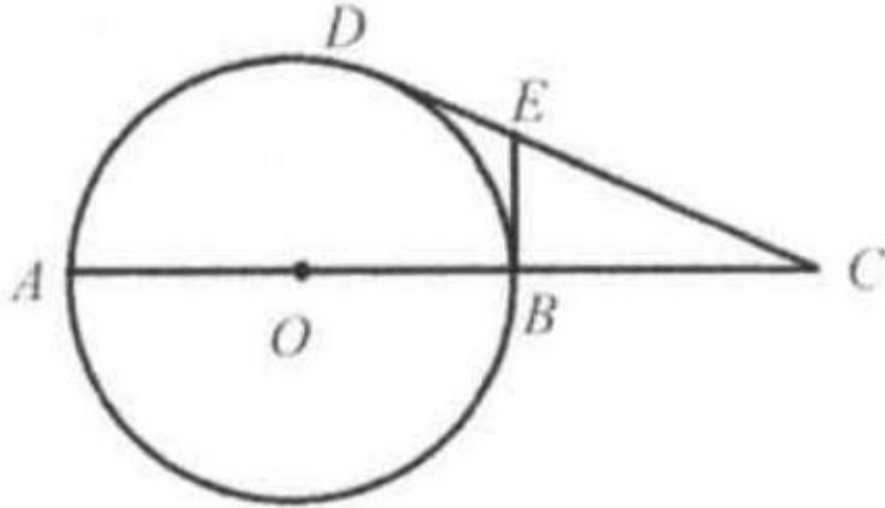


## Problem 6

### Problem

$AB$  is the diameter of the circle  $O$ . Extend  $AB$  to  $C$ .  $CD$  is tangent to the circle at  $D$ .  $BE$  is tangent to the circle at  $B$  and meets  $CD$  at  $E$ . Show that  $CA = \sqrt{3}CD$  if  $DE = \frac{1}{2}EC$ .



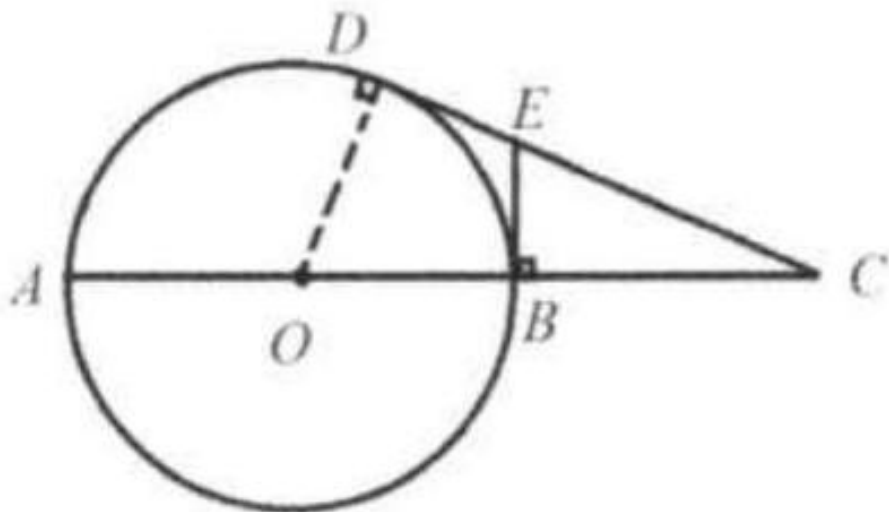
### Solution

Connect  $OB$ .

Since both  $CD$  and  $BE$  are tangents of circle,  $BE = DE$ .

Since  $DE = \frac{1}{2}EC$ ,  $BE = \frac{1}{2}EC$ . So  $\angle C = 30^\circ$ . Thus  $OC = 2OD$ .

Since  $OB = OD$ ,  $CB = OC - OB = OC - OD = 2OD -$



$$OD = OD = OB = OA.$$

$$\text{Thus } CA = 3CB.$$

We know that  $CD^2 = CA \times CB = \frac{1}{3}CA^2$ .

$$\text{So } CA^2 = 3CD^2 \Rightarrow CA = \sqrt{3}CD.$$