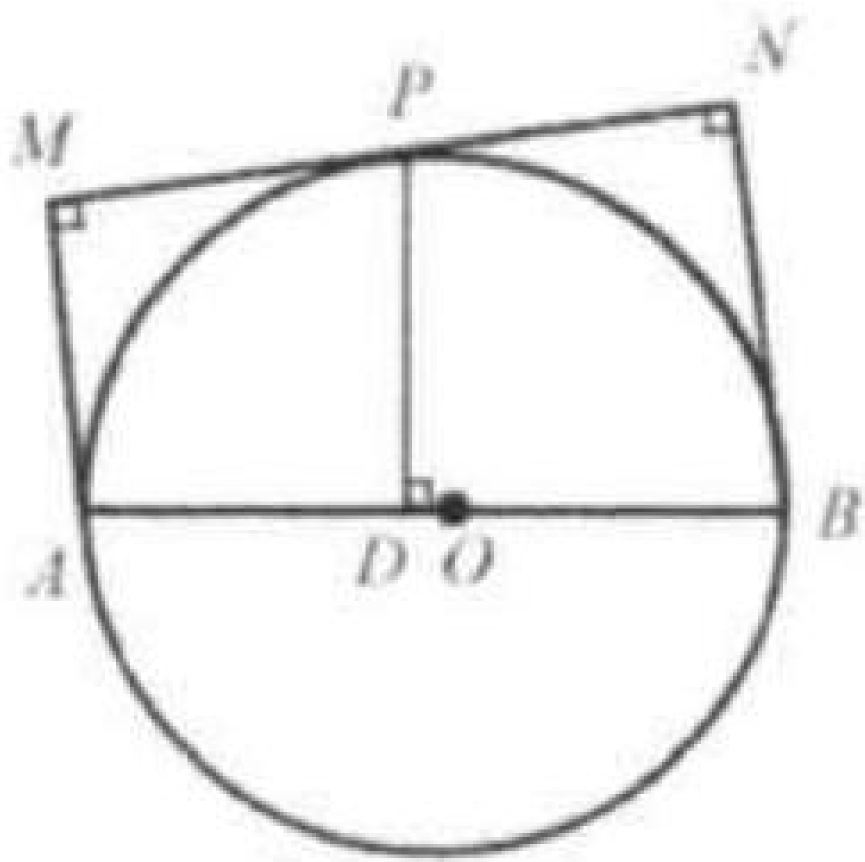
Example 5

In the adjoining figure A, B, P are tangent points on the circumference of the circle $O.AM \perp MN, \angle M = \angle N = 90°$. $PD \perp AB$. Show that $PD^2 = AM \times BN$.

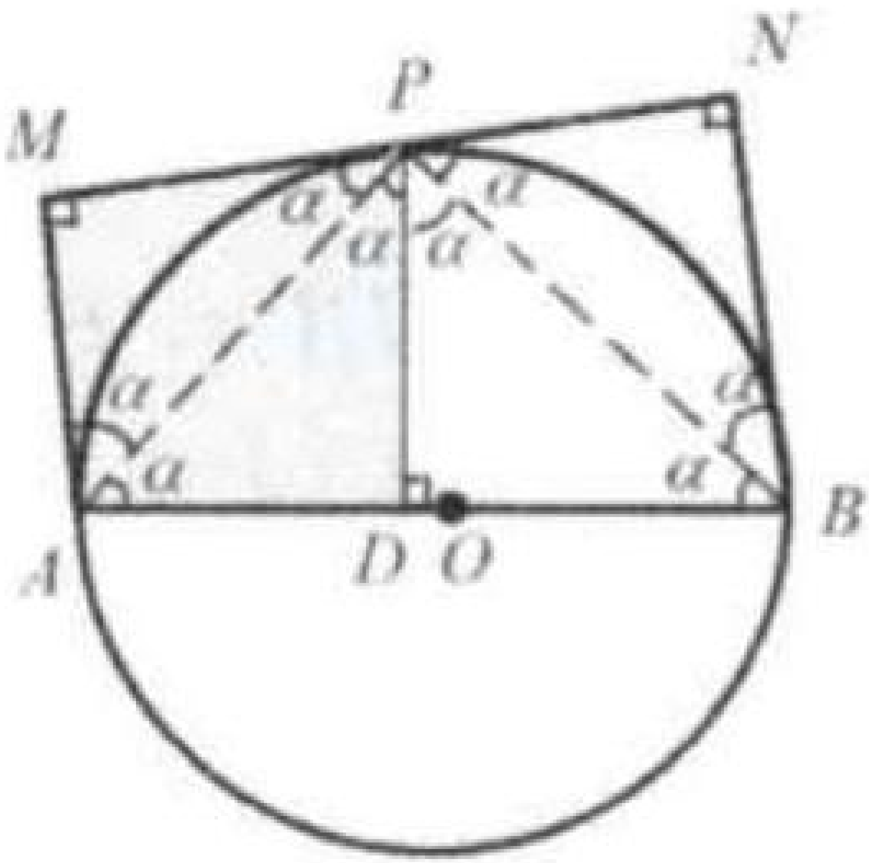
Solution: Connect AP, BP.



Since AB is the diameter, $\angle APB = 90°$.

Thus $PD^2 = AD \times DB$

Since $\angle M = 90°$, and $AM = PM, \angle MAP = \angle MPA = \alpha = 45°$.



Since $\angle N = 90°$, and $BN = PN, \angle NPB = \angle NBP = \alpha = 45°$.
So $\angle BAP = \angle BPN = \alpha = 45°$ (both angles face the same arc PB).
Since $\angle ADP = 90°, \angle DPA = \angle DAP = \alpha = 45°$.
Similarly $\angle DPB = \angle DBP = \alpha = 45°$.

$$\text{So } \triangle AMP \cong \triangle ADP \quad \Rightarrow \quad AM = AD$$

$$\text{So } \triangle BNP \cong \triangle BDP \quad \Rightarrow \quad BN = DB \text{ (3)}$$

Substituting (2) and (3) into (1): $PD^2 = AM \times BN$