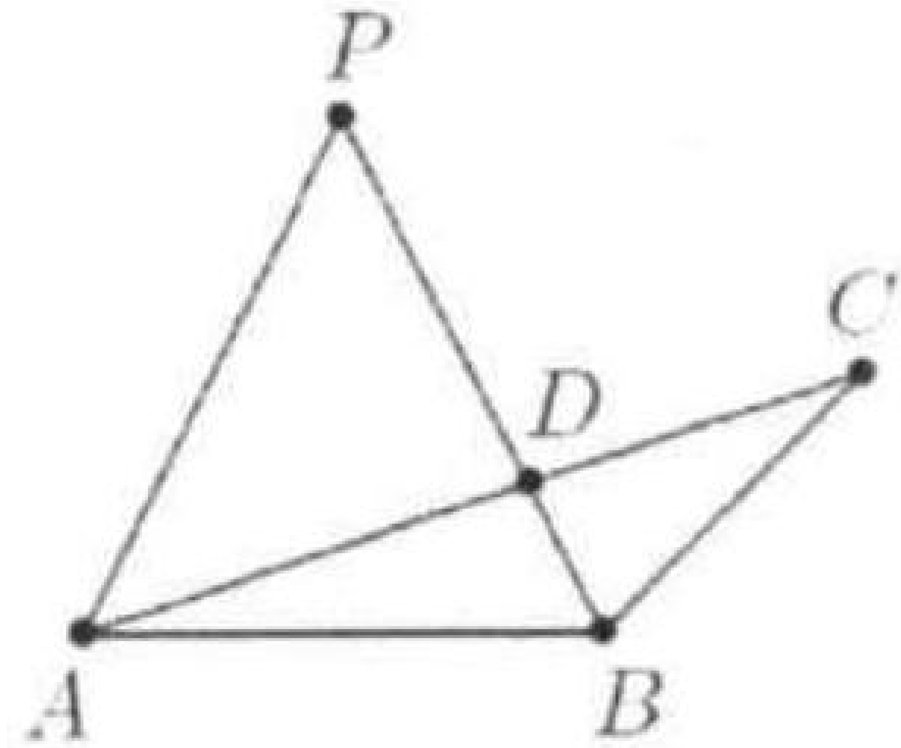
Problem

(AMC) Triangle ABC and point P in the same plane are given. Point P is equidistant from A and B, angle APB is twice angle ACB, and AC intersects BP at point D. If $PB = 3$ and $PD = 2$, then $AD \cdot CD =$

