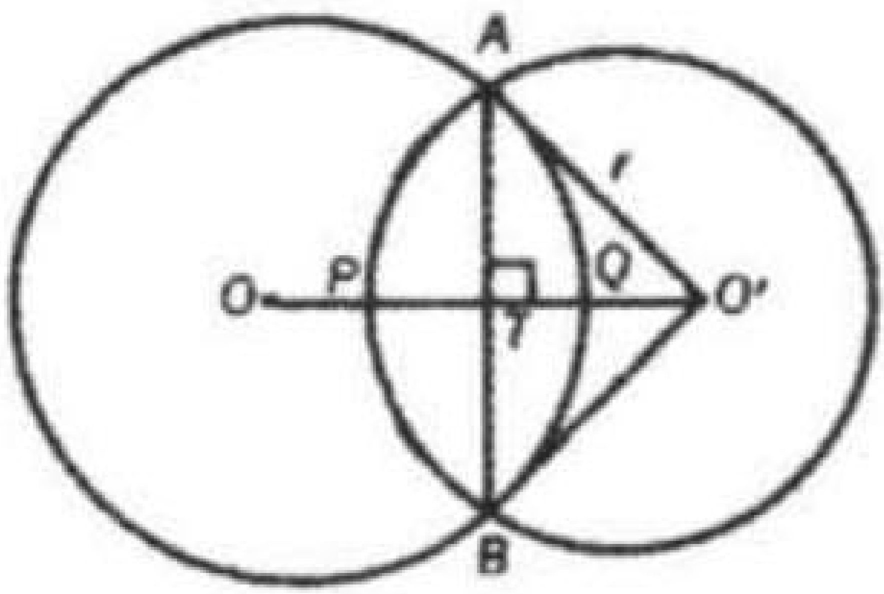
Problem 5

Problem

Two circles intersect in A and B, and the measure of the common chord $AB = 10$. The line joining the centers cuts the circles in P and Q. If $PQ = 3$ and the measure of the radius of one circle is 13 , find the radius of the other circle. (Note that the illustration is not drawn to scale.)



Solution

$\frac{41}{8}$.

Since $O'A = O'B$ and $OA = OB, OO'$ is the perpendicular bisector of AB. Therefore, in right $\triangle ATO$, since $AO = 13$ and $AT = 5$, we find $OT = 12$. Since $OQ = 13$ (also a radius of circle O), and $OT = 12, TQ = 1$. We Know that $PQ = 3$. $PT = PQ - TQ$; therefore, $PT = 2$. Let $O'A = O'P = r$, and $PT = 2, TO' = r - 2$.

Applying the Pythagorean Theorem in tight $\triangle ATO', (AT)^2 + (TO)^2 = (AO)^2$.

Substituting, $5^2 + (r-2)^2 = r^2$, and $r = \frac{29}{4}$. $PT = PQ + TQ$; therefore, $PT = 4$.

Again, let $O'A = O'P = r$ then $TO' = r - 4$.

Applying the Pythagorean Theorem in right $\triangle ATO'$,

$(AT)^2 + (TO)^2 = (AO)^2$.

Substituting, $5^2 + (r-4)^2 = r^2$, and $r = \frac{41}{8}$.