Computational Mathematics I
DCS-112

Lecture Notes

Billbrain Institute of Technology

Year: 1 - Semester: 1

Course Delivered by

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Course Overview

- This course is designed to provide a general overview of selected mathematical concepts with useful everyday life applications.

- The course is also designed to develop the ability to think critically, and to realize that the proper use of logic is a reasonable way to solve problems.

Learning Outcomes

By the end of the course, students should:

- ❑ Have the basic mathematical knowledge required for undertaking BIST
- ❑ Demonstrate understanding of the basic counting techniques and their applications for information systems students
- ❑ Demonstrate critical thinking in mathematical related situations.
- ❑ Perform problem solving using basic mathematical tools
- ❑ Relate mathematical problems to real world situations while coming up with appropriate solutions.

Topics to be covered

The topics to be covered as per the scope of this course include:

- Numbers
- Algebra
- Equations
- Logic & Set theory
- Statistics & Probability
- Coordinate Geometry
- Sets, Vectors, & Functions

Mode of Delivery & Assessment

Mode of Delivery

- Lectures
- Online learning management systems
- Class discussions and presentations
- Problem-based/case studies

Mode of Assessment

- Course work
  - Tests: 20%
  - Assignments: 20%
- Final written exam: 60%

Recommended Reading list

Below are some of the resources recommended for personal reading and revision.

1. William M, Beaver R.J, & Barbara M. B, (2013). *Introduction to Probability and Statistics. 14th Edition. Publisher: Brooks/Cole.*

2. Linda .B, Suzanne .C, (2000). *Core Maths for Advanced Level. Publisher: Nelson* Thornes.

3. Backhouse J.K., Houldsworth S.P.T, & Horril P.J.F. (1985). *Pure Mathematics.* Publisher: Longman.

4. Robert R. Stoll, (1979). *Set Theory and Logic.*

Topic 1: Numbers

Numbering: Terminologies

- Numbers: A Number can be defined as a symbol or word used to express value or quantity.

i.e the Arabic Hindu Numeral System - 0,1,2,3,4,5,6,7,8,9

- Digits: Name given to place or position of each numeral.

- Number Squence

…	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones
…	7	6	5	4	3	2	1

Topic 1: Numbers

Number categorisations include:

- Natural numbers
- Integers
- Prime numbers
- Rational and Irrational numbers
- Sequences
- Fractions
- Simple algebraic statements
- Estimates and approximations
- Ratios
- Direct and indirect proportions
- Percentage increases and decreases
- Simple and compound interest.

Topic 1: Numbers

Natural Numbers

❑ A natural number is a number that occurs commonly and obviously in nature. As such, it is a whole, non-negative number. The set of natural numbers, denoted ***N***, can be defined in either of two ways:

N = {0, 1, 2, 3, ...}

N = (1, 2, 3, 4, ...}

Properties of Natural Numbers

	Addition	Multiplication
Closure	$a + b$ is a natural number	$a \times b$ is a natural number
Associativity	$a + (b + c) = (a + b) + c$	$a \times (b \times c) = (a \times b) \times c$
Commutativity	$a + b = b + a$	$a \times b = b \times a$
Existence of an identity element	$a + 0 = a$	$a \times 1 = a$
Distributivity	$a \times (b + c) = (a \times b) + (a \times c)$	
No zero divisors:		if $ab = 0$, then either $a = 0$ or $b = 0$ (or both)

Topic 1: Numbers

Integers

- An **integer** is a whole number that can either be positive, negative, or zero. Examples of **integers** are: 5, 2, 0, -5, 8, 89, etc
- Integers can infinitely be represented on a number line to either side of 0 (positive or negative) as seen below

