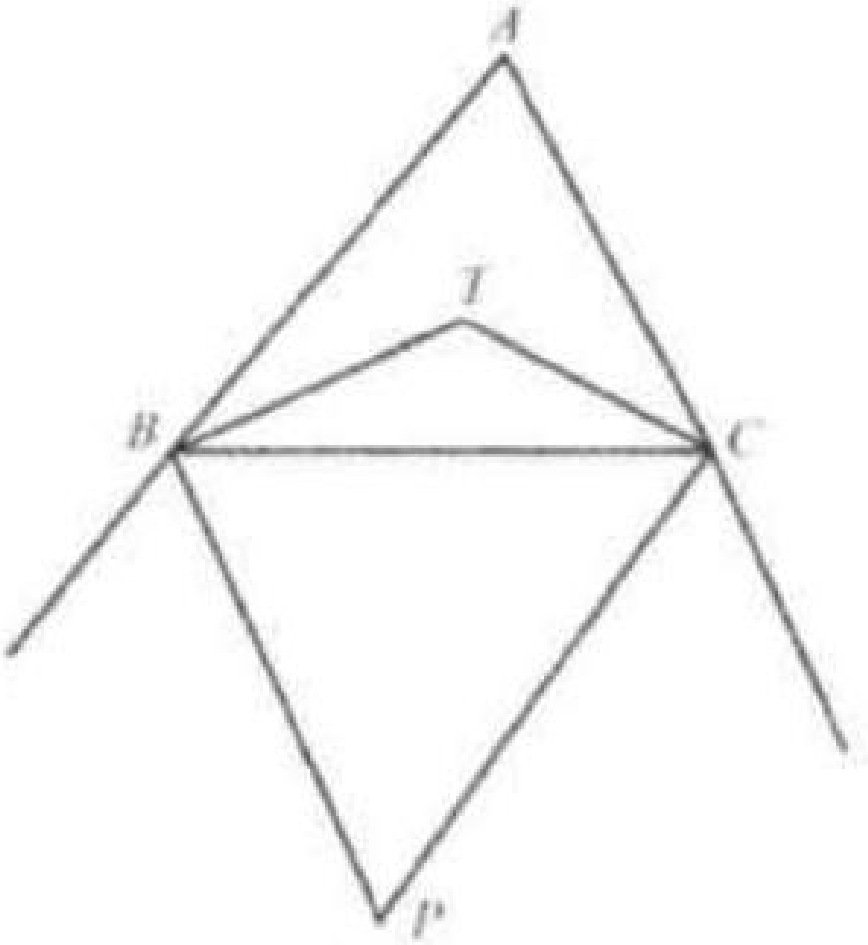
Problem 11

Problem

In $\triangle ABC$, the angle bisectors of $\angle B, \angle C$ meet at T, and the exterior angle bisectors of $\angle B, \angle C$ meet at P. Find $\angle BPC$ if $\angle A = 72°$.

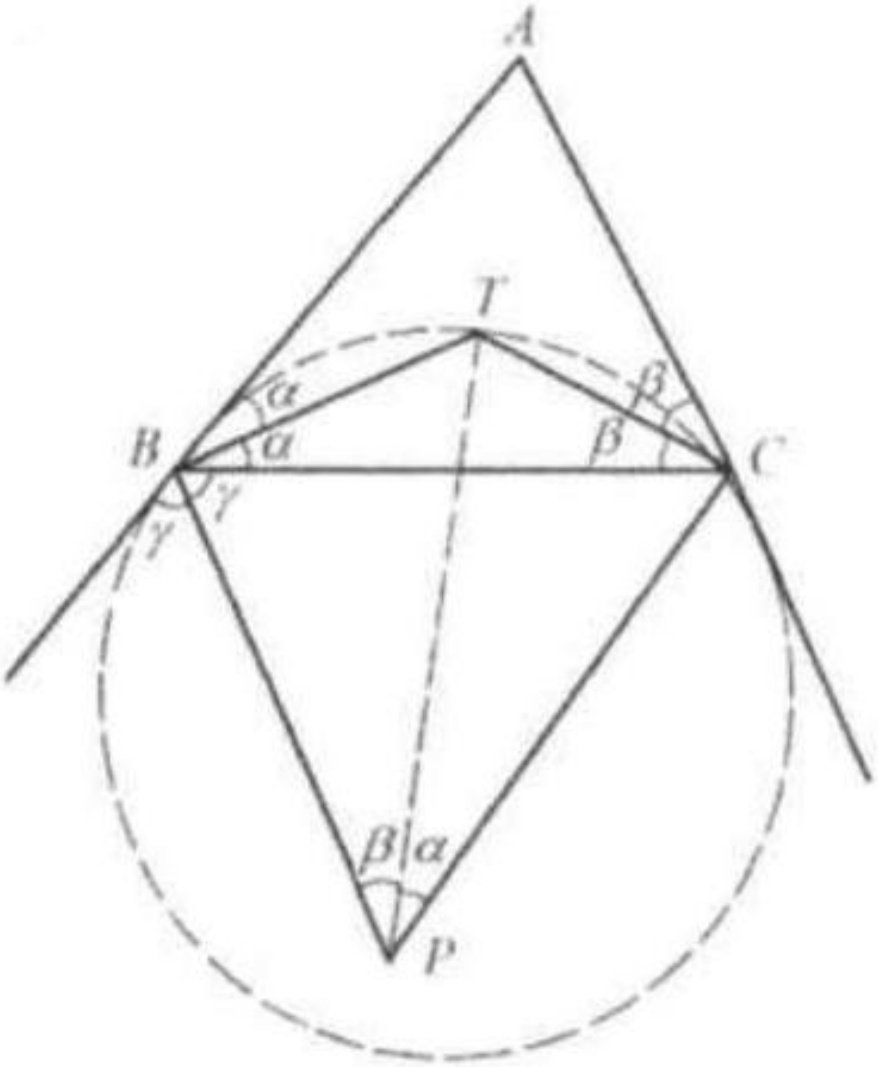


$\angle EFI = \angle EAI = \alpha$ (they face the same arc EI).

$\angle FEI = \angle FAI = \alpha$ (they face the same arc FI).
In $\triangle FEI, \angle FIE = 180^\circ - 2\alpha = 180^\circ - \angle A$
But $\angle FDE = \frac{1}{2}\angle FIE$. So $\angle FDE = 90^\circ - \frac{1}{2}\angle A$.

Solution

54°. Label $\angle TBC = \angle TBA = \alpha, \angle TCB = \angle TCA = \beta, \angle CBP = \gamma$.
Since $2\alpha + 2\gamma = 180^\circ, \alpha + \gamma = 90^\circ$.
Thus $\angle TBP = 90^\circ$.
Similarly, $\angle TCP = 90^\circ$.
So points B, P, C, and T are concyclic and TP is the diameter of the circle.
Therefore, $\angle BPT = \angle TCB = \beta, \angle CPT = \angle TBC = \alpha$.



$\angle BPC = \alpha + \beta = \frac{(180^\circ - \angle A)}{2} = \frac{(180^\circ - 72^\circ)}{2} = 54^\circ$.