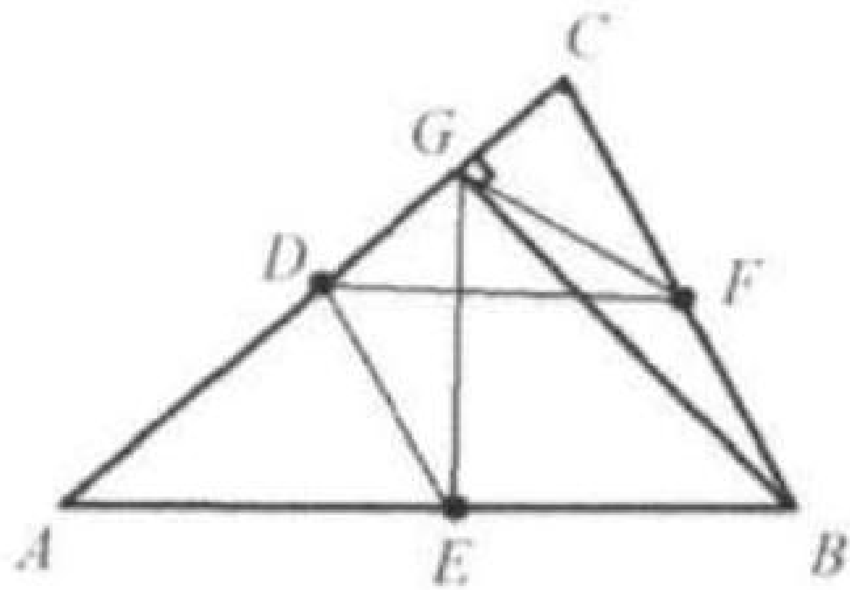
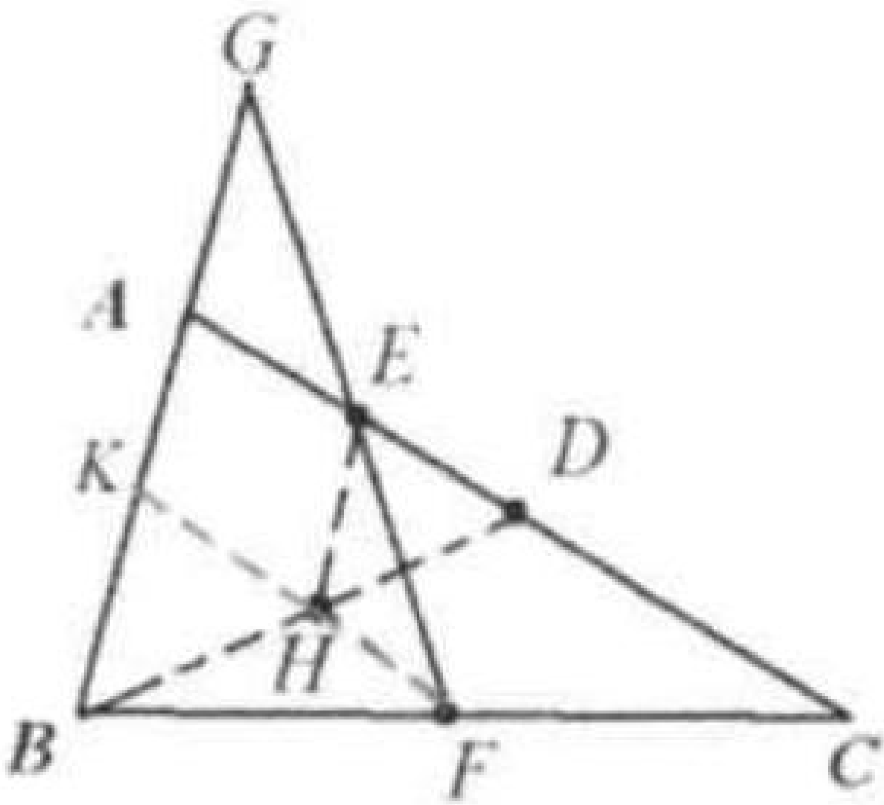
Problem

In any $\triangle ABC, D, E$, and F are midpoints of the sides AC, AB, and BC, respectively. BG is an altitude of $\triangle ABC$. Prove that $\angle EGF = \angle EDF$.



Solution

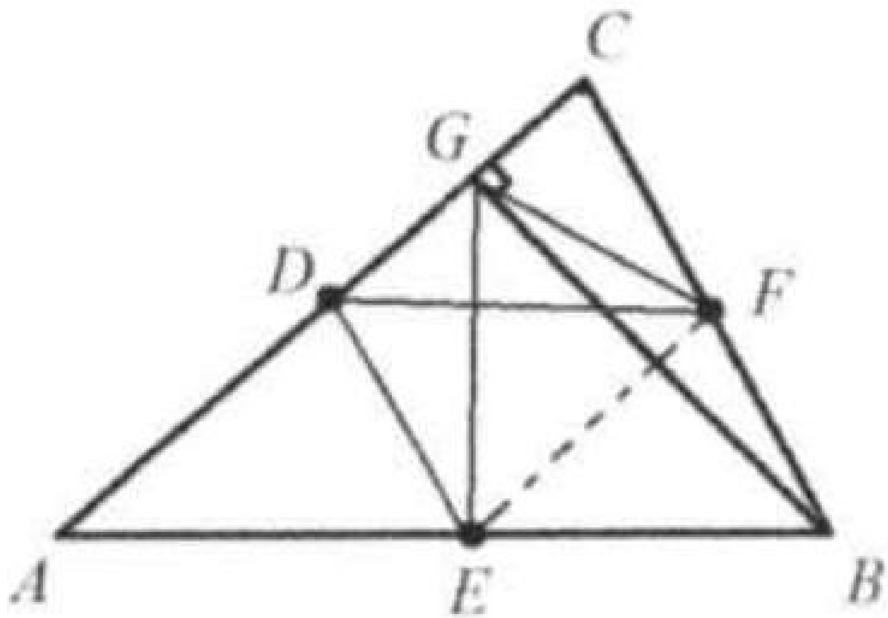


Connect EF. Since both E and F are midpoints of the sides AB and BC, respectively, $EF//AC//DG$.

Since D and E are midpoints of the sides AC and AB, DE is the midline of $\triangle ABC$. Thus $DE = CF$.

Since FG is the median of right $\triangle BGC, GF = CF$.

So $DE = GF$.



Quadrilateral $DGFE$ is an isosceles trapezoid.
Then $\angle DEF = \angle DFE$.
Thus $\triangle GFE \cong \triangle DEF(SAS)$, and $\angle EGF = \angle EDF$.