Example: Calculate $(-5) - (-2)$

Topic 1: Numbers

Prime Numbers

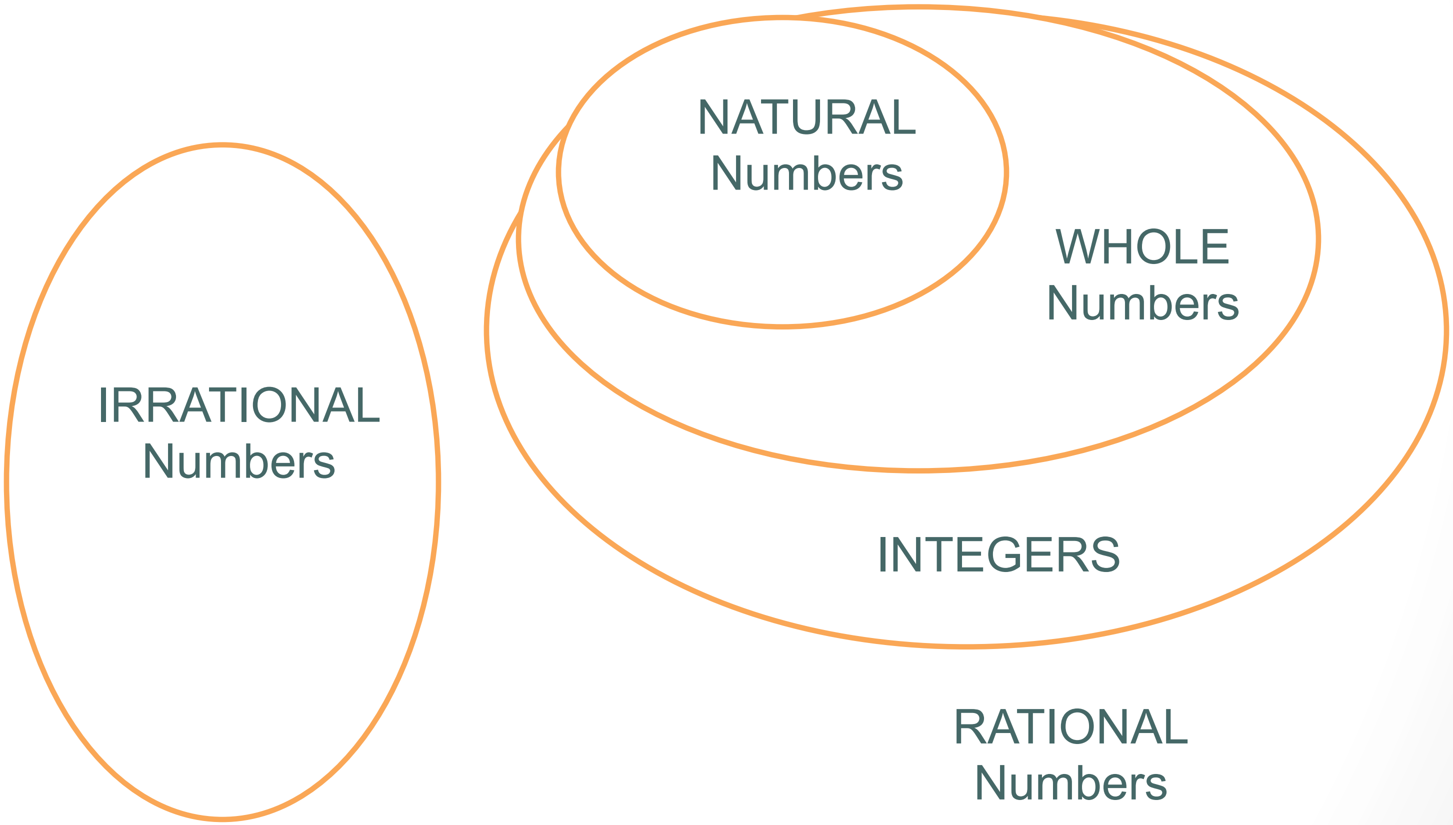
- A prime number is a whole number greater than 1, whose only two whole-number factors are 1 and itself. i.e: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, …

- For Example: 7 can only be divided evenly by 1 or 7, so it is a **prime number**. But 8 can be divided evenly by 1, 2, 4 and 8, so it is NOT a **prime number.**

- The numbers that do not qualify as prime numbers are categorised as composite **numbers**

Topic 1: Numbers

Rational & Irrational Numbers



Topic 1: Numbers

Rational Numbers

- A **number** is **rational** if we can write it as a fraction where the top **number** of the fraction and bottom **number** are both whole **numbers.**
- A fraction ***r* = *p/q***, where p and q are positive or negative integers, is called a rational number.
- We can suppose that:
  - (i) *p* and *q* have no common factor, as if they have a common factor we can divide each of them by it, and that,
  - (ii) *q* is positive, since *p(/-q) = (-p)/q;* *(-p)/(-q) = p/q*

Topic 1: Numbers

Irrational Numbers

- **An irrational number** is any **number** that cannot be written as a ratio of two integers (or cannot be expressed as a fraction).

