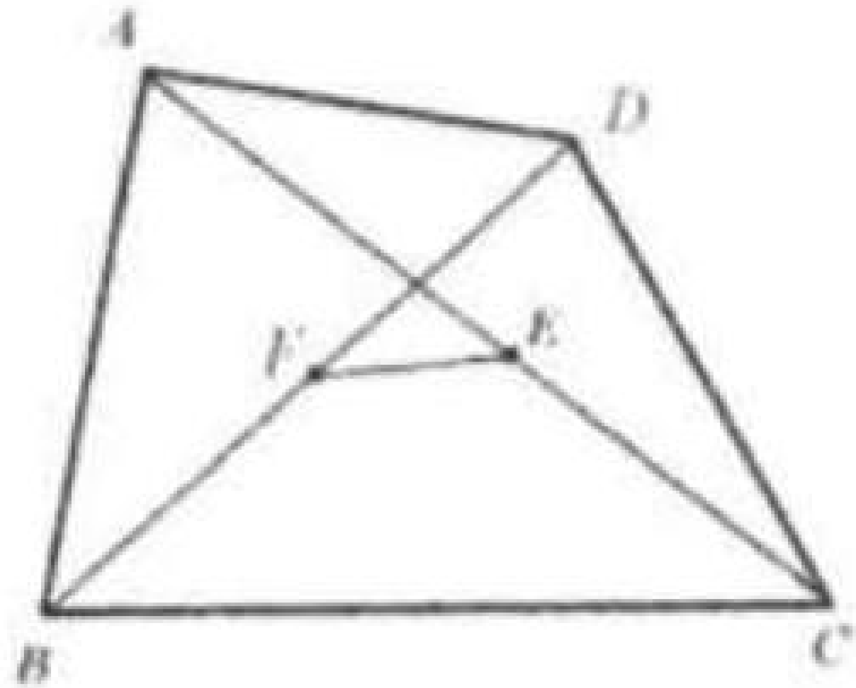
Problem 3

Problem

$ABCD$ is a convex quadrilateral. E and F are midpoints of diagonals BD, AC, respectively. Show that $EF > \frac{1}{2}(AB - CD)$

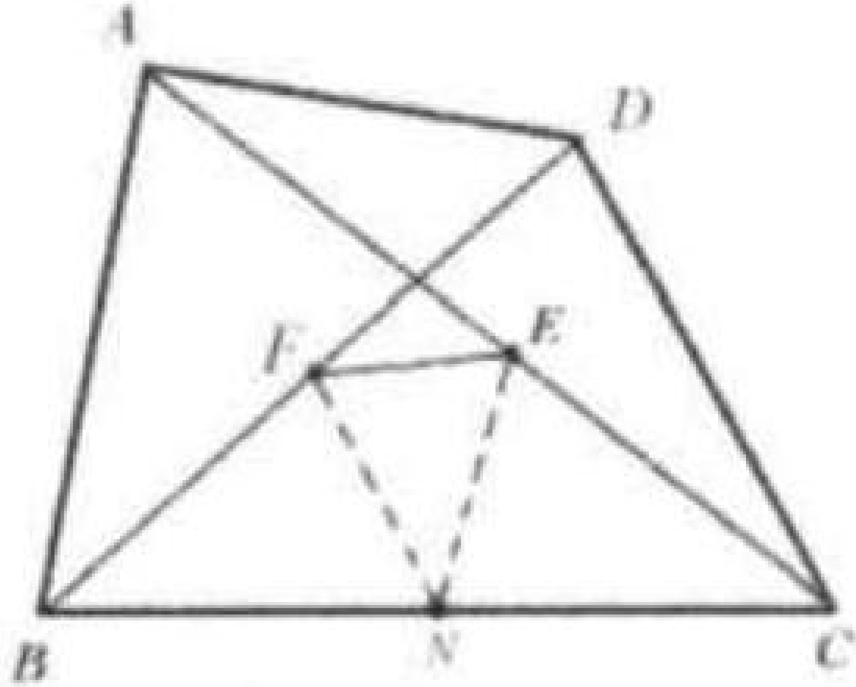


Solution

Take N, the midpoint BC.
Connect the midpoints of EN, and FN, respectively. By Theorem 2.1, in triangle $BDC, FN//DC$, and

$$FN = \frac{1}{2}DC$$

By Theorem 2.1, in triangle $CAB, EN//AB$, and



$$EN = \frac{1}{2}AB$$

(2) - (1): $EN - FN = \frac{1}{2}(AB - CD)$
By the triangle inequality theorem, $EN - FN < EF$.
(3) can be written as $EN - FN = \frac{1}{2}(AB - CD) < EN$, or $EF > \frac{1}{2}(AB - CD)$.