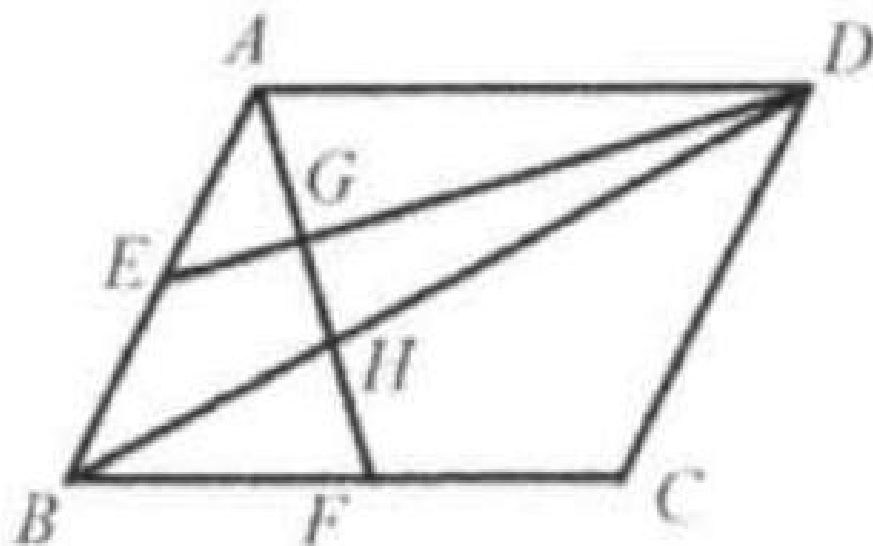


Problem 19

Problem

In parallelogram $ABCD$, shown here, points E and F are the midpoints of side AB and BC , respectively. AF meets DE at G and BD at H . Find the area of quadrilateral $BHGE$ if the area of $ABCD$ is 60 .

- (A) 10
- (B) 9
- (C) 8
- (D) 7
- (E) 5



Solution

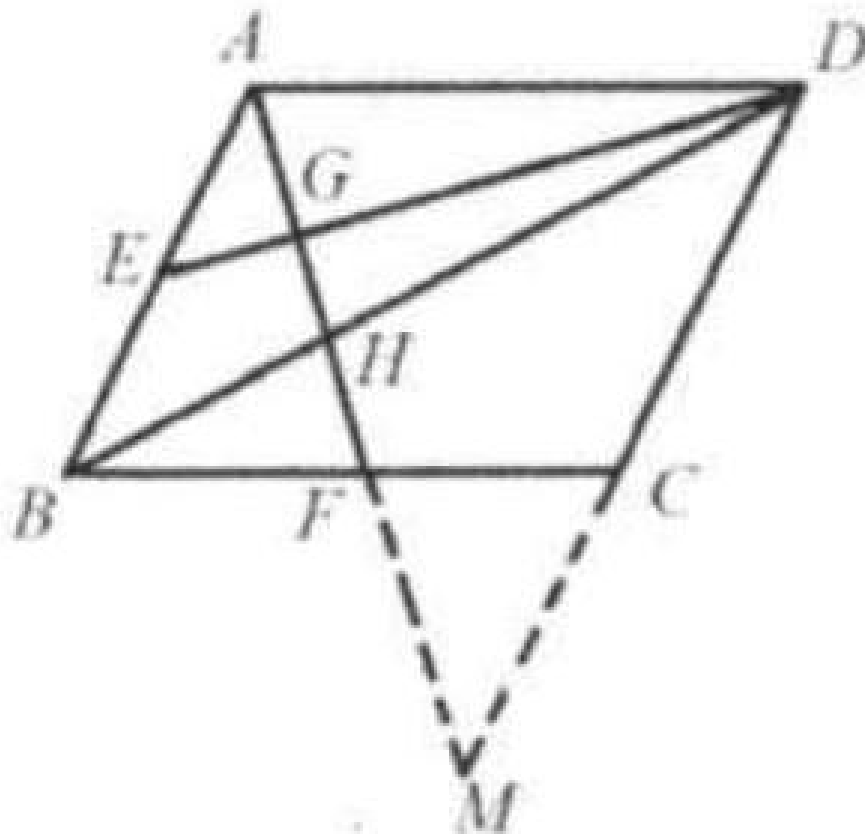
(D).

