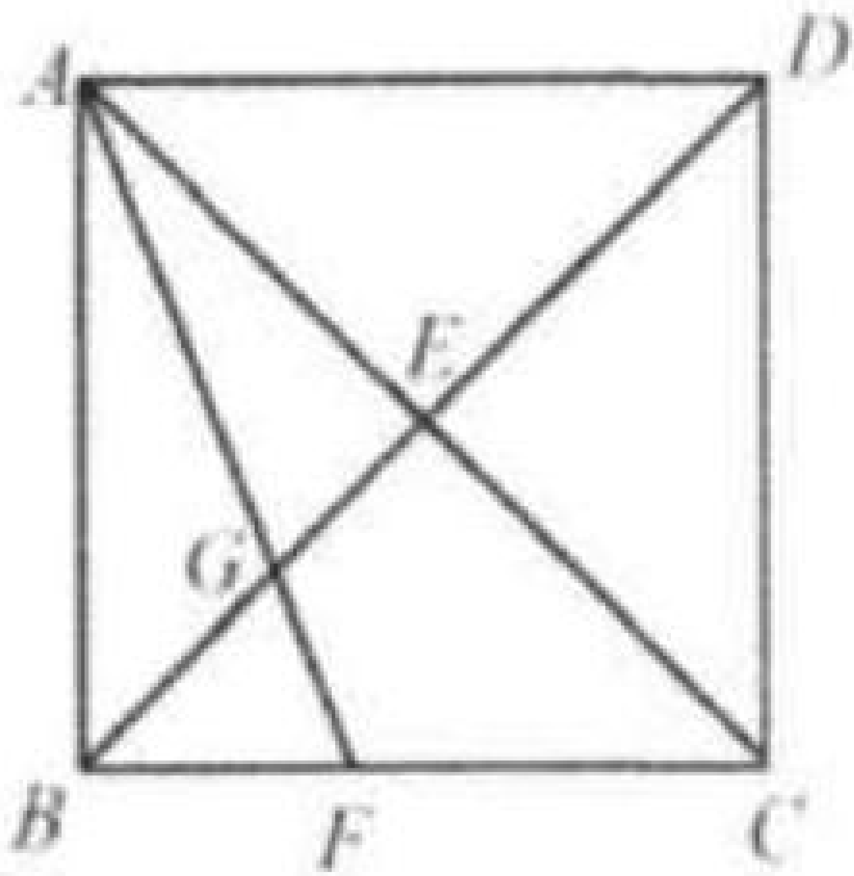
# Example 9

(1992 Shanghai Middle School Contest) In square $ABCD$, shown here, two diagonals meet at point $E$. The angle bisector of $\angle CAB$ meets $BD$ at $G$, and $BC$ at $F$. Find $FC$ if $GE = 24$.

Solution: 48. Method 1:



