"Machine Learning and Computational Statistics"

1st Homework

Exercise 1 (optional)**:**

(a) Define the parametric set of the **quadratic** functions $f_\vartheta: R \rightarrow R$ and give two instances of it.

(b) Define the parametric set of the **3rd degree polynomials** $f_\vartheta: R^2 \rightarrow R$ and give two instances of it.

(c) Define the parametric set of the **3rd degree polynomials** $f_\vartheta: R^3 \rightarrow R$ and give two instances of it.

Exercise 2:

Verify that for two l-dimensional column vectors $\boldsymbol{\theta}$ and $\boldsymbol{x}$ it holds: $(\boldsymbol{\theta}^{\mathrm{T}} \boldsymbol{x})\, \boldsymbol{x} = (\boldsymbol{x}\boldsymbol{x}^{\mathrm{T}})\, \boldsymbol{\theta}$.

Exercise 3:

Verify the following identities (see the slides for the definition of the involved quantities)

$$X^T X = \sum_{n=1}^{N} \boldsymbol{x}_n \boldsymbol{x}_n^T, \qquad X^T \boldsymbol{y} = \sum_{n=1}^{N} y_n \boldsymbol{x}_n$$

Exercise 4:

Consider the data set

X={ (x_i, y_i), $x_i \in R, y_i \in R$, i=1,...,5} = { (2, 2.01), (4, 4.01), (-2, -2.01), (-3, -3.01), (-1, -1.01) }.

(a) Draw the data points.
(b) Determine **manually** the best line $y=\theta_1 x + \theta_0$ that fits the data, with respect to the sum of error squares criterion, and write explicitly its equation.
(c) Draw the resulting line in the same plot with the data points.

Hint: Use the formulation described in slide 46 of the first lecture slides. That is, define the matrix X and the vector y for the present case and use the equation at the bottom of the slide to determine θ_1 and θ_0.

NOTE: Your answers should be brief.