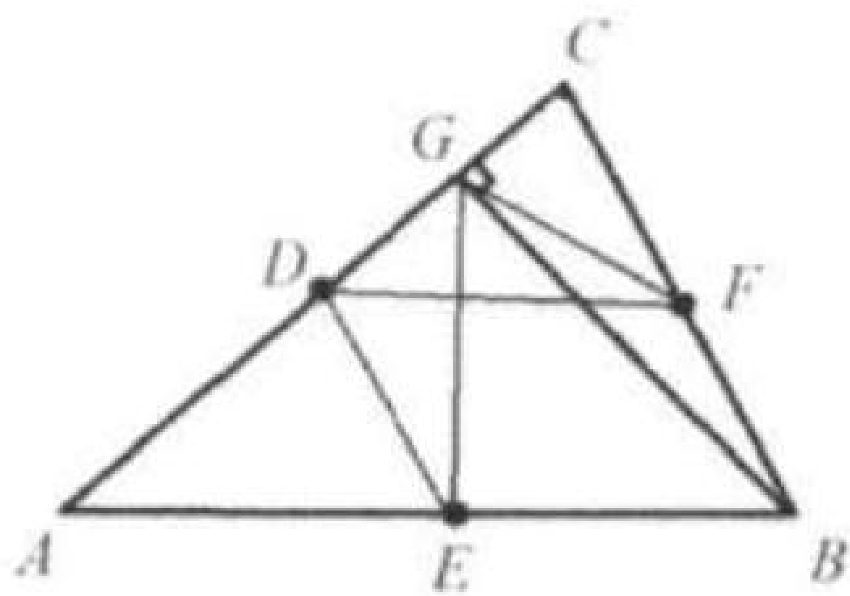
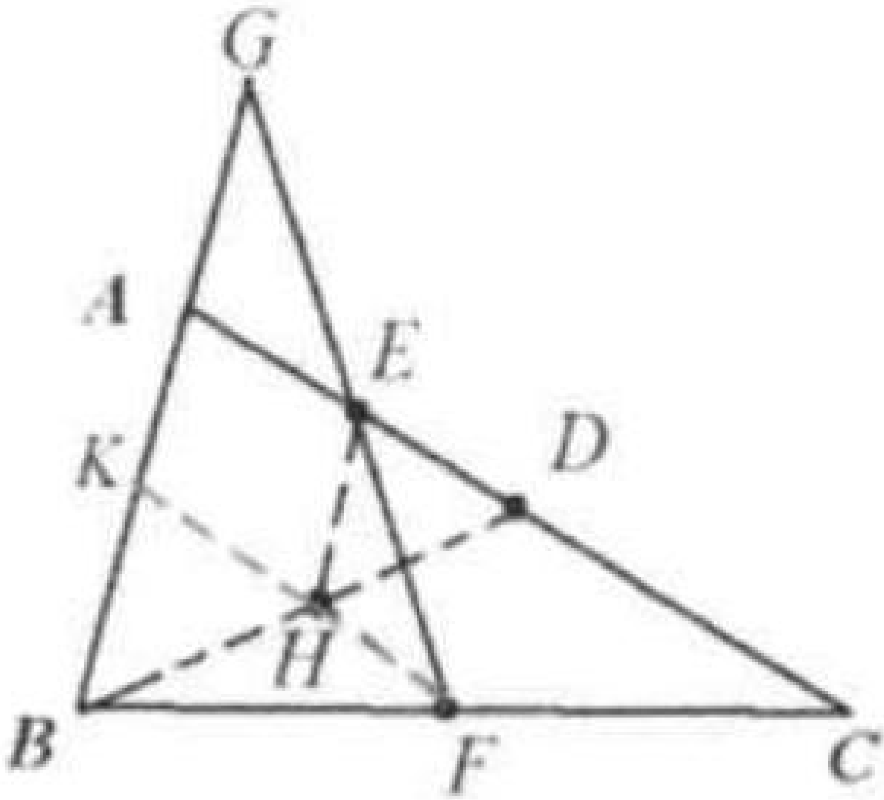
# Problem 7

## Problem

In any $\triangle ABC, D, E$, and $F$ are midpoints of the sides $AC, AB$, and $BC$, respectively. $BG$ is an altitude of $\triangle ABC$. Prove that $\angle EGF = \angle EDF$.



## Solution

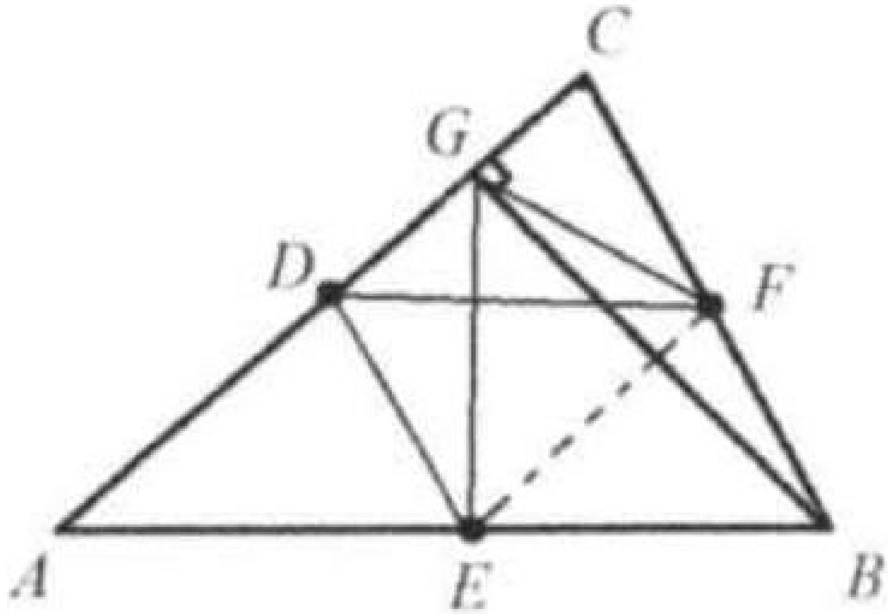



Connect $EF$. Since both $E$ and $F$ are midpoints of the sides $AB$ and $BC$, respectively, $EF//AC//DG$.

Since $D$ and $E$ are midpoints of the sides $AC$ and $AB, DE$ is the midline of $\triangle ABC$. Thus $DE = CF$.

Since $FG$ is the median of right $\triangle BGC, GF = CF$.

So $DE = GF$.




Quadrilateral $DGFE$ is an isosceles trapezoid.
Then $\angle DEF = \angle DFE$.
Thus $\triangle GFE \cong \triangle DEF (SAS)$, and $\angle EGF = \angle EDF$.