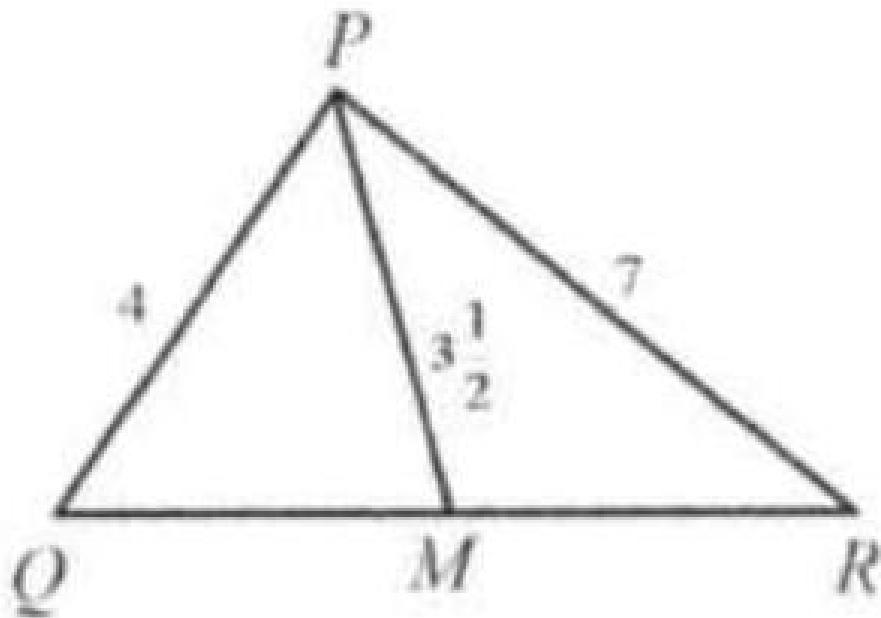


## Example 13

(AMC) The sides  $PQ$  and  $PR$  of triangle  $PQR$  are respectively of lengths 4 inches and 7 inches. The median  $PM$  is  $3\frac{1}{2}$  inches. Then  $QR$ , in inches, is:

- (A) 6
- (B) 7
- (C) 8
- (D) 9
- (E) 10



Solution: (D).

Drop the perpendiculars  $PH$  at  $H$ .

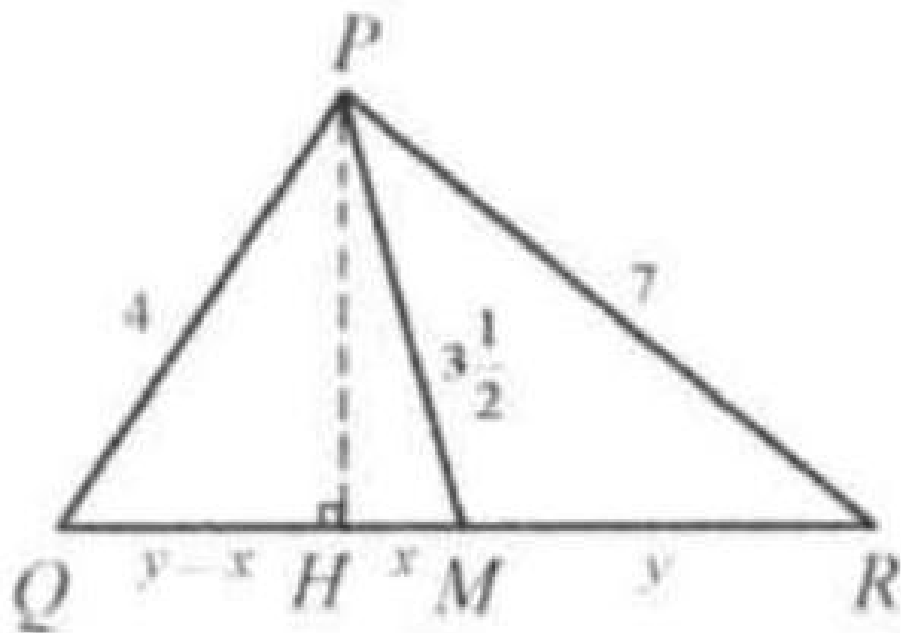
Let  $y$  denote half the length of  $QR$ , and let  $HM = x$ .  $MR = y$ . Then

$$QH = y - x.$$

Applying Pythagorean Theorem to  $\triangle QPH, \triangle MPH$  :

$$4^2 - (y - x)^2 = \left(3\frac{1}{2}\right)^2 - x^2$$

Applying Pythagorean Theorem to  $\triangle QPH, \triangle RPH$  :



$$4^2 - (y-x)^2 = (7)^2 - (x+y)^2$$

$$\text{From (1), we get } (y-x)^2 - x^2 = \frac{15}{4} \Rightarrow y^2 - 2xy = \frac{15}{4}$$

$$\text{From (2), we get } (y+x)^2 - (y-x)^2 = 33 \Rightarrow 4xy = 33 \quad (4) \text{ Solving the}$$

$$\text{system of (3) and (4): } y^2 = \frac{33}{2} + \frac{15}{4} - \frac{81}{4} \Rightarrow y = \frac{9}{2}. \text{ So } QR = 9.$$