

Digital Color Image Processing

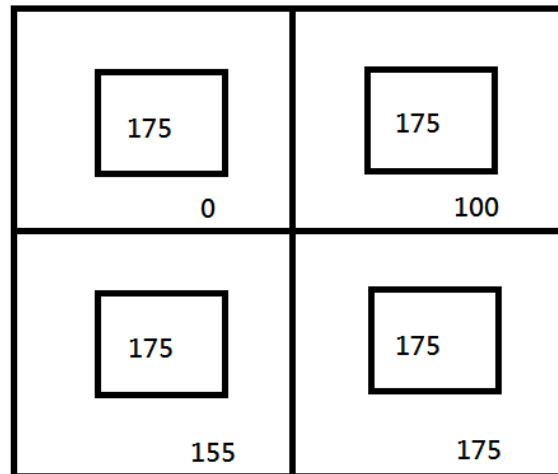
Computer Exercise

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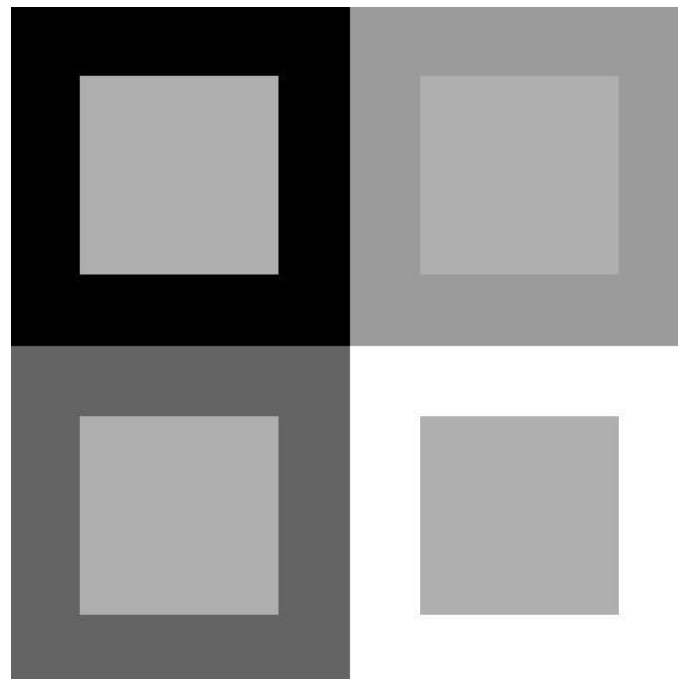
Computer Exercise 1.

