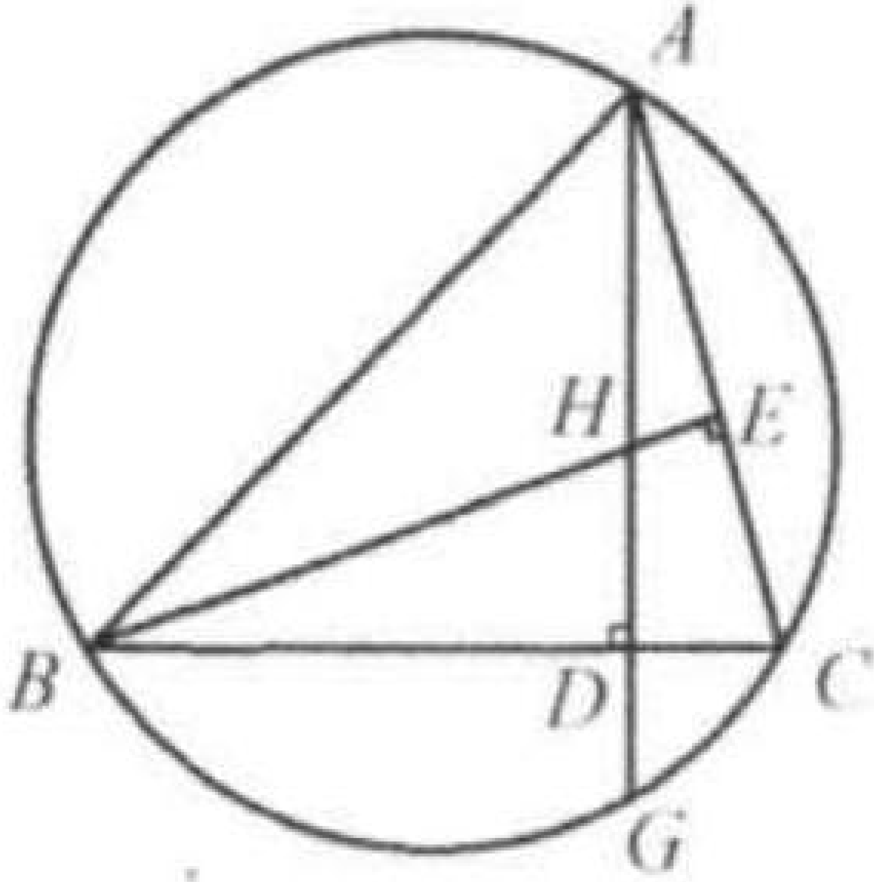
Problem

As shown in the figure, AD and BE are the heights of $\triangle ABC$ and they meet at H. Extend AD to meet the circumcircle O at G. Prove: $HD = DG$.



Solution

Connect BG.

Since BE and AD are the heights, $\angle HEC = \angle ADC = 90°$.

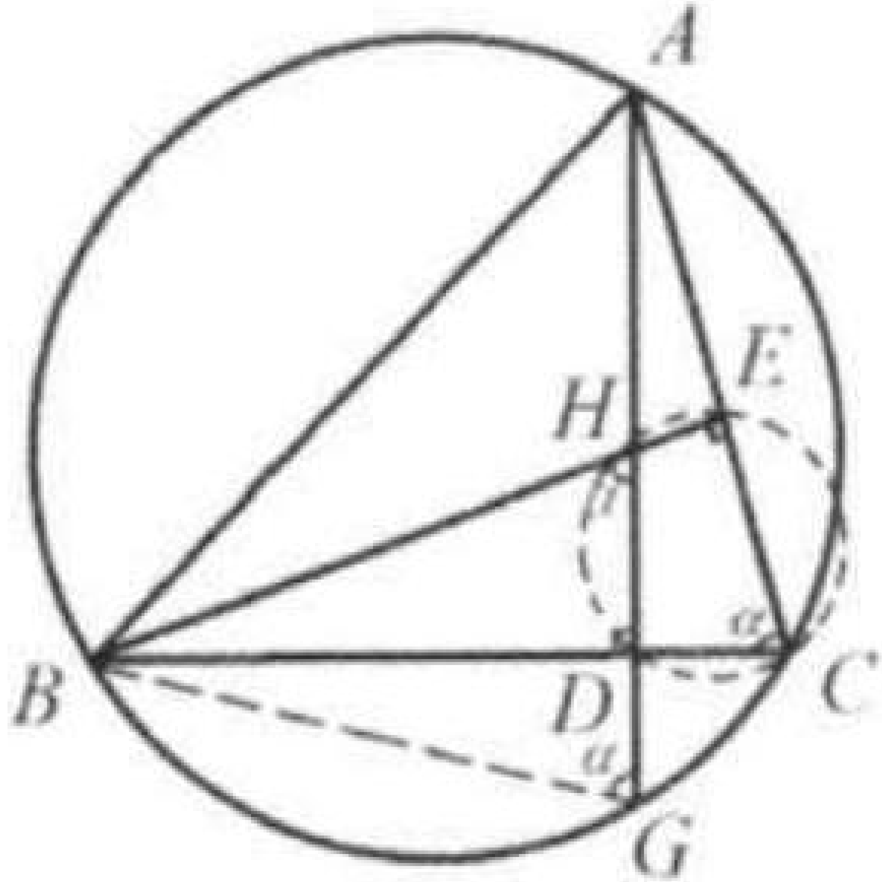
Thus points C, D, H, and E are concyclic. Therefore,

$\angle BHD = \angle BCE = \alpha$.

Note that $\angle AGB = \angle ACB$ (they face the same arc AB).

That is, $\angle BGH = \angle BHG = \alpha$.

Triangle BHG is an isosceles triangle and BD is the perpendicular bisector of HG and $HD = DG$.


A
E
H
β
α
B
D
C
α
G