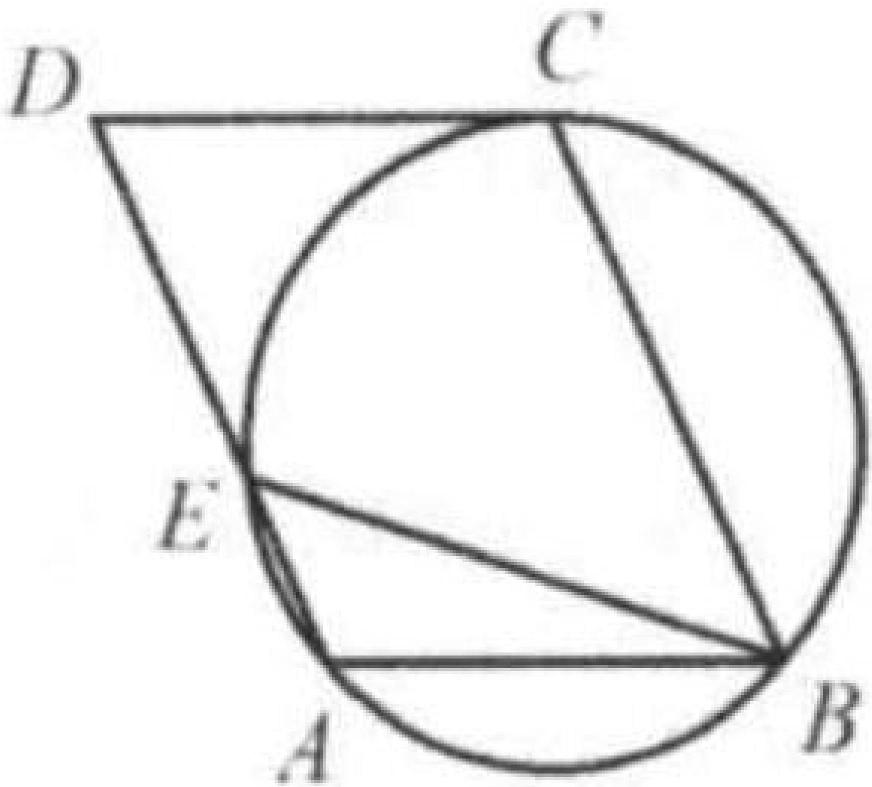
Example 9

$ABCD$ is a parallelogram. A circle is drawn such that it passes through A, B, C and meets the side DA at E. Find DE if $AB = 4$, and $BE = 5$.

Solution: $\frac{16}{5}$.

Method 1:



Connect AC, CE.

Since $ABCD$ is a parallelogram,

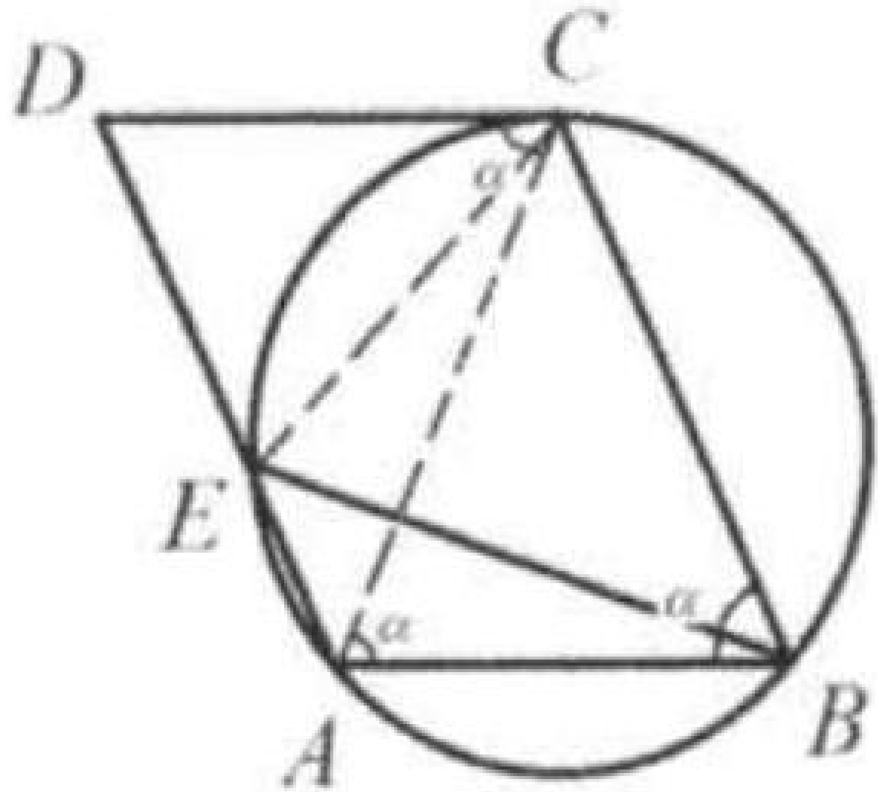
$BC = DA, AB = DC = 4. \angle ACD = \angle CBA = \alpha$ (alternate interior angles)

