Firm-Specific Human Capital and Life-Cycle Wage Profiles

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1 Introduction

Firm-specific human capital (FSHC) plays a crucial role in shaping life-cycle wage profiles by influencing both worker mobility and employer wage-setting power. Under perfect competition for human capital, firms take wages as given and life-cycle wage profiles reflect the accumulation of productivity. In contrast, FSHC binds workers to their employers and gives rise to wage-setting power. As a result, the accumulation of FSHC creates a life-cycle profile of monopsony power, thereby altering the dynamics of wage contracts and career trajectories depending on how firms dynamically exercise that monopsony power. This paper develops a joint theory of life-cycle wage dynamics in the presence of FSHC and under different contractual regimes that integrates insights from human capital theory, internal labor market sorting, dynamic contract theory, and monopsony models. In doing so, it explains not only how firms set wages dynamically when FSHC is present, but also which types of workers select into FSHC-intensive careers and why career stability becomes a central concern.

A key contribution of this paper is the synthesis of several strands of the literature. Classical work by Becker (1964) laid the foundation by distinguishing between general and firm-specific human capital and highlighting the hold-up problem that arises when workers accumulate non-transferable skills. Building on this idea, subsequent models—such as those developed by

Acemoglu and Pischke (1999)—have shown that labor market frictions can lead to underinvestment in firm-specific training. At the same time, the literature on worker sorting and internal labor markets (e.g., Jovanovic (1979); Topel (1991)) has documented how workers are sorted into firms based on their accumulation of specialized skills, yielding steep wage-tenure profiles in many industries. In contrast to these studies, which typically interpret steep tenure coefficients as a signal of continuous skill accumulation and selection, the model proposed in this paper argues that optimal wage contracts in the presence of FSHC should, in fact, exhibit flatter wage trajectories. This occurs because firms front-load wages to pre-compensate workers for the future loss of mobility—an effect that is amplified in environments where downward wage adjustments are not feasible.

The paper further extends the analysis by examining worker selection into different job types, and in particular highlights risk aversion, rather than ability or skill, as the primary driver of worker sorting into FSHC-intensive careers. Unlike classical models that emphasize productivity-based selection (e.g., Jovanovic, 1979), the framework developed here shows that risk-averse workers trade lower average wages for reduced wage volatility and job separation risk, a trade-off uniquely enabled by the monopsony power arising from FSHC. This is consistent with empirical evidence from studies such as Bonin et al. (2007) which demonstrate that risk-averse individuals systematically prefer stable employment, even at the cost of lower earnings.

Under downward wage rigidity the ability of firms to pay workers below their marginal product can be interpreted as providing an option value for the worker. This option value allows firms to absorb adverse productivity shocks without forcing separations, thus preserving the match. Such a mechanism is impossible in a framework based solely on general human capital under perfect competition, where wages are strictly tied to the marginal product. Recent empirical evidence substantiates this view. For example, Matschke (2022) documents cross-country evidence that nominal wages are significantly sticky downward, while a survey of European firms by Babecky et al. (2009) finds that both nominal and real wages adjust only minimally in adverse conditions. Furthermore, detailed job-level analyses by Hazell and Taska (2020) reveal that wage decreases between successive job postings are rare, and Schmitt-Grohé and Uribe (2022) provide U.S. evidence supporting the presence of asymmetric wage adjustments, with wages rising readily in expansions but exhibiting substantial downward rigidity during downturns.

In the model developed here, I compare two contractual regimes: one in which firms cannot commit to future wages (no commitment) and one in which they can (commitment). A key finding is that front-loading of wages arises in both settings, but for fundamentally different reasons. Under no commitment, the high initial wage compensates for the anticipated future exercise of monopsony power, as the firm expects to pay workers below their marginal product once FSHC binds them. Under commitment, however, front-loading emerges as an optimal response to workers' preferences for smooth consumption over time. Crucially, the analysis demonstrates that without FSHC, front-loading would not occur—wage profiles in careers based solely on general human capital (GHC) would simply reflect the competitive market determination of wages.

The novelty of this work lies not only in synthesizing disparate strands of the literature but also in its emphasis on career stability. The insight that monopsony power—by allowing firms to pay wages below the marginal product—can be reinterpreted as an option value for workers is a key contribution. This option value reduces wage volatility and lowers separation risk, providing a stabilizing force in the labor market. In this way, FSHC and associated monopsony power enable workers to trade lower average wages for increased stability—a feature that has important implications for wage inequality and career trajectories.

The model generates several testable predictions. First, front-loaded wage structures should be more prevalent in FSHC-intensive industries, such as manufacturing and finance, compared to GHC-intensive sectors. Second, worker sorting into these industries should be influenced by risk aversion and patience, with more risk-averse or patient workers selecting into careers where FSHC is dominant. Third, the option value associated with downward wage rigidity implies that unionization and collective bargaining may emerge as additional mechanisms to safeguard workers' consumption in non-commitment settings where firms extract large wage markdowns in later periods.

By explicitly modeling the role of commitment, worker sorting, and the option value of monopsony power in wage setting, this paper contributes to the literature on human capital investment, labor market frictions, and wage dynamics. It advances our understanding of how FSHC alters wage-setting incentives and highlights the importance of career stability in modern labor markets.

The remainder of the paper proceeds as follows: Section 2 reviews related literature. Sections 3–5 develop the theoretical model. Section 7 outlines empirical strategies, and Section 8 concludes.

2 Literature Review

This paper contributes to the intersection of labor economics, contract theory, and monopsony models by examining how firm-specific human capital (FSHC) influences wage structures under different contractual regimes. In what follows, I review several strands of literature that lay the foundation for this work. I begin with classical perspectives on human capital, wage dynamics, and worker sorting, then discuss models of intertemporal wage contracts and commitment, and finally review recent advances in monopsony power. Throughout, I highlight empirical evidence on wage front-loading and tenure effects, with special emphasis on how FSHC and monopsony power give rise to a trade-off between lower average wages and enhanced career stability—a feature that risk-averse workers highly value.

2.1 Human Capital, Wage Dynamics, and Worker Sorting

Seminal work by Becker (1964) distinguishes between general and firm-specific human capital. According to Becker, firms underinvest in firm-specific skills because workers may later extract rents from their employer after accumulating non-transferable abilities. This hold-up problem laid the groundwork for later models addressing wage dynamics in the presence of FSHC. Building on Becker's insight, Acemoglu and Pischke (1999) demonstrate that labor market frictions can amplify underinvestment in firm-specific training by limiting workers' outside options, thereby encouraging firms to subsidize such investments.

