Problem

(AMC) In $\triangle ABC$, we have $AB = 1$ and $AC = 2$. Side BC and the median from A to BC have the same length. What is BC ?

(A) $\frac{1+\sqrt{2}}{2}$
(B) $\frac{1+\sqrt{3}}{2}$
(C) $\sqrt{2}$
(D) $\frac{3}{2}$
(E) $\sqrt{3}$

Solution

(C).
By the median length formula:

$$\left(AD^2 + DC^2\right) + \left(AD^2 + BD^2\right) = AB^2 + AC^2$$
$$(2m)^2 + m^2 + (2m)^2 + m^2 = 1^2 + 2^2$$
$$10m^2 = 5 \Rightarrow m^2 = \frac{1}{2} \Rightarrow m = \frac{\sqrt{2}}{2}$$
$$BC = 2m = 2 \times \frac{\sqrt{2}}{2} = \sqrt{2}.$$