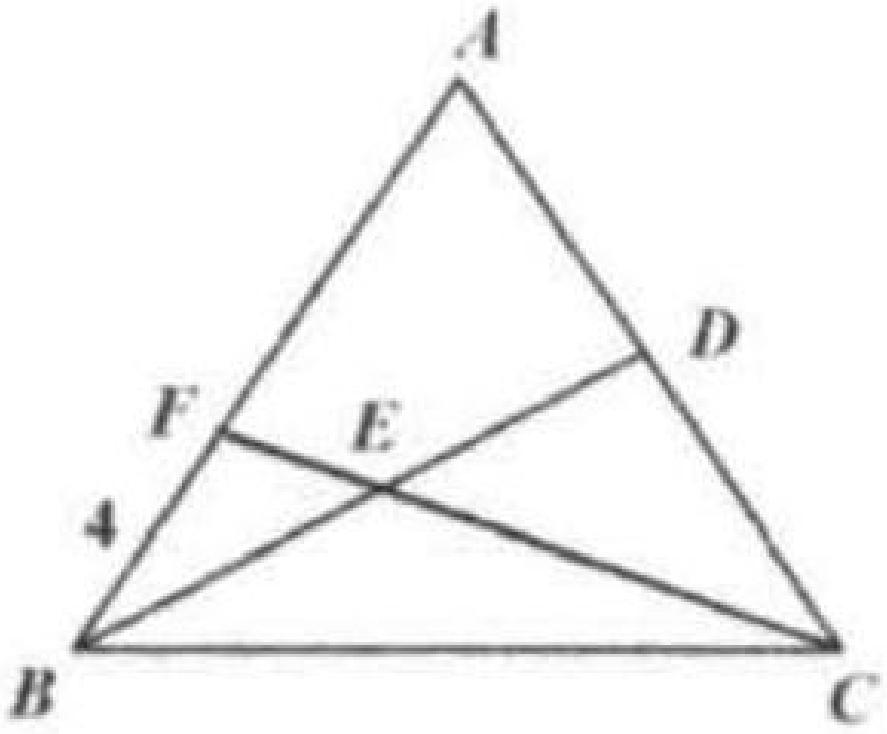
Problem 2

Problem

As shown in the figure, in triangle ABC, median BD intersects CF at E such that $BF = 4$ and $BE = ED$. Find the length of BA.

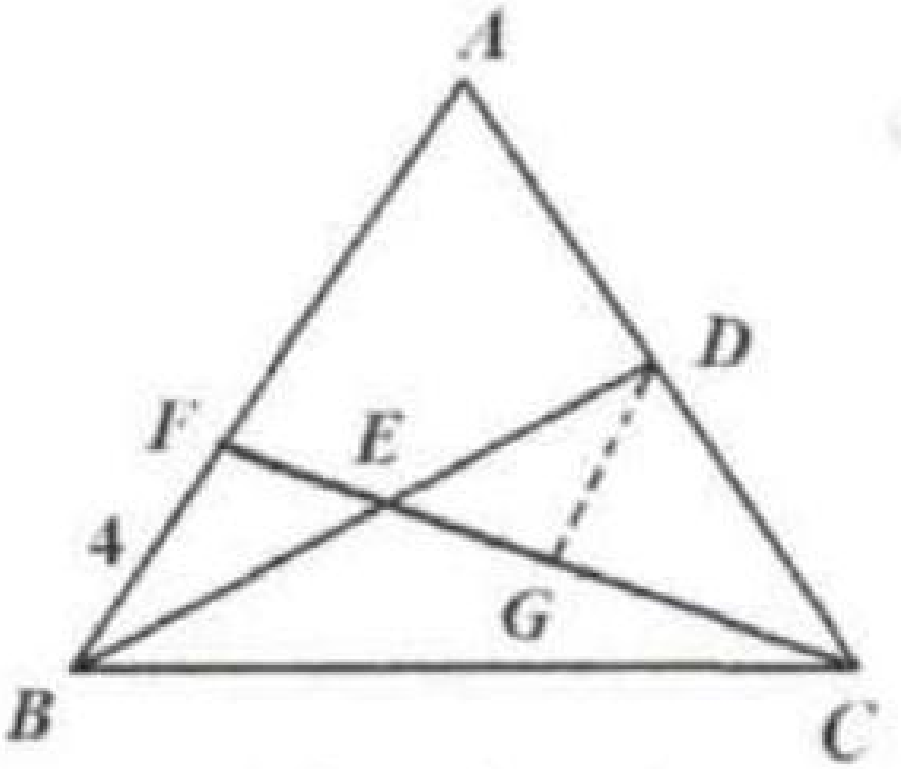


Solution

12.

Method 1:

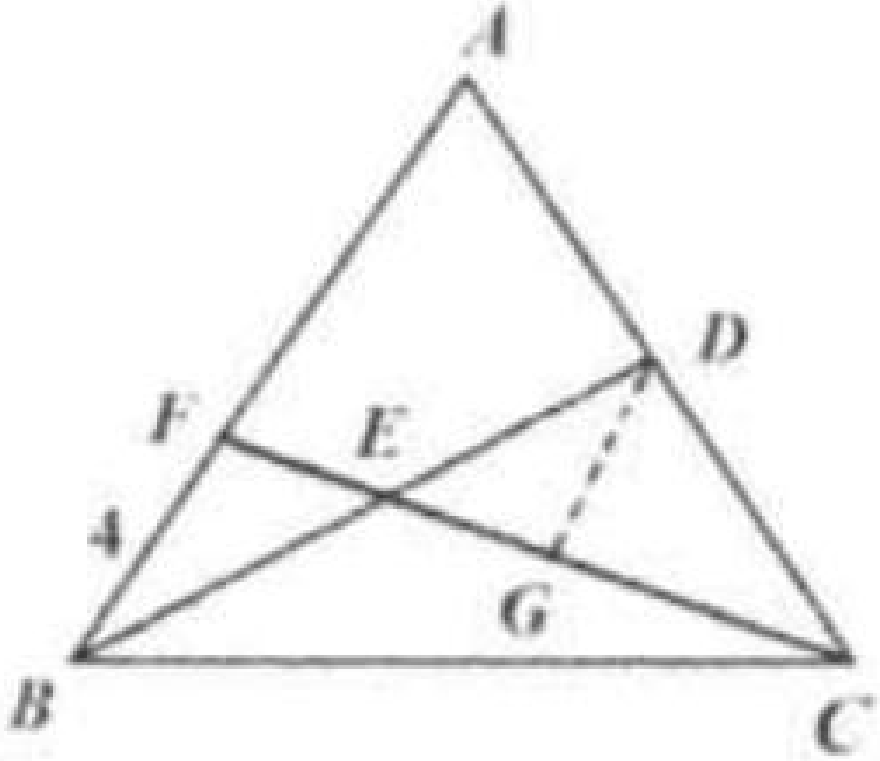
Draw $DG//AB$ to meet CF at G. Since D is the midpoint of AC, $AF = 2DG$.



Since $BE = ED, \angle EBF = \angle EDG$ (alternate interior angles) and $\angle BEF = \angle DEG$ (vertical angles), $\triangle EFB \cong \triangle EGD$ and $DG = BF = 4. AF = 2DG = 8. AB = 4 + 8 = 12.$

Method 2:

Pick up a point G on EC such that $FE = EG$. Connect D with G. Since $FE = EG$ and $BE = ED$ (diagonal bisects each other), $FDGB$ is a parallelogram. Thus $DG = 4, AF = 8, AB = 12.$


A
D
F
E
4
G
B
C