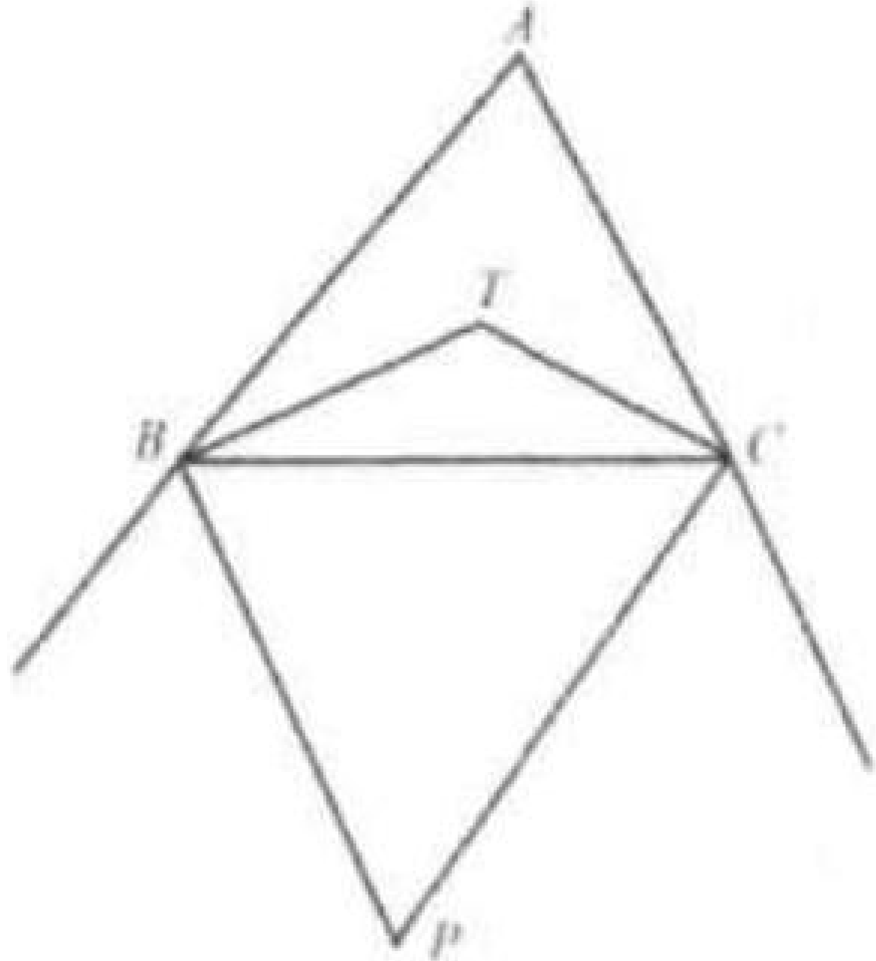
Example 3

In $\triangle ABC$, the angle bisectors of $\angle B, \angle C$ meet at T, and the exterior angle bisectors of $\angle B, \angle C$ meet at P. Show that
$\angle BPC = \frac{1}{2}(\angle ABC + \angle ACB)$.

Solution: Label $\angle TBC = \angle TBA = \alpha, \angle TCB = \angle TCA = \beta, \angle CBP =$



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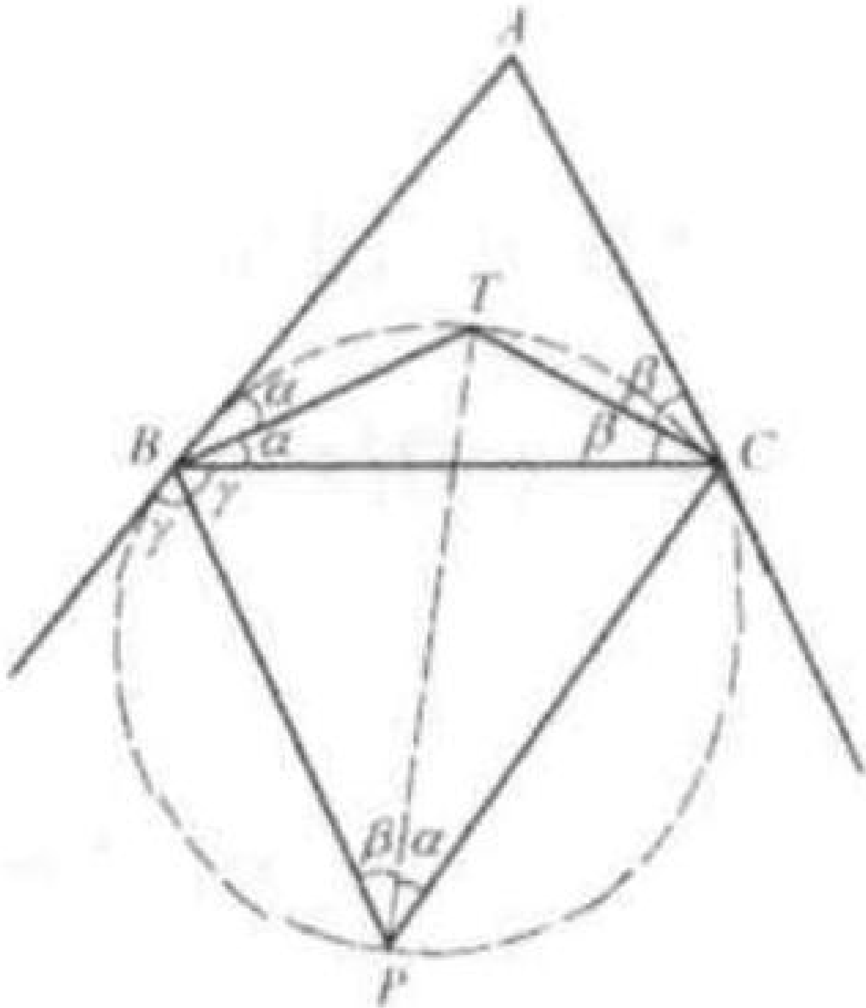
Since $2\alpha + 2\gamma = 180°, \alpha + \gamma = 90°$.
Thus $\angle TBP = 90°$.
Similarly, $\angle TCP = 90°$.
So points B, P, C, and T are concyclic and TP is the diameter of the circle.
Therefore, $\angle BPT = \angle TCB = \beta, \angle CPT = \angle TBC = \alpha$.



$$\angle BPC = \alpha + \beta = \angle TBC + \angle TCB$$
$$= \frac{1}{2}(\angle ABC + \angle ACB).$$