# RL: LEARN TO CUT PROJECT REPORT

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## ABSTRACT

Integer programming, in theory, is an NP-hard problem. In the paper Reinforcement Learning for Integer Programming: Learning to Cut [5], the author used a RL approach to solve the problem that significantly outperforms human-designed heuristics. In this report, we followed the work done by the authors, with some innovations and adaptations, to tackle IP. The adapted methods can (1) tackle even more difficult IP problems and (2) generalize to new instances.

## 1 Introduction and problems

The problem set up is the following:

$$\min\{c^T x : Ax \leq b, x \geq 0, x \in \mathbb{Z}^n\}$$

The Gomory cuts provide an effective method to solve integer programming. At each iteration, the Gomory cuts algorithm provides a set of possible cuts for us to cut the plane, which is adding more constraints. The goal is to find the optimality gap by adding more cuts, which is to optimize $OPT_{LP} - OPT_{Integer}$ [2] by adding more constraints to the integer programming.
Here, we can formulate the problem in the RL diagram:
Let $n$ be the number of variables:
At the $t = 0$, the state space is the set of all constraint: **S** = $\{A_t x \leq b_t\}$ where $\forall a_i \in A_t, a_i \in \mathbb{R}^n$ and $\forall b_i \in b_t, b_i \in \mathbb{R}$.
The action space is the possible new constraints/cuts generated by the Gomory cuts: **A** = $\{E_t x \leq d_t\}$ where $\forall e_i \in E_t, e_i \in \mathbb{R}^n$ and $\forall d_i \in d_t, d_i \in \mathbb{R}$.
The transition from $s_t$ to $s_{t+1}$ is simply adding the cut into the previous state as a new constraint.
Let $x^*_{LP}$ be the optimal solution, the reward is $C^T x^*_{LP}(t+1) - C^T x^*_{LP}(t) \geq 0$. We also use discounted reward with $\gamma$ to encourage the agent to find the solution early.

## 2 Related Work

This project mostly follows the paper Reinforcement Learning for Integer Programming: Learning to Cut [5].

## 3 Algorithm and Training method

### 3.1 Policy Architecture: LSTM and Attention

The policy $\pi$ samples an action from the action space each time: $a = \pi(s_t; \theta)$. The policy is a parametrized neural network.The core dilemma is that size of state and action space is not fixed. To solve this problem, we use the LSTM[4] and the Attention Mechanism[6][3]. First of all, we concatenate $A$ and $b$ and call it the constraint matrix, and concatenate $E$ and $d$ call it the candidate matrix. At time step $T = t$, the constraint matrix is a matrix with the shape of $\mathbb{R}^{(m+t)\times(n+1)}$, where $m + t$ is the number of constraints and $n$ is the number of variables. Similarly, for the candidate matrix, it is a matrix with the shape of $\mathbb{R}^{(k)\times(n+1)}$, where $k$ is the number of candidate Gomory cuts. The core idea of the algorithm is to calculate the attention score for each cuts in the candidate with all the constraint in the constraint matrix.[5] And the candidate with the highest attention score will be the most aligned with the set of constraints, which means the cuts with the highest reward.

The implementation is the following: we first use an LSTM with hidden dimension of 32 to embed the constraint matrix and candidate matrix. Here, we use the last hidden cell output of the LSTM as the final embedding. For the embedding, we pass it to a 2 hidden layer neural network with hidden unit of size 64 and Tanh nonlinear activation functions. The output dimension is $k = 16$. Let's call the embedded constraint matrix as $H$ and the embedded candidate matrix as $G$. The attention score matrix is calculated as

$$S_j = \frac{1}{N_t} \sum_{i=1}^{N_t} g_j^T h_i \forall\, h_i, g_j \text{ in H, G[5]}$$

The action is then simply the argmax over the Softmax of the attention score.

