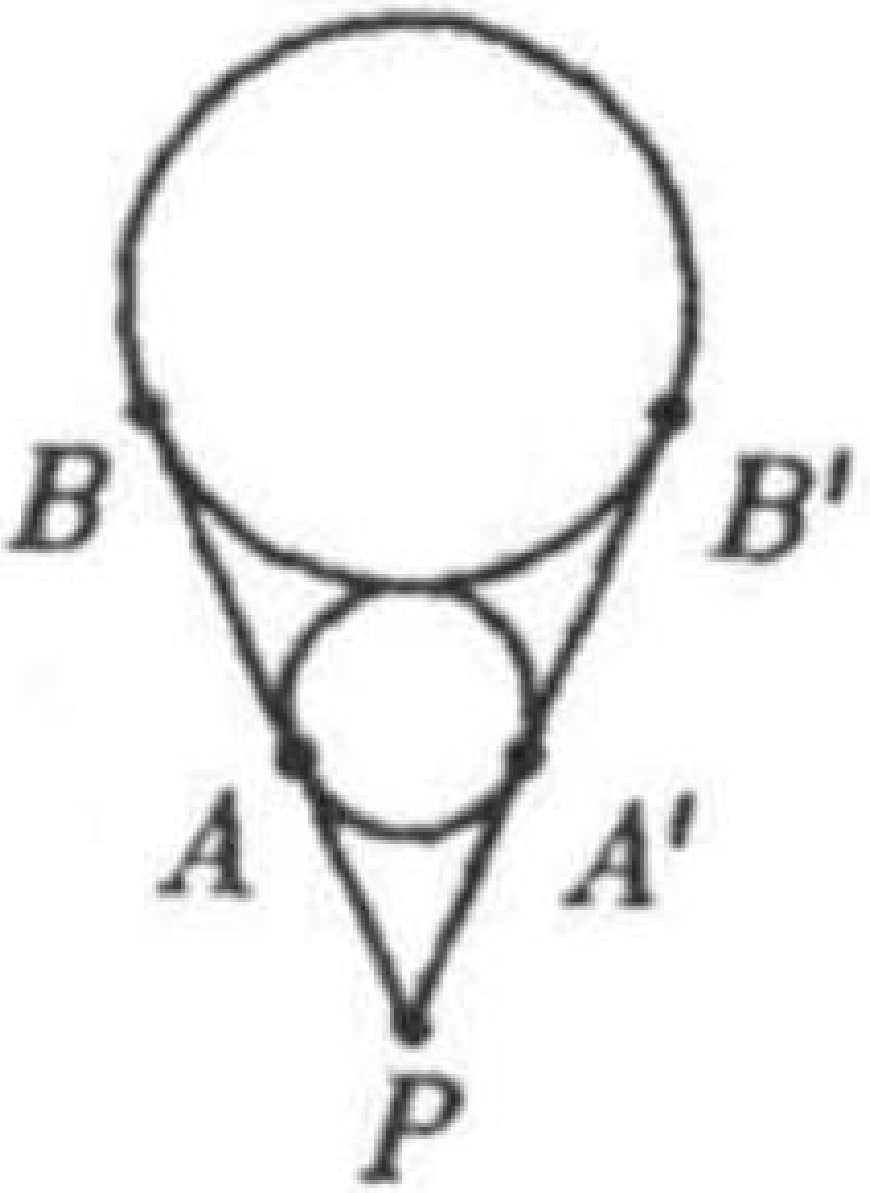
Problem 2

Problem

(AMC) Two circles are externally tangent. Lines PAB and $PA'B'$ are common tangents with A and A' on the smaller circle and B and B' on the larger circle. If $PA = AB = 4$, then the area of the smaller circle is

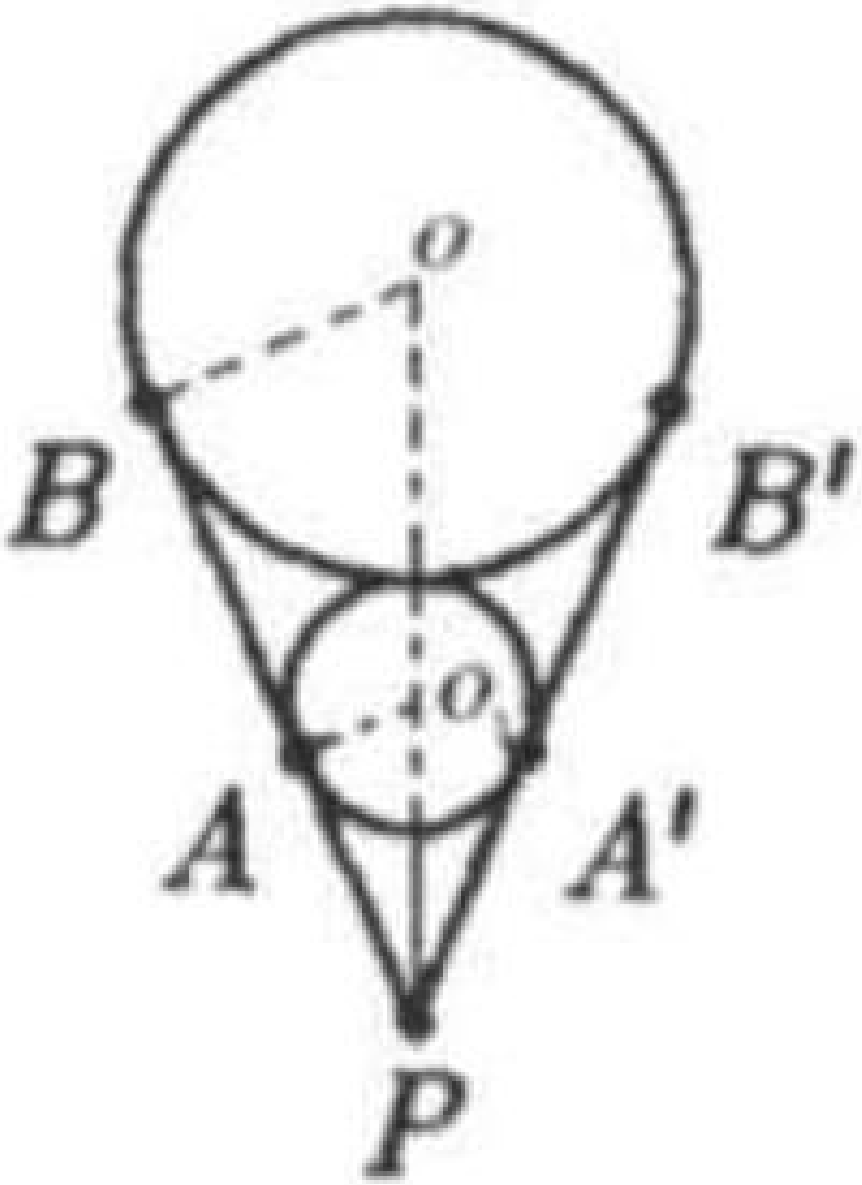
(A) 1.44π
(B) 2π
(C) 2.56π
(D) $\sqrt{8}\pi$
(E) 4π



Solution

(B) Connect PO, BO, AO_l. Since $PA = AB, OB \perp PB, O_1A \perp PB$,
$PB = PA + AB = 8, \quad OB = 2r.$
$PO = 2OO' = 2(R + r) = 2(2r + r) = 6r$
By the Pythagorean Theorem in right triangle POB, $PO^2 = OB^2 + PB^2$.

$$64 + 4r^2 = 36r^2 \quad \Rightarrow \quad r^2 = 2$$



The area of the smaller circle is 2π.