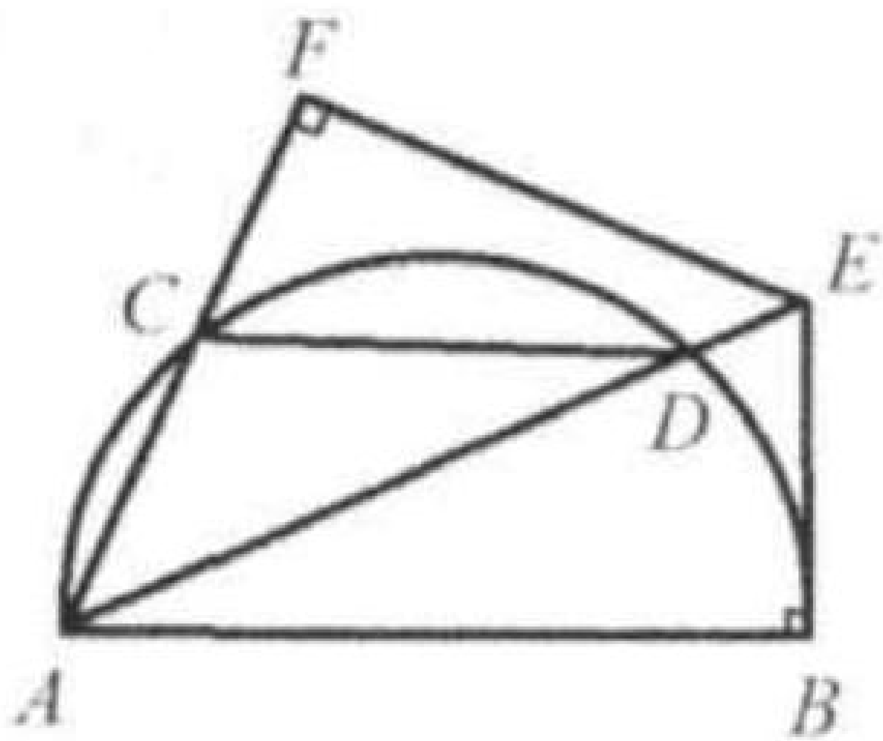
Problem 3

Problem

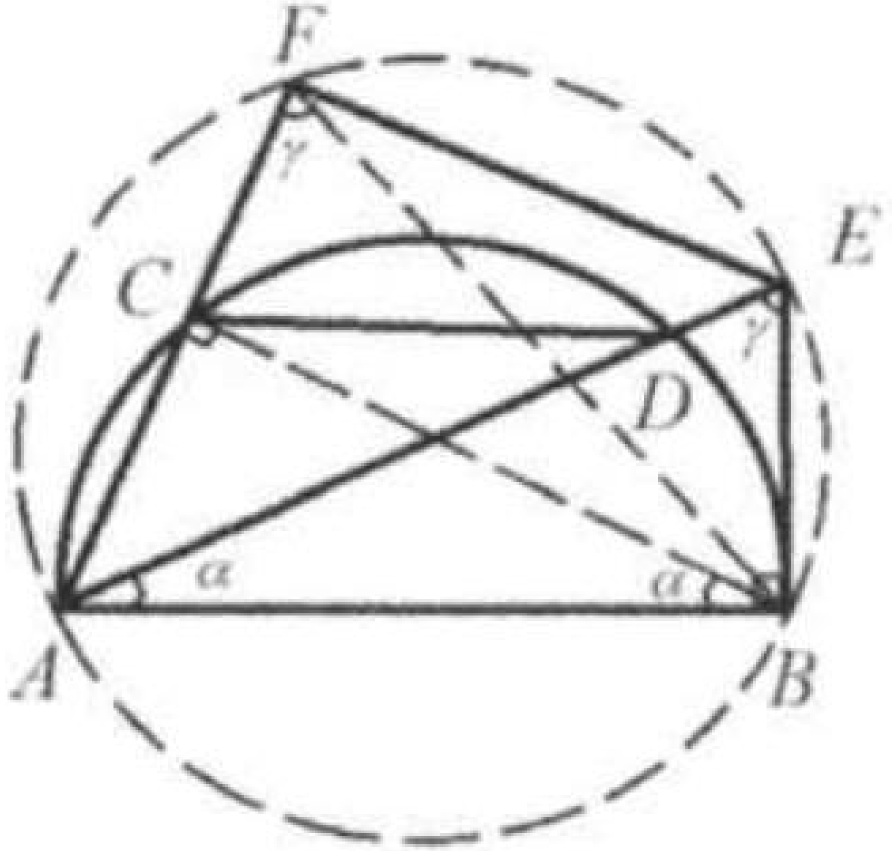
As shown in the figure, AB is the diameter of a semicircle and CD is a chord parallel to AB. Connect AD and extend it to meet BE of the perpendicular of AB at E. Draw $EF \perp AC$ and meets the extension of AC at F, F is the foot of perpendicular. Show that $AC = CF$.



Solution

Since $\angle F = \angle B = 90°$, points A, B, E, and F are concyclic.
$\angle AFB = \angle AEB = \gamma$.

Since $CD//AB$, arcs $AC = BD, \angle DAB = \angle CBA = \alpha$.
In Rt $\triangle ABE, \alpha + \gamma = 90°$.



In Rt $\triangle BCF, \angle FBC + \gamma = 90°$. So $\angle FBC = \alpha$. BC is the perpendicular bisector of AF in $\triangle BAF$. So $AC = CF$.