### 3.2 Policy Training: Policy Gradient

The policy gradient is:

$$\nabla_\phi \rho(\phi) = \mathbb{E}_\pi \big[ \sum_{t=0}^{\infty} Q^{\pi_\theta}(s_t, a_t) \nabla_\theta \log \pi(s_t, a_t; \theta) \big] [1]$$

To compute the gradient and update $\theta \leftarrow \theta + \alpha \nabla_\theta \rho(\theta)$, where $\alpha$ is the learning rate, we need to find the value of $Q^{\pi_\theta}(s_t, a_t)$. However, it is not analytically solvable, and thus requires us to estimate with either Monte Carlo estimation, or train a Value function as the baseline( Actor-Critic). In this problem, we simply sample one trajectory of cuts, which is one episode with discounted reward with $\gamma$, and compute the loss and gradient estimator $\hat{g}$ as the following:

For one trajectory of collected estimated $Q_s$ and a list of state action pairs $(s(t = i), a(t = i))$, the Loss is defined as $-\frac{1}{N} \sum_{i=1}^{N} Q_s(t = i) \log(\pi(s(t = i), a(t = i); \theta))$, and the gradient estimator $\hat{g}$ is:

$$-\frac{1}{N} \sum_{i=1}^{N} Q_s(t = i) \nabla_\theta \log(\pi(s(t = i), a(t = i); \theta) [1]$$

### 3.3 Evolution Strategies

For the evolution strategies, I did not strictly follow what was described in the paper [5]. The core idea of es is to calculate the noisy estimation of the performance of policy $\pi_\theta$. Thus we use the following noisy estimation of the $Q$ generated by policy $\pi_\theta$:

$$Q_i \leftarrow Q_i + \alpha \epsilon_i, \text{ where } \alpha = 0.15 \text{ and } \epsilon_i \sim \mathcal{N}(0, 1)$$

### 3.4 Curriculum Training

The easy configuration has 10 instances, where the hard configuration has 100 instances. To directly train the policy network on the hard configuration is extremely challenging. Instead, we propose the method of curriculum training: we generate three curriculums consists of 10, 40, 70 instances separately, and we train the same policy network on those three curriculum dataset first, and then train it on the hard configuration.

### 3.5 Data Processing

Data processing is extremely useful for deep learning tasks. In this task, we want to process both the constraint matrix and the candidate matrix. However, we do not want to lose the relative information about the constraint and the candidate, as numerical relationships are extremely useful. So we concatenate both the constraint and candidate matrix together and then standardize the matrix by subtract the mean and then divide the variance.

### 3.6 Pseudo code

Here are the algorithm for both easy config training and hard config training. Some terminology: $L_C, L_D, L_A, L_R$ refers to the list to record the constraint, candidates, actions, and rewards. $E_r$ refers to the cumulative episode rewards without discounting. $ND$ refers to not done, which is the condition that $x^*_{LP}(t)$ is not all integer value. $C$ is the state, which is the constraint matrix, and $D$ is the candidate space.

**Algorithm 1** Training [5]

```
Input: initialize a policy $\pi_\theta$, IP instance parameterized by c, A, b, timelimits T
Hyperparameter: learning rate $\alpha$, ES random rate $\sigma$
for $e_t \in$ episodes do
    reset environment, initialize t = 0, $E_r$ = 0, empty lists to record $L_C, L_D, L_A, L_R$
    $C(0) = [A, b]$, find $x^*_{LP}(0)$ and $D(0) = [E, d]$
    while ND and t < T do
        $s_t = \{C(t), D(t), x^*_{LP}(t)\}$
        standardize $C(t)$ and $D(t)$
        compute attention score with $att_{score}(t) = \pi_\theta(C(t), D(t))$
        sample action as $a(t)$ = argmax of softmax($att_{score}(t)$)
        find the reward $r(t)$
        generate $C(t+1)$ by append the cut to $C(t)$
        append $C(t), D(t), a(t), r(t)$ to $L_C, L_D, L_A, L_R$
        $E_r \leftarrow E_r + r(t)$
        $t \leftarrow t + 1$
    end while
    calculate discounted reward $d_r$
    calculate $Q_s$ with $d_r$ and ES
    use $L_C, L_D, L_A, L_R, Q_s$ to compute the gradient estimator $\hat{g}$
    $\theta \leftarrow \theta + \alpha\hat{g}$
end for
return $\theta$
```

The algorithm for curriculum training is:

**Algorithm 2** Curriclum Traning

```
Input: initialize a policy $\pi_\theta$, Curriclums, hard config
for curriculum($i$) $\in$ Curriclums do
    Train the network $\pi_\theta$ with the Training algorithm above
end for
Train the network $\pi_\theta$ with hard config
return $\theta$
```

