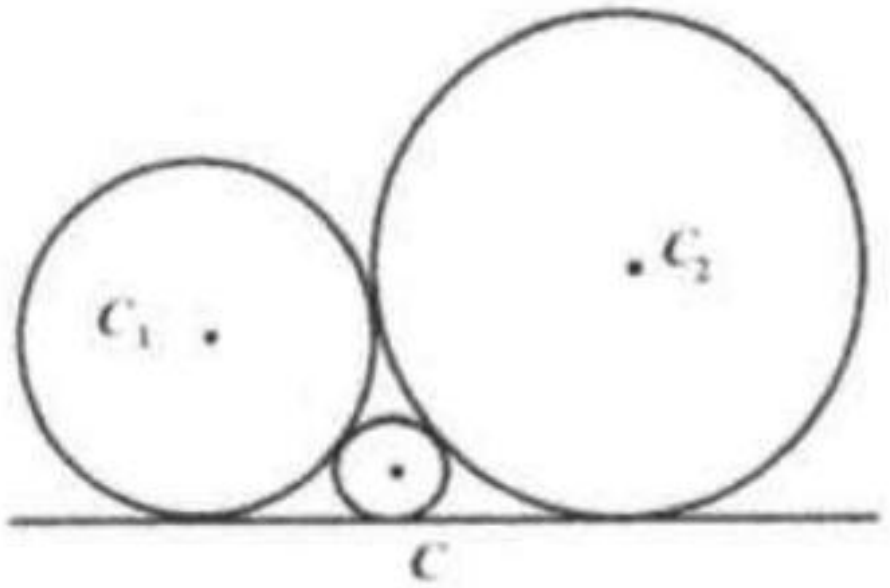
Example 7

Let r_1, r_2, and r be the radii of three circles as shown in the figure. Show that $\frac{1}{\sqrt{r}} = \frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}}$.

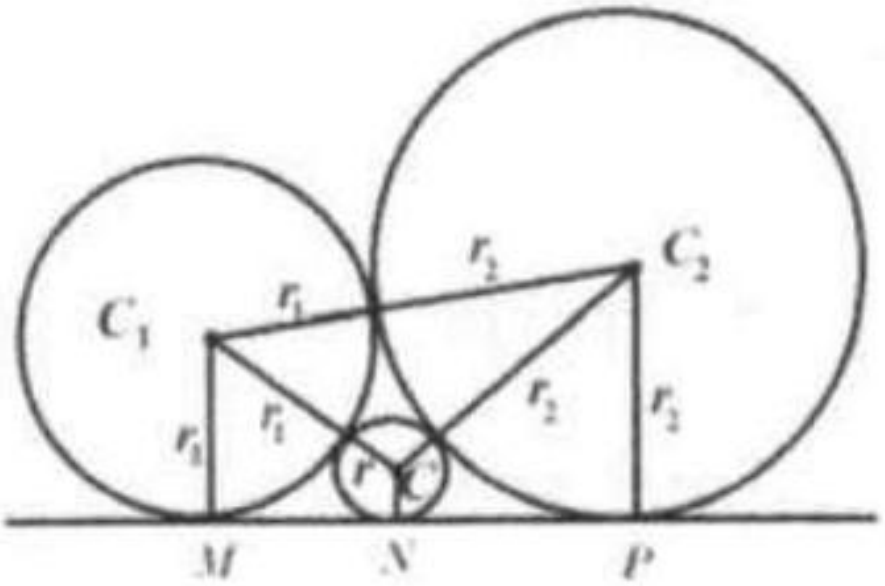
Proof:



Applying Pythagorean Theorem three times:

$$MN = \sqrt{(r_1 + r)^2 - (r_1 - r)^2} = 2\sqrt{r_1 r}$$
$$NP = 2\sqrt{r_2 r}$$
$$MP = 2\sqrt{r_2 r_1}.$$

We see that $\sqrt{r_1 r} + \sqrt{r_2 r} = \sqrt{r_1 r_2}$.



$$\Rightarrow \quad \frac{1}{\sqrt{r}} = \frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}}.$$