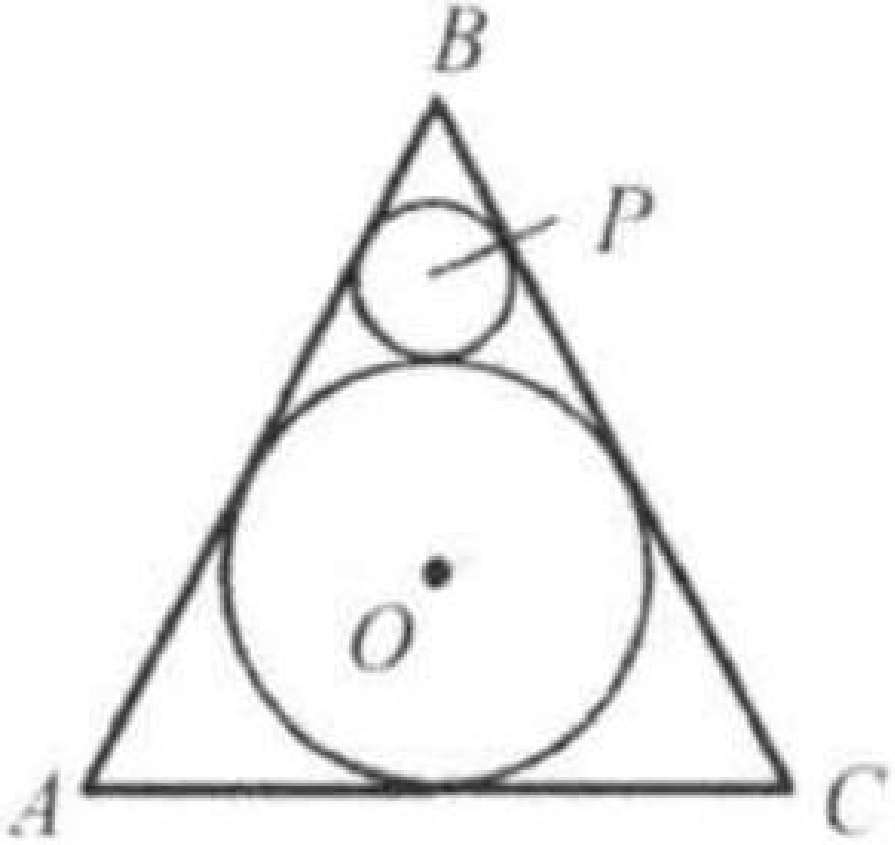
Problem 1

Problem

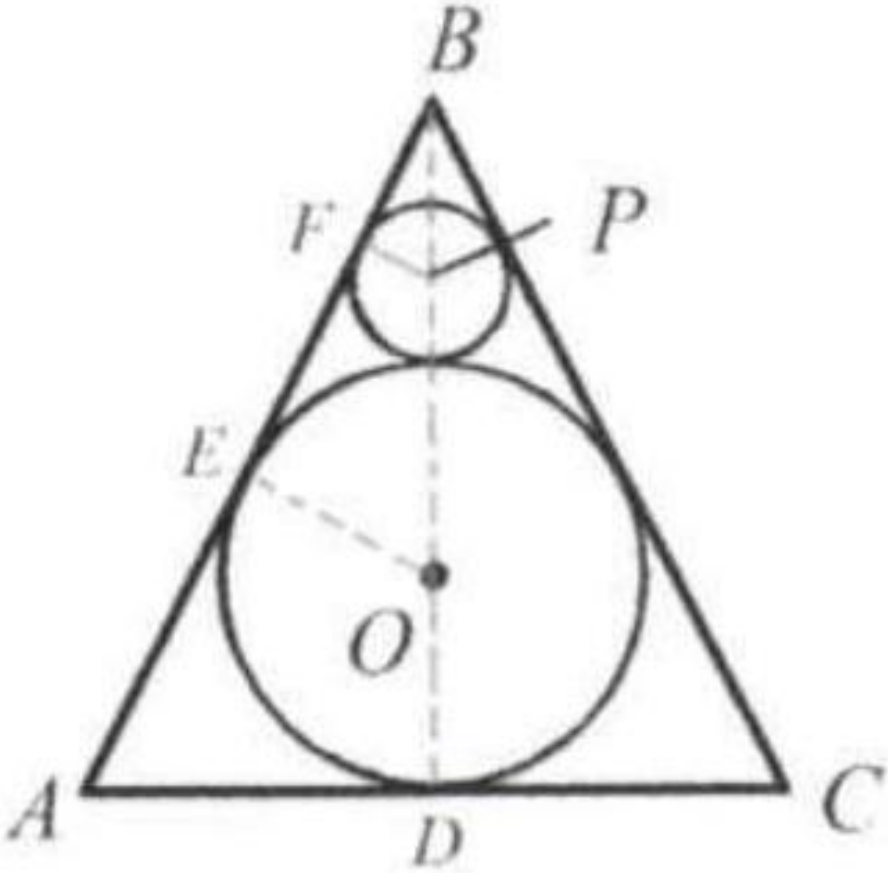
Circle O of radius 45 is inscribed in equilateral triangle ABC. Circle P is tangent to circle O and segments AB and BC. Find the area of circle P.

(A) 245π
(B) 625π
(C) 225
(D) 225π
(E) 700



Solution

(D).
Draw $PF \perp AB$ and $OE \perp AB$.
Connect BD.
Triangle BEO is a $30° - 60° - 90°$ triangle and $BO = 2OE$. $OE = 45$ and $BO = 2OE = 90$, and $BD = 90 + 45 = 135$.
Let $PF = r$.



Triangle BFP is a $30° - 60° - 90°$ triangle and $BP = 2r$.
In other word, $BP = BO - 45 - r$.
So $2r = BO - 45 - r \quad \Rightarrow 3r = 90 - 45 = 45$.
So $r = 15$.
The area is $\pi(15)^2 = 225\pi$.