

# Batch RNN

June 28, 2022

```
[1]: import numpy as np
import matplotlib.pyplot as plt
import torch
from torchvision import datasets
from torchvision.transforms import ToTensor
from torch.utils.data import TensorDataset, DataLoader
import sys
import copy
```

```
[2]: def tanh(x):
    return np.tanh(x)

def d_tanh(x):
    return 1 - np.square(tanh(x))

def sigmoid(y):
    y[y < -700] = -700
    return 1.0/(1.0+np.exp(-y))

def d_sigmoid(x):
    z = sigmoid(x)*(1-sigmoid(x))
    return z

def softmax(x):
    m = nn.Softmax(dim=0)
    return m(torch.tensor(x)).numpy()

def d_softmax(z):
    '''
    return the jacobian of the softmax
    '''
    z = softmax(z)
    return np.diag(z) - np.outer(z,z)

def MSE(a,b):
    return np.sum(np.square(a-b), axis = 1)
```

```
def d_MSE(output_activations, y):
    return (output_activations - y)
```

[228]: `class RNN:`

```
def __init__(self, input_dim, hidden_dim, output_dim, lr):
    self.input_dim = input_dim
    self.hidden_dim = hidden_dim
    self.output_dim = output_dim
    self.w_in = np.random.uniform(-1,1, (hidden_dim, input_dim))
    self.w_hidden = np.random.uniform(-1,1, (hidden_dim, hidden_dim))
    self.b_hidden = np.random.uniform(-1,1, (hidden_dim, 1))
    self.w_out = np.random.uniform(-1,1, (output_dim, hidden_dim))
    self.b_out = np.random.uniform(-1,1, (output_dim, 1))
    self.loss = []
    self.lr = lr
    self.epsilon = 0.000001

def forward(self, x):
    hidden_state = np.zeros((self.hidden_dim,1))
    T = x.shape[-1]
    prediction = np.zeros((T,1))

    print(T)
    for i in range(T):
        x_in = self.w_in @ x[:,i].reshape(self.input_dim,1)
        x_hid = self.w_hidden @ hidden_state + self.b_hidden
        hid_out = sigmoid(x_in + x_hid)
        hidden_state = hid_out
        x_out = self.w_out @ hid_out + self.b_out
        prediction[i] = x_out
    return prediction

def update(self, x, y):

    dldb_hidden, dldb_out, dldw_in, dldw_hidden, dldw_out, loss = self.
    ↪backprop(x, y)
    self.w_in -= self.lr * dldw_in
    self.w_hidden -= self.lr * dldw_hidden
    self.b_hidden -= self.lr * dldb_hidden
    self.w_out -= self.lr * dldw_out
    self.b_out -= self.lr * dldb_out
    self.loss.append(loss)

def gradient_approximation(self, x, y):
    nable_w_in = np.zeros(self.w_in.shape)
    nable_w_hidden = np.zeros(self.w_hidden.shape)
```

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nable_w_out = np.zeros(self.w_out.shape)
nable_b_hidden = np.zeros(self.b_hidden.shape)
nable_b_out = np.zeros(self.b_out.shape)

aprox1_w_in = np.zeros(self.w_in.shape)
aprox2_w_in = np.zeros(self.w_in.shape)
aprox1_w_hidden = np.zeros(self.w_hidden.shape)
aprox2_w_hidden = np.zeros(self.w_hidden.shape)
aprox1_w_out = np.zeros(self.w_out.shape)
aprox2_w_out = np.zeros(self.w_out.shape)
aprox1_b_hidden = np.zeros(self.b_hidden.shape)
aprox2_b_hidden = np.zeros(self.b_hidden.shape)
aprox1_b_out = np.zeros(self.b_out.shape)
aprox2_b_out = np.zeros(self.b_out.shape)

for k in range(len(self.b_hidden)):
    aprox1_b_hidden = copy.deepcopy(self.b_hidden)
    aprox2_b_hidden = copy.deepcopy(self.b_hidden)
    aprox1_b_hidden[k] += self.epsilon
    aprox2_b_hidden[k] -= self.epsilon

    hidden_state_1 = np.zeros((batch_size,self.hidden_dim,1))
    loss1 = np.zeros((batch_size,1))
    T = x.shape[-1]
    for i in range(T):
        x_in = self.w_in @ np.expand_dims(x[:, :, i], axis = 2)
        x_hid = self.w_hidden @ hidden_state_1 + aprox1_b_hidden
        hid_out = sigmoid(x_in + x_hid)
        hidden_state_1 = hid_out
        x_out = self.w_out @ hid_out + self.b_out
        loss1 += MSE(x_out,np.expand_dims(y[:, :, i],axis = 2))

    hidden_state_2 = np.zeros((batch_size,self.hidden_dim,1))
    loss2 = np.zeros((batch_size,1))
    T = x.shape[-1]

    for i in range(T):
        x_in = self.w_in @ np.expand_dims(x[:, :, i],axis = 2)
        x_hid = self.w_hidden @ hidden_state_2 + aprox2_b_hidden
        hid_out = sigmoid(x_in + x_hid)
        hidden_state_2 = hid_out
        x_out = self.w_out @ hid_out + self.b_out
        loss2 += MSE(x_out,np.expand_dims(y[:, :, i],axis = 2))

    nable_b_hidden[k] = (np.sum((loss1 - loss2) / (2*self.epsilon))) / ↵
↵batch_size

```

```

for k in range(len(self.w_in)):
    for j in range(len(self.w_in[k])):
        aprox1_w_in = copy.deepcopy(self.w_in)
        aprox2_w_in = copy.deepcopy(self.w_in)
        aprox1_w_in[k][j] += self.epsilon
        aprox2_w_in[k][j] -= self.epsilon

    hidden_state_1 = np.zeros((batch_size,self.hidden_dim,1))
    loss1 = np.zeros((batch_size,1))
    T = x.shape[-1]
    for i in range(T):
        x_in = aprox1_w_in @ np.expand_dims(x[:, :, i], axis = 2)
        x_hid = self.w_hidden @ hidden_state_1 + self.b_hidden
        hid_out = sigmoid(x_in + x_hid)
        hidden_state_1 = hid_out
        x_out = self.w_out @ hid_out + self.b_out
        loss1 += MSE(x_out, np.expand_dims(y[:, :, i], axis = 2))

    hidden_state_2 = np.zeros((batch_size,self.hidden_dim,1))
    loss2 = np.zeros((batch_size,1))
    T = x.shape[-1]

    for i in range(T):
        x_in = aprox2_w_in @ np.expand_dims(x[:, :, i], axis = 2)
        x_hid = self.w_hidden @ hidden_state_2 + self.b_hidden
        hid_out = sigmoid(x_in + x_hid)
        hidden_state_2 = hid_out
        x_out = self.w_out @ hid_out + self.b_out
        loss2 += MSE(x_out, np.expand_dims(y[:, :, i], axis = 2))

    nable_w_in[k][j] = (np.sum((loss1 - loss2) / (2*self.epsilon)))
    ↪ / batch_size

for k in range(len(self.w_hidden)):
    for j in range(len(self.w_hidden[k])):
        aprox1_w_hidden = copy.deepcopy(self.w_hidden)
        aprox2_w_hidden = copy.deepcopy(self.w_hidden)
        aprox1_w_hidden[k][j] += self.epsilon
        aprox2_w_hidden[k][j] -= self.epsilon

    hidden_state_1 = np.zeros((batch_size,self.hidden_dim,1))
    loss1 = np.zeros((batch_size,1))
    T = x.shape[-1]
    for i in range(T):
        x_in = self.w_in @ np.expand_dims(x[:, :, i], axis = 2)

