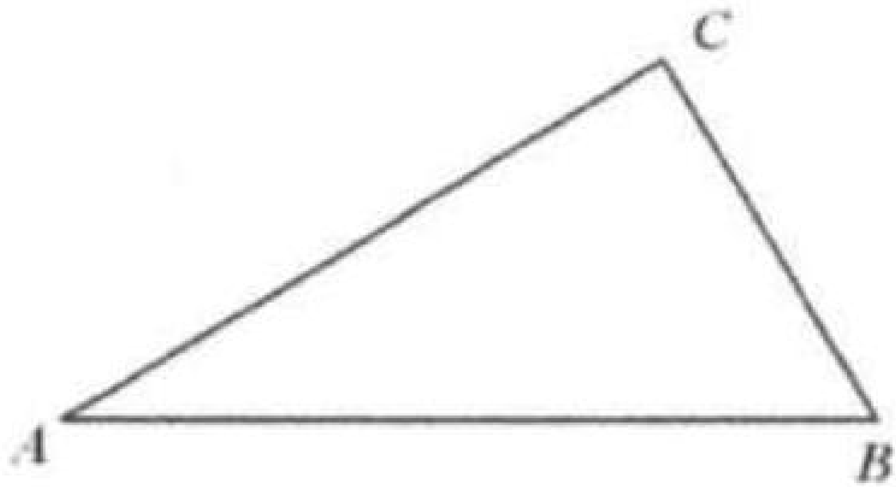
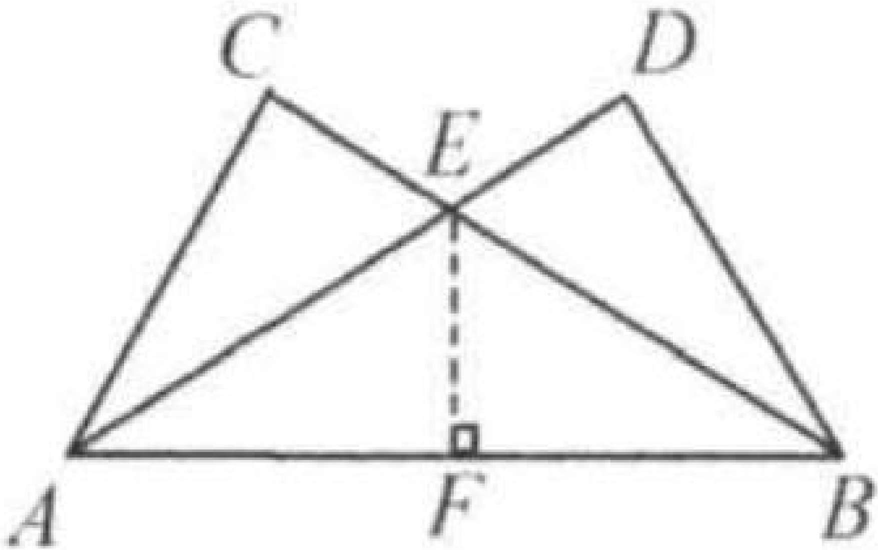
Problem

In $\triangle ABC, \angle B = 2\angle A$ and $AB = 2BC$. Show that $AB^2 = AC^2 + BC^2$.



Solution

(D).
Method 1 (official solution):



In the adjoining figure MV is an altitude of $\triangle AMV$ ($30° - 60° - 90°$ triangle), and MV has length $2\sqrt{3}$. The required area of triangle ABV is $\frac{1}{2}(AB)(MV)$

$= \frac{1}{2} \times 12 \times 2\sqrt{3} = 12\sqrt{3}.$

Method 2 (our solution):

In the adjoining figure we draw EF, an altitude of $\triangle AEB$. EF divides the figure into four congruent triangles. Since $\triangle ABC$ is a $30° - 60° - 90°$ triangle, thus $AB = 12, AC = 6$ and $BC = 6\sqrt{3}$. The area of $\triangle ABC$ is $\frac{1}{2}(AC)(BC) = \frac{1}{2} \times 6 \times 6\sqrt{3} = 18\sqrt{3}$.

The required area of triangle ABE, therefore is $\frac{2}{3} \times 18\sqrt{3} = 12\sqrt{3}$.