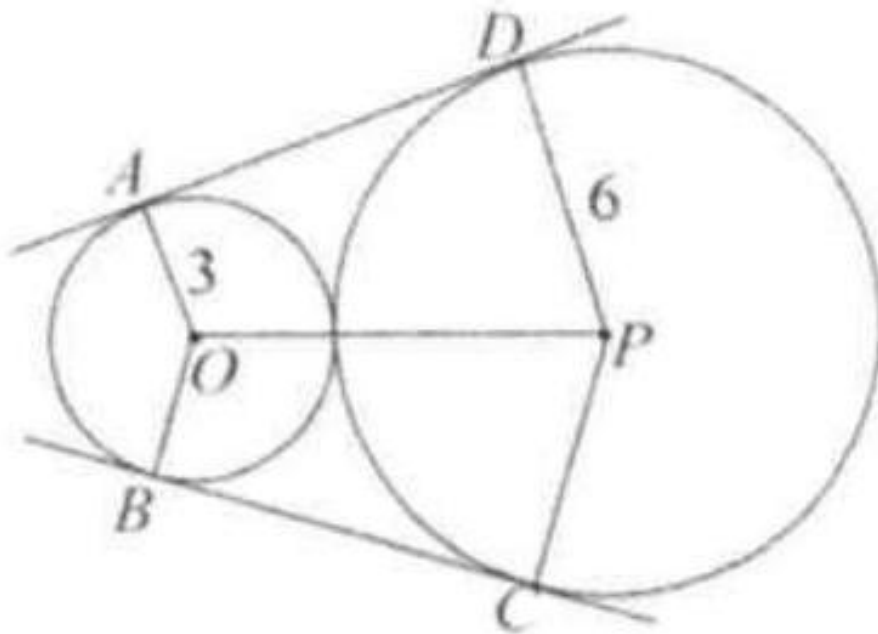# Example 3

As shown in the figure, circles O and P have radii 3 and 6, and are externally tangent. Points A and B are on the circle O, and points C and D are on the circle P. AD and BC are common external tangents to both circles. Find the area of trapezoid $AOPD$.

(A) $54\sqrt{2}$
(B) $27\sqrt{2}$
(C) 54
(D) $54\sqrt{3}$
(E) $18\sqrt{3}$

Solution: (B).

We draw a line through O parallel to AD intersecting PD at F. So $AOFD$ is a rectangle and OPF is a right triangle. $DF = 3, FP = 3$, and $OP = 3 + 6 = 9$.
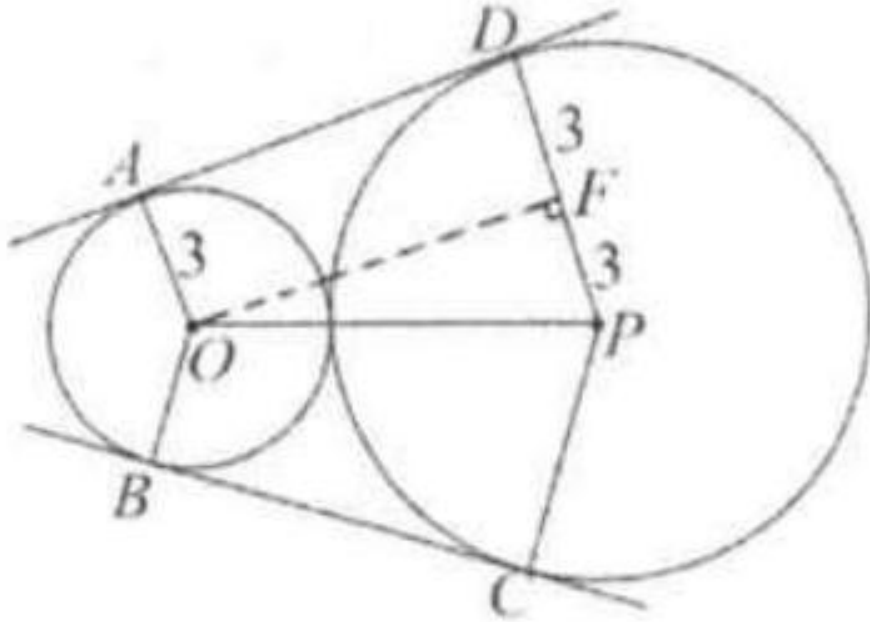
Applying Pythagorean Theorem to triangle OPF :
$$OF^2 + FP^2 = OP^2 \Rightarrow OF^2 = OP^2 - +FP^2 = 81 - 9 = 72 \quad \Rightarrow OF = \sqrt{72} = 6\sqrt{2}.$$
The area of trapezoid $AOPD$ is $12\frac{(3+6)\times 6\sqrt{2}}{2} = 27\sqrt{2}.$