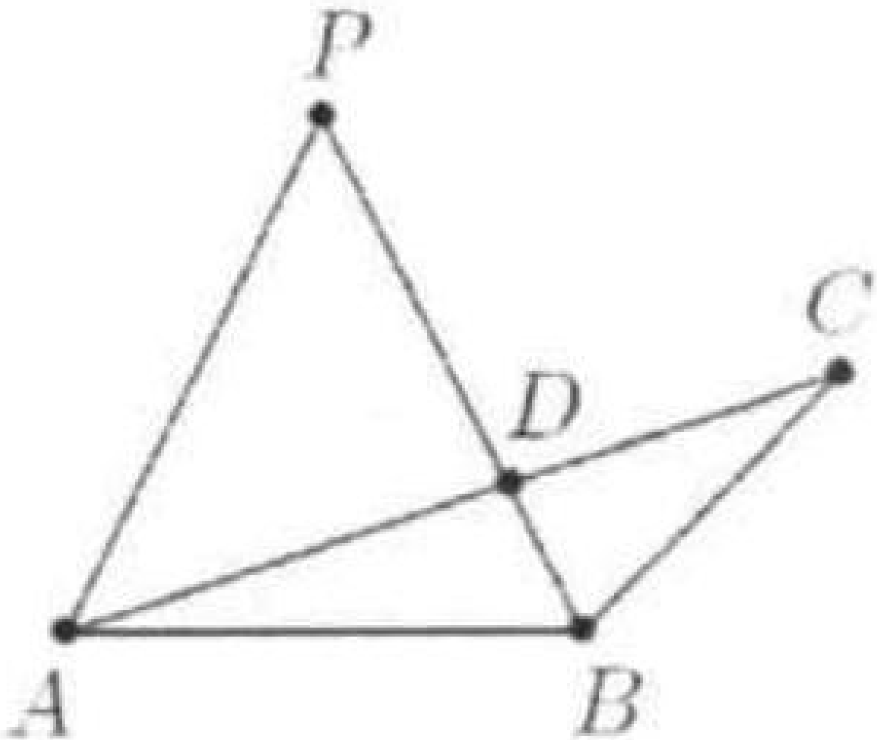
Example 8

(2001 China Middle School Math Contest) As shown in the figure, triangle ABC and point P in the same plane are given. $PA = PB. \angle APB = 2\angle ACB$. AC intersects BP at point D. If $PB = 4$ and $PD = 3$, then $AD \cdot CD =$
(A) 6
(B) 7
(C) 12
(D) 16

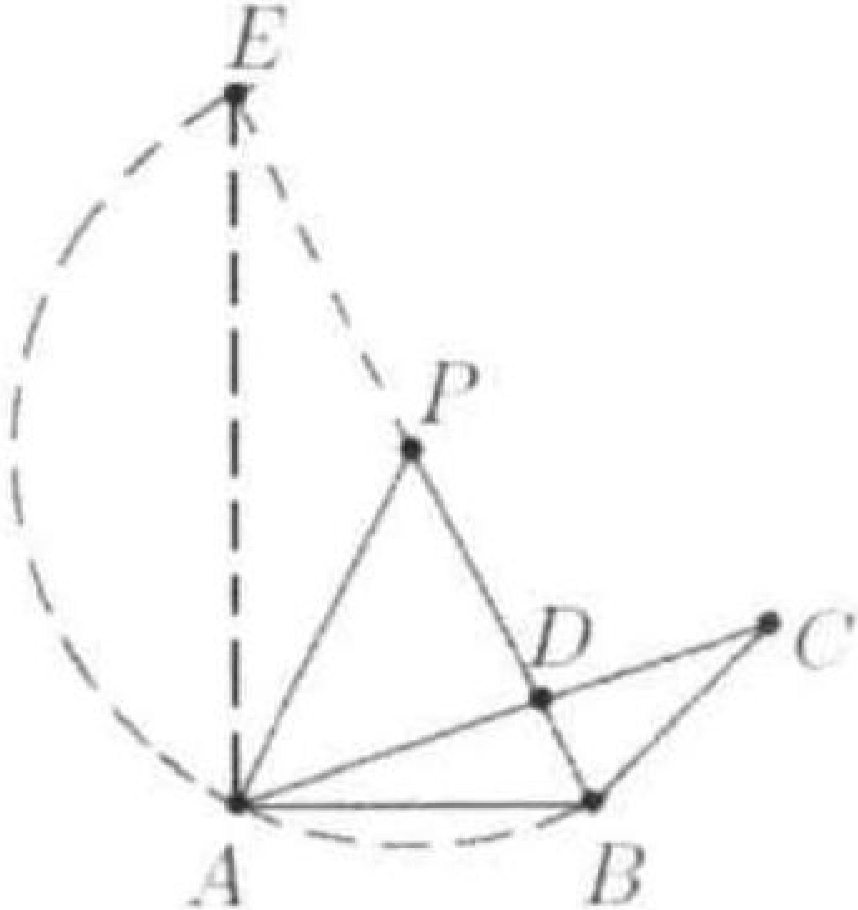
Solution: (B).



Extend BP to E such that $PE = PB$. Since $PA = PB = PE$, points A, B, and E are concyclic. Construct this semicircle with center P as shown in the figure to the right.

So we have $\angle AEB = \frac{1}{2}\angle APB = \angle ACB$. We also know that $\angle ADE = \angle BDC$

(vertical angles). Therefore $\triangle AED \sim \triangle DCB$.
$\frac{AD}{BD} = \frac{ED}{DC} \Rightarrow AD \cdot CD = ED \cdot BD = (PE + PD)(PB-$



$PD) = (4+3)(4-3) = 7.$