } k = 0 \end{cases}$$

$$X(z) = \sum_{k=0}^{\infty} \delta(k) \cdot z^{-k} = 1 \cdot z^0 = 1$$

Ist der Impuls um k_0 verschoben, ändert sich d. Z-Transf. zu:

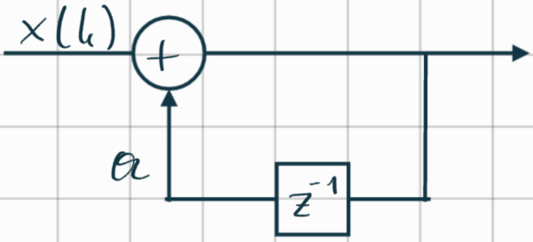
$$X(z) = \sum_{k=0}^{\infty} \delta(k-k_0) \cdot z^{-k} = 1 \cdot z^{-k_0}$$

6 Digitale Filter

6.1 Blockschaltbild

$$y(k) = u(k) + \underset{\uparrow}{a} \cdot y(k-1)$$

Gedächnisfaktor



$$z^{-1} \rightarrow e^{-s \cdot k \cdot T_A}$$

6.2 Blockdiagramm f. Differenzengleichung n-ter O.

$$G(z) = \frac{Y(z)}{U(z)} = \frac{\sum_{m=0}^{M} d_m \cdot z^{-m}}{\sum_{n=0}^{N} c_n \cdot z^{-n}} = \frac{d_0 + d_1 \cdot z^{-1} + d_2 \cdot z^{-2} + \dots}{c_0 + c_1 \cdot z^{-1} + c_2 \cdot z^{-2} + \dots}$$

$Y(z)$ ... Ausgangssignal (Z-Transformiert)

$U(z)$ ... Eingangssignal (Z-Transformiert)

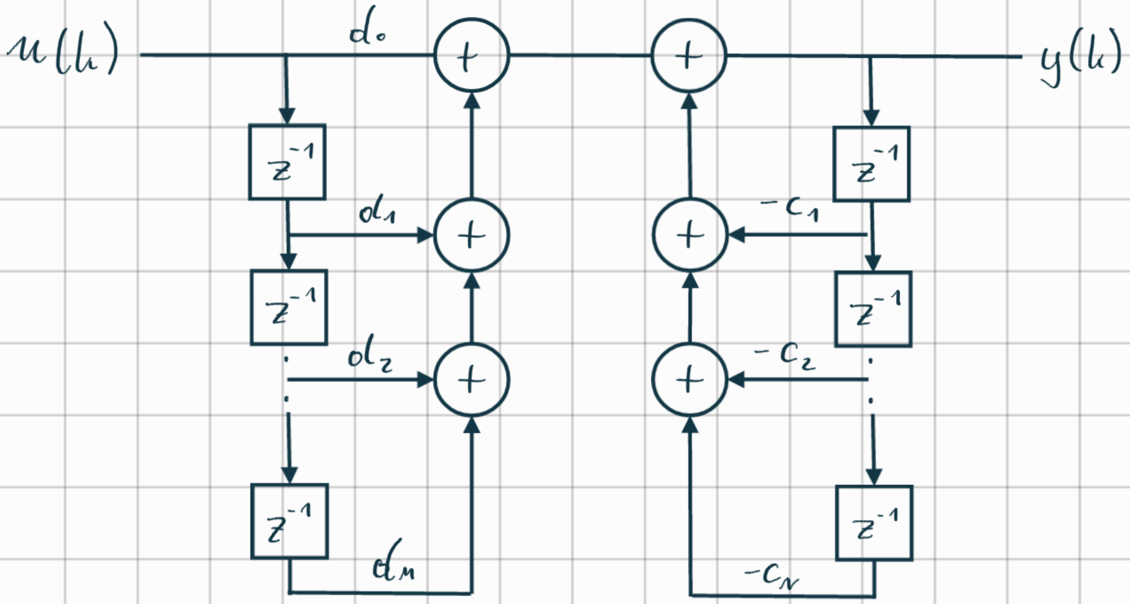
$$\frac{Y(z)}{U(z)} = \frac{\sum_{m=0}^{M} d_m \cdot z^{-m}}{\sum_{n=0}^{N} c_n \cdot z^{-n}}$$