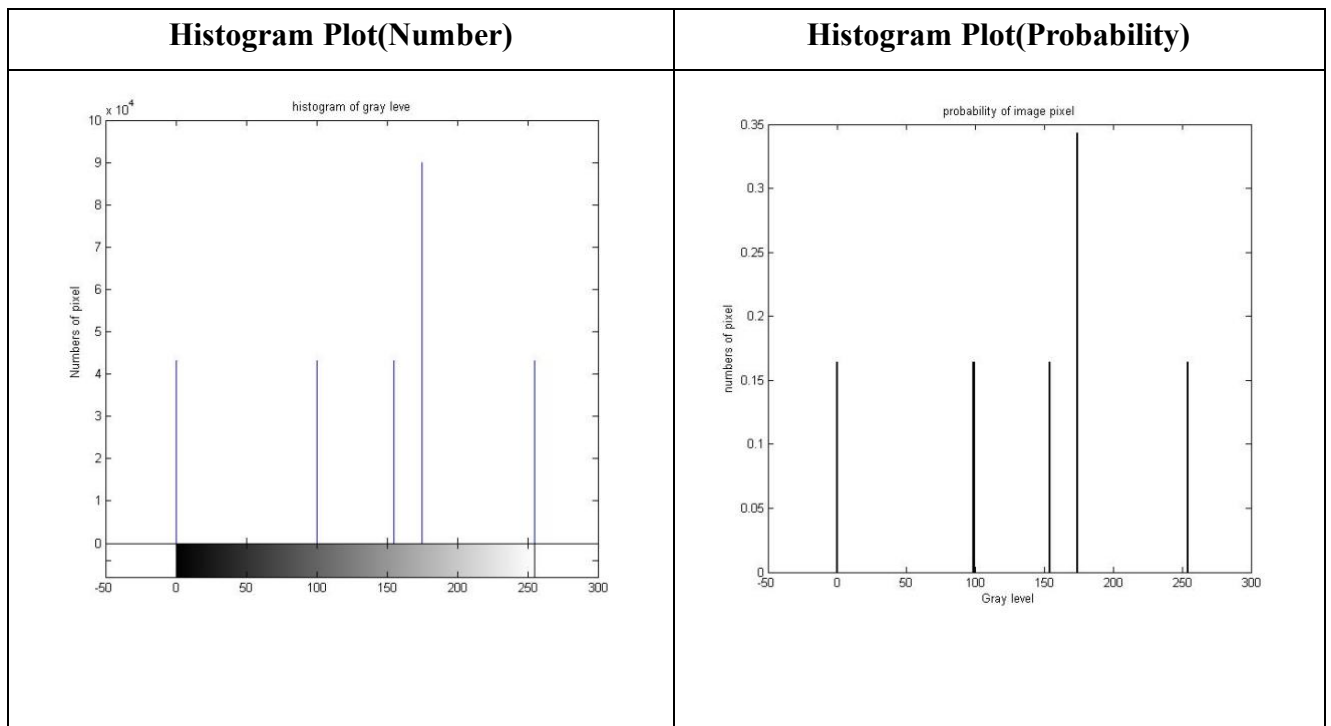
- Simultaneous Contrast

示意圖



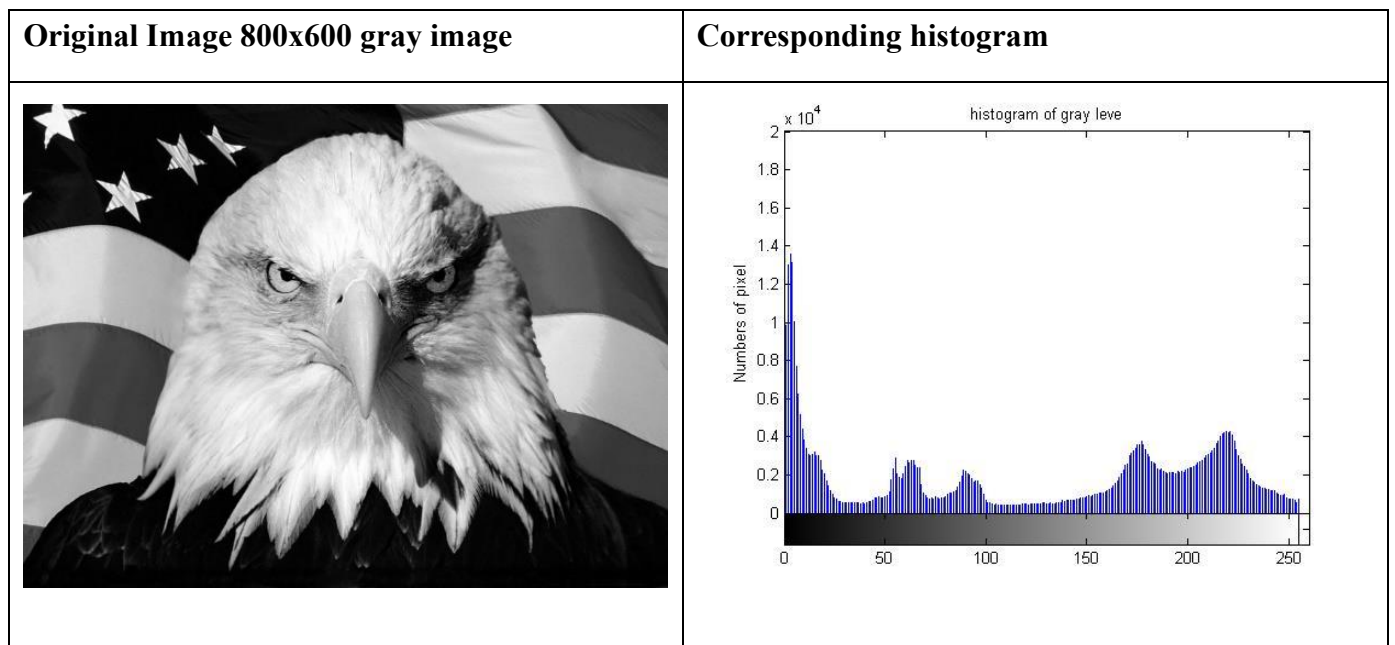
實際圖

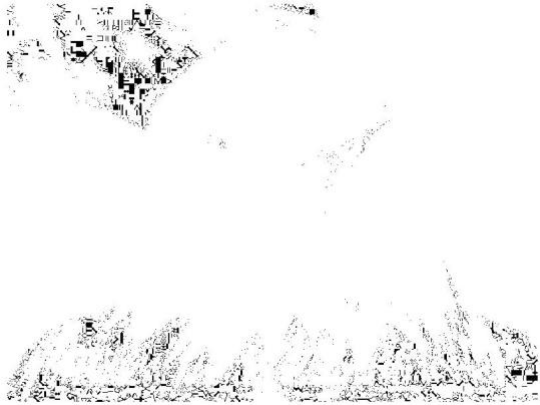





Computer Exercise 2.

- Histogram Plots of Digital Image & its segemetation



Gray level = 3	Gray level = [1,2,3,4,5]
	
在灰階值 = 3 時，可以粗略的老鷹圖像描繪出來。	在灰階值等於[1,2,3,4,5]時，可將更清晰的老鷹圖像，以及部分的美國國旗描繪出來。

Computer Exercise 3.

- Edge Detection

Mask1:H = $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	
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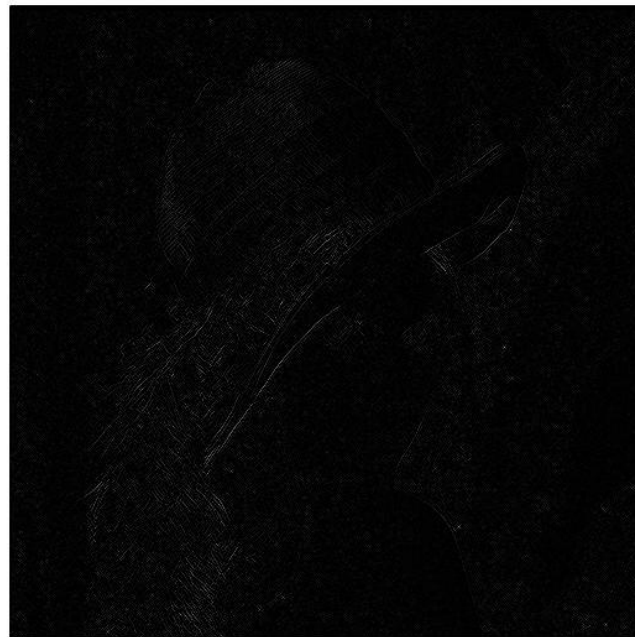
Mask2:H =

