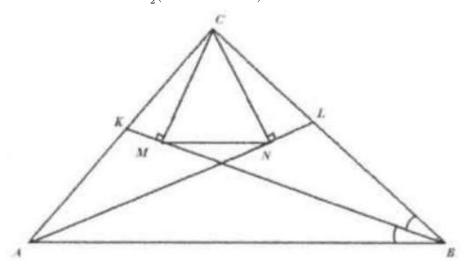
Problem 7

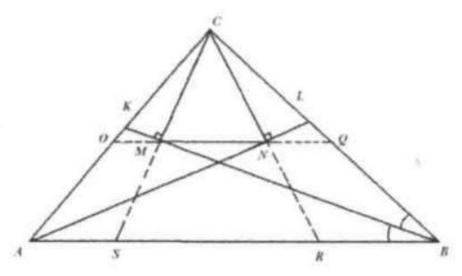
Problem

In triangle ABC, the angle bisector of angle A intersects BC at point L, and the angle bisector of angle B intersects AC at point K. Let M and N be the feet of the perpendiculars from C to BK and AL, respectively. Show that $MN = \frac{1}{2}(BC + AC - AB).$



Solution

Extend MN such that it intersects lines AC and BC at point O and Q, respectively. Extend CM to meet AB at, say, S. then triangle BCM and triangle BSM are congruent. Hence BS = BC = 120. Similarly, extend CN to meet AB at, say, R, and triangle ACN and triangle ARN are congruent. Hence AR = AC. So CM = MS, and CN = NR. So MN is the midline of triangle CSR.



 $MN = \frac{SR}{2}.$ SR = AR - AS = AC - (AB - BS) = AC - (AB - BC) = BC + AC - AB. Thus $MN = \frac{1}{2}(BC + AC - AB).$