Batch RNN

June 28, 2022

[470]:
```
import numpy as np
import matplotlib.pyplot as plt
import torch
from torchvision import datasets
from torchvision.transforms import ToTensor
from torch.utils.data import TensorDataset, DataLoader
import sys
import copy
```

[2]:
```
def tanh(x):
    return np.tanh(x)

def d_tanh(x):
    return 1 - np.square(tanh(x))

def sigmoid(y):
    y[y < -700] = -700
    return 1.0/(1.0+np.exp(-y))

def d_sigmoid(x):
    z = sigmoid(x)*(1-sigmoid(x))
    return z


def softmax(x):
    m = nn.Softmax(dim=0)
    return m(torch.tensor(x)).numpy()

def d_softmax(z):
    '''
    return the jacobian of the softmax
    '''
    z = softmax(z)
    return np.diag(z) - np.outer(z,z)

def MSE(a,b):
    return np.sum(np.square(a-b), axis = 1)
```

```python
def d_MSE(output_activations, y):
    return (output_activations - y)
```

[228]:
```python
class RNN:

    def __init__(self,input_dim,hidden_dim,output_dim,lr):
        self.input_dim = input_dim
        self.hidden_dim = hidden_dim
        self.output_dim = output_dim
        self.w_in = np.random.uniform(-1,1, (hidden_dim, input_dim))
        self.w_hidden = np.random.uniform(-1,1, (hidden_dim, hidden_dim))
        self.b_hidden = np.random.uniform(-1,1, (hidden_dim, 1))
        self.w_out = np.random.uniform(-1,1, (output_dim, hidden_dim))
        self.b_out = np.random.uniform(-1,1, (output_dim, 1))
        self.loss = []
        self.lr = lr
        self.epsilon = 0.000001

    def forward(self, x):
        hidden_state = np.zeros((self.hidden_dim,1))
        T = x.shape[-1]
        prediction = np.zeros((T,1))

        for i in range(T):
            x_in = self.w_in @ x[:,i].reshape(self.input_dim,1)
            x_hid = self.w_hidden @ hidden_state + self.b_hidden
            hid_out = sigmoid(x_in + x_hid)
            hidden_state = hid_out
            x_out = self.w_out @ hid_out + self.b_out
            prediction[i] = x_out
        return prediction

    def update(self, x, y):

        dldb_hidden,dldb_out,dldw_in,dldw_hidden,dldw_out,loss = self.
↪backprop(x, y)
        self.w_in -=  self.lr * dldw_in
        self.w_hidden -=  self.lr * dldw_hidden
        self.b_hidden -=  self.lr * dldb_hidden
        self.w_out -=  self.lr * dldw_out
        self.b_out -=  self.lr * dldb_out
        self.loss.append(loss)

    def gradient_approximation(self, x, y):
        nable_w_in = np.zeros(self.w_in.shape)
        nable_w_hidden = np.zeros(self.w_hidden.shape)
        nable_w_out = np.zeros(self.w_out.shape)
```

```python
        nable_b_hidden = np.zeros(self.b_hidden.shape)
        nable_b_out = np.zeros(self.b_out.shape)

        aprox1_w_in = np.zeros(self.w_in.shape)
        aprox2_w_in = np.zeros(self.w_in.shape)
        aprox1_w_hidden = np.zeros(self.w_hidden.shape)
        aprox2_w_hidden = np.zeros(self.w_hidden.shape)
        aprox1_w_out = np.zeros(self.w_out.shape)
        aprox2_w_out = np.zeros(self.w_out.shape)
        aprox1_b_hidden = np.zeros(self.b_hidden.shape)
        aprox2_b_hidden = np.zeros(self.b_hidden.shape)
        aprox1_b_out = np.zeros(self.b_out.shape)
        aprox2_b_out = np.zeros(self.b_out.shape)

        for k in range(len(self.b_hidden)):
            aprox1_b_hidden = copy.deepcopy(self.b_hidden)
            aprox2_b_hidden = copy.deepcopy(self.b_hidden)
            aprox1_b_hidden[k] += self.epsilon
            aprox2_b_hidden[k] -= self.epsilon

            hidden_state_1 = np.zeros((batch_size,self.hidden_dim,1))
            loss1 = np.zeros((batch_size,1))
            T = x.shape[-1]
            for i in range(T):
                x_in = self.w_in @ np.expand_dims(x[:,:,i],axis = 2)
                x_hid = self.w_hidden @ hidden_state_1 + aprox1_b_hidden
                hid_out = sigmoid(x_in + x_hid)
                hidden_state_1 = hid_out
                x_out = self.w_out @ hid_out + self.b_out
                loss1 += MSE(x_out,np.expand_dims(y[:,:,i],axis = 2))

            hidden_state_2 = np.zeros((batch_size,self.hidden_dim,1))
            loss2 = np.zeros((batch_size,1))
            T = x.shape[-1]

            for i in range(T):
                x_in = self.w_in @ np.expand_dims(x[:,:,i],axis = 2)
                x_hid = self.w_hidden @ hidden_state_2 + aprox2_b_hidden
                hid_out = sigmoid(x_in + x_hid)
                hidden_state_2 = hid_out
                x_out = self.w_out @ hid_out + self.b_out
                loss2 += MSE(x_out,np.expand_dims(y[:,:,i],axis = 2))

            nable_b_hidden[k] = (np.sum((loss1 - loss2) / (2*self.epsilon))) /␣
↪batch_size
```

```
        for k in range(len(self.w_in)):