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        x_hid = aprox1_w_hidden @ hidden_state_1 + self.b_hidden
        hid_out = sigmoid(x_in + x_hid)
        hidden_state_1 = hid_out
        x_out = self.w_out @ hid_out + self.b_out
        loss1 += MSE(x_out,np.expand_dims(y[:, :, i],axis = 2))

hidden_state_2 = np.zeros((batch_size,self.hidden_dim,1))
loss2 = np.zeros((batch_size,1))
T = x.shape[-1]

for i in range(T):
    x_in = self.w_in @ np.expand_dims(x[:, :, i],axis = 2)
    x_hid = aprox2_w_hidden @ hidden_state_2 + self.b_hidden
    hid_out = sigmoid(x_in + x_hid)
    hidden_state_2 = hid_out
    x_out = self.w_out @ hid_out + self.b_out
    loss2 += MSE(x_out,np.expand_dims(y[:, :, i],axis = 2))

    nable_w_hidden[k][j] = (np.sum((loss1 - loss2) / (2*self.
→epsilon)))) / batch_size

for k in range(len(self.w_out)):
    for j in range(len(self.w_out[k])):
        aprox1_w_out = copy.deepcopy(self.w_out)
        aprox2_w_out = copy.deepcopy(self.w_out)
        aprox1_w_out[k][j] += self.epsilon
        aprox2_w_out[k][j] -= self.epsilon

hidden_state_1 = np.zeros((batch_size,self.hidden_dim,1))
loss1 = np.zeros((batch_size,1))
T = x.shape[-1]
for i in range(T):
    x_in = self.w_in @ np.expand_dims(x[:, :, i],axis = 2)
    x_hid = self.w_hidden @ hidden_state_1 + self.b_hidden
    hid_out = sigmoid(x_in + x_hid)
    hidden_state_1 = hid_out
    x_out = aprox1_w_out @ hid_out + self.b_out
    loss1 += MSE(x_out,np.expand_dims(y[:, :, i],axis = 2))

hidden_state_2 = np.zeros((batch_size,self.hidden_dim,1))
loss2 = np.zeros((batch_size,1))
T = x.shape[-1]

for i in range(T):
    x_in = self.w_in @ np.expand_dims(x[:, :, i],axis = 2)
    x_hid = self.w_hidden @ hidden_state_2 + self.b_hidden
    hid_out = sigmoid(x_in + x_hid)

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        hidden_state_2 = hid_out
        x_out = aprox2_w_out @ hid_out + self.b_out
        loss2 += MSE(x_out,np.expand_dims(y[:, :, i],axis = 2))

        nable_w_out[k][j] = (np.sum((loss1 - loss2) / (2*self.
→epsilon)))) / batch_size

    for k in range(len(self.b_out)):
        aprox1_b_out = copy.deepcopy(self.b_out)
        aprox2_b_out = copy.deepcopy(self.b_out)
        aprox1_b_out[k] += self.epsilon
        aprox2_b_out[k] -= self.epsilon

    hidden_state_1 = np.zeros((batch_size,self.hidden_dim,1))
    loss1 = np.zeros((batch_size,1))
    T = x.shape[-1]
    for i in range(T):
        x_in = self.w_in @ np.expand_dims(x[:, :, i],axis = 2)
        x_hid = self.w_hidden @ hidden_state_1 + self.b_hidden
        hid_out = sigmoid(x_in + x_hid)
        hidden_state_1 = hid_out
        x_out = self.w_out @ hid_out + aprox1_b_out
        loss1 += MSE(x_out,np.expand_dims(y[:, :, i],axis = 2))

    hidden_state_2 = np.zeros((batch_size,self.hidden_dim,1))
    loss2 = np.zeros((batch_size,1))
    T = x.shape[-1]

    for i in range(T):
        x_in = self.w_in @ np.expand_dims(x[:, :, i],axis = 2)
        x_hid = self.w_hidden @ hidden_state_2 + self.b_hidden
        hid_out = sigmoid(x_in + x_hid)
        hidden_state_2 = hid_out
        x_out = self.w_out @ hid_out + aprox2_b_out
        loss2 += MSE(x_out,np.expand_dims(y[:, :, i],axis = 2))

    nable_b_out[k] = (np.sum((loss1 - loss2) / (2*self.epsilon))) /
→batch_size

    return nable_b_hidden,nable_b_out,nable_w_in,nable_w_hidden,nable_w_out

def backprop(self, x, y):
    hidden_state = np.zeros((batch_size,self.hidden_dim,1))
    hidden_out = []
    hidden_out.append(hidden_state)
    hidden_z = []

```

```

y_out = []
dldw_in = np.zeros((batch_size,) + self.w_in.shape)
dldw_hidden = np.zeros((batch_size,) + self.w_hidden.shape)
dlldb_hidden = np.zeros((batch_size,) + self.b_hidden.shape)
dldw_out = np.zeros((batch_size,) + self.w_out.shape)
dlldb_out = np.zeros((batch_size,) + self.b_out.shape)
loss = 0
T = x.shape[-1]

for i in range(T):
    x_in = self.w_in @ np.expand_dims(x[:, :, i], axis = 2)
    x_hid = self.w_hidden @ hidden_state + self.b_hidden
    hidden_z.append(x_in+x_hid)
    # hid_out = tanh(x_in + x_hid)
    hid_out = sigmoid(x_in + x_hid)
    hidden_out.append(hid_out)
    hidden_state = hid_out
    x_out = self.w_out @ hid_out + self.b_out
    y_out.append(x_out)
    loss += MSE(x_out,np.expand_dims(y[:, :, i],axis = 2))