$$Y(z) \cdot \sum_{n=0}^{N} c_n \cdot z^{-n} = U(z) \cdot \sum_{m=0}^{M} d_m \cdot z^{-m}$$

$$Y(z) \cdot (c_0 + c_1 \cdot z^{-1} + c_2 \cdot z^{-2} + \dots) = U(z) \cdot (d_0 + d_1 \cdot z^{-1} + d_2 \cdot z^{-2} + \dots)$$

$$Y(z) \cdot c_0 = U(z) \cdot \sum_{m=0}^{M} d_m \cdot z^{-m} - Y(z) \cdot \sum_{n=1}^{N} c_n \cdot z^{-n}$$

IIR-Filter

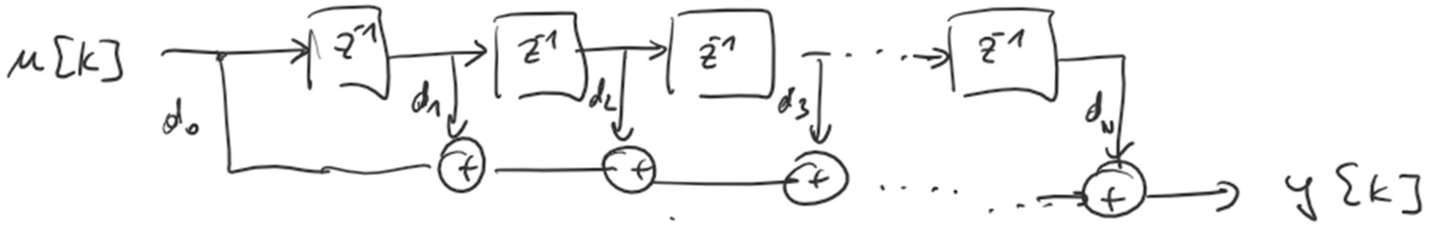


Digitale Filter:

Struktur FIR-Filter

FIR ... Finite Impulse Response.

Direkt-Struktur

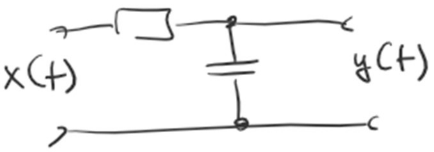


6.3 Entwurf digitaler Filter durch die Impulsantwort

$u[k] = \delta[k]$  $\rightarrow \quad y[k] = d_k$

Impulsantwort eines Analogfilters wird abgetastet und ergibt die Filterkoeffizienten

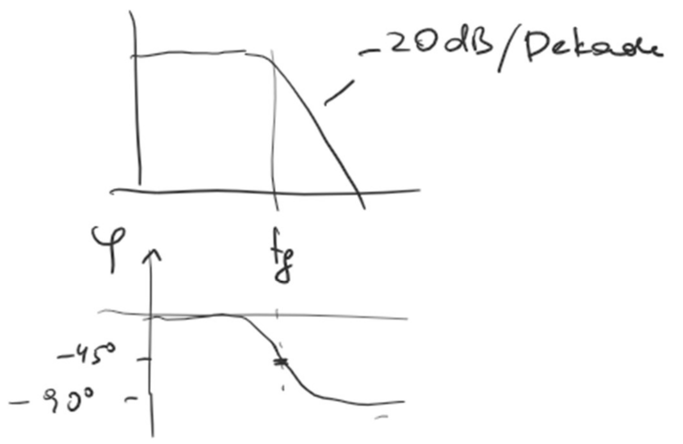
<u>Bsp</u> Tiefpass 1. Ordnung



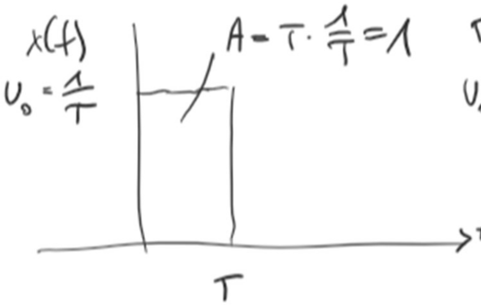
$$H(j\omega) = \frac{1}{1+j\omega RC} = \frac{1}{1+j\frac{f}{f_g}}$$