            for j in range(len(self.w_in[k])):
                aprox1_w_in = copy.deepcopy(self.w_in)
                aprox2_w_in = copy.deepcopy(self.w_in)
                aprox1_w_in[k][j] += self.epsilon
                aprox2_w_in[k][j] -= self.epsilon

                hidden_state_1 = np.zeros((batch_size,self.hidden_dim,1))
                loss1 = np.zeros((batch_size,1))
                T = x.shape[-1]
                for i in range(T):
                    x_in = aprox1_w_in @ np.expand_dims(x[:,:,i],axis = 2)
                    x_hid = self.w_hidden @ hidden_state_1 + self.b_hidden
                    hid_out = sigmoid(x_in + x_hid)
                    hidden_state_1 = hid_out
                    x_out = self.w_out @ hid_out + self.b_out
                    loss1 += MSE(x_out,np.expand_dims(y[:,:,i],axis = 2))

                hidden_state_2 = np.zeros((batch_size,self.hidden_dim,1))
                loss2 = np.zeros((batch_size,1))
                T = x.shape[-1]

                for i in range(T):
                    x_in = aprox2_w_in @ np.expand_dims(x[:,:,i],axis = 2)
                    x_hid = self.w_hidden @ hidden_state_2 + self.b_hidden
                    hid_out = sigmoid(x_in + x_hid)
                    hidden_state_2 = hid_out
                    x_out = self.w_out @ hid_out + + self.b_out
                    loss2 += MSE(x_out,np.expand_dims(y[:,:,i],axis = 2))

                nable_w_in[k][j] = (np.sum((loss1 - loss2) / (2*self.epsilon)))␣
↪/ batch_size


        for k in range(len(self.w_hidden)):
            for j in range(len(self.w_hidden[k])):
                aprox1_w_hidden = copy.deepcopy(self.w_hidden)
                aprox2_w_hidden = copy.deepcopy(self.w_hidden)
                aprox1_w_hidden[k][j] += self.epsilon
                aprox2_w_hidden[k][j] -= self.epsilon

                hidden_state_1 = np.zeros((batch_size,self.hidden_dim,1))
                loss1 = np.zeros((batch_size,1))
                T = x.shape[-1]
                for i in range(T):
                    x_in = self.w_in @ np.expand_dims(x[:,:,i],axis = 2)
                    x_hid = aprox1_w_hidden @ hidden_state_1 + self.b_hidden
```

```
                    hid_out = sigmoid(x_in + x_hid)
                    hidden_state_1 = hid_out
                    x_out = self.w_out @ hid_out + self.b_out
                    loss1 += MSE(x_out,np.expand_dims(y[:,:,i],axis = 2))

                hidden_state_2 = np.zeros((batch_size,self.hidden_dim,1))
                loss2 = np.zeros((batch_size,1))
                T = x.shape[-1]

                for i in range(T):
                    x_in = self.w_in @ np.expand_dims(x[:,:,i],axis = 2)
                    x_hid = aprox2_w_hidden @ hidden_state_2 + self.b_hidden
                    hid_out = sigmoid(x_in + x_hid)
                    hidden_state_2 = hid_out
                    x_out = self.w_out @ hid_out + + self.b_out
                    loss2 += MSE(x_out,np.expand_dims(y[:,:,i],axis = 2))

                nable_w_hidden[k][j] = (np.sum((loss1 - loss2) / (2*self.
↪epsilon))) / batch_size

        for k in range(len(self.w_out)):
            for j in range(len(self.w_out[k])):
                aprox1_w_out = copy.deepcopy(self.w_out)
                aprox2_w_out = copy.deepcopy(self.w_out)
                aprox1_w_out[k][j] += self.epsilon
                aprox2_w_out[k][j] -= self.epsilon

                hidden_state_1 = np.zeros((batch_size,self.hidden_dim,1))
                loss1 = np.zeros((batch_size,1))
                T = x.shape[-1]
                for i in range(T):
                    x_in = self.w_in @ np.expand_dims(x[:,:,i],axis = 2)
                    x_hid = self.w_hidden @ hidden_state_1 + self.b_hidden
                    hid_out = sigmoid(x_in + x_hid)
                    hidden_state_1 = hid_out
                    x_out = aprox1_w_out @ hid_out + self.b_out
                    loss1 += MSE(x_out,np.expand_dims(y[:,:,i],axis = 2))

                hidden_state_2 = np.zeros((batch_size,self.hidden_dim,1))
                loss2 = np.zeros((batch_size,1))
                T = x.shape[-1]

                for i in range(T):
                    x_in = self.w_in @ np.expand_dims(x[:,:,i],axis = 2)
                    x_hid = self.w_hidden @ hidden_state_2 + self.b_hidden
                    hid_out = sigmoid(x_in + x_hid)
                    hidden_state_2 = hid_out
```

```
                    x_out = aprox2_w_out @ hid_out + + self.b_out
                    loss2 += MSE(x_out,np.expand_dims(y[:,:,i],axis = 2))

                nable_w_out[k][j] = (np.sum((loss1 - loss2) / (2*self.
