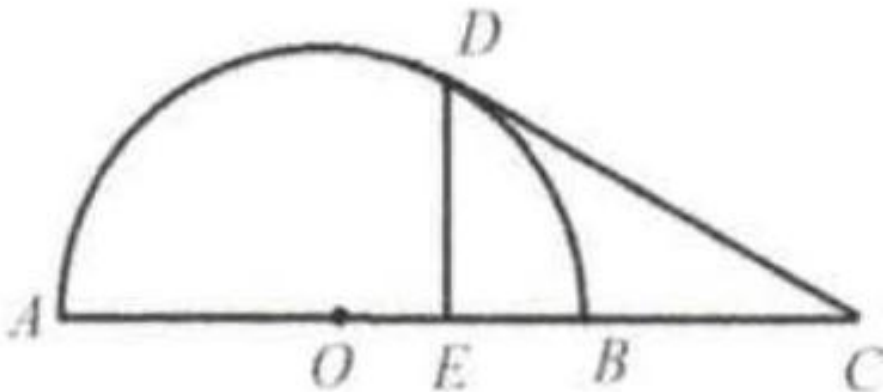


Example 13

AB is the diameter of semicircle O . Extend AB to C . DC is tangent to the semicircle at D . $DE \perp AB$. E is the foot of the perpendicular from D to AB . $CD = 2$, $AE = 4BE$. Find the length of BC .

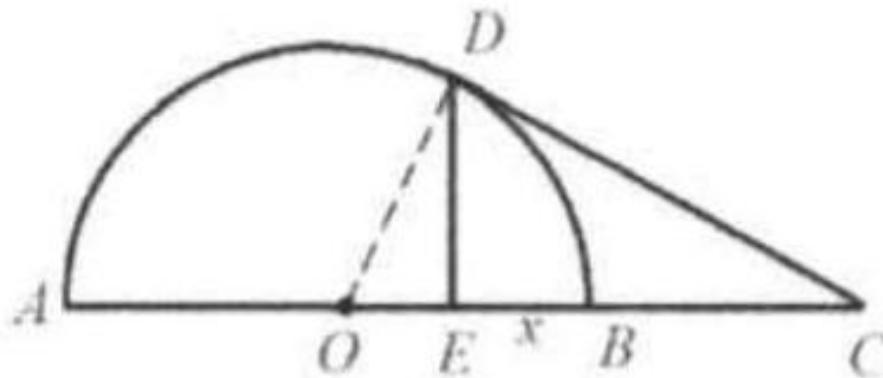


Solution: 1.

Connect OD .

Let $BE = x$.

$$OB = \frac{AB}{2} = \frac{AE+BE}{2} = \frac{5BE}{2} = \frac{5x}{2}$$



$$OE = OB - BE = \frac{AB}{2} - x = \frac{5x}{2} - x = \frac{3x}{2}$$

We know that

$$OD^2 = OC \times OE$$

$$\begin{aligned}
&\Rightarrow \left(\frac{5x}{2}\right)^2 = \left(\frac{5x}{2} + BC\right) \times \frac{3x}{2} \\
&\quad \Rightarrow \frac{25x}{6} = \frac{5x}{2} + BC \\
\Rightarrow BC &= \frac{25x}{6} - \frac{5x}{2} = \frac{25x-15x}{6} = \frac{5x}{3} \\
&\quad \text{We also know that} \\
&\quad CD^2 = AC \times BC \\
&\quad \Rightarrow 4 = \left(5x + \frac{5x}{3}\right) \times \frac{5x}{3} \\
\Rightarrow 4 &= \frac{9x^2}{100} \quad \Rightarrow x^2 = \frac{9}{25} \Rightarrow x = \frac{3}{5}. \\
&\quad \text{Thus } BC = \frac{5x}{3} = \frac{5}{3} \times \frac{3}{5} = 1.
\end{aligned}$$