$$f_g = \frac{1}{2\pi RC}$$

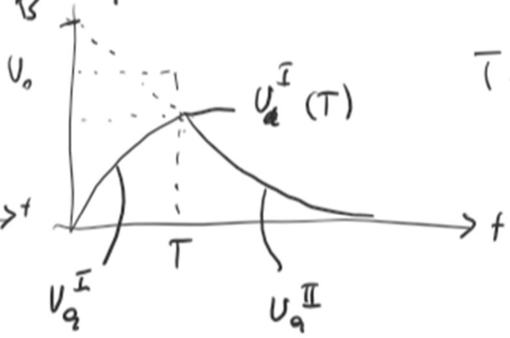
$$\tau = RC$$



Impulsantwort $h(t)$ für das analoge Filter berechnen



$U_a = U_c$



$T \to 0$

$$U_a^{I} = U_0\left(1 - e^{-\frac{t}{\tau}}\right) \qquad 0 \le t \le T$$

$$U_a^{II} = B \cdot e^{-\frac{t}{\tau}} \qquad t \ge T$$

Verschieben U_a^{II} um T

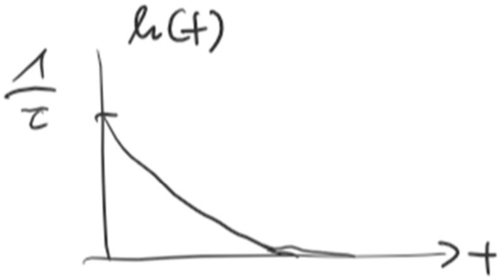
$$U_a^{II} = U_a^{I}(T) \cdot e^{-\frac{(t-T)}{\tau}} = \underbrace{U_0\left(1 - e^{-\frac{T}{\tau}}\right)}_{U_a^{I}(T)} \cdot e^{-\frac{(t-T)}{\tau}} =$$

$$= U_0\left(1 - e^{-\frac{T}{\tau}}\right) \cdot e^{-\frac{t}{\tau}} \cdot e^{\frac{T}{\tau}} = U_0\left(e^{\frac{T}{\tau}} - 1\right) e^{-\frac{t}{\tau}} \qquad t > T$$

$$U_0 = \frac{1}{T}$$

$$U_a^{II}(t) = \frac{1}{T}\left(\underbrace{e^{\frac{T}{\tau}} - 1}_{0 \text{ für } T \to 0}\right) e^{-\frac{t}{\tau}} \qquad \lim_{T \to 0} \frac{e^{\frac{T}{\tau}} - 1}{T} = \lim_{T \to 0} \frac{\frac{1}{\tau} \cdot e^{\frac{T}{\tau}}}{1} = \frac{1}{\tau}$$

$$U_a^{II}(t) = \frac{1}{\tau} \cdot e^{-\frac{t}{\tau}}$$



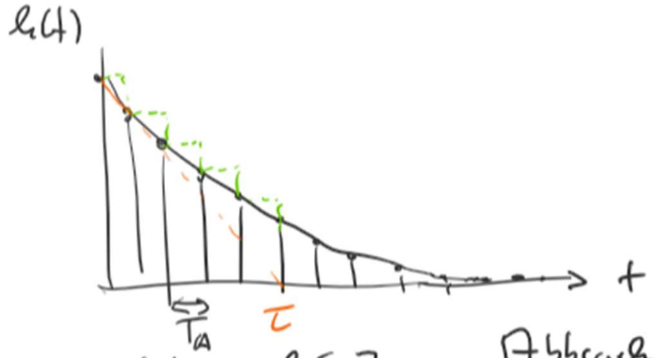
$$h(t) = \frac{1}{\tau} \cdot e^{-\frac{t}{\tau}} = \omega_g \cdot e^{-\omega_g \cdot t}$$