The roll out of the policy:

**Algorithm 3** Roll out of policy

```
Input: a trained policy $\pi_\theta$, IP instance parameterized by c, A, b, time limits T
reset environment, initialize t = 0, $E_r$ = 0
$C(0) = [A, b]$, find $x^*_{LP}(0)$ and $D(0) = [E, d]$
while ND and t < T do
    $s_t = \{C(t), D(t), x^*_{LP}(t)\}$
    standardize $C(t)$ and $D(t)$
    compute attention score with $att_{score}(t) = \pi_\theta(C(t), D(t))$
    sample action as $a(t)$ = argmax of softmax($att_{score}(t)$)
    find the reward $r(t)$
    generate $C(t+1)$ by append the cut to $C(t)$
    $E_r \leftarrow E_r + r(t)$
    $t \leftarrow t + 1$
end while
return $E_r$
```

# 4 Result: Training and Testing

## 4.1 easy configuration

For the easy configuration, the final converged reward is $1.115$. The moving average is the following:

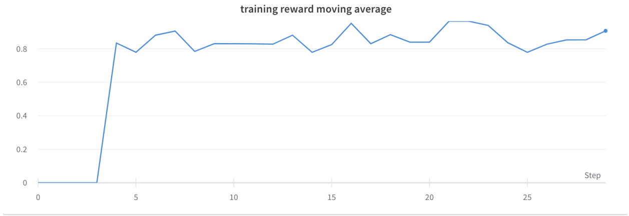


Figure 1: Averaged Moving Reward within 5 steps for easy configuration

## 4.2 hard configuration

With curriculum training on instances of 10, 40, 70, sequentially, the model have a converged final reward of $1.07268$, with a converged moving average reward above $1$:

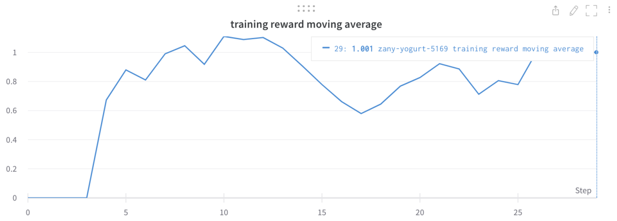


Figure 2: Averaged Moving Reward within 5 steps for hard configuration

## 4.3 Very hard configuration

Here is the result of training on 200 instances. We generate non trivial result.

