Econ 712: Term Project

Leon Huetsch

December 3, 2019

1 Partial Equilibrium

1.1 Recursive Problem of the Agent

Bellman equation:

$$\begin{aligned} v(a,y) = \max_{(c,a')\geq 0} u(c) + \beta \sum_{y'\in\mathcal{Y}} \pi\left(y'|y\right) v(a',y') \\ s.t. \;\; c + a' = y + (1+r)a \end{aligned}$$

where $\beta = \frac{1}{1+\rho}$ and y' follows an AR(1) process given by $\log(y') = \delta \log(y) + \sqrt{1-\delta^2}\varepsilon$ with $\varepsilon \sim N(0, \sigma_Y^2)$.

Plugging in the budget constraint yields the Bellman equation in one variable only.

$$v(a,y) = \max_{a'\geq 0} u(y + (1+r)a - a') + \beta \sum_{y'\in\mathcal{Y}} \pi\left(y'|y\right) v(a',y') \tag{1}$$

First order condition withr respect to savings a' and the Envelope condition yield

$$\begin{aligned} u_c(y + (1+r)a - a') = \beta \sum_{y'\in\mathcal{Y}} \pi\left(y'|y\right) v_1(a',y') \\ v_1(a,y) = u_c(y + (1+r)a - a')(1+r) \end{aligned}$$

Combining the two equations at the optimal asset holding decision $a'(a,y)$ yields the stochastic Euler equation

$$u_c(y+(1+r)a-a'(a,y)) = \beta \sum_{y'\in\mathcal{Y}} \pi\left(y'|y\right) u_c\left(y' + (1+r)a'(a,y) - a'(a'(a,y),y')\right)(1+r) \tag{2}$$

1.2 Infinite Horizon VFI and Simulation

The files blabla.m and simulationbla.m in the folder Part1_PE/Functions/ contain the value function iteration and simulation for the infinite horizon case, respectively.

1.3 Finite Horizon VFI and Simulation

The files blabla.m and simulationbla.m in the folder Part1_PE/Functions/ contain the value function iteration and simulation for the finite horizon case, respectively.

1.4 Infinite Horizon Case

Figures 1 and 2 show the policy function for consumption in the infinite horizon case with low and high variance of the income process, respectively.

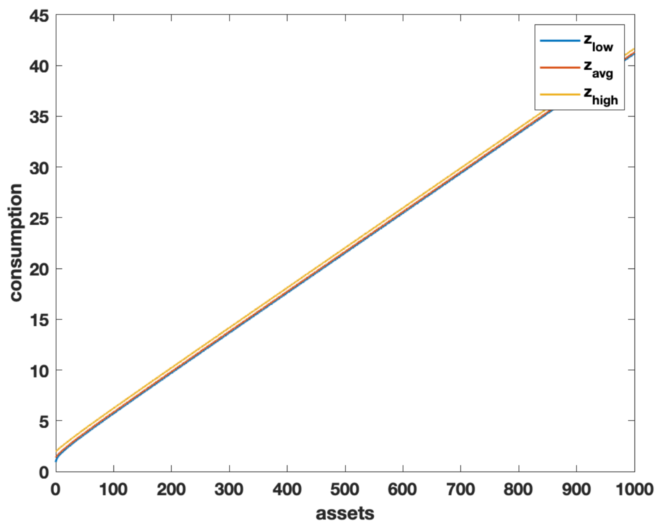

Figure 1: Consumption function with low variance $\sigma_Y = 0.2$

As expected consumption increases with higher income, shown in the plot by an upward shift of the policy function for higher realizations of z. However, the increase in consumption is very low.

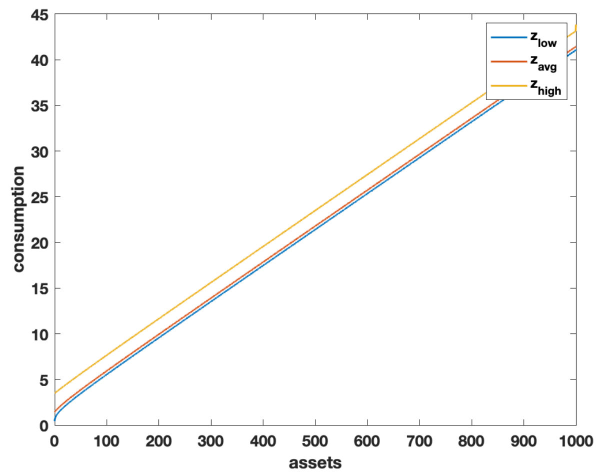

Figure 2: Consumption function with high variance $\sigma_Y = 0.4$

With high income volatility consumption also increases with more income. However, consumption increases more, meaning With low income, z_{low}, consumption is higher in the case of lower income volatility. The ratio of consumption between the high and low variance case is approximately 3 and constant across asset holdings and realization of the income shock. That is, households increase their consumption by the same factor independent of

