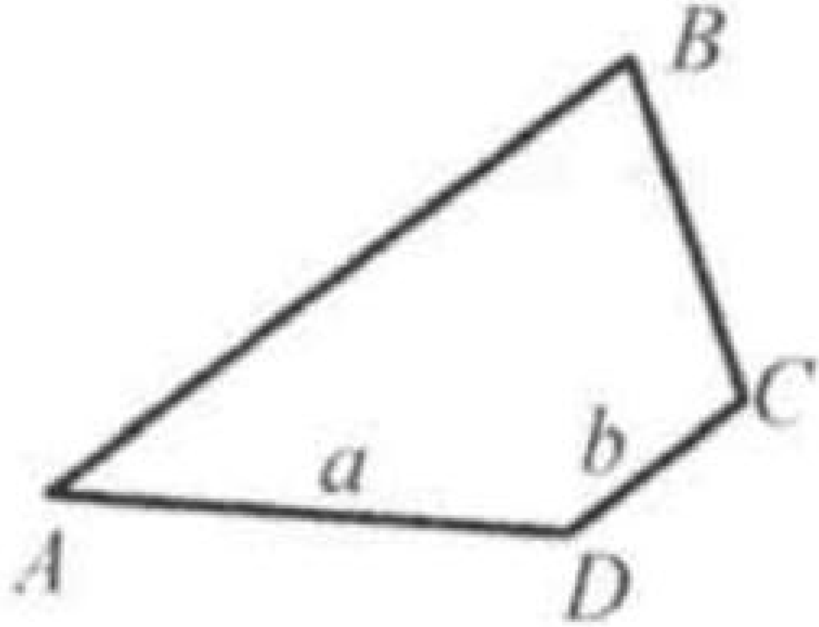
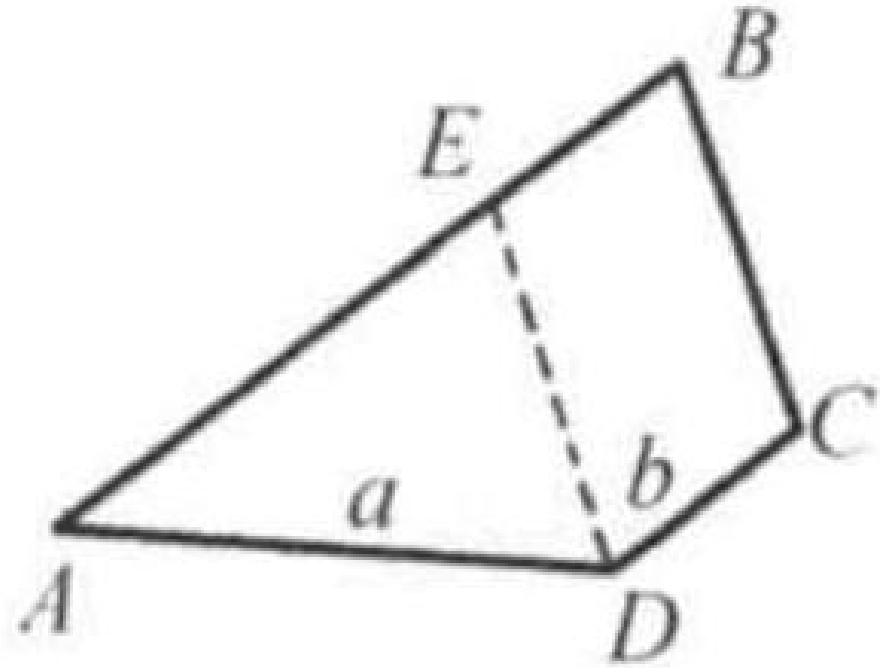
Example 10

(AMC) In the accompanying figure, segments AB and CD are parallel, the measure of angle D is twice that of angle B, and the measures of segments AD and CD are a and b respectively. Then the measure of AB is equal to

(A) $\frac{1}{2}a + 2b$
(B) $\frac{3}{2}b + \frac{3}{4}a$
(C) $2a - b$
(D) $4b - \frac{1}{2}a$
(E) $a + b$.