Abtastfrequenz wählen

$$f_a \gg 2 f_g \qquad f_g = 1\,Hz$$

$$f_a = 10\,Hz \gg 2 \cdot 1\,Hz$$

Anzahl und Werte der Abtastpunkte der Impulsantwort bestimmen



$h(t) \circ\!\!-\!\!\bullet\ h[n]$ ... Abbruch bei $h(t_{max}) = 0.01 \cdot h(0)$

$h[n]$ aus $h(t)$ berechnen $\triangleq$ Filterkoeffizienten

$h(t) = \frac{1}{\tau} \cdot e^{-\frac{t}{\tau}} \Rightarrow$ Normierungsfaktor : T_A

$$h'(t) = \frac{T_A}{\tau} \cdot e^{-\frac{t}{\tau}}$$

$$h(t_{max}) = 0.01\ h(0)$$

$$\frac{1}{\tau} \cdot e^{-\frac{t_{max}}{\tau}} = 0.01 \frac{1}{\tau}$$

$$-\frac{t_{max}}{\tau} = \ln 0{,}01$$

$$t_{max} = \tau \ln 100 = 4{,}605\,\tau$$

$$t_{max} = N_{max} \cdot T_A \Rightarrow N_{max} = \frac{t_{max}}{T_A} = \frac{4{,}605\,\tau}{T_A} = \frac{4{,}605 \frac{1}{f_g}}{2\pi \frac{1}{f_a}} = 0{,}733 \frac{f_A}{f_g}$$

$$N_{max} = 0{,}733 \cdot \frac{10\,Hz}{1\,Hz} = 7{,}33 \rightarrow N_{max} = 7 \text{ (oder 8)}$$

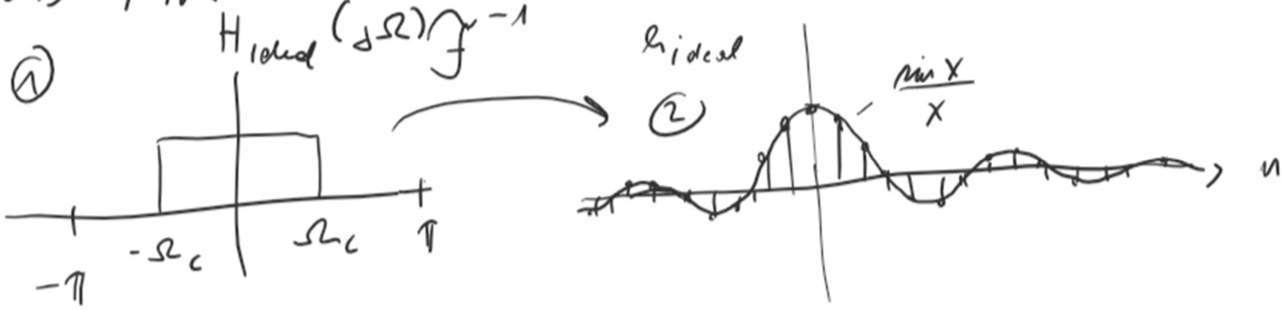
$$h[n] = \frac{T_A}{\tau}\, e^{-\frac{T_A \cdot n}{\tau}}$$

$$h[n] = 0{,}628 \cdot e^{-0{,}628 \cdot n}$$

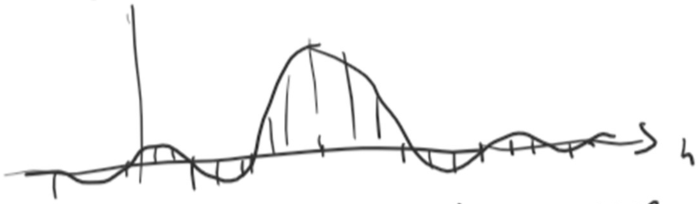
$h[0] = 0{,}628$
$h[1] = 0{,}335$
$h[2] = 0{,}179$
$h[3] = 0{,}095$
$h[4] = 0{,}051$
$h[5] = 0{,}0272$
$h[6] = 0{,}0145$
$h[7] = 0{,}0077$

