Baden-Wuerttemberg
Cooperative State University Stuttgart
Course: STG-TINF18A
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Analysis II for computer scientists

Problem sheet 2

If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is.

John von Neumann (Hungarian mathematician, 1903-1957)

Problem 5 (**T**hings **t**o **t**hink)
Correct or incorrect? If you think the assertion is correct then justify your answer. Provide a counterexample otherwise.

(i) Every function $f\colon \mathbb{R}^2 \to \mathbb{R}$ is partially differentiable.

(ii) What is the difference between $f'(x_0)$, $\frac{df}{dx}(x_0)$, $\frac{\partial f}{\partial x}(x_0)$ and $f_x(x_0)$?

(iii) If f is partially differentiable, then it is partially continuous.

(iv) If f is partially differentiable, then it is continuous.

(v) What does Schwarz' theorem tell us? Why is it useful in practice?

(vi) Every continuous function $f\colon \mathbb{R}^2 \to \mathbb{R}$ has a tangent plane at every point.

(vii) State in your own words, what differentiability of a single variable function means. Also remark on the relation to approximating functions.

(viii) What is the jacobian matrix of a multi-variable function? What is its total derivative?

(ix) Why is the function

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2}, & (x,y) \neq \mathbb{0}, \\ 0, & \text{else}, \end{cases}$$

not totally differentiable?

Problem 6 (Partial derivatives)
Determine all partial derivatives of order 1 of the following functions (together with the maximum domains).

$(a)\quad f(x,y) = e^{-x}\cos(y)$

$(b)\quad f(x,y) = 2e^x + \ln(y) + 5x^3y$

$(c)\quad f(x,y) = \sqrt{1-x^2-y^2}$

$(d)\quad f(x,y) = \arcsin\left(\frac{y}{x}\right)$

$(e)\quad f(x,y) = \ln\frac{1}{\|(x,y)\|_2}$

$(f)\quad f(x,y,z) = e^{x-y}\cos(5z)$

Problem 7 (Partial differential equations (PDEs))

(a) Show, that the temperatur distribution given by the function

$$v : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R},\ \ (t,x) \mapsto \frac{1}{\sqrt{t}}e^{-x^2/4t}$$

satisfies the (homogeneous) heat equation

$$\frac{\partial v}{\partial t}(t,x) - \frac{\partial^2 v}{\partial x^2}(t,x) = 0,\ \ (t,x) \in \mathbb{R}_+ \times \mathbb{R}.$$

(b) Let $g\colon \mathbb{R} \to \mathbb{R}$ be differentiable. Show that the function

$$\omega : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}, (t, x) \mapsto g(x - t)$$

satisfies the transport equation

$$\frac{\partial \omega}{\partial t}(t, x) = -\frac{\partial \omega}{\partial x}(t, x), \;\; (t, x) \in \mathbb{R}_+ \times \mathbb{R}.$$

Problem 8 (Total differentiability)
Calculate the jacobian matrices of the following functions and the tangent planes in the given points.

(a) $f\colon \mathbb{R}^2 \to \mathbb{R}, \;\; (x, y) \to f(x, y) = e^{-(x^2+y^2)}$ at $P(0.15, 0.15)$.

(b) $f\colon \mathbb{R}^2 \to \mathbb{R}, \;\; (x, y) \to f(x, y) = (x^2 + 3y^2)e^{1-x^2-y^2}$ at $P(0.5, 0.5)$.

(c) $f\colon \mathbb{R}^2 \to \mathbb{R}, \;\; (x, y) \to f(x, y) = 2e^x + \ln(y) + 5x^3y$ at $P(1, 1)$.

Problem 9 (Schwarz' theorem)
Validate Schwarz' theorem by explicitly calculating the mixed second order derivatives of

$$f : \mathbb{R}^2 \to \mathbb{R}, \;\; (x, y) \mapsto f(x, y) = \sin(x^2 + 2y).$$

The problems are due on 01.10.2019.