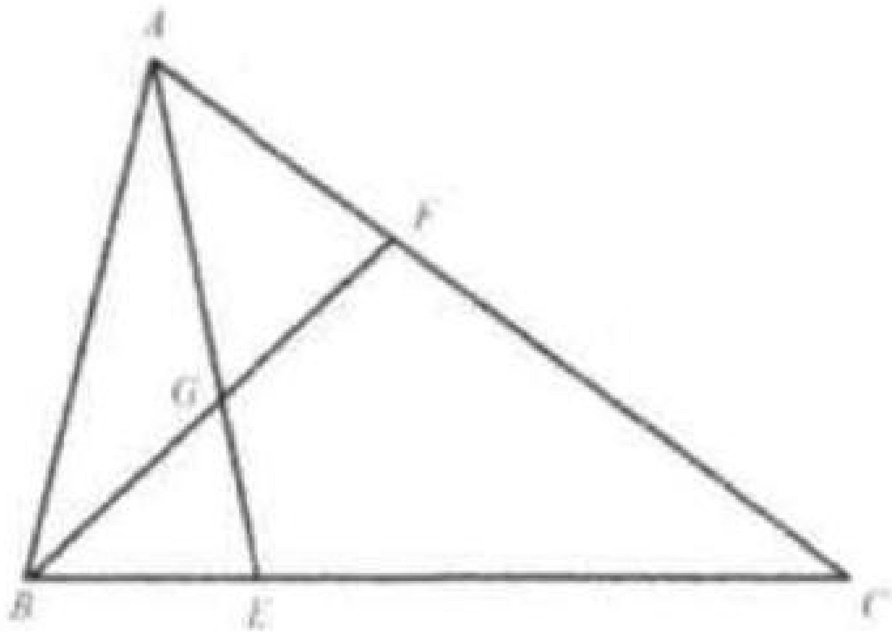
Problem 3

Problem

(AMC) In triangle ABC, point F divides side AC in the ratio 1:2. Let E be the point of intersection of side BC and AG where G is the midpoint of BF. Then point E divides side BC in the ratio

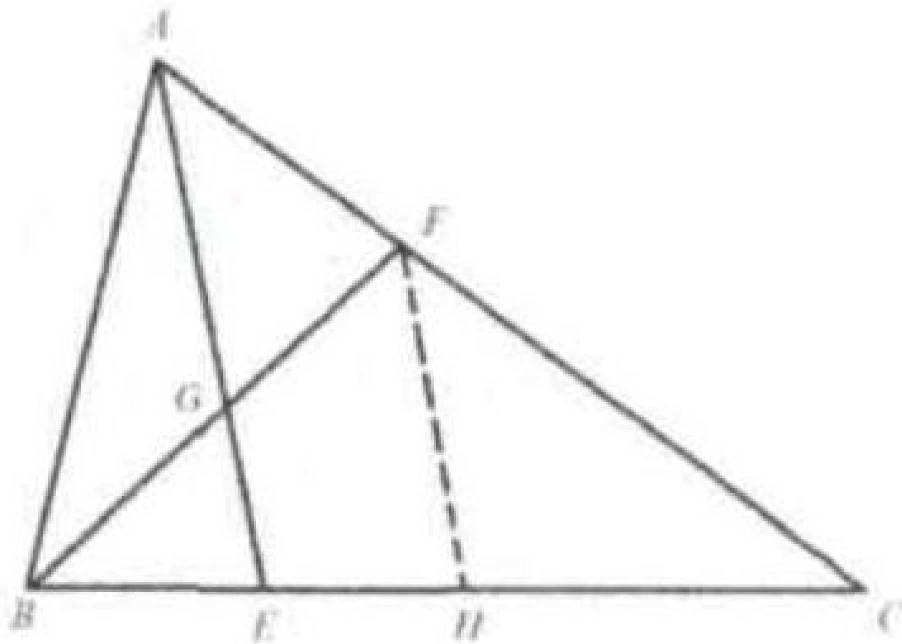
(A) $1:4$
(B) $1:3$
(C) $2:5$
(D) $4:11$
(E) $3:8$



Solution

(B).
Method 1:

Draw FH parallel to line AGE (see figure). Then $BE = EH$ because $BG = GF$ and a line (GE) parallel to the base (HF) of a triangle (HFB) divides the other two sides proportionally. By the same reasoning applied to triangle AEC with line FH parallel to base AE, we see that $HC = 2EH$, because $FC = 2AF$ is given. Therefore $EC = EH + HC = 3EH = 3BE$, and E



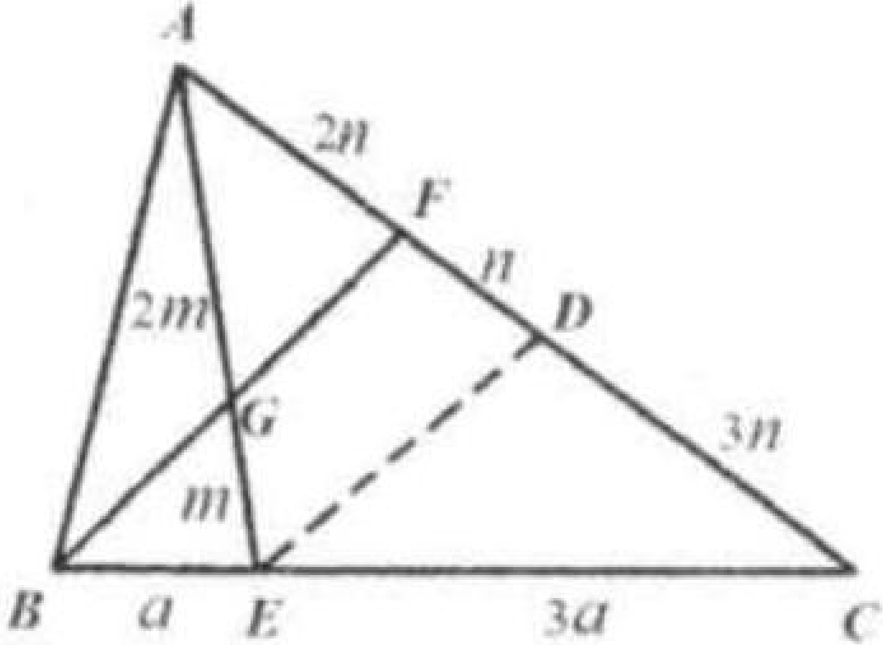
divides side BC in the ratio 1:3.

Method 2:

Applying Menelaus Theorem to $\triangle AFG$ with the transversal points C, B, and E :

$$\frac{AC}{FC} \cdot \frac{FB}{GB} \cdot \frac{GE}{AE} = 1 \Rightarrow \frac{3}{2} \cdot \frac{2}{1} \cdot \frac{GE}{AE} = 1 \Rightarrow$$
$$\frac{GE}{AE} = \frac{1}{3}$$

Draw $ED//BF$ as shown in the figure. Let AF be $2n$ then FD



$= n$ and $DC = 3n.E$ divides side BC in the ratio $1 : 3$ as well.