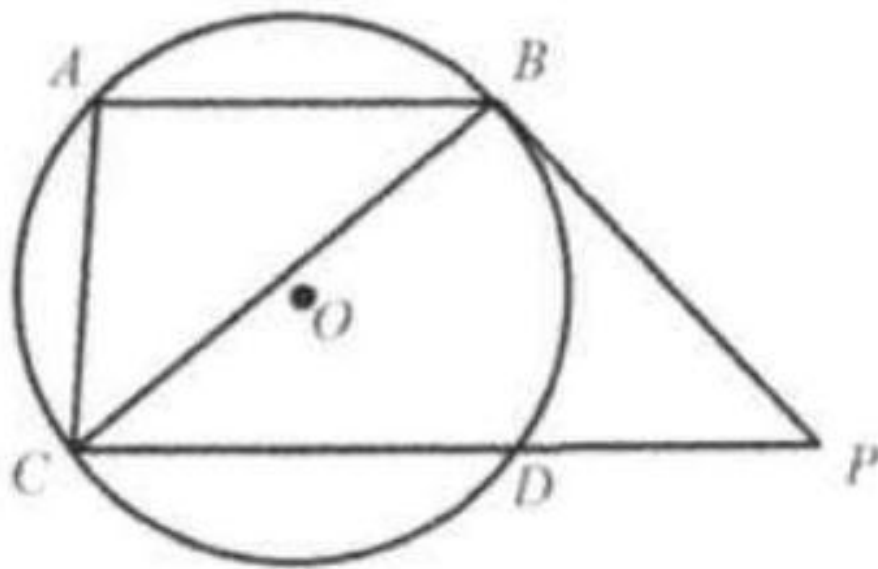


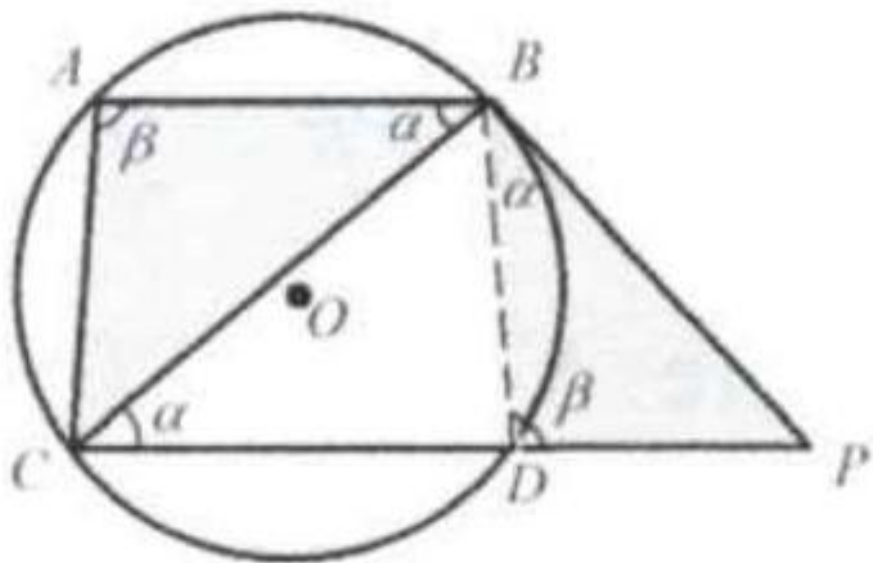
## Problem

Triangle  $ABC$  is inscribed in the circle  $O$ . Draw  $CD \parallel AB$ . Draw tangent line through  $B$  to meet the extension of  $CD$  at  $P$ . Show that  $PB \times CA = CB \times PD$ .



## Solution

Connect  $BD$ . Since  $BP$  is tangent to circle  $O$ ,  $\angle PBD = \angle BCD = \alpha$  (both angles face the same arc  $BD$  ).  
Since  $AB \parallel CD$ ,  $\angle BCD = \angle CBA = \alpha$ .  
So  $\angle PBD = \angle CBA = \alpha$ .  
Since points  $A$ ,  $B$ ,  $D$ , and  $C$  are concyclic,  $\angle PDB = \angle CAB = \beta$ .  
Thus  $\triangle PDB \sim \triangle CAB$ .



$$\text{Thus } \frac{AC}{PD} = \frac{CB}{PB} \Rightarrow PB \times CA = CB \times PD.$$