**Research Project**
**Economics 712**
**Fall 2019**
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**Due December 6, 2019 at Noon**

# Part I
# Partial Equilibrium

## 1 The Model

Consider an individual with preferences

$$u(c) = E_0 \sum_{t=0}^{T} \beta^t U(c_t)$$

with $\beta = \frac{1}{1+\rho} \in (0,1)$. The individual might either be finitely or infinitely lived.

The individual has stochastic income process $\{y_t\}_{t=0}^{T}$ where $y_t \in Y = \{y_1, \ldots, y_N\}$. The income process is Markov. Let $\pi(y'|y)$ denote the probability that tomorrow's endowment takes the value $y'$ if today's endowment takes the value $y$. The agents' budget constraint at period $t$ reads as

$$c_t + a_{t+1} = y_t + (1+r)a_t$$

For now suppose that the interest rate $r$ is exogenously given, so that $r$ is a parameter. In order to make the problem computable we first have to specify the parameters of the model.

### 1.1 Preferences

Let us start with preferences: we will assume that preferences can be represented by a period utility function that is of CRRA form:

$$U(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$$

with $\sigma > 0$ (and $\sigma = 1$ is understood to be the log-case).

### 1.2 Discount Rate and Interest Rate

With respect to the time discount rate, let us assume that agents, in the presence of certainty equivalence, would choose a declining consumption profile, i.e. assume that $\rho > r$. Structure your program in such a way that adjusting $(\rho, r)$ is easy.

### 1.3 Income Process

With respect to the income process we consider a discretized version of a simple AR(1) process. In particular, suppose log-income follows a process

$$\log(y_{t+1}) = \delta \log(y_t) + (1-\delta^2)^{\frac{1}{2}} \varepsilon_t$$

where the persistence parameter $\delta \in [0,1)$ and $\varepsilon_t$ is normally distributed with zero mean and variance $\sigma_y^2$. Note that for this process

$$\begin{aligned} Var(\log(y_t)) &= \sigma_y^2 && (1) \\ Corr(\log(y_{t+1}), \log(y_t)) &= \frac{Cov(\log(y_{t+1}), \log(y_t))}{\sqrt{Var(\log(y_{t+1})) * Var(\log(y_t))}} = \delta. && (2) \end{aligned}$$

Discretize this process into an $N$-state Markov chain using Tauchen's procedure (if you are familiar with it, you could also try Rouwenhorst's method, which gives better results for highly persistent processes). Ideally choose $N \geq 11$. Note that once you have a Markov chain for $\log(y)$, it is straightforward to exponentiate the states to obtain a Markov chain for income levels $y$. Normalize the income levels such that expected income (and thus average income in an economy with many agents) is equal to 1.

If you can't manage to deal with the Tauchen (or Rouwenhorst) procedure, choose $N = 2$, and let income levels and the transition matrix be given by $Y = \{1-\sigma_y, 1+\sigma_y\}$ and

$$\pi = \begin{pmatrix} \frac{1+\delta}{2} & \frac{1-\delta}{2} \\ \frac{1-\delta}{2} & \frac{1+\delta}{2} \end{pmatrix}$$

Parameter values for $\delta, \sigma_y$ will be given below.

### 1.4 Borrowing Constraints

We consider a tight borrowing constraint $a_{t+1} \geq 0$.

### 1.5 Initial Conditions

The initial asset position of the household is $a_0 = 0$ and the initial income realization is $y_0 = y_l$, where $y_l$ is the lowest realization the Markov chain can take.

## 2 Exercises

1. Formulate the problem of the agent recursively, i.e. write down Bellman's equation and derive the stochastic Euler equation.

2. For $T = \infty$ write a computer program that computes the value function $v(a, y)$ and the policy functions $a'(a, y)$ and $c(a, y)$ for given choice of the utility function as well as given parameterization of the income process. Also include in your program a subroutine that simulates paths of consumption and asset holdings for the first 61 periods of an agent's life. Structure your program in such a way that it easy to generate $M$ simulated paths of 61 periods, where $M$ might be a reasonably large number (that is, we might want to simulate 1000 or so households, for 61 time periods each).

3. Repeat the same exercise for $T = 60$, (i.e. an economy with finitely lived agents) where now we aim for a sequence of functions $\{v_t(a, y), a'_t(a, y), c_t(a, y)\}_{t=0}^{60}$. Remember that here you can iterate backwards from $v_{T+1}(a, y) = 0$ for all $(a, y) \in A \times Y$. Note that model age 0 should be interpreted as real age 20, so that people live from age 20 to 80 in real time.

4. Let $T = \infty$, $\sigma = 1$ and $\delta = 0.8$, $\sigma_y \in \{0.2, 0.4\}, \rho = 4\%$ and $r = 2\%$. Plot the consumption function. Interpret the difference between small and large income shocks.

