Baden-Wuerttemberg
Cooperative State University Stuttgart
Course: STG-TINF19A
Dipl.-Ing. Tim Lindemann

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Analysis II for computer scientists

Problem sheet 3

Mathematics consists in proving the most obvious thing in the least obvious way.

George Pólya (Hungarian mathematician, 1887-1985)

Problem 10 (**T**hings **t**o **t**hink)
Correct or incorrect? If you think the assertion is correct then justify your answer. Provide a counterexample otherwise.

(i) The multi-dimensional chain rule serves to find derivatives of a function in multiple variables.

(ii) The multi-dimensional chain rule is a method to calculate the total derivative of a function.

(iii) What is implicit differentiation and what is it used for?

(iv) The directional derivative points in the direction of the gradient.

(v) What is the directional derivative of a function f in (x_0, y_0) w.r.t. to a vector orthogonal to $\nabla f(x_0, y_0)$?

(vi) Why is a partial derivative a special type of directional derivative?

(vii) Argue why the following equation holds for partially differentiable functions f and g:

$$\nabla\left[(f\cdot g)(x,y)\right] = g(x,y)\cdot\nabla f(x,y) + f(x,y)\cdot\nabla g(x,y).$$

(viii) Prove that the gradient points in the direction of steepest ascent. Illustrate the situation in an x-y-diagram.

Problem 11 (Chain rule)
Calculate the following derivatives using the multi-dimensional chain rule.

$$\begin{array}{llll}
(a) & f(x,y) = x^2y + y^3, & x = x(t) = t^2, & y = y(t) = e^t, \\
(b) & f(x,y) = (x-y)^2, & x = x(t) = 2\cos(t), & y = y(t) = 2\sin(t), \quad 0 \le t < 2\pi, \\
(c) & f(x,y) = \sin(x+y), & x = x(t) = t, & y = y(t) = t^2.
\end{array}$$

Problem 12 (Chain rule)
Let the function f be given by $f(x,y) = ye^{2x-y}$, where $x = x(t)$ and $y = y(t)$ are themselves differentiable functions depending on the single parameter t. We also know the following data:

$$x(0) = 2, \qquad y(0) = 4, \qquad x'(0) = -1, \qquad y'(0) = 4.$$

Calculate the derivative of the function given by $F(t) = f(x(t), y(t))$ in the point $t = 0$.

Problem 13 (Implicit differentiation)
What is the slope of the curve with characteristic equation

$$\left(x^2+y^2\right)^2-2x\left(x^2+y^2\right)-y^2=0$$

in the point $P=(0,1)$?

Problem 14 (Directional derivatives & gradient)

(a) Consider the function

$$f:\mathbb{R}^2\to\mathbb{R},\ \ (x,y)\mapsto f(x,y)=(3x+x\cdot y)^2\,.$$

Calculate the gradient of f in a general point (x_0,y_0). Also determine the directional derivative w.r.t. $v^\top=(2,4)^\top$. What is the direction of steepest ascent in the point $(1,2)$?

(b) Consider the function

$$f:\mathbb{R}^3\to\mathbb{R},\ \ (x,y,z)\mapsto f(x,y,z)=y\cdot\cos(z)+\frac{\ln(1+x^2)}{y}.$$

Calculate the gradient of f in a general point (x_0,y_0,z_0). Also determine the directional derivative w.r.t. $v^\top=(3,-1,2)^\top$. What is the direction of steepest descent in the point $(1,2,3)$?

Problem 15 (The mountain climber)
On a certain mountain the elevation z (in m) above a point (x,y) in the horizontal $x-y$-plane at sea level is

$$z=2500-3x^2-4y^2.$$

The positive x-axis points east and the positive y-axis points north. Suppose that a climber is at the point $(15,-10,1425)$.

(a) If the climber walks due west, will the climber ascend or descend? At what rate?

(b) If the climber walks southeast, will the climber ascend or descend? At what rate?

(c) In what direction should the climber walk to travel a level path? In what direction should he walk to get down as quick as possible?

The problems are due on 08.10.2019.