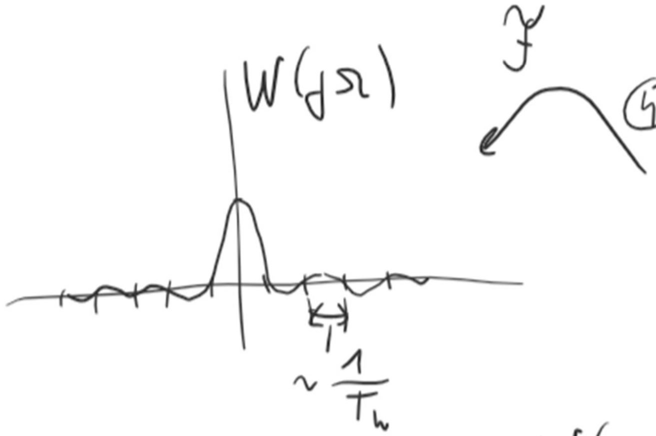
6.5 FIR-Entwurf mit Fenstermethode

①

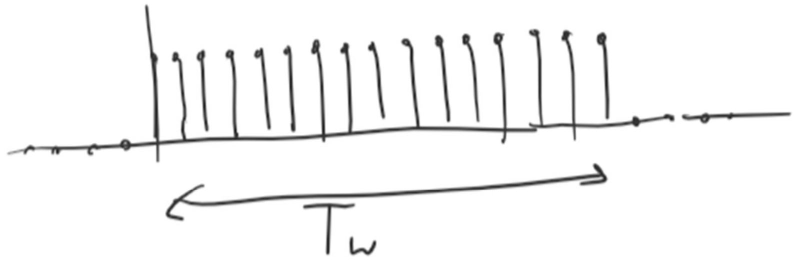


③ right shift

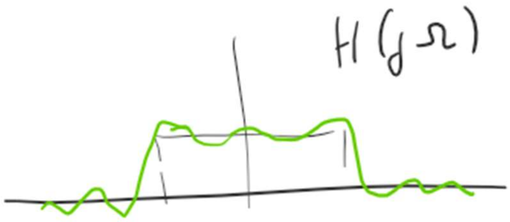




④ Fensterfunktion wird verwendet

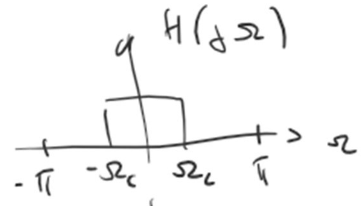


Faltung von $H_{ideal}(j\Omega)$ mit $W(j\Omega)$

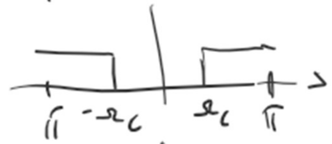


Filtertypen:

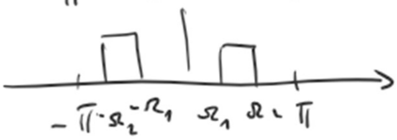
- Tiefpass



- Hochpass



- Bandpass



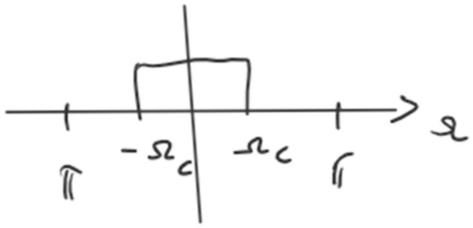
- Bandsperre

Impulsantwort – idealer Filter

allg.:

$$h[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} H(e^{j\Omega})\, e^{j\Omega n}\, d\Omega \quad \ldots \text{ inverse Fouriertransformation}$$