↪epsilon))) / batch_size

        for k in range(len(self.b_out)):
            aprox1_b_out = copy.deepcopy(self.b_out)
            aprox2_b_out = copy.deepcopy(self.b_out)
            aprox1_b_out[k] += self.epsilon
            aprox2_b_out[k] -= self.epsilon

            hidden_state_1 = np.zeros((batch_size,self.hidden_dim,1))
            loss1 = np.zeros((batch_size,1))
            T = x.shape[-1]
            for i in range(T):
                x_in = self.w_in @ np.expand_dims(x[:,:,i],axis = 2)
                x_hid = self.w_hidden @ hidden_state_1 + self.b_hidden
                hid_out = sigmoid(x_in + x_hid)
                hidden_state_1 = hid_out
                x_out = self.w_out @ hid_out + aprox1_b_out
                loss1 += MSE(x_out,np.expand_dims(y[:,:,i],axis = 2))


            hidden_state_2 = np.zeros((batch_size,self.hidden_dim,1))
            loss2 = np.zeros((batch_size,1))
            T = x.shape[-1]

            for i in range(T):
                x_in = self.w_in @ np.expand_dims(x[:,:,i],axis = 2)
                x_hid = self.w_hidden @ hidden_state_2 + self.b_hidden
                hid_out = sigmoid(x_in + x_hid)
                hidden_state_2 = hid_out
                x_out = self.w_out @ hid_out + aprox2_b_out
                loss2 += MSE(x_out,np.expand_dims(y[:,:,i],axis = 2))

            nable_b_out[k] = (np.sum((loss1 - loss2) / (2*self.epsilon))) /␣
↪batch_size

        return nable_b_hidden,nable_b_out,nable_w_in,nable_w_hidden,nable_w_out

    def backprop(self, x, y):
        hidden_state = np.zeros((batch_size,self.hidden_dim,1))
        hidden_out = []
        hidden_out.append(hidden_state)
        hidden_z = []
        y_out = []
```

```
        dldw_in = np.zeros((batch_size,) + self.w_in.shape)
        dldw_hidden = np.zeros((batch_size,) + self.w_hidden.shape)
        dldb_hidden = np.zeros((batch_size,) + self.b_hidden.shape)
        dldw_out = np.zeros((batch_size,) + self.w_out.shape)
        dldb_out = np.zeros((batch_size,) + self.b_out.shape)
        loss = 0
        T = x.shape[-1]

        for i in range(T):
            x_in = self.w_in @ np.expand_dims(x[:,:,i],axis = 2)
            x_hid = self.w_hidden @ hidden_state + self.b_hidden
            hidden_z.append(x_in+x_hid)
            # hid_out = tanh(x_in + x_hid)
            hid_out = sigmoid(x_in + x_hid)
            hidden_out.append(hid_out)
            hidden_state = hid_out
            x_out = self.w_out @ hid_out + self.b_out
            y_out.append(x_out)
            loss += MSE(x_out,np.expand_dims(y[:,:,i],axis = 2))

        '''
        dldw_out
        '''


        for i in range(1,T+1):

            dldb_out += d_MSE(y_out[-i],np.expand_dims(y[:,:,-i],axis = 2))
            dldw_out += d_MSE(y_out[-i],np.expand_dims(y[:,:,-i],axis = 2)) @␣
↪np.transpose(hidden_out[-i],(0, 2, 1))

        '''
        dldw_hidden
        dldw_input
        '''

        for i in range(1,T+1):

            delta = d_MSE(y_out[-i],np.expand_dims(y[:,:,-i],axis = 2))
            delta = np.transpose(self.w_out) @ delta * d_sigmoid(hidden_z[-i])
            dldb_hidden += delta
            dldw_hidden += delta @ np.transpose(hidden_out[-i-1],(0, 2, 1))
            dldw_in += delta @ np.transpose(np.expand_dims(x[:,:,-i],axis =␣
↪2),(0, 2, 1))

            for k in range(i+1,T+1):
```

```
                delta = np.transpose(self.w_hidden) @ delta *␣
↪d_sigmoid(hidden_z[-k])
                dldb_hidden += delta
                dldw_hidden += delta @ np.transpose(hidden_out[-k-1],(0, 2, 1))
                dldw_in += delta @ np.transpose(np.expand_dims(x[:,:,-k],axis =␣
↪2),(0, 2, 1))

        dldb_hidden = 2 * dldb_hidden.sum(axis = 0) / batch_size
        dldb_out = 2 * dldb_out.sum(axis = 0) / batch_size
        dldw_in = 2 * dldw_in.sum(axis = 0) / batch_size
        dldw_hidden = 2 * dldw_hidden.sum(axis = 0) / batch_size
        dldw_out = 2 * dldw_out.sum(axis = 0) / batch_size
        loss = loss.sum(axis = 0) / batch_size

        return (dldb_hidden,dldb_out,dldw_in,dldw_hidden,dldw_out,loss)
```

[479]:
```
input = []
target = []
T = 50
Num = 50


for i in range(Num):

    #parameters = np.random.randint(1,5,(3,1))
    p = np.random.uniform(1,5)
    fix = np.array([0.0,1.0])
    parameters = np.insert(fix,0,p).reshape(3,1)
    x = np.tile(parameters,(T))
    time = np.linspace(0, 5, T)
    frequency = x[0][0]
    theta = x[1][0]
    amplitude = x[2][0]
    y = amplitude * np.sin(frequency * time + theta)
    input.append(x)
    target.append(y)

batch_size = 5
input = np.array(input)
target = np.array(target).reshape(Num,T)
batch_input = np.split(input, Num/batch_size)
batch_target = np.split(target, Num/batch_size)
```

[480]:
```
%%time
sinrnn = RNN(3,100,1,0.0001)
Loss = []
```

```
for i in range(500):
    idx = 0
    for input,target in zip(batch_input,batch_target):
        #nable_b_hidden,nable_b_out,nable_w_in,nable_w_hidden,nable_w_out =␣
 ↪sinrnn.gradient_approximation(input,np.expand_dims(target,axis = 1))
        #dldb_hidden,dldb_out,dldw_in,dldw_hidden,dldw_out,loss = sinrnn.
