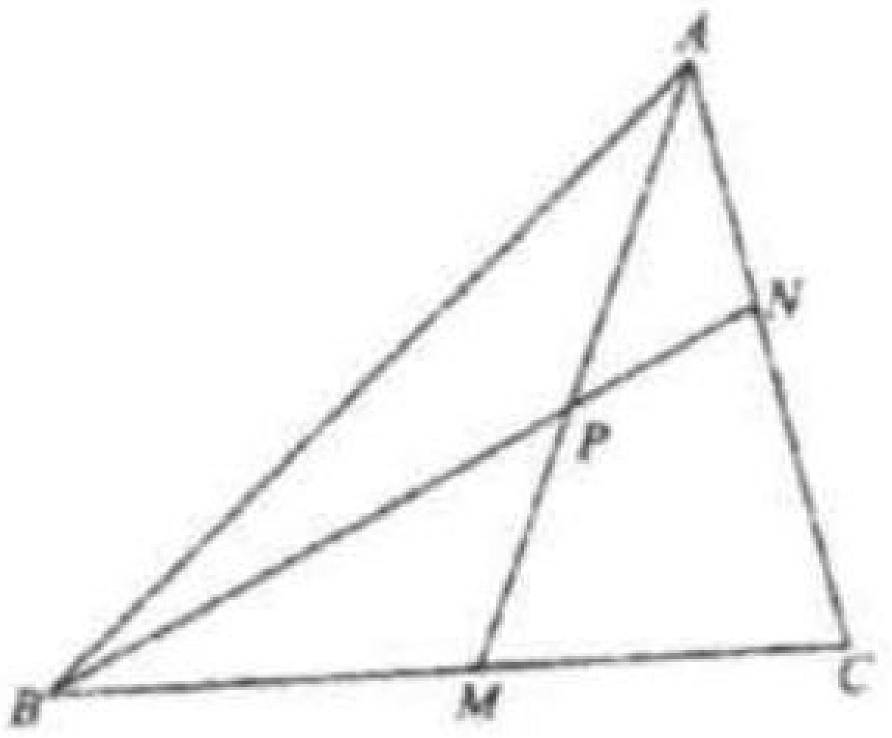
Problem 2

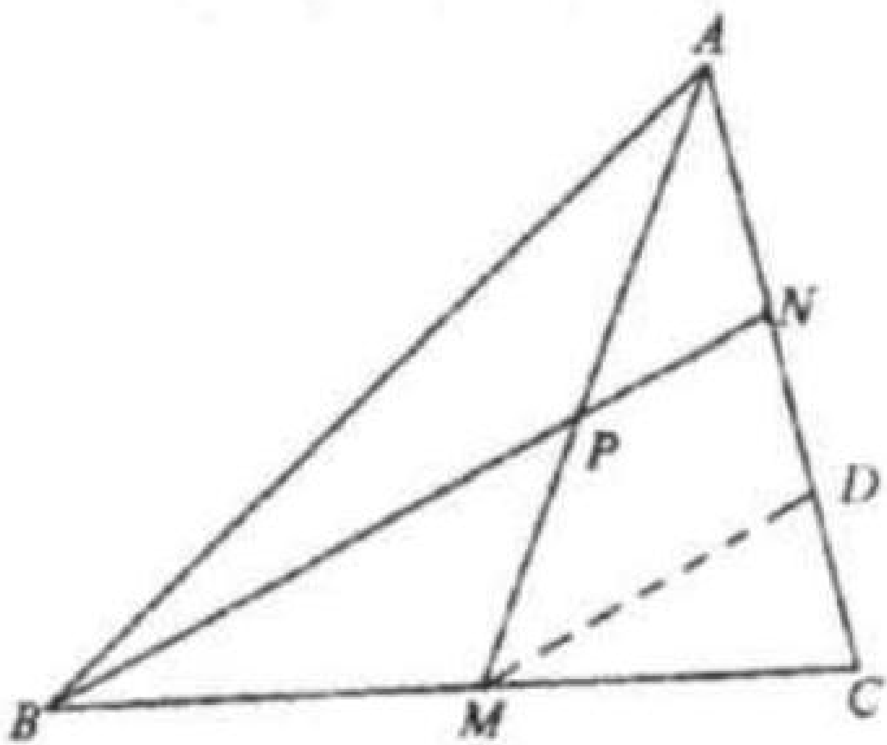
Problem

As shown in the figure, in triangle ABC, M is the midpoint of BC. $AN = \frac{1}{3}AC$. Connect BN and denote the point where BN meets AM as P. Show that $BP = 3PN$.



Solution

Method 1:
Take D, the midpoint of NC. Connect MD.
Since M is the midpoint of BC, N is the midpoint of $NC, MD \mathbin{//} BN$ and $MD = \frac{1}{2}BN$.



In triangle CBN, since $MB = MC$ and $MD//BN, DC = DN$ and
$2MD = BN$

In triangle $AMD, AN = ND = DC, PN//MD$, so $2PN = MD$

