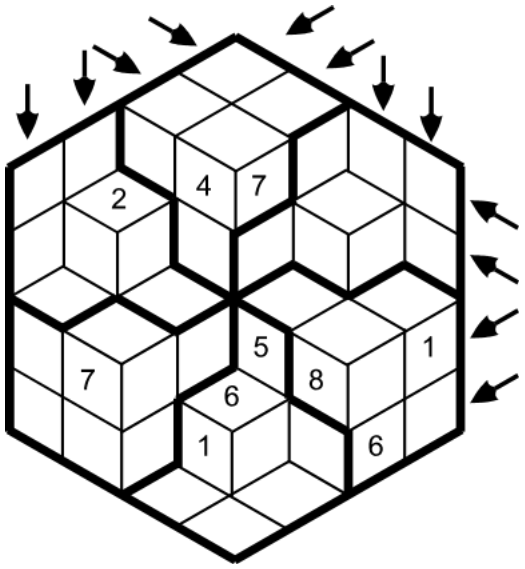
Recursive Backtracking

Backtracking can be thought of as a selective tree/graph traversal method. The tree is a way of representing some initial star position (the parent node) and a final goal state (one of the leaves). Backtracking allows us to deal with situations in which a brute-force approach would explode into an impossible number of choices to consider. Backtracking is a sort of refined brute At each node, we eliminate choices that are obviously not possible and proceed to recursively check only those that have po This way, at each depth of the tree, we mitigate the number of choices to consider in the future.



Suppose you get to a bad leaf. You can backtrack to continue the search for a good leaf by revoking your most recent choice trying out the next option in that set of options. If you run out of options, revoke the choice that got you here, and try another at that node. If you end up at the root with no options left, there are no good leaves to be found.

Backtracking is essential for solving constraint satisfaction problems, such as crosswords, verbal arithmetic, Sudoku, and ma puzzles. It is also used in solving the knapsack problem, parsing texts and other combinatorial optimization problems.

What's interesting about backtracking is that we back up only as far as needed to reach a previous decision point with an as- unexplored alternative. In general, that will be at the most recent decision point. Eventually, more and more of these decision will have been fully explored, and we will have to backtrack further and further. If we backtrack all the way to our initial state a explored all alternatives from there, we can conclude the particular problem is unsolvable. In such a case, we will have done work of the exhaustive recursion and known that there is no viable solution possible.

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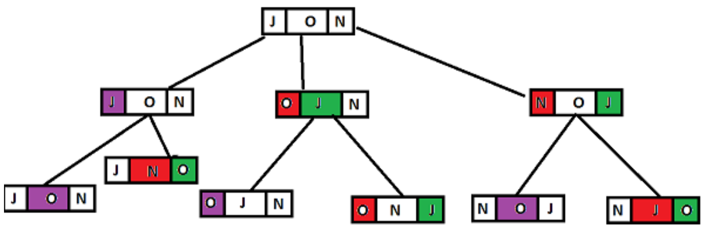
Permutations

A permutation of a given set of items is a certain rearrangement of the elements. It can be shown that an array A of length N
permutations. For example the array ['J','O','N'] has the following permutations:

```
['J', 'O', 'N']
['J', 'N', 'O']
['O', 'J', 'N']
['O', 'N', 'J']
['N', 'J', 'O']
['N', 'O', 'J']
```

The backtracking algorithm applied here is fairly straight forward because the calls are not subject to any constraint. We are
backtracking from an unwanted result, we are merely backtracking to return to a previous state without filtering out unwanted
This is elaborated a little bit more in the picture and code below:



diag

As shown in the diagram the algorithm is based on swapping. When implemented, the backtracking part is swapping back th
to their previous place after the permutation has been printed.

The following python code shows how this is done:

Python

```python
def permutation(list, start, end):
    '''This prints all the permutations of a given list
       it takes the list,the starting and ending indices as input'''
    if (start == end):
        print list
    else:
        for i in range(start, end + 1):
            list[start], list[i] = list[i], list[start]  # The swapping
            permutation(list, start + 1, end)
            list[start], list[i] = list[i], list[start]  # Backtracking


permutation([1, 2, 3], 0, 2)  # The first index of a list is zero
```

It outputs

```
>>>
[1, 2, 3]
[1, 3, 2]
[2, 1, 3]
[2, 3, 1]
[3, 2, 1]
[3, 1, 2]
```

TRY IT YOURSELF

Submit your answer

There are N integers with 77 digits such that the sum of any three consecutive digits within the integer is at most 7. Find N and input the last three digits as your answer.

