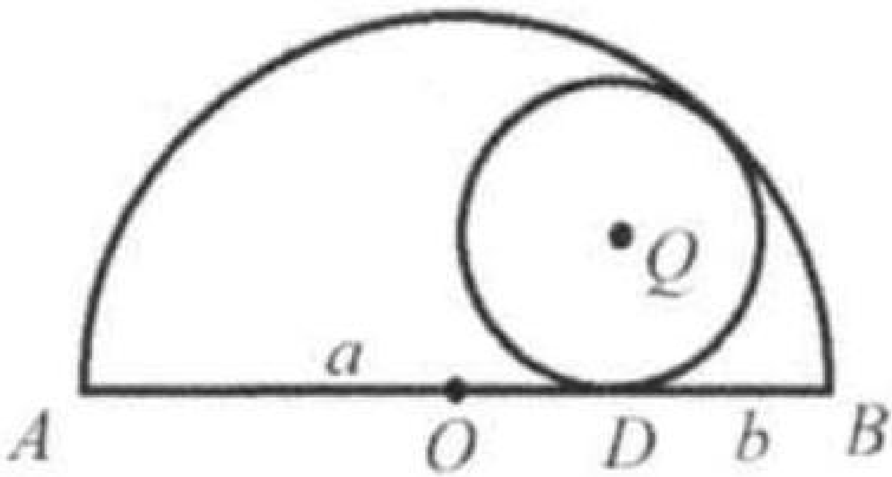
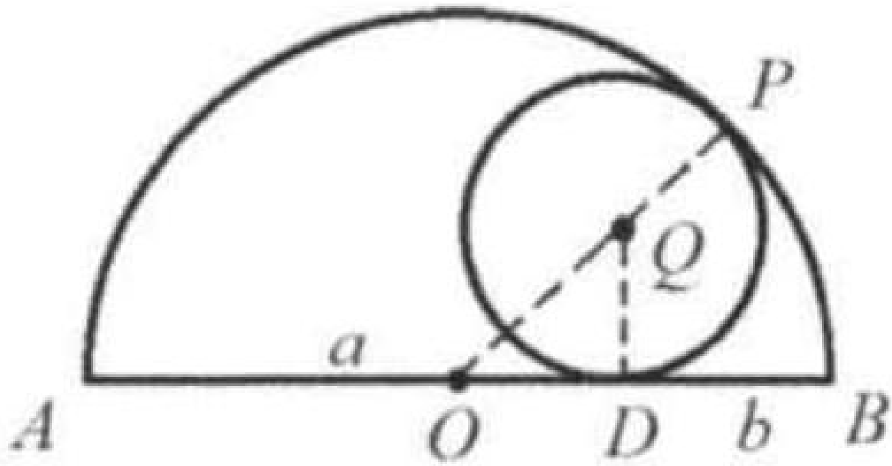
Example 6

In the diagram, $AB = a + b$ cm is the diameter of semicircle O. Circle Q has the radius of r and is inscribed in circle O, and is tangent to AB at D. Let $AD = a$ and $DB = b(a > b)$. Find r in terms of a and b.



Solution: Connect OQ and extend it to meet the semicircle O at P. Connect QD.

Since P is the tangent point of circle Q and semicircle O,



points O, Q, P are collinear.

$OP = OA = OB = \frac{1}{2}(a+b)$

$OQ = OP - QP = \frac{1}{2}(a+b) - r$

$OD = OD - DB = \frac{1}{2}(a-b).$

In $\triangle ODQ, OQ^2 = QD^2 + OD^2$, or

$\left[\frac{1}{2}(a+b) - r\right]^2 = r^2 + \left[\frac{1}{2}(a-b)\right]^2.$

Solving we get $r = \frac{ab}{a+b}$ or $\frac{1}{a} + \frac{1}{b} = \frac{1}{r}$.