 ↪backprop(input,np.expand_dims(target,axis = 1))
        #print('approximation',(nable_w_hidden / dldw_hidden).sum())
        sinrnn.update(input,np.expand_dims(target,axis = 1))
        Loss.append(np.sum(sinrnn.loss))
    print('epoch',i,': ',np.sum(sinrnn.loss))
    sinrnn.loss.clear()
```

```
epoch 0 :  384.96272671316905
epoch 1 :  317.3553270089092
epoch 2 :  297.8930150586428
epoch 3 :  284.2806773588231
epoch 4 :  274.323033908126
epoch 5 :  266.78721260860516
epoch 6 :  260.98492895343463
epoch 7 :  256.4408307366314
epoch 8 :  252.79029919712232
epoch 9 :  249.77860928144597
epoch 10 :  247.24154617060128
epoch 11 :  245.07318332293607
epoch 12 :  243.20071353003752
epoch 13 :  241.56971989930193
epoch 14 :  240.13696631860316
epoch 15 :  238.86722971692166
epoch 16 :  237.73177371657448
epoch 17 :  236.70730289292837
epoch 18 :  235.77503224337556
epoch 19 :  234.91983346922345
epoch 20 :  234.12949061726087
epoch 21 :  233.3940839561098
epoch 22 :  232.7054982038124
epoch 23 :  232.05703840300407
epoch 24 :  231.44313307432913
epoch 25 :  230.8591055536442
epoch 26 :  230.3009975431707
epoch 27 :  229.7654322755983
epoch 28 :  229.24950765118027
epoch 29 :  228.75071208877748
epoch 30 :  228.26685766198142
epoch 31 :  227.79602646553258
epoch 32 :  227.33652717696705
```

```
epoch 33 :  226.88685953163002
epoch 34 :  226.44568498536253
epoch 35 :  226.01180225078173
epoch 36 :  225.58412669891223
epoch 37 :  225.16167284627122
epoch 38 :  224.74353931889536
epoch 39 :  224.32889581409353
epoch 40 :  223.91697167871067
epoch 41 :  223.50704579722614
epoch 42 :  223.09843753980613
epoch 43 :  222.6904985636521
epoch 44 :  222.28260529370215
epoch 45 :  221.87415193319325
epoch 46 :  221.46454387243182
epoch 47 :  221.05319137659356
epoch 48 :  220.63950344141452
epoch 49 :  220.22288171003808
epoch 50 :  219.80271434578083
epoch 51 :  219.3783697550583
epoch 52 :  218.94919005336817
epoch 53 :  218.51448416700026
epoch 54 :  218.0735204671442
epoch 55 :  217.62551884651882
epoch 56 :  217.16964218015588
epoch 57 :  216.70498717564166
epoch 58 :  216.2305747367672
epoch 59 :  215.74534017452268
epoch 60 :  215.24812395744607
epoch 61 :  214.73766428567427
epoch 62 :  214.2125937248688
epoch 63 :  213.6714436168122
epoch 64 :  213.11266219208636
epoch 65 :  212.5346554092338
epoch 66 :  211.93586347518033
epoch 67 :  211.31489007609636
epoch 68 :  210.67070351888853
epoch 69 :  210.00292488003817
epoch 70 :  209.31220077538597
epoch 71 :  208.6006199970453
epoch 72 :  207.8720744737025
epoch 73 :  207.13240843639625
epoch 74 :  206.3891964607837
epoch 75 :  205.65109132172108
epoch 76 :  204.92686720747037
epoch 77 :  204.22443382864157
epoch 78 :  203.5500873394855
epoch 79 :  202.90809923457567