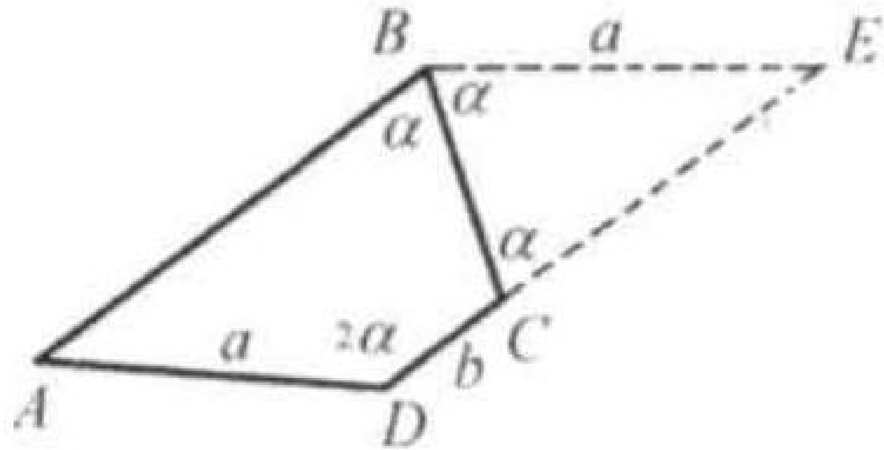
Solution: (E).
Method 1:
Let the bisector of $\angle D$ intersect AB at E (see figure). Then the alternate interior angles AED and EDC as well as $\angle ADE$ are equal to angle B, so $\triangle AED$ is isosceles with equal angles at P and D. This means that
$AE = AD = a$.
Since $EBCD$ is a parallelogram, we have $EB = DC = b$; so AB



$= AE + EB = a + b.$

Method 2:

Extend DC to E and connect BE. This results in $ABED$ being a parallelogram, and so $AB = DE, BE = a$, and $\angle ADC = \angle ABE$.

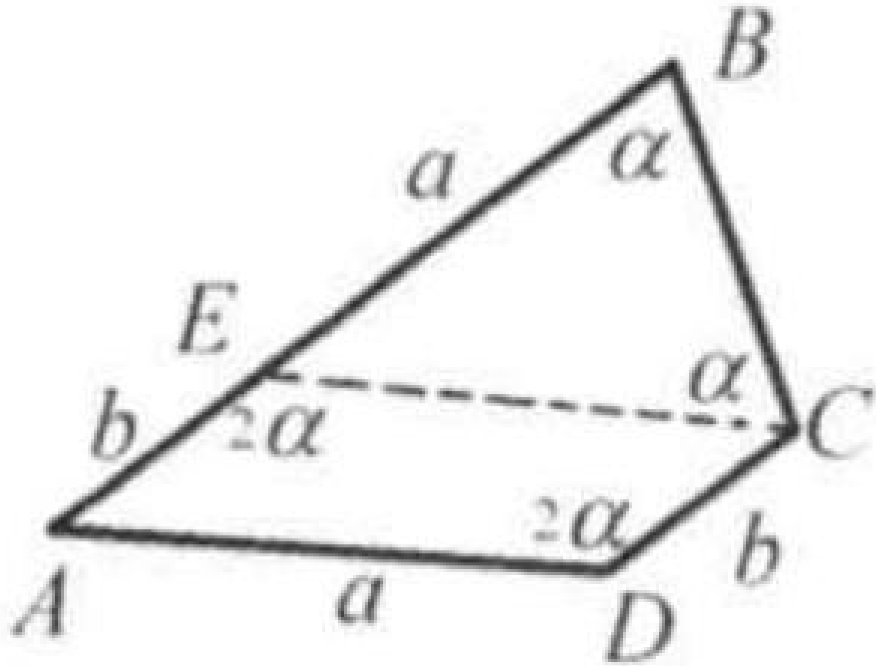


Since $\angle ABC = \alpha$, then $\angle CBE = \alpha$. Since AB//DE, $\angle ECB = \angle EBC$. Thus, $BE = EC = a$, and so $AB = a + b$.

Method 3:

Draw $EC//AD$ to meet AB at E. This gives us parallelogram $AECD$ where

$AD = EC = a$ and $\angle ADC = \angle AEC$.
Since $\angle AEC = 2\alpha = \angle EBC + \angle ECB$, so $\angle ECB = \alpha$. Thus,



$BE = EC = a$, and so $AB = a + b$.