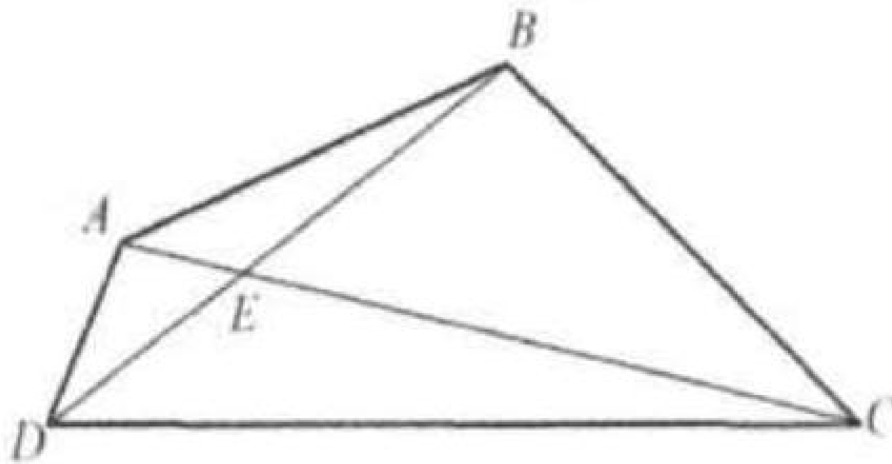
Problem 9

Problem

Diagonals AC and BD of quadrilateral $ABCD$ meet at E. If $AE = 2, BE = 5, CE = 10, DE = 4$, and $BC = 15/2$, find AB.



Solution

$\frac{1}{2}\sqrt{171}$.

As shown in the figure, since $BE/AE = CE/DE = 5/2, \triangle AED \sim \triangle BEC$.

Therefore, $BE/AE = BC/AD$, or $\frac{5}{2} = \frac{\frac{15}{2}}{AD}$.

Thus, $AD = 3$.

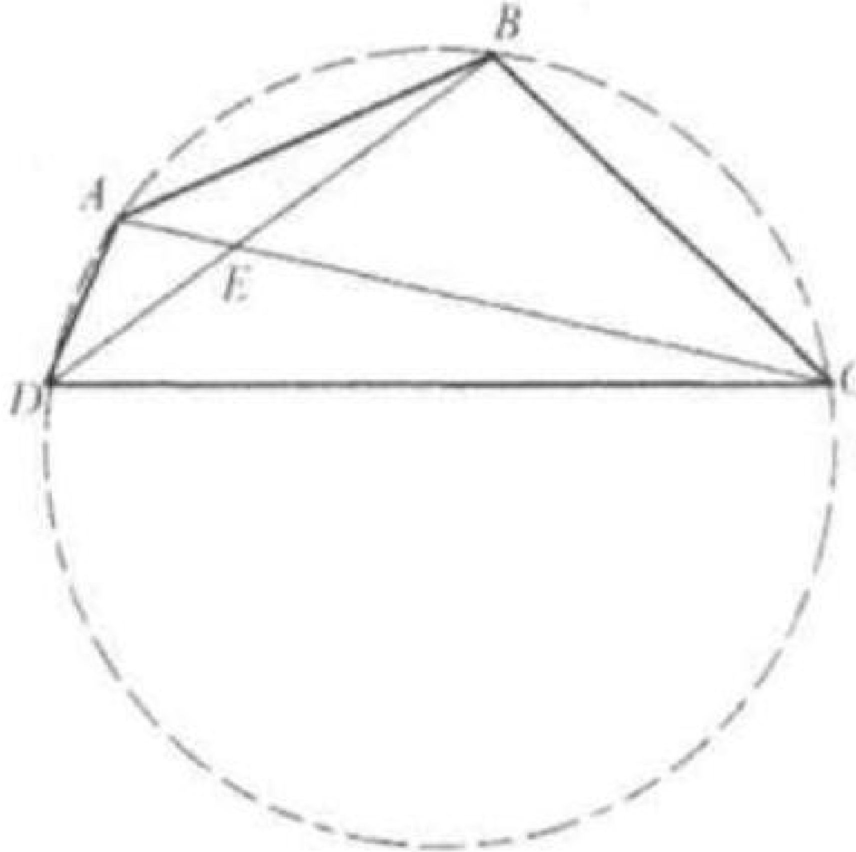
Similarly, $\triangle AEB \sim \triangle DEC$.

Therefore, $AE/DE = AB/DC$ or $1/2 = AB/DC$.

Thus, $DC = 2(AB)$.

Also, $\angle \text{L}BAC = \angle BDC$. Therefore, quadrilateral ABCD is cyclic.

Now, applying Ptolemy's Theorem to cyclic quadrilateral



$ABCD$, $(AB)(DC) + (AD)(BC) = (AC)(BD)$.
Substituting, we find that $AB = \frac{1}{2}\sqrt{171}$.