$h[n]$ … Tiefpass



$$h[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} H(e^{j\Omega})\, e^{j\Omega n}\, d\Omega$$

$$H(e^{j\Omega}) : \begin{cases} 1 & \text{für } |\Omega| < \Omega_c \\ 0 & \text{for } \Omega_c < |\Omega| < \pi \end{cases}$$

$$h[n] = \frac{1}{2\pi} \int_{-\Omega_c}^{+\Omega_c} e^{j\Omega n}\, d\Omega$$

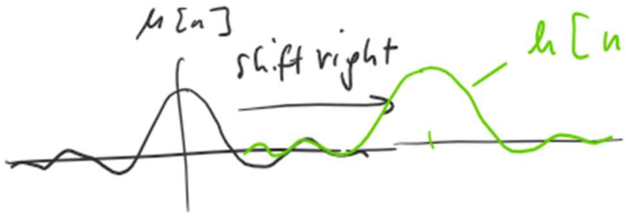
$$h[n] = \frac{1}{2\pi} \frac{1}{jn} \cdot e^{j\Omega n} \Big|_{-\Omega_c}^{+\Omega_c} =$$

$$h[n] = \frac{1}{2\pi j n} \cdot \left(e^{j\Omega_c n} - e^{-j\Omega_c n}\right) = \frac{1}{\pi n} \cdot \sin(\Omega_c \cdot n) \quad n \neq 0$$

$$\boxed{\sin(x) = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right)}$$

$n = 0 \Rightarrow$ L'Hospital

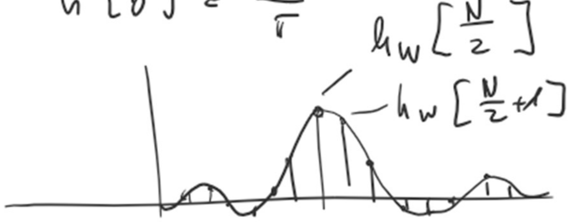
$$h[0] = \lim_{n \to 0} \frac{\sin \Omega_c \cdot n}{\pi \cdot n} \Rightarrow \frac{\Omega_c \cdot \cos \Omega_c \cdot n}{\pi} = \frac{\Omega_c}{\pi}$$



$\frac{N}{2}$ … mittlerer Index

$$h[n] = \frac{1}{\pi \cdot n} \sin(\Omega_c n) \quad \text{für } n \neq 0$$

$$h[0] = \frac{\Omega_c}{\pi}$$



$h_w = h[n - \frac{N}{2}]$ $\quad n = 0$ bis N

N … Ordnung, $\quad N+1$ Koeffizienten

$h[0], \ldots, h[N]$

Zusammenfassung:

1) Definition des idealen Frequenzganges
2) $h[n]$ mit $\mathcal{F}^{-1}$ berechnen
3) right shift
4) Fensterung von $h[n]$ => $h_w[n]$ berechnen für $n=0$ bis N
5) Filter realisieren

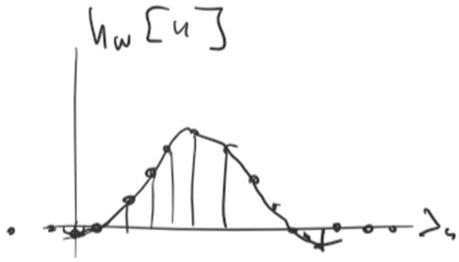
Bsp FIR-Tiefpassfilter

geg Ordnung $N=10$

$f_A = 20\,kHz$

$f_g = f_c = 2{,}5\,kHz$

Fenster: Rechteck



$N=10$ => 11 Filterkoeffizienten

$$h_w[n] = \begin{cases} \dfrac{\sin(\Omega_c \cdot (n-5))}{\pi(n-5)} & n \neq 5 \\ \dfrac{\Omega_c}{\pi} & n=5 \end{cases}$$

$$\Omega_c = \frac{2\pi f_c}{f_A} = \frac{2\pi \cdot 2.5\,kHz}{20\,kHz} = 0{,}25\,\pi$$

$h[0] = \dfrac{\sin \Omega_c \cdot (-5)}{-5\pi} = -0{,}045016 = h[10]$ wegen Symmetrie