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$



Mask3:H =

$$\begin{bmatrix} 1 & -2 & -1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$



<p>Add Edge-Enhanced Image to the Original Image</p> <p>Mask4:H =</p> $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ <p>+</p> $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$ $= \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	
<p>Sobel</p> <p>該算子包含兩組 3x3 的矩陣，分別為橫向及縱向，將之與圖像作平面卷積，即可分別得出橫向及縱向的亮度差分近似值。</p>	
<p>Discussion</p>	<p>由上述幾組圖片可以看到，相較於 Sobel 邊緣偵測，單單用一個遮罩進行邊緣偵測的結果(Mask1、2、3)，因為沒有經過梯度的運算，故其結果並不會這麼明顯。</p>

Computer Exercise 4.



● Invariant Color Space

(1) RGB to C space

$$C_1 = \tan^{-1} \left(\frac{R}{\max(G, B)} \right)$$

$$C_2 = \tan^{-1} \left(\frac{G}{\max(R, B)} \right)$$

$$C_3 = \tan^{-1} \left(\frac{B}{\max(R, G)} \right)$$



Original	C space
	

(2) RGB to l_1, l_2, l_3 Color Space

$$l_1 = \frac{|R - G|}{|R - G| + |G - B| + |R - B|}$$

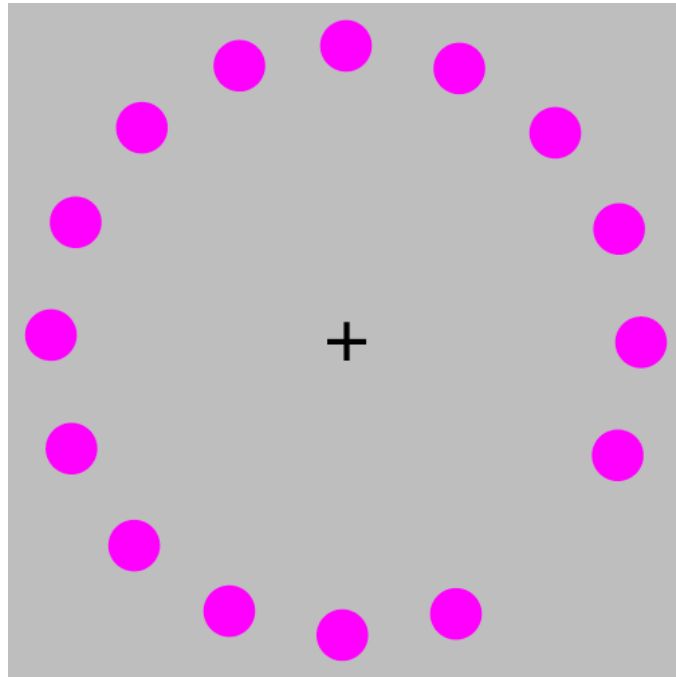
$$l_2 = \frac{|R - B|}{|R - G| + |G - B| + |R - B|}$$

$$l_3 = \frac{|B - G|}{|R - G| + |G - B| + |R - B|}$$

Original	l_1, l_2, l_3 Color Space
	

Computer Exercise 5.

● Lilac Chaser



寫一個程式，使周圍有許多同色的圓形，並有個空缺，並使之旋轉，眼睛盯著中間十字後，在旋轉時，空缺處會產生某色球的互補色。

Lilac Chaser 是一種視覺錯覺，通常會在周圍圓盤上加入模糊的效果，主要產生錯覺的原因為飛現象(phi phenomenon)和視覺暫留所引起的視覺錯覺。

心得➡在盯著十字中心後約莫五秒鐘，就能看到紫紅色的互補色(綠色)在周圍旋轉。

Computer Exercise 6.

- Color Simultaneous Contrast

RGB → HSV → RGB

simple:

RGB to HSV Conversion Formular

$$R' = R/255$$

$$G' = G/255$$

$$B' = B/255$$

$$C_{max} = \max(R', G', B')$$

$$C_{min} = \min(R', G', B')$$

$$\Delta = C_{max} - C_{min}$$

Hue calculation:

$$H = \begin{cases} 0^\circ & \Delta = 0 \\ 60^\circ \times \left(\frac{G' - B'}{\Delta} \bmod 6 \right) & , C_{max} = R' \\ 60^\circ \times \left(\frac{B' - R'}{\Delta} + 2 \right) & , C_{max} = G' \\ 60^\circ \times \left(\frac{R' - G'}{\Delta} + 4 \right) & , C_{max} = B' \end{cases}$$

Saturation calculation:

$$S = \begin{cases} 0 & , C_{max} = 0 \\ \frac{\Delta}{C_{max}} & , C_{max} \neq 0 \end{cases}$$

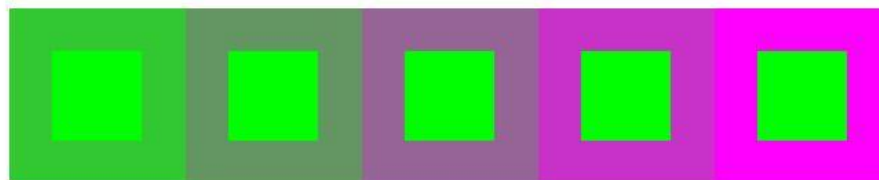
Value calculation:

$$V = C_{max}$$

➔在此 Color Space 的互補色可在 RGB Color Space 很快求出

➔(R,G,B) 的互補色為(255-R,255-G,255-B)

