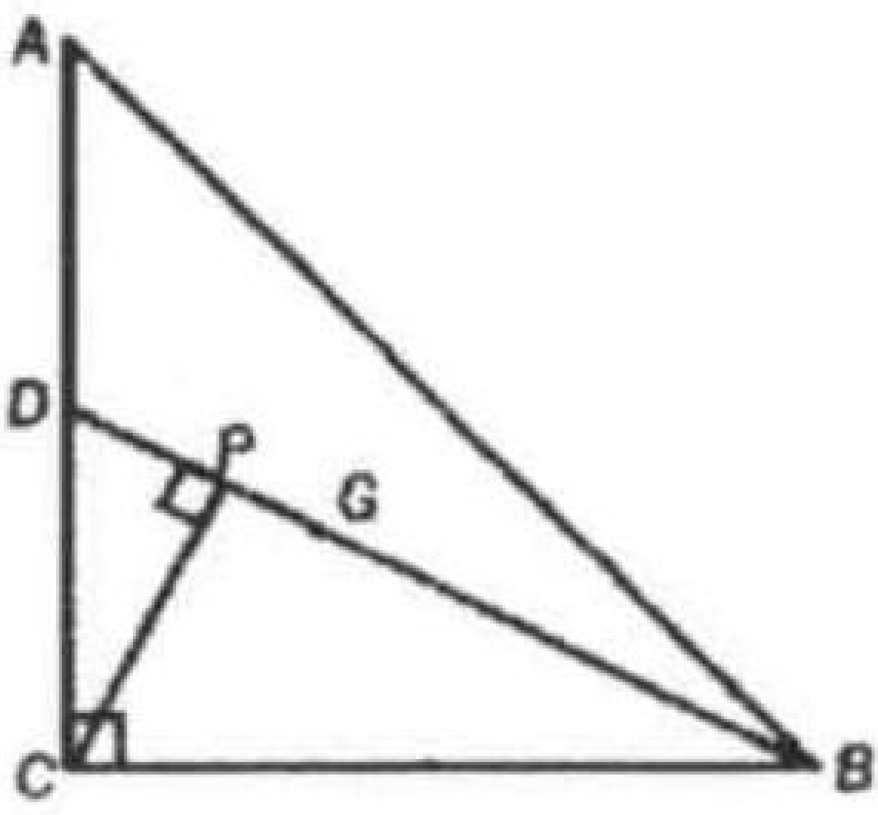
Problem 5

Problem

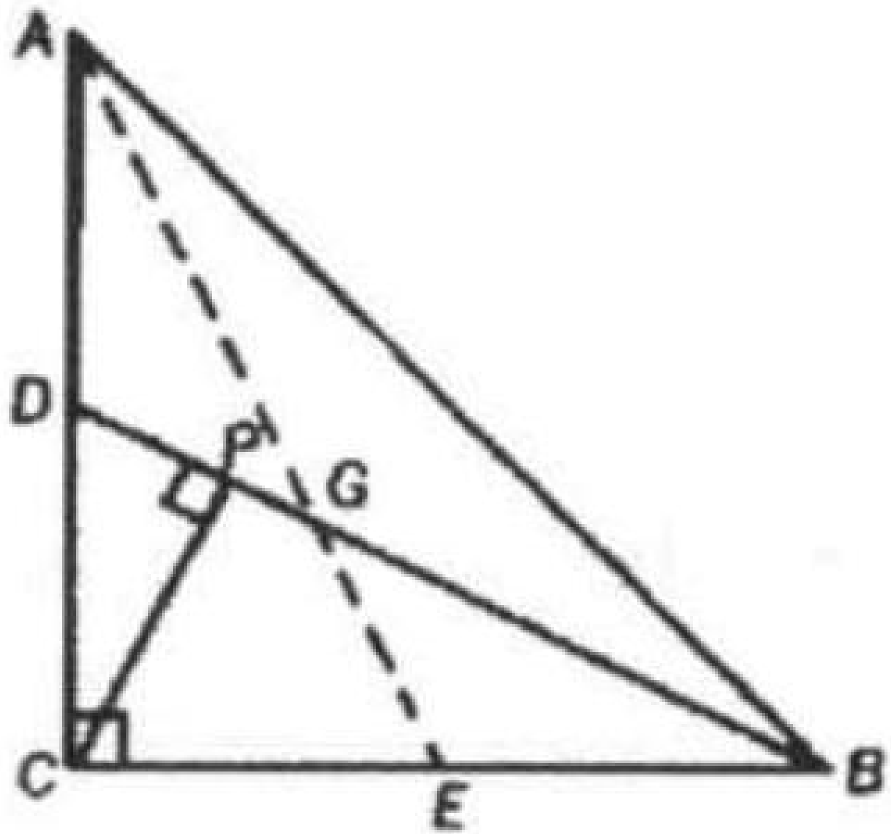
In $\triangle ABC$, angle C is a right angle. AC and BC are each equal to 1. D is the midpoint of $AC.BD$ is drawn, and a line perpendicular to BD at P is drawn from C. Find the distance from P to the intersection of the medians of $\triangle ABC$.



Solution

$\frac{1}{15}\sqrt{5}$.

Applying the Pythagorean Theorem to $\triangle DCB$ gives us
$(DC)^2 + (CB)^2 = (DB)^2$.
$1/4 + 1 = (DB)^2, DB = \frac{1}{2}\sqrt{5}$.
Since the centroid of a triangle trisects each of the medians,
$DG = \frac{1}{3}DB = \frac{1}{3}\left(\frac{1}{2}\sqrt{5}\right) = \frac{1}{6}\sqrt{5}$
Consider right $\triangle DCB$ where CP is the altitude drawn upon the hypotenuse.
Therefore, $DB/DC = DC/DP. \frac{\frac{1}{2}\sqrt{5}}{\frac{1}{2}} = \frac{\frac{1}{2}}{DP}, DP = \frac{\sqrt{5}}{10}$



Thus, $PG = DG - DP$, and $PG = \frac{1}{6}\sqrt{5} - \frac{1}{10}\sqrt{5} = \frac{1}{15}\sqrt{5}$.