Example 10

In triangle $ABC, \angle A = 2\angle B, AB = 4$, and $BC = 2\sqrt{3}$. Find the value of AC.

Solution: 2. Method 1:
Draw the angle bisector AD.
$\triangle ABC \sim \triangle DAC(\angle CAD = \angle ABD$ and $\angle C = \angle C)$.

$$\frac{CD}{x} = \frac{x}{2\sqrt{3}}$$

By the angle bisector theorem, we have:

$$\frac{x}{CD} = \frac{4}{2\sqrt{3} - CD}$$

Solve for x using (1) and (2), we get $x = 2$.
Method 2:
Draw the angle bisector AD.
$\triangle ABC \sim \triangle DAC(\angle CAD = \angle ABD$ and $\angle C = \angle C)$.

$$\frac{CD}{x} = \frac{x}{2\sqrt{3}} = \frac{AD}{4}$$

We also know that $\angle DAB = \angle DBA$, and $AD = DB$.
(1) becomes: $\frac{CD}{x} = \frac{x}{2\sqrt{3}} = \frac{DB}{4} = \frac{2\sqrt{3}-CD}{4} = \frac{2\sqrt{3}}{x+4}$ or $\frac{x}{2\sqrt{3}} = \frac{2\sqrt{3}}{x+4}$
Solve for x in (2), we get $x = 2$.