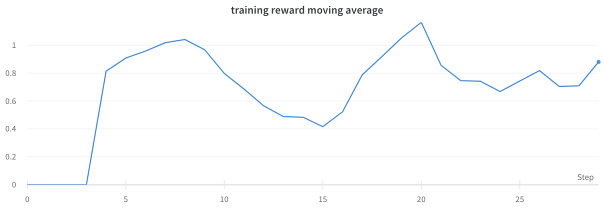
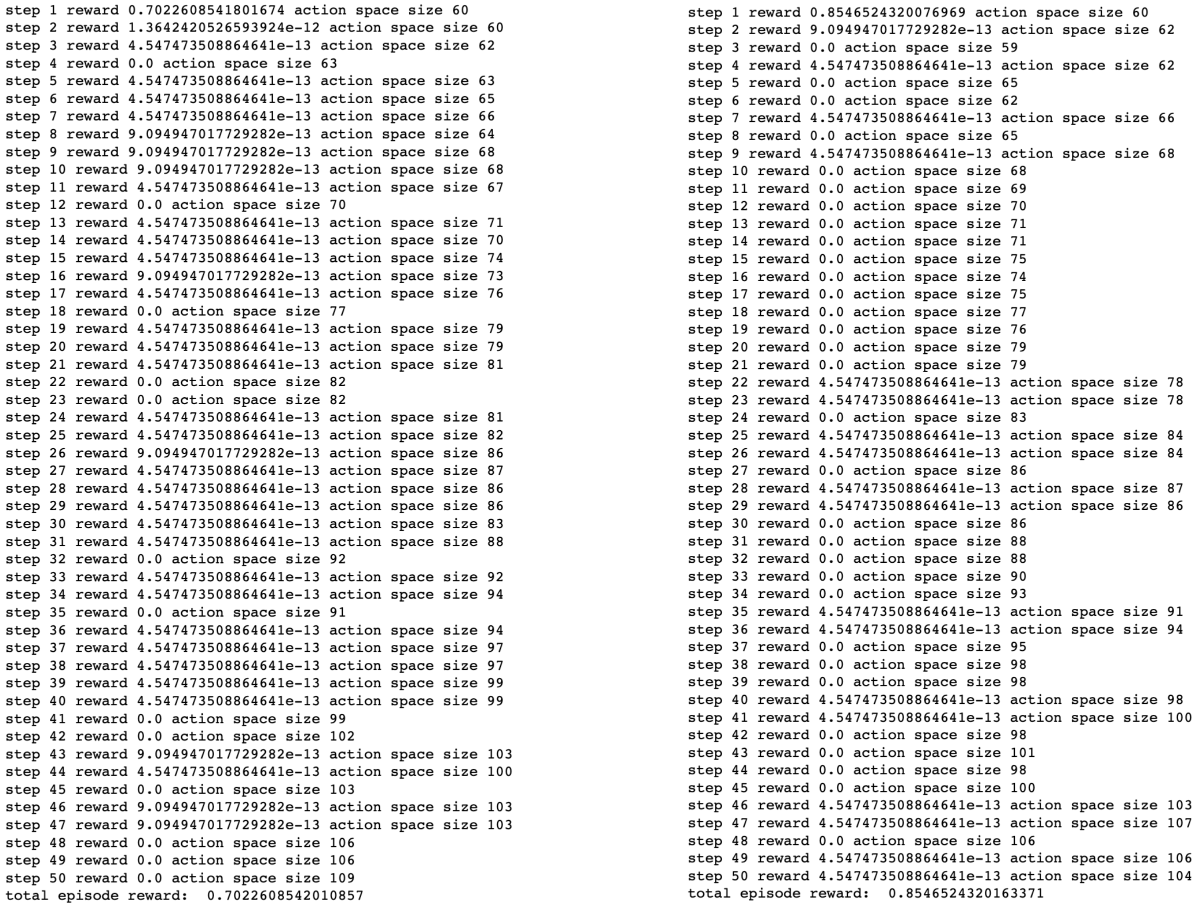


Figure 3: Averaged Moving Reward within 10 steps for super hard configuration

## 4.4 Testing configuration

Here we provide the complete cut log of both easy model and hard model when cutting an instance out of the training dataset. We can see that both models indeed prefers early cut(step 1).



```
step 1 reward 0.7022608541801674 action space size 60
step 2 reward 1.3642420526593924e-12 action space size 60
step 3 reward 4.547473508864641e-13 action space size 62
step 4 reward 0.0 action space size 63
step 5 reward 4.547473508864641e-13 action space size 63
step 6 reward 4.547473508864641e-13 action space size 65
step 7 reward 4.547473508864641e-13 action space size 66
step 8 reward 9.094947017729282e-13 action space size 64
step 9 reward 9.094947017729282e-13 action space size 68
step 10 reward 9.094947017729282e-13 action space size 68
step 11 reward 4.547473508864641e-13 action space size 67
step 12 reward 0.0 action space size 70
step 13 reward 4.547473508864641e-13 action space size 71
step 14 reward 4.547473508864641e-13 action space size 70
step 15 reward 4.547473508864641e-13 action space size 74
step 16 reward 9.094947017729282e-13 action space size 73
step 17 reward 4.547473508864641e-13 action space size 76
step 18 reward 0.0 action space size 77
step 19 reward 4.547473508864641e-13 action space size 79
step 20 reward 4.547473508864641e-13 action space size 79
step 21 reward 4.547473508864641e-13 action space size 81
step 22 reward 0.0 action space size 82
step 23 reward 0.0 action space size 82
step 24 reward 4.547473508864641e-13 action space size 81
step 25 reward 4.547473508864641e-13 action space size 82
step 26 reward 9.094947017729282e-13 action space size 86
step 27 reward 4.547473508864641e-13 action space size 87
step 28 reward 4.547473508864641e-13 action space size 86
step 29 reward 4.547473508864641e-13 action space size 86
step 30 reward 4.547473508864641e-13 action space size 83
step 31 reward 4.547473508864641e-13 action space size 88
step 32 reward 0.0 action space size 92
step 33 reward 4.547473508864641e-13 action space size 92
step 34 reward 4.547473508864641e-13 action space size 94
step 35 reward 0.0 action space size 91
step 36 reward 4.547473508864641e-13 action space size 94
step 37 reward 4.547473508864641e-13 action space size 97
step 38 reward 4.547473508864641e-13 action space size 97
step 39 reward 4.547473508864641e-13 action space size 99
step 40 reward 4.547473508864641e-13 action space size 99
step 41 reward 0.0 action space size 99
step 42 reward 0.0 action space size 102
step 43 reward 9.094947017729282e-13 action space size 103
step 44 reward 4.547473508864641e-13 action space size 100
step 45 reward 0.0 action space size 103
step 46 reward 9.094947017729282e-13 action space size 103
step 47 reward 9.094947017729282e-13 action space size 103
step 48 reward 0.0 action space size 106
step 49 reward 0.0 action space size 106
step 50 reward 0.0 action space size 109
total episode reward:  0.7022608542010857
```