✓ Final Result

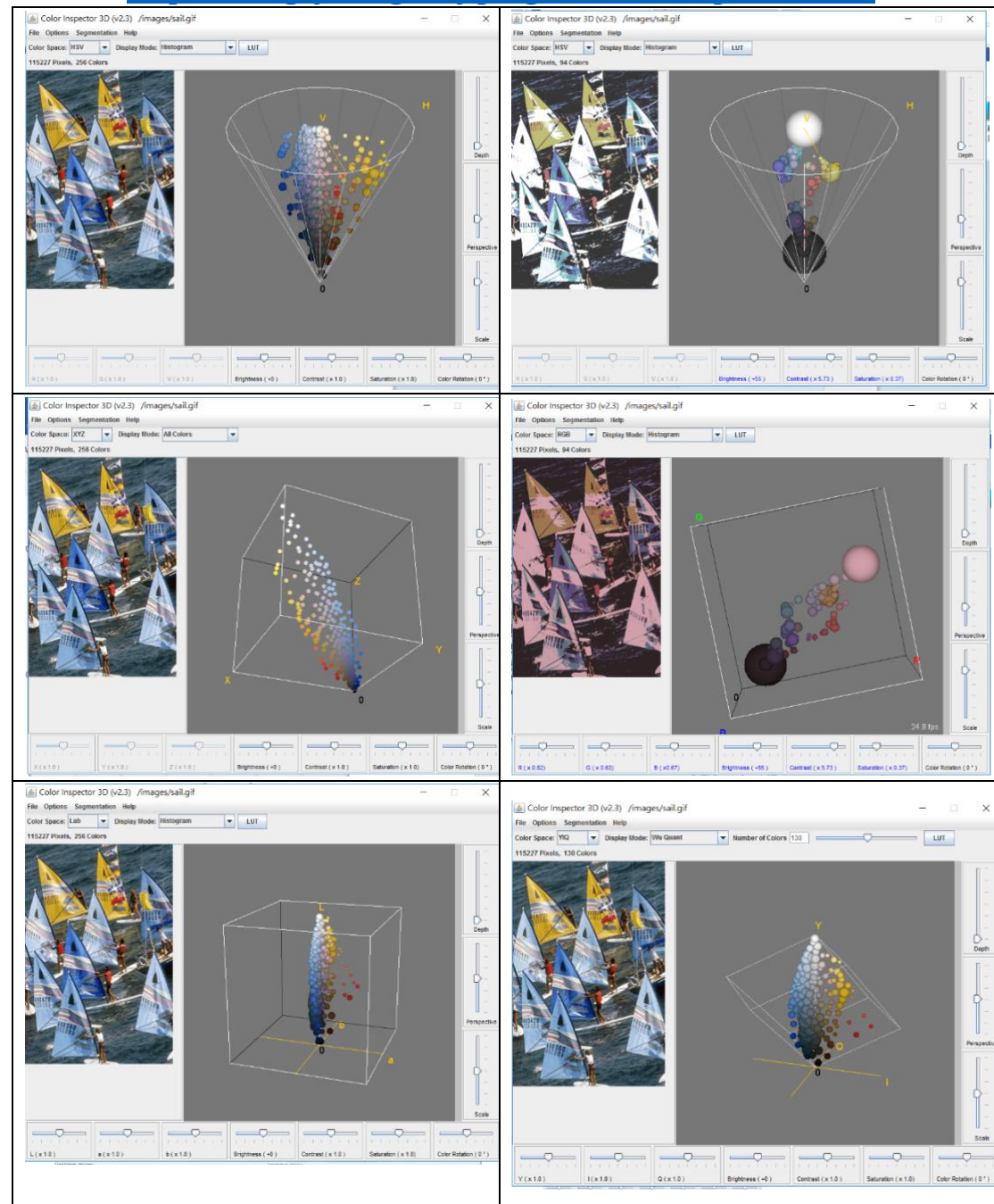


- 由上圖可以看到，影像對比的增強變化，固定住中間的方塊皆為綠色 (0,255,0)，可以發現有左至右，RGB 分別為(50,200,50)、(100,150,100)、(150,100,150)、(200,50,200)、(255,0,255)，顏色的對比越來越明顯。

Computer Exercise 7

- Color Mapping using 3-D Color Inspector/Color histogram

Source: <https://imagej.nih.gov/ij/plugins/color-inspector.html>



- 此 Color Inspector 是由 java 寫成的，可以讀入不同的圖片，並在不同的色彩空間對於色彩的長條圖進行拉伸，藉此觀察圖片的變化情形，或者在每種色彩空間，都可以對於圖片的 HSV 進行調整。在長條圖或立方體或圓錐等等的不同色彩空間立體表示，觀察圖片的變化。

Computer Exercise 8.

