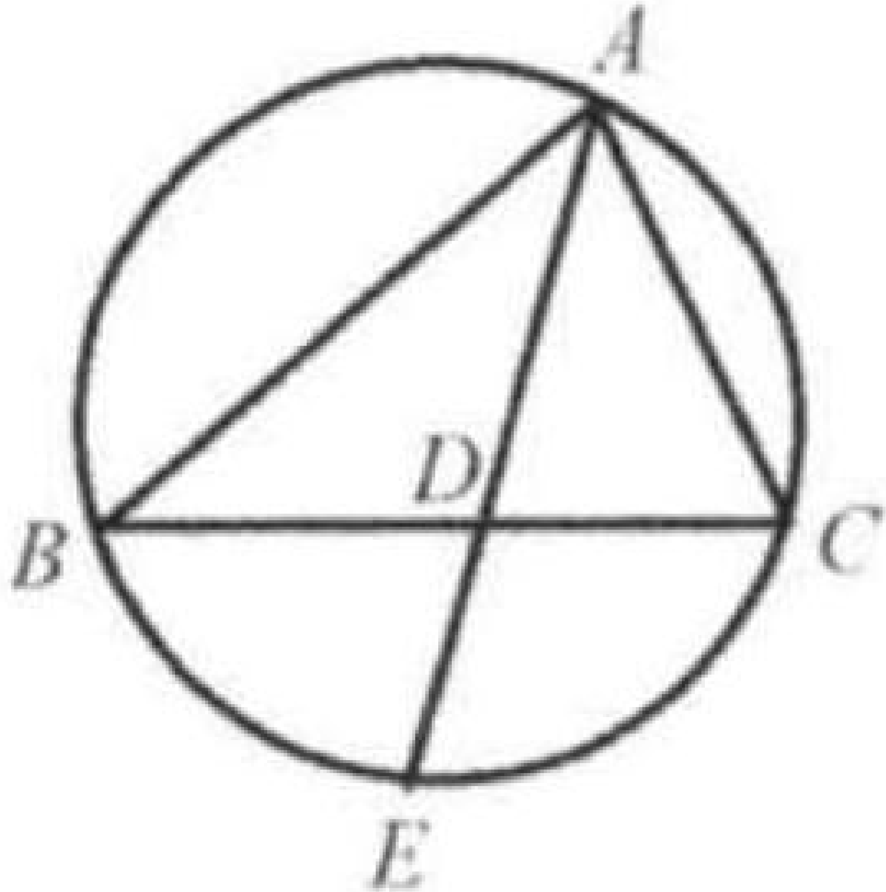
Example 5

In triangle ABC, AD is the angle bisector of $\angle A$. Extend AD to meet the circumcircle at E. Show that $AB \times AC = AD \times AE$.

Solution: Method 1:

Connect CE.

