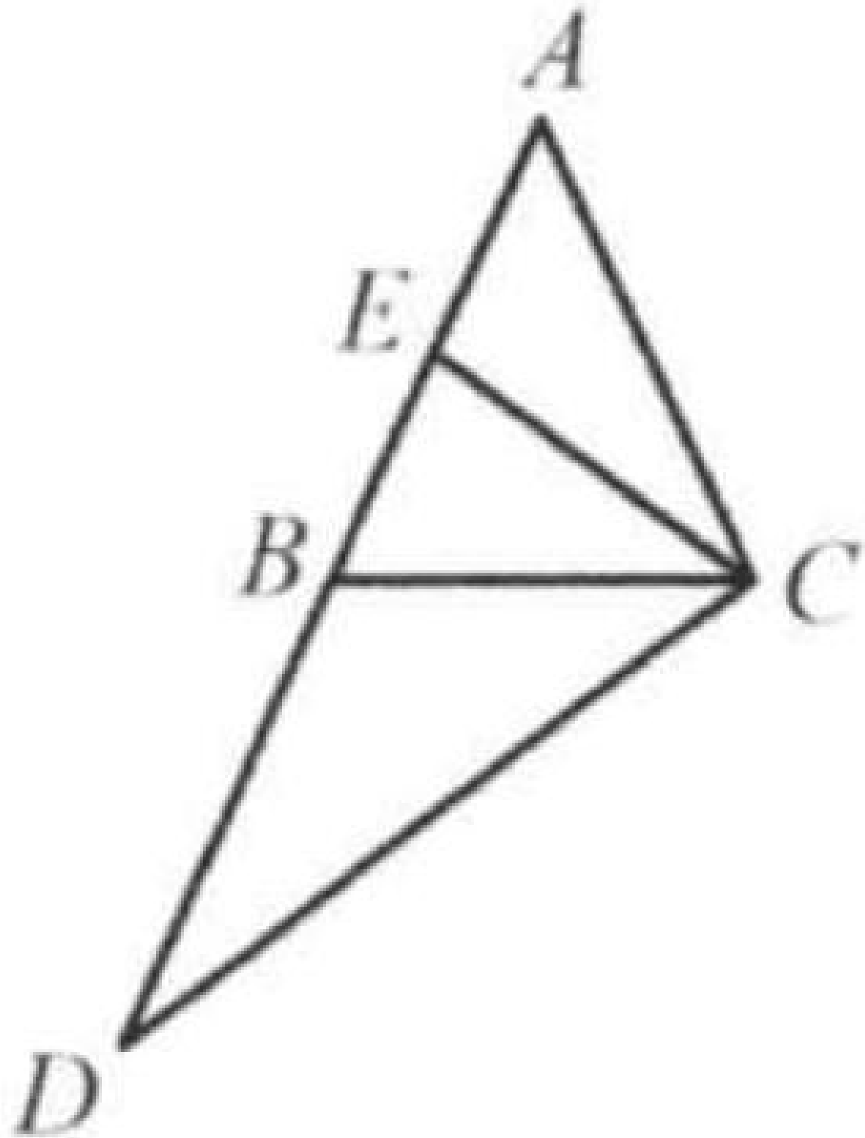
Problem 4

Problem

In $\triangle ABC, AB = AC$. E is the midpoint of AB. Extend AB to D such that $BD = BA$. Prove: $CD = 2CE$.



Solution

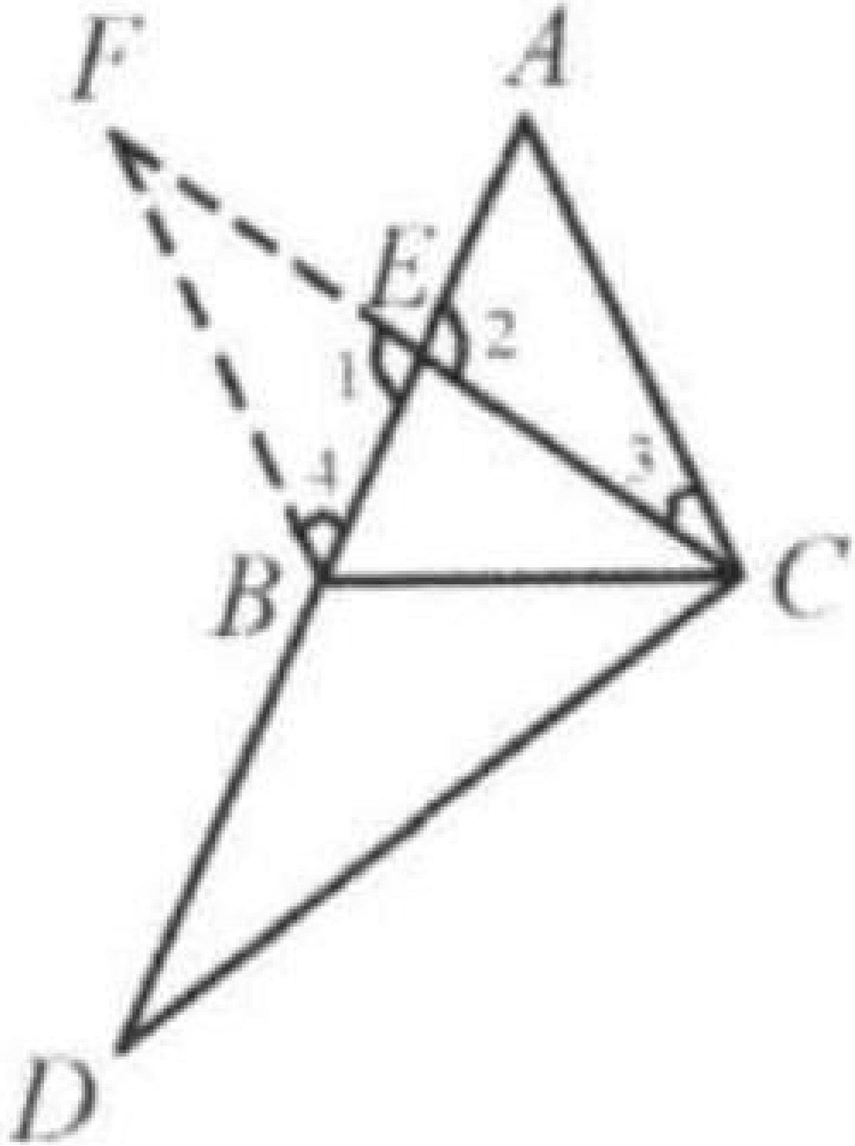
Method 1:
Extend CE to F such that $CE = EF$. Since $AE = EB$ and $\angle 1 = \angle 2, \triangle AEC \cong \triangle BEF$.
Thus, $\angle 3 = \angle F, \angle 4 = \angle A, BF = AC$. Since $AB = AC = BD$, therefore

