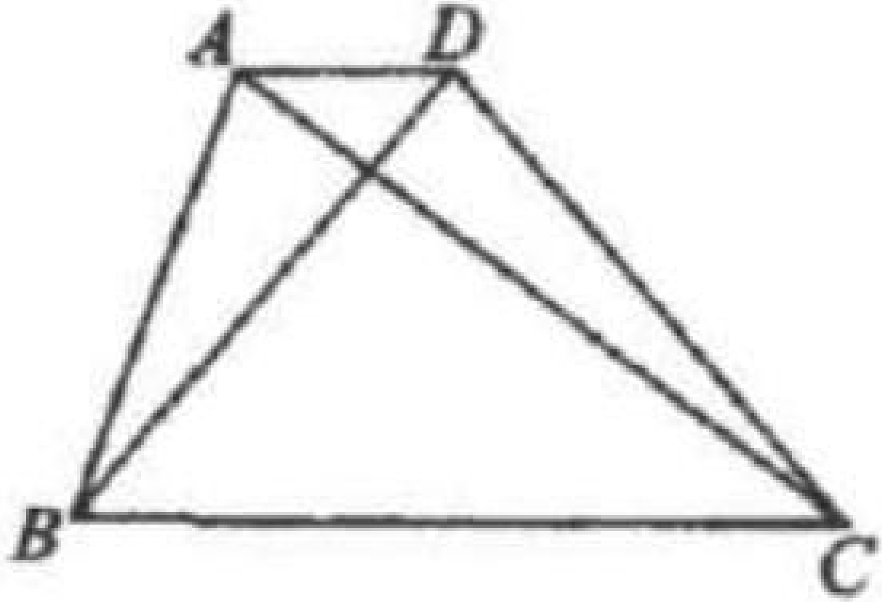
Problem

In a convex quadrilateral $ABCD, AD//BC$. Show that $AC \perp BD$ if $AC^2 + BD^2 = (AD + BC)^2$.

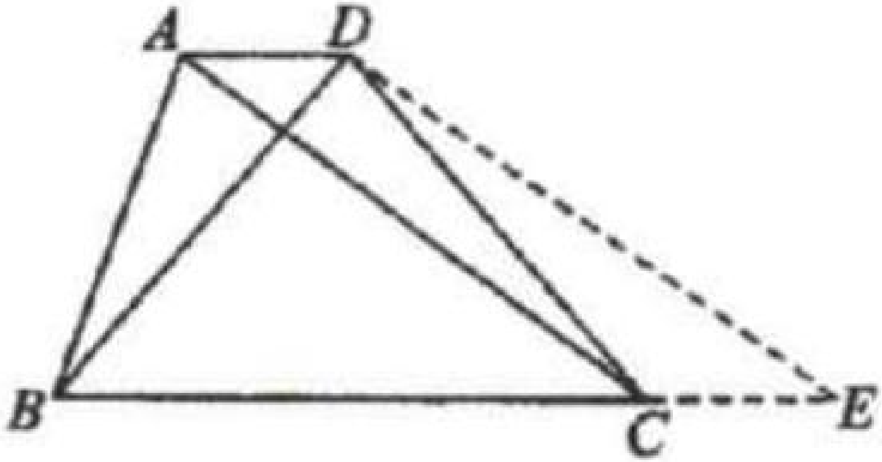


Solution

Draw DE so that $DE//AC$ and DE meets the extension of BC at E. Then $\triangle CED \cong \triangle DAC$ and $DE = AC, CE = AD$.

In $\triangle BDE, BE = BC + CE = BC + AD. ADCE$ is a parallelogram and $DE = AC$.

Since $AC^2 + BD^2 = (AD + BC)^2$, or $DE^2 + BD^2 = BE^2$,



by the converse of the Pythagorean theorem, $\angle BDE = 90^\circ$.
Therefore $BD \perp DE$. We also know that $AC//DE$, so $AC \perp BD$.