

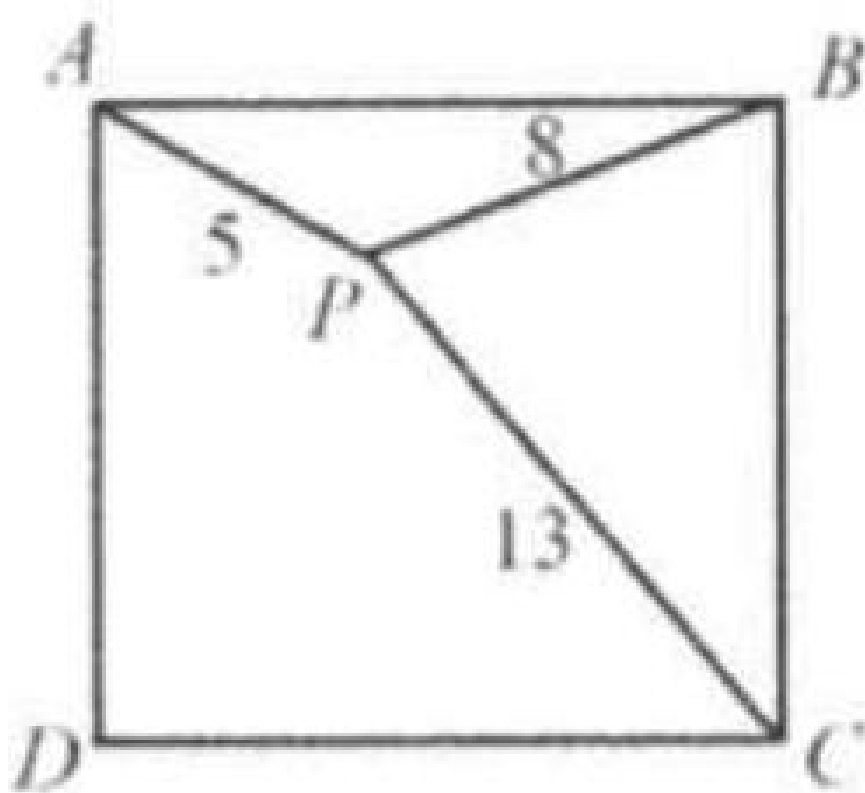
## Example 9

Let  $P$  be a point inside the square  $ABCD$ . Find the area of the square if  $PA = 5$ ,  $PB = 8$ , and  $PC = 13$ .

- (A) 153
- (B) 126
- (C) 128
- (D) 130
- (E) 132

Solution: (A).

Draw  $PE \perp AB$ ,  $PF \perp BC$ , as shown in the figure. Let the side length be  $a$ ,  $PE = x$ , and  $PF = y$ .



Applying the Pythagorean Theorem on  $\triangle AEP$ ,  $\triangle CPF$ , and  $\triangle BPF$ , we have

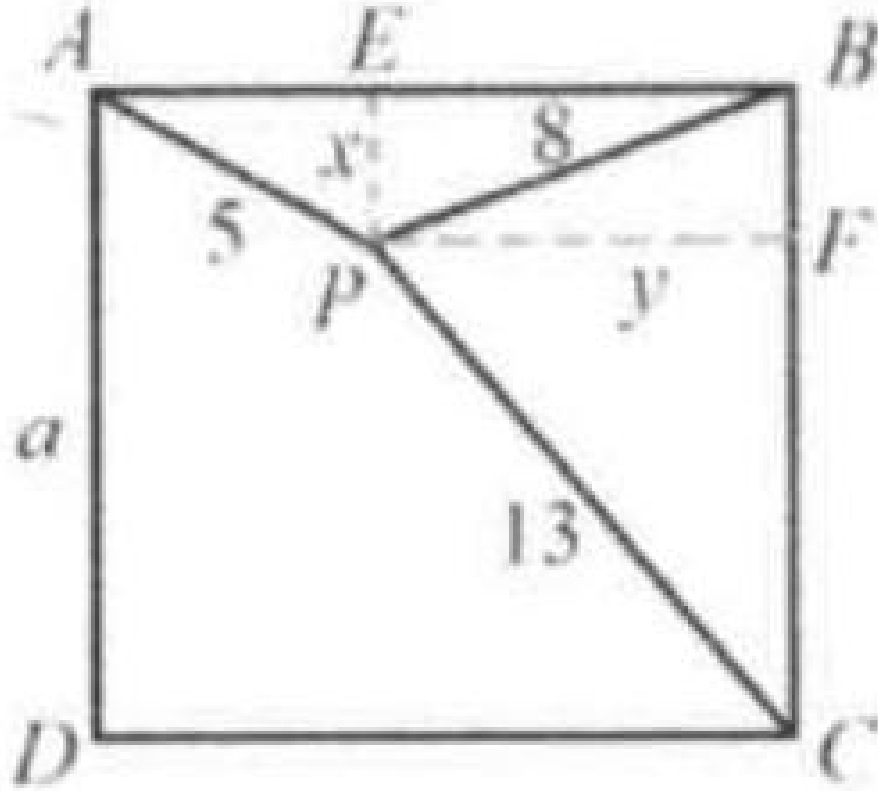
$$x^2 + (a - y)^2 = 25$$

$$(a - x)^2 + y^2 = 169$$

$$x^2 + y^2 = 64$$

$$(1) - (3): a^2 - 2ay = -39 \Rightarrow 4a^2y^2 = (a^2 + 39)^2$$

$$(2) - (3): a^2 - 2ax = 105 \Rightarrow 4a^2x^2 = (a^2 - 105)^2$$



$$(4) + (5): 4a^2(x^2 + y^2) = (a^2 + 39)^2 + (a^2 - 105)^2$$

$$\text{Substituting (3) into (6): } 4a^2 \times 64 = (a^2 + 39)^2 + (a^2 - 105)^2$$

$$\Rightarrow a^4 - 194a^2 - 6273 = 0.$$

$$\text{So } a^2 = 41 \text{ or } 153.$$

$$\text{Since } PC < AC, \text{ or } 13 < \sqrt{2}a, 2a^2 > 169. \text{ Thus } a^2 = 153.$$

$$\text{So the area is } 153.$$