5. Repeat question 4., but for finite horizon $T = 60$. Contrast the consumption function for young households ($t = 0$), with the consumption function for old households (say $t = 55$)

6. Do you get a hump in life cycle consumption when you simulate the model with $T = 60$ (to answer this question for the model take the average of a fairly large ($\geq 1000$) number of simulated time paths for $T = 60$)? Experiment with potential ways to cure your failure or to explain your success (in particular, look at your income process, the borrowing constraint and the relation between $\rho$ and $r$).

7. For your favorite parameterization of 6., now assume that the deterministic part $\bar{y}_t$ of the income process for the first 45 years is defined by the first column of the file incprofile.txt. Thus the income process of the household is now given by

$$y_t = \bar{y}_t \tilde{y}_t$$

where $\log(\tilde{y}_t)$ follows the AR(1) process defined above. For years 46-61 assume that the household receives social security benefits equal to $\theta$ times average income at age 45, that is

$$\bar{y}_t = \theta \bar{y}_{45}$$

with $\theta = 0.7$. In addition, introduce mortality risk into the model with $T = 60$. The conditional probability $\psi_j$ of surviving from age $j$ to $j + 1$ is given by the $j$-th row of the file survs.txt. Therefore the effective time discount factor of a household of age $j$ for $j + 1$ is given by $\psi_j \beta$. Also assume that there are no annuity markets (that is, the household cannot insure against this mortality risk). However, social security pays benefits as long as the household survives. Repeat question 6. and compare your results.

8. The file consprofile.txt contains an empirical life cycle profile of non-durable consumption (deflated by family size) that Fernandez-Villaverde and Krueger (2007) estimated from CEX consumption data. The data are for ages 22 to 88, in quarter year increments, and the first observation is normalized to 1. The model in part 7 delivers a mean consumption profile (the average across a large number of simulations), for age 20 to 80, in yearly increments. Compare the empirical and the model-generated consumption profile. What could you do to improve the fit between the model and the data?

9. In this part we assess the degree of consumption insurance. One popular measure of consumption insurance due to Blundell, Pistaferri and Preston (2008) is consumption insurance coefficient:

$$\phi = 1 - \frac{Cov(\Delta \log(c_{it}), \Delta \log(y_{it}))}{Var(\Delta \log(y_{it}))} \tag{3}$$

where the covariance and the variance in the formula are cross-sectional moments. Use your simulated data for either the infinite or finite horizon economy to calculate this coefficient, and document how it varies with the persistence of the income process $\delta$. That is, compute the coefficient in an economy with $\delta = 0$ and $\delta = .99$. If you do this for the life cycle economy, the insurance coefficients will vary quite strongly with age (see Kaplan and Violante, 2010). Thus compute the consumption insurance coefficient for each age separately (by using simulated data for that specific age only).

# Part II
# General Equilibrium

## 3 Computing Stationary Equilibria in the Aiyagari Model

1. Incorporate your programs from before for $T = \infty$ into general equilibrium to compute equilibria for the Aiyagari (1994 QJE) economy. In particular, the algorithm goes like this

   (a) Guess an interest rate $r \in (-\delta, \rho)$

   (b) Use the first order conditions for the firm to determine $K(r)$ and $w(r)$

(c) Solve the household problem for given $r$ and $w(r)$. Here you will use your programs from project 1

(d) Use the optimal decision rule $a'(a, y)$ together with the exogenous Markov chain $\pi$ to find an invariant distribution $\Phi$ associated with $a'(a, y)$ and $\pi$. For this you better first check that $a'(a, y)$ intersects the 45-degree line for $a$ large enough. Note that if you have discretized the state space for assets, finding $\Phi_r$ amounts to finding the eigenvector (normalized to length one) associated with the largest eigenvalue of the transition matrix $Q$ generated by $a'(a, y)$ and $\pi$

(e) Compute

$$Ea(r) = \int a'(a, y) d\Phi_r$$

Again, if you have discretized the state space, the integral really is a sum

(f) Compute

$$d(r) = K(r) - Ea(r)$$

If $d(r) = 0$ you have found a stationary recursive equilibrium, if not, update your guess for $r$ and start with $a$.

2. Use your programs to reproduce or correct *selected* results from Aiyagari's (1994) Table 2 (i.e. use some of his parameterizations, including his specification of the borrowing constraint). Note that you will not be able to reproduce the exact numbers of Aiyagari, but you should be able to obtain similar numerical comparative statics (how does $r$ change with persistence and variance of income shocks, and with prudence $\sigma$). For other parameter choices use Ayiagari's values.

3. Choose one parameterization of the model from the previous question. Now introduce three additional features into the model:

   (a) Now households can decide whether to work or not, $l_t \in \{0, 1\}$. In every period where they work ($l_t = 1$), they receive income according to the stochastic process from the previous questions, and their period utility is given by

   $$U(c_t) - \kappa.$$

   If they do not work they receive no labor income in that period and their utility function is as before, $U(c_t)$.

   (b) We introduce a Universal Basic Income (UBI) system: everyone gets a transfer equal to a fraction $\lambda$ of average labor productivity $\bar{y} = 1$,

   $$b = \lambda.$$

   This is an unconditional transfer that does not depend on whether a person works, and thus the total cost of the program is equal to $\lambda$, given that everyone receives it and the total mass of the population is 1. The parameter $\lambda$ measures the size (generosity) of the system.

