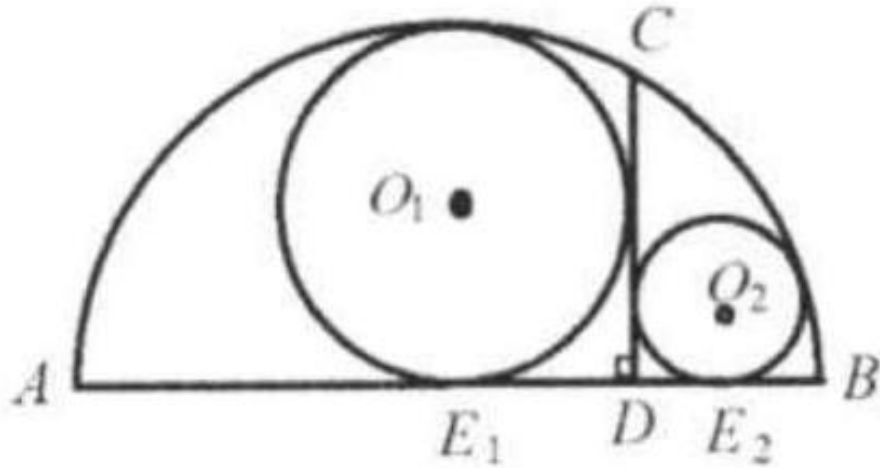


## Problem 8

### Problem

$AB$  is the diameter of the semicircle as shown in the figure.  $CD \perp AB$ , with the foot at  $D$ . Two circles  $O_1$  ( $r_1 = 12$ ) and  $O_2$  ( $r_2 = 8$ ) inscribed in the semicircle and are tangent to  $CD$ . The tangent points to  $AB$  are  $E_1$  and  $E_2$ , respectively. Find the length of  $AB$ .



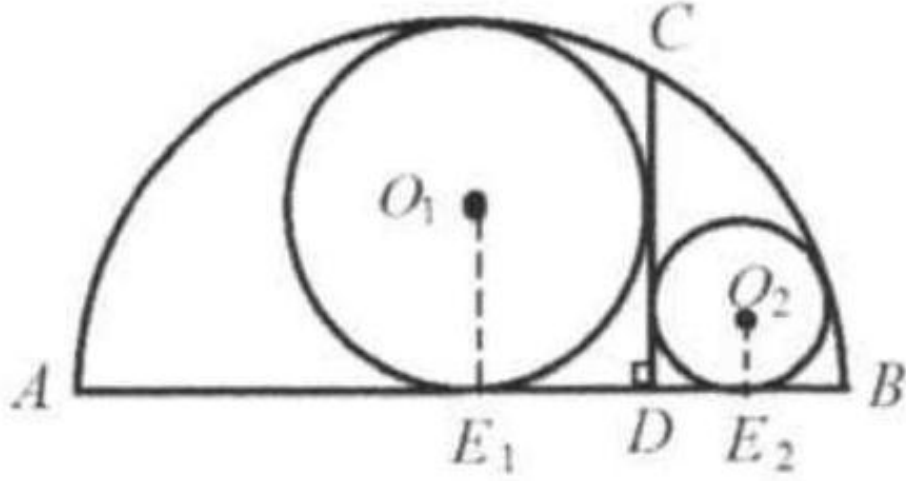
### Solution

Connect  $O_1E_1$  and  $O_2E_2$ .

$$E_1D = O_1E_1 = 12, DE_2 = O_2E_2 = 8.$$

Let  $AD = x, DB = y$ .

$$\text{Then } AE_1 = x - 12, E_1B = y + 12, AE_2 = x + 8, E_2B = y - 8.$$



Then we have  $\frac{1}{AE_1} + \frac{1}{E_1B} = \frac{1}{O_1E_1}$

$$\frac{1}{AE_2} + \frac{1}{E_2B} = \frac{1}{O_2E_2}$$

Or  $\frac{1}{x-12} + \frac{1}{y+12} = \frac{1}{12}$

$$\frac{1}{x+8} + \frac{1}{y-8} = \frac{1}{8}$$

Solving we get  $x = 32, y = 18$ .

$$AB = x + y = 50.$$