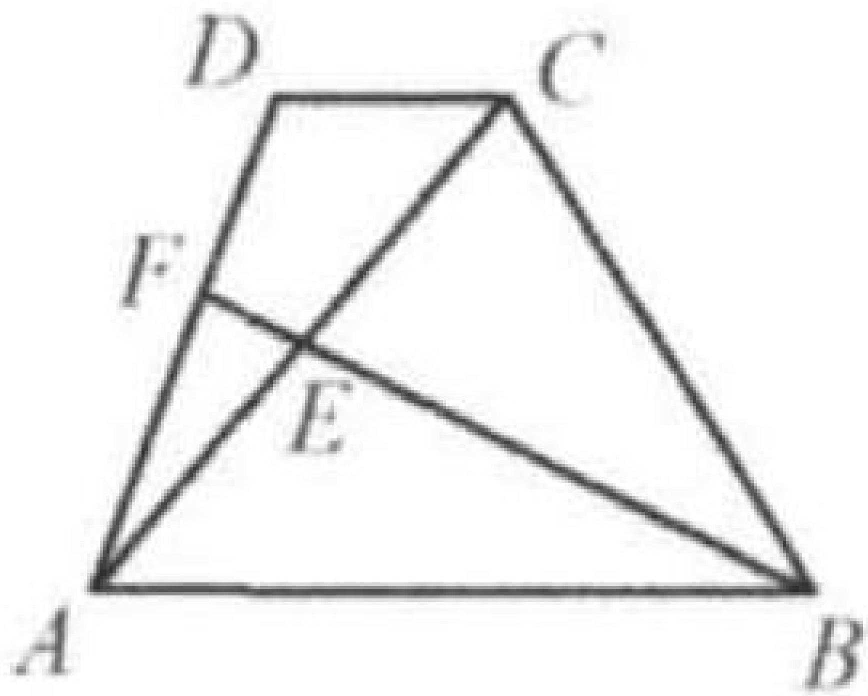
Problem 10

Problem

In trapezoid $ABCD, AB = 3CD$ and $AB//CD$. E is the midpoint of the diagonal AC. BE meets AD at F. Find the value of $AF : FD$.

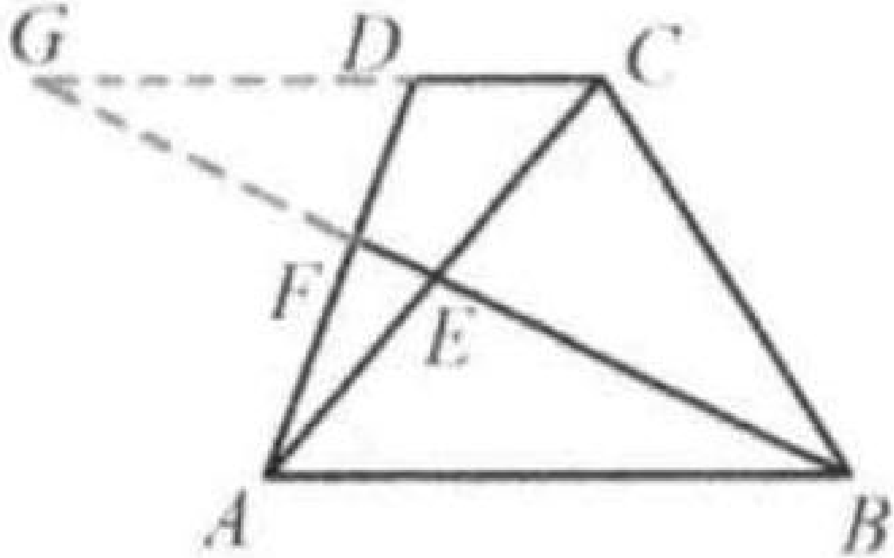
(A) $\frac{5}{3}$
(B) $\frac{3}{2}$
(C) $\frac{10}{7}$
(D)
(E) $\frac{12}{5}$.



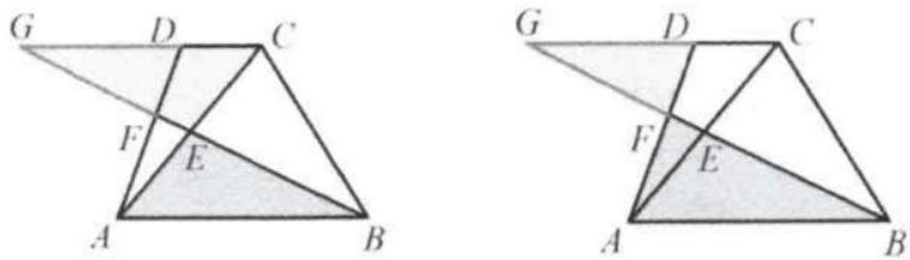
Solution

(B). Method 1:

Extend CD and BF to meet at G. Since $CG//AB, \triangle ABE \sim \triangle CGE$. So

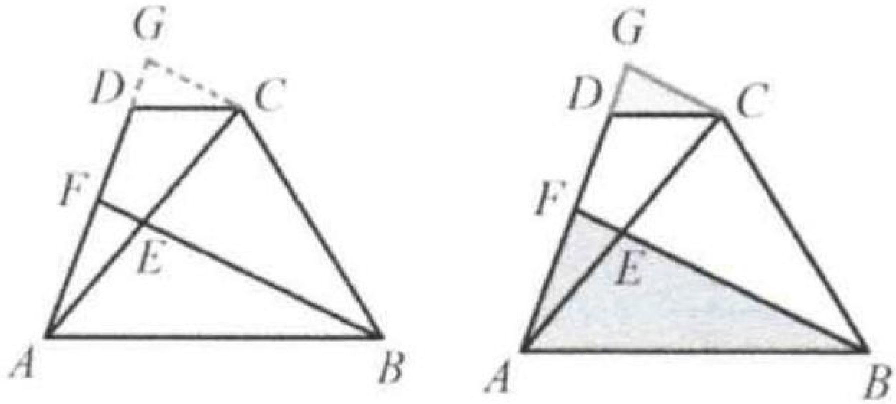


$\frac{CG}{AB} = \frac{CE}{AE} = 1 \Rightarrow CG = AB \Rightarrow$
$GD + CD = 3CD \Rightarrow GD = 2CD$
Since $DG//AB, \triangle ABF \sim \triangle DGE$.



So
$\frac{AF}{FD} = \frac{AB}{DG} \quad \Rightarrow \frac{AF}{FD} = \frac{3CD}{2CD} = \frac{3}{2}$.
Method 2:
Draw $CG//BF$ to meet the extension of AD at G. Since $AB//CD, CG//BF$,

$$\triangle ABF \sim \triangle DCG. \text{ So } \frac{AF}{DG} = \frac{AB}{CD} = 3$$
$$\Rightarrow AF = 3DG.$$



Since $CG//EF, AE = EC, AF = FG$. Therefore $FD = 2DG, \frac{AF}{FD} = \frac{3}{2}$.