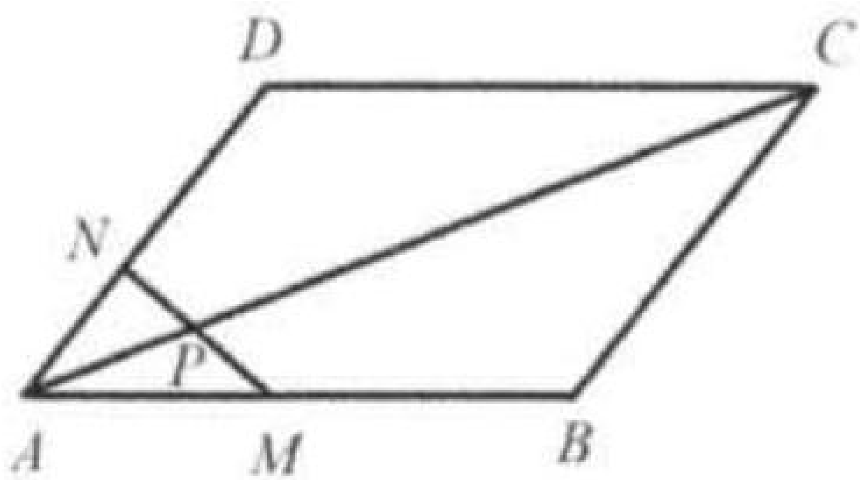
# Problem 21

## Problem

In parallelogram $ABCD$, point $M$ is on $AB$ so that $\frac{AM}{MB} = \frac{17}{1000}$, and point $N$ is on $AD$ so that $\frac{AN}{ND} = \frac{17}{2009}$. Let $P$ be the point of intersection of $AC$ and $MN$. Find $\frac{PC}{PA}$.




## Solution

178.

Extend $NM$ through $M$ to $E$ and to meet the extension of $CB$ at $E$. We label the line segments as shown in the figures.

We know that $AD//CE$. So $\triangle AMN \sim \triangle BME$ (Figure 1). $\frac{AN}{BE} = \frac{AM}{MB} \quad \Rightarrow$

$$\frac{17y}{BE} = \frac{17x}{1000x} \Rightarrow \quad BE = 1000y.$$

We know that $AN//CE$. So $\triangle APN \sim \triangle CPE$ (Figure 2).

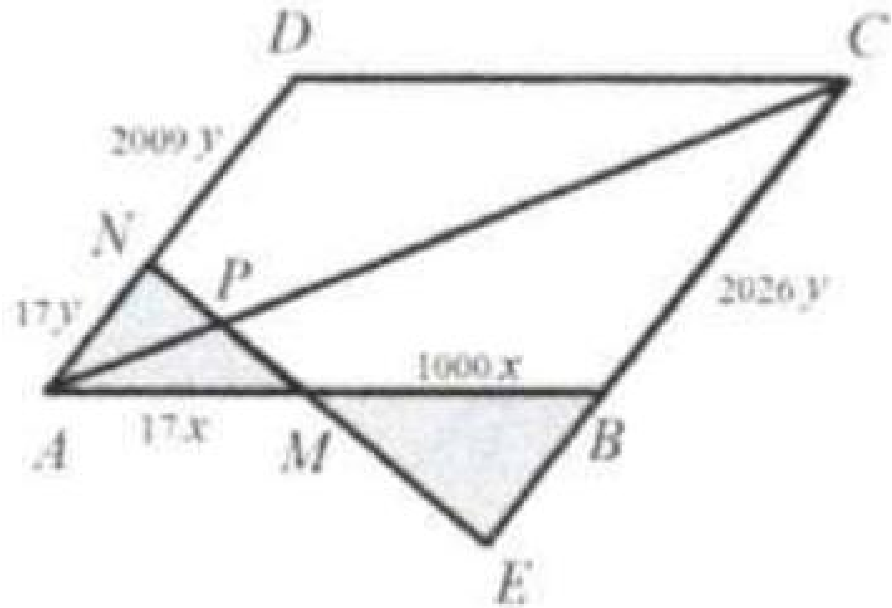



Figure 1

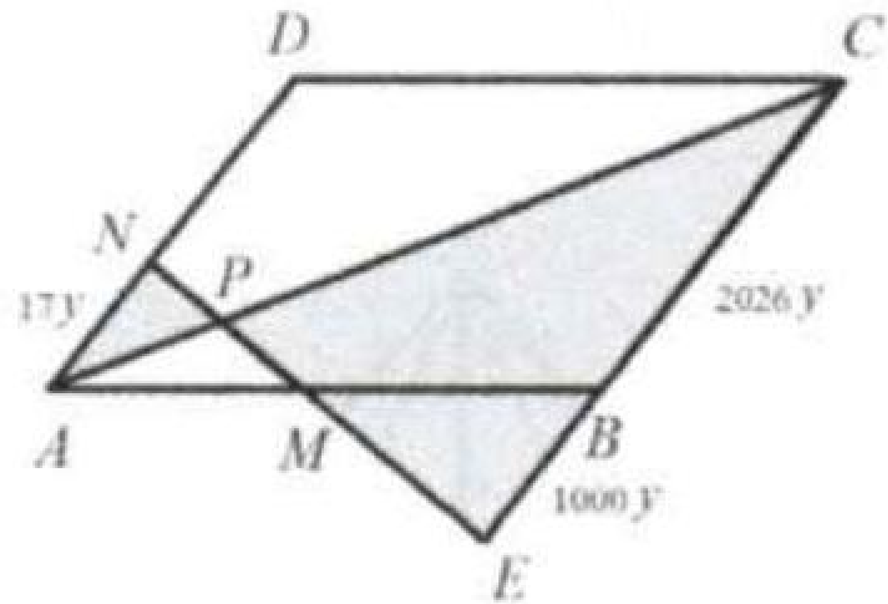



Figure 2

$\frac{PC}{PA} = \frac{CE}{AN} = \frac{2026y+1000y}{17y} = 178.$