'''
dldw_out
'''

for i in range(1,T+1):

    dlldb_out += d_MSE(y_out[-i],np.expand_dims(y[:, :, -i],axis = 2))
    dldw_out += d_MSE(y_out[-i],np.expand_dims(y[:, :, -i],axis = 2)) @
→np.transpose(hidden_out[-i],(0, 2, 1))

'''
dldw_hidden
dldw_input
'''

for i in range(1,T+1):

    delta = d_MSE(y_out[-i],np.expand_dims(y[:, :, -i],axis = 2))
    delta = np.transpose(self.w_out) @ delta * d_sigmoid(hidden_z[-i])
    dlldb_hidden += delta
    dldw_hidden += delta @ np.transpose(hidden_out[-i-1],(0, 2, 1))
    dldw_in += delta @ np.transpose(np.expand_dims(x[:, :, -i],axis =
→2),(0, 2, 1))

    for k in range(i+1,T+1):

```

```

        delta = np.transpose(self.w_hidden) @ delta * ↳
        d_sigmoid(hidden_z[-k])
        dldb_hidden += delta
        dldw_hidden += delta @ np.transpose(hidden_out[-k-1], (0, 2, 1))
        dldw_in += delta @ np.transpose(np.expand_dims(x[:, :, -k], axis = ↳
        ↳2), (0, 2, 1))

        dldb_hidden = 2 * dldb_hidden.sum(axis = 0) / batch_size
        dldb_out = 2 * dldb_out.sum(axis = 0) / batch_size
        dldw_in = 2 * dldw_in.sum(axis = 0) / batch_size
        dldw_hidden = 2 * dldw_hidden.sum(axis = 0) / batch_size
        dldw_out = 2 * dldw_out.sum(axis = 0) / batch_size
        loss = loss.sum(axis = 0) / batch_size

        return (dldb_hidden, dldb_out, dldw_in, dldw_hidden, dldw_out, loss)

```

```

[451]: input = []
target = []
T = 50
Num = 50

for i in range(Num):

    #parameters = np.random.randint(1,5, (3,1))
    p = np.random.randint(1,5)
    fix = np.array([0,1])
    parameters = np.insert(fix,0,p).reshape(3,1)
    x = np.tile(parameters,(T))
    time = np.linspace(0, 5, T)
    frequency = x[0][0]
    theta = x[1][0]
    amplitude = x[2][0]
    y = amplitude * np.sin(frequency * time + theta)
    input.append(x)
    target.append(y)

batch_size = 5
input = np.array(input)
target = np.array(target).reshape(Num,T)
batch_input = np.split(input, Num/batch_size)
batch_target = np.split(target, Num/batch_size)

```

```

[452]: %%time
sinrnn = RNN(3,100,1,0.0001)

```



```

Loss = []

for i in range(500):
    idx = 0
    for input,target in zip(batch_input,batch_target):
        #nable_b_hidden,nable_b_out,nable_w_in,nable_w_hidden,nable_w_out =
        ↪sinrnn.gradient_approximation(input,np.expand_dims(target,axis = 1))
        #dldb_hidden,dldb_out,dldw_in,dldw_hidden,dldw_out,loss = sinrnn.
        ↪backprop(input,np.expand_dims(target,axis = 1))
        #print('approximation',(nable_w_hidden / dldw_hidden).sum())
        sinrnn.update(input,np.expand_dims(target,axis = 1))
        Loss.append(np.sum(sinrnn.loss))
    print('epoch',i,': ',np.sum(sinrnn.loss))
    sinrnn.loss.clear()

```

```

epoch 0 : 334.9824358477371
epoch 1 : 317.020180067336
epoch 2 : 310.7517708382896
epoch 3 : 305.3482641386281
epoch 4 : 300.55079581290954
epoch 5 : 296.21560518859985
epoch 6 : 292.2566088383144
epoch 7 : 288.6173669323599
epoch 8 : 285.2573350802092
epoch 9 : 282.1450676824884
epoch 10 : 279.25479370421317
epoch 11 : 276.5646312984585
epoch 12 : 274.0556042678642
epoch 13 : 271.7110562755419
epoch 14 : 269.51626724885153
epoch 15 : 267.458176860891
epoch 16 : 265.5251684381629
epoch 17 : 263.70689010276925
epoch 18 : 261.9941013642727
epoch 19 : 260.37853895069577
epoch 20 : 258.8527984039949
epoch 21 : 257.41022930964544
epoch 22 : 256.0448426869183
epoch 23 : 254.75122938283153
epoch 24 : 253.52448846230814
epoch 25 : 252.36016465848894
epoch 26 : 251.25419398652951
epoch 27 : 250.2028566553452
epoch 28 : 249.20273644633826
epoch 29 : 248.25068577092142
epoch 30 : 247.34379567042947
epoch 31 : 246.47937008139144

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epoch 32 : 245.65490375363268  
epoch 33 : 244.86806327550772  
epoch 34 : 244.1166707270789  
epoch 35 : 243.39868954604324  
epoch 36 : 242.7122122509855  
epoch 37 : 242.05544972098593  
epoch 38 : 241.4267217791304  
epoch 39 : 240.8244488698756  
epoch 40 : 240.24714465665602  
epoch 41 : 239.69340939696042  
epoch 42 : 239.16192397786642  
epoch 43 : 238.65144451629834  
epoch 44 : 238.16079744566537  
epoch 45 : 237.6888750246519  
epoch 46 : 237.23463121529306  
epoch 47 : 236.79707788658945  
epoch 48 : 236.37528130720023  
epoch 49 : 235.96835889657802  
epoch 50 : 235.5754762085634  
epoch 51 : 235.1958441251992  
epoch 52 : 234.82871624153677  
epoch 53 : 234.47338642466582  
epoch 54 : 234.12918653221203  
epoch 55 : 233.7954842772243  
epoch 56 : 233.47168122777825  
epoch 57 : 233.1572109308216  
epoch 58 : 232.8515371508167  
epoch 59 : 232.55415221463232  
epoch 60 : 232.26457545492502  
epoch 61 : 231.98235174494792  
epoch 62 : 231.70705011834937  
epoch 63 : 231.4382624680798  
epoch 64 : 231.17560231903235  
epoch 65 : 230.91870366949456  
epoch 66 : 230.66721989690353  
epoch 67 : 230.42082272377107  
epoch 68 : 230.17920123998476  
epoch 69 : 229.94206097800304  
epoch 70 : 229.70912303774335  
epoch 71 : 229.48012325822265  
epoch 72 : 229.25481143324274  
epoch 73 : 229.03295056862996  
epoch 74 : 228.8143161787358  
epoch 75 : 228.59869562008564  
epoch 76 : 228.38588746022955  
epoch 77 : 228.1757008800017  
epoch 78 : 227.9679551075352  
epoch 79 : 227.76247888251174