$h[1] = \dfrac{\sin \Omega_c \cdot (-4)}{-4\pi} = 0 \qquad = h[9]$

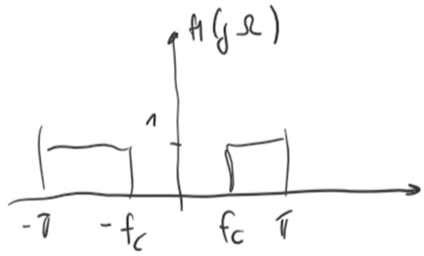
$h[2] = 0{,}075 \qquad = h[8]$

$h[3] = 0{,}159155 \qquad = h[7]$

$h[4] = 0{,}225079 \qquad = h[6]$

$h[5] = 0{,}25$

Hochpassfilter



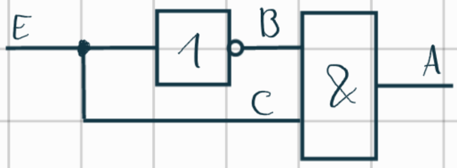
$f_A = 20\,kHz$

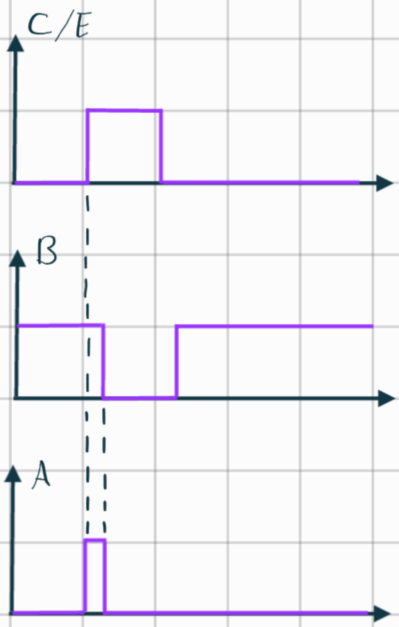
$f_c = 2{,}5\,kHz$

Taktflankenselektion

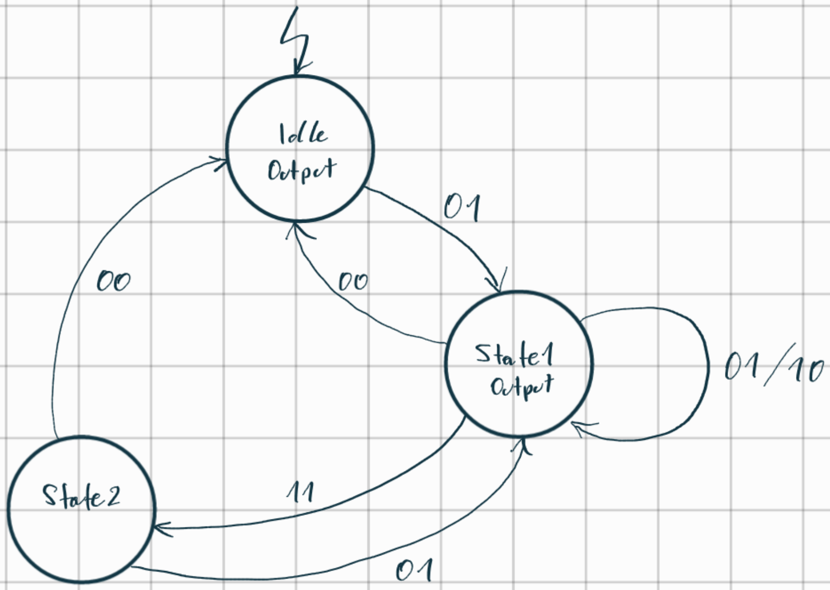
für z.B. D-FF

UND-Gatter für pos. Flanken
ODER-Gatter für neg. Flanken





State Machine



C:

```
TState STATE;
switch (STATE)
{
    case IDLE:
        if (Data == 01)
            STATE = STATE1;
        break;
    case STATE1:
        :
        break;

    case STATE2:
        :
        break;
}
```

Task-Scheduler

- präemptiv -> alles muss gesichert werden
    - kann während der Laufzeit Tasks wechseln
- kooperativ
- zeitgesteuert

counter

Task -> NextTime ≤ counter

-> Execute

NextTime += Task -> Intervall