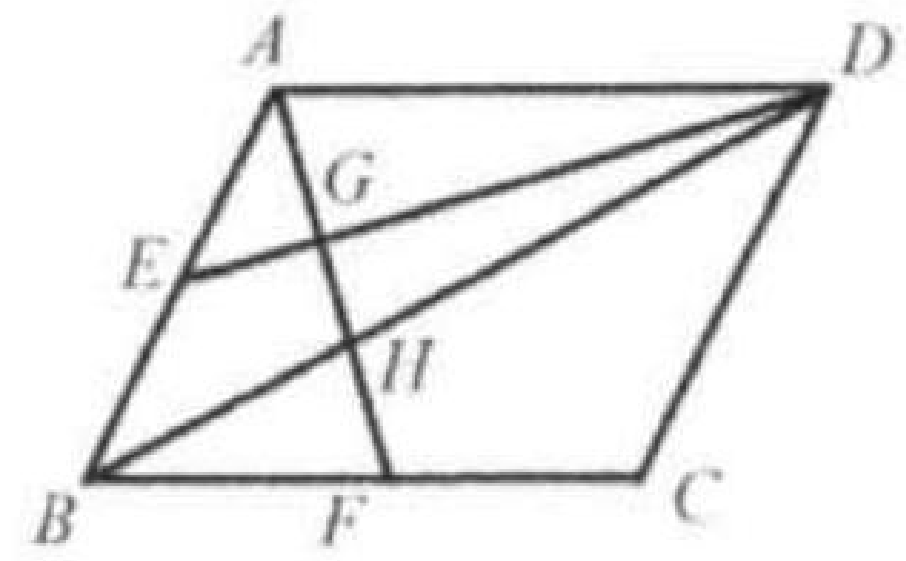
Problem

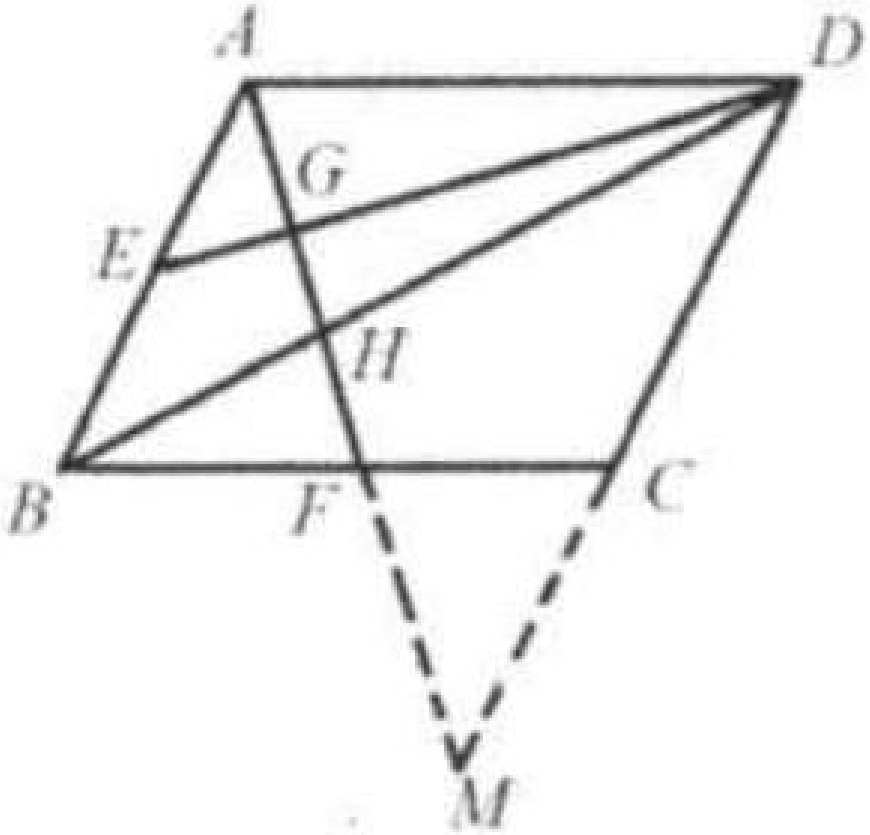
In parallelogram $ABCD$, shown here, points E and F are the midpoints of side AB and BC, respectively. AF meets DE at G and BD at H. Find the area of quadrilateral $BHGE$ if the area of $ABCD$ is 60 .

