Άσκηση 1

(a) $f(x) = b_0 + b_1 x_1^2 + b_2 x_1$ με $b_1 \neq 0$

$f_\theta : x \in R \to y \in R$

$x = [x_1] \quad x \in R$

$\theta = [b_0, b_1, b_2] \quad \theta \in R^3$

Instance 1: $f_{\theta_1}(x) = 2x_1^2 + 0{,}5x_1 + 1 \qquad \theta_1 = [1, 2, 0{,}5]$

Instance 2: $f_{\theta_2}(x) = 18x_1^2 \qquad \theta_2 = [0, 18, 0]$

(b) $f(x) = b_0 x_1^3 + b_1 x_2^3 + b_2 x_1^2 x_2 + b_3 x_2^2 x_1 + b_4 x_1 \cdot x_2 + b_5$

με $b_0, b_1 \neq 0$

$f_\theta : R^2 \to R, \quad \theta \in R^6$

$x = [x_1, x_2]$

$\theta = [b_0, b_1, b_2, b_3, b_4, b_5]$

Instance 1: $f_{\theta_1}(x) = x_1^3 + 2x_2^3 + 4x_1x_2$

Instance 2: $f_{\theta_2}(x) = 2x_1^3 + 3x_2^3 + 4x_1^2x_2 + x_2^2x_1 + 8x_1x_2 + 312$

(c) $f(x) = b_0 x_1^3 + b_1 x_2^3 + b_3 x_3^3 + b_4 x_1^2 x_2 x_3 + b_5 x_2^2 x_1 x_3 + b_6 x_3^2 x_1 x_2$
$+ b_7 x_1 x_2 x_3 + b_8$ με $b_0, b_1, b_2 \neq 0$

$x = [x_1, x_2, x_3]$

$\theta = [b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8]$

$f_\theta : R^3 \to R, \quad \theta \in R^9$

Instance 1: $f_{\theta_1}(x) = x_1^3 + x_2^3 + x_3^3 \qquad \theta_1 = [1, 1, 1, 0, 0, 0, 0, 0, 0]$

Instance 2: $f_{\theta_2}(x) = 2x_1^3 + 3x_2^3 + 0{,}5x_3^3 + 4x_1^2 x_2 x_3 + 8$

$\theta_2 = [2, 3, 0{,}5, 4, 0, 0, 0, 0, 8]$

Άσκηση 2

Verify that $(\Theta^T \cdot x) \cdot x = (x \cdot x^T) \cdot \Theta$

1ο μέλος

$$(\Theta^T \cdot x) \cdot x = \left([\Theta_1, \Theta_2, \dots, \Theta_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} =$$

$$= [\Theta_1 x_1 + \Theta_2 x_2 + \dots + \Theta_n x_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} =$$

$$= \left[\sum_{n=1}^{N} x_n \cdot \Theta_n \right] \cdot \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} =$$

$$= \begin{bmatrix} x_1 \cdot \sum_{n=1}^{N} x_n \Theta_n \\ x_2 \cdot \sum_{n=1} x_n \Theta_n \\ \vdots \\ x_n \sum_{n=1}^{N} x_n \Theta_n \end{bmatrix} \quad (1)$$

2ο μέλος

$$(x \cdot x^T) \cdot \Theta = \left(\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} [x_1, x_2 \dots x_n] \right) \begin{bmatrix} \Theta_1 \\ \Theta_2 \\ \vdots \\ \Theta_n \end{bmatrix} =$$

$$= \begin{bmatrix} x_1 x_1, & x_1 x_2, & \dots, & x_1 x_n \\ x_2 x_1, & x_2 x_2, & \dots, & x_2 x_n \\ \vdots & & \ddots & \\ x_n x_1, & x_n x_2, & \dots, & x_n x_n \end{bmatrix} \begin{bmatrix} \Theta_1 \\ \Theta_2 \\ \vdots \\ \Theta_n \end{bmatrix} =$$

$$= \begin{bmatrix} x_1 x_1 \theta_1 + x_1 x_2 \theta_2 + \dots + x_1 x_n \theta_n \\ x_2 x_1 \theta_1 + x_2 x_2 \theta_2 + \dots + x_2 x_n \theta_n \\ \vdots \qquad \ddots \\ x_n x_1 \theta_1 + x_n x_2 \theta_2 + \dots + x_n x_n \theta_n \end{bmatrix} =$$

$$= \begin{bmatrix} x_1 \sum_{n=1}^{N} x_i \theta_n \\ x_2 \sum_{n=1}^{N} x_n \theta_n \\ \vdots \\ x_n \sum_{n=1}^{N} x_n \theta_n \end{bmatrix} \quad (2)$$

Παρατηρούμε ότι (1)=(2), άρα η ισότητα <u>ισχύει</u>

<u>Άσκηση 3</u>

(a) $$X^T \cdot X = [x_1, x_2, \dots, x_n] \begin{bmatrix} x_1^T \\ x_2^T \\ \vdots \\ x_n^T \end{bmatrix} =$$

