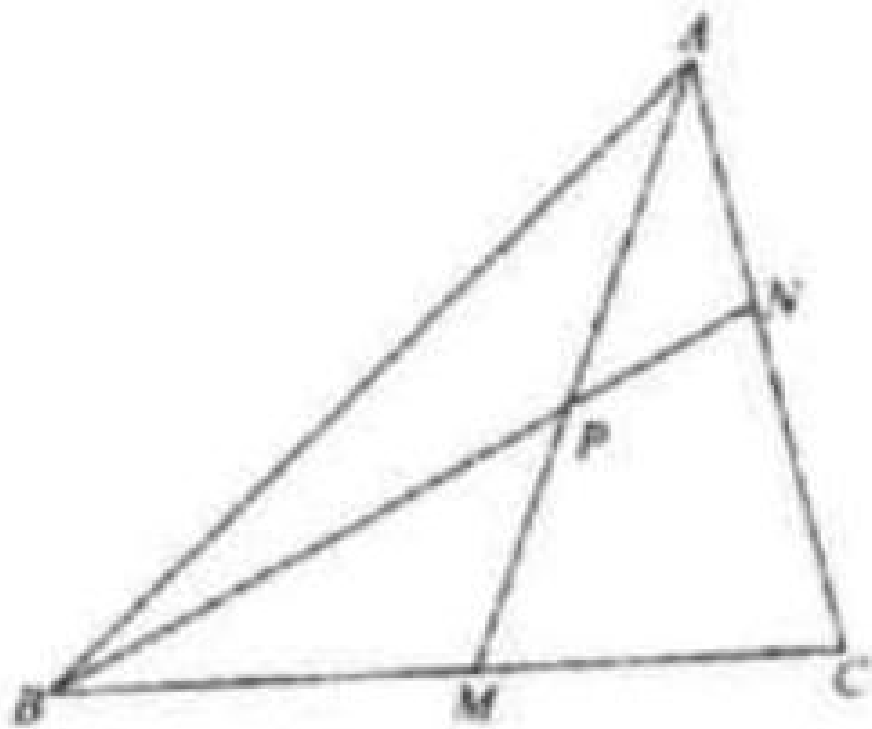


Problem 16

Problem

As shown in the figure, in triangle ABC , M is the midpoint of BC . $AN = \frac{1}{3}AC$. Connect BN and denote the point where BN meets AM to be P . Show that $BP = 3PN$.

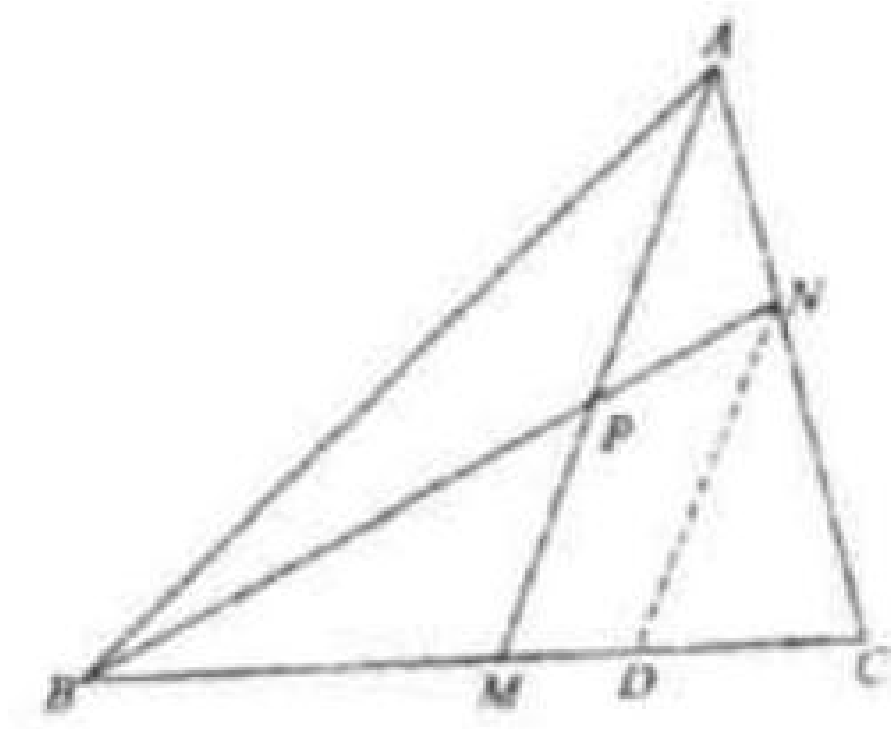


Solution

This problem is the same as Example 4. Here, we show two new, different ways to solve it.

Method 1:

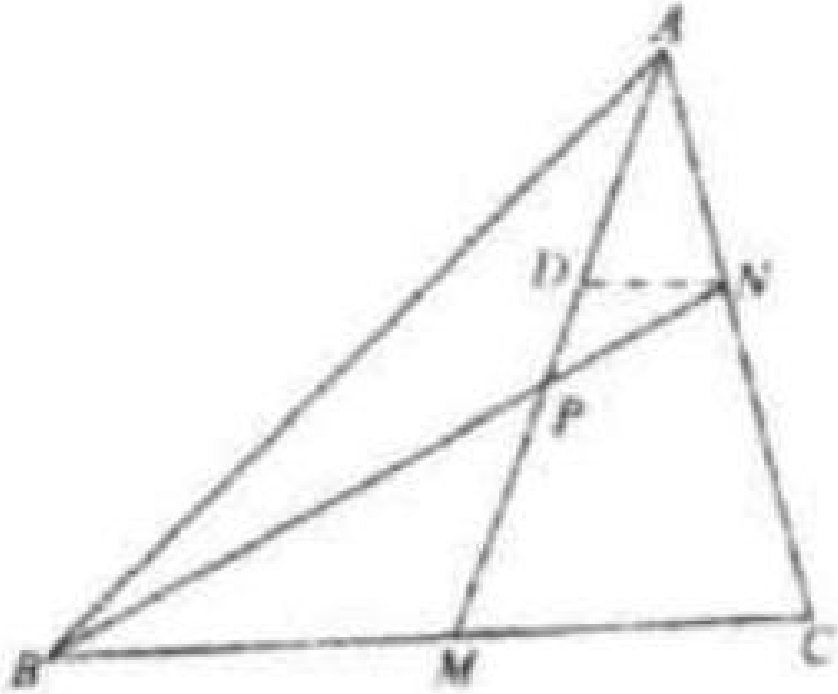
Draw a line through N parallel to AM to meet CM at D .
 Since $\frac{AN}{AC} = \frac{1}{3}$, $\frac{MD}{MC} = \frac{1}{3}$.



We know that $BM = MC$, so $\frac{MD}{BM} = \frac{1}{3}$.
 We also know that $MP \parallel DN$, so $\frac{PN}{BP} = \frac{1}{3}$
 $\Rightarrow BP = 3PN$.

Method 2:

Draw a line through N parallel to BC to meet AM at D .



Since $\triangle ADN \sim \triangle AMC$, $\frac{AN}{AC} = \frac{1}{3}$.
 Since $MB = MC$, $3DN = MC = MB$.
 Since $\triangle PDN \sim \triangle PMA$, $\frac{DN}{MC} = \frac{PN}{PB} = \frac{1}{3}$
 $\Rightarrow BP = 3PN$.