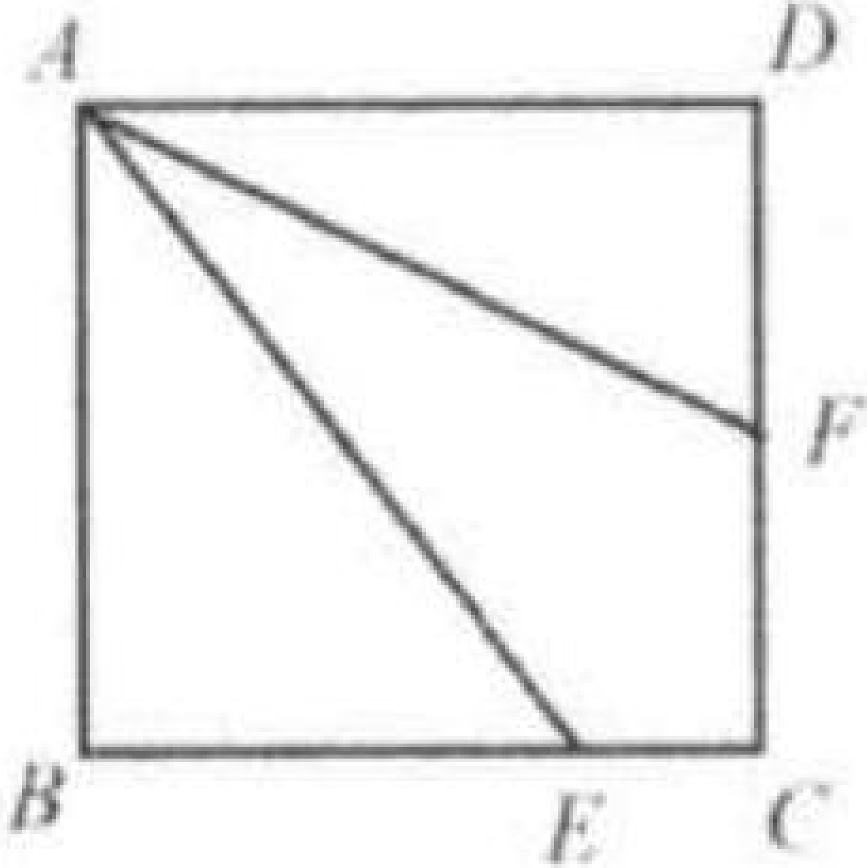
Example 9

As shown in the figure below, in square $ABCD, F$ is the midpoint of DC. E is a point on BC such that $AE = DC + CE$. Show that AF bisects $\angle DAE$.

Solution: Method 1:



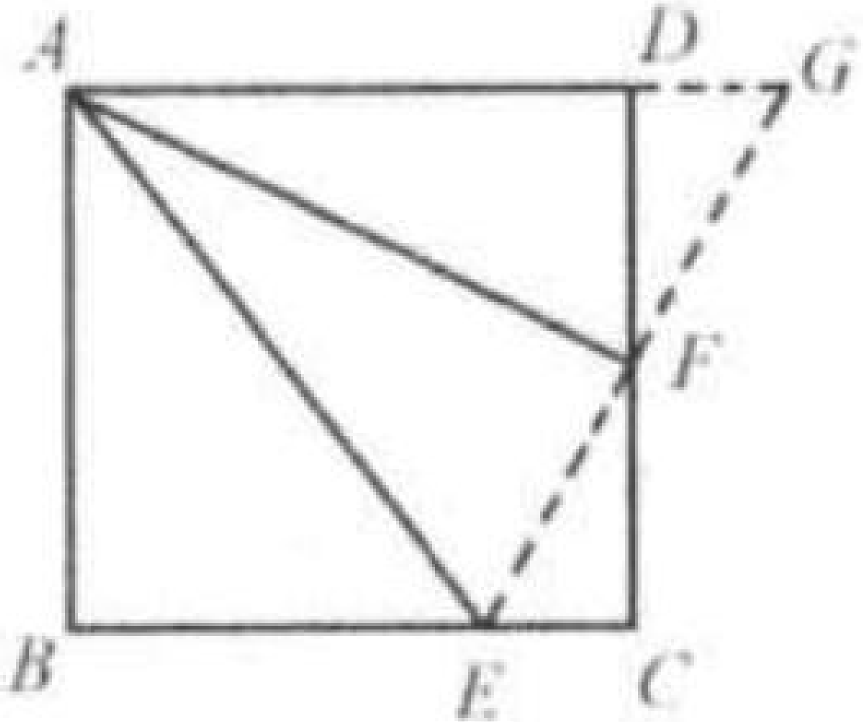
Connect EF and extend it to meet the extension of AD at G. look at triangles FDG and FCE.

We see that $FD = FC, \angle DFG = \angle CFE, \angle C = \angle FDG$.

Thus, $\triangle FDG \cong \triangle FCE$ and $DG = CE, EF = FG$.

So $AG = AD + DG = DC + CE = AE$.

Since $EF = FG, AE = AG, AF = AF, \triangle AFG \cong \triangle AFE$.

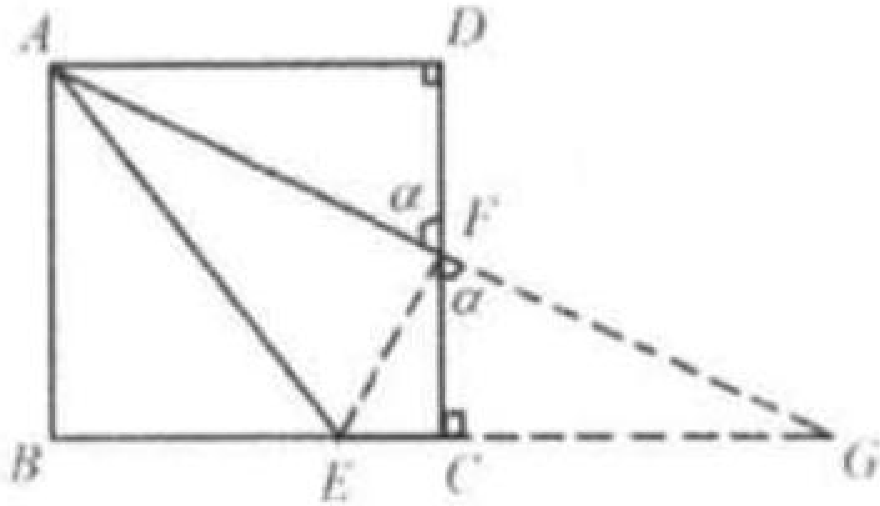


Therefore, $\angle EAF = \angle GAF, AF$ bisects $\angle DAE$.

Method 2:

Extend AF to meet the extension of BC at G.

Connect EF. Triangles ADF and GDF are congruent.



So $AD = CG . \angle DAF = \angle CGF$.

$AE = DC + CE = GC + CE = GE$.

So triangle AEG is an isosceles triangle with $AE = GE$ or $\angle EAF = \angle GGF$.

So $\angle DAF = \angle EAF$. AF bisects $\angle DAE$.