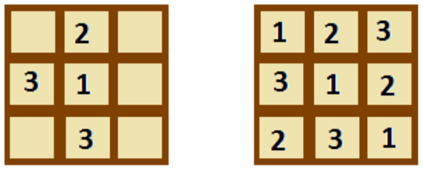
Image Credit: [brisbanepowerhouse.org]

Mini Sudoku

Sudoku is a logic puzzle in which the goal is to fill grid with digits so that each column, each row, and each of the sub-grids th compose the grid contains all of the digits from 1 to n.The same single integer may not appear twice in the same row , colum sub-grid.

Let us look at a simplified 3×3 mini version of the original Sudoku puzzle. Here, each cell is a subgrid containing 1 elemen trivial distinct. This means we only need to check if the rows and columns contain the integers 1,2 and 3 with no repetitions.

Below is an example of a mini Sudoku puzzle(left) and its solution (right)



	2	
3	1	
	3	

1	2	3
3	1	2
2	3	1

It should be obvious by now that this puzzle is ripe for recursive backtracking. Let us now lay out pseudocode that will help u: it.

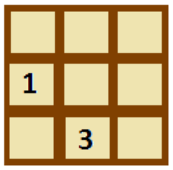
```
function solve( board )
    if the board contains no invalid cells, ie.cells that violate the rules:
        if it is also completely filled out then
            return true
    for each cell in the board
        if the cell is empty
            for each number in {1,2,3}
                replace the current cell with number
                if solve(board) and the board is valid
                    return true
                else
```

```
                    backtrack
        return false
```

The code above is a classic example of backtracking. The function returns true if a given board **can** be solved. It also solves given board so the scope of the variable `board` should be outside the function.

EXAMPLE

Implement an actual mini 3×3 solver and use it to print the solution\s to the puzzle below



Example solution in python

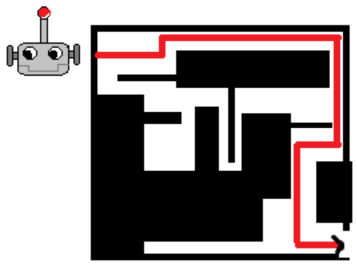
Python

```python
from itertools import *
from copy import copy



def is_distinct( list ):
    '''Auxiliary function to is_solved
    checks if all elements in a list are distinct
    (ignores 0s though)
    '''
    used = []
    for i in list:
        if i == 0:
            continue
        if i in used:
            return False
        used.append(i)
    return True


def is_valid( brd ):
    '''Checks if a 3x3 mini-Sudoku is valid.'''
    for i in range(3):
        row = [brd[i][0],brd[i][1],brd[i][2]]
        if not is_distinct(row):
            return False
        col = [brd[0][i],brd[1][i],brd[2][i]]
        if not is_distinct(col):
            return False
    return True

def solve( brd , empties = 9):
    '''
      Solves a mini-Sudoku
      brd is the board
      empty is the number of empty cells
    '''

    if empties == 0:
        #Base case
        return is_valid( brd )
    for row,col in product(range(3),repeat=2):
        #Run through every cell
        cell = brd[row][col]
        if cell != 0:
            #If its not empty jump
            continue
        brd2 = copy( brd )
        for test in [1,2,3]:
            brd2[row][col] = test
            if is_valid(brd2) and solve(brd2,empties-1):
                return True
            #BackTrack
            brd2[row][col] = 0
    return False

Board = [ [ 0 , 0 , 0 ],
          [ 1 , 0 , 0 ],
          [ 0 , 3 , 1 ] ]
solve( Board , 9 - 3 )


for row in Board:#Prints a solution
    print row
```

It outputs

```
>>>
[3, 1, 2]
[1, 2, 3]
[2, 3, 1]
```

Path Finding



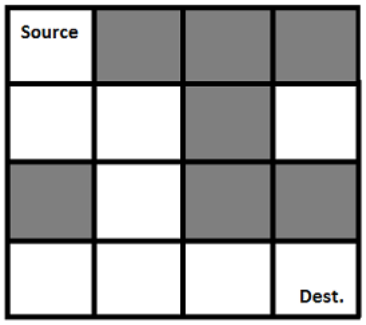
A more practical and well known example of backtracking is path finding. A robot can for example plan its path in a maze by recurring over the paths and backtracking from the ones that lead no where. This of course requires us to represent the maz way that the algorithm is compatible with. A common method is to use a $2-d$ matrix and values within it to represent obsta paths. Below is a simplified version of the maze solving problem that should help clarify the backtracking algorithm.