epoch 80 : 227.5591099492433  
epoch 81 : 227.3576945772968  
epoch 82 : 227.1580871084769  
epoch 83 : 226.96014952907802  
epoch 84 : 226.76375106640668  
epoch 85 : 226.56876780866267  
epoch 86 : 226.3750823473415  
epoch 87 : 226.18258344139886  
epoch 88 : 225.99116570248214  
epoch 89 : 225.80072930060084  
epoch 90 : 225.6111796896632  
epoch 91 : 225.422427352361  
epoch 92 : 225.2343875639332  
epoch 93 : 225.04698017437985  
epoch 94 : 224.86012940873596  
epoch 95 : 224.67376368504364  
epoch 96 : 224.4878154496858  
epoch 97 : 224.30222102975847  
epoch 98 : 224.1169205021672  
epoch 99 : 223.9318575791306  
epoch 100 : 223.74697950976199  
epoch 101 : 223.56223699737671  
epoch 102 : 223.3775841321384  
epoch 103 : 223.19297833860878  
epoch 104 : 223.0083803377047  
epoch 105 : 222.82375412249036  
epoch 106 : 222.63906694714197  
epoch 107 : 222.45428932831476  
epoch 108 : 222.269395058025  
epoch 109 : 222.08436122701974  
epoch 110 : 221.89916825746036  
epoch 111 : 221.71379994358554  
epoch 112 : 221.52824349884327  
epoch 113 : 221.34248960781068  
epoch 114 : 221.1565324810327  
epoch 115 : 220.9703699107354  
epoch 116 : 220.7840033251962  
epoch 117 : 220.59743783939615  
epoch 118 : 220.41068229944284  
epoch 119 : 220.22374931814338  
epoch 120 : 220.03665529903873  
epoch 121 : 219.849420446184  
epoch 122 : 219.66206875699183  
epoch 123 : 219.47462799554177  
epoch 124 : 219.28712964392184  
epoch 125 : 219.0996088293942  
epoch 126 : 218.9121042254848  
epoch 127 : 218.7246579254711

epoch 128 : 218.53731528719396  
epoch 129 : 218.350124748628  
epoch 130 : 218.16313761421125  
epoch 131 : 217.97640781253656  
epoch 132 : 217.7899916266324  
epoch 133 : 217.60394739868946  
epoch 134 : 217.41833521169565  
epoch 135 : 217.23321655101327  
epoch 136 : 217.04865394943567  
epoch 137 : 216.86471061968842  
epoch 138 : 216.68145007866102  
epoch 139 : 216.49893576787485  
epoch 140 : 216.31723067478123  
epoch 141 : 216.13639695945676  
epoch 142 : 215.9564955911109  
epoch 143 : 215.77758599855594  
epoch 144 : 215.59972573842583  
epoch 145 : 215.42297018447934  
epoch 146 : 215.24737224081107  
epoch 147 : 215.07298208123723  
epoch 148 : 214.8998469165451  
epoch 149 : 214.72801079071888  
epoch 150 : 214.55751440669707  
epoch 151 : 214.38839498169574  
epoch 152 : 214.22068613165976  
epoch 153 : 214.0544177839939  
epoch 154 : 213.88961611737713  
epoch 155 : 213.72630352718568  
epoch 156 : 213.5644986148384  
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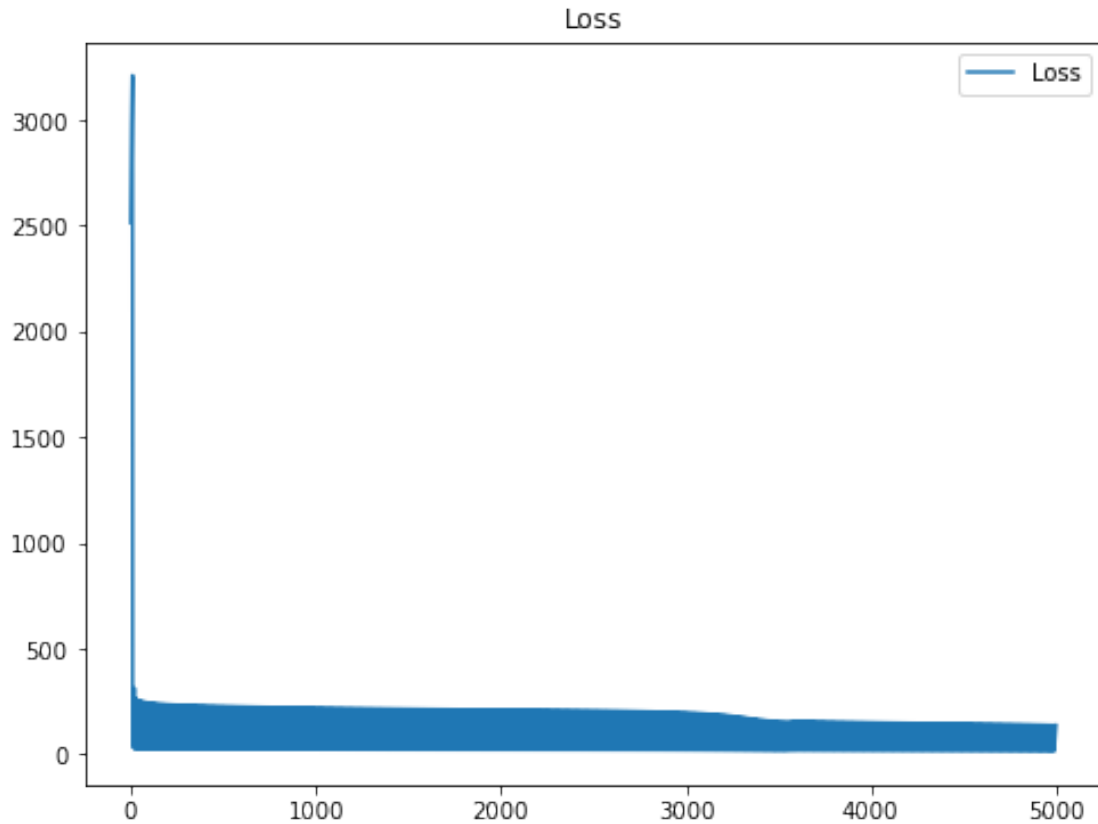


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epoch 416 : 176.61209003695419  
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epoch 491 : 173.2434721897706
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epoch 493 : 199.95052721858528
epoch 494 : 192.72702321310553
epoch 495 : 177.72826336107744
epoch 496 : 186.2203261103288
epoch 497 : 185.57193153774045
epoch 498 : 181.06365078598904
epoch 499 : 172.89573858264708
CPU times: user 12min 16s, sys: 1.37 s, total: 12min 17s
Wall time: 12min 17s
```

