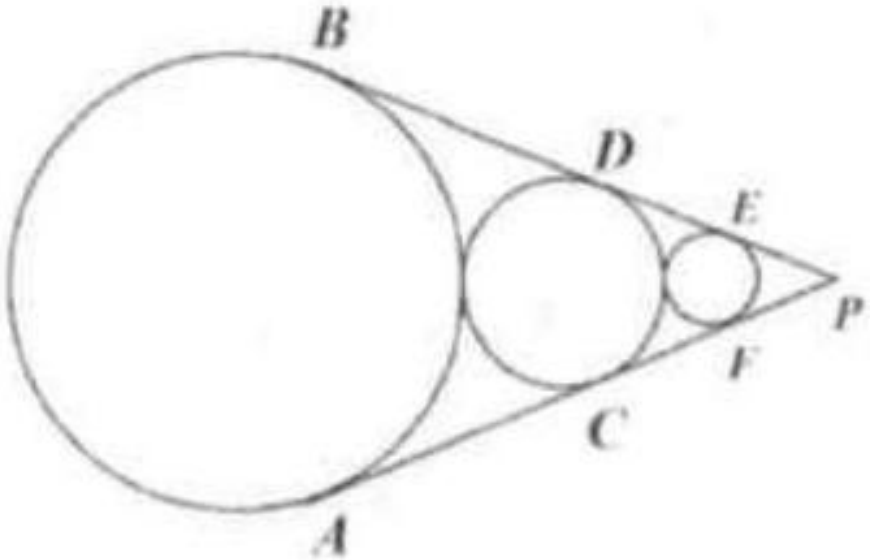
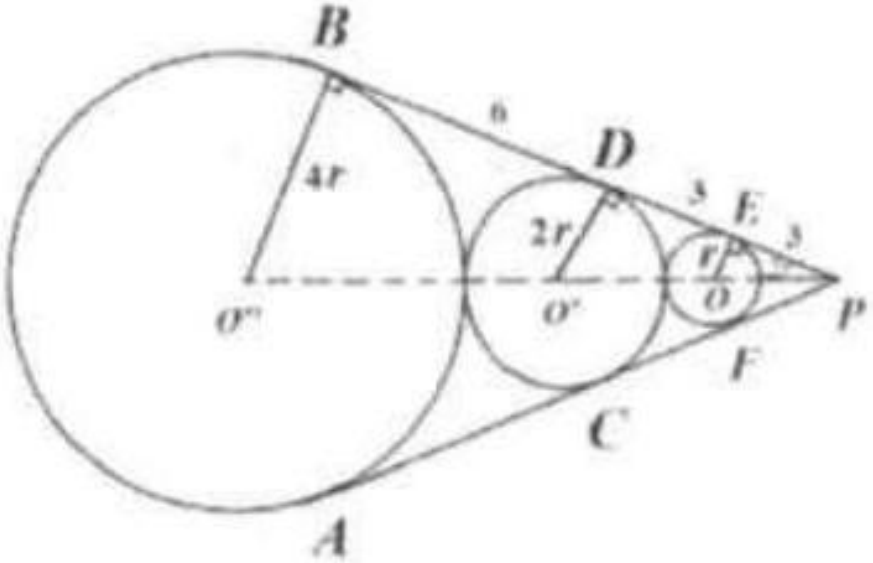
Example 2

As shown in the figure, three circles are externally tangent. PCA and PDB are common tangents. D and E are the tangent points and are also the midpoints of PB and PD respectively. Find the area of the largest circle if $PB = 12$.

(A) 14π
(B) 18π
(C) 25π
(D) 24π
(E) 14π



Solution: (B).
Connect PO'', BO'', DO', and OE.$PE = DE = 3$.
$BD = 6$.$O''B \perp PB, O'D \perp PB, OE \perp PB$.
$OE = r$.$OD' = 2r$.$O''B = 4r$.$O''O' = O'P = 6r$.
By the Pythagorean Theorem in right triangle $PO''B$,



$PO''^2 = PB^2 + O''B^2 \Rightarrow 144 + 16r^2 = 144r^2 \quad \Rightarrow (4r)^2 = 18.$
The area of the largest circle is 18π.