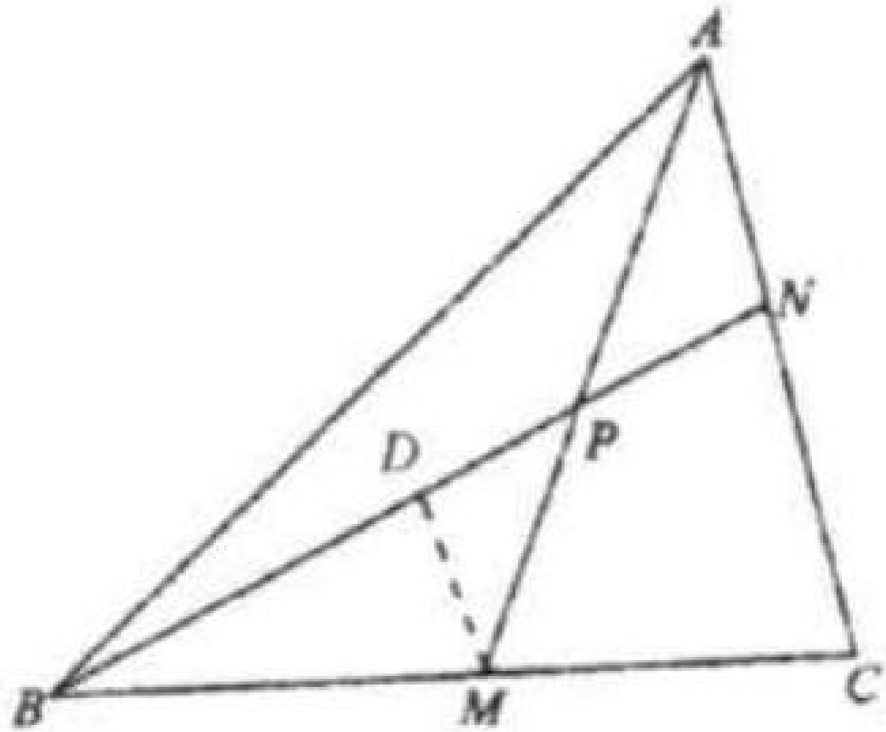
Substituting (2) into (1):

$$4PN = BN \quad \Rightarrow \quad 4PN = BP + PN \Rightarrow BP = 3PN.$$

Method 2:

Take D, the midpoint of BN. Connect MD.

Since M is the midpoint of BC, N is the midpoint of BN,



$MD // NC$ and $MD = \frac{1}{2} NC$.

In triangle CBN, since $MB = MC$ and $MD // CN, 2DM = CN$ and $DM = AN$.

We also see that $\triangle ANP \sim \triangle MDP$, so $DP = PN$.

$$BN = BD + DN = 2DN$$
$$= 4PN = 3PN + PN = BP + PN.$$
$$\Rightarrow \quad BP = 3PN.$$