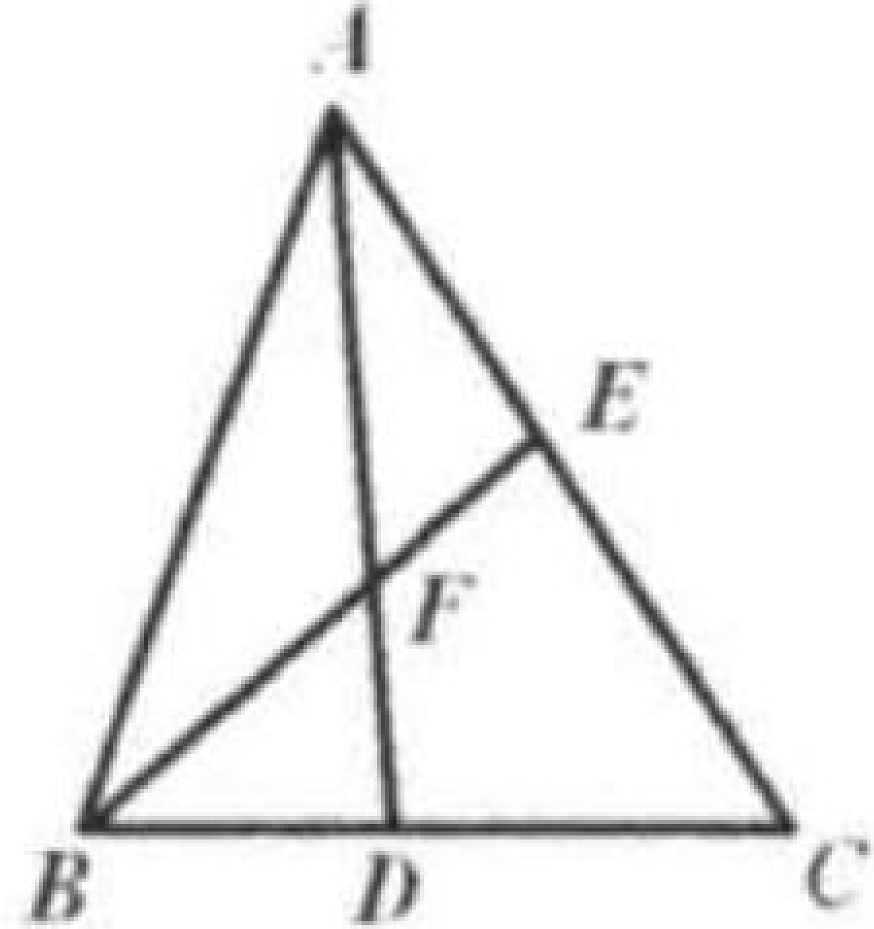
Example 5

Point E is selected on side AC of triangle ABC in such a way that $AE : EC = 3 : 4$ and point D is selected on side BC so that $BD : DC = 2 : 3$. The point of intersection of AD and BE is F. Then $\frac{AF}{FD} \times \frac{BF}{FE}$ is

(A) $\frac{7}{3}$
(B) $\frac{14}{9}$
(C) $\frac{35}{12}$
(D) $\frac{56}{15}$
(E) $\frac{3}{1}$



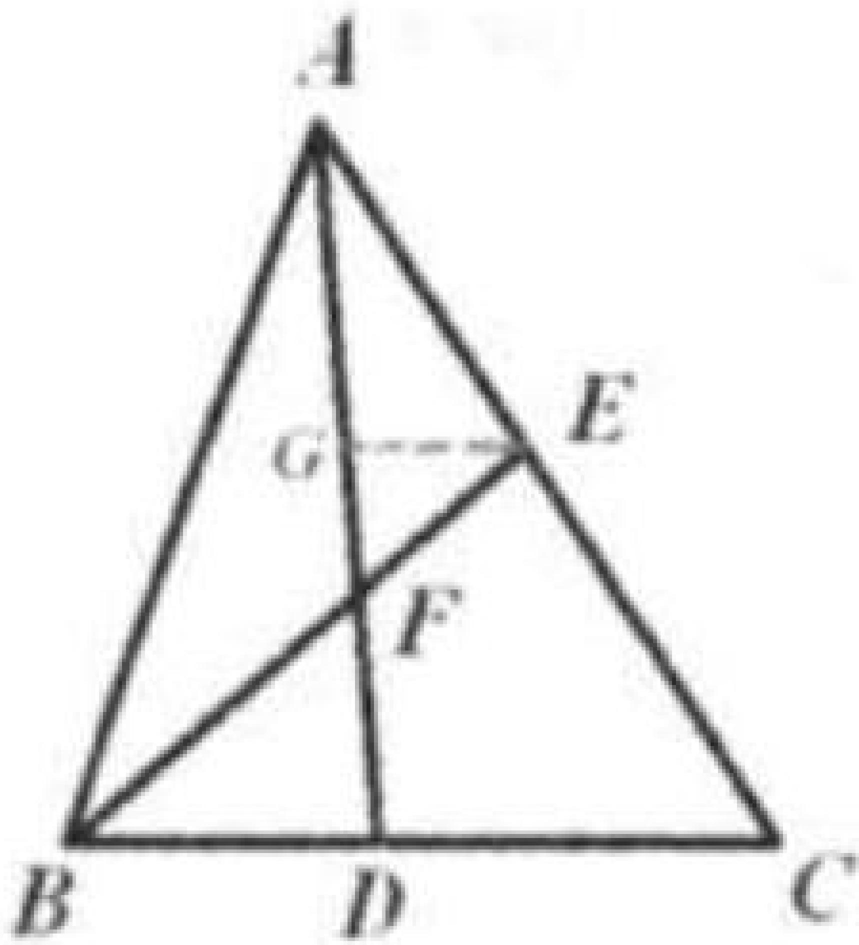
Solution: (C).

Draw $EG//BC$ such that EG meets AD at G.

By similar triangles, $\frac{GE}{BD} = \frac{GE/DC}{BD/DC} = \frac{AE/AC}{BD/DC} = \frac{3/7}{2/3} = \frac{9}{14}$.

Therefore $\frac{FG}{DF} = \frac{FE}{BF} = \frac{GE}{BD} = \frac{9}{14}$.

We also have $\frac{DG}{DF} = \frac{23}{14}, \frac{AG}{AD} = \frac{3}{7}$, so $\frac{GD}{AD} = \frac{4}{7}$



$\Rightarrow GD = \frac{4}{7}AD.$

Thus $\frac{AD}{DF} = \frac{\frac{7}{4}GD}{DF} = \frac{7}{4} \cdot \frac{23}{14} = \frac{23}{8}$.

We also know that $\frac{AE}{DF} = \frac{15}{8}$.

Therefore $\frac{AF}{FD} \times \frac{BF}{FE} = \frac{15}{8} \times \frac{14}{9} = \frac{35}{12}$.