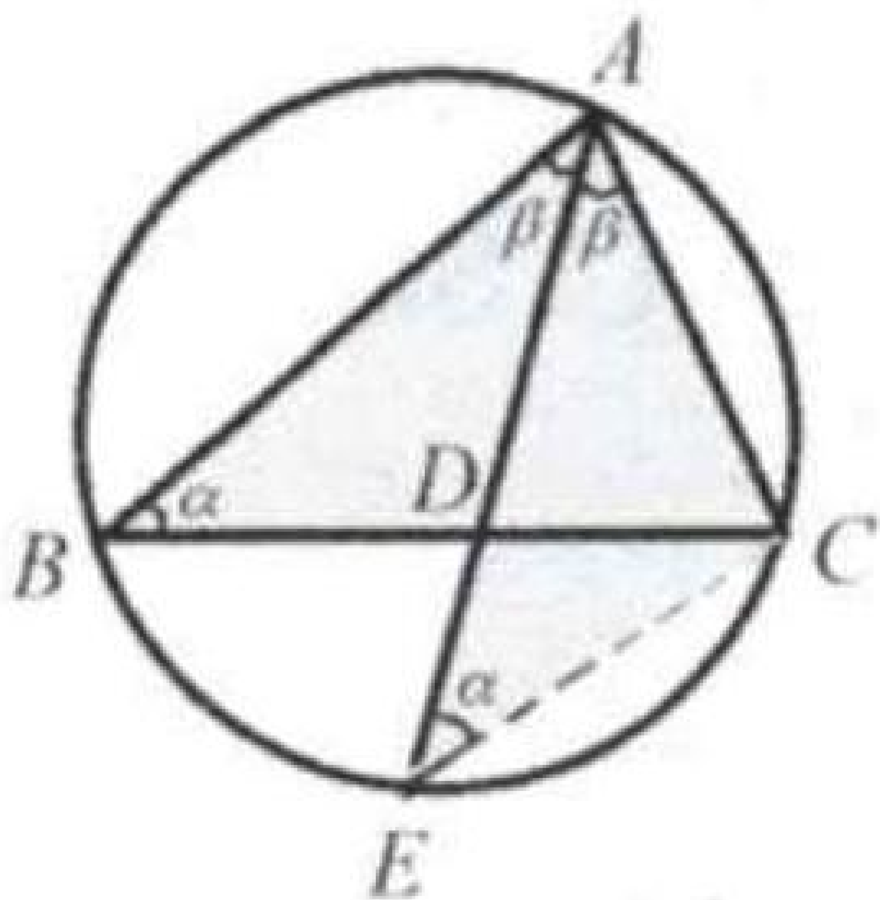


$\angle B = \angle E = \alpha$ (Both face the same arc AC).

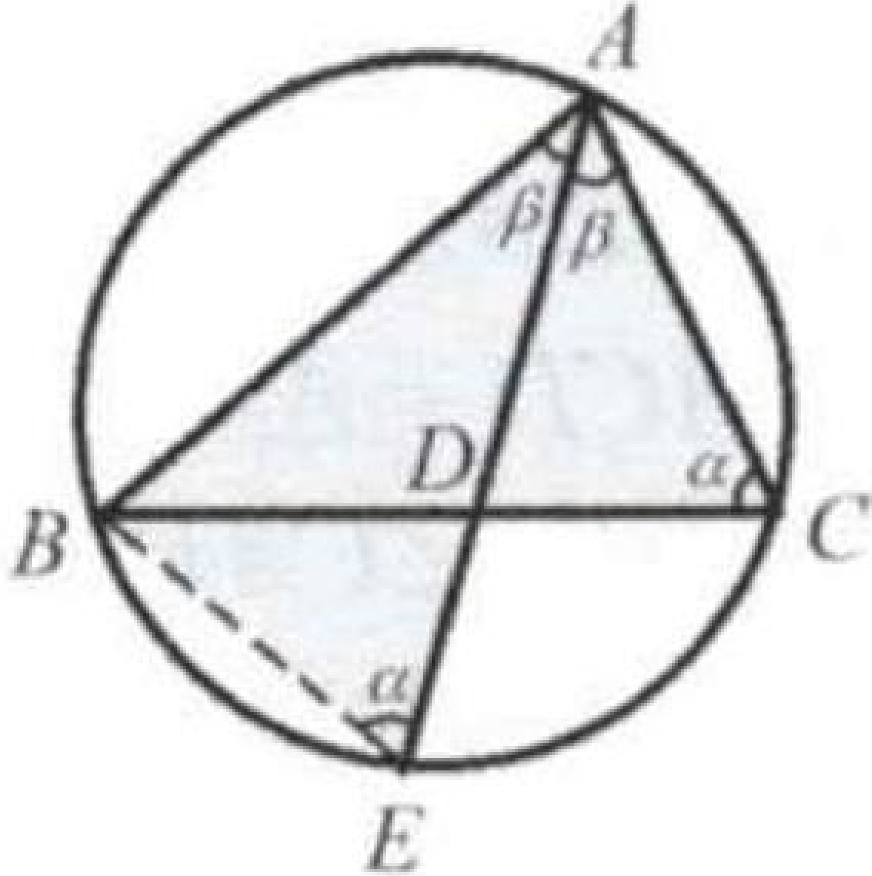
Since AD is the angle bisector of $\angle A, \angle EAC = \angle BAD = \beta$.

So $\triangle EAC \sim \triangle BAD$

$\frac{AC}{AD} = \frac{AE}{AB} \quad \Rightarrow \quad AB \times AC = AD \times AE.$



Method 2:
Connect BE.
$\angle C = \angle E = \alpha$ (Both face the same arc AB).
Since AD is the angle bisector of $\angle A, \angle DAC = \angle EAB = \beta$.
So $\triangle EAB \sim \triangle CAD$
$\frac{AC}{AD} = \frac{AE}{AB} \quad \Rightarrow \quad AB \times AC = AD \times AE.$


A
β
β
D
α
B
C
α
E