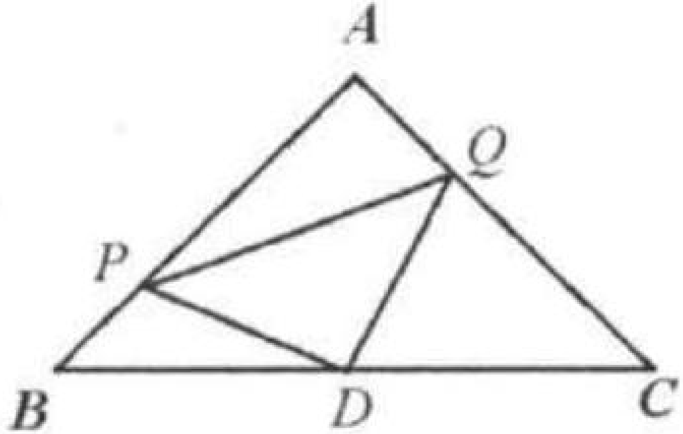
Example 12

$\triangle ABC$ is an isosceles right triangle with $\angle A = 90°$. Points P and Q are points on sides AB and AC, respectively. $BP = AQ$. Show that $\triangle PDQ$ is also an isosceles right triangle if D is the midpoint of BC.

Solution: Draw the median AD.

Since $\triangle ABC$ is an isosceles right triangle and D is the midpoint of $BC, AD \perp BC, \angle ADC = 90°$.

By Theorem 1.3, $AD = BD = DC. \angle DAQ = \angle A = 45°$.


A
Q
P
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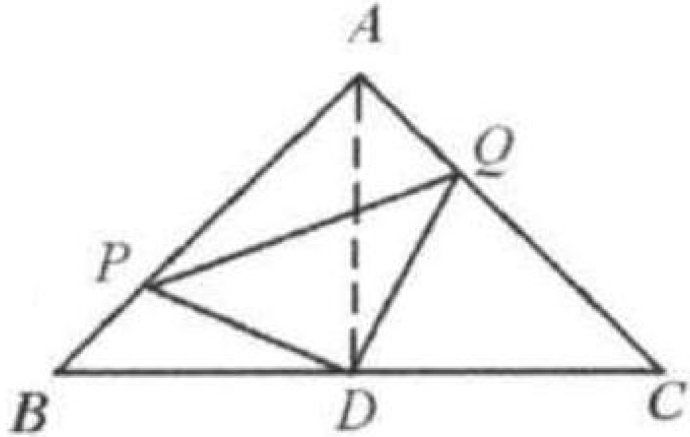
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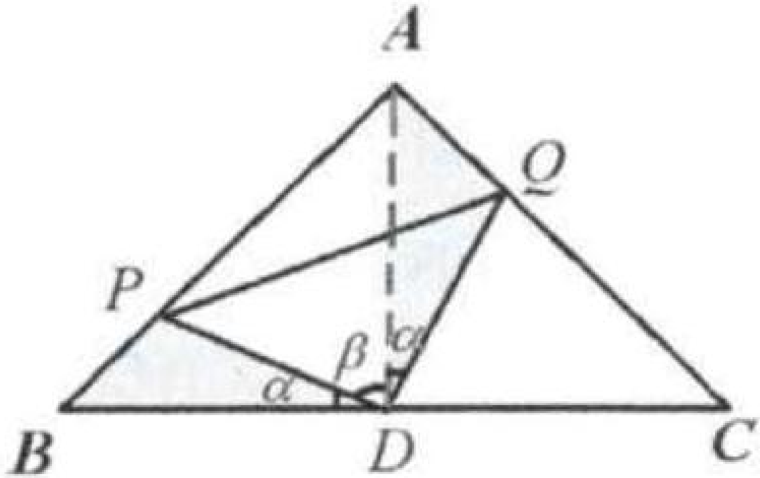
A
Q
P
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α
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C

Since $BP = AQ, \triangle BPD \cong \triangle AQD$.
Thus, $PD = QD. \angle ADQ = \angle BDP = \alpha$.
We see that $\angle ADQ = \alpha + \beta = 90°$.
So $\angle PDQ = \alpha + \beta = 90°$.
We also know that $PD = QD$.
Thus $\triangle PDQ$ is an isosceles right triangle.