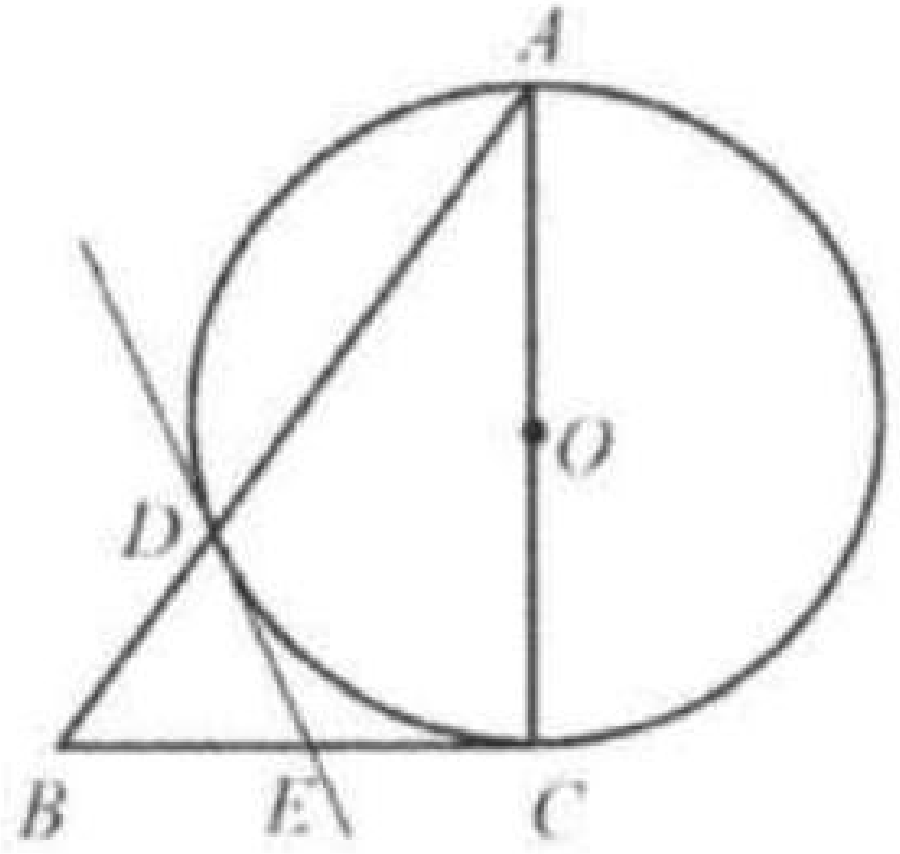
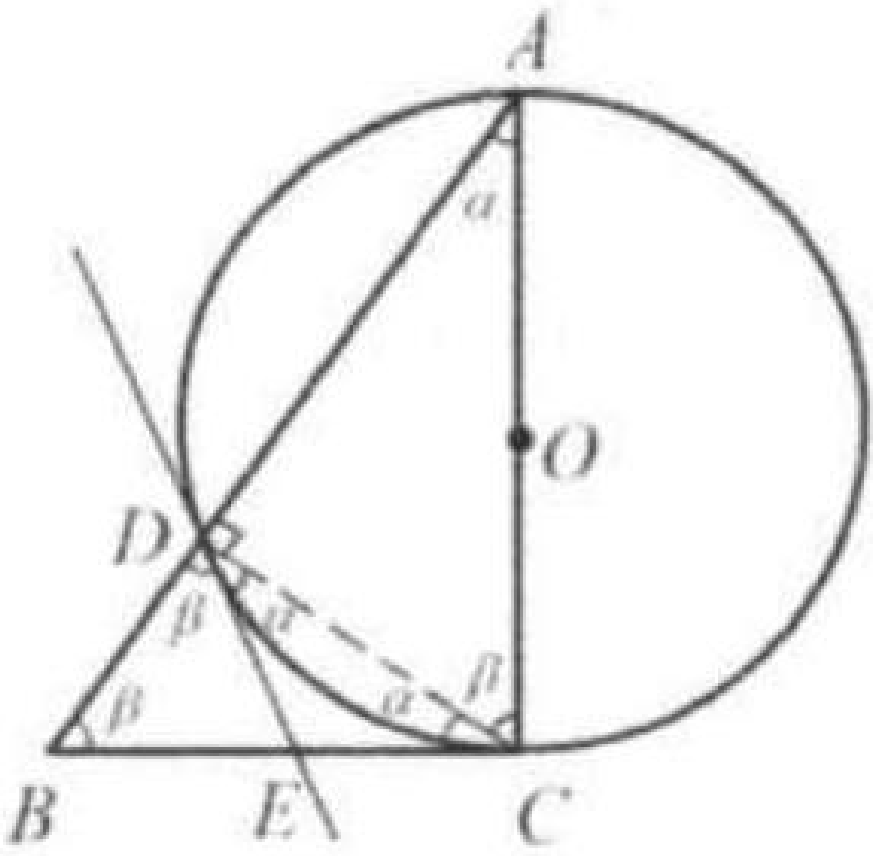
Problem

As shown in the figure, $\triangle ABC$ is a right triangle with $\angle C = 90°$.AC is the diameter of circle O. Circle O meets the hypotenuse AB at D. Draw the tangent through D to the circle to meet the leg BC at E. Prove: $BE = EC$.



Solution

Connect CD.
Since AC is the diameter, $\angle ADC = 90°$.
Let $\angle A = \alpha, \angle ACD = \beta$.
Then $\angle EDC = \angle ECD = \angle A = \alpha$ (they all face the same arc DC).
Since $\angle ACB = 90°, \angle B = \beta$.
Since $\angle BDC = 90°, \angle BDE = \beta$.
Note that both EC and ED are tangent to the circle O, ED



$= EC.$
Thus $DE = BE = EC.$