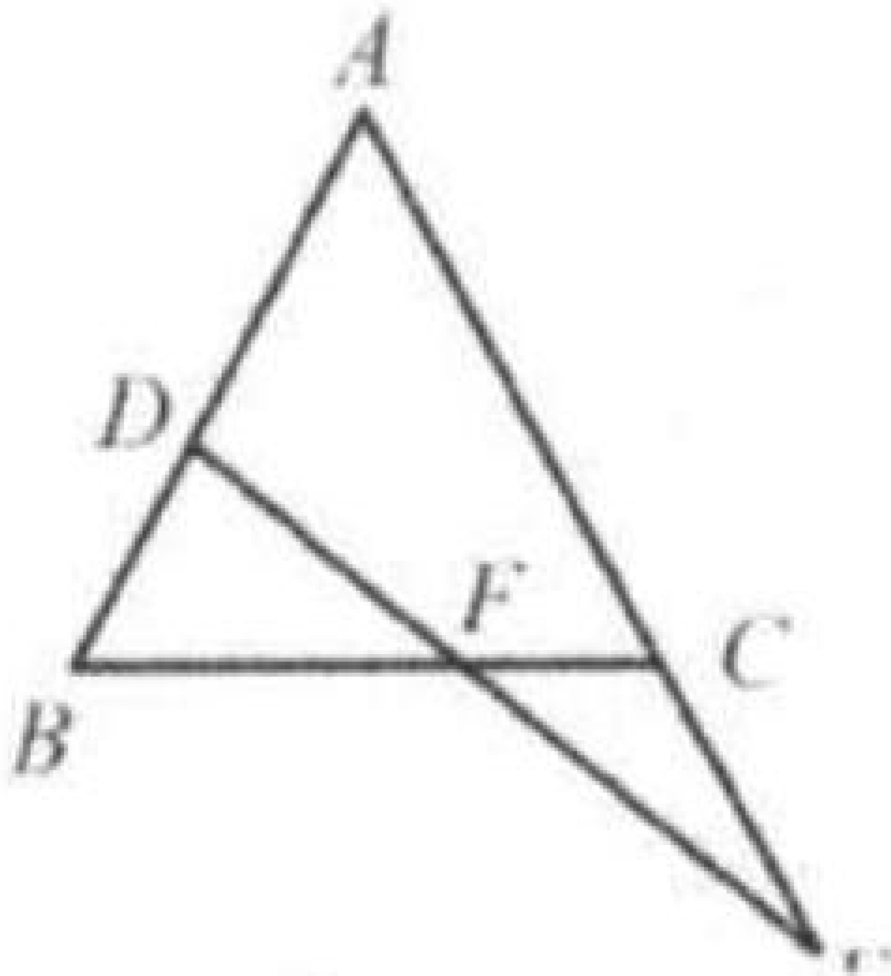
Example 17

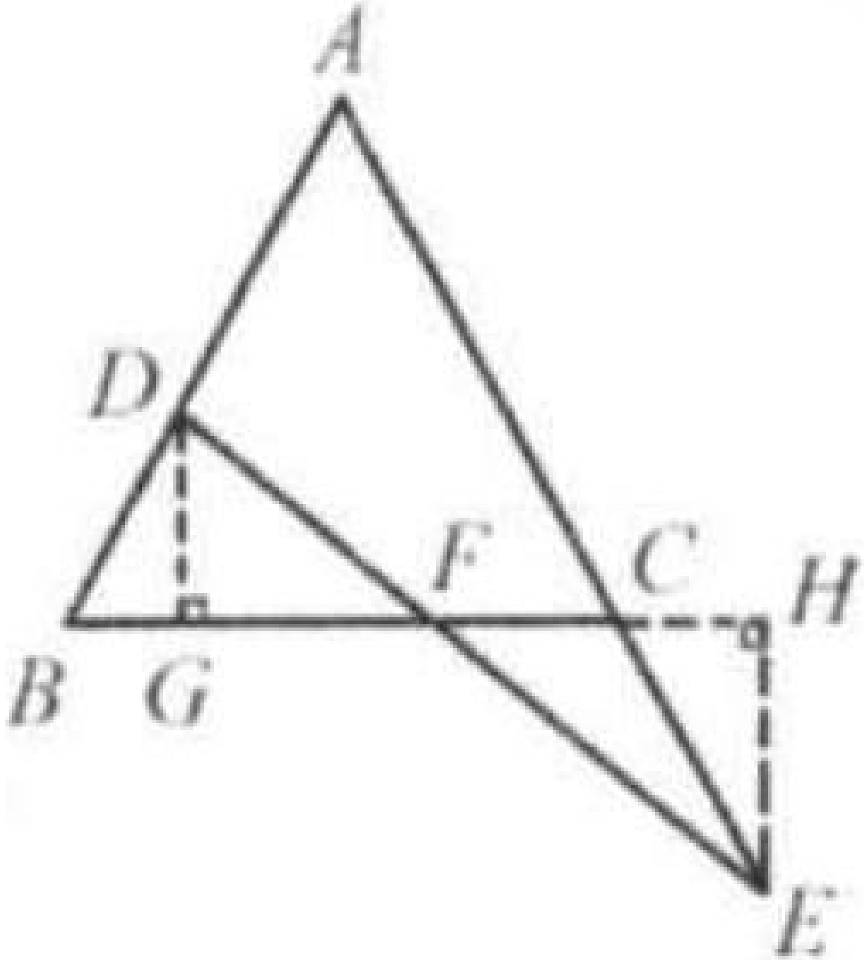
Triangle ABC is an isosceles triangle. D is on AB. Extend AC to E and connect DE so that $BD = CE$. Prove: $DF = FE$.

Proof: Draw $DG \perp BC$, extend BC, and then draw EH to meet the extension of BC at H such that $EH \perp BC$.



We see that $DB = BE, \angle B = \angle ACB = \angle ECH, \angle DGB = \angle EHC = 90°$.

Thus $\triangle DBG \cong \triangle EHC$.
So $DG = EH$.



In $\triangle DGF$ and $\triangle EHF, \angle GDF = \angle HEF$ (alternate interior angles of parallel lines of DG and EH), $\angle DGF = \angle EHF = 90°, DG = EH$. Thus $\triangle DGF \cong \triangle EHF$ and $DF = EF$.