Parallel to these developments, the literature on worker sorting and internal labor markets (e.g., Jovanovic (1979); Topel (1991)) documents how workers are sorted into firms based on their accumulation of specialized skills, which typically yields steep wage-tenure profiles. Traditionally, these studies interpret steep wage trajectories as evidence of continuous skill accumulation and selection—where only those workers who benefit most from FSHC remain with the firm. In contrast, this paper proposes that if firms optimally front-load wages to pre-compensate workers for the future loss of mobility induced by FSHC, the marginal wage growth with tenure is reduced. Moreover, risk-averse workers or those with a strong preference for career stability are more likely to self-select into FSHC-intensive occupations. Thus, while previous literature emphasizes rising wages with tenure, I argue that the optimal wage contract in the presence of FSHC can generate flatter wage profiles by trading lower average wages for enhanced career stability.

2.2 Wage Contracts, Commitment, and Empirical Evidence on Wage Front-Loading

A foundational contribution to the study of intertemporal wage contracts is provided by Harris and Holmström (1982). Their model of implicit contracts shows how firms smooth wages over time to insure workers against consumption volatility by front-loading wages as a risk-sharing mechanism. Complementary empirical work by Beaudry and DiNardo (1991) provides evidence for forward-looking wage setting, and Golosov, Kocherlakota, and Tsyvinski (2003) analyze optimal dynamic contracts under commitment. Although these models capture the intertemporal optimization of wages, they largely abstract from issues of worker retention and the accumulation of firm-specific human capital.

Empirical studies have consistently documented wage front-loading and steep tenure coefficients. Traditional evidence (e.g., Topel (1991)) suggests that in industries where FSHC is presumed to be important, wages rise sharply with tenure because workers accumulate non-transferable skills and are retained due to high switching costs. However, this paper posits a contrasting prediction: if firms optimally account for the future loss of mobility induced by FSHC, they will front-load wages—resulting in a substantial portion of compensation being paid early. This optimal design flattens the marginal wage increase over time. Moreover, recent empirical evidence substantiates the prevalence of downward wage rigidity. For example, Matschke (2022) documents cross-country evidence that nominal wages are notably sticky downward, and a survey of European firms by Babecky et al. (2009) finds that both nominal and real wages adjust only minimally in response to adverse conditions. Additionally, detailed job-level analyses by Hazell and Taska (2020) reveal that wage decreases between successive postings are rare, and Schmitt-Grohé and Uribe (2022) provide U.S. evidence of asymmetric wage adjustments, with downward movements being particularly constrained. Together, these findings support the notion that the trade-off inherent in optimal wage contracts—lower average wages in exchange for enhanced match stability—is a real and empirically relevant phenomenon.

2.3 Monopsony Power, Recent Advances, and the Option Value of Downward Wage Rigidity

The literature on monopsony power has evolved substantially over the past two decades. Early work by Manning (2003) formalized the idea that firms, due to limited worker options, systematically pay wages below workers' marginal products. Empirical studies, such as those by Webber (2015), find that wage markdowns are larger in more concentrated labor markets, underscoring the role of monopsony in wage suppression.

More recent contributions (e.g., Azar, Mollner, and Shepherd (2018);

Kaufman, Olson, and Rees (2020)) provide further evidence that wage suppression intensifies in markets with higher concentration. New theoretical developments have introduced heterogeneity in worker mobility and examined how the interplay between FSHC and monopsony power can amplify wage suppression. For instance, research by Bagenal et al. (2017) shows that when workers invest in non-transferable skills, their increased switching costs magnify the monopsonistic power of firms.

This paper extends these insights by explicitly modeling how FSHC influences intertemporal wage contracts under both commitment and no-commitment regimes. In the proposed framework, front-loading arises either as a means of intertemporal consumption smoothing (when commitment is feasible) or as compensation for future monopsony exploitation (when commitment is absent). Importantly, the paper introduces a novel interpretation: the ability of firms to pay wages below the marginal product—enabled by FSHC and monopsony power—can be seen as providing an option value for workers. This option value allows firms to absorb adverse productivity shocks without forcing separations, thereby preserving long-term matches. For risk-averse workers, this trade-off—lower average wages in exchange for reduced wage volatility and lower separation risk—is particularly attractive. This mechanism, which is absent in frameworks based solely on general human capital under perfect competition, represents a key novel contribution of the paper.

In summary, this literature review integrates classical human capital theories, insights from worker sorting and internal labor markets, models of intertemporal wage contracts, and recent advances in monopsony power. While each strand has contributed to our understanding of wage dynamics and tenure effects, the model presented in this paper uniquely positions FSHC as the unifying determinant that drives wage front-loading across both commitment and no-commitment regimes. This synthesis not only bridges existing gaps in the literature but also highlights a novel trade-off: firms can offer lower average wages in exchange for enhanced career stability—a benefit that risk-averse workers highly value. This trade-off, interpreted as an option value stemming from downward wage rigidity, generates novel, testable predictions for wage structure and worker mobility.

3 A Simple Two-Period Model

This section introduces a two-period model to analyze how firm-specific human capital (FSHC) influences wage-setting dynamics under different contractual regimes. The primary goal is to understand why front-loading of wages arises in both commitment and no-commitment settings, but for fundamentally different reasons. First, I analyze the no-commitment case, establishing it as a benchmark where dynamic monopsony power drives front-loading. I then extend the model to allow for commitment, demonstrating that front-loading persists but instead serves as an optimal consumption-smoothing mechanism. Importantly, we show that FSHC is crucial for front-loading in either regime, as it limits worker mobility and enables second-period underpayment.

The section is structured as follows. First, I introduce the model setup, specifying worker preferences, firm production, and the role of FSHC. Next, I solve for the optimal wage contract under no commitment, highlighting how monopsony power leads to front-loading. I then derive the optimal contract under commitment, demonstrating that front-loading occurs for different reasons. Finally, I summarize the key insights from both contractual settings.

3.1 Model Setup

I consider a worker who lives for two periods, $t = 1, 2$, and supplies labor to a firm. In period 1, the worker decides whether to enter a firm that offers firm-specific human capital (FSHC) accumulation or a firm where general human capital (GHC) accumulates. The key distinction is that FSHC can only utilized by the specific firm it corresponds to and is non-transferable, making outside options less valuable in period 2. GHC can be used be all firms and is fully transferable.

Worker Preferences Workers have time-separable preferences over consumption, represented by a constant relative risk aversion (CRRA) utility function:

$$U(c_1, c_2) = u(c_1) + \beta_W u(c_2), \quad u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}, \tag{1}$$

where β_W is the worker's discount factor and $\sigma > 0$ denotes the intertemporal elasticity of substitution (IES). The parameter σ governs the worker's willingness to substitute consumption across periods: a higher σ implies a lower IES, meaning the worker prefers smoother consumption and requires greater front-loading.

Human Capital. The worker enters the labor market with general human capital $h_{g,1}$ in period 1. She then accumulates either general or firm-specific human capital on the job, depending on which firm she entered in period 1, which determines productivity in period 2.

