Example 7

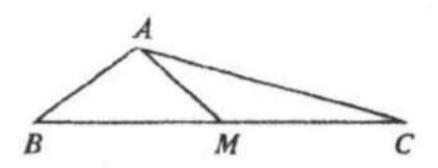
(1975 AMC) In the adjoining figure triangle ABC is such that AB=4 and AC = 8. If M is the midpoint of BC and AM = 3, what is the length of BC?

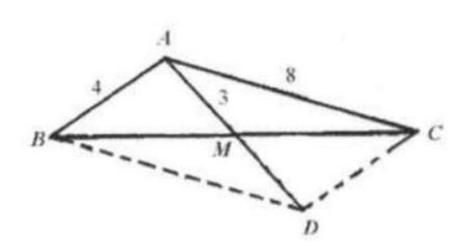
(A) $2\sqrt{26}$ (B) $2\sqrt{31}$

(C) 9

(D) $4 + 2\sqrt{13}$

(E) not enough information given to solve the problem





Solution: (B). Extend AM to D such that AM = MD. Connect BD and CD.ABDC is a parallelogram.

We know that the sum of the squares of the sides of a parallelogram is equal to the sum of the squares of its diagonals.

Applying this to the parallelogram having AB and AC as adjacent sides yields $AD^2 + BC^2 = AB^2 + CD^2 + AC^2 + BD^2 \Rightarrow AD^2 + BC^2 = 2\left(AB^2 + AC^2\right)$ $BC^2 = 2\left(4^2 + 8^2\right) - 6^2 = 124$. $BC = 2\sqrt{31}$.