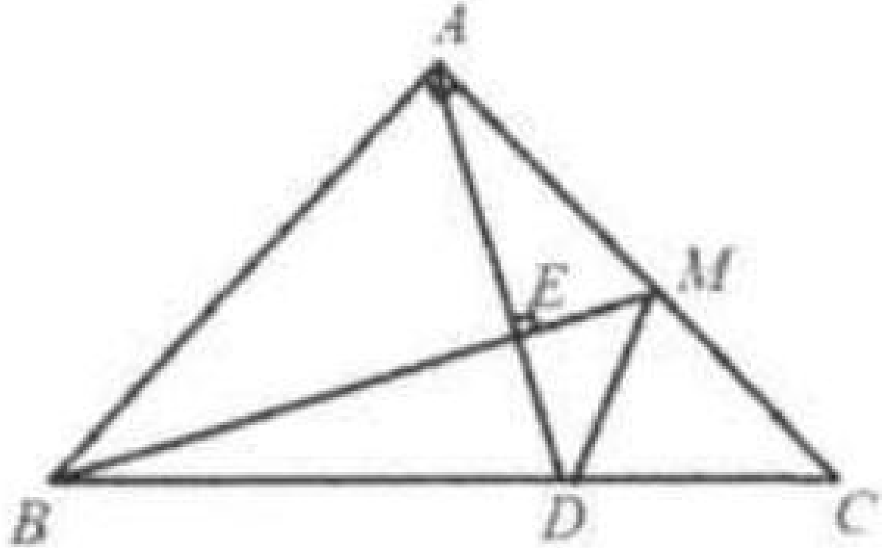
Problem 17

Problem

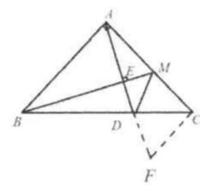
Given $\triangle ABC, \angle A = 90°. AB = AC. M$ is the midpoint of $AC. AE \perp BM$ with the feet at E. Extend AE to meet BC at D. Prove that $\angle AMB = \angle CMD$.



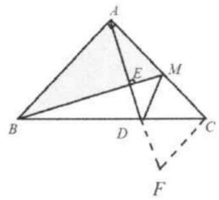
Solution

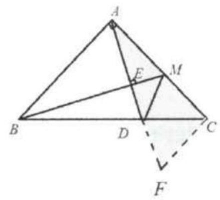
Draw $CF // AB$. Extend AD to meet CF at F.
Since $\angle ABM + \angle AMB = 90°$ and $\angle MAE + \angle AMB = 90°$,
$\angle ABM = \angle CAF$.
Since $AB = AC, \angle ABM = \angle CAF$. Rt $\triangle ABM \cong$ Rt $\triangle CAF$, $\angle AMB = \angle F$.

Since $\angle ABM + \angle AMB = 90^\circ$ and $\angle MAE + \angle AMB = 90^\circ$,
$\angle ABM = \angle CAF$.

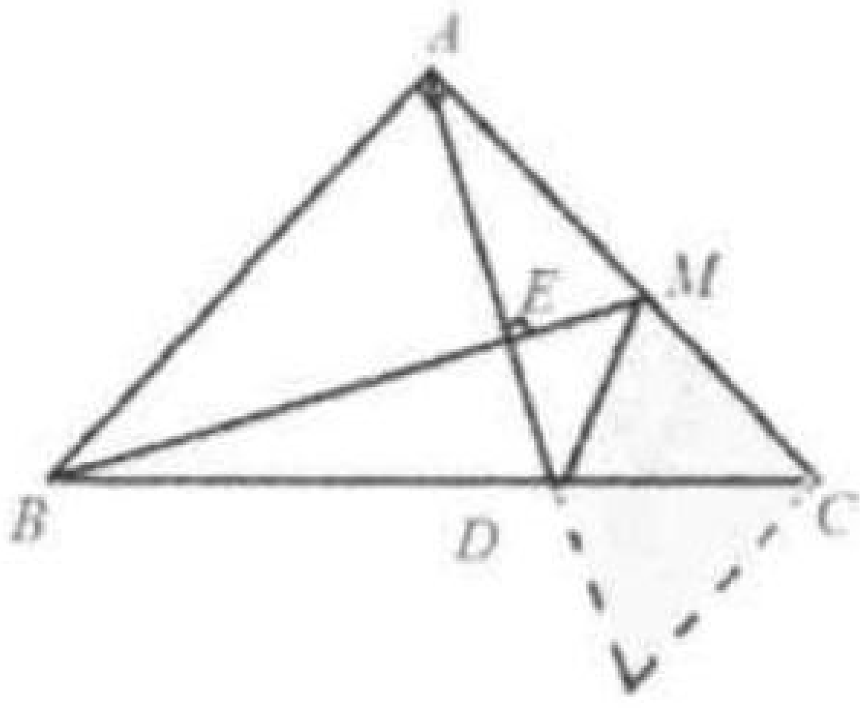


Since $AB = AC$, $\angle ABM = \angle CAF$. Rt$\Delta ABM \cong$ RtΔCAF,
$\angle AMB = \angle F$.





Since
MC = AM = CF, $\angle MCD = 45^\circ = \angle FCD$, CF = CF, $\triangle MCD \cong \triangle FCD$.



$\angle CMD = \angle F$.
Therefore, $\angle AMB = \angle CMD$.