- If a whole number is not a perfect square, then its square root is an irrational number.
- Example:

$\sqrt{2}$, $\sqrt{5}$, $\sqrt{7}$

These are whole number that are not a perfect square.

Topic 1: Numbers

Sequences

- The term **sequence,** as used in arithmetic is an ordered list of numbers
- In a **sequence**, each term is equal to the previous term, plus (or minus) a constant. The constant is called the common difference (d).

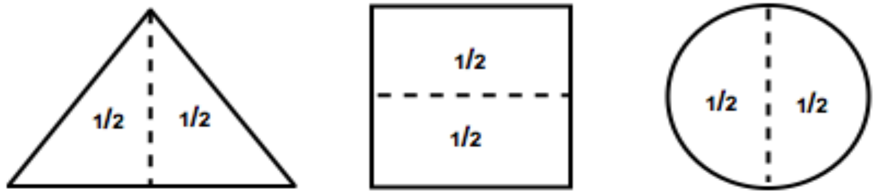
Increasing Arithmetic Sequence	Decreasing Arithmetic Sequence
* Common difference is positive!	* Common difference is negative!
5 , 9 , 13 , 17 , . . . +4 +4 +4	20 , 17 , 14 , 11 , . . . −3 −3 −3

Topic 1: Numbers

Fractions

- ❑ A Fraction is part of a whole unit or quantity.
- ❑For Example ½ is a half fraction of 1.   ½ + ½ = 1

Illustration of fractions using objects



- ❑Fractions can also be represented as **DECIMAL NUMBERS.**
- ❑Written on one line as whole number, the position of period determines power of decimal.

	Tenths	Hundredths	Thousandths
.	0	0	5

Topic 1: Numbers

Simple Algebra

Common terms as used in Algebra:

- Variable – A variable is a letter or symbol that represents a number (unknown quantity). ($8 + n = 12$)

- Algebraic expression – a group of numbers, symbols, and variables that express an operation or a series of operations.

Topic 1: Numbers

Simple Algebra (*Basic Example Solutions*)

Single Equation 1:

$$2(4y+1)=3y$$
$$8y+2=3y$$
$$5y=-2$$
$$y=-\frac{2}{5}$$

Single Equation 2:

$$-3(x-6)+4(x+1)=7x-10$$
$$-3x+18+4x+4=7x-10$$
$$x+22=7x-10$$
$$\underline{-7x\quad -22\quad -7x\quad -22}$$
$$-6x\quad = -32$$
$$-6x=-32$$
$$x=\frac{-32}{-6}=\frac{16}{3}$$

Simultaneous Equations

$$3x+\quad 3y\ =\ 24$$
$$\underline{-6x+(-3y)=-39}$$
$$-3x\qquad =-15$$
$$\frac{-3x}{-3}=\frac{-15}{-3}$$
$$x=5$$

Topic 1: Numbers

Simple Algebra (Sample Questions to try out)

Equations with Fractions

1) $\frac{f}{5} + 2 = 8$

2) $\frac{w}{3} - 5 = 2$

3) $\frac{x}{8} + 3 = 12$

4) $\frac{5t}{4} + 3 = 18$

5) $\frac{3y}{2} - 1 = 8$

6) $\frac{2x}{3} + 5 = 12$

7) $\frac{t}{5} + 3 = 1$

8) $\frac{x + 3}{2} = 5$

9) $\frac{t - 5}{2} = 3$

Simultaneous Equations

1. $4x + y = 17$
   $2x + y = 9$

2. $5x + 2y = 13$
   $x + 2y = 9$

3. $3x + 2y = 11$
   $2x - 2y = 14$

4. $3x - 4y = 17$
   $x - 4y = 3$

5. $2x + 5y = 37$
   $y = 11 - 2x$

6. $4x - 3y = 7$
   $x = 13 - 3y$

Topic 1: Numbers

Estimates and Approximations

- **Key terms defined?**

Estimation: An intelligent guess made about something based on some information. It can also be termed as a calculated approximation of a result which is usable even if input data may be incomplete or uncertain

Approximation: is a guess that is nearly exact.

