Baden-Wuerttemberg
Cooperative State University Stuttgart
Course: STG-TINF18A
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Analysis II for computer scientists

Problem sheet 1

Wer Grenzen überschreitet, versucht, in eine neue Dimension vorzustoßen.

Daniel Mühlemann (1959-), German translator and aphorist

Problem 1 (**T**hings **t**o **t**hink)
Correct or incorrect? If you think the assertion is correct then justify your answer. Provide a counterexample otherwise.

(i) A function of several variables is given by a rule f such that $f : \mathbb{R} \mapsto \mathbb{R}^n$.

(ii) All level sets of functions in 2 variables are non-empty.

(iii) The euclidian norm of a vector can be negative.

(iv) If $\lim\limits_{n\to\infty} x_n = x$ holds then a sequence $(x_n, y_n)_{n\in\mathbb{N}}$ is convergent for the euclidian norm.

(v) What is the shape of the unit-ball corresponding to the maximum norm in $\mathbb{R}^2$?

(vi) Consider a function $f : \mathbb{R}^2 \to \mathbb{R}$. If we have

(a) $x_n \to x_0 \implies f(x_n, y) \to f(x_0, y)$ for all $y \in \mathbb{R}$ and

(b) $y_n \to y_0 \implies f(x, y_n) \to f(x, y_0)$ for all $x \in \mathbb{R}$

then f is continuous.

(vii) What is the taxicab norm? Describe the shape of the sphere of radius 1 in $\mathbb{R}^3$.

Problem 2 (Level sets & coordinate curves)
Consider the function f defined by $f : \mathbb{R}^2 \to \mathbb{R},\ (x, y) \mapsto \sqrt{x^2 + y^2}$.

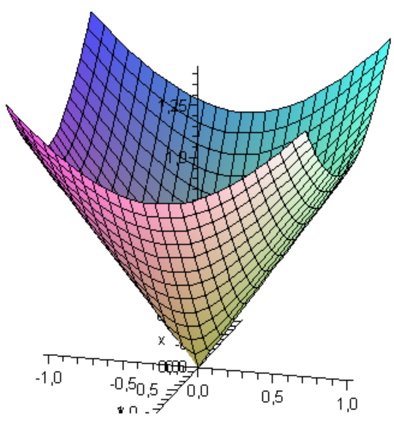

Figure 1: The graph of f.

(i) Determine the level sets $L_k(f)$ for $k = 0, 1, 2, 3, 4$. Use an x-y-diagram to illustrate the level sets.

(ii) Determine the coordinate curves for fixed $x = 0, 1, 2, 3, 4$ and $y = 0, 1, 2, 3, 4$, respectively. Plot these curves into corresponding 2-dimensional diagrams.

Problem 3 (Level sets)
Sketch the level sets of the following functions in an x-y-diagram:

(i) $L_k(f)$ $(k = 0, 3, 8)$ for f given by $f(x, y) = x^2 + y^2 - 2y$.

(ii) $L_k(f)$ $(k = -12, -6, 0, 6, 12)$ for f given by $f(x, y) = 3x + 6y$.

(iii) $L_k(f)$ $(k = 0, 1, \sqrt{2}, \sqrt{3})$ for f given by $f(x, y) = \sqrt{y - x^2}$.

Problem 4 (Continuity)

(i) Prove that the function f defined by

$$f : \mathbb{R}^2 \to \mathbb{R}, \ \ (x, y) \mapsto \begin{cases} \frac{\sin(xy)}{x}, & x \neq 0, \\ y, & x = 0. \end{cases}$$

is continuous.

(ii) Check whether the function f defined by

$$f : \mathbb{R}^2 \to \mathbb{R}, \ \ (x, y) \mapsto \begin{cases} \frac{xy}{\sqrt{|x|+y^2}}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0). \end{cases}$$

is continuous.

The problems are due on 10.09.2018.