$\angle ABC = \angle ACD = \alpha$ (they face the same arc AC).

Thus $AC = BC$.

Since $AE // BC, AECB$ is an isosceles trapezoid. So $BE = AC = 5$.

Since $AD = BC = AC = 5$.



Therefore, $DC^2 = DA \times DE \Rightarrow 4^2 = 5 \times DE \Rightarrow DE = \frac{4^2}{5} = \frac{16}{5}$.

Method 2:

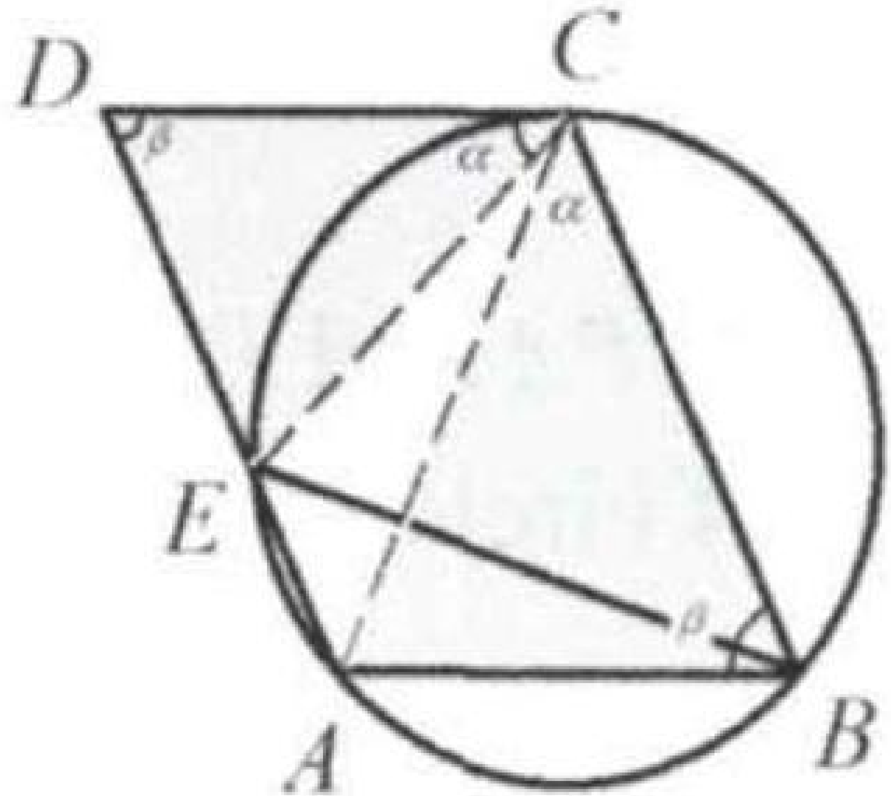
Connect AC, CE.

Since $ABCD$ is a parallelogram, $AB = DC = 4. \angle D = \angle CBA = \beta$ (alternate interior angles).

Since $AE//BC, AECB$ is an isosceles trapezoid. So $AB = CE = 4$, and $BE = AC = 5$.

$\angle DCE = \angle ACD = \alpha$ (they face the arcs of the same length arcs CE, AB).

Then triangles CDE and CBA are similar to each other, so the following equality holds true: $\frac{CE}{DE} = \frac{AC}{AB}$



$$\Rightarrow \quad \frac{4}{DE} = \frac{5}{4}$$
$$\Rightarrow \quad DE = \frac{4^2}{5} = \frac{16}{5}.$$