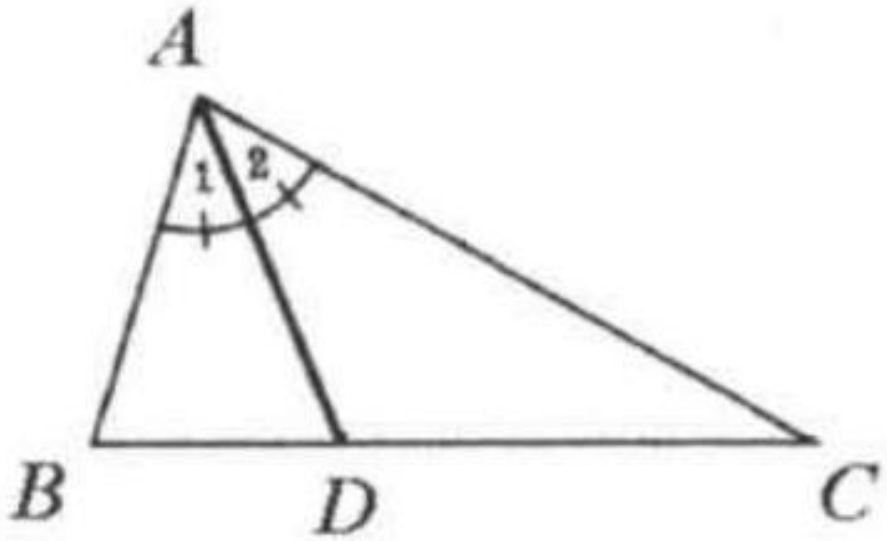
# Example 2

Show that The angle bisector of a triangle divides the opposite side into segments that are proportional to the adjacent sides.

$$\frac{AB}{AC} = \frac{BD}{CD} \quad \text{or} \quad \frac{AB}{BD} = \frac{AC}{CD}$$

Proof:




Draw $AH \perp BC$ at $H$. The ratio of the areas $\triangle ABD$ and

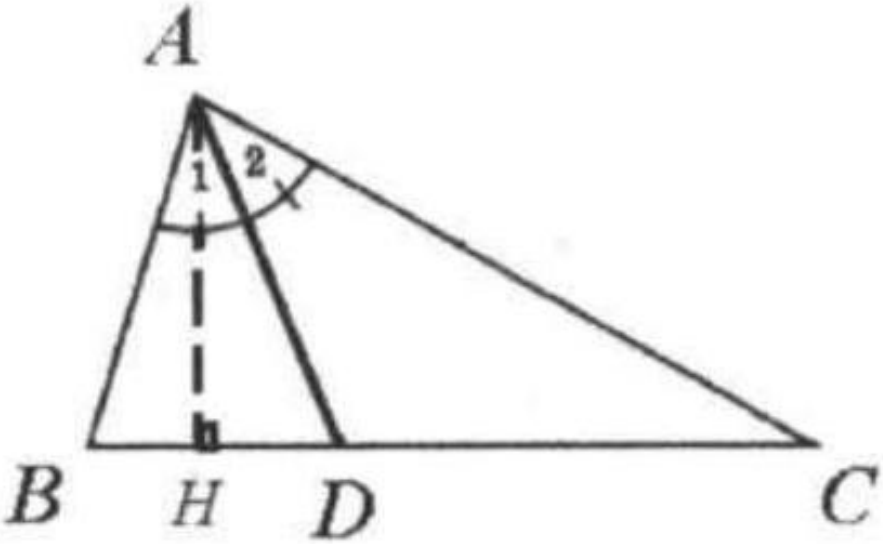
$$ADC \text{ is } \frac{S_{\triangle ABD}}{S_{\triangle ADC}} = \frac{\frac{1}{2}BD \times AH}{\frac{1}{2}CD \times AH} = \frac{BD}{CD}.$$

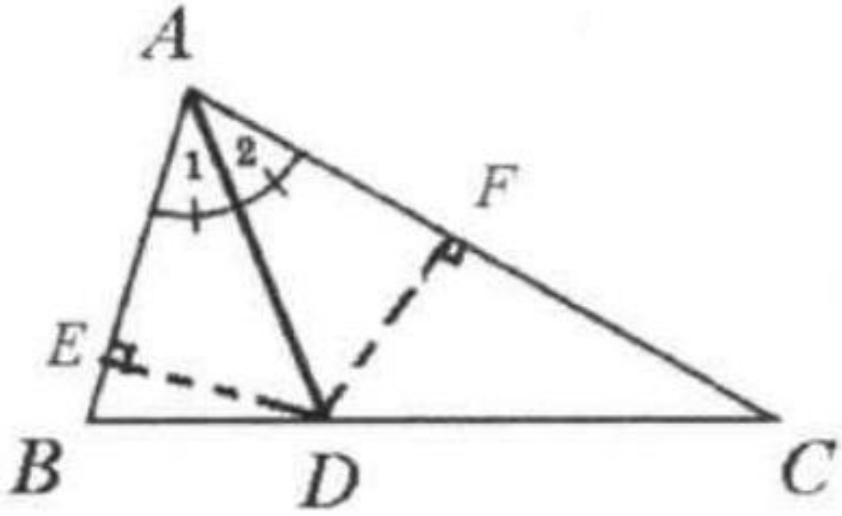
Draw $DE \perp AB, DF \perp AC$.

By Theorem 3.2, $DE = DF$.

The ratio of the areas $\triangle ABD$ and $ADC$ is

We also know that $\frac{S_{\triangle ABD}}{S_{\triangle ADC}} = \frac{\frac{1}{2}AB \times DE}{\frac{1}{2}AC \times DF} = \frac{AB}{AC}$




Therefore: $\frac{AB}{AC} = \frac{BD}{CD}$.