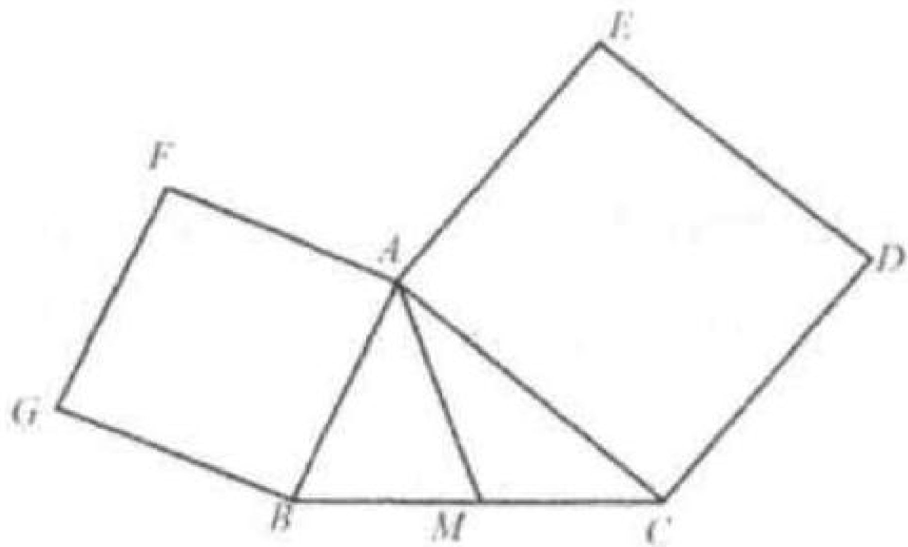
Example 5

In $\triangle ABC, AM$ is the median. $ABGF$ and $ACDE$ are squares. Prove: $EF = 2AM$.

Solution: Extend AM to N such that $AM = MN$.

Connect BN.



Since $BM = CM, AM = MN$, and $\angle AMC = \angle NMB$, we have

$\triangle AMC \cong \triangle NMB$.

Thus $BN = AC, \angle MNB = \angle MAC = \alpha$,

Connect EF.

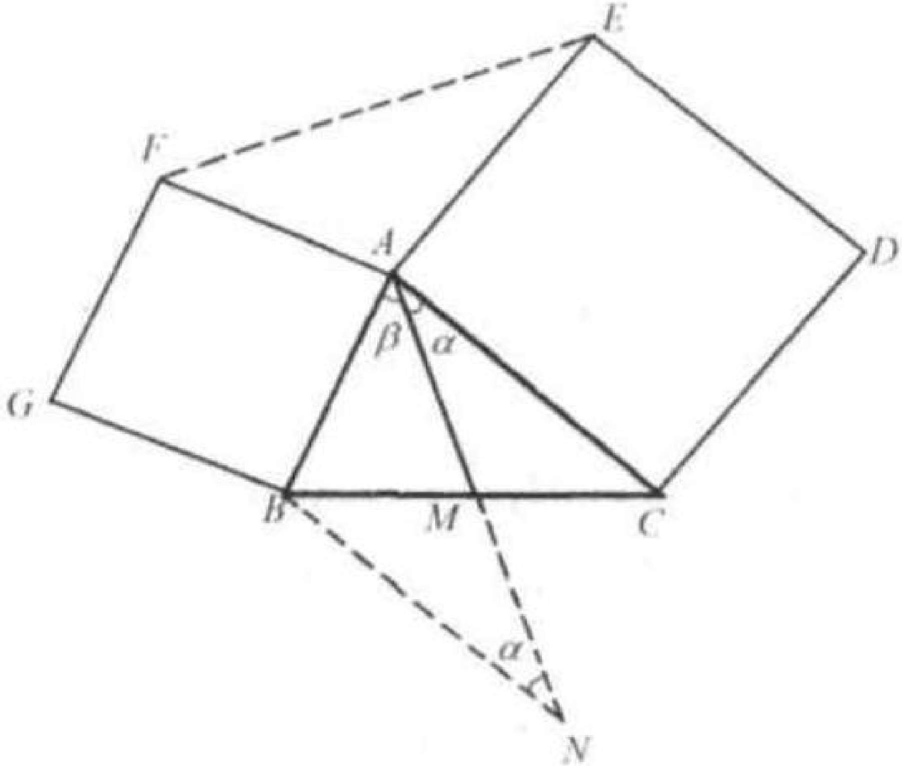
$$\angle EAF = 360^\circ - 90^\circ - 90^\circ - \angle BAM - \angle MAC$$

$$= 180^\circ - \beta - \alpha.$$

$$\angle ABN = 180^\circ - \angle BAM - \angle MNB.$$

Since $\angle MNB = \angle MAC = \alpha$,

$$\angle ABN = 180^\circ - \angle BAM - \angle MAC == 180^\circ - \beta$$



$-\alpha$.

Therefore, $\angle ABN = \angle EAF$.

In $\triangle EAF$ and $\triangle NBA, AF = AB, AE = AC = BN$, and $\angle ABN = \angle EAF$.

Thus $\triangle EAF \cong \triangle NBA$. So $EF = AN = 2AM$.