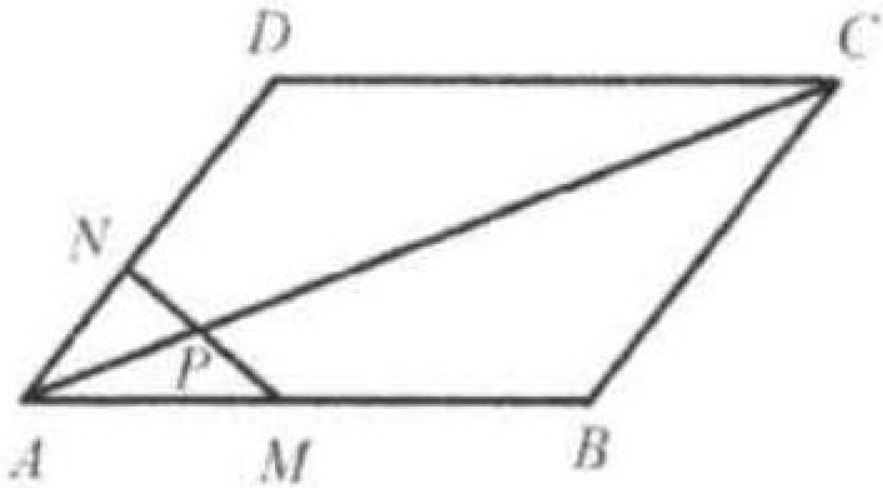
# Problem

(2009 AIME) In parallelogram $ABCD$, point $M$ is on $AB$ so that $\frac{AM}{AB} = \frac{17}{1000}$, and point $N$ is on $AD$ so that $\frac{AN}{AD} = \frac{17}{2009}$. Let $P$ be the point of intersection of $AC$ and $MN$. Find $\frac{AC}{AP}$.



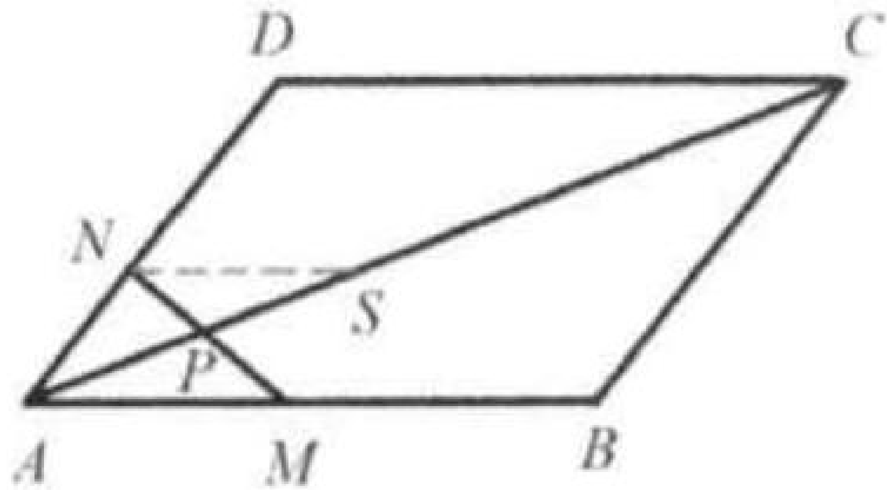

# Solution

177. Method 1:

Let point $S$ be on $AC$ such that $NS$ is parallel to $AB$.

Because $\triangle ASN$ is similar to $\triangle ACD, AS/AC = (AP + PS)/AC$
$= AN/AD = 17/2009.$

Because $\triangle PSN$ is similar to $\triangle PAM, PS/AP = SN/AM$




$= \frac{\frac{17}{2009}CD}{\frac{17}{1000}AB} = \frac{1000}{2009}$, and so $\frac{PS}{AP} + 1 = \frac{3009}{2009}$.

Hence $\frac{\frac{17}{2009}AC}{AP} = \frac{3009}{2009}$, and $\frac{AC}{AP} = 177$.

Method 2:

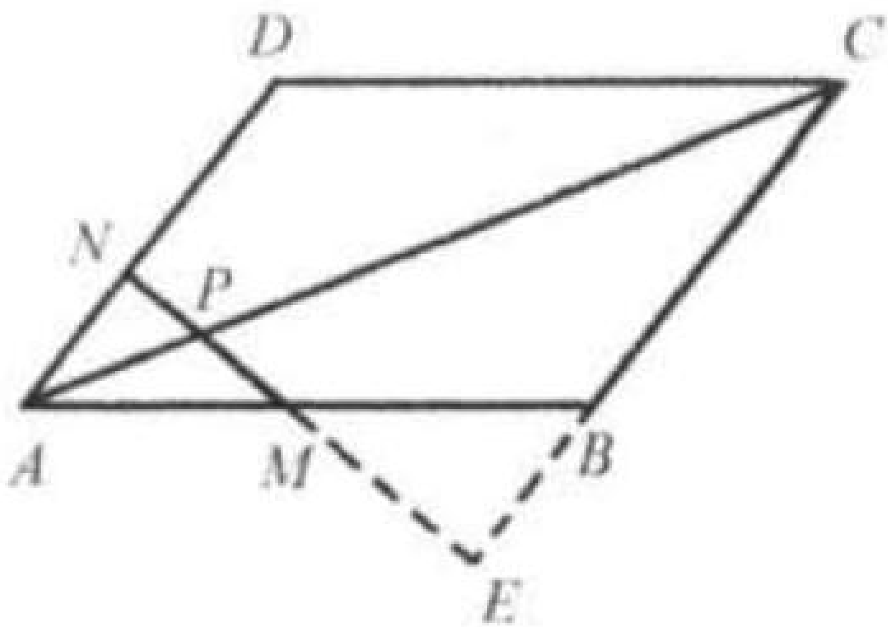
Extend $NM$ through $M$ to $E$ and to meet the extension of $CB$ at $E$.

