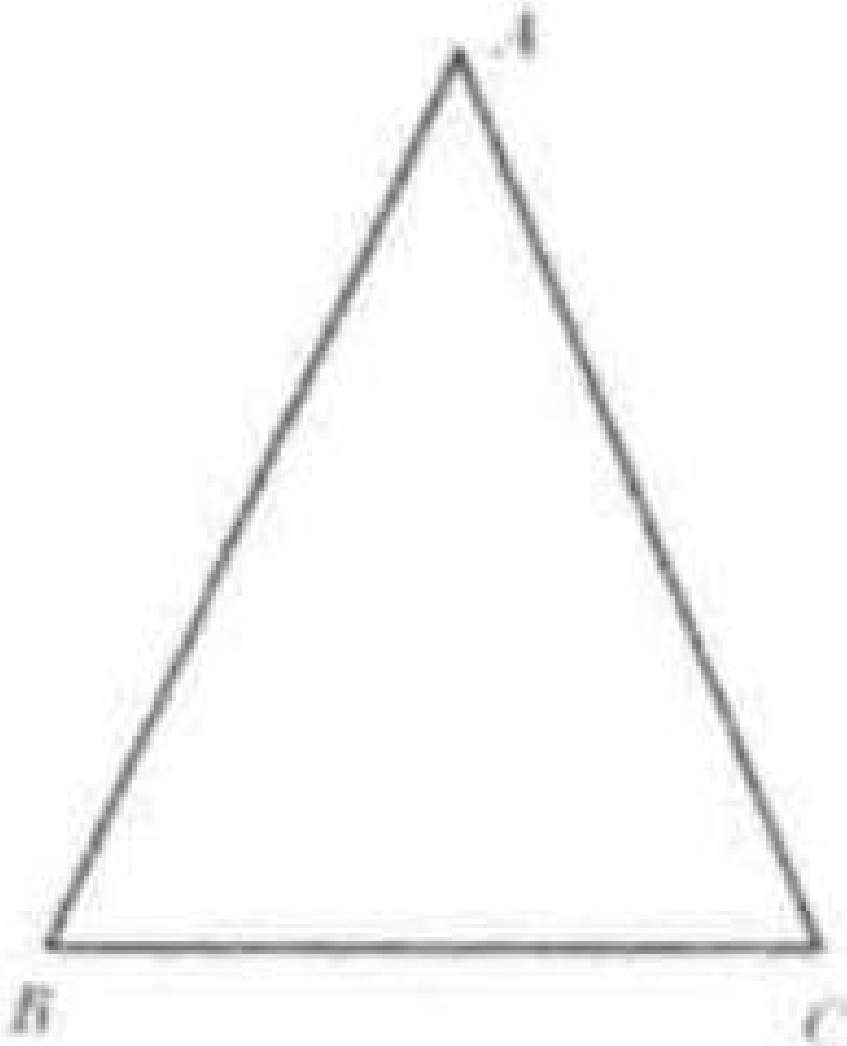


Example 11

$\triangle ABC$ is an isosceles triangle with $AB = AC$ and $\angle CAB = 36^\circ$. Find the value of $BC : AB$.

Solution: $\frac{\sqrt{5}-1}{2}$.



Let $AB = AC = 1$. Since $\angle A = 36^\circ$, $\angle B = \angle C = \frac{180^\circ - \angle A}{2} = 72^\circ$.

Draw the angle bisector CD of $\angle C$ where D is on AB .

$$\angle BCD = \frac{1}{2} \angle ACB = 36^\circ.$$

So in $\triangle BCD$, $\angle BCD = \angle BAC = 36^\circ$ and $\angle CBD = \angle ABC = 72^\circ$.

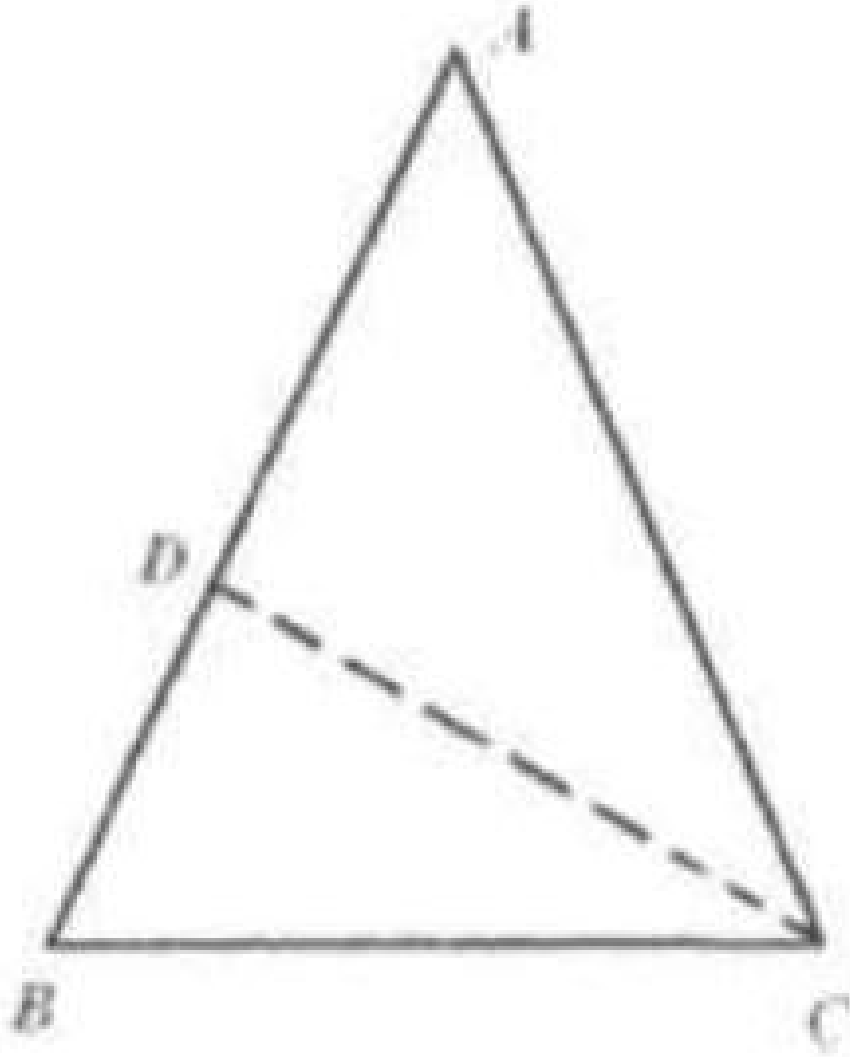
Therefore, $\triangle BCD \sim \triangle BAC$.

Let $BC = x$. Since $\angle BDC = \angle DBC = 72^\circ$, $BC = CD$, so $CD = x$.

Since $\angle DCA = 36^\circ = \angle A$, $CD = DA$.

Therefore $AD = x$ and $BD = 1 - x$.

Since $\triangle BCD \sim \triangle BAC$, $\frac{BC}{BD} = \frac{AB}{BC}$.



Therefore we have $\frac{x}{1-x} = \frac{1}{x}$. Cross-multiplying gives us $x^2 + x - 1 = 0$.
Solve for x : $x = \frac{\sqrt{5}-1}{2}$. Hence $\frac{BC}{AB} = \frac{x}{1} = \frac{\sqrt{5}-1}{2}$.