- **General Human Capital (GHC)**: In GHC firms, the worker accumulates GHC on the job at rate g: $h_{g,2} = h_{g,1}(1+g)$.
- **Firm-Specific Human Capital (FSHC)**: In FSHC firms, the worker accumulates FSHC on the job: $h_{f,2} = h_f$.

Firm Production and Outside Options. Firms produce with labor and operate a linear production in both types of human capital:

$$Y_t = h_{g,t} + h_{f,t}. \tag{2}$$

Since GHC can be used all firms, labor markets for GHC are perfectly competitive and wages in GHC firms are equal to their marginal product of labor given by a worker's GHC:

$$w_{\text{out},t} = h_{g,t}. \tag{3}$$

In contrast, there are no markets for FSHC as only one firm can use FSHC in production and, as a result, firms have monopsony power with respect to FSHC. The focus is on studying the wage profiles FSHC firms offer when operating in the context of a labor market for GHC which serves as outside option for workers.

The Contracting Problem. The firm offers a wage contract $\{w_1, w_2\}$ to maximize the discounted sum of profits from a match:

$$\max_{w_1,w_2} \Pi = (h_{g,1} - w_1) + \beta_F(h_{g,1} + h_f - w_2), \tag{4}$$

where β_F is the firm's discount factor, subject to:

- **Participation constraint (PC):** The worker must weakly prefer the FSHC contract over the GHC option in period 1:

$$u(w_1) + \beta_W u(w_2) \geq u(h_{g,1}) + \beta_W u(h_{g,2}). \tag{5}$$

- **Retention constraint (RC):** The firm must ensure that the worker does not leave in period 2:

$$w_2 \geq h_{g,1}, \tag{6}$$

  where w_2 is the worker's outside option in the labor market for their GHC in period 2.

3.2 Wage Contract Under No Commitment

If the firm cannot credibly commit to a period 2 wage when hiring workers in period 1, it sets wages period-by-period. Both parties anticipate that in period 2, the firm optimally exploits its monopsony power fully, paying the lowest wage necessary to retain the worker $w_2 = h_{g,1}$. Profit maximization requires (PC) to hold with equality, the firm then sets the lowest w_1 that satisfies (PC) which yields for the optimal wage contract without commitment:

$$\boxed{\begin{aligned} & w_1^{NC} = h_{g,1}\left[1 + \beta_W\left((1+g)^{1-\sigma} - 1\right)\right]^{\frac{1}{1-\sigma}}, \\ & w_2^{NC} = h_{g,1}. \end{aligned}} \tag{7}$$

Note, for $\sigma \to 0$, we get linear utility, $u(c) = c$, and the period 1 wage simplifies to:

$$w_1^{NC} = h_{g,1}\Big(1 + \beta_W\, g\Big). \tag{8}$$

A higher σ implies a lower IES, meaning the worker strongly prefers a stable wage path and requires greater front-loading to compensate for monopsony-driven suppression of w_2. Conversely, as $\sigma \to 0$ (high IES), the worker is more willing to tolerate wage fluctuations, reducing the need for front-loading.

3.3 Wage Contract Under Commitment

If the firm can credibly commit to a period 2 wage, it offers a binding wage contract $\{w_1, w_2\}$ to the worker in period 1. As before, profit maximization requires the PC to hold with equality. Setting up and solving the lagrangian yields for the optimal wage contract under commitment:

$$\boxed{\begin{aligned} & w_1^{C} = h_{g,1}\left[\frac{1 + \beta_W\ (1+g)^{1-\sigma}}{1 + \beta_W\left(\frac{\beta_W}{\beta_F}\right)^{\frac{1-\sigma}{\sigma}}}\right]^{\frac{1}{1-\sigma}}, \\ & w_2^{C} = w_1\left(\frac{\beta_W}{\beta_F}\right)^{\frac{1}{\sigma}}. \end{aligned}} \tag{9}$$

Intuition: Front-loading reflects efficient risk-sharing:

- Workers value stable consumption ($\sigma > 0$).
- Firms extract surplus via discounting ($\beta_F > \beta_W$).

3.4 Key Takeaways: Front-Loading in Both Regimes

The model yields the following insights:

- FSHC is crucial for front-loading—without it, wages would always match outside options, eliminating intertemporal wage adjustments.
- Front-loading occurs under both regimes but for different reasons:
  - Under no commitment, front-loading compensates for future monopsony power.
  - Under commitment, front-loading emerges from consumption smoothing.
- In GHC careers, front-loading never occurs, because firms cannot underpay in period 2 when outside options remain available.

These findings highlight that FSHC changes wage-setting incentives fundamentally, shaping both firm strategies and worker career choices.

4 Worker Heterogeneity and Career Sorting

Next steps:

- Need to check positive profits for firm above. When do profits become negative, what is the constraint here? Make point that 2. period monopsony power has to be viewed dynamically - important for policy. Generally good angle to introduce policy relevance here, should be in intro: Calls for dynamic view of MP -> specially when thinking about policy and possible regulation, static view is wrong then.
- Write more general case with GHC rate g1 and g2 in both firms to understand IES effect better, want rising wage path.
- Section 4 is incomplete and trivial. 1) incomplete: need ability to be unobserved by firm, then firm offers contract for expected ability.

Separating equilibrium? selection has to be consistent with firm expectation. 2) trivial. Perhaps existence of separating is interesting? Otherwise get rid of 4, include some of that heterogeneity perhaps in 3, and then go straight for shocks as main application/extension to career stability - actually that is more interesting anyway and better structured, do not want too many sections without clear motivation studying just different extensions.

This section extends the baseline model by introducing worker heterogeneity to analyze how individual characteristics influence career sorting between FSHC-intensive and GHC-intensive jobs. The central question is: which workers choose careers in firms that rely on FSHC, and which workers opt for GHC-intensive careers?

Workers differ along three key dimensions:

- Patience (Discount Factor, β_W): More patient workers place a higher value on future earnings and may be more willing to accept front-loaded wage structures.
- Ability (a): Higher-ability workers accumulate general human capital at a faster rate, making GHC careers more attractive.
- Intertemporal Elasticity of Substitution (IES, σ): A worker's willingness to shift consumption across time influences their preference for front-loaded wage structures.

This section is structured as follows. First, I formalize how heterogeneity in these characteristics affects the worker's decision between FSHC and GHC careers. I then derive the sorting equilibrium, followed by testable predictions.

