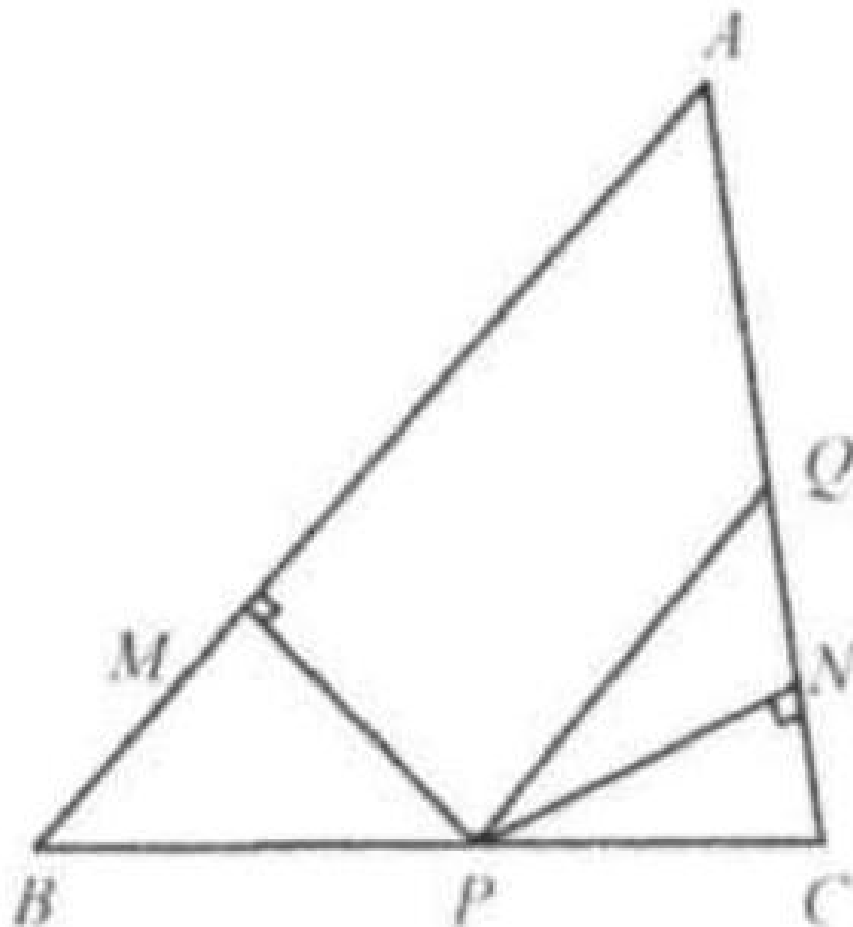


Problem 3

Problem

In scalene triangle ABC , $AQ = PQ$, $MP = PN$, $PM \perp AB$, $PN \perp AC$. The correct one of the followings is

- (1) $AN = AM$;
- (2) $QP \parallel AM$;
- (3) $\triangle BMP \cong \triangle QNP$.
- (A) all are correct
- (B) only (1) and (2) are correct
- (C) only (2) and (3) are correct
- (D) only (1) is correct
- (E) none is correct



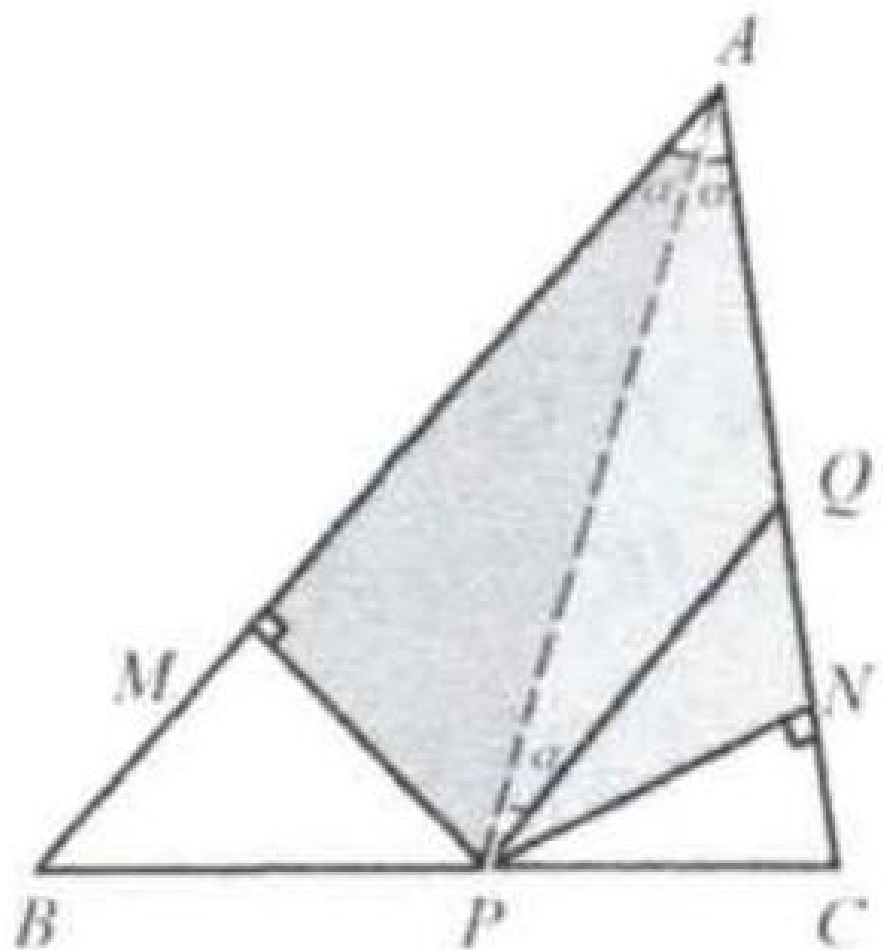
Solution

(B). Connect AP . Since $MP = PN$ and $PM \perp AB$, AP is the angle bisector of $\angle A$. We also know that $AQ = PQ$.

So $\angle APQ = \angle QAP = \angle PAM = \alpha$.

Thus, $QP \parallel AM$.

Since $\triangle APM$ and $\triangle APN$ are congruent ($AN = AM$, $MP = PN$, and $\angle AMP = \angle ANP = 90^\circ$),



So $AN = AM$.

If $\triangle BMP \cong \triangle QNP$, then $BP - PQ = AQ$, $\angle B = \angle QPC = \angle PQC$. Then we will have $PC = QC$, $BC = AC$. Triangle ABC is not a scalene triangle anymore.

So only (1) and (2) are correct.