UNIVERSITÀ DI PISA

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Logistics

Project report
Wireless set covering problem

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Contents

1. Mathematical formulation

1.1 First ILP model

For the first formulation we have the following:
Sets:

- I: Set of all towns indexed by i, where i = 1,2,...,25;
- J: Set of all towers indexed by j where j = 1,2,...,25.

Parameters:

- C: Activation cost for each tower j, as fixed cost 150.000;
- $a_{i,j}$: Coverage parameter,where i, j = 1,2,...,25. If town i is covered by tower j, then a_{ij}=1, otherwise it is 0;
- R_i : where R is the matrix of expected annual revenue if wireless internet service is provided to the town i (in 1000's of dollars). R_i are its element such that $R_1 = 34$, $R_2 = 43$, ..., $R_{25} = 44$.

$$\begin{matrix} 34 & 43 & 62 & 42 & 34 \\ 64 & 43 & 71 & 48 & 65 \\ 57 & 57 & 51 & 61 & 30 \\ 32 & 38 & 70 & 56 & 40 \\ 68 & 73 & 30 & 56 & 44 \end{matrix}$$

Decision variables:

- $x_j : \begin{cases} 1 & \text{if tower at location j is built.} \\ 0 & \text{otherwise} \end{cases} \quad \forall j \in J$
- $y_i : \begin{cases} 1 & \text{if town at location i is covered.} \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in I$

Objective function:

$$max \sum_{i=1}^{I} R_i \cdot y_i - C \sum_{j=1}^{J} x_j$$

the objective is to identify the optimal tower placement strategy that maximizes the company's first-year profit. This optimization problem inherently involves a trade-off between revenue generation and infrastructure costs. The model incorporates revenue generation by calculating the expected annual revenue for each geographic area covered. This calculation is achieved by multiplying the potential revenue of each area with a binary decision variable. This decision variable reflects the coverage in that specific area. Subsequently, the model integrates the total cost associated with tower construction. This is accomplished by multiplying the fixed cost of building a tower with the binary decision variable. This objective function prioritizes preventing the double-counting of revenue for serviced towns.

Constraints:

$$y_i \leq \sum_{j=1}^{J} a_{ij} \cdot x_j \quad \forall i \in I$$

The constraint states that if a tower is built at location j ($x_j = 1$), then the sum of $a_{ij} \cdot x_j$ over all towers must be greater than or equal to y_i for each town i. In other words, if a tower is built at location j, it must cover all towns that it can potentially cover.

This constraint ensures that the coverage decision (y_i) is consistent with the tower placement decision (x_j). It prevents situations where a tower is built but fails to cover the towns it is supposed to serve.

1.2 Second ILP model

The second ILP model can be simplified as shown because of the decision binary variable we set as y_i. Since this denotes the coverage of the towns, we can say that all the towns need to be covered, as each town's coverage must be equal to 1. In terms of the objective function we will be now minimizing the cost of covering all towns.

Objective function:

$$min \sum_{j=1}^{J} C \cdot x_j$$

Constraints:

$$y_i = 1 \quad \forall i \in I$$

Riformulating the constraint of the first ILP model:

$$1 \leq \sum_{j=1}^{J} a_{ij} \cdot x_j \quad \forall i \in I$$

1.3 Third ILP model

The third ILP problem follows the same structure and logic that the first one has, the only difference is that we have an additional restriction where we cannot have both the first town y_1 and the last town y_{25} covered at the same time, the model needs to decide if it's more profitable to cover one or the other, the rest of the formulation remains the same.

Objective function:

$$max \sum_{i=1}^{I} R_i \cdot y_i - C \sum_{j=1}^{J} x_j$$

Constraints:

$$y_i \leq \sum_{j=1}^{J} a_{ij} \cdot x_j \quad \forall i \in I$$

$$y_1 + y_{25} \leq 1$$

2. AMPL Results

2.1 First solution

Towers Built (highlighted in *yellow*): **4**
The towns covered by the towers are highlighted in *green*.
Objective: 377.000

34	43	62	42	34
64	43	71	48	65
57	57	51	61	30
32	38	70	56	40
68	73	30	56	44

2.2 Second solution

Towers Built (highlighted in *yellow*): **7**
The towns covered by the towers are highlighted in *green*.
Objective: 1.050.000

34	43	62	42	34
64	43	71	48	65
57	57	51	61	30
32	38	70	56	40
68	73	30	56	44

2.3 Third solution

Towers Built (highlighted in *yellow*): **4**
The towns covered by the towers are highlighted in *green*.
Objective: 377.000

34	43	62	42	34
64	43	71	48	65
57	57	51	61	30
32	38	70	56	40
68	73	30	56	44

The output is the same as in Point (1) because it does not cover either y_1 or y_{25}, which are not covered because it is not the optimal solution.