Image Warpping

$$\begin{aligned}x' &= (x - c_x) \cos \theta + (y - c_y) \sin \theta + c_x \\y' &= -(x - c_x) \sin \theta + (y - c_y) \cos \theta + c_y\end{aligned}$$

(C_x, C_y) is the rotation center

$$\text{ssd} = \theta = \frac{r}{2}$$

in matrix form

$$\begin{bmatrix} x' - C_x \\ y' - C_y \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x - C_x \\ y - C_y \end{bmatrix}$$

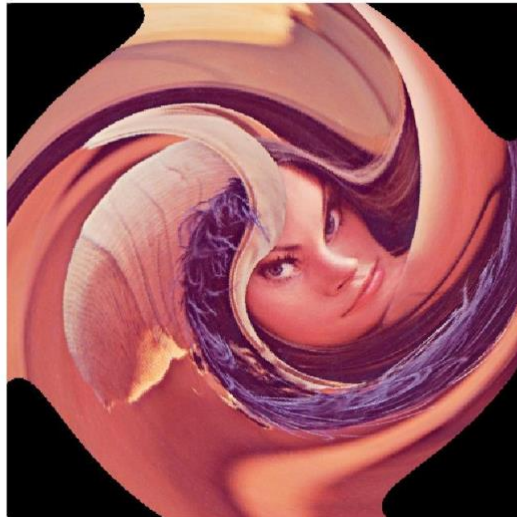
➔直接投影會產生空隙，如圖



➔故反過來 mapping，就不會有點空隙

➔The Result of Lena


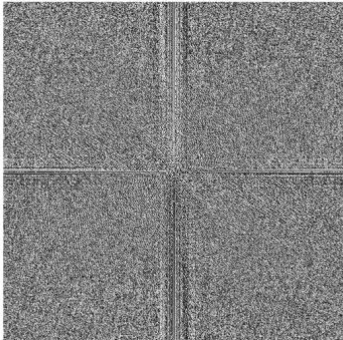
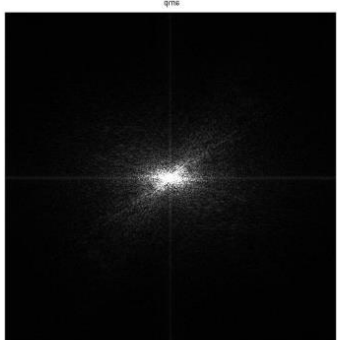
➔此兩張圖除了有無空隙外，扭曲的結果也不一樣，主要原因為解析度不同所造成，Result 是以 512x512 的解析度，而上圖是以 256x256 的解析度進行投影。

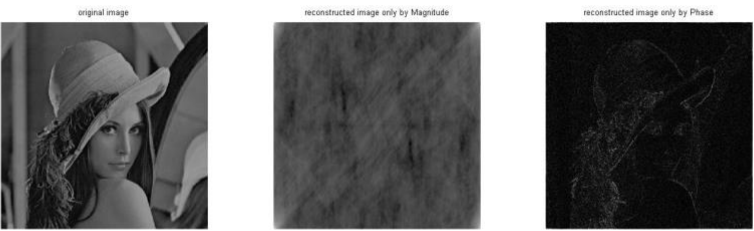


Ref : Slabaugh, Greg, Richard Boyes, and Xiaoyun Yang. "Multicore image processing with openmp [applications corner]." *IEEE Signal Processing Magazine* 27.2 (2010): 134-138.


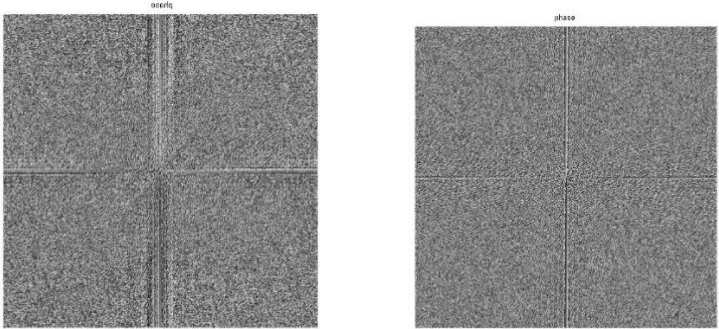
Computer Exercise 9.

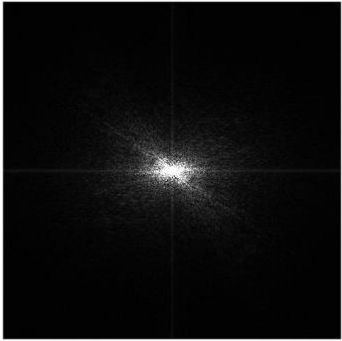
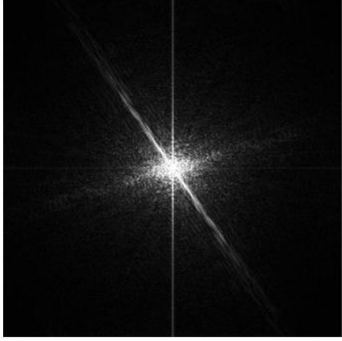


1. Do 2DFFT & IFFT of Lena image. 求得 Amplitude only & phase only image

Original Image		
Phase & Amplitude		

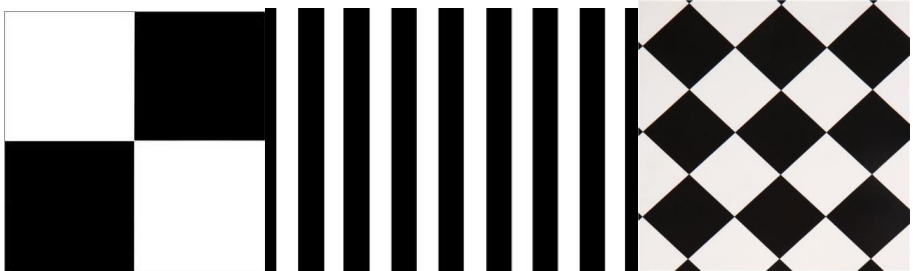
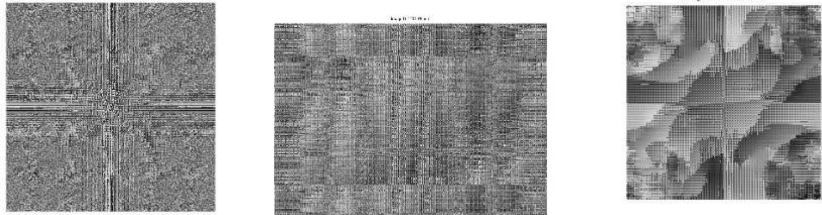