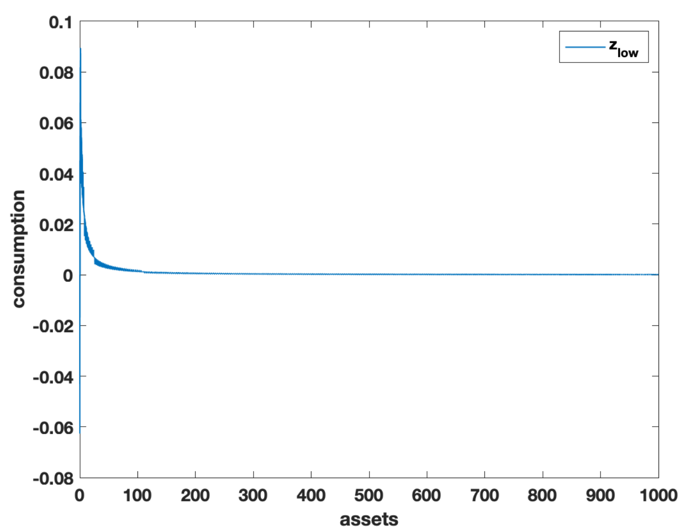

Figure 3: Consumption function with low variance $\sigma_Y = 0.2$

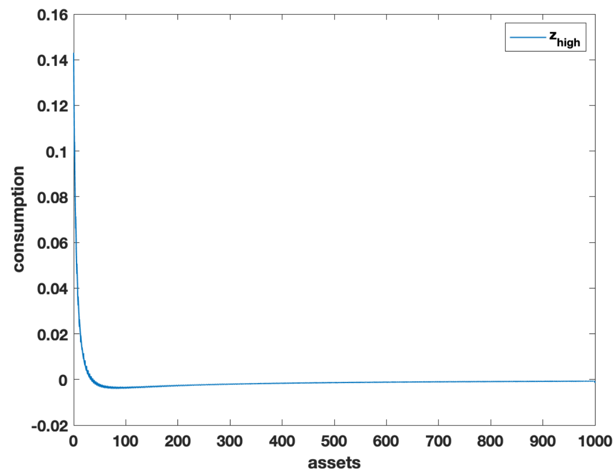

Figure 4: Consumption function with high variance $\sigma_Y = 0.4$

1.5 Finite Horizon Case

Figures 1 and 2 show the policy function for consumption in the finite horizon case with low and high variance of the income process, respectively.

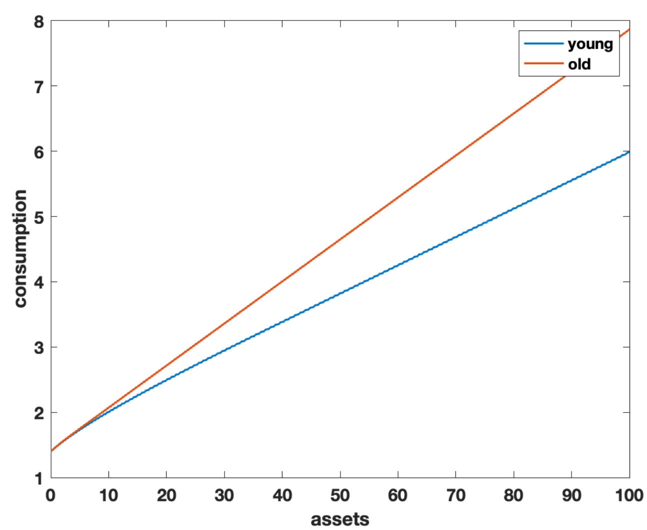

Figure 5: Consumption function with low variance $\sigma_Y = 0.2$ and average income

Young households consume less and save more with the same available resources, as they face a longer lifespan and, thus, more future consumption periods which increases their precautionary savings motive.

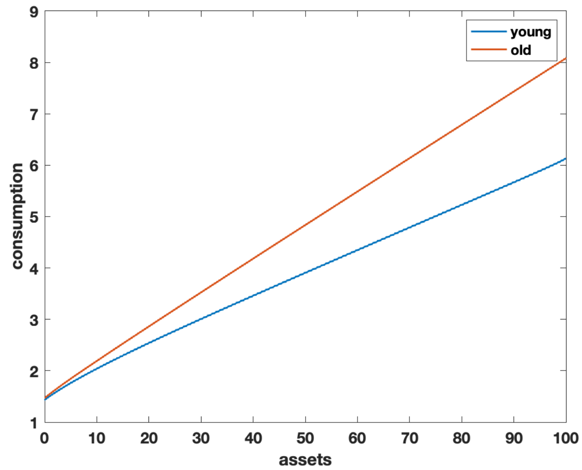

Figure 6: Consumption function with high variance $\sigma_Y = 0.4$ and average income

The same holds true with high income volatility. The two plots are very similar in levels. However medium income is higher

1.6 Hump in Life Cycle

Consumption in the previous section is basically flat over the life cycle. The discount rate, $\rho = 0.04$, is larger than the interest rate, $r = 0.02$, thus, agents in principle would like to have a declining consumption profile. However, they are not allowed to borrow as the borrowing constraint is set to 0.

Relaxing the borrowing constraint yields a declining consumption profile, not a hump, though. The first difficulty with implementation is to make sure agents cannot choose a Ponzi scheme and never pay back their debt. Since borrowing was permitted, this has not been an issue so far.