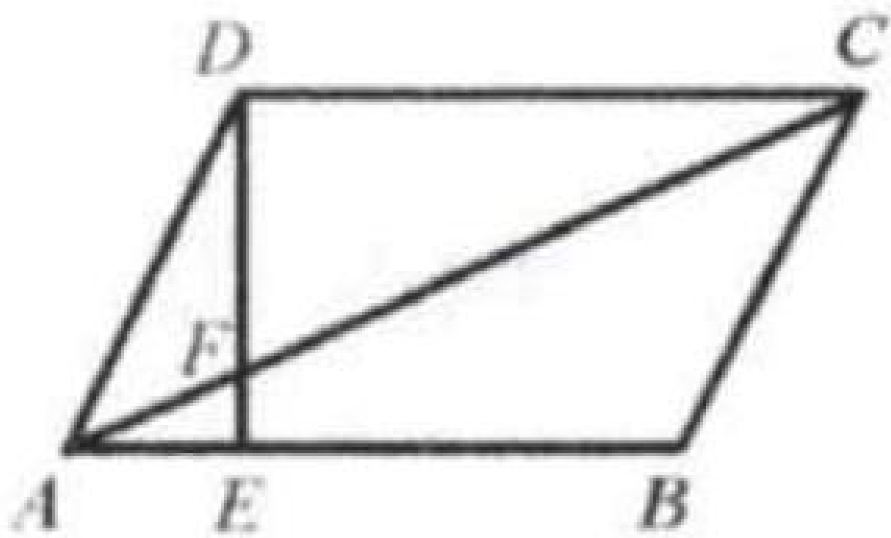
Example 10

$ABCD$ is a parallelogram. $DE \perp AB$ at $E.AD = \frac{1}{2}FC$. Show that $\angle DAB = 3\angle ACD$.

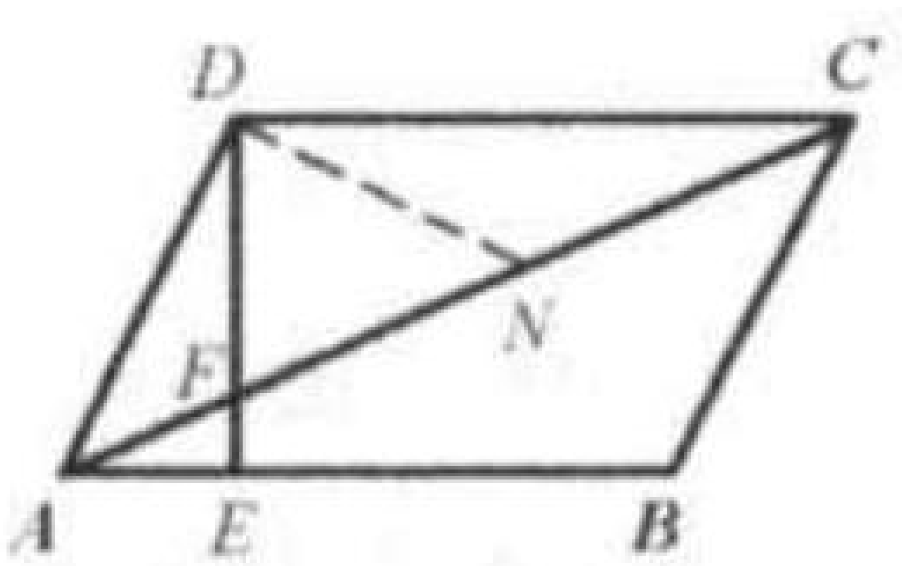
Solution:



Draw DN, the median of triangle CDF. Since DN is the median, by Theorem 1.3, $DN = FN = NC$.

Since $AD = \frac{1}{2}FC, AD = DN$.

Thus triangle AND is an isosceles triangle with $\angle DAN = \angle DNA$.



We also know that $DN = NC$, so $\angle NDC = \angle NCD$.
$\angle DNA$ is the exterior angle of triangle DNC. So $\angle DNA = \angle NDC + \angle NCD$
$= 2\angle NCD$.
Note that $\angle CAB = \angle NCD$.
Therefore $\angle DAB = \angle DCA + \angle CAB = \angle DNA + \angle NCD$
$= 2\angle NCD + \angle NCD = 3\angle NCD = 3\angle ACD$.