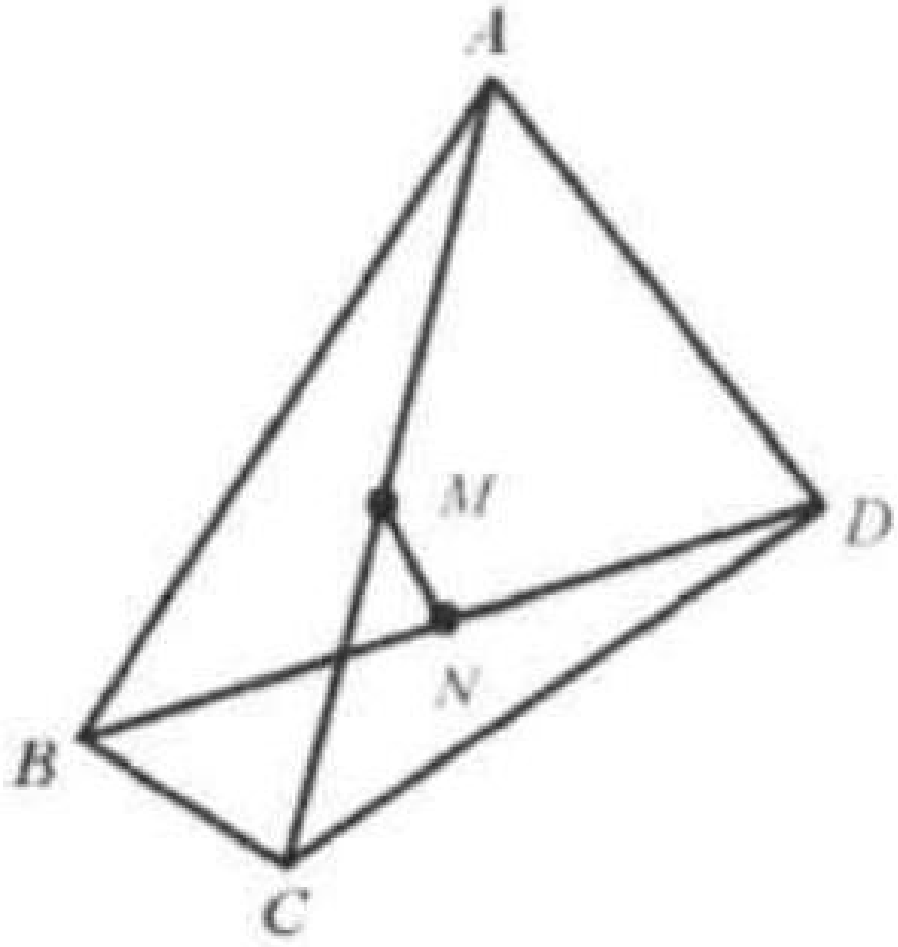
Problem 9

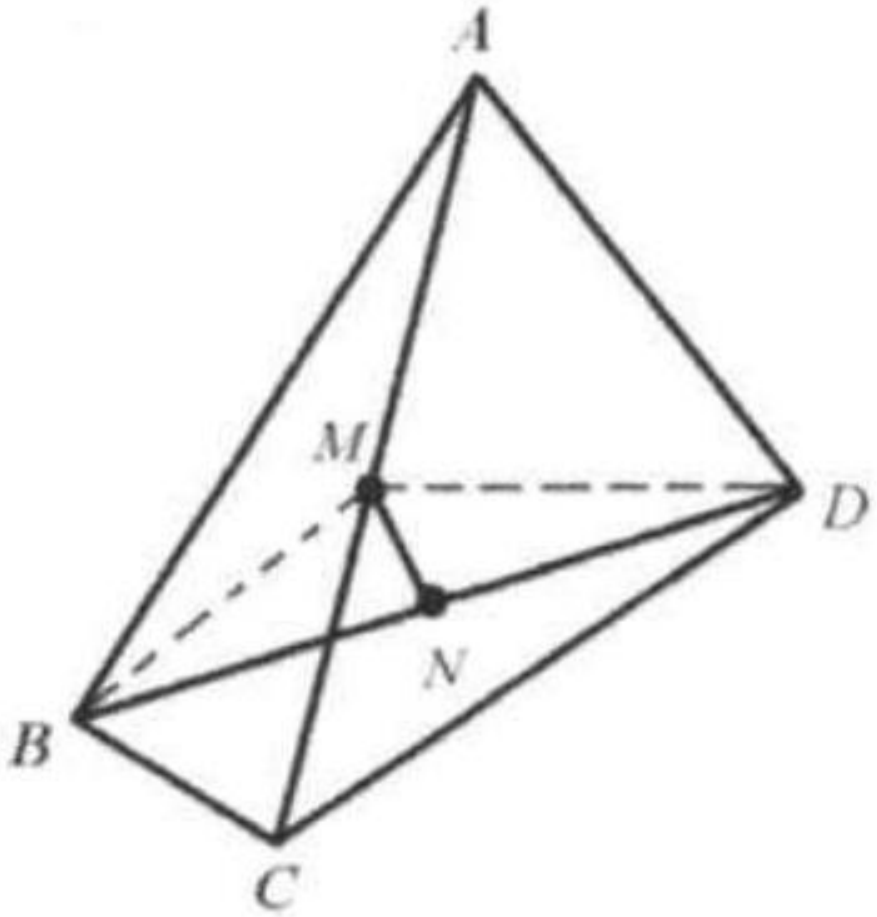
Problem

Both $\triangle ABC$ and $\triangle ADC$ are right triangles sharing the hypotenuse AC with $\angle ABC = \angle ADC = 90°$. Points M and n are the midpoints on sides AC and BD, respectively. Show that $MN \perp BD$.



Solution

Draw MB, the median of triangle ABC. Since MB is the median, by Theorem 1.3, $MB = MA = MC$
Draw MD, the median of triangle ADC. Since MD is the median, by Theorem 1.3, $MD = MA = MC$. So $MB = MD$.



Since $MB = MD$ and $BN = ND, MN$ is the perpendicular bisector of BD.
Thus $MN \perp BD$.