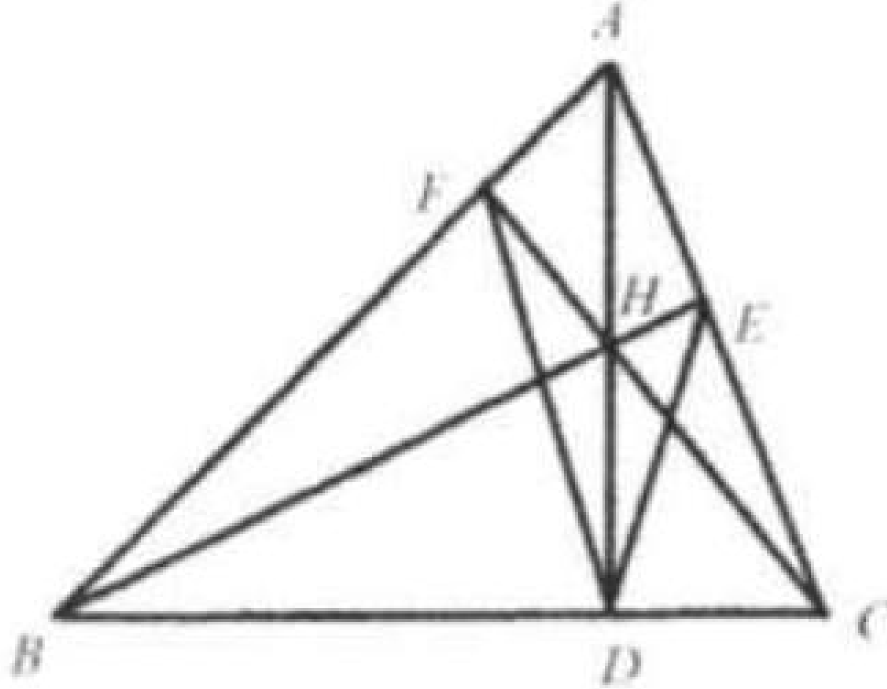
Example 20

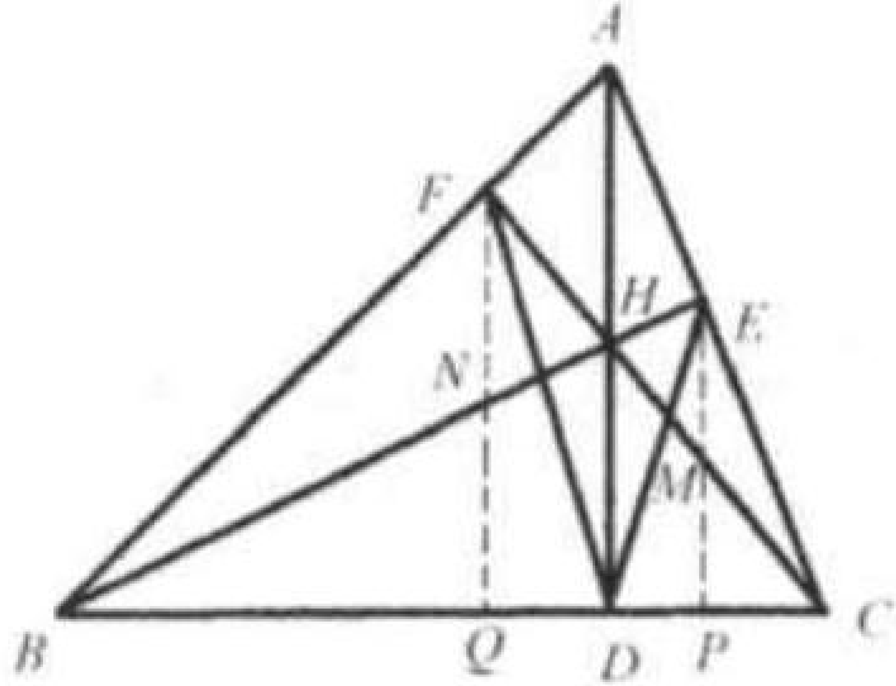
(1994 Canadian Mathematical Olympiad) Let ABC be an acute angled triangle. Let AD be the altitude on BC, and let H be any interior point on AD. Lines BH and CH, when extended, intersect AC and AB at E and F, respectively. Prove that $\angle EDH = \angle FDH$.



Solution: Draw $EP \perp BC$ to meet CF at M and BC at P.
Draw $FQ \perp BC$ to meet BE at N and BC at Q.
$EP//AD//FQ$

$$\frac{FN}{FQ} = \frac{AH}{AD} = \frac{EM}{EP}, \text{ or } \frac{FN}{EM} = \frac{FQ}{EP}$$

We see that $\triangle EMH \sim \triangle NHF, DP, DQ$ are the heights of $\triangle EMH$ and $\triangle NHF$, respectively. So we have



$$\frac{FN}{EM} = \frac{DQ}{DP}$$

From (1) and (2), $\frac{FQ}{EP} = \frac{DQ}{DP}$. Thus $\triangle FQD \sim \triangle EPD$, and $\angle FDQ = \angle EDP$. So $\angle EDH = \angle FDH$.