(A) 10
(B) 9
(C) 8
(D) 7
(E) 5

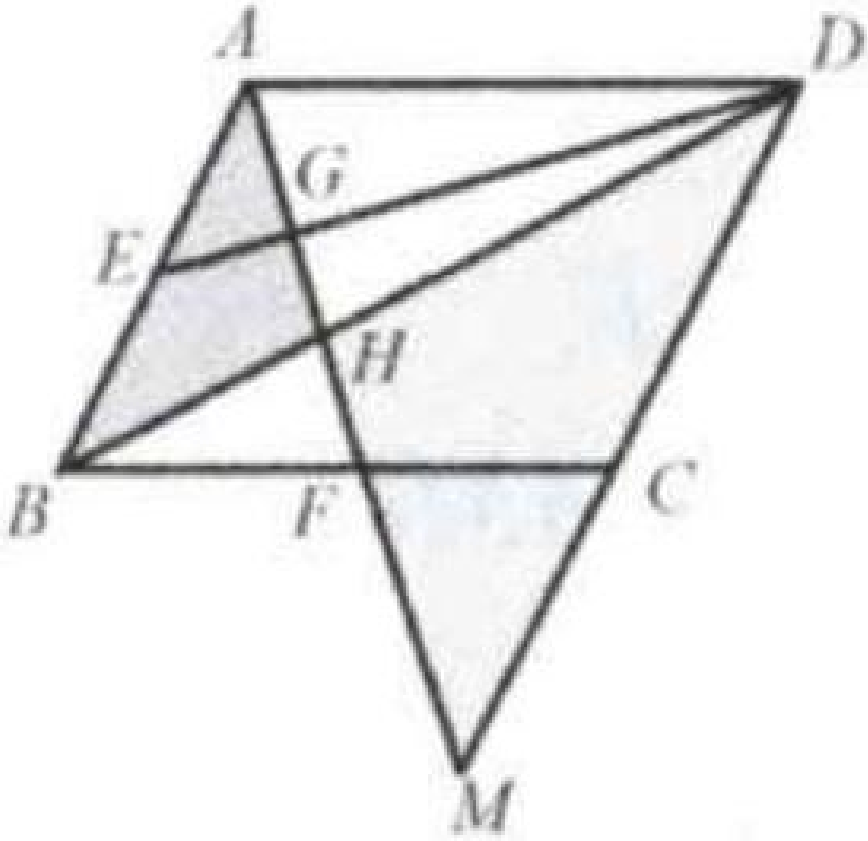


Solution

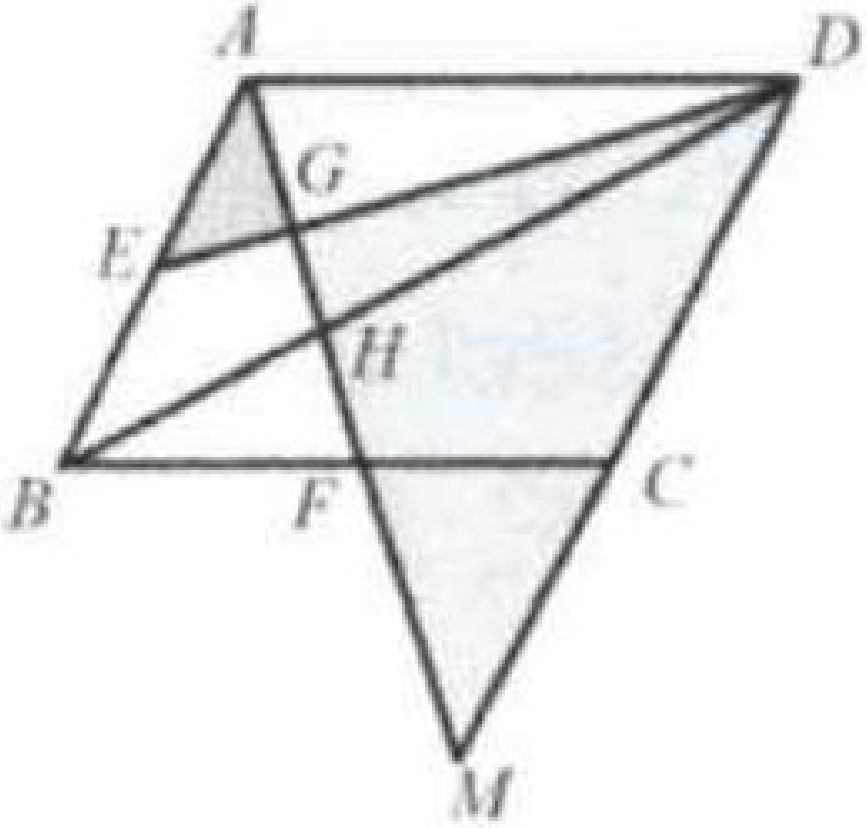
(D).
Extend AF and DC to meet at M.
$MC = AB = CD.AF = FM.$
Triangle ABH is similar to triangle MDH.
$\frac{AH}{HM} = \frac{AB}{DM} = \frac{1}{2}$



Thus $\frac{AH}{HM} = \frac{1}{3}$, and $\frac{AH}{AF} = \frac{2}{3}$.
$S_{\triangle ABH} = \frac{2}{3} S_{\triangle ABF} = \frac{2}{3} \times \frac{1}{4} S_{ABCD} = 10$
Triangle AEG is similar to triangle MDG.



$\frac{EG}{DG} = \frac{AE}{MD} = \frac{1}{4}$. Thus $\frac{EG}{ED} = \frac{1}{5}$.
$S_{\triangle AEG} = \frac{1}{5} S_{\triangle ADE} = \frac{1}{5} \times \frac{1}{4} S_{ABCD} = 3$.
Therefore $S_{BHGE} = S_{\triangle ABH} - S_{\triangle AEG} = 10 - 3 = 7$.


A
D
G
E
H
B
F
C
M