epoch 80 :  202.30054599624916
```

```
epoch 81 :  201.72714576605443
epoch 82 :  201.18478523459024
epoch 83 :  200.66631597160656
epoch 84 :  200.15799632832352
epoch 85 :  199.63468451912033
epoch 86 :  199.0525187775265
epoch 87 :  198.34886123741248
epoch 88 :  197.54609662846735
epoch 89 :  197.5855381928708
epoch 90 :  201.23771233742588
epoch 91 :  202.49627483650525
epoch 92 :  201.60125097463
epoch 93 :  201.14363840751147
epoch 94 :  200.59735452245062
epoch 95 :  200.07152924006377
epoch 96 :  199.55553279400783
epoch 97 :  199.0411855870473
epoch 98 :  198.5222516602989
epoch 99 :  197.99436633617267
epoch 100 :  197.45530523071133
epoch 101 :  196.90488402740394
epoch 102 :  196.3447167873849
epoch 103 :  195.77793182028256
epoch 104 :  195.20887462522793
epoch 105 :  194.6427956060561
epoch 106 :  194.08550765835588
epoch 107 :  193.54299657372954
epoch 108 :  193.02097098417593
epoch 109 :  192.52434754390853
epoch 110 :  192.0566846441998
epoch 111 :  191.61961121383052
epoch 112 :  191.21235207203156
epoch 113 :  190.83151980829902
epoch 114 :  190.47138428644107
epoch 115 :  190.12476924840908
epoch 116 :  189.78450442130486
epoch 117 :  189.44504129896555
epoch 118 :  189.10362998685315
epoch 119 :  188.76056179901684
epoch 120 :  188.41839306145766
epoch 121 :  188.0805177197531
epoch 122 :  187.7496769405347
epoch 123 :  187.42692059994573
epoch 124 :  187.11129248970803
epoch 125 :  186.80023916414308
epoch 126 :  186.4905101944382
epoch 127 :  186.17916423727888
epoch 128 :  185.8642789100091
```

```
epoch 129 :  185.54511758514744
epoch 130 :  185.2217703939583
epoch 131 :  184.89451638104754
epoch 132 :  184.5632303667304
epoch 133 :  184.22706825233695
epoch 134 :  183.88448489737593
epoch 135 :  183.5334705774882
epoch 136 :  183.17181077949158
epoch 137 :  182.79720528313044
epoch 138 :  182.4071938309334
epoch 139 :  181.99895272892294
epoch 140 :  181.56907793433362
epoch 141 :  181.11343576972865
epoch 142 :  180.6270879931638
epoch 143 :  180.1042586894007
epoch 144 :  179.53834806669855
epoch 145 :  178.92208172205235
epoch 146 :  178.24793831200304
epoch 147 :  177.50898199177732
epoch 148 :  176.7001551977624
epoch 149 :  175.81998710999778
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epoch 151 :  173.87058877946976
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epoch 165 :  161.3798584733346
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epoch 176 :  147.7113370834704
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epoch 199 :  123.37915923635545
epoch 200 :  122.18265122405693
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epoch 307 :  11.597127240917857
epoch 308 :  11.417733530888675
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epoch 378 :  5.017093568144808