```
[417]: plt.figure(figsize = (8, 6))
plt.plot(Loss, label = 'Loss', markersize = 3)
plt.title("Loss")
plt.legend()
plt.show()
```



```
[453]: for i in range(500,1000):
        for input,target in zip(batch_input,batch_target):
            sinrnn.update(input,np.expand_dims(target,axis = 1))
            Loss.append(np.sum(sinrnn.loss))
        print('epoch',i,': ',np.sum(sinrnn.loss))
        sinrnn.loss.clear()
```

```
epoch 500 : 198.24944756470492
epoch 501 : 202.5444883848307
epoch 502 : 197.45436722823044
epoch 503 : 189.84922773637317
epoch 504 : 173.92292426693962
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epoch 512 : 174.01212389699123
epoch 513 : 201.08760487224978
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epoch 681 : 176.84121166242568  
epoch 682 : 155.2717228657098  
epoch 683 : 174.61563350175584  
epoch 684 : 171.8190789807155  
epoch 685 : 165.72719952965548  
epoch 686 : 155.5963102106594  
epoch 687 : 175.58565089275749  
epoch 688 : 173.96801089050112  
epoch 689 : 155.35534178155416  
epoch 690 : 175.63808683101982  
epoch 691 : 172.42672692460422  
epoch 692 : 156.54768587633774  
epoch 693 : 165.89139983594498  
epoch 694 : 154.84353142553175  
epoch 695 : 170.427185609472  
epoch 696 : 164.6925400185089  
epoch 697 : 174.58431335632068  
epoch 698 : 171.07546403698373  
epoch 699 : 153.7133514685843  
epoch 700 : 171.7418094762678  
epoch 701 : 165.39627594354494  
epoch 702 : 165.43857495776672  
epoch 703 : 154.96342952608742  
epoch 704 : 159.8230701020177  
epoch 705 : 150.6000382221719



epoch 706 : 181.93742507137426  
epoch 707 : 182.41730663437897  
epoch 708 : 163.96935625200985  
epoch 709 : 156.59686810777893  
epoch 710 : 147.88151647425747  
epoch 711 : 175.5872597666983  
epoch 712 : 173.54111575187468  
epoch 713 : 150.5397001008146  
epoch 714 : 151.28278592060533  
epoch 715 : 158.39745056289368  
epoch 716 : 145.97909203803522  
epoch 717 : 162.39422001049533  
epoch 718 : 152.60706251310907  
epoch 719 : 171.8079386571859  
epoch 720 : 167.62090827791837  
epoch 721 : 142.72373762270277  
epoch 722 : 146.34042546741858  
epoch 723 : 146.69400928470156  
epoch 724 : 154.38933180354363  
epoch 725 : 139.11752693909733  
epoch 726 : 131.03274160260813  
epoch 727 : 189.0189730595796  
epoch 728 : 178.53091186511762  
epoch 729 : 163.1476672373317  
epoch 730 : 142.64297544600737  
epoch 731 : 128.11716185749466  
epoch 732 : 136.83137645083772  
epoch 733 : 121.41924192417783  
epoch 734 : 122.4388979638049  
epoch 735 : 169.85563873177958  
epoch 736 : 151.76063674293923  
epoch 737 : 129.52792486582064  
epoch 738 : 108.70872656763629  
epoch 739 : 102.7525614397106  
epoch 740 : 138.2697728137293  
epoch 741 : 111.91631119531486  
epoch 742 : 94.52835736905982  
epoch 743 : 164.09790162070541  
epoch 744 : 127.70558724374699  
epoch 745 : 101.00350429652215  
epoch 746 : 90.10970480247838  
epoch 747 : 96.93143971155395  
epoch 748 : 90.13637814078223  
epoch 749 : 143.32456867872463  
epoch 750 : 84.51492085036955  
epoch 751 : 148.44882426891465  
epoch 752 : 88.8342577996097  
epoch 753 : 71.3522864505098

epoch 754 : 103.51414040559322  
epoch 755 : 161.93477290786197  
epoch 756 : 91.66505430555681  
epoch 757 : 76.83855334047722  
epoch 758 : 154.63450286102469  
epoch 759 : 92.49427305177014  
epoch 760 : 76.67900781351172  
epoch 761 : 92.20215625176728  
epoch 762 : 59.69493440391055  
epoch 763 : 75.52805040994704  
epoch 764 : 163.52066366740308  
epoch 765 : 75.17587154096012  
epoch 766 : 176.61438757350925  
epoch 767 : 92.85180559050062  
epoch 768 : 64.10451236791923  
epoch 769 : 126.48681177709969  
epoch 770 : 66.10536851221059  
epoch 771 : 60.173437299732925  
epoch 772 : 58.71204387432381  
epoch 773 : 193.79603231659752  
epoch 774 : 96.72198832607884  
epoch 775 : 77.40874438142156  
epoch 776 : 48.37631819452589  
epoch 777 : 122.38807527005088  
epoch 778 : 56.045528001436764  
epoch 779 : 133.35268464632873  
epoch 780 : 59.760472175719066  
epoch 781 : 30.836265500012342  
epoch 782 : 47.759342283963214  
epoch 783 : 46.54761769263891  
epoch 784 : 67.69296862391147  
epoch 785 : 103.67331795785023  
epoch 786 : 36.08745701943293  
epoch 787 : 130.05046988854005  
epoch 788 : 51.774415563647906  
epoch 789 : 29.271750371818094  
epoch 790 : 47.352054838674526  
epoch 791 : 21.4783427590489  
epoch 792 : 80.12992897281212  
epoch 793 : 28.47490745891056  
epoch 794 : 29.026854495421905  
epoch 795 : 42.54173313293042  
epoch 796 : 24.00400765607066  
epoch 797 : 39.03306624919102  
epoch 798 : 28.38324670581524  
epoch 799 : 31.146181999902783  
epoch 800 : 28.76069057659184  
epoch 801 : 27.638057970939727