4.1 Worker Decision-Making

Workers compare the lifetime utility of an FSHC career and a GHC career:

$$U^{\text{FSHC}} = u(w_{1,\text{FSHC}}) + \beta_W u(w_{2,\text{FSHC}}), \tag{10}$$

$$U^{\text{GHC}} = u(h_{g,1}) + \beta_W u(h_{g,2}). \tag{11}$$

A worker chooses an FSHC career if:

$$u(w_{1,\text{FSHC}}) + \beta_W u(w_{2,\text{FSHC}}) \geq u(h_{g,1}) + \beta_W u(h_{g,2}). \tag{12}$$

At the threshold a^*, the worker is indifferent between the two career paths, so I solve for the point where the utilities are equal. Using the CRRA utility function and solving for the sorting condition:

$$w_{1,\text{FSHC}}^{1-\sigma} + \beta_W w_{2,\text{FSHC}}^{1-\sigma} = h_{g,1}^{1-\sigma} + \beta_W h_{g,2}^{1-\sigma}. \tag{13}$$

For log utility ($\sigma = 1$), one obtains an explicit sorting threshold by taking the logarithm on both sides and rearranging:

$$a^* = \frac{1}{\alpha}\left(\ln w_{1,\text{FSHC}} - \ln h_{g,1} + \beta_W [\ln w_{2,\text{FSHC}} - \ln h_{g,1}]\right). \tag{14}$$

This threshold defines the boundary between workers who sort into FSHC careers ($a < a^*$) and those who choose GHC careers ($a \geq a^*$). The equation highlights that sorting depends on the difference in wage structures between the two career paths and the worker's time preference.

4.2 Testable Predictions

The model generates several predictions about career sorting:

- Impatient workers (low β_W) are more likely to enter FSHC careers due to the high initial wages.
- High-ability workers (high a) are more likely to choose GHC careers, since they accumulate general skills at a faster rate.

- Workers with low IES (high σ) prefer FSHC careers, as they strongly dislike earnings volatility and value front-loaded wages.

These predictions are empirically testable using longitudinal worker-firm datasets that measure ability, patience, and intertemporal preferences.

5 Productivity Shocks and Job Stability

This section extends the model by introducing match-specific productivity shocks in period 2. The goal is to analyze how these shocks affect job stability and to assess whether workers in FSHC-intensive careers experience different separation rates compared to those in GHC-intensive careers.

The key insight is that FSHC careers provide greater stability in response to shocks, as firms have an incentive to retain trained workers. In contrast, wages in GHC careers adjust fully to market conditions, leading to higher job separation rates when productivity declines.

5.1 Shock Structure

I introduce match-specific shocks that affect productivity in period 2:

- GHC Career: Productivity is subject to a shock $\epsilon \sim F(\epsilon)$, reducing productivity to $h_{g,2}(1-\epsilon)$.
- FSHC Career: Productivity is subject to a shock $\zeta \sim G(\zeta)$, reducing firm-specific human capital to $h_f(1-\zeta)$.

Wages in GHC careers adjust flexibly to productivity shocks:

$$w_{2,\text{GHC}} = h_{g,2}(1-\epsilon). \tag{15}$$

In contrast, wages in FSHC careers are fixed in period 1, making separation more likely when productivity declines. A worker separates if:

$$h_{g,1} + h_f(1-\zeta) < w_{2,\text{FSHC}} \quad \Rightarrow \quad \zeta > \zeta^*. \tag{16}$$

The threshold ζ^* defines the shock level at which separation occurs.

5.2 Risk Aversion and Job Stability

To better understand worker sorting under uncertainty, I introduce Epstein-Zin preferences, which separate risk aversion (γ) from the intertemporal elasticity of substitution (ψ):

$$U_{\text{FSHC}} = \left((w_1)^{1-\gamma} + \beta_W \left[G(\zeta^*)(w_2^*)^{1-\gamma} + (1 - G(\zeta^*))(h_{g,1})^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}. \tag{17}$$

Key result: Workers with high risk aversion (γ) sort into FSHC careers, as they value the stability provided by firm-specific human capital.

5.3 Implications for Job Stability

The model suggests the following testable implications:

- FSHC careers have lower job separation rates, as firms retain trained workers even after negative shocks.
- GHC careers exhibit higher turnover, since wages fully adjust to shocks, making layoffs more frequent.
- More risk-averse workers sort into FSHC careers, as they prefer stability over flexible wage adjustments.

These predictions can be tested using employer-employee matched datasets that track job separations across FSHC and GHC industries.

6 Empirical Strategy and Predictions

To validate the model's predictions, I propose an empirical strategy that examines sectoral differences in wage structures, job mobility, and unionization patterns. The key hypothesis is that FSHC-intensive sectors exhibit front-loading of wages, independent of unionization, due to their monopsony-like features.

6.1 Measuring FSHC Intensity

A critical challenge in empirical work on FSHC is the measurement of FSHC intensity across industries. Several proxies can be used:

- Training Investment: Sectors that invest heavily in firm-specific training programs (e.g., manufacturing, finance) exhibit greater FSHC intensity.
- Internal Promotion Rates: Firms with high rates of internal promotions rather than external hiring tend to rely more on FSHC.
- Turnover and Retention Rates: Higher retention and lower voluntary turnover suggest stronger FSHC dependence.
- Wage Growth Profiles: Sectors with FSHC intensity should show steeper early-career wage growth, consistent with front-loading predictions.

Using administrative datasets such as SOEP (Germany) or LEHD (U.S.), I can analyze sectoral variation in these proxies and relate them to wage structures.

6.2 Unionization and Bargaining Power

The second key prediction is that unionization arises as a response to monopsony power, independent of FSHC intensity. While FSHC leads to front-loading in all cases, unionization specifically emerges when workers perceive significant underpayment in period 2.

Empirical tests will examine:

- Unionization Rates vs. FSHC Intensity: If unionization is a response to monopsony, it should be strongest in sectors with high monopsony power but low FSHC intensity.
- Wage Growth in Unionized vs. Non-Unionized Sectors: If unionization counteracts monopsony, wage suppression in period 2 should be mitigated in unionized sectors.

- Unionization and Worker Sorting: More patient workers (higher β_W) should be more likely to sort into unionized FSHC careers, as unions effectively increase long-run earnings predictability.

By linking these empirical predictions to the model's core mechanisms, the paper provides a roadmap for testing the role of FSHC, commitment, and monopsony power in shaping labor market outcomes.

7 Empirical Outlook

7.1 Key Predictions

The model generates three testable predictions:

- **Prediction 1**: FSHC-intensive sectors exhibit steeper front-loaded wage profiles.
- **Prediction 2**: Low-ability, impatient, and risk-averse workers disproportionately sort into FSHC careers.
- **Prediction 3**: FSHC careers have lower turnover rates, especially after productivity shocks.

7.2 Empirical Tests

Test 1: Wage Profiles and FSHC Intensity

- **Regression Model**:

  $$\text{Wage Growth}_{it} = \alpha + \beta \cdot \text{FSHC Intensity}_j + \gamma \cdot X_{it} + \epsilon_{it},$$

  where i = worker, t = time, j = sector.