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epoch 416 :  3.6430922603528804
```

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epoch 460 :  2.6453236350372595
epoch 461 :  2.6276700485161033
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epoch 463 :  2.592928329132403
epoch 464 :  2.5758369834699755
```

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epoch 465 :  2.5589299372541876
epoch 466 :  2.542205739036521
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epoch 470 :  2.477111134740107
epoch 471 :  2.4612823094251945
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epoch 473 :  2.430152297708571
epoch 474 :  2.4148501202528454
epoch 475 :  2.399722986018511
epoch 476 :  2.3847712680475404
epoch 477 :  2.36999591328064
epoch 478 :  2.3553986584686575
epoch 479 :  2.3409823461890777
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epoch 482 :  2.2988756560962202
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epoch 488 :  2.221761807138025
epoch 489 :  2.2106406876846245
epoch 490 :  2.2005675352673926
epoch 491 :  2.192076908277077
epoch 492 :  2.1860886708752094
epoch 493 :  2.1842541055723355
epoch 494 :  2.1896994986282494
epoch 495 :  2.208732588644157
epoch 496 :  2.255005036600102
epoch 497 :  2.360162356860622
epoch 498 :  2.601639591278877
epoch 499 :  3.171622111761611
CPU times: user 12min 9s, sys: 1.6 s, total: 12min 11s
Wall time: 12min 11s
```
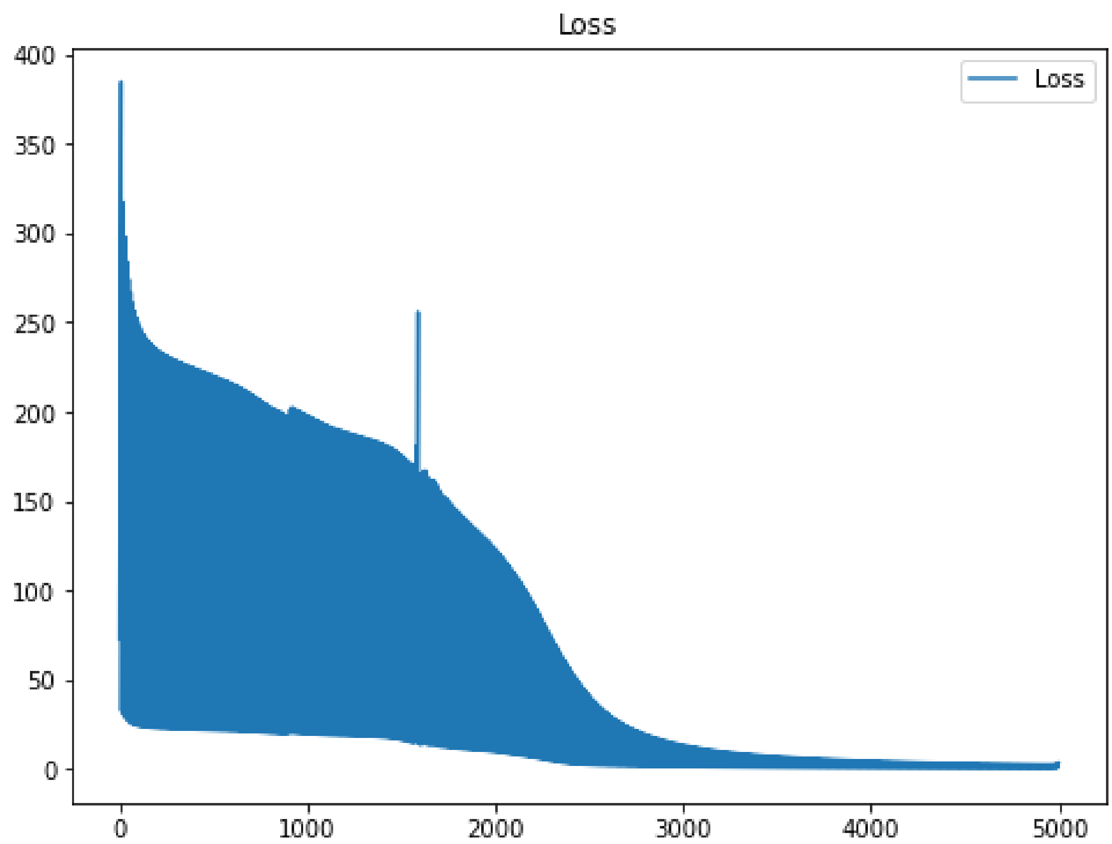

[481]:

```
plt.figure(figsize = (8, 6))
plt.plot(Loss, label = 'Loss', markersize = 3)
plt.title("Loss")
plt.legend()
plt.show()
```



[453]:
```
for i in range(500,1000):
    for input,target in zip(batch_input,batch_target):
        sinrnn.update(input,np.expand_dims(target,axis = 1))
        Loss.append(np.sum(sinrnn.loss))
    print('epoch',i,': ',np.sum(sinrnn.loss))
    sinrnn.loss.clear()
```

```
epoch 500 :  198.24944756470492
epoch 501 :  202.5444883848307
epoch 502 :  197.45436722823044
epoch 503 :  189.84922773637317
epoch 504 :  173.92292426693962
epoch 505 :  184.4960952117848
epoch 506 :  181.6608038112666
epoch 507 :  176.2926666643467
epoch 508 :  184.8534715946752
epoch 509 :  180.4805952762221
epoch 510 :  176.84409165695405
epoch 511 :  181.71584382087377
epoch 512 :  174.01212389699123
epoch 513 :  201.08760487224978
```

```
epoch 514 :  204.56618449056907
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epoch 523 :  174.94470148982015
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epoch 543 :  183.90735008346488
epoch 544 :  178.60397953638386
epoch 545 :  176.72492288247003
epoch 546 :  167.70911395335517
epoch 547 :  176.43872217796675
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epoch 549 :  169.0886068866996