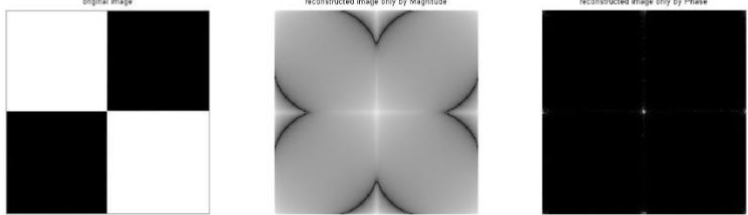
2DFFT	<div data-bbox="533 322 1289 551">  </div>
<p>● 在 IFFT(phase)中還依稀看得出原圖，而在 IFFT(Amplitude)則是完全看不出來，故在圖 phase 較 amp 重要。</p>	

2.IFFT(One image(Lena) + another image)

Original image	
Phase left:lena right:man	<div data-bbox="472 1361 1193 1688">  </div>

<p>Amplitude left: Lena right: Man</p>	<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p>amp</p>  </div> <div style="text-align: center;"> <p>amp</p>  </div> </div>
<p>Lena's Phase + Man's Amplitude</p>	<p style="text-align: center;">Image C: Magnitude</p> 
<p>Lena's Amplitude + Man's Phase</p>	<p style="text-align: center;">Image D: Magnitude</p> 
<p>● 不論是哪張圖片，因為 Phase 的資訊較為重要，故只要有採用該張圖片的 Phase 加另一圖片的 Amplitude，結果會呈現出 Phase 為主的原圖資訊。</p>	

3. When the image is stripe lines or checker board, the amp. is more important than phase info.

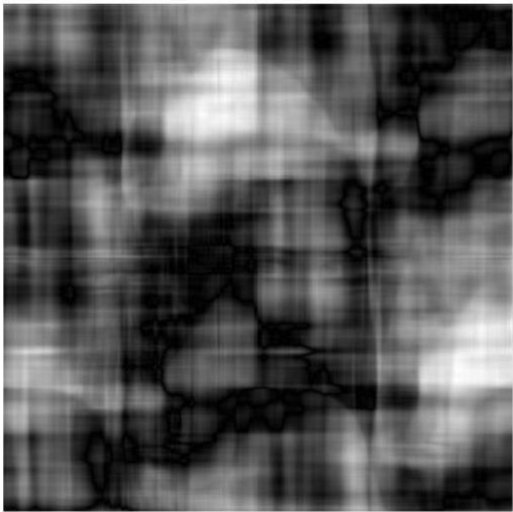
Original image	
Phase	
Amp.	
The Result	

	<div data-bbox="507 353 1279 1079"> </div>
	<ul style="list-style-type: none"> ● 在上述例子，amp info is more important than phase info.，原因為高頻的成分較多，所以 amp 較重要。

4.Lena's phase + Stripe's (or square)Amplitude.

➔Lena 出不來

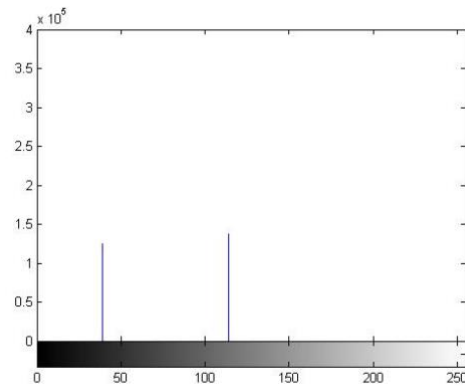
<p>Original Image</p>	
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Lena's Phase + Square's amp	<div data-bbox="486 212 1000 745"> <p>Image C: Magnitude</p>  </div>
<p>● 在上圖還稍微看的到 Lena 的輪廓，如欲使影像更清晰，可用 Lena 的影像只給 Phase → 2DFFT → Phase + Amp → 2D IFFT，此方法進行疊代，，利用 phase 重建影像。</p>	

Computer Exercise 10.

1. Do computer program for multilevel threshold for Lena's Image (256 Gray level → 2,3,4 levels). Plot Histogram & Threshold

2 levels & Histogram Z0 = 39 Z1 = 114 P0 = 0.4793 P1 = 0.5207 Threshold = 79	<p>Solution</p> <p><i>A.1. Bilevel Thresholding Solutions</i></p> <p>(i)</p> $c_d = \begin{vmatrix} m_0 & m_1 \\ m_1 & m_2 \end{vmatrix};$ $c_0 = (1/c_d) \begin{vmatrix} -m_2 & m_1 \\ -m_3 & m_2 \end{vmatrix};$ $c_1 = (1/c_d) \begin{vmatrix} m_0 & -m_2 \\ m_1 & -m_3 \end{vmatrix}.$ <p>(ii)</p> $z_0 = \left(\frac{1}{2}\right) \left[-c_1 - (c_1^2 - 4c_0)^{1/2} \right];$ $z_1 = \left(\frac{1}{2}\right) \left[-c_1 + (c_1^2 - 4c_0)^{1/2} \right].$ <p>(iii)</p> $p_d = \begin{vmatrix} 1 & 1 \\ z_0 & z_1 \end{vmatrix};$ $p_0 = (1/p_d) \begin{vmatrix} 1 & 1 \\ m_1 & z_1 \end{vmatrix};$ $p_1 = 1 - p_0.$
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3 levels
& Histogram
Z0 = 24
Z1 = 82
Z2 = 134
P0 = 0.2523
P1 = 0.5414
P2 = 0.2063
Threshold
t1 = 50
t2 = 107