- **Variables**:
  - Dependent: Wage Growth_{it} (log wage difference between early and late career).

- Independent: FSHC Intensity_j (e.g., sector-specific training investment from OECD data).
- Controls (X_{it}): Tenure, education, firm size.

- **Expected Result**: $\beta < 0$ (steeper front-loading in FSHC sectors).

Test 2: Worker Sorting

- **Regression Model**:

$$\Pr(\text{FSHC Career}_i = 1) = \Phi(\alpha + \beta_1 \cdot \text{Impatience}_i + \beta_2 \cdot \text{Ability}_i + \beta_3 \cdot \text{Risk Aversion}_i + \gamma \cdot X_i),$$

where Φ is the probit link function.

- **Variables**:
  - Dependent: FSHC Career_i (1 if worker is in FSHC sector, 0 otherwise).
  - Independents:
    * Impatience_i: Discount rate from time preference experiments.
    * Ability_i: Standardized test scores or educational attainment.
    * Risk Aversion_i: Survey-based risk tolerance measures.

- **Expected Result**: $\beta_1 > 0$, $\beta_2 < 0$, $\beta_3 > 0$.

Test 3: Job Stability and Shocks

- **Regression Model**:

$$\text{Separation}_{ijt} = \alpha + \beta \cdot \text{FSHC}_j \cdot \text{Shock}_{jt} + \gamma \cdot X_{ijt} + \epsilon_{ijt}.$$

- **Variables**:
  - Dependent: Separation_{ijt} (1 if worker i leaves firm j in year t).
  - Independent: Interaction of FSHC_j (sector dummy) and Shock_{jt} (e.g., sectoral productivity decline).

- **Expected Result**: $\beta < 0$ (FSHC reduces separation risk post-shock).

7.3 Data Requirements

1. German Socio-Economic Panel (SOEP) Contains longitudinal data on wages, job transitions, and worker characteristics (e.g., risk preferences, education). Enables tests of wage profiles (Test 1) and sorting mechanisms (Test 2) through linked career histories.

2. Longitudinal Employer-Household Dynamics (LEHD) Provides matched employer-employee records with tenure, wage growth, and firm identifiers. Critical for measuring FSHC intensity (e.g., firm-specific training duration) and testing stability (Test 3).

3. OECD Sectoral Productivity Data Includes sector-level productivity shocks and R&D expenditure. Allows identification of exogenous shocks for Test 3.

4. Survey of Consumer Finances (SCF) Measures individual risk aversion and time preferences via hypothetical gambles. Key for operationalizing Risk Aversion_i and Impatience_i in Test 2.

8 Conclusion

This paper demonstrates that firm-specific human capital fundamentally shapes labor market outcomes through three channels:

- **Wage Profiles**: Front-loading arises under both commitment and no-commitment.
- **Worker Sorting**: Heterogeneity in patience, ability, and risk aversion drives career choices.
- **Job Stability**: FSHC careers offer implicit insurance against shocks.

Future research could test these predictions using linked employer-employee data and explore policy implications for workforce development and labor market regulation.

A Model Setup (Section 3.1)

Time Horizon: Two periods, $t = 1, 2$.

Worker Preferences: The worker's utility function is

$$U = u(w_1) + \beta_W\, u(w_2), \quad \text{with } u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad \sigma > 0, \tag{18}$$

where β_W is the worker's discount factor.

Human Capital:

- *General Human Capital (GHC):* In period 1, productivity is $h_{g,1}$. In period 2, it grows at rate g:

$$h_{g,2} = h_{g,1}(1+g).$$

- *Firm-Specific Human Capital (FSHC):* In period 1, assume no firm-specific capital (normalized to 0). In period 2, the worker accumulates an increment h_f that is only valuable at the current firm.

Firm Production: In each period, output is given by

$$y_t = h_{g,t} + h_{f,t}, \tag{19}$$

with $h_{f,1} = 0$ and $h_{f,2} = h_f$.

Outside Options: In GHC careers, wages equal marginal product:

$$w_{out,t} = h_{g,t}.$$

Constraints in FSHC Careers: Because FSHC is non-transferable, the worker's outside option in period 2 is $h_{g,1}$ (i.e. the productivity in period 1). Thus, the firm must offer a wage high enough to ensure retention.

Key Constraints:

1. **Retention Constraint:** The worker must prefer staying at the firm in period 2:

$$w_2 \geq h_{g,1}.$$

In our derivations I take this as binding.

2. **Participation Constraint (PC):** The worker's lifetime utility from accepting the FSHC contract must equal his reservation utility, taken from the GHC career:

$$u(w_1) + \beta_W \, u(w_2) = u(h_{g,1}) + \beta_W \, u(h_{g,2}). \tag{20}$$

B Wage Contracts in the Simple Model With and Without Commitment

B.1 Wage Contract Under No Commitment

In the no-commitment regime the firm sets wages sequentially and in period 2 it exploits its monopsony power. Thus, the firm chooses:

$$w_2^{NC} = h_{g,1}.$$

The firm's profit is

$$\begin{aligned}
\Pi^{NC} &= \underbrace{(h_{g,1} - w_1)}_{\text{Period 1}} + \beta_F \underbrace{(h_{g,1} + h_f - w_2^{NC})}_{\text{Period 2}} \\
&= (h_{g,1} - w_1) + \beta_F \, (h_{g,1} + h_f - h_{g,1}) \\
&= (h_{g,1} - w_1) + \beta_F \, h_f, \qquad (21)
\end{aligned}$$

where β_F is the firm's discount factor.

Since the firm's period 2 wage is fixed by the retention constraint, it minimizes wage payments by choosing the lowest w_1 that satisfies the worker's participation constraint (20). In the no-commitment case, substitute $w_2^{NC} = h_{g,1}$ into (20):

$$u(w_1) + \beta_W \, u(h_{g,1}) = u(h_{g,1}) + \beta_W \, u(h_{g,2}). \tag{22}$$

Solve for $u(w_1)$:

$$u(w_1) = u(h_{g,1}) + \beta_W \Big[u(h_{g,2}) - u(h_{g,1}) \Big]. \tag{23}$$

Using the CRRA utility function,

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

I have

$$u(h_{g,1}) = \frac{h_{g,1}^{1-\sigma}}{1-\sigma} \quad \text{and} \quad u(h_{g,2}) = \frac{h_{g,2}^{1-\sigma}}{1-\sigma}.$$

Thus, (23) becomes

$$\frac{w_1^{1-\sigma}}{1-\sigma} = \frac{h_{g,1}^{1-\sigma}}{1-\sigma} + \beta_W \left[\frac{h_{g,2}^{1-\sigma} - h_{g,1}^{1-\sigma}}{1-\sigma} \right]. \tag{24}$$

