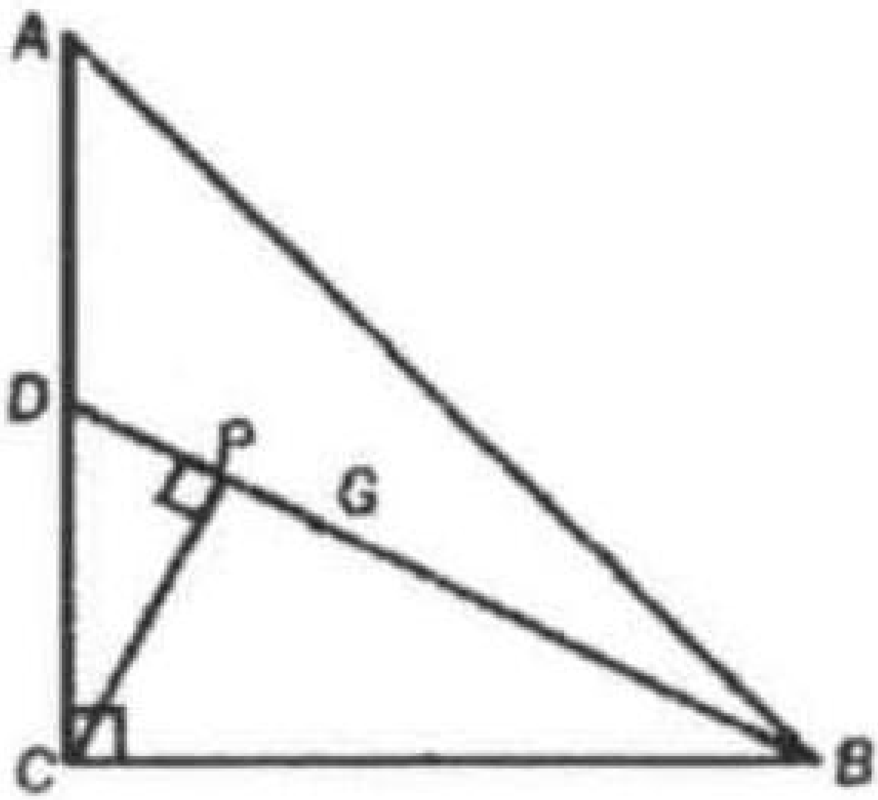
Problem

In $\triangle ABC$, angle C is a right angle. AC and BC are each equal to 1. D is the midpoint of $AC.BD$ is drawn, and a line perpendicular to BD at P is drawn from C. Find the distance from P to the intersection of the medians of $\triangle ABC$.