(a) Testing Easy Model

```
step 1 reward 0.8546524320076969 action space size 60
step 2 reward 9.094947017729282e-13 action space size 62
step 3 reward 0.0 action space size 59
step 4 reward 4.547473508864641e-13 action space size 62
step 5 reward 0.0 action space size 65
step 6 reward 0.0 action space size 62
step 7 reward 4.547473508864641e-13 action space size 66
step 8 reward 0.0 action space size 65
step 9 reward 4.547473508864641e-13 action space size 68
step 10 reward 0.0 action space size 68
step 11 reward 0.0 action space size 69
step 12 reward 0.0 action space size 70
step 13 reward 0.0 action space size 71
step 14 reward 0.0 action space size 71
step 15 reward 0.0 action space size 75
step 16 reward 0.0 action space size 74
step 17 reward 0.0 action space size 75
step 18 reward 0.0 action space size 77
step 19 reward 0.0 action space size 76
step 20 reward 0.0 action space size 79
step 21 reward 0.0 action space size 79
step 22 reward 4.547473508864641e-13 action space size 78
step 23 reward 4.547473508864641e-13 action space size 78
step 24 reward 0.0 action space size 83
step 25 reward 4.547473508864641e-13 action space size 84
step 26 reward 4.547473508864641e-13 action space size 84
step 27 reward 0.0 action space size 86
step 28 reward 4.547473508864641e-13 action space size 87
step 29 reward 4.547473508864641e-13 action space size 86
step 30 reward 0.0 action space size 86
step 31 reward 0.0 action space size 88
step 32 reward 0.0 action space size 88
step 33 reward 0.0 action space size 90
step 34 reward 0.0 action space size 93
step 35 reward 4.547473508864641e-13 action space size 91
step 36 reward 4.547473508864641e-13 action space size 94
step 37 reward 0.0 action space size 95
step 38 reward 0.0 action space size 98
step 39 reward 0.0 action space size 98
step 40 reward 4.547473508864641e-13 action space size 98
step 41 reward 4.547473508864641e-13 action space size 100
step 42 reward 0.0 action space size 98
step 43 reward 0.0 action space size 101
step 44 reward 0.0 action space size 98
step 45 reward 0.0 action space size 100
step 46 reward 4.547473508864641e-13 action space size 103
step 47 reward 4.547473508864641e-13 action space size 107
step 48 reward 0.0 action space size 106
step 49 reward 4.547473508864641e-13 action space size 106
step 50 reward 4.547473508864641e-13 action space size 104
total episode reward:  0.8546524320163371
```

(b) Testing Hard Model

Figure 4: Performance of trained model on test dataset

## 5 Conclusion

In this project, we explored how to solve IP under the RL paradigm. The model can find a way to deal with the unfixed size of state space and action space with the attention mechanism. The current model can even more generalize to different set of instances and even different number of instances. We discovered a systematic way to train harder configurations: curriculum training. However, if the number of variable changes, we need to train a new model. A future direction could be finding a new method to embed different size of variables, such as sequentially embedding the instances with an LSTM of input size 1.

## References

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