Multiplying both sides by $1-\sigma$ yields

$$w_1^{1-\sigma} = h_{g,1}^{1-\sigma} + \beta_W \Big[h_{g,2}^{1-\sigma} - h_{g,1}^{1-\sigma} \Big]. \tag{25}$$

Taking the $(1/(1-\sigma))$th power, the optimal period 1 wage is

$$w_1^{NC} = \Big[h_{g,1}^{1-\sigma} + \beta_W \Big(h_{g,2}^{1-\sigma} - h_{g,1}^{1-\sigma} \Big) \Big]^{\frac{1}{1-\sigma}}. \tag{26}$$

Substitute $h_{g,2} = h_{g,1}(1+g)$ into (26):

$$w_1^{NC} = \Big[h_{g,1}^{1-\sigma} + \beta_W \Big((h_{g,1}(1+g))^{1-\sigma} - h_{g,1}^{1-\sigma} \Big) \Big]^{\frac{1}{1-\sigma}}.$$

Factor out $h_{g,1}^{1-\sigma}$:

$$w_1^{NC} = \Big[h_{g,1}^{1-\sigma} \left(1 + \beta_W \left[(1+g)^{1-\sigma} - 1 \right] \right) \Big]^{\frac{1}{1-\sigma}}.$$

Thus,

$$w_1^{NC} = h_{g,1} \left[1 + \beta_W \left((1+g)^{1-\sigma} - 1 \right) \right]^{\frac{1}{1-\sigma}}. \tag{27}$$

Special Case: Linear Utility ($\sigma = 0$)

For linear utility, $u(c) = c$. In this case, note that the CRRA utility form is not defined at $\sigma = 0$, but taking the limit as $\sigma \to 0$ (or directly using linear utility), equation (25) becomes:

$$w_1 = h_{g,1} + \beta_W \Big[h_{g,1}(1+g) - h_{g,1} \Big] = h_{g,1} + \beta_W\, h_{g,1}\, g.$$

Thus, I obtain:

$$w_1^{NC} = h_{g,1} \Big(1 + \beta_W\, g \Big). \tag{28}$$

B.2 Optimal Wage Contract Under Commitment (Section 3.3)

In this section, I derive the optimal wage contract when the firm can pre-commit to the wage pair (w_1, w_2) in period 1. The firm maximizes profit subject to the worker's participation constraint.

Firm's Problem

The firm's profit is

$$\Pi = (y_1 - w_1) + \beta_F\, (y_2 - w_2),$$

where I assume

$$y_1 = h_{g,1}, \quad y_2 = h_{g,1} + h_f.$$

The worker's utility is

$$U = u(w_1) + \beta_W\, u(w_2),$$

with the CRRA utility function

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad u'(c) = c^{-\sigma},$$

and the participation constraint (which must hold with equality) is

$$u(w_1) + \beta_W\, u(w_2) = u(h_{g,1}) + \beta_W\, u(h_{g,2}).$$

Lagrangian Formulation

The firm chooses (w_1, w_2) to maximize profit while satisfying the participation constraint. The Lagrangian is

$$\mathcal{L} = \Big[(y_1 - w_1) + \beta_F\,(y_2 - w_2)\Big] + \lambda\Big[u(w_1) + \beta_W\, u(w_2) - u(h_{g,1}) - \beta_W\, u(h_{g,2})\Big].$$

First-Order Conditions

Differentiate with respect to w_1:

$$\frac{\partial \mathcal{L}}{\partial w_1} = -1 + \lambda\, u'(w_1) = 0 \quad\Longrightarrow\quad \lambda = \frac{1}{u'(w_1)}.$$

Differentiate with respect to w_2:

$$\frac{\partial \mathcal{L}}{\partial w_2} = -\beta_F + \lambda\, \beta_W\, u'(w_2) = 0 \quad\Longrightarrow\quad \lambda = \frac{\beta_F}{\beta_W\, u'(w_2)}.$$

Equate the two expressions for λ:

$$\frac{1}{u'(w_1)} = \frac{\beta_F}{\beta_W\, u'(w_2)} \quad\Longrightarrow\quad \beta_W\, u'(w_2) = \beta_F\, u'(w_1).$$

Since $u'(c) = c^{-\sigma}$, this becomes

$$\beta_W\, w_2^{-\sigma} = \beta_F\, w_1^{-\sigma}.$$

Rearrange to obtain:

$$\left(\frac{w_2}{w_1}\right)^{-\sigma} = \frac{\beta_F}{\beta_W} \quad\Longrightarrow\quad \left(\frac{w_2}{w_1}\right)^{\sigma} = \frac{\beta_W}{\beta_F}.$$

Taking the $1/\sigma$ power, I get the relation:

$$\frac{w_2}{w_1} = \left(\frac{\beta_W}{\beta_F}\right)^{1/\sigma},$$

or equivalently,

$$w_2 = w_1 \left(\frac{\beta_W}{\beta_F}\right)^{1/\sigma}. \tag{1}$$

Substitute into the Participation Constraint

The participation constraint is

$$u(w_1) + \beta_W\, u(w_2) = u(h_{g,1}) + \beta_W\, u(h_{g,2}).$$

Using CRRA utility, this becomes

$$\frac{w_1^{1-\sigma}}{1-\sigma} + \beta_W\, \frac{w_2^{1-\sigma}}{1-\sigma} = \frac{h_{g,1}^{1-\sigma}}{1-\sigma} + \beta_W\, \frac{h_{g,2}^{1-\sigma}}{1-\sigma}.$$

Multiplying through by $(1-\sigma)$ yields

$$w_1^{1-\sigma} + \beta_W\, w_2^{1-\sigma} = h_{g,1}^{1-\sigma} + \beta_W\, h_{g,2}^{1-\sigma}.$$

Substitute equation (1) into the above:

$$w_1^{1-\sigma} + \beta_W \left[w_1 \left(\frac{\beta_W}{\beta_F}\right)^{1/\sigma}\right]^{1-\sigma} = h_{g,1}^{1-\sigma} + \beta_W\, h_{g,2}^{1-\sigma}.$$

Simplify the second term:

$$w_1^{1-\sigma} + \beta_W\, w_1^{1-\sigma} \left(\frac{\beta_W}{\beta_F}\right)^{\frac{1-\sigma}{\sigma}} = h_{g,1}^{1-\sigma} + \beta_W\, h_{g,2}^{1-\sigma}.$$

Factor out $w_1^{1-\sigma}$:

$$w_1^{1-\sigma} \left[1 + \beta_W \left(\frac{\beta_W}{\beta_F}\right)^{\frac{1-\sigma}{\sigma}}\right] = h_{g,1}^{1-\sigma} + \beta_W\, h_{g,2}^{1-\sigma}.$$

Solve for $w_1^{1-\sigma}$:

$$w_1^{1-\sigma} = \frac{h_{g,1}^{1-\sigma} + \beta_W\, h_{g,2}^{1-\sigma}}{1 + \beta_W \left(\frac{\beta_W}{\beta_F}\right)^{\frac{1-\sigma}{\sigma}}}. \tag{2}$$

