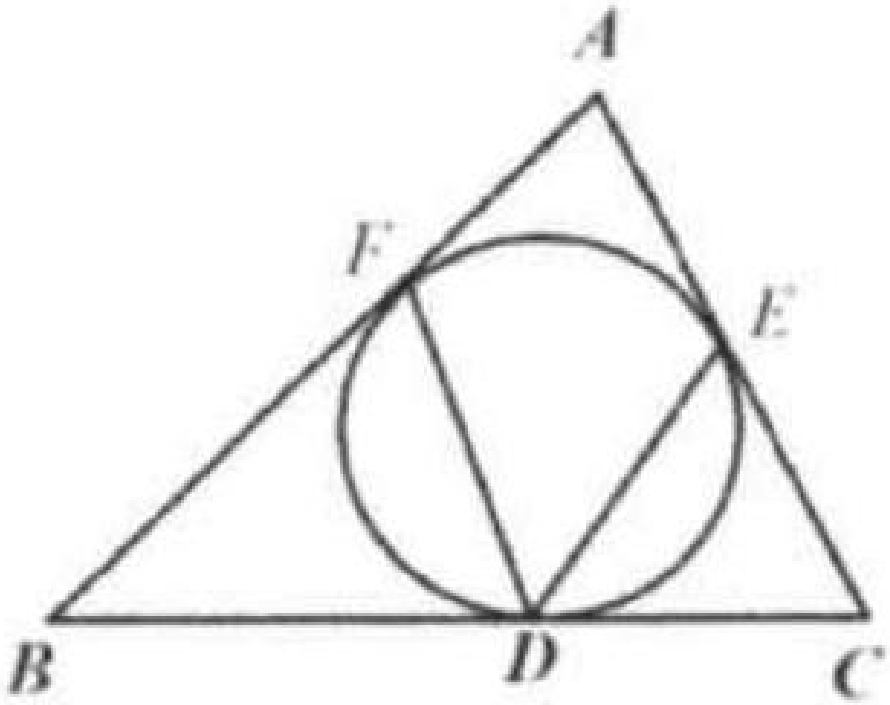
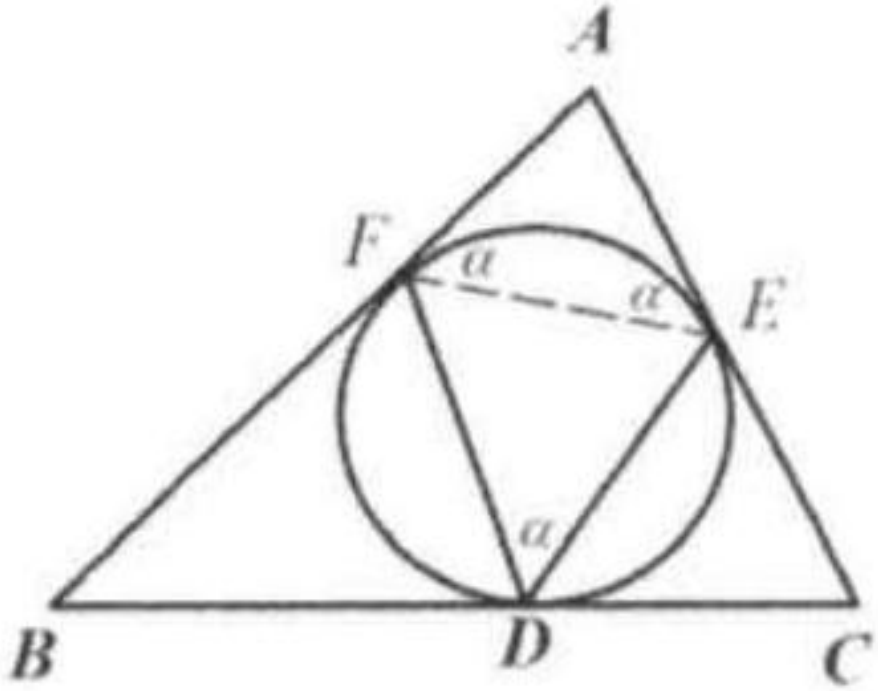
Problem

A circle is inscribed in triangle ABC. The tangent points are D, E, F as shown. Show that $\angle FDE = 90° - \frac{1}{2}\angle A$.



Solution

Connect EF.
Since both AF and AE are tangent to circle O, $AF = AE$ and
$\angle AFE = \angle AEF = \alpha$.
Thus $\angle FDE = \alpha$($\angle FDE, \angle AFE$, and $\angle AEF$ face the same arc FE).
In $\triangle AEF, \angle A = 180° - 2\alpha \quad \Rightarrow \quad 2\alpha = 180° - \angle A$
That is $\alpha = \angle FDE = 90° - \frac{1}{2}\angle A$.


A
F
E
B
D
C
α
α
α