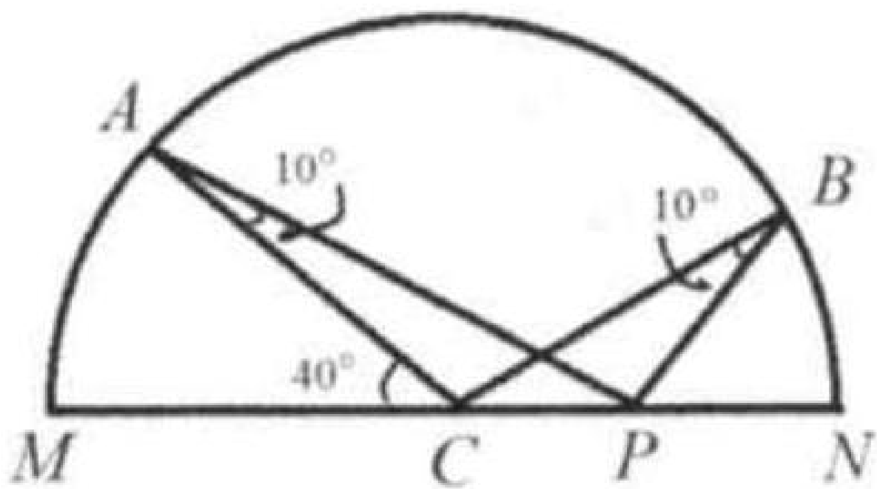
Example 9

(AMC) Distinct points A and B are on a semicircle with diameter MN and center C. The point P is on CN and $\angle CAP = \angle CBP = 10°$. If $MA = 40°$, then BN equals
(A) $10°$
(B) $15°$
(C) $20°$
(D) $25°$
(E) $30°$

Solution: (C).



Method 1:
In $\triangle ACP$ and $\triangle BCP$ we have (in the order given) the condition angle-side-side.
Since these triangles are not congruent ($\angle CPA \neq \angle CPB$), we must have that $\angle CPA$ and $\angle CPB$ are supplementary.
From $\triangle ACP$ we compute
$\angle CPA = 180° - 10° - (180° - 40°) = 30°$.
Thus $\angle CPB = 150°$ and $BN = \angle PCB = 180° - 10° - 150° = 20°$.
Method 2: $\angle CPA = 30°(\text{arc } AC)$
$\angle CBA = 30°(\text{arc } AC)$
$\angle CAB = \angle CBA(\text{arc } BC = \text{arc } AC)$

$$\angle PAB = 30^\circ - 10^\circ = 20^\circ.$$
$$\angle BCP = \angle PAB = 20^\circ \text{ (same arc } PB \text{).}$$