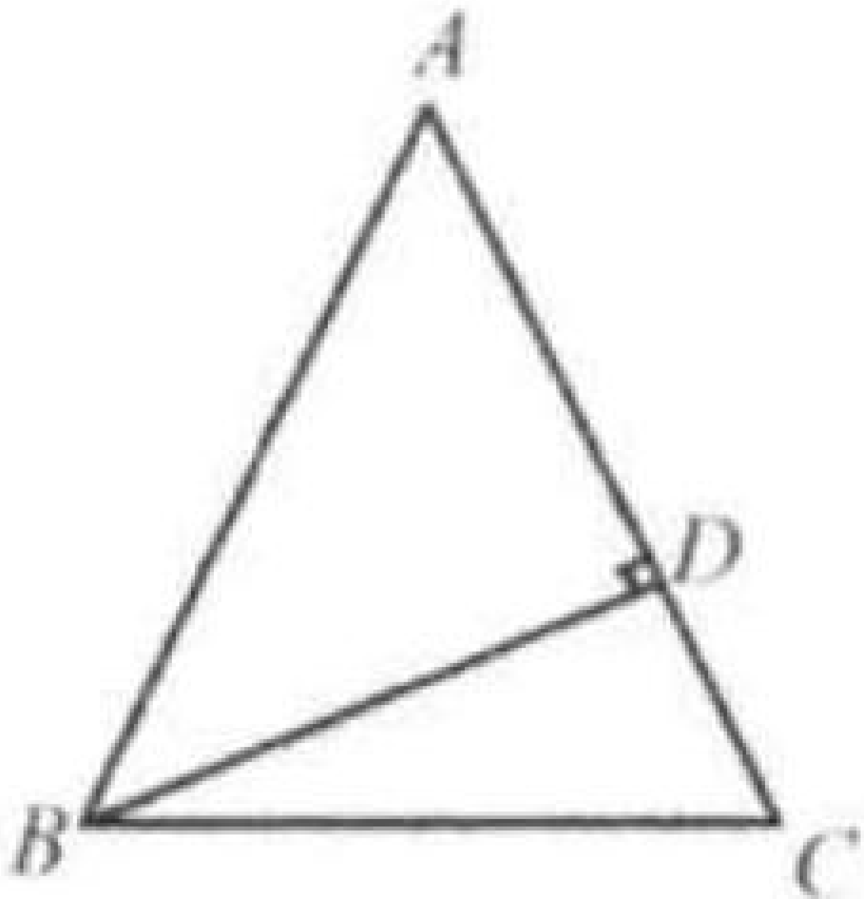
Example 1

Given $\triangle ABC, AB = AC, BD \perp AC$. Prove: $\angle CBD = \frac{1}{2}\angle A$.

Solution: Method 1:

Draw $CE \perp AB$. E is the foot of the perpendicular from C to AB.
Points A, E, F, and E are concyclic ($\angle AEF + \angle ADF = 90^\circ +$