The Simplified Path Finding Problem

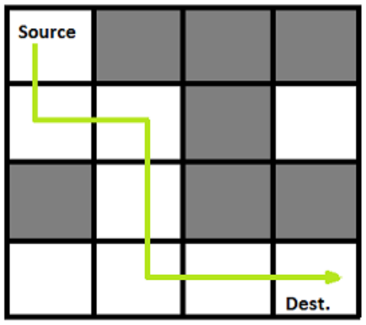
Given an $N \times N$ matrix of blocks with a source upper left block, we want to find a path from the source to the destination(lower right block). We can only move downwards and to the left. Also a path is given by 1 and a wall is given by 0.

The following is an example of of a maze(the black cells are inaccessible)

```
{ 1 , 0 , 0 , 0 }
{ 1 , 1 , 0 , 0 }
{ 0 , 1 , 0 , 0 }
{ 0 , 1 , 1 , 1 }
```



The solution is the follows:



We can now outline a backtracking algorithm that returns an array containing the path in a coordinate form . For example, fo
picture above, the solution is $(0,0) \rightarrow (1,0) \rightarrow (1,1) \rightarrow (2,1) \rightarrow (3,1) \rightarrow (3,2) \rightarrow (3,3)$

Python

```
If we have reached the destination point
        return an array containing only the position of the destination
else
       a,move in the forwards direction and check if this leads to a solution
       b,if  option a does not work, then move down
      c, if either work, add the current position to the solution obtained at either a or b
```

An implementation in python looks like the following

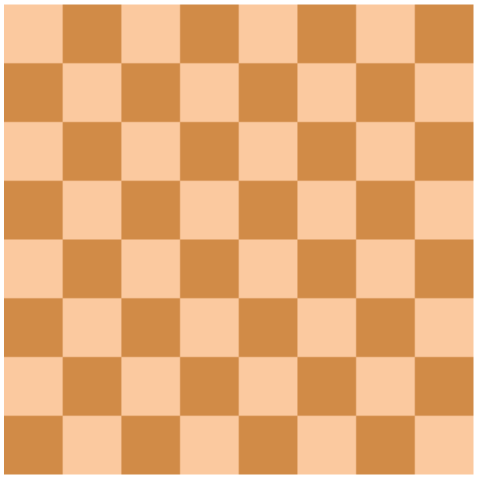
Python

```python
def solveMaze( Maze , position , N ):
    # returns a list of the paths taken
    if position == ( N - 1 , N - 1 ):
        return [ ( N - 1 , N - 1 ) ]
    x , y = position
    if x + 1 < N and Maze[x+1][y] == 1:
        a = solveMaze( Maze , ( x + 1 , y ) , N )
        if a != None:
            return [ (x , y ) ] + a

    if y + 1 < N and Maze[x][y+1] == 1:
        b = solveMaze( Maze , (x , y + 1) , N )
        if  b != None:
            return [ ( x , y ) ] + b


Maze = [[ 1 , 0 , 1, 0 , 0],
        [ 1 , 1 , 0, 1 , 0],
        [ 0 , 1 , 0, 1 , 0],
        [ 0 , 1 , 0, 0 , 0],
        [ 1 , 1 , 1, 1 , 1]
        ]
```

```

print solveMaze(Maze,(0,0),5)
```

8-Queens

Contrary to the permutations problem, here we will see an example of backtracking that involves checking a lot of constraints
does not sound good but having a large number of constraints actually allows us to significantly reduce the search space wh
are backtracking. This also means a substantial improvement in run time and performance.



An example of a solution

Image Credit: [wikipedia]

A very common example of backtracking in computer science is the problem of placing N queens on a checkers board in a
no two queens attack each other. A checker board consists of 8×8 cells. Queens can move vertically, horizontally and diag
The problem is computing the number of solutions, not enumerating each individual solution.

C++

```
function solve( board ){
     //Solves the 8queen problem
     if the number of queens on the board equals 8
        return true;
    for position in board.empty_spaces()
        board.placeQueen( position )
        if there are conflicts in the board
            board.removeQueen(position) //Backtrack
            continue
        else
            if solve(board)
                return true
            else
                return false;
    return false;
}
```

Because of the nature of chess, when covering the chess board we cut the search space whenever we find a square where
cannot put another queen given our configuration. A backtrack search is most effective here because it eliminates around 95
search space. The pseudo-code above shows the specifics of how this can be done.

