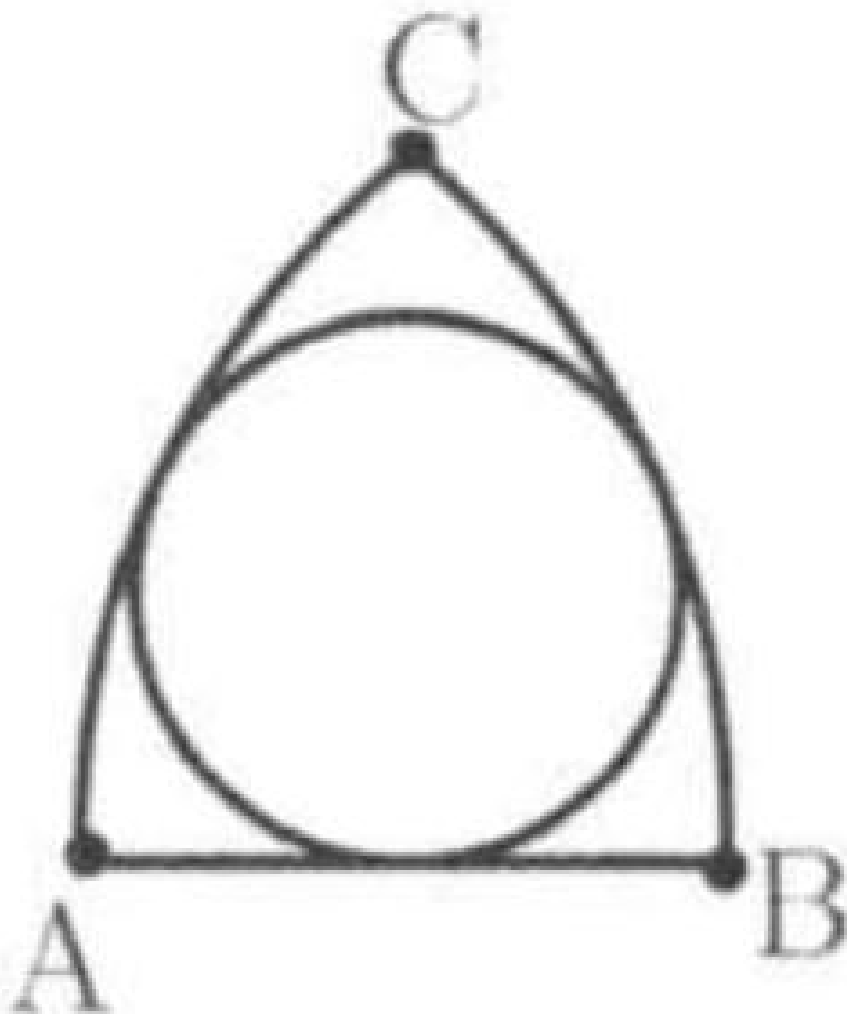


Problem 10

Problem

(AMC) If circular arcs AC and BC have centers at B and A , respectively, then there exists a circle tangent to both arcs AC and BC , and to AB . If the length of arc BC is 12, then the circumference of the circle is

- (A) 24
- (B) 25
- (C) 26
- (D) 27
- (E) 28



Solution

(D). Method 1 (official solution):

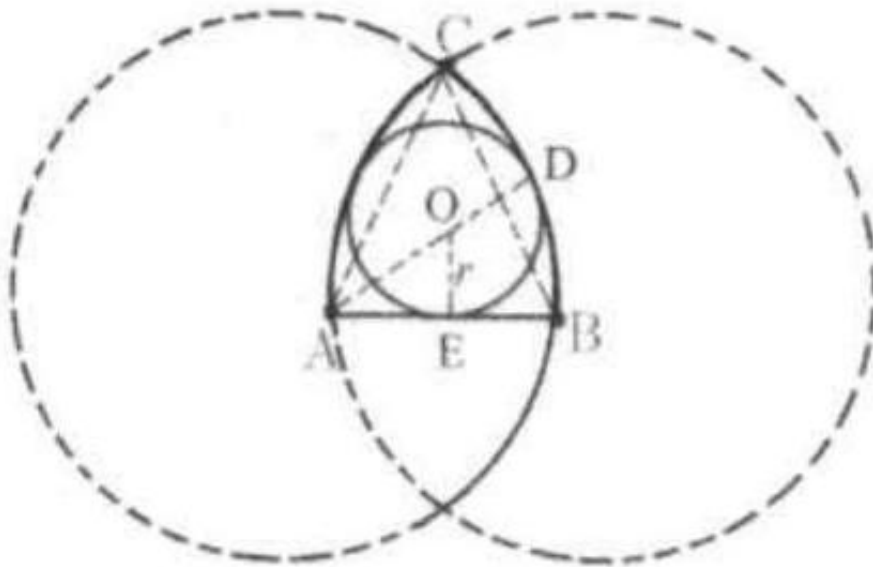
Construct the circle with center A and radius AB . Let F be the point of tangency of the two circles. Draw AF , and let E be the point of intersection of AF and the given circle.

By the Power of a Point Theorem, $AD^2 = AF \cdot AE$. Let r be the radius of the smaller circle. Since AF and AB are radii of the larger circle, $AF = AB$ and $AE = AF - EF = AB - 2r$. Because $AD = AB/2$, substitution into the first equation yields $(AB/2)^2 = AB \cdot (AB - 2r)$; or, equivalently, $r/AB = 3/8$.

Therefore $c = (3/8) \cdot 72 = 27$.

Construct the circles with centers A and B with radius AB . Triangle ABC is an equilateral triangle.

Applying Pythagorean Theorem to triangle AOE ,



$$\begin{aligned}
 r^2 &= AO^2 - AE^2 = (AD - r)^2 - \left(\frac{1}{2}AB\right)^2 \\
 &= (AB - r)^2 - \left(\frac{1}{2}AB\right)^2 \\
 r &= \frac{3AB}{8} = \frac{3 \times 36}{8\pi} = \frac{27}{2\pi}. \text{ The circumference is } 2\pi r = 2\pi \times \frac{27}{2\pi} = 27.
 \end{aligned}$$