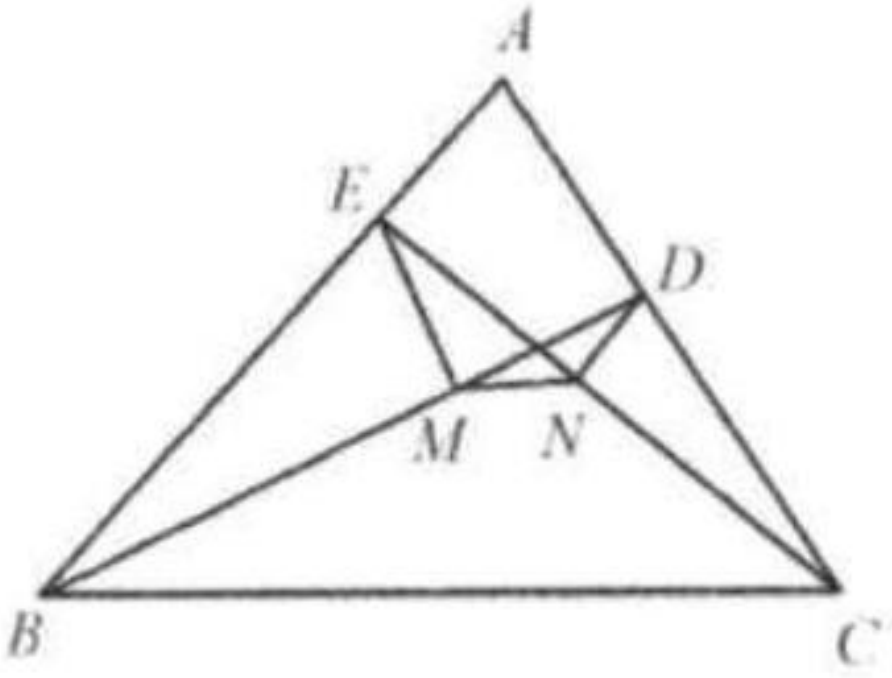
Problem 6

Problem

As shown in the figure, BD, CE are the altitudes on AC, AB of $\triangle ABC$, respectively. $EM \perp BD$ at $M, DN \perp CE$ at N. Show that $MN//BC$.



Solution

$\angle BEC = \angle BDC = 90°$.
Thus points B, C, D, and E are concyclic.
Draw the circle as shown.
So $\angle CED = \angle CBD = \alpha$ (they face the same arc CD).
$\angle EMD = \angle END = 90°$.
Thus points E, M, N, and D are concyclic.
So $\angle NED = \angle NMD = \alpha$ (they face the same chord DN).

Since $\angle CBD = \angle NMD = \alpha, MN//BC$.