Ofcourse when actually writing an implementation we worry about data structures and efficient means of actually representin
problem. The python code below shows an example of how an implementation of the backtracking search can be tackled.

EXAMPLE

8-Queen's problem

Python

```python
from itertools import *
import copy

class Board:
    ''' A class to represent the checker board'''
    def __init__(self , n): #Initializes the class
        self.board = [ [ None  for i in range(8) ] for i in range(8) ]
        self.pieces = set()

    def __str__(self):
        '''Allows us to print the board'''
        S = ''
        for i in self.board:
            S += str(i) + '\n'
        return S

    def PlaceQueen(self,row,column):
        '''Places a queen at row,column'''
        self.pieces.add( (row , column) )
        self.board[row][column] = 'Q'

    def RemoveQueen(self,row,column):
        '''Removes a queen from a given 'row' and 'column' '''
        self.board[row][column] = None
        self.pieces.remove( ( row , column ) )

    def isAttacking( self,  piece1 , piece2  ):
        '''Checks if piece1 attacks piece2'''
        if piece1[0] == piece2[0] or piece1[1] == piece2[1]: #Check if they are in same row or col
            return True
        '''Time to check if they are attacking diagonally
         This can be done efficiently via simple algebra
         The two pices are on the same diagonal if they
         satisfy an equation of a line containing the two points'''
        x1 , y1 , x2 , y2 = piece1[1] , piece1[0] ,piece2[1] ,piece2[0]
        m = float(y2 - y1) / (x2 - x1)
        if abs(m) != 1.0:
            return False
        else:
            b = y2 - m * x2
            return y1 == m * x1 + b

    def isAttackingAny( self , piece ):
        '''Checks if piece is being atacked by
           any other piece in the board'''
        for piece1 in self.pieces:
            if self.isAttacking(piece,piece1):
                return True
        return False


def NQueens( board , n ):
    if n == 0:
        return [board]
    solutions = []
    i = 0
    for piece1 in product(range(8),repeat=2):
        if piece1 in board.pieces:
            continue
        if not board.isAttackingAny(piece1):
            i += 1
            fresh = copy.deepcopy(board)
            fresh.PlaceQueen(piece1[0] , piece1[1] )
            solutions += NQueens( fresh , n - 1 )
    return solutions
```

EXAMPLE

8-Queen's problem

Python

```
'''
   Contributed by: Bipin Oli
'''
def isValid(row,pos):
    if pos>=N or row>=N: return False
    if pos in sol: return False
    for i in range(len(sol)):
        if pos == sol[i]+row-i or pos == sol[i]-(row-i):
            return False
    return True

def solve(pos,Qc):
    Qc-=1
    sol.append(pos)
    if Qc==0: return True
    #Try
    for i in range(N):
        if(isValid(N-Qc,i)):
           if(solve(i,Qc)): return True
    sol.pop()
    Qc+=1
    return False

Queens = int(input().strip())
N=Queens
sol=[]
for i in range(Queens):
    if(solve(i,N)):
        print(sol)
    sol=[]
```

Problem Solving

TRY IT YOURSELF

Submit your answer

Form a cycle with a permutation of the first n positive integers. The cycle is called *Prime Cycle* if all neighboring pairs sum up to be a prime. The two **distinct** prime cycles for $n = 6$ are:

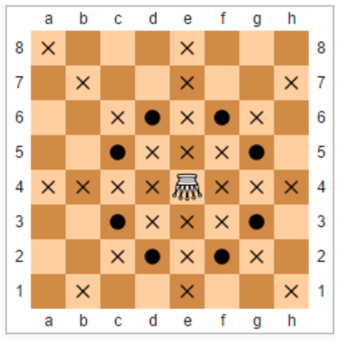
- $1, 4, 3, 2, 5, 6$
- $1, 6, 5, 2, 3, 4$

The permutation $3, 2, 5, 6, 1, 4$ is considered the same as the first sequence.

How many distinct prime cycles are there for $n = 16$?

TRY IT YOURSELF

Submit your answer



Squares an Amazon can threaten

An *Amazon* is a chess piece that combines the power of a knight and a queen.

In how many ways can you place 12 Amazons in a 12×12 chess board such that no Amazons attack each other?

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