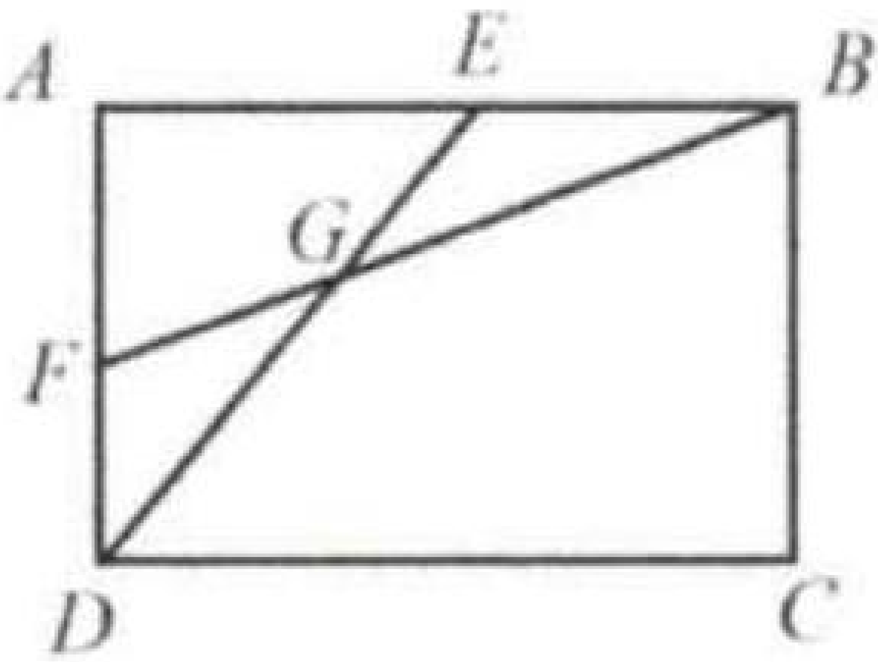
Example 8

$ABCD$ is a rectangle with $AB = 2BC$. E and F are the midpoints of AB and AD, respectively. DE and BF meet at G. What is the ratio of the area of $GBCD$ to the area of $ABCD$?

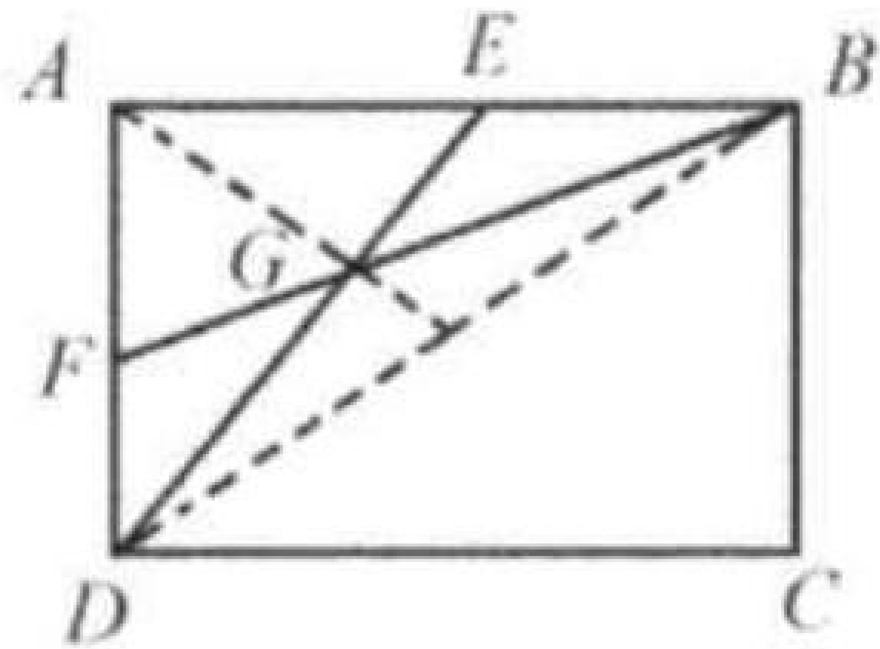
(A) $\frac{5}{2}$
(B) $\frac{2}{5}$
(C) $\frac{2}{3}$
(D) $\frac{4}{5}$
(E) $\frac{3}{5}$



Solution: (C).

We connect BD and extend AG to meet BD at H.

Since the medians DE and BF meet at G, G is the centroid of triangle ABD and the six small triangles formed have the same area.



Each of the small triangles has an area that is $\frac{1}{12}$ of the area of the rectangle $ABCD$, so the area of triangle $DBG, S_{DGB} = \frac{2}{12} \times S_{ABCD}$.

The area of triangle $DBC, S_{DBC} = \frac{1}{2} \times S_{ABCD}$.

$S_{DCBC} = S_{DCB} + S_{DBC} = \frac{2}{12}S_{ABCD} + \frac{1}{2}S_{ABCD} = \left(\frac{2}{12} + \frac{1}{2}\right) \times S_{ABCD}$.

The ration of the area of $GBCD$ to the area of $ABCD$.

$S_{DCBC} = \frac{S_{DCBC}}{S_{ABCD}} = \frac{2}{12} + \frac{1}{2} = \frac{2+6}{12} = \frac{2}{3}$.