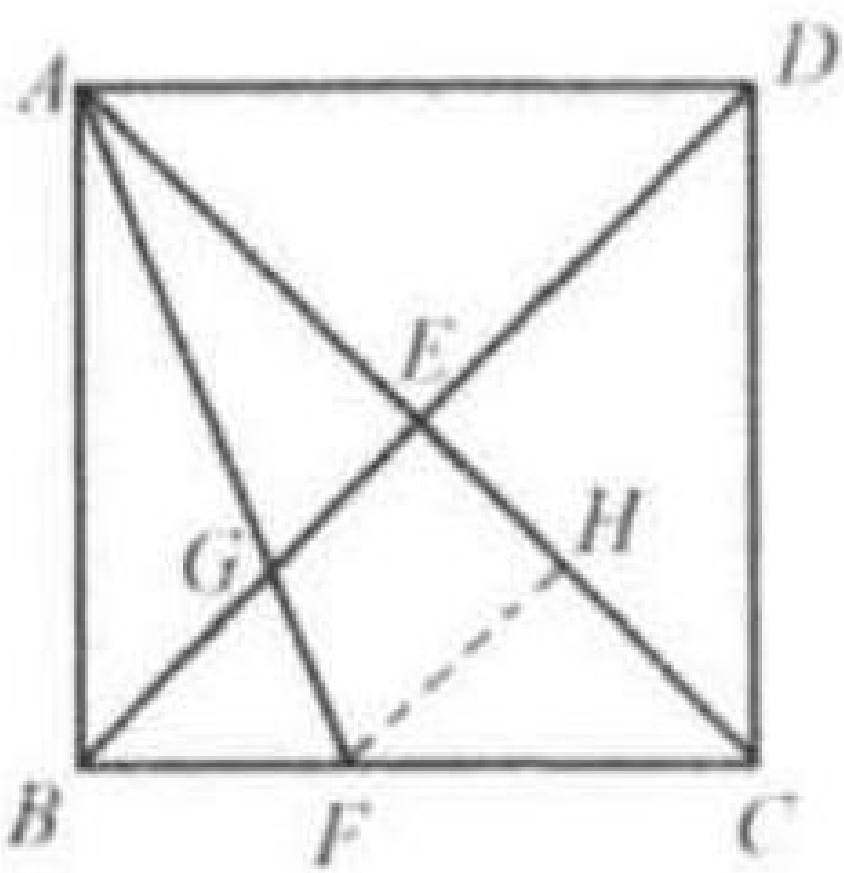

Draw $FH//BD$ to meet $AC$ at $H.\angle HFC = \angle DBC = \angle HCF = 45°$.

$HC = FH$.

Thus $\frac{GE}{FH} = \frac{AE}{AH}$

Since $AF$ is the angle bisector of $\angle CAB, \angle BAF = \angle HAF, \angle BFA$

$+\angle BAF = \angle HFA + \angle HAF = 90^\circ \quad \Rightarrow \quad \angle BFA = \angle HFA.$




We know that $AF = AF$.

So $\triangle ABF \cong \triangle AHF, AH = AB$.

Let $BF = x$. Then $HC = FH = x, FC = \sqrt{2}x$.

$DC = (1+\sqrt{2})x,$

$AC = (2+\sqrt{2})x, AE = \frac{2+\sqrt{2}}{2}x.$

(1) becomes: $\frac{24}{x} = \frac{\frac{2+\sqrt{2}}{2}x}{AC-HC} \quad \Rightarrow \quad \frac{24}{x} = \frac{\frac{2+\sqrt{2}}{2}x}{(2+\sqrt{2})x-x}.$

Solving we get $x = 24\sqrt{2}$.

$FC = \sqrt{2}x = 48.$

Method 2:

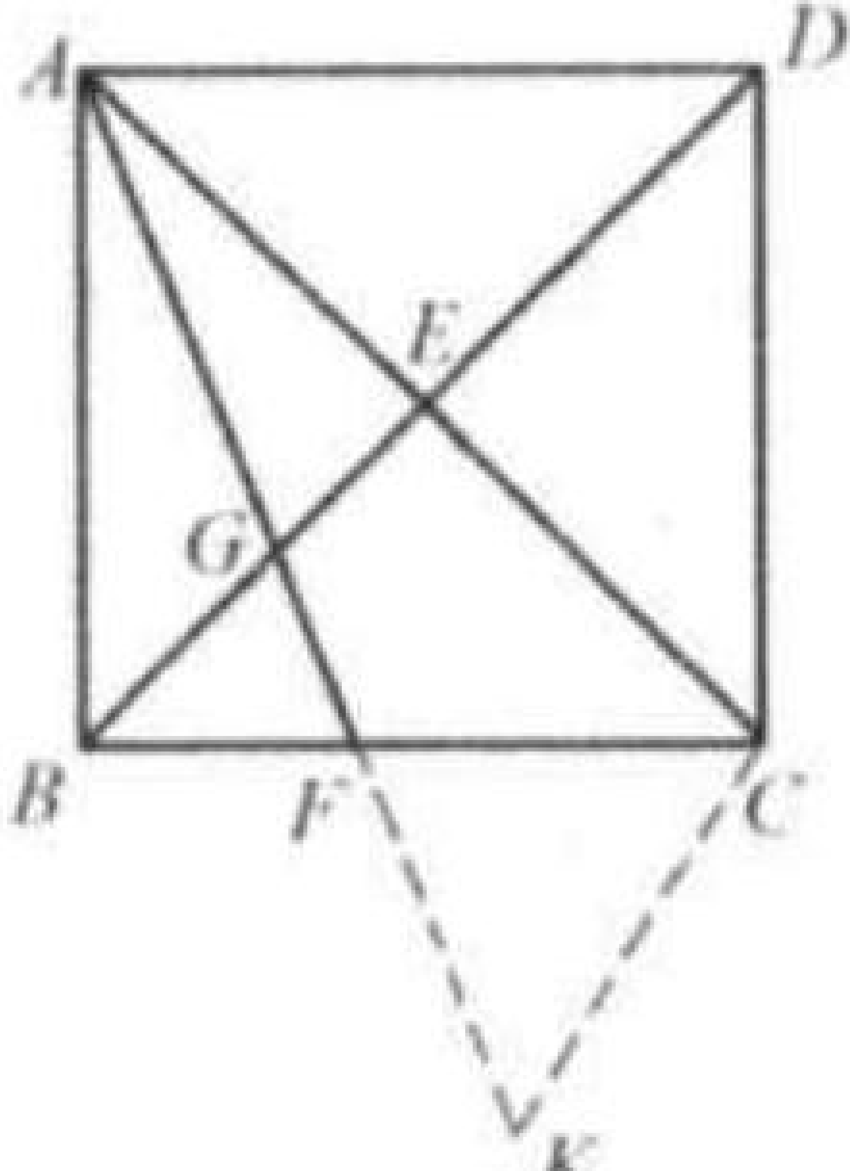
Draw $CK//GE$ to meet the extension of $AF$ at $K$.