We label the line segments as shown in the figure 1. We know that $AD//CE$.

So $\triangle AMN \sim \triangle BME$ (Figure 1).

$\frac{AN}{BE} = \frac{AM}{MB} \Rightarrow \quad \frac{17y}{BE} = \frac{17x}{983x} \Rightarrow BE = 983y$.

We know that $AN//CE$. So $\triangle APN \sim \triangle CPE$ (Figure 2).



$$\frac{AN}{CE} = \frac{AP}{PC} \quad \Rightarrow \frac{17y}{(2009+983)y} = \frac{AP}{AC-AP} \quad \Rightarrow$$
$$\frac{AC}{AP} - 1 = \frac{2992}{17} \quad \Rightarrow \quad \frac{AC}{AP} = \frac{2992}{17} + 1 = 177.$$

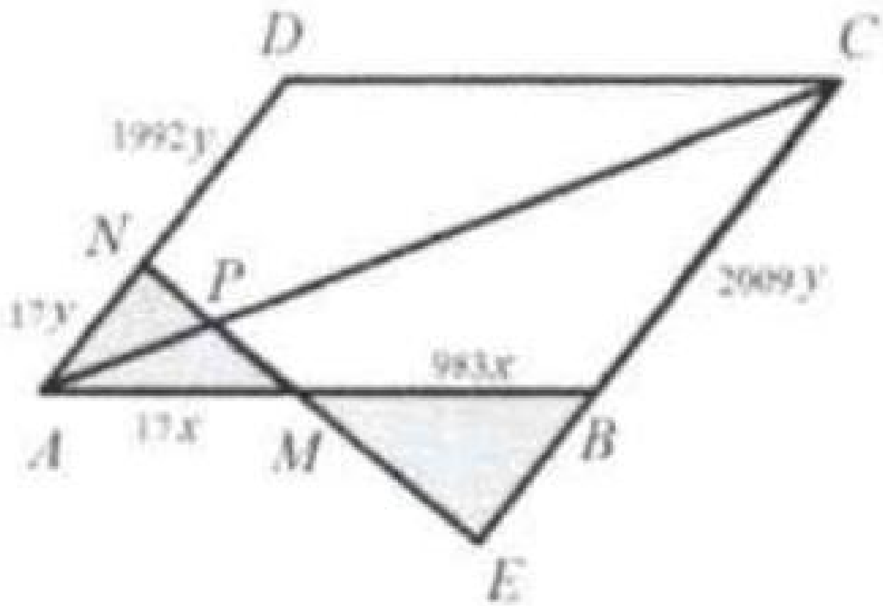



Figure 1

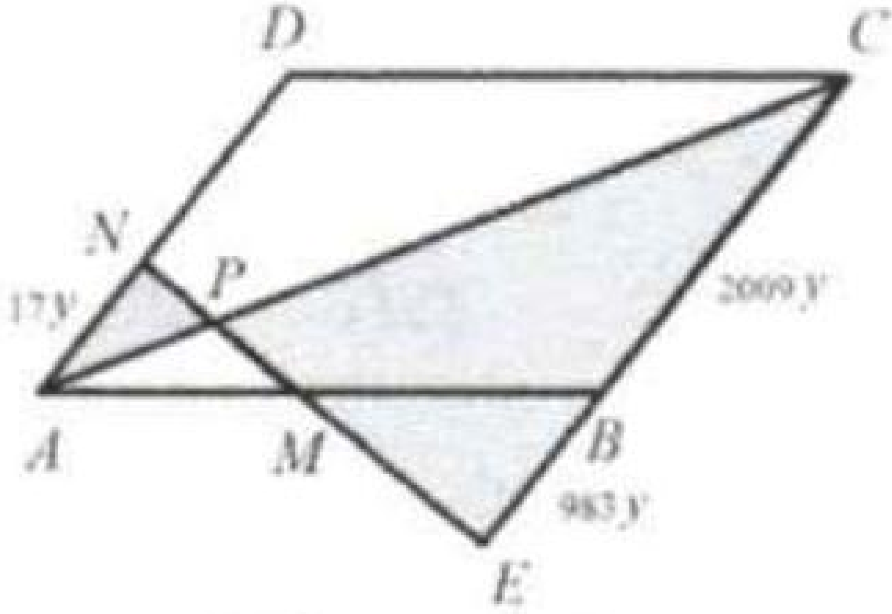



Figure 2