(c) The UBI is financed by a proportional labor income tax with tax rate $\tau$. The tax is chosen to clear the government budget, such that

$$\lambda = \tau w \int yl(a,y)d\Phi$$

where $l(a,y) \in \{0,1\}$ is the labor supply of a household with assets $(a,y)$. As a consequence, total labor income of a household is

$$b + (1-\tau)wyl.$$

Now calibrate the economy without the UBI (that is, $\lambda = \tau = 0$) has 80% of the population working, that is, find $\kappa$ such that in the associated stationary equilibrium

$$\int l(a,y)d\Phi = 0.8$$

Then evaluate Andrew Young's proposal of a UBI: Compare the stationary equilibrium with $\lambda = \tau = 0$ to a stationary equilibrium where $\lambda = 0.2$ and the tax rate $\tau$ adjusts to clear the government budget. What happens to macroeconomic aggregates $(Y, K, C)$, to equilibrium prices $(w, r)$ and the equilibrium distributions for earnings $(1-\tau)wyl$, income $(1-\tau)wyl + ra$, assets $a$ and consumption $c$. For the distributions, you may want to calculate Gini coefficients or if possible, plot the Lorenz curves, under the two different specifications. Is UBI welfare-improving? To answer this question you may want to compare the value functions $v(a,y)$ under the two policies for some combinations of $(a,y)$, or aggregate (utilitarian) social welfare

$$\int v(a,y)d\Phi.$$

## Part III

# Optional 1: Computing Transition Paths in the Aiyagari Model: The Great Recession

1. Again choose $T = \infty$ and your preferred parameterization from the last question. Compute the transition path induced by an unexpected Great Recession. Specifically, in the economy without UBI, start with a stationary equilibrium in which aggregate TFP is equal to 1. Now suppose that for 10 years TFP falls by 10 % and then recovers back to 1. This event is perfectly unexpected, but once it occurs the path of TFP is perfectly foreseen. That is, in the production function

$$Y_t = A_t F(K_t, L_t)$$

the productivity term equals $A_t = 1$ for $t = 0$ and $t > 11$, and $A_t = 0.9$ for $1 \leq t \leq 10$. The change in TFP induces a transition path from the initial steady state to the final steady state (which is equal to the initial steady state) that should look like a recession. Note that you have already computed the initial (final) steady state, so you know where the transition starts and where it ends. Plot the time paths of aggregate TFP, output, capital and consumption (in percentage deviations from the initial steady state). How could you alter the model to generate more amplification of the TFP shocks?

2. Repeat the same question, but now in the economy with a UBI from above. Comment on the differences between the results.

# Part IV

# Optional 2: General Equilibrium in Life Cycle Economies

Repeat the exercises in the previous two sections, but for a version of the model in which agents live for $T = 61$ periods, face an income process and mortality risk as described in the previous project. They have no bequest motive. In each period there are 61 generations of different sizes (since some households will have died prior to age 61). The government collects assets of those that died with positive assets and redistributes them in a lump-sum fashion among the households alive. Newborn agents draw their income shock from the stationary distribution $\Pi$ and start with $a_0 = 0$ asset holdings. Note that the algorithm for computing the stationary equilibrium is identical to the one in 2. The only differences are that $\Phi(a, y, age)$ has an age-dimension now, the consumer problem is solved by backward iteration and the stationary distribution is solved forward:

$$\Phi(a, y, age = 0) = \begin{cases} \Pi(y) \text{ if } a = 0 \\ 0 \text{ otherwise} \end{cases}$$

and for $age > 0$ one determines $\Phi(a, y, age)$ from $\Phi(a, y, age-1)$ and $a'(a, y, age-1)$ and $\pi(y'|y)$.

# Part V

# Optional 3: The Great Recession with Aggregate Shocks

Repeat the Great Recession experiment, but in an economy where aggregate TFP $A_t$ is stochastic and follows a two state Markov process with states $A_t \in$

$\{ 0.9, 1\}$. and transition matrix with small probability of a great recession (say a probability of 5% of going from high to low TFP, and a probability of 95% of recovery to 1, conditional on $A = 0.9$). Note you can do this either in the infinite horizon or the life cycle economy.

**Remark 1** *If you have problems formulating the recursive problem of the household, the market clearing conditions or the aggregate law of motion in the finite horizon economy you may consult Conesa and Krueger, JME 2006 or the chapter by Victor Rios-Rull in the Marimon-Scott volume, who set up very similar problems.*

**Remark 2** *If you feel you are missing information (parameter values, details of the model), either email me or make assumptions yourself that appear plausible to you. Real research is all about making these types of assumptions and justifying them to others.*

**Remark 3** *If you want to look at papers studying UBI in quantitative models, look at the papers by Andre Victor Luduvice (2019) "The Macroeconomic Effects of Universal Basic Income Programs", or by Nezih Guner, Remzi Kaygusuz and Gustavo Ventura (2019) "Rethinking the Welfare State"*