$BF = BD$. $\angle DBC = \angle A + \angle ACB = \angle A + \angle ABC$ or $\angle FBC = \angle 4 + \angle ABC = \angle A + \angle ABC$.

Thus, $\angle DBC = \angle FBC$. Since $BC = BC, \triangle FBC \cong \triangle DBC$.

Therefore $CF = CD$.

Since $CE = EF = 1/2CF = 1/2CD, CD = 2CE$.



Method 2:

Extend CE to F such that $CE = EF$. Since $AE = EB$ and

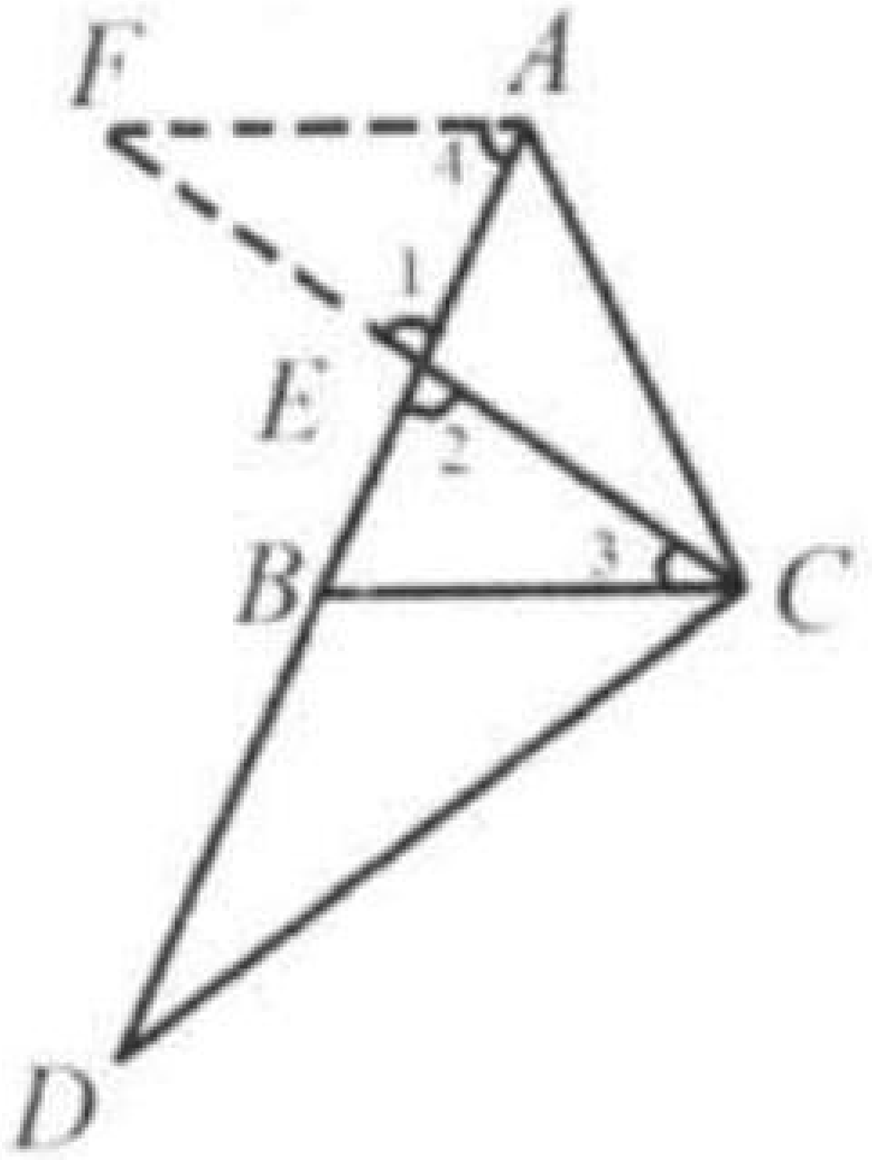
$\angle 1 = \angle 2, \triangle AEF \cong \triangle BEC$.

Thus, $\angle 3 = \angle F, \angle 4 = \angle CBE, AF = BC$. Since $AB = AC = BD, AC = BD$.

$$\angle DBC = \angle CAB + \angle ACB = \angle CAB + \angle ABC$$

$$\angle CAF = \angle CAB + \angle 4 = \angle CAB + \angle ABC.$$

Thus, $\angle DBC = \angle CAF$. Since $AF = BC, AC = BD$, so $\triangle FAC \cong \triangle CBD$.



Therefore $CF = CD$.

Since $CE = EF = 1/2CF = 1/2CD, CD = 2CE$.