Extend AF and DC to meet at M .

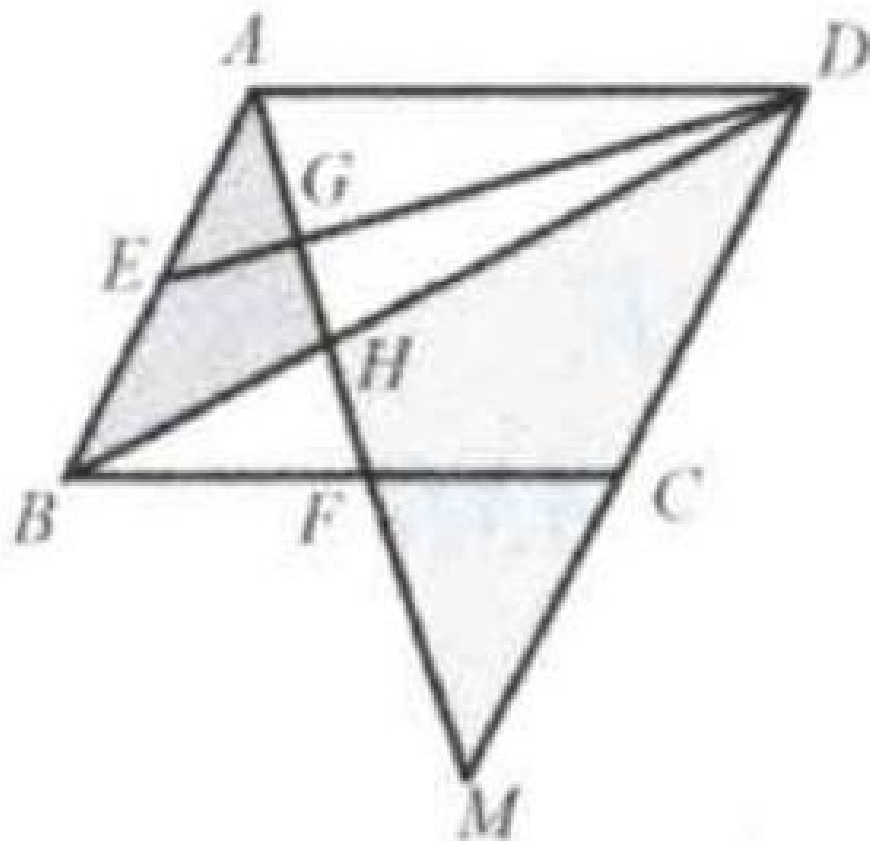
$$MC = AB = CD, AF = FM.$$

Triangle ABH is similar to triangle MDH .

$$\frac{AH}{HM} = \frac{AB}{DM} = \frac{1}{2}$$



Thus $\frac{AH}{HM} = \frac{1}{3}$, and $\frac{AH}{AF} = \frac{2}{3}$.
 $S_{\triangle ABH} = \frac{2}{3}S_{\triangle ABF} = \frac{2}{3} \times \frac{1}{4}S_{ABCD} = 10$
 Triangle AEG is similar to triangle MDG .



$$\frac{EG}{DG} = \frac{AE}{MD} = \frac{1}{4}. \text{ Thus } \frac{EG}{ED} = \frac{1}{5}.$$

$$S_{\triangle AEG} = \frac{1}{5} S_{\triangle ADE} = \frac{1}{5} \times \frac{1}{4} S_{ABCD} = 3.$$

Therefore $S_{BHGE} = S_{\triangle ABH} - S_{\triangle AEG} = 10 - 3 = 7.$

