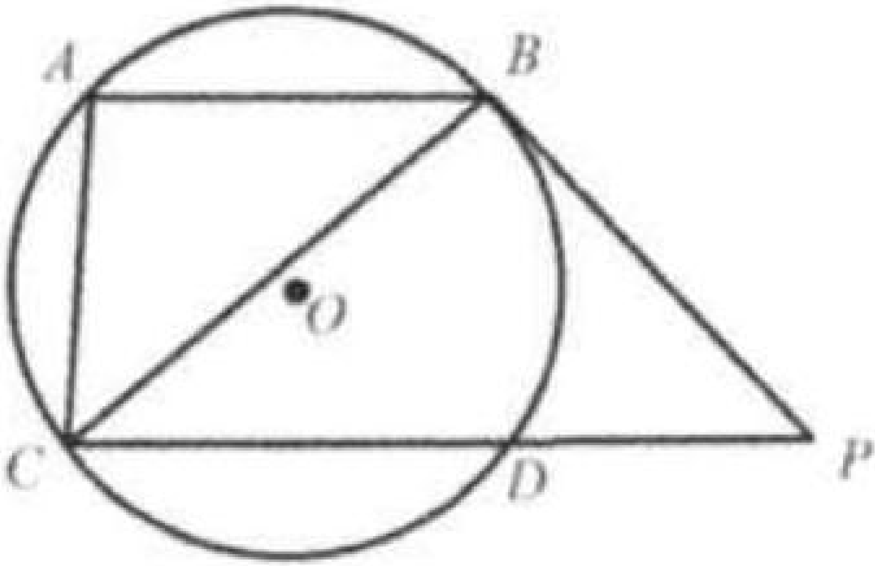
Problem 4

Problem

Triangle ABC is inscribed in the circle O. Draw $CD//AB$. Draw tangent line through B to meet the extension of CD at P. Show that $PB \times CA = CB \times PD$.



Solution

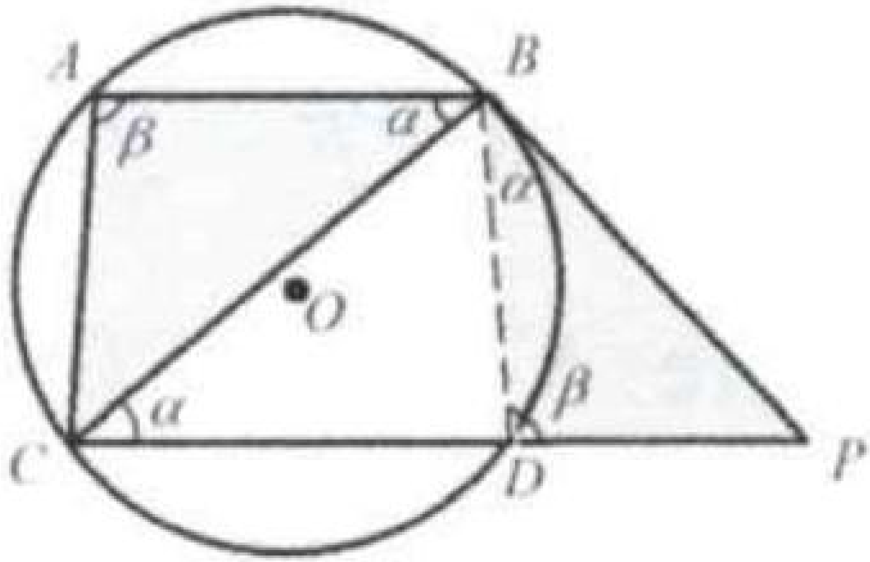
Connect BD. Since BP is tangent to circle $O, \angle PBD = \angle BCD = \alpha$ (both angles face the same arc BD).

Since $AB//CD, \angle BCD = \angle CBA = \alpha$.

So $\angle PBD = \angle CBA = \alpha$.

Since points A, B, D, and C are concyclic, $\angle PDB = \angle CAB = \beta$.

Thus $\triangle PDB \sim \triangle CAB$.



Thus $\frac{AC}{PD} = \frac{CB}{PB} \quad \Rightarrow \quad PB \times CA = CB \times PD.$