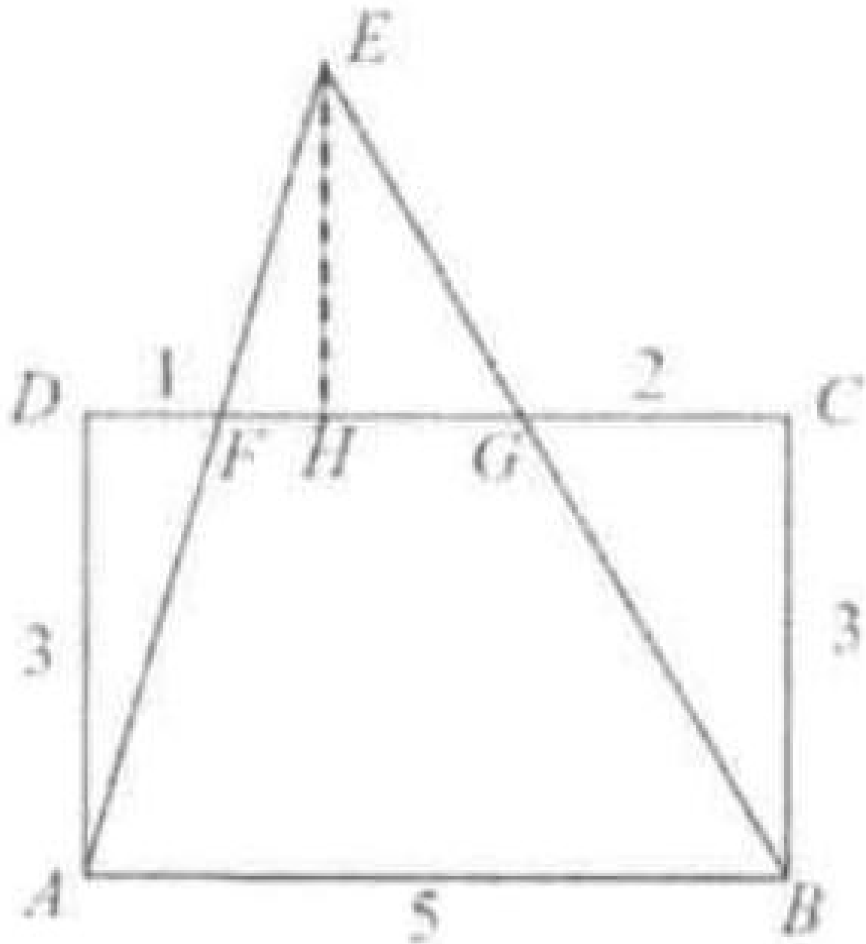


Solution

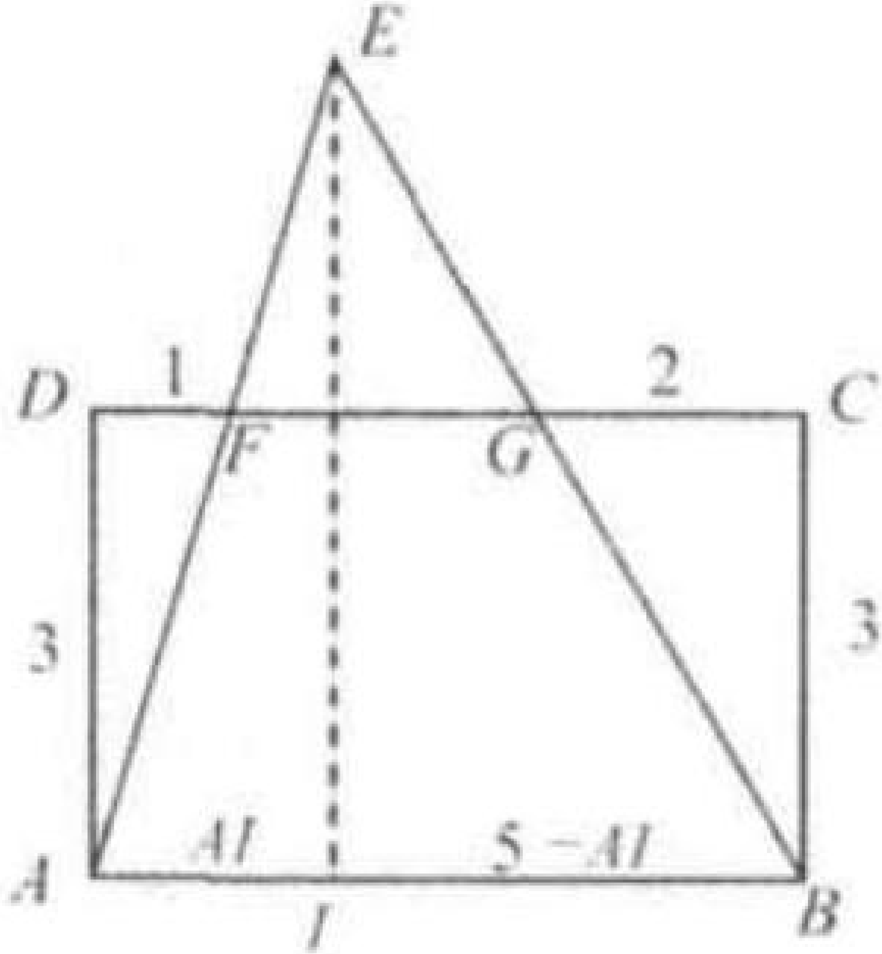
(D).

Method 1:

Let H be the foot of the perpendicular from E to DC. Since $CD = AB = 5$, $FG = 2$, and $\triangle FEG$ is similar to $\triangle AEB$, we have $\frac{EH}{EH+3} = \frac{2}{5}$, so $5EH = 2EH + 6$, and $EH = 2$. Hence Area $(\triangle AEB) = \frac{1}{2}(2+3) \cdot 5 = \frac{25}{2}$.



Method 2:
Let I be the foot of the perpendicular from E to AB. Since $\triangle EIA$ is similar to $\triangle ADF$ and $\triangle EIB$ is similar to $\triangle BCG$, we have $AI/EI = 1/3$ and $(5 - AI)/EI = 2/3$.
Adding gives $5/EI = 1$, so $EI = 5$. The area of the triangle is $(1/2) \cdot 5 \cdot 5 = 25/2$.


E
D
1
2
C
F
G
3
3
AI
5 − AI
A
I
B