"Machine Learning and Computational Statistics"

5th Homework

Exercise 1:

Consider the Erlang distribution $p(x) = \theta^2 x \exp(-\theta x)u(x)$, (where $u(x)$=1(0), if x≥0 (<0)), whose mean equals to $2/\theta$.

(a) Given a set of N measurements $x_1,\ldots, x_N$, for the random variable x that follows the Erlang distribution, prove that the ML estimate of θ is
$\vartheta_{ML} = 2N / \Sigma_{i=1}^{N} x_i$

(b) For N=5 and x_1=2, x_2=2.2, x_3=2.7, x_4=2.4, x_5=2.6, estimate the mean of the random variable x.

Exercise 2:

Consider again the Erlang distribution $p(x)=\theta^2 x \exp(-\theta x)u(x)$, (where $u(x)$=1(0), if x≥0 (<0)). Given

- a set of N measurements $x_1,\ldots, x_N$, for the random variable x that follows the Erlang distribution, and
- the a priori probability for the parameter θ is a normal distribution, $N(\theta_0,\sigma_0^2)$ (where θ_0, σ_0^2 are known)

(a) Compute the MAP estimate of the parameter θ.

(b) How this estimate becomes for the case were (i) N→∞, (ii) σ_0^2>> and (c) σ_0^2 <<? Give a short justification.

Exercise 3 (python code + text):

Consider the attached image (it is an image taken by the Huble telescope).

(a) Read and depict the image. Let A be the MxN array corresponding to the image

(b) Produce 15 **noisy** versions of the image adding Gaussian noise with zero mean and variance 256. ***In reality, these versions may be different images of exactly the same part of the sky, taken at different times*** (to produce a noisy version of A: **(i)** create an MxN array, B, each one of its entries steming from a zero mean, unit variance normal distribution, **(ii)** multiply the array with $\sqrt{256}$., **(iii)** the noisy version C of A is produced as C=A+B).

(c) Average the noisy versions of the images and compare with the original one *A*. Report and justify your findings.

Hint: To read and show an image, use the python commands

```
A = mpimg.imread('image_name')
plt.imshow(A)
plt.show()
```

at the beginning insert the instruction:

```
import matplotlib.image as mpimg
```