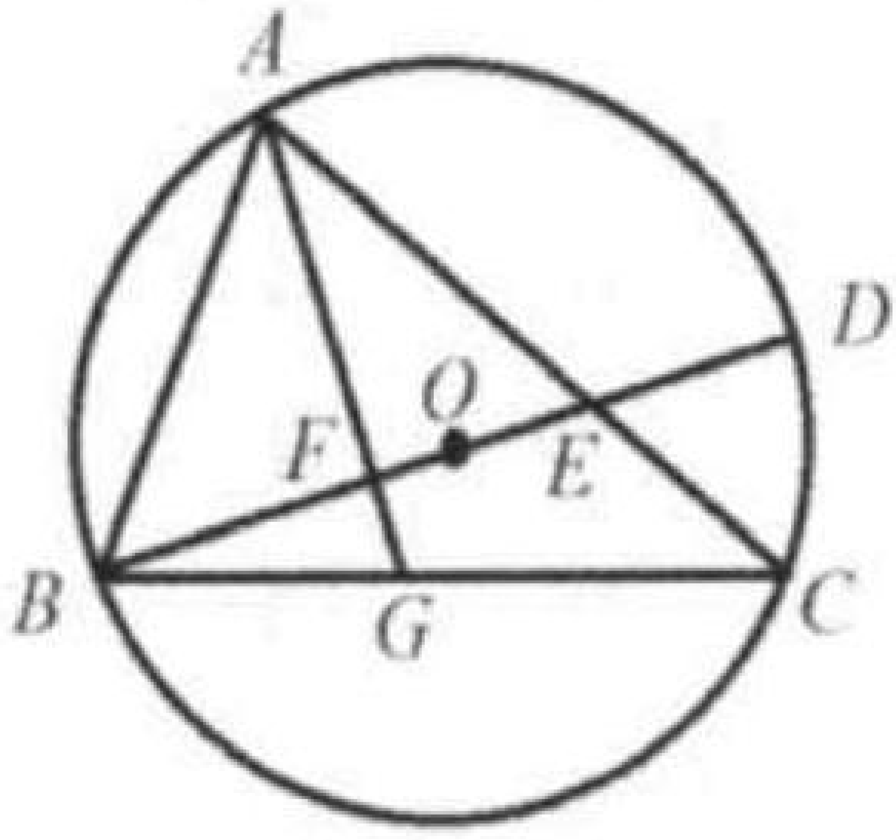
Example 6

Triangle ABC is inscribed in the circle O. The diameter BD meets AC at E. Draw $AF \perp BD, F$ is the foot of the perpendicular from A to BD. Extend AF to meet BC at G. Show that $AB^2 = BG \times BC$.

Solution: Connect AD. Since points A, B, C, and D are concyclic,

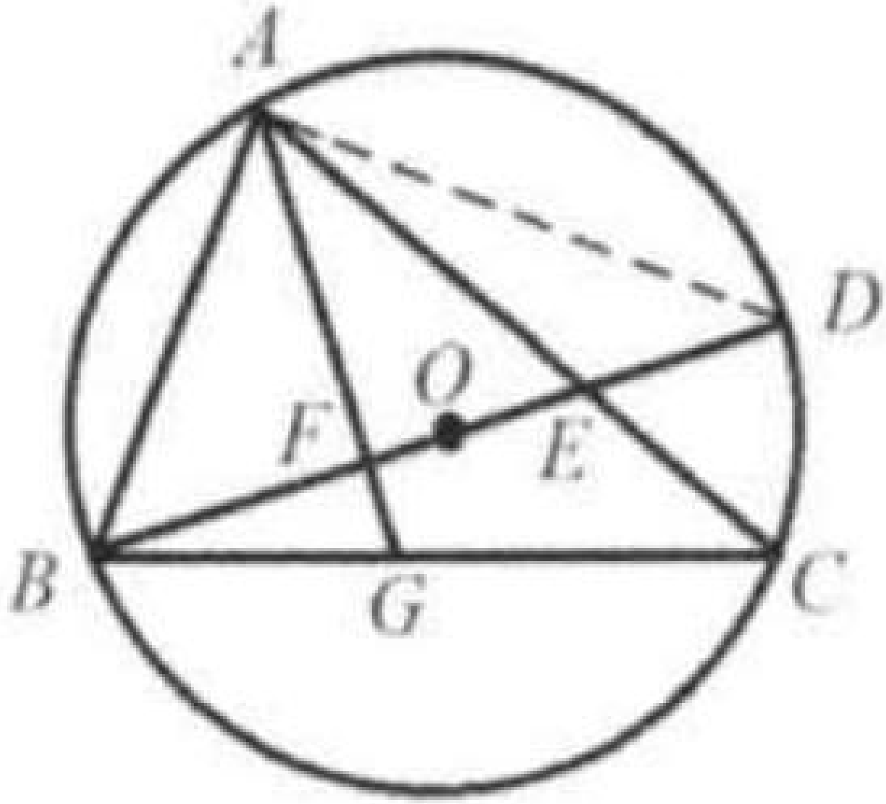


$\angle C = \angle D$.

Since BD is the diameter, $\angle BAD = 90°$.

$\angle D = 90° - \angle ABD$.

Since $AF \perp BD$, in Rt $\triangle BAF, \angle BAF = 90° - \angle ABD$.



Thus $\angle C = \angle BAF$.
Since $\angle ABC = \angle ABG, \triangle ABG \sim \triangle CBA$.
Thus $\frac{AB}{CB} = \frac{BG}{BA} \quad \Rightarrow \quad AB^2 = BG \times BC$.