Taking the $\frac{1}{1-\sigma}$ power, I have

$$w_1 = \left[\frac{h_{g,1}^{1-\sigma} + \beta_W\, h_{g,2}^{1-\sigma}}{1 + \beta_W \left(\frac{\beta_W}{\beta_F}\right)^{\frac{1-\sigma}{\sigma}}}\right]^{\frac{1}{1-\sigma}}.$$

Finally, using (1), the optimal period 2 wage is

$$w_2 = w_1 \left(\frac{\beta_W}{\beta_F}\right)^{1/\sigma}.$$

Final Result

The optimal wage contract under commitment is:

$$\boxed{\begin{aligned} w_1 &= \left[\frac{h_{g,1}^{1-\sigma} + \beta_W\, h_{g,2}^{1-\sigma}}{1 + \beta_W \left(\frac{\beta_W}{\beta_F}\right)^{\frac{1-\sigma}{\sigma}}}\right]^{\frac{1}{1-\sigma}}, \\ w_2 &= w_1 \left(\frac{\beta_W}{\beta_F}\right)^{\frac{1}{\sigma}}. \end{aligned}}$$

This contract maximizes the firm's profit under commitment, subject to the worker's participation constraint.

B.3 Concluding Remarks

In summary, I have derived the optimal wage contracts in a two-period model with firm-specific human capital. Under the no-commitment regime (Section B.1), the firm sets $w_2^{NC} = h_{g,1}$ and the period 1 wage is determined solely by the worker's participation constraint:

$$w_1^{NC} = \left[h_{g,1}^{1-\sigma} + \beta_W \Big([h_{g,1}(1+g)]^{1-\sigma} - h_{g,1}^{1-\sigma} \Big) \right]^{\frac{1}{1-\sigma}}.$$

Under commitment (Section B.2), the firm can credibly commit to a higher period 2 wage by sharing the surplus from FSHC:

$$w_2^C = h_{g,1} + \frac{\beta_F}{1+\beta_F}\, h_f,$$

with the corresponding w_1^C determined from the participation constraint as in (**??**). These derivations spell out each step and provide a self-contained reference for the model.

C Worker Heterogeneity and Career Sorting

This section extends the baseline model by introducing worker heterogeneity into the career decision. Workers choose between an FSHC career and a GHC career based on their lifetime utility. In particular, I assume that workers differ in ability, denoted by a, which affects the accumulation of general human capital in the GHC career. Specifically, I assume that for a worker with ability a, the period 2 general human capital is given by

$$h_{g,2} = h_{g,1}\big(1 + g(a)\big),$$

with $g'(a) > 0$. Higher-ability workers accumulate GHC at a faster rate, making the GHC career more attractive.

C.1 Worker Decision-Making

Let the lifetime utility from an FSHC career be

$$U_{\text{FSHC}} = u(w_{1,\text{FSHC}}) + \beta_W\, u(w_{2,\text{FSHC}}),$$

and from a GHC career be

$$U_{\text{GHC}} = u(h_{g,1}) + \beta_W \, u\Big(h_{g,1}\big(1 + g(a)\big)\Big).$$

A worker will choose the FSHC career if

$$U_{\text{FSHC}} \geq U_{\text{GHC}}.$$

Define the indifference threshold a^* by the equality

$$u(w_{1,\text{FSHC}}) + \beta_W \, u(w_{2,\text{FSHC}}) = u(h_{g,1}) + \beta_W \, u\Big(h_{g,1}\big(1 + g(a^*)\big)\Big). \tag{29}$$

C.2 Sorting Equilibrium: General CRRA Utility ($\sigma \neq 1$)

Using the CRRA utility function,

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

the indifference condition (29) becomes

$$\frac{w_{1,\text{FSHC}}^{1-\sigma}}{1-\sigma} + \beta_W \, \frac{w_{2,\text{FSHC}}^{1-\sigma}}{1-\sigma} = \frac{h_{g,1}^{1-\sigma}}{1-\sigma} + \beta_W \, \frac{\Big[h_{g,1}\big(1 + g(a^*)\big)\Big]^{1-\sigma}}{1-\sigma}.$$

Multiplying both sides by $1 - \sigma$ yields

$$w_{1,\text{FSHC}}^{1-\sigma} + \beta_W \, w_{2,\text{FSHC}}^{1-\sigma} = h_{g,1}^{1-\sigma} + \beta_W \, \Big[h_{g,1}\big(1 + g(a^*)\big)\Big]^{1-\sigma}. \tag{30}$$

Rearrange to isolate the term involving a^*:

$$\Big[h_{g,1}\big(1 + g(a^*)\big)\Big]^{1-\sigma} = \frac{w_{1,\text{FSHC}}^{1-\sigma} + \beta_W \, w_{2,\text{FSHC}}^{1-\sigma} - h_{g,1}^{1-\sigma}}{\beta_W}.$$

Taking the $1/(1-\sigma)$ power on both sides yields

$$h_{g,1}\big(1 + g(a^*)\big) = \left[\frac{w_{1,\text{FSHC}}^{1-\sigma} + \beta_W \, w_{2,\text{FSHC}}^{1-\sigma} - h_{g,1}^{1-\sigma}}{\beta_W}\right]^{\frac{1}{1-\sigma}}.$$

Dividing by $h_{g,1}$ gives

$$1 + g(a^*) = \frac{1}{h_{g,1}} \left[\frac{w_{1,\mathrm{FSHC}}^{1-\sigma} + \beta_W \, w_{2,\mathrm{FSHC}}^{1-\sigma} - h_{g,1}^{1-\sigma}}{\beta_W} \right]^{\frac{1}{1-\sigma}}. \tag{31}$$

If I assume a linear relationship for the growth function, say

$$g(a) = \alpha \, a,$$

with $\alpha > 0$, then the threshold ability a^* is given by

$$\alpha \, a^* = \frac{1}{h_{g,1}} \left[\frac{w_{1,\mathrm{FSHC}}^{1-\sigma} + \beta_W \, w_{2,\mathrm{FSHC}}^{1-\sigma} - h_{g,1}^{1-\sigma}}{\beta_W} \right]^{\frac{1}{1-\sigma}} - 1,$$

or equivalently,

$$a^* = \frac{1}{\alpha} \left\{ \frac{1}{h_{g,1}} \left[\frac{w_{1,\mathrm{FSHC}}^{1-\sigma} + \beta_W \, w_{2,\mathrm{FSHC}}^{1-\sigma} - h_{g,1}^{1-\sigma}}{\beta_W} \right]^{\frac{1}{1-\sigma}} - 1 \right\}. \tag{32}$$