epoch 802 : 26.798066796245003  
epoch 803 : 26.12963309328024  
epoch 804 : 25.565660097836314  
epoch 805 : 25.072215369139883  
epoch 806 : 24.63150241376434  
epoch 807 : 24.232802199131196  
epoch 808 : 23.868747540355617  
epoch 809 : 23.533817010108773  
epoch 810 : 23.223675471641524  
epoch 811 : 22.93483432758819  
epoch 812 : 22.66444335643515  
epoch 813 : 22.41014748495363  
epoch 814 : 22.16998235387007  
epoch 815 : 21.94229600579118  
epoch 816 : 21.725689131223596  
epoch 817 : 21.518968809248936  
epoch 818 : 21.321112208619997  
epoch 819 : 21.131237746814065  
epoch 820 : 20.948581917251094  
epoch 821 : 20.772480488715146  
epoch 822 : 20.602353124528026  
epoch 823 : 20.437690710168006  
epoch 824 : 20.27804484963332  
epoch 825 : 20.1230191148157  
epoch 826 : 19.972261723193494  
epoch 827 : 19.825459387078755  
epoch 828 : 19.68233212910502  
epoch 829 : 19.542628898284406  
epoch 830 : 19.40612385180753  
epoch 831 : 19.272613192235955  
epoch 832 : 19.141912469240282  
epoch 833 : 19.01385427081259  
epoch 834 : 18.888286241722252  
epoch 835 : 18.765069377497117  
epoch 836 : 18.6440765508638  
epoch 837 : 18.525191234738692  
epoch 838 : 18.4083063917778  
epoch 839 : 18.293323505413685  
epoch 840 : 18.180151731389255  
epoch 841 : 18.068707152205704  
epoch 842 : 17.95891211969956  
epoch 843 : 17.850694673368103  
epoch 844 : 17.743988023963116  
epoch 845 : 17.638730093558774  
epoch 846 : 17.534863104635047  
epoch 847 : 17.432333211851972  
epoch 848 : 17.331090171165883  
epoch 849 : 17.231087041700018

epoch 850 : 17.132279916474488  
epoch 851 : 17.034627678635946  
epoch 852 : 16.938091780314377  
epoch 853 : 16.84263604161031  
epoch 854 : 16.748226467562834  
epoch 855 : 16.654831081216578  
epoch 856 : 16.562419771149088  
epoch 857 : 16.47096415203105  
epoch 858 : 16.380437436937388  
epoch 859 : 16.290814320312258  
epoch 860 : 16.202070870574083  
epoch 861 : 16.114184431504817  
epoch 862 : 16.027133531604477  
epoch 863 : 15.940897800737845  
epoch 864 : 15.855457893410428  
epoch 865 : 15.770795418118434  
epoch 866 : 15.686892872252113  
epoch 867 : 15.603733582079405  
epoch 868 : 15.521301647394088  
epoch 869 : 15.439581890430881  
epoch 870 : 15.358559808705664  
epoch 871 : 15.278221531453765  
epoch 872 : 15.198553779376159  
epoch 873 : 15.119543827417795  
epoch 874 : 15.041179470349972  
epoch 875 : 14.963448990906002  
epoch 876 : 14.88634113029167  
epoch 877 : 14.80984506086105  
epoch 878 : 14.733950360789656  
epoch 879 : 14.658646990594631  
epoch 880 : 14.583925271342144  
epoch 881 : 14.509775864417234  
epoch 882 : 14.436189752728833  
epoch 883 : 14.363158223239287  
epoch 884 : 14.290672850709006  
epoch 885 : 14.218725482569523  
epoch 886 : 14.147308224828569  
epoch 887 : 14.076413428929332  
epoch 888 : 14.006033679496587  
epoch 889 : 13.93616178288425  
epoch 890 : 13.866790756488303  
epoch 891 : 13.797913818736207  
epoch 892 : 13.72952437972918  
epoch 893 : 13.661616032467606  
epoch 894 : 13.59418254463064  
epoch 895 : 13.527217850848192  
epoch 896 : 13.460716045464185  
epoch 897 : 13.394671375703311

epoch 898 : 13.329078235269302  
epoch 899 : 13.263931158292898  
epoch 900 : 13.199224813638944  
epoch 901 : 13.134953999528456  
epoch 902 : 13.071113638459769  
epoch 903 : 13.007698772407709  
epoch 904 : 12.944704558280419  
epoch 905 : 12.88212626361604  
epoch 906 : 12.819959262501257  
epoch 907 : 12.758199031704066  
epoch 908 : 12.696841146994105  
epoch 909 : 12.635881279651187  
epoch 910 : 12.575315193138696  
epoch 911 : 12.51513873994406  
epoch 912 : 12.455347858554417  
epoch 913 : 12.395938570593268  
epoch 914 : 12.33690697806313  
epoch 915 : 12.27824926073595  
epoch 916 : 12.219961673638196  
epoch 917 : 12.162040544664903  
epoch 918 : 12.104482272274709  
epoch 919 : 12.047283323307115  
epoch 920 : 11.990440230863587  
epoch 921 : 11.933949592300744  
epoch 922 : 11.87780806728462  
epoch 923 : 11.822012375937565  
epoch 924 : 11.766559297045953  
epoch 925 : 11.711445666349528  
epoch 926 : 11.656668374881448  
epoch 927 : 11.602224367389718  
epoch 928 : 11.548110640798166  
epoch 929 : 11.49432424273988  
epoch 930 : 11.440862270136025  
epoch 931 : 11.387721867826066  
epoch 932 : 11.33490022725368  
epoch 933 : 11.282394585188579  
epoch 934 : 11.230202222504614  
epoch 935 : 11.178320462986997  
epoch 936 : 11.126746672199129  
epoch 937 : 11.07547825636216  
epoch 938 : 11.024512661304795  
epoch 939 : 10.97384737141067  
epoch 940 : 10.923479908641028  
epoch 941 : 10.873407831550168  
epoch 942 : 10.82362873436825  
epoch 943 : 10.774140246085215  
epoch 944 : 10.724940029586095  
epoch 945 : 10.676025780797543

