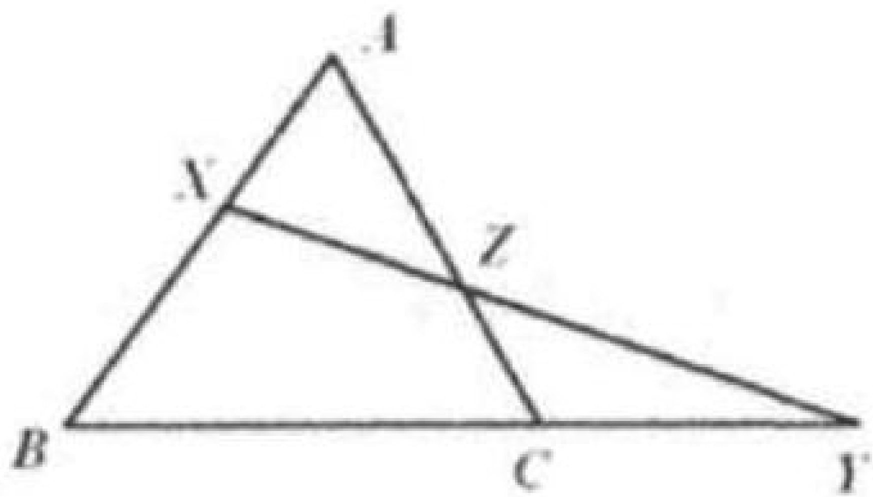
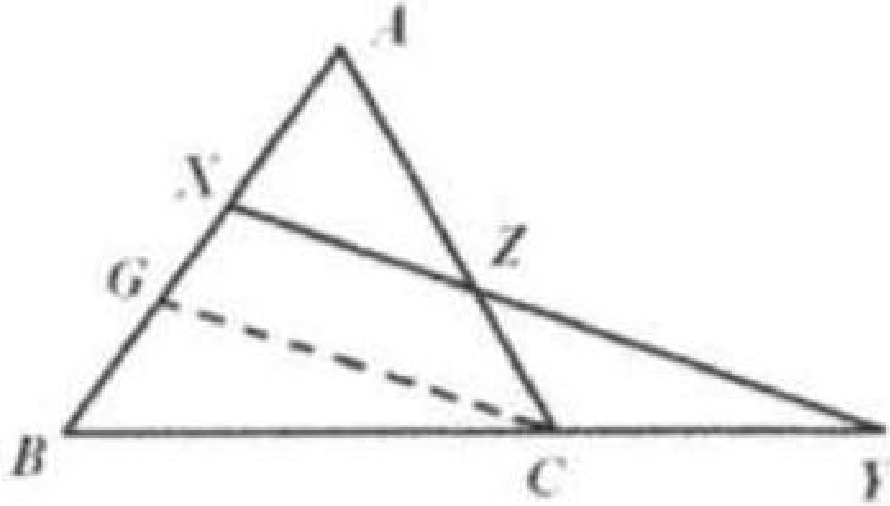
Example 15

Prove the Menelaus' Theorem: A line intersects the sides or extension of the sides AB, BC and CA of $\triangle ABC$ at X, Y and Z, respectively. The following holds $\frac{AX}{BX} \cdot \frac{BY}{CY} \cdot \frac{CZ}{AZ} = 1$

Proof:



Draw $CG//XY$ to meet AB at G.
Since $\triangle BYX \sim \triangle BCG$, we have $\frac{BY}{CY} = \frac{BX}{GX}$



Since $\triangle AGC \sim \triangle AXZ$, we have $\frac{CZ}{AZ} = \frac{GX}{AX}$ (2)

(1) $\times$ (2): $\frac{BY}{CY} \times \frac{CZ}{AZ} = \frac{BX}{GX} \times \frac{GX}{AX} = \frac{BX}{AX}$, or $\frac{AX}{BX} \cdot \frac{BY}{CY} \cdot \frac{CZ}{AZ} = 1$.