Estimate: To estimate is to judge tentatively or approximately the value, worth, or significance of:

Point estimation: is estimation in which a single value is assigned to a parameter.

Topic 1: Numbers

Estimates and Approximations

- Significant figures:

- Rounding Off

There are three major ways to round off numbers

- To the nearest 10, 100, 1000, etc
- To a certain number of significant figures
- To a certain number of decimal places

-

Estimatation Example

Qns: How many days will it take you to complete task *x*?

Ans: **ideally** *n* days

Topic 1: Numbers

Approximation

Rules of Applicable to Estimation & Approximation
-0 between two whole numbers or digits is significant
-0 at the beginning of a number is not significant
-If the digit after the required significant figure is >= 5, round of by adding 1 to the previous digit.
-Rounding off to the required significant figure turns all values to the right of the rounded off value to zero

Examples: Write the following numbers to 2 significant

a) 0403670=
b) 0.052407=
c) 34.08945=

Topic 1: Numbers

Approximation

Examples

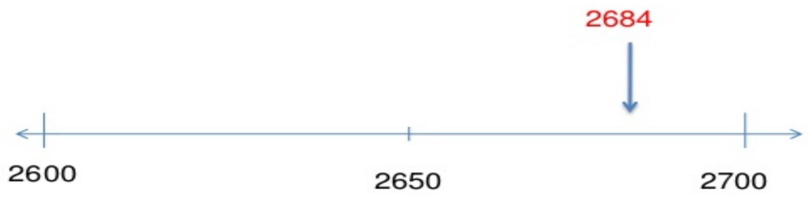
1. Given 0.074, find the first significant figure.

The first significant number is 7. Look at the next number; 4. Is it >=5 to increase the first value (7)? **No.** So the 4 is eliminated.

You remain with 0.07.
Therefore 0.074 to 1 significant figure is: 0.07

Another way of looking at approximation:

2. Given 2684, approximate to the nearest whole number. In this case a number line is used.



Ratios, Rates and Proportions

Terms and their definitions

- A ***ratio*** is the comparison of two numbers written as a fraction.
- In a ***ratio***, if the numerator and denominator are measured in different units then the ratio is called a ***rate***.
- A ***unit rate*** is a rate per one given unit, like 60 miles per 1 hour.
- When Ratios are written in this order, *a* and *d* are the ***extremes***, or outside values, of the proportion, and *b* and *c* are the ***means***, or middle values, of the proportion.

Topic 1: Numbers

Ratios Expounded.

- There are two different types of ratios
  - Part-to-Part Ratios
    - Example: John has 2 dogs and 8 cats. The ratio is: ***2 to 8*** or ***1 to 4***
  - Part –to-Whole Ratios
    - Example: John has 2 dogs, and a total pet number of 10. In this case, the ratio is: ***2 to 10*** or ***1 to 5***

Examples

1. e are 12 apples and 3 bananas

   So the ratio of apples to bananas is

   4:1, 4 to 1, OR 4/1

2. There are 15 oranges and 12 apples

   So the ratio of oranges to apples is

   5:4, 5 to 4, OR 5/4

Topic 1: Numbers

Rates Expounded.

