Econ 712: Term Project

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November 8, 2019

Find the Codes for all questions at the end.

Problem 1. Formulate the problem of the agent recursively and derive the stochastic Euler equation.

Solution 1. Bellman equation:

$$v(a, y) = \max_{(c,a')\geq 0} u(c) + \beta \sum_{y' \in \mathcal{Y}} \pi\left(y'|y\right) v(a', y')$$
$$s.t. \quad c + a' = y + (1+r)a$$

where $\beta = \frac{1}{1+\rho}$ and y' follows an AR(1) process given by $\log(y') = \delta \log(y) + \sqrt{1-\delta^2}\varepsilon$ with $\varepsilon \sim N(0, \sigma_Y^2)$.

Plugging in the budget constraint yields the Bellman equation in one variable only.

$$v(a, y) = \max_{a' \geq 0} u(y + (1+r)a - a') + \beta \sum_{y' \in \mathcal{Y}} \pi\left(y'|y\right) v(a', y') \tag{1}$$

First order condition withr respect to savings a' and the Envelope condition yield

$$u_c(y + (1+r)a - a') = \beta \sum_{y' \in \mathcal{Y}} \pi\left(y'|y\right) v_1(a', y')$$
$$v_1(a, y) = u_c(y + (1+r)a - a')(1+r)$$

Combining the two equations at the optimal asset holding decision $a'(a, y)$ yields the Euler equation

$$u_c(y + (1+r)a - a'(a, y)) = \beta \sum_{y' \in \mathcal{Y}} \pi\left(y'|y\right) u_c\left(y' + (1+r)a'(a, y) - a'(a'(a, y), y')\right)(1+r) \tag{2}$$