epoch 550 :  168.70257653326462
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epoch 552 :  180.14106418269301
epoch 553 :  176.38138332177317
epoch 554 :  197.75603433370344
epoch 555 :  202.45918032594003
epoch 556 :  196.23859431011766
epoch 557 :  186.5965134981833
epoch 558 :  167.9752625322221
epoch 559 :  183.32585478050504
epoch 560 :  184.19404644186034
epoch 561 :  175.8827440123618
```

```
epoch 562 :  167.40344985822344
epoch 563 :  193.20202696999078
epoch 564 :  198.07641062082675
epoch 565 :  188.81564245734327
epoch 566 :  170.4653649300191
epoch 567 :  185.54429933266772
epoch 568 :  188.14160175300586
epoch 569 :  168.38014366917847
epoch 570 :  188.2316379856375
epoch 571 :  192.98477059621817
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epoch 574 :  167.83445259399238
epoch 575 :  189.9982050996111
epoch 576 :  195.41831014289696
epoch 577 :  183.9798219040802
epoch 578 :  165.71554285368546
epoch 579 :  185.57036032225938
epoch 580 :  188.92634926376797
epoch 581 :  169.91016033617257
epoch 582 :  184.4591722771166
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epoch 585 :  185.36817306802558
epoch 586 :  188.79760423147707
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epoch 592 :  191.62295779832039
epoch 593 :  175.92598722036107
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epoch 597 :  190.63898115006486
epoch 598 :  173.72105549381132
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epoch 600 :  171.23371679665968
epoch 601 :  183.14393095410298
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epoch 603 :  164.84538531461126
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epoch 606 :  164.72813132654682
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epoch 608 :  184.7066998003348
epoch 609 :  164.48831090434518
```

```
epoch 610 :  182.42572425574437
epoch 611 :  184.1404888840592
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epoch 613 :  182.71990758694696
epoch 614 :  184.71320256747794
epoch 615 :  164.32903466215944
epoch 616 :  180.94280317545605
epoch 617 :  181.49150970183283
epoch 618 :  162.8212539402514
epoch 619 :  181.15531960544803
epoch 620 :  180.15076706310697
epoch 621 :  166.80941821164575
epoch 622 :  170.80729735695883
epoch 623 :  162.32855723246823
epoch 624 :  180.07962071511085
epoch 625 :  176.52124535576183
epoch 626 :  168.37624734925475
epoch 627 :  175.69587002713445
epoch 628 :  167.3153235858682
epoch 629 :  181.05645734820217
epoch 630 :  182.3390571874286
epoch 631 :  161.4569987870986
epoch 632 :  183.63560400986017
epoch 633 :  186.52210372690502
epoch 634 :  166.2628558899888
epoch 635 :  174.48058017552086
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epoch 637 :  181.35341209196983