epoch 946 : 10.627395227875798  
epoch 947 : 10.57904613040562  
epoch 948 : 10.53097627864062  
epoch 949 : 10.483183492750806  
epoch 950 : 10.43566562211013  
epoch 951 : 10.388420544588  
epoch 952 : 10.341446165885426  
epoch 953 : 10.294740418862673  
epoch 954 : 10.248301262920867  
epoch 955 : 10.202126683367696  
epoch 956 : 10.156214690837725  
epoch 957 : 10.110563320693094  
epoch 958 : 10.065170632480621  
epoch 959 : 10.020034709368485  
epoch 960 : 9.975153657634369  
epoch 961 : 9.930525606134157  
epoch 962 : 9.886148705821581  
epoch 963 : 9.842021129249895  
epoch 964 : 9.798141070114795  
epoch 965 : 9.754506742790351  
epoch 966 : 9.711116381893568  
epoch 967 : 9.667968241851549  
epoch 968 : 9.625060596488186  
epoch 969 : 9.582391738618094  
epoch 970 : 9.53995997965621  
epoch 971 : 9.497763649234097  
epoch 972 : 9.45580109483434  
epoch 973 : 9.414070681427189  
epoch 974 : 9.372570791124874  
epoch 975 : 9.331299822841892  
epoch 976 : 9.290256191965314  
epoch 977 : 9.249438330036622  
epoch 978 : 9.208844684441008  
epoch 979 : 9.168473718104483  
epoch 980 : 9.12832390920383  
epoch 981 : 9.088393750878744  
epoch 982 : 9.048681750958165  
epoch 983 : 9.009186431689944  
epoch 984 : 8.969906329479397  
epoch 985 : 8.930839994638285  
epoch 986 : 8.891985991133435  
epoch 987 : 8.853342896354162  
epoch 988 : 8.814909300869118  
epoch 989 : 8.776683808211263  
epoch 990 : 8.738665034648177  
epoch 991 : 8.700851608973675  
epoch 992 : 8.66324217229667  
epoch 993 : 8.625835377840684

```
epoch 994 : 8.588629890744192
epoch 995 : 8.55162438787303
epoch 996 : 8.514817557629488
epoch 997 : 8.478208099775838
epoch 998 : 8.44179472525303
epoch 999 : 8.40557615601341
```

```
[454]: for i in range(1000,1100):
        for input,target in zip(batch_input,batch_target):
            sinrnn.update(input,np.expand_dims(target,axis = 1))
            Loss.append(np.sum(sinrnn.loss))
        print('epoch',i,': ',np.sum(sinrnn.loss))
        sinrnn.loss.clear()
```

```
epoch 1000 : 8.369551124851567
epoch 1001 : 8.333718375240734
epoch 1002 : 8.298076661178376
epoch 1003 : 8.2626247470275
epoch 1004 : 8.227361407370738
epoch 1005 : 8.192285426860868
epoch 1006 : 8.157395600082634
epoch 1007 : 8.122690731409032
epoch 1008 : 8.088169634872038
epoch 1009 : 8.05383113402614
epoch 1010 : 8.019674061825016
epoch 1011 : 7.985697260491291
epoch 1012 : 7.951899581400612
epoch 1013 : 7.9182798849582
epoch 1014 : 7.884837040486128
epoch 1015 : 7.851569926109107
epoch 1016 : 7.818477428645586
epoch 1017 : 7.785558443500238
epoch 1018 : 7.752811874556473
epoch 1019 : 7.720236634079584
epoch 1020 : 7.68783164261159
epoch 1021 : 7.655595828878558
epoch 1022 : 7.623528129690331
epoch 1023 : 7.59162748985353
epoch 1024 : 7.559892862075372
epoch 1025 : 7.528323206878424
epoch 1026 : 7.496917492514179
epoch 1027 : 7.465674694874331
epoch 1028 : 7.434593797413401
epoch 1029 : 7.40367379106465
epoch 1030 : 7.3729136741597
epoch 1031 : 7.342312452355105
epoch 1032 : 7.311869138552231
epoch 1033 : 7.281582752826394
```

epoch 1034 : 7.251452322350665  
epoch 1035 : 7.221476881329678  
epoch 1036 : 7.191655470923588  
epoch 1037 : 7.161987139185431  
epoch 1038 : 7.132470940990924  
epoch 1039 : 7.103105937973963  
epoch 1040 : 7.07389119846139  
epoch 1041 : 7.044825797411509  
epoch 1042 : 7.01590881635055  
epoch 1043 : 6.987139343312101  
epoch 1044 : 6.958516472778855  
epoch 1045 : 6.930039305621855  
epoch 1046 : 6.901706949044912  
epoch 1047 : 6.873518516526929  
epoch 1048 : 6.845473127768236  
epoch 1049 : 6.8175699086318895  
epoch 1050 : 6.789807991094798  
epoch 1051 : 6.762186513192992  
epoch 1052 : 6.73470461896819  
epoch 1053 : 6.707361458420271  
epoch 1054 : 6.6801561874534165  
epoch 1055 : 6.653087967829156  
epoch 1056 : 6.626155967117231  
epoch 1057 : 6.599359358646694  
epoch 1058 : 6.572697321459034  
epoch 1059 : 6.546169040263231  
epoch 1060 : 6.519773705387875  
epoch 1061 : 6.493510512736131  
epoch 1062 : 6.467378663743195  
epoch 1063 : 6.441377365330209  
epoch 1064 : 6.415505829861706  
epoch 1065 : 6.389763275103627  
epoch 1066 : 6.364148924178853  
epoch 1067 : 6.338662005528443  
epoch 1068 : 6.3133017528683695  
epoch 1069 : 6.288067405150234  
epoch 1070 : 6.262958206519575  
epoch 1071 : 6.237973406278218  
epoch 1072 : 6.213112258843941  
epoch 1073 : 6.188374023711059  
epoch 1074 : 6.163757965414807  
epoch 1075 : 6.1392633534894365  
epoch 1076 : 6.11488946243561  
epoch 1077 : 6.090635571679436  
epoch 1078 : 6.066500965536986  
epoch 1079 : 6.0424849331800035  
epoch 1080 : 6.018586768597809  
epoch 1081 : 5.994805770562401



```

epoch 1082 : 5.971141242593151
epoch 1083 : 5.94759249292348
epoch 1084 : 5.924158834464693
epoch 1085 : 5.900839584772832
epoch 1086 : 5.8776340660147595
epoch 1087 : 5.85454160493402
epoch 1088 : 5.83156153281959
epoch 1089 : 5.808693185470748
epoch 1090 : 5.785935903165999
epoch 1091 : 5.763289030630205
epoch 1092 : 5.740751917003277
epoch 1093 : 5.718323915808038
epoch 1094 : 5.696004384918736
epoch 1095 : 5.673792686529069
epoch 1096 : 5.651688187124758
epoch 1097 : 5.629690257447733
epoch 1098 : 5.607798272469859
epoch 1099 : 5.586011611360936

```

```

[459]: for i in range(1100,1500):
        for input,target in zip(batch_input,batch_target):
            sinrnn.update(input,np.expand_dims(target,axis = 1))
            Loss.append(np.sum(sinrnn.loss))
        print('epoch',i,': ',np.sum(sinrnn.loss))
        sinrnn.loss.clear()

```

```

epoch 1100 : 5.564329657460245
epoch 1101 : 5.542751798244087
epoch 1102 : 5.5212774252995995
epoch 1103 : 5.499905934293321
epoch 1104 : 5.478636724944222
epoch 1105 : 5.457469200992622
epoch 1106 : 5.436402770172233
epoch 1107 : 5.415436844184663
epoch 1108 : 5.3945708386667075
epoch 1109 : 5.373804173166446
epoch 1110 : 5.35313627111287
epoch 1111 : 5.332566559789582
epoch 1112 : 5.312094470307067
epoch 1113 : 5.2917194375769325
epoch 1114 : 5.271440900282671
epoch 1115 : 5.2512583008548495
epoch 1116 : 5.231171085443072
epoch 1117 : 5.21117870389121
epoch 1118 : 5.1912806097101605
epoch 1119 : 5.171476260051957
epoch 1120 : 5.151765115684695
epoch 1121 : 5.13214664096604