Since $AE = EC, CK//GE, CK = 2GF = 48$.

$\angle KAC = 22.5^\circ$.

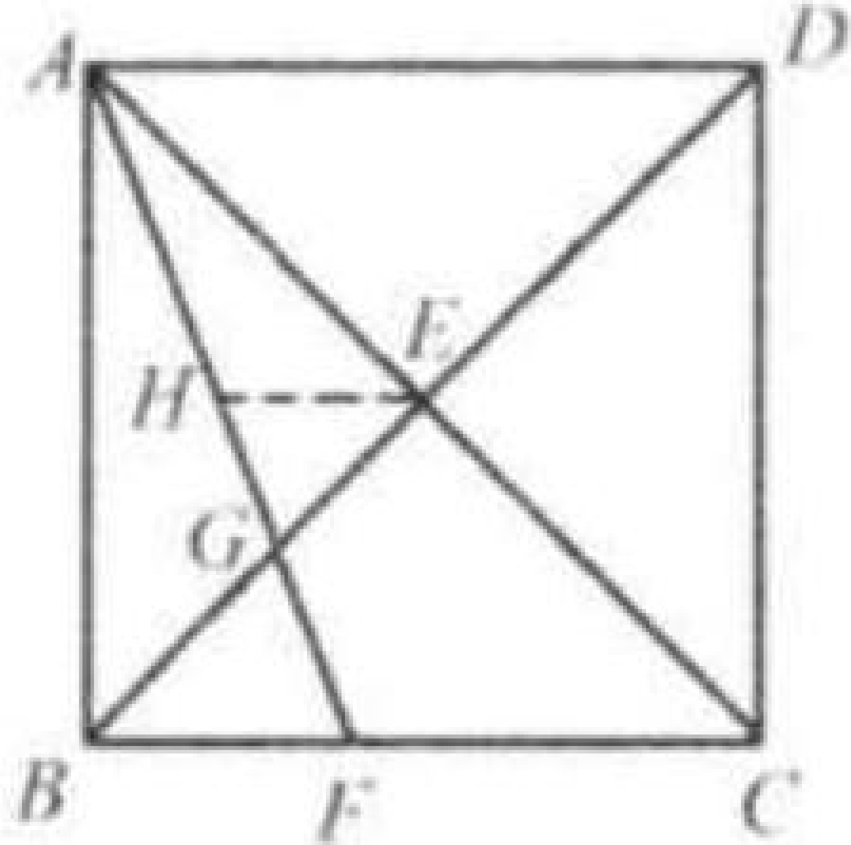
$\angle ACK = 90^\circ$. So $\angle K = 67.5^\circ$.

$\angle FCK = 45^\circ$. So $\angle CFK = 67.5^\circ$.




Thus $FC = CK = 48$.
Method 3:
Draw $EH//FC$ to meet $AF$ at $H$.
Since $AE = EC, EH//FC, FC = 2EH$.
Since $\angle BAF = 22.5°, \angle AFB = 90° - 22.5° = 67.5°$.

Since $EH//FC//BF, \angle EHG = \angle BFG = 67.5°$




$\angle EGF$ is the exterior angle of triangle $ABG$. So
$\angle EGF = 22.5° + 45° = 67.5° = \angle EHG$.
Triangle EHG is an isosceles triangle with $EH = EG = 24$.
So $FC = 2EH = 48$.