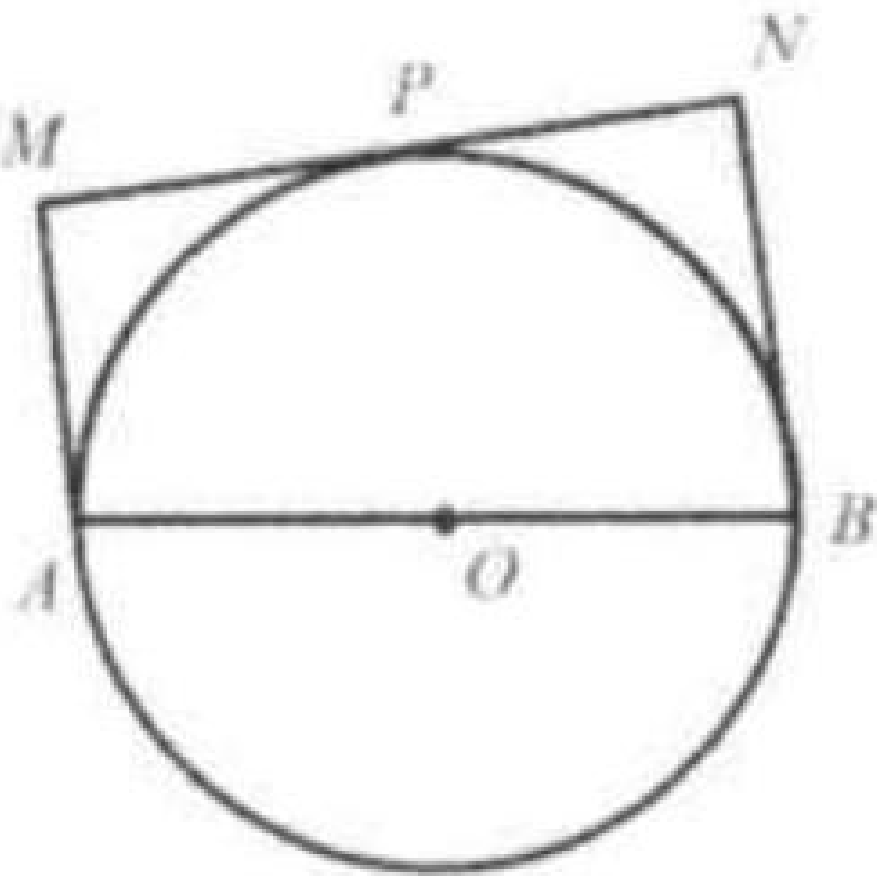


Problem 5

Problem

In the adjoining figure AM and BN are parallel to each other and are tangent to the circle O , with A and B the points of tangency. MPN is a third tangent with P as point of tangency. Show that the radius of the circle is

$$r = \sqrt{AM \times BN}.$$



Solution

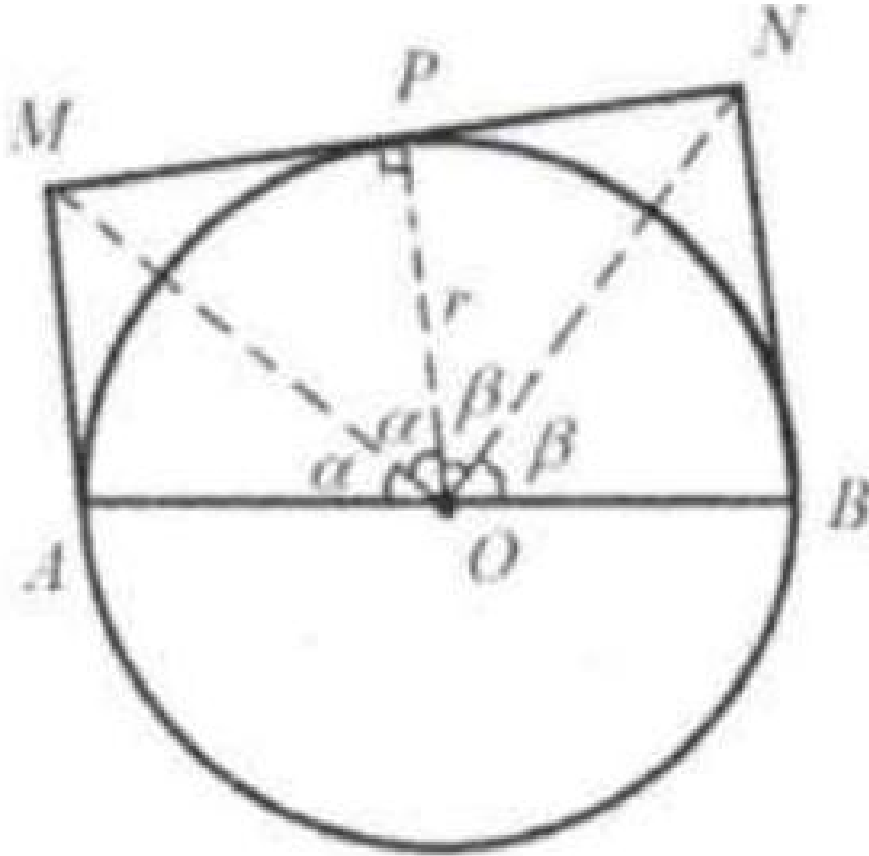
Connect OM , OP , ON .

Since A and P are two tangent points, $AM = PM$.

$OA = OP = r$. $OM = OM$.

So $\triangle AOM \cong \triangle POM$. Then $\angle AOM = \angle POM = \alpha$.

Similarly, $\angle PON = \angle BON = \beta$.



Since $2\alpha + 2\beta = 180^\circ$, $\alpha + \beta = \angle MON = 90^\circ$,

Thus $PO^2 = MP \times PN = AM \times BN$, or

$$r^2 = AM \times BN \Rightarrow r = \sqrt{AM \times BN}$$