- A rate is a ratio that compares two different quantities or measurements.
- Rates can be simplified
- Rates us the words per and for
  - Example: Driving 55 miles **per** hour $\Rightarrow$ ***Miles/Hour***
  - Example: 3 tickets for $1 $\Rightarrow$ ***Tickets/Dollar***

Example

- Suppose a boat travels 12 miles in 2/3 hours. Determine the unit rate, (in this case it's speed).

$$\boxed{\frac{12 miles}{2/3 hours}} \Rightarrow \boxed{12 \div \frac{2}{3}} \Rightarrow \boxed{12 * \frac{3}{2} = 18} = 18\text{mph}$$

Topic 1: Numbers

Direct and Indirect Proportions

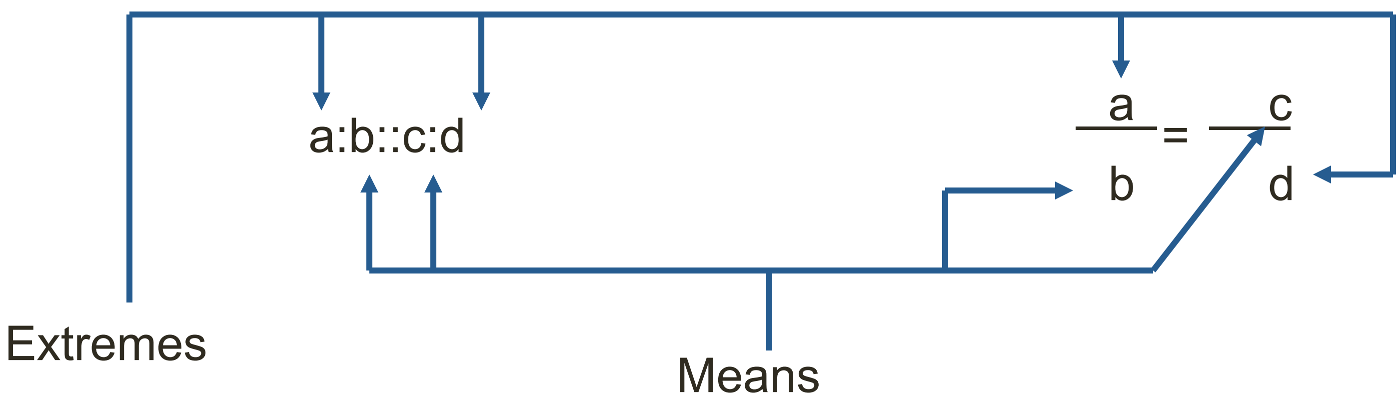
- An equation in which two ratios are equal is called a ***proportion***.
- A proportion can be represented in the following ways. i.e:
  - Using colon notation: *a:b::c:d*
  - OR as the more recognizable (and useable) equivalence of two fractions:

$$\frac{a}{b} = \frac{c}{d}$$

- Writing the units when comparing each unit of a rate is called ***unit analysis***.

Direct and Indirect Proportions

When Ratios are written in this order, *a* and *d* are the ***extremes***, or outside values, of the proportion, and *b* and *c* are the ***means***, or middle values, of the proportion.



Direct and Indirect Proportions

Direct proportions

There is direct proportion between two values when one is multiple of the other.

For example: ***1cm = 10mm***

To convert cm to mm, the multiplier is always 10. Direct proportion is used to calculate the cost of petrol or exchange rates of foreign money.

The symbol for direct proportion is $\propto$.

The statement ‘t is directly proportional to r’ can be written using the proportionality symbol: $t \propto r$

Direct and Indirect Proportions

Indirect / Inverse **proportions**

If one value is inversely proportional to another then it is written using the proportionality symbol in different w y. Inverse proportion occurs when one value increases and the other decreases. For example, more workers on job would reduce the time to complete the task.

They are inversely proportional.

The statement ‘b is inversely proportional to m’ is written: $b \propto \frac{1}{m}$

Equations involving inverse proportions can be used to calculate other values.

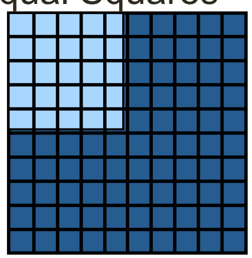
Using: $g = \frac{36}{w}$ (so ***g*** is inversely proportional to ***w***).

Percentage Increases and Decreases

1. Percent

- Used to show how many parts of a total are taken out.
- Short way of saying “by the hundred or hundredths part of the whole”.
- The symbol % is used to indicate percent.
- Often displayed as diagrams.