Solution

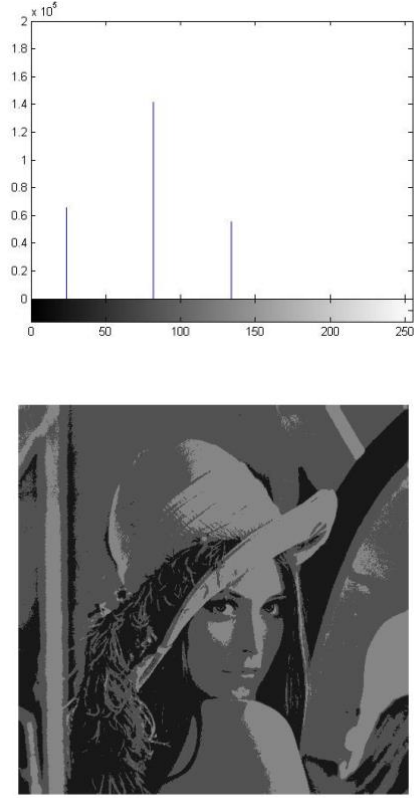
A.2. Trilevel Thresholding Solutions

$$\begin{aligned}
 \text{(i)} \quad c_d &= \begin{bmatrix} m_0 & m_1 & m_2 \\ m_1 & m_2 & m_3 \\ m_2 & m_3 & m_4 \end{bmatrix}; \\
 c_0 &= (1/c_d) \begin{bmatrix} -m_3 & m_1 & m_2 \\ -m_4 & m_2 & m_3 \\ -m_5 & m_3 & m_4 \end{bmatrix}; \\
 c_1 &= (1/c_d) \begin{bmatrix} m_0 & -m_3 & m_2 \\ m_1 & -m_4 & m_3 \\ m_2 & -m_5 & m_4 \end{bmatrix}; \\
 c_2 &= (1/c_d) \begin{bmatrix} m_0 & m_1 & -m_3 \\ m_1 & m_2 & -m_4 \\ m_2 & m_3 & -m_5 \end{bmatrix}. \\
 \text{(ii)} \quad z_0 &= -c_2/3 - A - B; \\
 z_1 &= -c_2/3 - W_1 A - W_2 B; \\
 z_2 &= -c_2/3 - W_2 A - W_1 B,
 \end{aligned}$$

where A , B , W_1 , and W_2 are as follows:

$$\begin{aligned}
 A &= \left((c_0/2 - c_1 c_2/6 + c_2^3/27) \right. \\
 &\quad \left. - \left[(c_0/2 - c_1 c_2/6 + c_2^3/27)^2 + (c_1/3 - c_2^2/9) \right]^{1/2} \right)^{1/3}; \\
 B &= -(c_1/3 - c_2^2/9)/A; \\
 W_1 &= -1/2 + i(\sqrt{3}/2); \\
 W_2 &= -1/2 - i(\sqrt{3}/2); \\
 i &= \sqrt{-1}.
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad p_d &= \begin{bmatrix} 1 & 1 & 1 \\ z_0 & z_1 & z_2 \\ z_0^2 & z_1^2 & z_2^2 \end{bmatrix}; \\
 p_0 &= (1/p_d) \begin{bmatrix} m_0 & 1 & 1 \\ m_1 & z_1 & z_2 \\ m_2 & z_1^2 & z_2^2 \end{bmatrix}; \\
 p_1 &= (1/p_d) \begin{bmatrix} 1 & m_0 & 1 \\ z_0 & m_1 & z_2 \\ z_0^2 & m_2 & z_2^2 \end{bmatrix}; \\
 p_2 &= 1 - p_0 - p_1.
 \end{aligned}$$

	
4level & Histogram Z0 = 18 Z1 = 57 Z2 = 101 Z3 = 142 P0 = 0.1709 P1 = 0.3213 P2 = 0.3986 P3 = 0.1091 Threshold t1 = 32 t2 = 80 t3 = 125	<p>A.3. Quarterlevel Thresholding Solutions</p> <p>(i)</p> $c_d = \begin{bmatrix} m_0 & m_1 & m_2 & m_3 \\ m_1 & m_2 & m_3 & m_4 \\ m_2 & m_3 & m_4 & m_5 \\ m_3 & m_4 & m_5 & m_6 \end{bmatrix};$ $c_0 = (1/c_d) \begin{bmatrix} -m_4 & m_1 & m_2 & m_3 \\ -m_5 & m_2 & m_3 & m_4 \\ -m_6 & m_3 & m_4 & m_5 \\ -m_7 & m_4 & m_5 & m_6 \end{bmatrix};$ $c_1 = (1/c_d) \begin{bmatrix} m_0 & -m_4 & m_2 & m_3 \\ m_1 & -m_5 & m_3 & m_4 \\ m_2 & -m_6 & m_4 & m_5 \\ m_3 & -m_7 & m_5 & m_6 \end{bmatrix};$ $c_2 = (1/c_d) \begin{bmatrix} m_0 & m_1 & -m_4 & m_5 \\ m_1 & m_2 & -m_5 & m_4 \\ m_2 & m_3 & -m_6 & m_5 \\ m_3 & m_4 & -m_7 & m_6 \end{bmatrix};$ $c_3 = (1/c_d) \begin{bmatrix} m_0 & m_1 & m_2 & -m_4 \\ m_1 & m_2 & m_3 & -m_5 \\ m_2 & m_3 & m_4 & -m_6 \\ m_3 & m_4 & m_5 & -m_7 \end{bmatrix};$ <p>(ii)</p> $z_0 = \left(\frac{1}{4}\right) \left\{ -(c_0/2 + A) - [(c_0/2 + A)^2 - 4(Y + B)]^{1/2} \right\};$ $z_1 = \left(\frac{1}{4}\right) \left\{ -(c_0/2 + A) + [(c_0/2 + A)^2 - 4(Y + B)]^{1/2} \right\};$ $z_2 = \left(\frac{1}{4}\right) \left\{ -(c_0/2 - A) - [(c_0/2 - A)^2 - 4(Y - B)]^{1/2} \right\};$ $z_3 = \left(\frac{1}{4}\right) \left\{ -(c_0/2 - A) + [(c_0/2 - A)^2 - 4(Y - B)]^{1/2} \right\};$ $A = \left(\frac{1}{2}\right) (c_3^2 - 4c_2 + 8Y)^{1/2};$ $B = (c_3Y - c_1)/2A$ $C = [G - (G^2 + H^3)^{1/2}]^{1/3}$ $D = -H/C;$ $G = \left(\frac{1}{432}\right) (72c_0c_2 + 9c_1c_2c_3 - 27c_1^2 - 27c_0c_3^2 - 2c_2^3);$ $H = \left(\frac{1}{36}\right) (3c_1c_3 - 12c_0 - c_2^2).$