```

epoch 1122 : 5.112620303817441  
epoch 1123 : 5.093185575700584  
epoch 1124 : 5.073841931590739  
epoch 1125 : 5.05458884995209  
epoch 1126 : 5.035425812713302  
epoch 1127 : 5.016352305242855  
epoch 1128 : 4.997367816324502  
epoch 1129 : 4.978471838132992  
epoch 1130 : 4.959663866210089  
epoch 1131 : 4.940943399440364  
epoch 1132 : 4.92230994002683  
epoch 1133 : 4.903762993469823  
epoch 1134 : 4.885302068539736  
epoch 1135 : 4.866926677257371  
epoch 1136 : 4.84863633486762  
epoch 1137 : 4.830430559819352  
epoch 1138 : 4.812308873739871  
epoch 1139 : 4.794270801414079  
epoch 1140 : 4.77631587076105  
epoch 1141 : 4.758443612812451  
epoch 1142 : 4.7406535616889265  
epoch 1143 : 4.722945254579484  
epoch 1144 : 4.705318231718223  
epoch 1145 : 4.687772036363436  
epoch 1146 : 4.670306214775401  
epoch 1147 : 4.652920316195229  
epoch 1148 : 4.635613892823081  
epoch 1149 : 4.618386499797712  
epoch 1150 : 4.601237695172956  
epoch 1151 : 4.584167039901642  
epoch 1152 : 4.567174097809238  
epoch 1153 : 4.550258435576587  
epoch 1154 : 4.5334196227182595  
epoch 1155 : 4.516657231562147  
epoch 1156 : 4.499970837229959  
epoch 1157 : 4.483360017615723  
epoch 1158 : 4.466824353367269  
epoch 1159 : 4.450363427864629  
epoch 1160 : 4.4339768272023585  
epoch 1161 : 4.417664140168147  
epoch 1162 : 4.4014249582243075  
epoch 1163 : 4.385258875488529  
epoch 1164 : 4.369165488714068  
epoch 1165 : 4.353144397270953  
epoch 1166 : 4.337195203127266  
epoch 1167 : 4.321317510829596  
epoch 1168 : 4.305510927485048  
epoch 1169 : 4.2897750627434315

epoch 1170 : 4.274109528776527  
epoch 1171 : 4.258513940262269  
epoch 1172 : 4.242987914364825  
epoch 1173 : 4.2275310707169975  
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```

```
[458]: #parameters = np.ones((3,1))
T = 50
parameters = np.array([6,0,1]).reshape(3,1)
x = np.tile(parameters,(T))
```

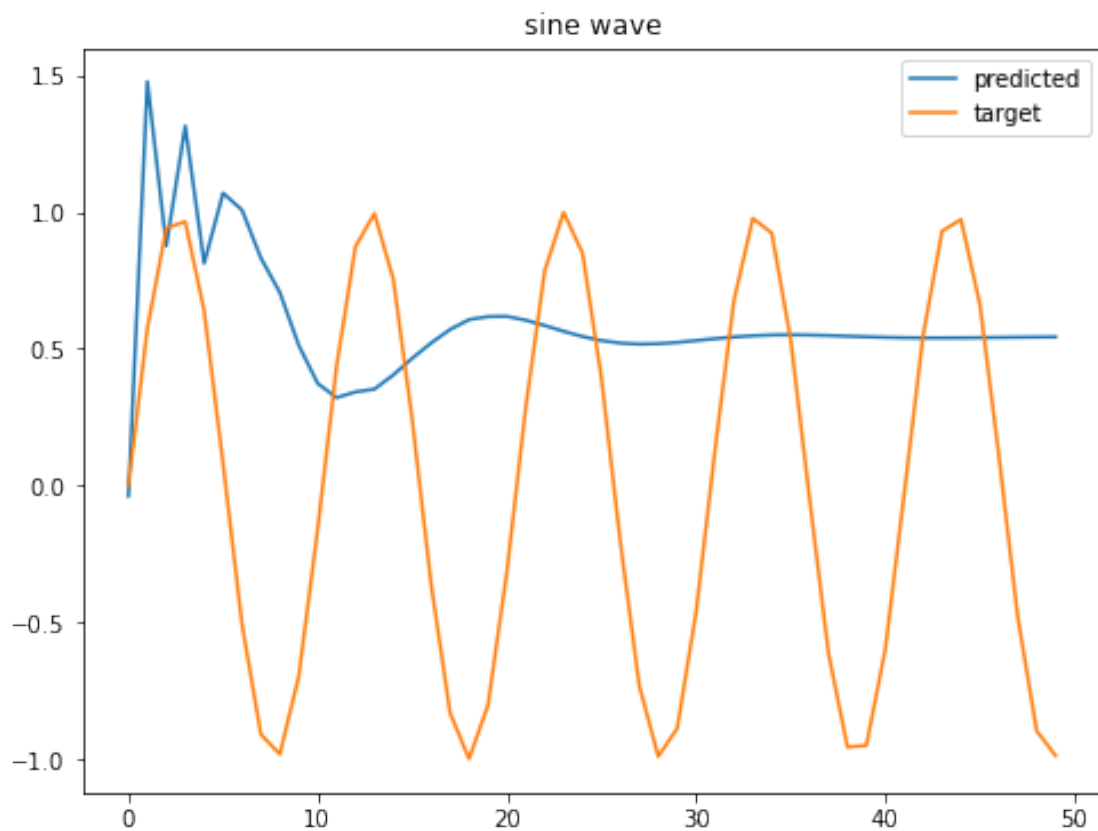
```

time = np.linspace(0, 5, T)
frequency = x[0][0]
theta = x[1][0]
amplitude = x[2][0]
y = amplitude * np.sin(frequency * time + theta)

predicted = sinrnn.forward(x)
target = y
plt.figure(figsize = (8, 6))
plt.plot(predicted, label = 'predicted', markersize = 3)
plt.plot(target, label = 'target', markersize = 3)
plt.title("sine wave")
plt.legend()
plt.show()

```

50



```

[390]: parameters = np.random.randint(3,5)
fix = np.array([0,1])
x = np.insert(fix,0,parameters).reshape(3,1)

```

[390]: 3

```
[395]: x = np.insert(fix,0,parameters).reshape(3,1)
```

```
[396]: x.shape
```

[396]: (3, 1)

```
[ ]:
```