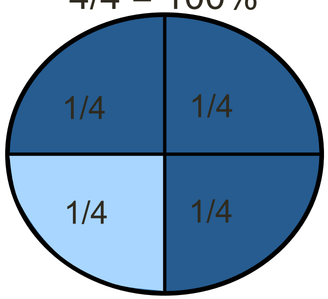
100 Equal Squares = 100%



25% or 25/100

or

4/4 = 100%



25/100 = 25%

To change a decimal to a %, move decimal point two places to right and write percent sign.

.15 = 15% .55 = 55%
.853 = 85.3% 1.02 = 102%

“Zeros may be added at the end to hold a place”.

.8 = 80%

Topic 1: Numbers

Percentage Increases and Decreases

2. Percentage

- Refers to value of any percent of a given number.
- First number is called "base".
- Second number called "rate"... Refers to percent taken from base.
- Third number called "percentage".

Rule: The product of the base, times the rate, equals the percentage.

Percentage = Base x Rate or $P = B \times R$

NOTE: Rate must always be in decimal form.

To find the formula for a desired quantity, cover it and the remaining factors indicate the correct operation.

Only three types of percent problems exist.

1. Find the amount or rate. $R = P \times B$
2. Find the percentage. $P = \frac{R}{B}$
3. Find the base. $B = \frac{R}{P}$

Topic 1: Numbers

Simple and Compound Interest

❑ What's the Difference?

If you walk into a bank and open up a savings account you will earn interest on the money you deposit in the bank.

- If the interest is calculated once a year then the interest is called "**simple interest**".
- If the interest is calculated more than once per year, then it is called "**compound interest**".

Topic 1: Numbers

Simple and Compound Interest

❑ Simple Interest

- **Simple interest** is the interest that is computed on the *original principal* only.
- If I denotes the *interest* on a *principal* P at an interest rate of R per year for t years, then we have $I = PRT$
- The accumulated amount A, is the sum of the principal and interest after T years is given by

$$A = P + I = P + PRT$$

$$= P\,(I + RT) \text{ and is a linear function of } T.$$

Example:

Suppose an investment of $7,000 is invested at 7.5%, what is the total simple interest accumulated after 3 years?

➢ $I = PRT$ ⇨ $I = (7{,}000 \times 0.075 \times 3)$ ⇨ $I = \$1575$

Simple and Compound Interest

Compound Interest

- Frequently, interest earned is *periodically* added to the principal and thereafter *earns interest itself* at the same rate. This is called compound interest.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Where:

A = Amount (sometimes represented as FV; to mean final value)

P = Principal Amount

r = Interest rate (decimal)

n = Number of times the interest is compounded per year

t = Time (number of years)

Simple and Compound Interest

Compound Interest

Example 1: If you deposit $4000 into an account paying 6% annual interest compounded quarterly, how much money will be in the account after 5 years?

Solutions

$$FV = 4000\left(1 + \frac{0.06}{4}\right)^{4(5)}$$

$$FV = 4000(1.015)^{20}$$

$$FV = 4000(1.346855007)$$

$$FV = 5387.42$$

After 5 years there will be $5387.42 in the account

Topic 1: Numbers

Simple and Compound Interest

Compound Interest

Example 2: Example 2: How much money would you need to deposit today at 9% annual interest compounded monthly to have $12000 in the account after 6 years?

Solutions

$$12000 = P\left(1+\frac{0.09}{12}\right)^{12(6)}$$

$$12000 = P(1.0075)^{72}$$

$$12000 = P(1.712552707)$$

$$P = 7007.08$$

You would need to deposit $7007.08 to have $12000 in 6 years.

Topic 1: Numbers

Simple and Compound Interest

❑ Compound Interest – Practice Questions

1. If you deposit $5000 into an account paying 6% annual interest compounded monthly, how long until there is $8000 in the account

2. If you deposit $8000 into an account paying 7% annual interest compounded quarterly, how long until there is $12400 in the account?

3. At 3% annual interest compounded monthly, how long will it take to double your money?

END OF TOPIC