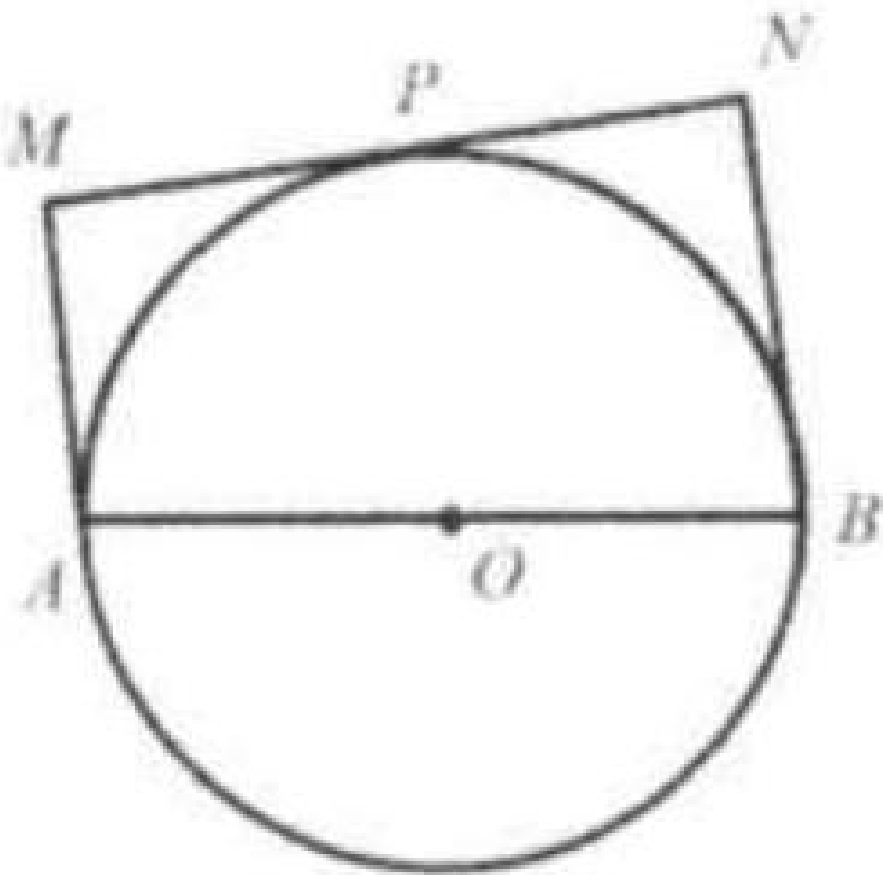


## Problem

In the adjoining figure  $AM$  and  $BN$  are parallel to each other and are tangent to the circle  $O$ , with  $A$  and  $B$  the points of tangency.  $MPN$  is a third tangent with  $P$  as point of tangency. Show that the radius of the circle is  $r = \sqrt{AM \times BN}$ .



## Solution

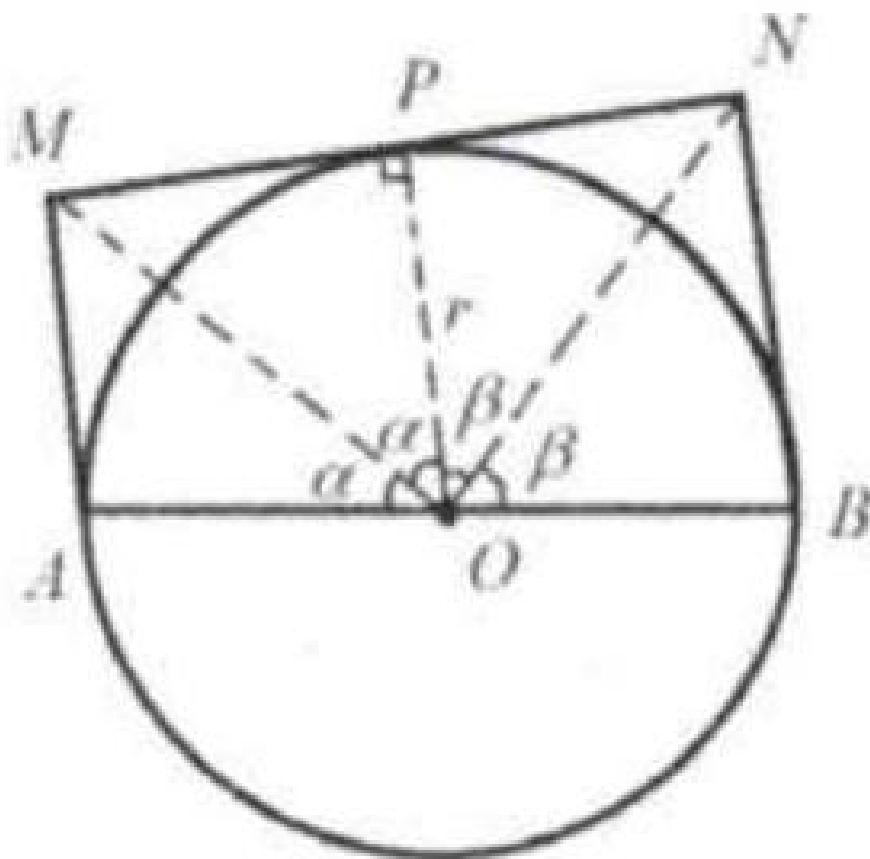
Connect OM, OP, ON .

Since  $A$  and  $P$  are two tangent points,  $AM = PM$ .

$OA = OP = r$ .  $OM = OM$ .

So  $\triangle AOM \cong \triangle POM$ . Then  $\angle AOM = \angle POM = \alpha$ .

Similarly,  $\angle PON = \angle BON = \beta$ .



Since  $2\alpha + 2\beta = 180^\circ$ ,  $\alpha + \beta = \angle MON = 90^\circ$ ,  
 Thus  $PO^2 = MP \times PN = AM \times BN$ , or  
 $r^2 = AM \times BN \Rightarrow r = \sqrt{AM \times BN}$