όπου τα x είναι διανύσματα

$$= [x_1 x_1^T + x_2 x_2^T + \dots + x_n x_n^T] = \sum_{n=1}^{N} x_n x_n^T$$

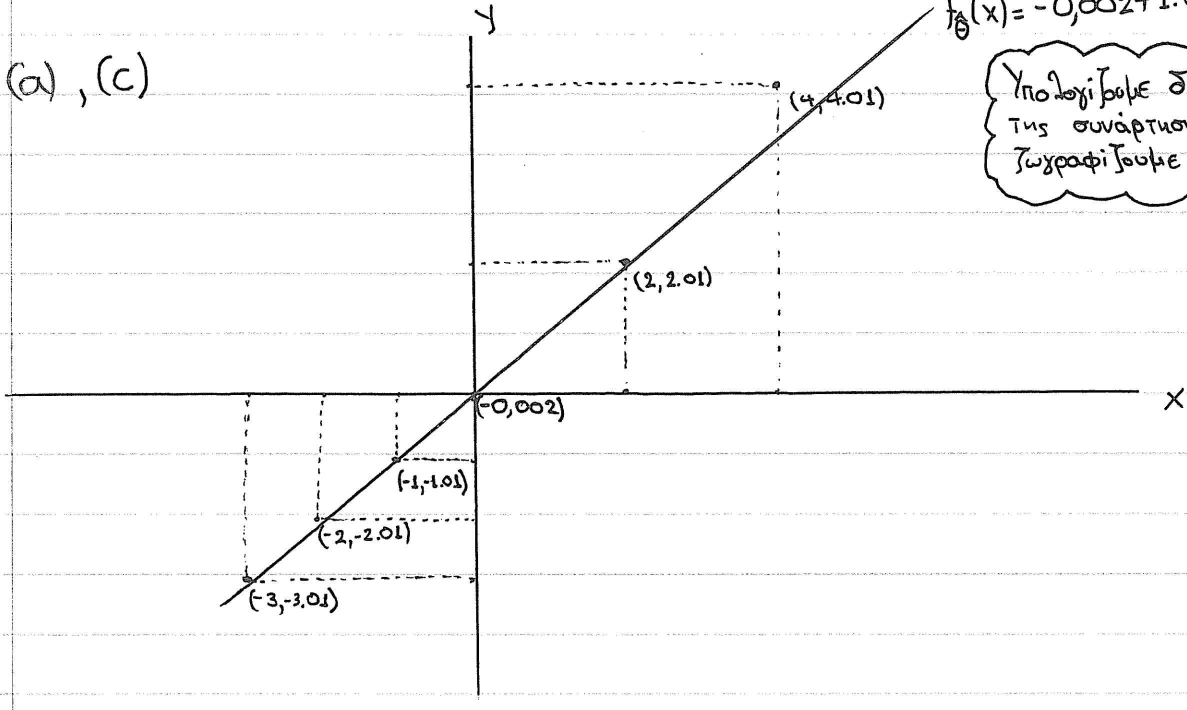
(b) $$X^T \cdot y = [x_1, x_2, \dots, x_n] \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = [x_1 y_1 + x_2 y_2 + \dots + x_n y_n]$$

$$= \sum_{n=1}^{N} y_n \cdot x_n$$

Άσκηση 4

(a), (c)

$f_{\hat\theta}(x) = -0{,}002 + 1.003x$



Υπολογίζουμε δύο σημεία της συνάρτησης και ζωγραφίζουμε την ευθεία

(b)

$$y = \begin{bmatrix} 2.01 \\ 4.01 \\ -2.01 \\ -3.01 \\ -1.01 \end{bmatrix} \qquad X = \begin{bmatrix} 1 & 2 \\ 1 & 4 \\ 1 & -2 \\ 1 & -3 \\ 1 & -1 \end{bmatrix} \qquad X^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 4 & -2 & -3 & -1 \end{bmatrix}$$

$$\hat\theta = (X^T X)^{-1} X^T \cdot y$$

$$(X^T \cdot X)^{-1} = \left(\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 4 & -2 & -3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 1 & 4 \\ 1 & -2 \\ 1 & -3 \\ 1 & -1 \end{bmatrix} \right)^{-1} = \begin{bmatrix} 5 & 0 \\ 0 & 34 \end{bmatrix}^{-1} = \frac{1}{5 \cdot 34 - 0} \begin{bmatrix} 5 & 0 \\ 0 & 34 \end{bmatrix}$$

$$= \begin{bmatrix} 1/5 & 0 \\ 0 & 1/34 \end{bmatrix}$$

$$(X^T X)^{-1} \cdot X^T \cdot y = \begin{bmatrix} 1/5 & 0 \\ 0 & 1/34 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 4 & -2 & -3 & -1 \end{bmatrix} \begin{bmatrix} 2.01 \\ 4.01 \\ -2.01 \\ -3.01 \\ -1.01 \end{bmatrix}$$

$$= \begin{bmatrix} 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 2/34 & 4/34 & -2/34 & -3/34 & -1/34 \end{bmatrix} \begin{bmatrix} 2.01 \\ 4.01 \\ -2.01 \\ -3.01 \\ -1.01 \end{bmatrix} = \begin{bmatrix} -0{,}002 \\ 1.003 \end{bmatrix} = \hat\theta$$