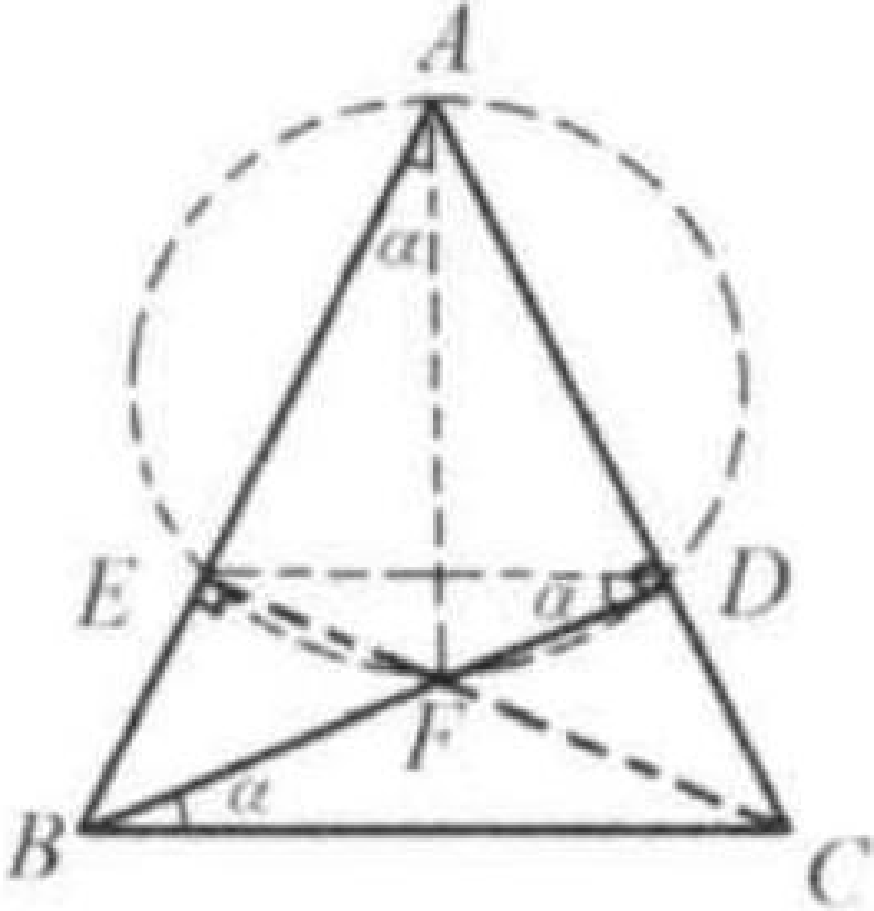


$90^\circ = 180^\circ$).
Connect AF and $ED . \angle EAF = \angle DAF = \alpha$.

$\angle EDF = \angle EAF = \alpha$. (both angles face the same arc EF).

Note that $ED//BC$. Thus $\angle EDB = \angle CBD = \alpha$.

That is, $\angle CBD = \frac{1}{2}\angle A$.

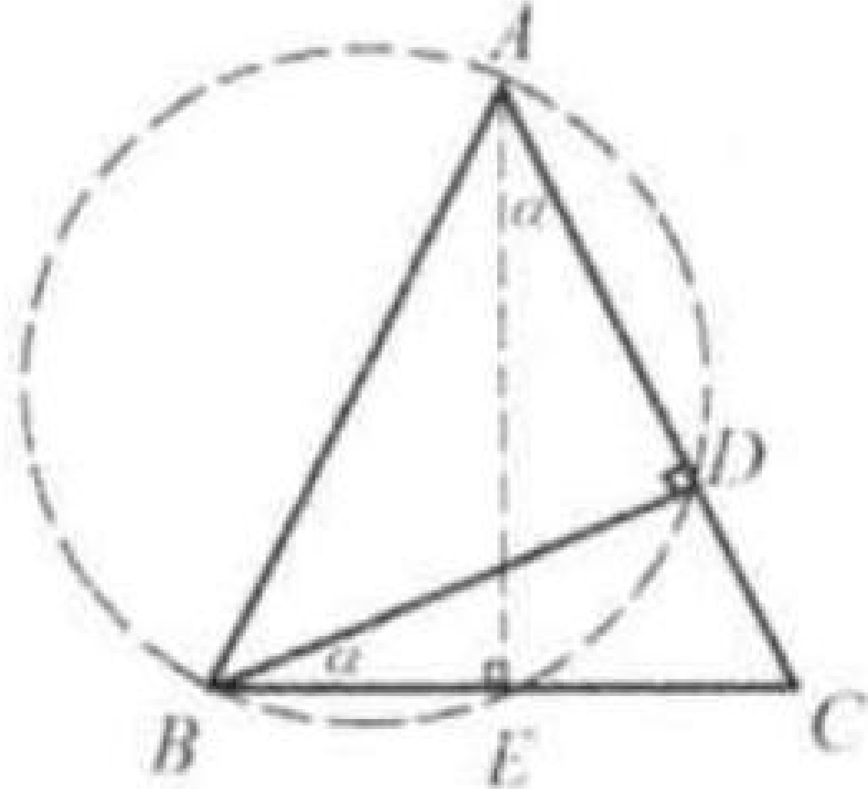


Method 2:

Draw $AE \perp BC$. E is the foot of the perpendicular from A to BC.

Points A, B, E, and D are concyclic ($\angle ADB = \angle AEB = 90°$).

$\angle DAE = \angle DBE = \alpha$ (both angles face the same arc DE).



Note that $\triangle ABC$ is an isosceles triangle, AE is also the angle bisector of $\angle A$. Thus $\angle CBD = \angle DBE = \alpha = \frac{1}{2}\angle A$.