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epoch 639 :  161.5591508520815
epoch 640 :  177.41427808645753
epoch 641 :  175.20456035602757
epoch 642 :  168.5745539450775
epoch 643 :  160.61479983719377
epoch 644 :  180.86154587654949
epoch 645 :  179.9082031147004
epoch 646 :  160.1098816820844
epoch 647 :  182.11140650635772
epoch 648 :  183.21233121625386
epoch 649 :  161.87127221942768
epoch 650 :  175.52241446388607
epoch 651 :  171.67856624532217
epoch 652 :  174.00805607586574
epoch 653 :  166.7632181822241
epoch 654 :  177.4543594457402
epoch 655 :  175.73890521022702
epoch 656 :  168.42411858660788
epoch 657 :  159.1917738743095
```

```
epoch 658 :  183.1166219598456
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epoch 704 :  159.8230701020177
epoch 705 :  150.6000382221719
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epoch 723 :  146.69400928470156
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epoch 806 :  24.63150241376434
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epoch 878 : 14.733950360789656
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epoch 946 :  10.627395227875798
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epoch 993 :  8.625835377840684
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epoch 994 :  8.588629890744192
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epoch 996 :  8.514817557629488
epoch 997 :  8.478208099775838
epoch 998 :  8.44179472525303
epoch 999 :  8.40557615601341
```
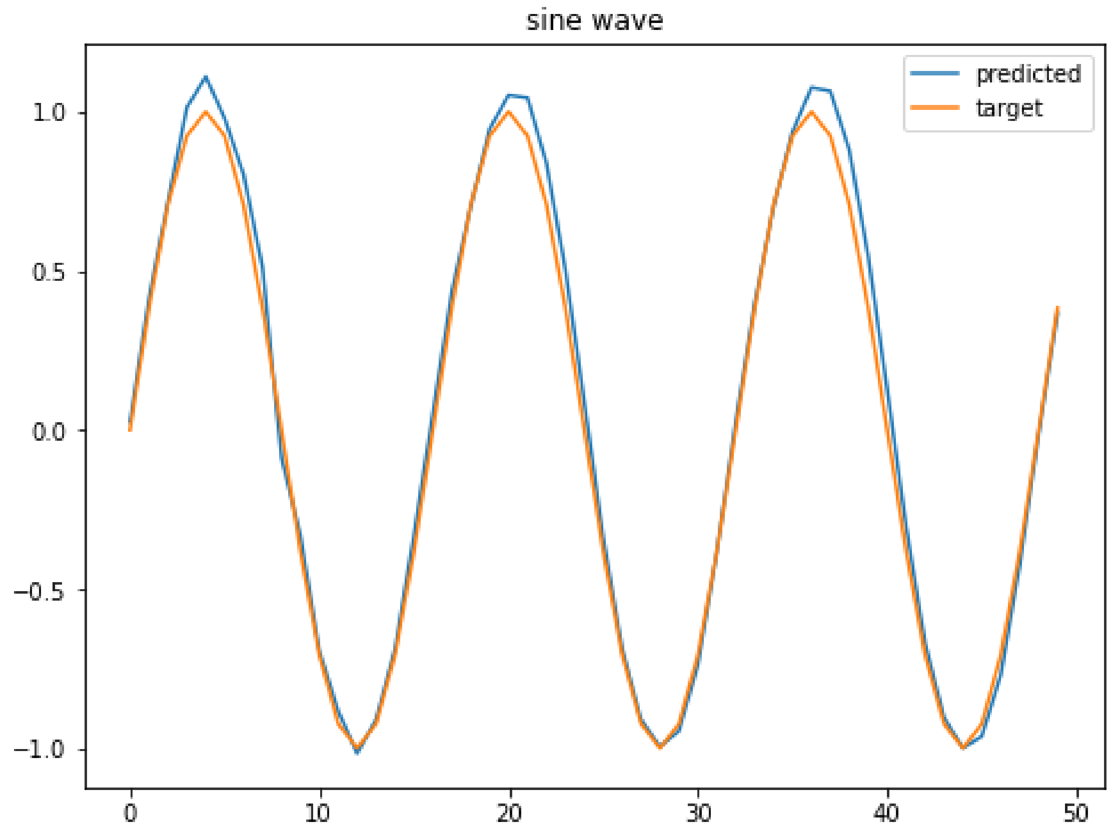

[493]:

```
#parameters = np.ones((3,1))
T = 50
p = np.random.uniform(1,5)
fix = np.array([0.0,1.0])
parameters = np.insert(fix,0,p).reshape(3,1)
x = np.tile(parameters,(T))
time = np.linspace(0, 5, T)
frequency = x[0][0]
theta = x[1][0]
amplitude = x[2][0]
y = amplitude * np.sin(frequency * time + theta)
print('frequency: ',p)


predicted = sinrnn.forward(x)
target = y
plt.figure(figsize = (8, 6))
plt.plot(predicted, label = 'predicted', markersize = 3)
plt.plot(target, label = 'target', markersize = 3)
plt.title("sine wave")
plt.legend()
plt.show()
```

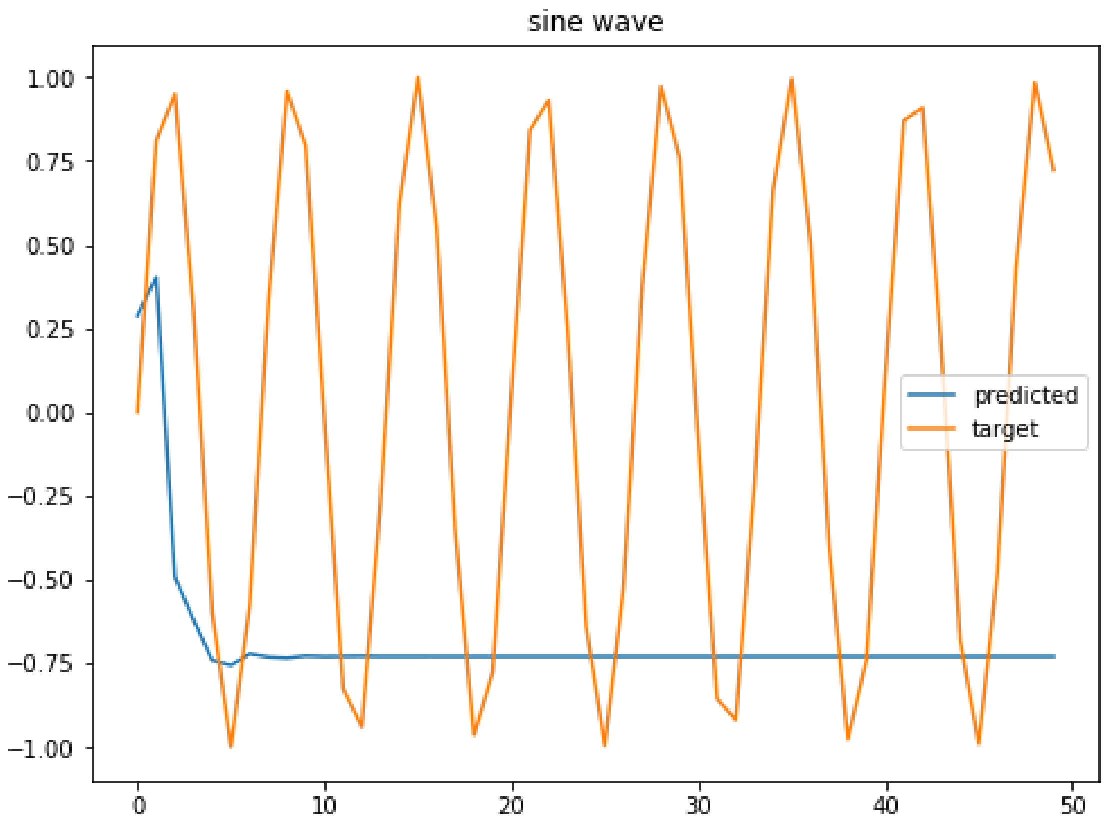
```
frequency:  3.848427718978435
50
```



[495]:
```
T = 50
p = np.random.uniform(1,10)
fix = np.array([0.0,1.0])
parameters = np.insert(fix,0,p).reshape(3,1)
x = np.tile(parameters,(T))
time = np.linspace(0, 5, T)
frequency = x[0][0]
theta = x[1][0]
amplitude = x[2][0]
y = amplitude * np.sin(frequency * time + theta)
print('frequency: ',p)


predicted = sinrnn.forward(x)
target = y
plt.figure(figsize = (8, 6))
plt.plot(predicted, label = 'predicted', markersize = 3)
plt.plot(target, label = 'target', markersize = 3)
plt.title("sine wave")
plt.legend()
```

```
plt.show()
```

```
frequency:  9.263371130938035
50
```




```
[ ]:
```