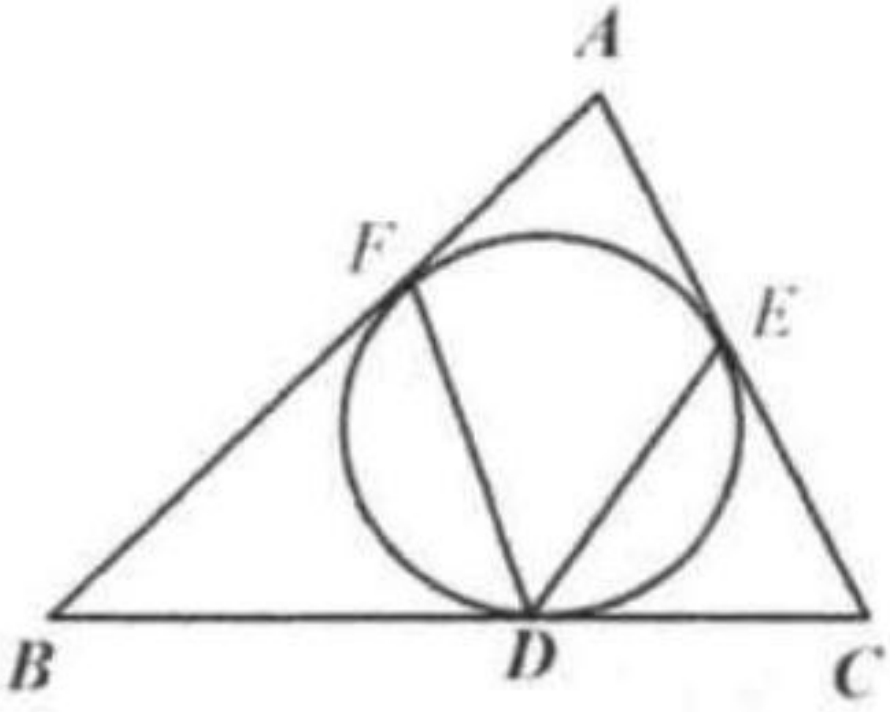
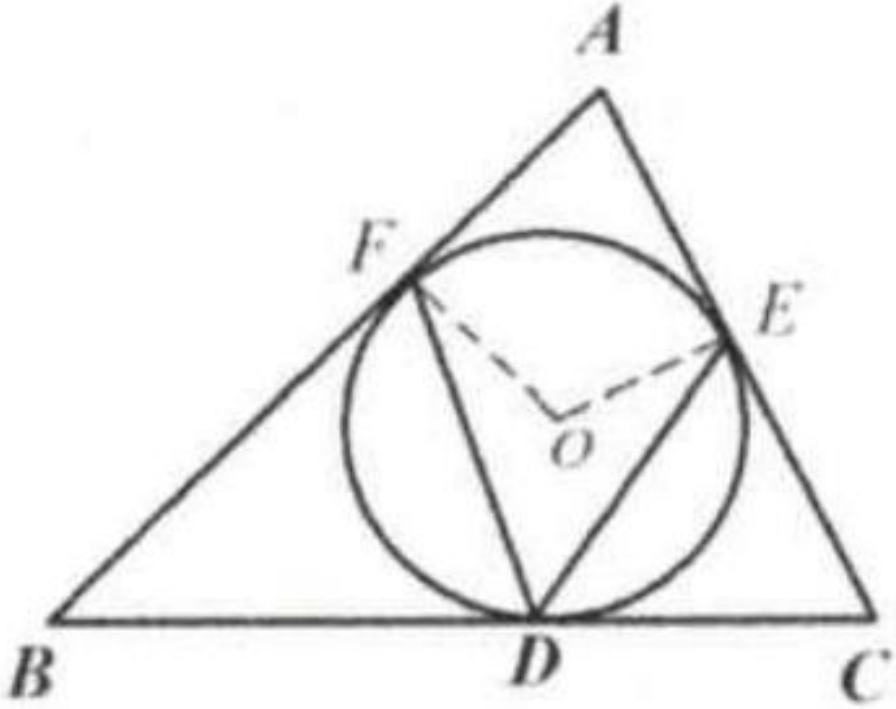
Problem

A circle is inscribed in triangle ABC. The tangent points are D, E, F as shown. Show that $\angle FDE = 90° - \frac{1}{2}\angle A$.



Solution

Connect OE, OF.
Since both AF and AE are tangent to circle
$O, AF \perp OF, AE \perp OE, \angle AFO = \angle AEO = 90°$.
In quadrilateral AFOE, $\angle A + \angle FOE = 360° - 2 \times 90° = 180°$.
So $\angle FOE = 180° - \angle A$
But $\angle FDE = \frac{1}{2}\angle FOE$. So $\angle FDE = 90° - \frac{1}{2}\angle A$.


A
F
E
O
B
D
C