C.3 Special Case: Log Utility ($\sigma = 1$)

For log utility, $u(c) = \ln(c)$, the indifference condition (29) becomes

$$\ln(w_{1,\mathrm{FSHC}}) + \beta_W \, \ln(w_{2,\mathrm{FSHC}}) = \ln(h_{g,1}) + \beta_W \, \ln\Big[h_{g,1}\big(1 + g(a^*)\big)\Big].$$

Rearrange the terms:

$$\ln(w_{1,\mathrm{FSHC}}) + \beta_W \, \ln(w_{2,\mathrm{FSHC}}) - (1 + \beta_W) \ln(h_{g,1}) = \beta_W \, \ln\big(1 + g(a^*)\big).$$

Solving for $\ln\big(1 + g(a^*)\big)$ yields

$$\ln\big(1 + g(a^*)\big) = \frac{1}{\beta_W} \left[\ln(w_{1,\mathrm{FSHC}}) + \beta_W \, \ln(w_{2,\mathrm{FSHC}}) - (1 + \beta_W) \ln(h_{g,1})\right].$$

Exponentiating both sides, I obtain

$$1+g(a^*) = \exp\left\{\frac{1}{\beta_W}\left[\ln(w_{1,\text{FSHC}}) + \beta_W\ \ln(w_{2,\text{FSHC}}) - (1+\beta_W)\ln(h_{g,1})\right]\right\}.$$

Assuming again a linear growth function $g(a) = \alpha\, a$, the threshold is

$$a^* = \frac{1}{\alpha}\left\{\exp\left[\frac{1}{\beta_W}\Big(\ln(w_{1,\text{FSHC}}) + \beta_W\ \ln(w_{2,\text{FSHC}}) - (1+\beta_W)\ln(h_{g,1})\Big)\right] - 1\right\}. \tag{33}$$

C.4 Summary

A worker will choose the FSHC career if their ability a is less than the threshold a^* given by (32) (or (33) for log utility). In other words, workers with lower ability (who have slower GHC accumulation) are more likely to choose an FSHC career, whereas higher-ability workers prefer the GHC career.

This derivation provides a complete step-by-step explanation of the sorting condition. Testable predictions can be derived from how a^* depends on the wage contract parameters $w_{1,\text{FSHC}}$ and $w_{2,\text{FSHC}}$, the baseline GHC $h_{g,1}$, the discount factor β_W, and the sensitivity parameter α.

D Productivity Shocks and Job Stability

This section extends the model by introducing match-specific productivity shocks in period 2. I analyze how such shocks affect job stability and derive the threshold for separation in an FSHC career. I then incorporate risk aversion to obtain the worker's expected lifetime utility under uncertainty.

D.1 Shock Structure

In the presence of productivity shocks, realized output in period 2 becomes stochastic.

GHC Career: In a GHC career, assume that period 2 general human capital is subject to a shock ϵ, so that the realized productivity is

$$h_{g,2}^{\text{real}} = h_{g,2}\,(1-\epsilon).$$

Since wages in GHC careers adjust fully, I have

$$w_{2,\text{GHC}} = h_{g,2}\,(1-\epsilon).$$

FSHC Career: In an FSHC career, the worker accumulates firm-specific human capital h_f in period 2. The potential (pre-shock) productivity is

$$y_2^{\text{FSHC}} = h_{g,1} + h_f.$$

However, suppose that due to a match-specific shock ζ, only a fraction $(1-\zeta)$ of h_f is realized. Then the effective productivity is

$$y_2^{\text{FSHC,real}} = h_{g,1} + h_f\,(1-\zeta).$$

In FSHC careers, the wage contract is fixed in period 1 at a predetermined level $w_{2,\text{FSHC}}$. For the worker to remain in the job, the realized productivity must cover this wage. That is, the worker stays if

$$y_2^{\text{FSHC,real}} \geq w_{2,\text{FSHC}},$$

and separates if

$$y_2^{\text{FSHC,real}} < w_{2,\text{FSHC}}.$$

Substitute the expression for $y_2^{\text{FSHC,real}}$:

$$h_{g,1} + h_f\,(1-\zeta) < w_{2,\text{FSHC}}.$$

Rearrange this inequality:

$$h_f\,(1-\zeta) < w_{2,\text{FSHC}} - h_{g,1},$$

$$1 - \zeta < \frac{w_{2,\mathrm{FSHC}} - h_{g,1}}{h_f},$$

$$\zeta > 1 - \frac{w_{2,\mathrm{FSHC}} - h_{g,1}}{h_f} \equiv \zeta^*.$$

Thus, the threshold shock value is

$$\boxed{\zeta^* = 1 - \frac{w_{2,\mathrm{FSHC}} - h_{g,1}}{h_f}.}$$

A worker in an FSHC career will separate if $\zeta > \zeta^*$.

D.2 Risk Aversion and Expected Utility

Assume that the worker has CRRA utility

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

and let β_W denote the worker's discount factor. In an FSHC career, the wage in period 1, w_1, is received with certainty. However, the period 2 outcome depends on the shock:

- With probability $G(\zeta^*)$, the shock is not severe (i.e. $\zeta \leq \zeta^*$) and the match continues, so the worker receives the contract wage $w_{2,\mathrm{FSHC}}$.
- With probability $1 - G(\zeta^*)$, the shock is severe (i.e. $\zeta > \zeta^*$) and the worker separates, reverting to the outside option $h_{g,1}$.

Thus, the expected utility from period 2 is

$$E\big[u(w_2)\big] = G(\zeta^*)\, u(w_{2,\mathrm{FSHC}}) + \Big[1 - G(\zeta^*)\Big]\, u(h_{g,1}).$$

Accordingly, the worker's lifetime expected utility in an FSHC career is

$$\boxed{U_{\mathrm{FSHC}} = u(w_1) + \beta_W \Big\{ G(\zeta^*)\, u(w_{2,\mathrm{FSHC}}) + \Big[1 - G(\zeta^*)\Big]\, u(h_{g,1}) \Big\}.}$$

D.3 Implications

This derivation shows that the probability of separation in an FSHC career depends on the threshold ζ^*, which in turn depends on the predetermined wage $w_{2,\mathrm{FSHC}}$, the baseline productivity $h_{g,1}$, and the magnitude of firm-specific human capital h_f. A higher $w_{2,\mathrm{FSHC}}$ (relative to $h_{g,1}$) or a lower h_f results in a higher threshold ζ^*, reducing the likelihood of separation. Conversely, if the contract wage is set too low, ζ^* is lower, increasing the risk of separation.

Furthermore, risk aversion (through the curvature of $u(\cdot)$) amplifies the effect of potential separation on the worker's overall welfare. This mechanism provides a basis for explaining observed differences in job stability between FSHC and GHC careers, and it has testable implications for how variations in wage-setting and shock distributions affect worker outcomes.