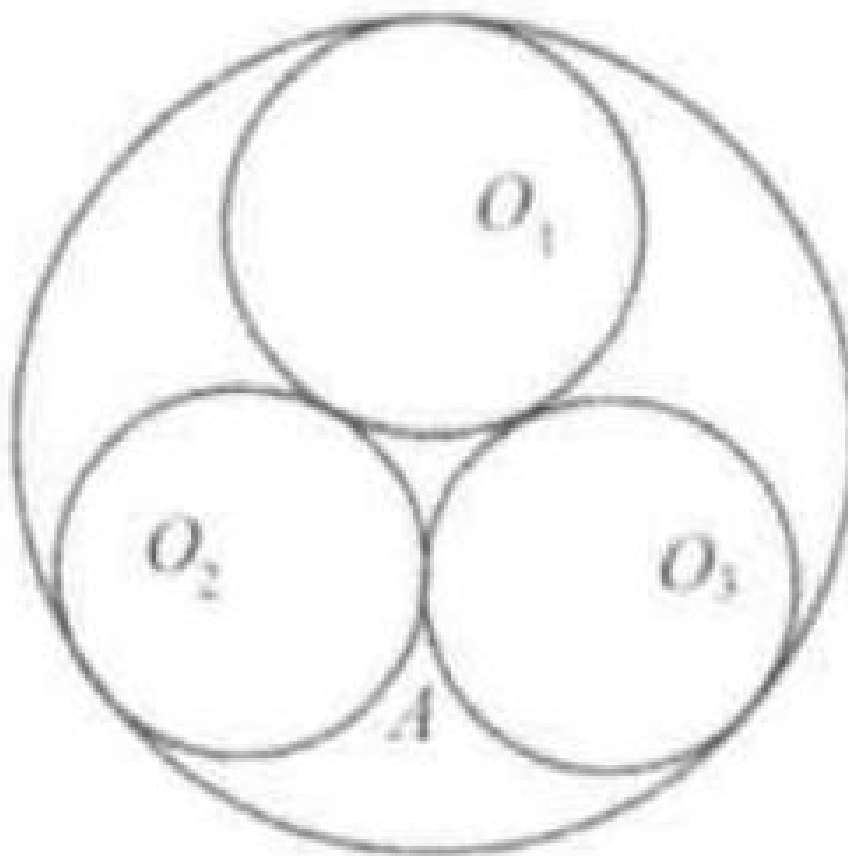# Example 8

(1994 China Middle School Math Contest) Circles O_1, O_2, and O_3, are externally tangent to each other and internally tangent to circle O. Circles O_2, and O_3 are congruent. Circle O_1 has radius 8 . Circle O_2 has radius 5 . What is the radius of circle O ?
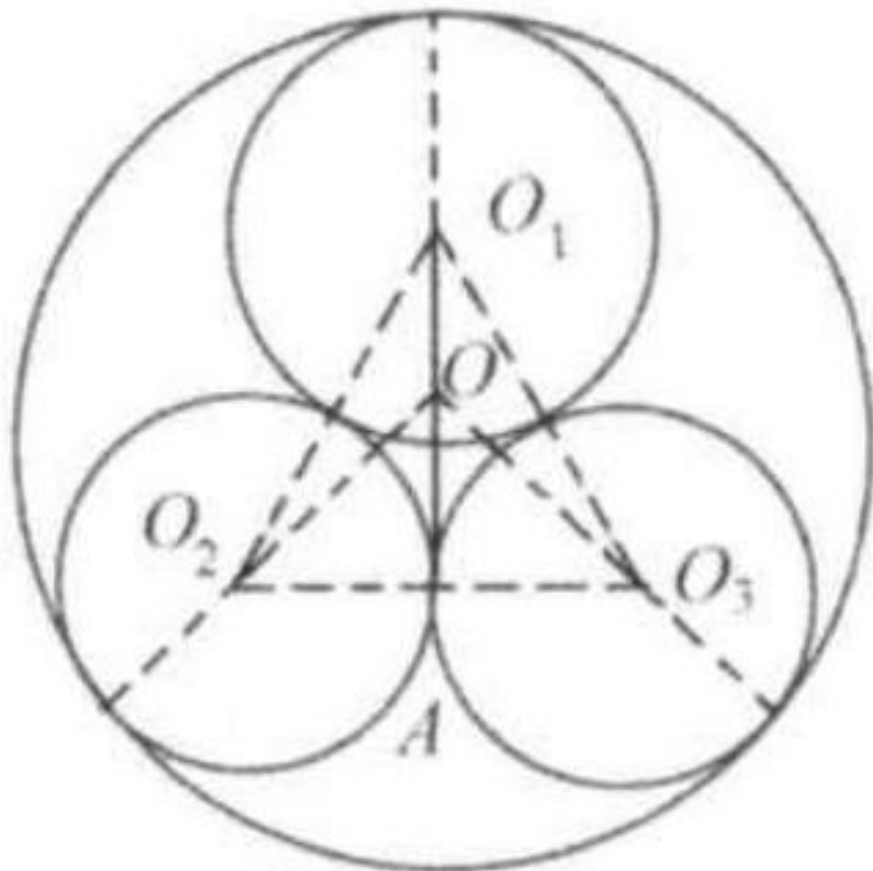

Solution: $\frac{40}{3}$ or $13\frac{1}{3}$.
Connect O_1, O_2, and O_3 as shown. The center of circle O must be on the line segment O_1A. Let r be the radius of circle O.
Applying Pythagorean Theorem to Rt ΔO_1O_2A :

$$O_1A^2 = O_1O_2{}^2 - O_2A^2 \Rightarrow O_1A^2 = (8+5)^2 - 5^2$$
$$\Rightarrow \quad O_1A = \sqrt{(8+5)^2 - 5^2} = 12$$

Applying Pythagorean Theorem to Rt $\triangle OO_2\,A$:
$$O_2A^2 = OA^2 + O_2A^2 \Rightarrow \quad (r-5)^2 = (12+8-r)^2 + 5^2$$
$$\Rightarrow \quad r = \tfrac{40}{3} = 13\tfrac{1}{3}.$$