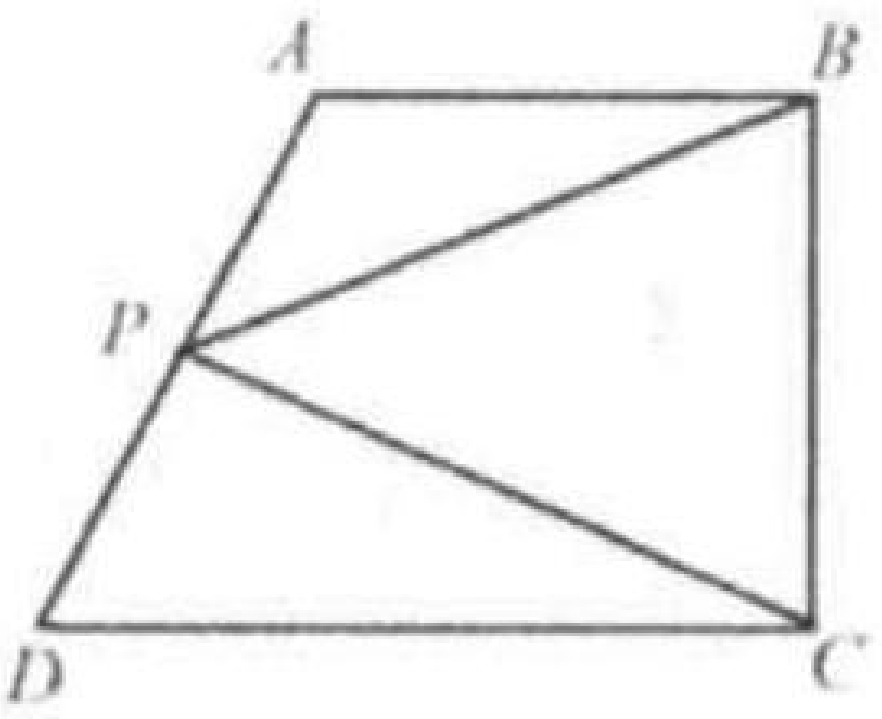
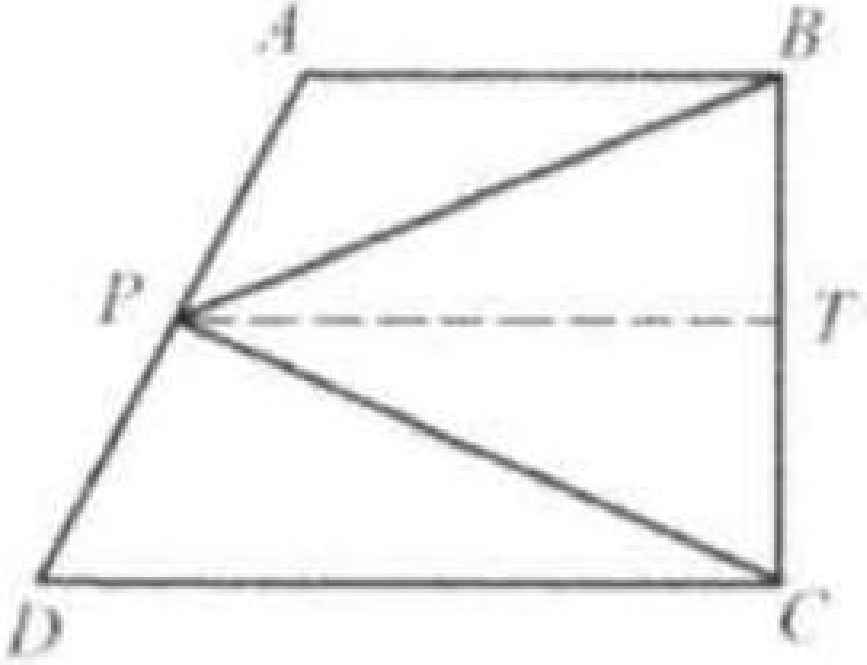
Example 4

(Phillips Academy Prize Exam) Bases AB and DC of a trapezoid $ABCD$ are perpendicular to BC at B and C respectively. From P, the midpoint of side AD, lines are drawn to B and C. Prove: $PB = PC$.

Solution:



Given: Trapezoid $ABCD$ with $AB \perp BC, DC \perp BC$, and P is the midpoint of AD. Let T be the midpoint of BC. Since PT is the median of trapezoid $ABCD$, then $PT//AB$, making PT the perpendicular bisector of BC. Thus $PB = PC$ because points on a perpendicular bisector are equidistance from the endpoints of a segment.


A
B
P
T
D
C