(A) 5
(B) 6
(C) 7
(D) 8
(E) 9



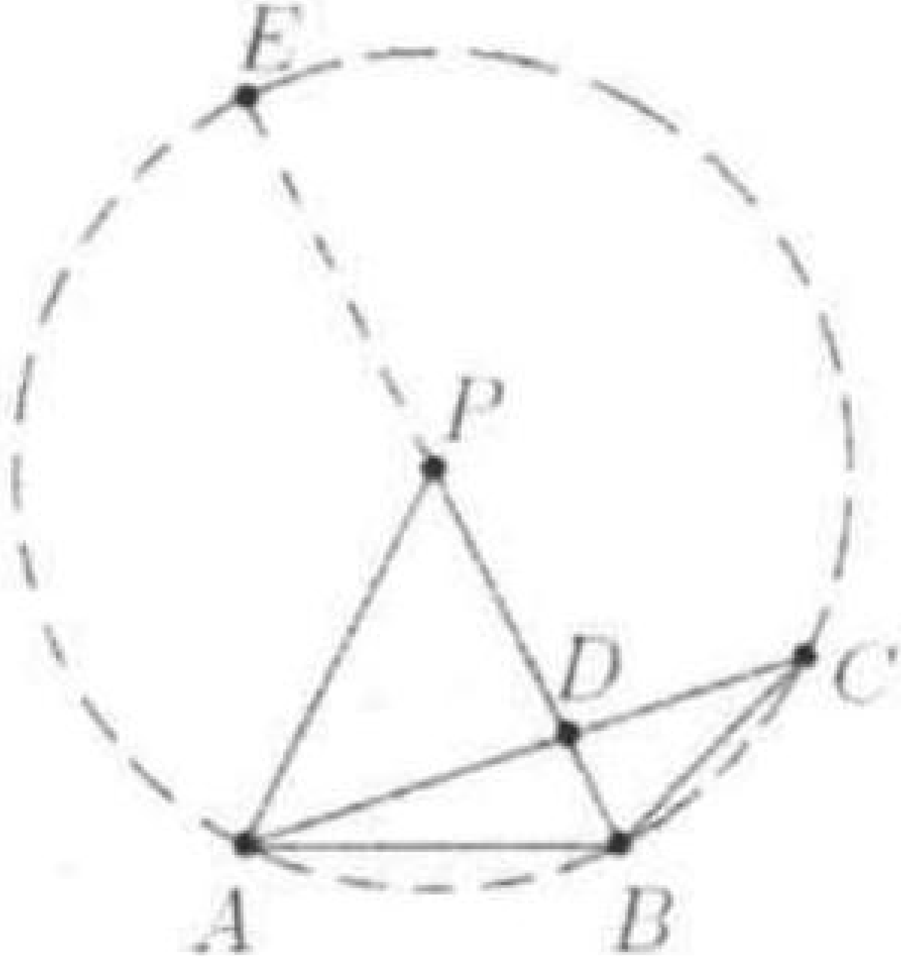
Solution

(A).

Method 1:

Construct a circle with center P and radius PA. Point C then lies on the circle, since the angle ACB is half angle APB.

Extend BP through P to get diameter BE. Since A, B, C, and E are concyclic, using the Power of a Point formula, we have:

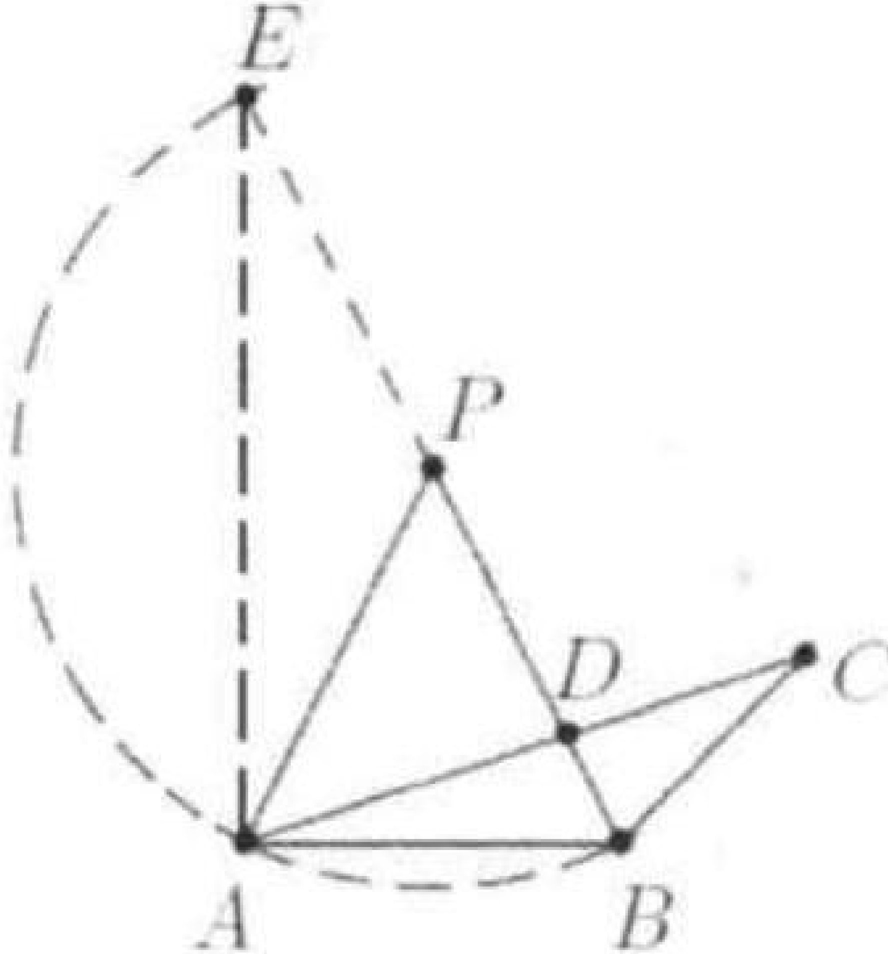


$AD \cdot CD = ED \cdot BD$
$= (PE + PD)(PB - PD) = (3 + 2)(3 - 2) = 5.$

Method 2:

Extend BP to E such that $PE = PB$. Since $PA = PB = PE$, points A, B, and E are concyclic. Construct this semicircle with center P as shown in the figure to the right.

So we have $\angle AEB = \frac{1}{2}\angle APB = \angle ACB$. We also know that



$\angle ADE = \angle BDC$ (vertical angles). Therefore $\triangle AED \sim \triangle DCB$.
$\frac{AD}{BD} = \frac{ED}{DC} \Rightarrow AD \cdot CD = ED \cdot BD = (PE + PD)(PB - PD)$
$= (3 + 2)(3 - 2) = 5.$