(iii)

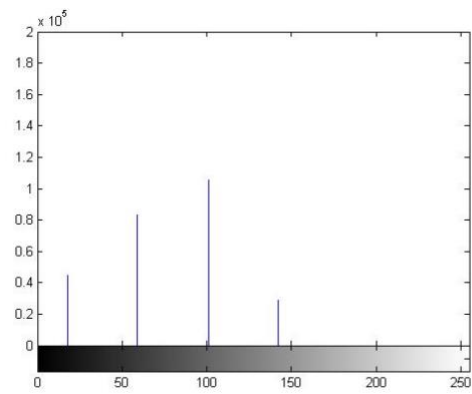
$$p_d = \begin{vmatrix} 1 & 1 & 1 & 1 \\ z_0 & z_1 & z_2 & z_3 \\ z_0^2 & z_1^2 & z_2^2 & z_3^2 \\ z_0^3 & z_1^3 & z_2^3 & z_3^3 \end{vmatrix};$$

$$p_0 = (1/p_d) \begin{vmatrix} 1 & 1 & 1 & 1 \\ m_1 & z_1 & z_2 & z_3 \\ m_2 & z_1^2 & z_2^2 & z_3^2 \\ m_3 & z_1^3 & z_2^3 & z_3^3 \end{vmatrix};$$

$$p_1 = (1/p_d) \begin{vmatrix} 1 & 1 & 1 & 1 \\ z_0 & m_1 & z_2 & z_3 \\ z_0^2 & m_2 & z_2^2 & z_3^2 \\ z_0^3 & m_3 & z_2^3 & z_3^3 \end{vmatrix};$$

$$p_2 = (1/p_d) \begin{vmatrix} 1 & 1 & 1 & 1 \\ z_0 & z_1 & m_1 & z_3 \\ z_0^2 & z_1^2 & m_2 & z_3^2 \\ z_0^3 & z_1^3 & m_3 & z_3^3 \end{vmatrix};$$

$$p_3 = 1 - p_0 - p_1 - p_2.$$



2. Illustrate the above example's answer.

● Bilevel

如圖

$Z_0 = 12.2698$, $Z_1 = 37.6060$ → 四捨五入取最接近值 $Z_0 = 12$, $Z_1 = 38$

$P_0 = 0.4984$, $P_1 = 0.5016$

故取 $t = 27$ 結果如下：

I =												
	12	12	12	12	12	12	38	38	38	38	38	38
	12	12	12	12	12	12	38	38	38	38	38	38
	12	12	12	12	12	12	38	38	38	38	38	38
	12	12	12	12	12	12	38	38	38	38	38	38

● Trilevel

如圖

$Z_0 = 10.0405$, $Z_1 = 25.0021$, $Z_2 = 39.8442$

→ 四捨五入取 $Z_0 = 10$, $Z_1 = 25$, $Z_2 = 40$

$P_0 = 0.3607$, $P_1 = 0.2773$, $P_2 = 0.3620$

故取 $t_1 = 18$, $t_2 = 30$

I =												
	10	10	10	10	25	25	40	25	40	40	40	40
	10	10	10	10	25	25	25	25	40	40	40	40
	10	10	10	10	25	25	25	25	40	40	40	40
	10	10	10	10	25	25	40	25	40	40	40	40

● Quaterlevel

如圖

$Z_0 = 9.5171$, $Z_1 = 19.3371$, $Z_2 = 30.9802$, $Z_3 = 40.4078$

→ 四捨五入取 $Z_0 = 10$, $Z_1 = 19$, $Z_2 = 31$, $Z_3 = 40$

$P_0 = 0.3109$, $P_1 = 0.1912$, $P_2 = 0.1903$, $P_3 = 0.3075$

故取 $t_1 = 11$, $t_2 = 27$, $t_3 = 37$

I =												
	10	10	10	10	19	19	31	31	40	40	40	40
	19	10	10	10	19	19	31	31	40	40	40	40
	10	10	10	10	19	19	31	31	40	40	40	40
	10	10	10	10	19	19	31	31	40	40	40	40