

Analysis II for computer scientists

Problem sheet 6

Problem 26 (Things to think)

Correct or incorrect? If you think the assertion is correct then justify your answer. Provide a counterexample otherwise.

- (i) What is a line search and what is it used for?
- (ii) Name advantages of the steepest descent method vs. Newton's method and vice versa.
- (iii) Assume that the steepest descent direction v satisfies $\partial_v f(x) < 0$. Then the steepest descent method with unit steps produces a descent step.
- (iv) How many iterations are necessary to optimize a quadratic function by the Newton method accurately?
- (v) What is the derivative of $f(x, y) = 0.5\|F(x, y)\|_2^2$ for a (totally) differentiable function $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$?
- (vi) When and why is the Newton direction a descending direction for $f(x, y) = 0.5\|F(x, y)\|_2^2$ with a (totally) differentiable function $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$? What is this insight used for?

Problem 27 (Steepest descent method)

- (a) Carry out two iterations of the steepest descent method starting at $(1, 1)$ to minimize

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}, \quad (x, y) \mapsto f(x, y) = x^2 + 3y^2 - 2xy.$$

At step 1, apply a line search to guarantee that the function values decrease. Are the function values decreasing during the 2nd iteration (with a unary step) as well?

- (b) Solve the non-linear system

$$F(x, y, z) = \begin{pmatrix} f_1(x, y, z) = 3x - \cos(yz) - \frac{3}{2} \\ f_2(x, y, z) = 4x^2 - 625y^2 + 2y - 1 \\ f_3(x, y, z) = e^{-xy} + 20z + \frac{10\pi-3}{3} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

using the steepest descent method. Therefore, apply one step with step size 0.001 (initial iterate: $(0, 0, 0)$).

Hint:

Minimize the function $f(x, y, z) = 0.5 \cdot \|F(x, y, z)\|_2^2$.

Problem 28 (Newton's method)

- (a) Carry out two iterations of Newton's method to find the minimum of the function

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}, \quad (x, y) \mapsto f(x, y) = x^3 + 3xy^2 - 27x - 12y.$$

starting at $(0, 2)$. Are the function values decreasing during these iterations?

- (b) In the following we want to find the intersection points of a circle and a hyperbola (compare Figure 1).

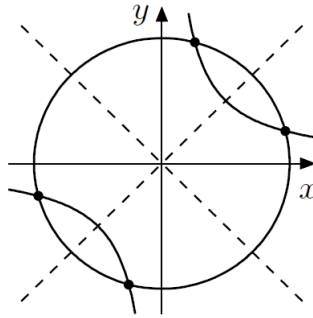


Figure 1: Intersection of a circle and a hyperbola

Therefore, carry out 2 Newton iterations for the non-linear system of equations

$$F(x, y) = \begin{pmatrix} f_1(x, y) = x^2 + y^2 - 4 \\ f_2(x, y) = xy - 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

using the initial iterate $(2, 1)$. Also calculate the exact solution analytically.

The problems are due on 22.10.2019.