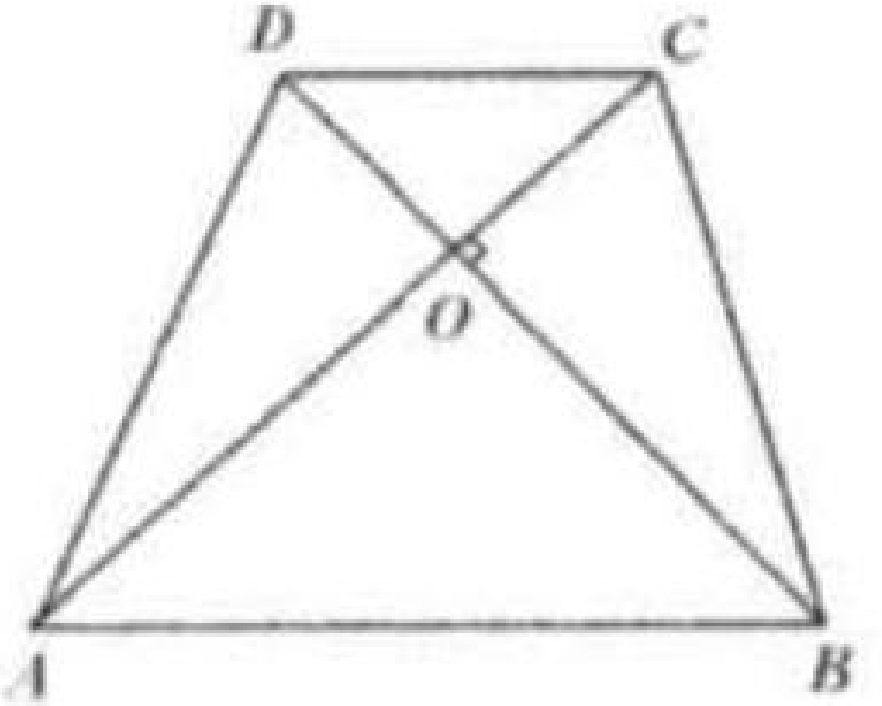
Example 14

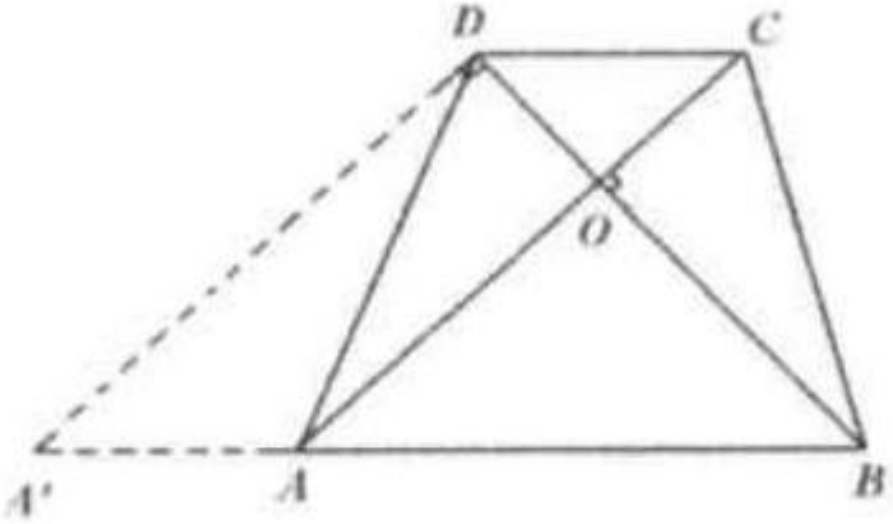
As shown in the figure, AC and BD are two diagonals of trapezoid $ABCD$ and $AC \perp BD$. Show that $AC^2 + BD^2 = (AB + DC)^2$.

Solution: Draw $DA'//CA$ and meets the extension of BA at A.



$A'ACD$ is a parallelogram with $A'D//AC$ and $A'D = AC$. Therefore, we know that $\angle A'DB$ is a right angle.

$$A'D^2 + DB^2 = A'B^2 \Rightarrow$$
$$AC^2 + BD^2 = (A'A + AB)^2 = (DC + AB)^2$$


D
C
O
A'
A
B