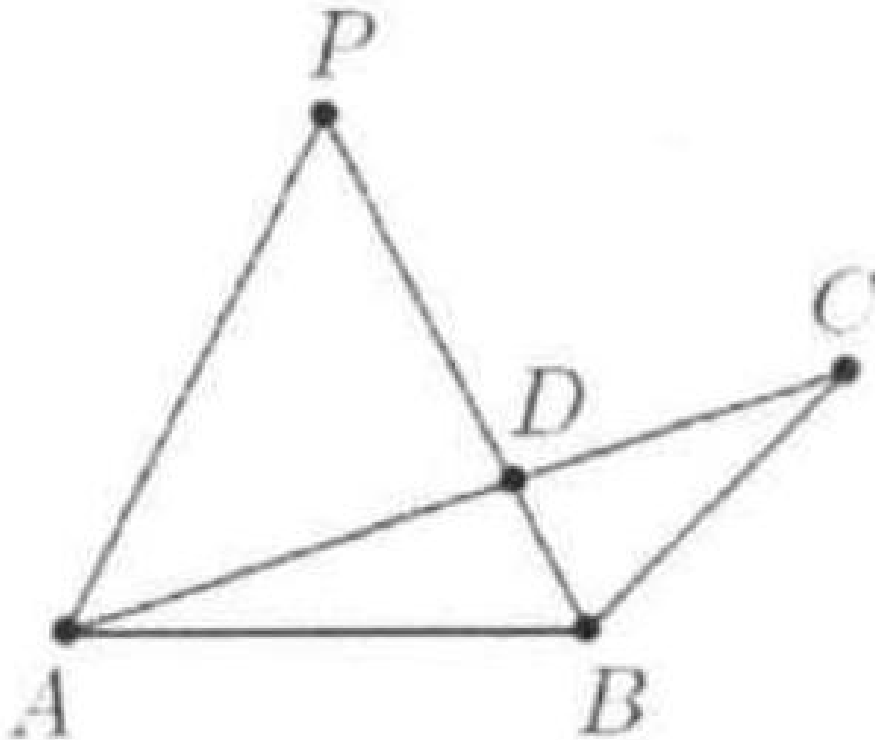


Problem 8

Problem

(AMC) Triangle ABC and point P in the same plane are given. Point P is equidistant from A and B , angle APB is twice angle ACB , and AC intersects BP at point D . If $PB = 3$ and $PD = 2$, then $AD \cdot CD =$

- (A) 5
- (B) 6
- (C) 7
- (D) 8
- (E) 9



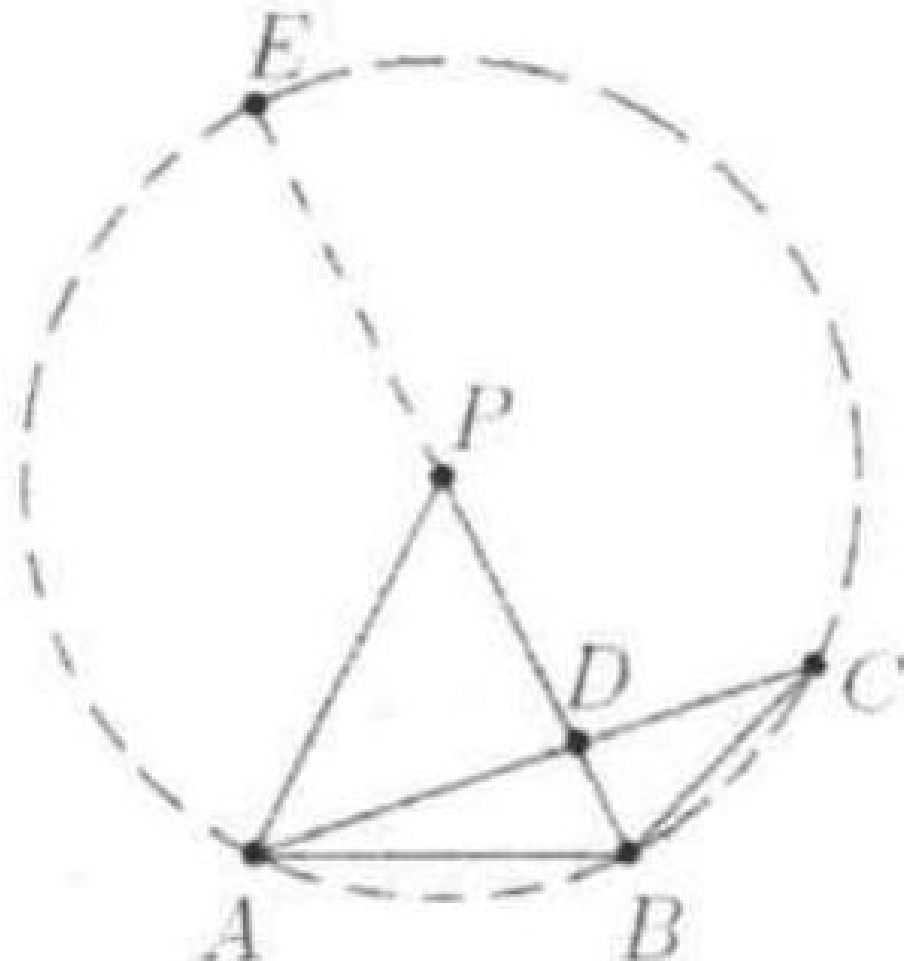
Solution

(A).

Method 1:

Construct a circle with center P and radius PA . Point C then lies on the circle, since the angle ACB is half angle APB .

Extend BP through P to get diameter BE . Since A, B, C , and E are concyclic, using the Power of a Point formula, we have:



$$\begin{aligned} AD \cdot CD &= ED \cdot BD \\ &= (PE + PD)(PB - PD) = (3 + 2)(3 - 2) = 5. \end{aligned}$$

Method 2:

Extend BP to E such that $PE = PB$. Since $PA = PB = PE$, points A, B , and E are concyclic. Construct this semicircle with center P as shown in the figure to the right.

$\angle ADE = \angle BDC$ (vertical angles). Therefore $\triangle AED \sim \triangle DCB$.
 $\frac{AD}{BD} = \frac{ED}{DC} \Rightarrow AD \cdot CD = ED \cdot BD = (PE + PD)(PB - PD)$
 $= (3 + 2)(3 - 2) = 5$.