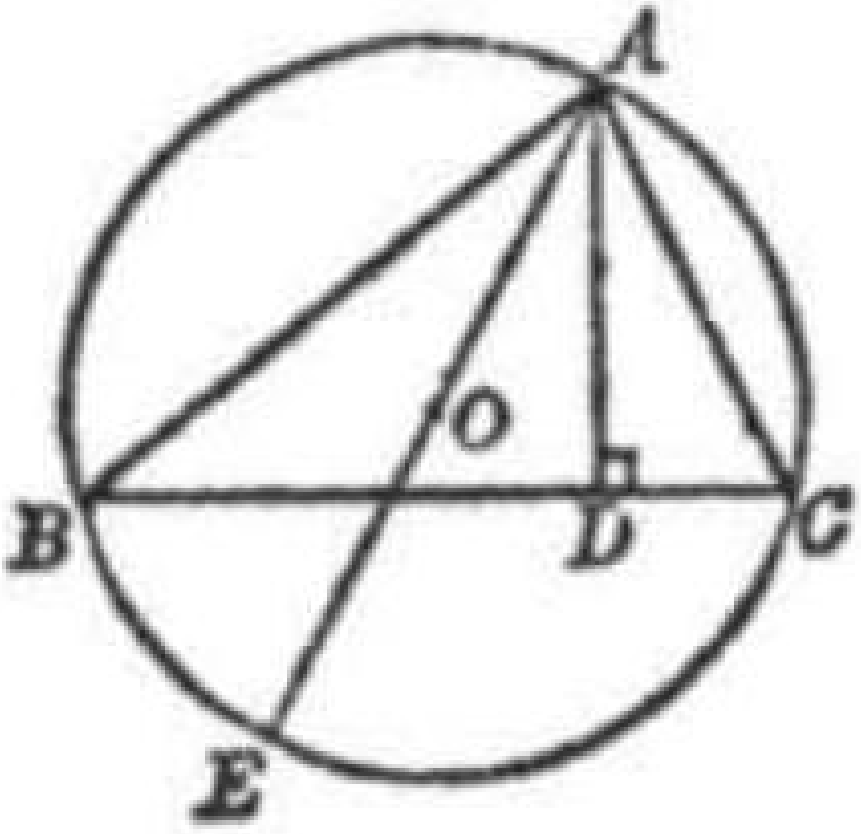
Problem 5

Problem

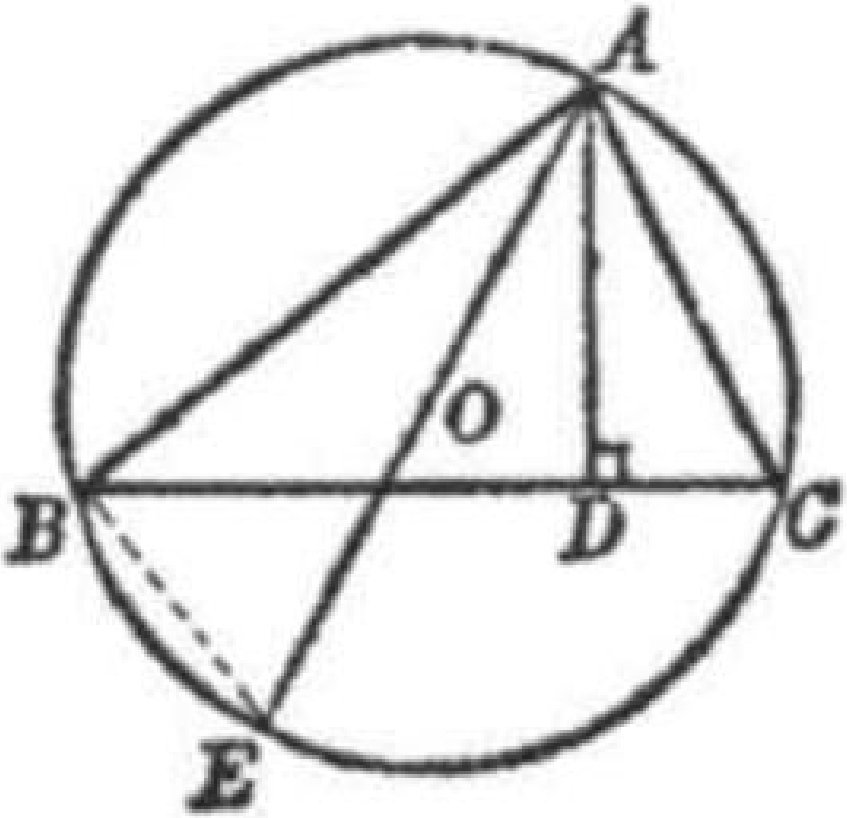
In $\triangle ABC, AE$ is the diameter of the circumcircle. AD is the altitude on BC.
Show that $AB \times AC = AD \times AE$.



Solution

Method 1:

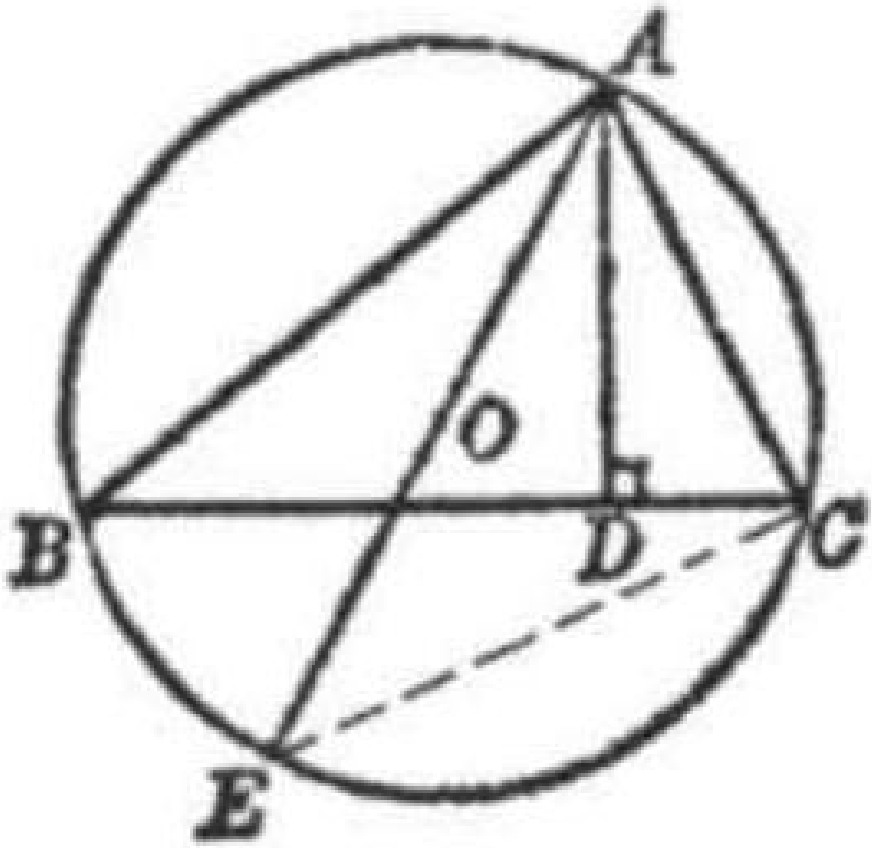
Connect BE. Triangle ABE is a right triangle.
Since $\angle ACB$ and $\angle AEB$ face the same arc, $\angle ACB = \angle AEB$.
We also know that $\angle ABE = \angle ADC = 90°$.
Therefore $\triangle ACD$ and $\triangle ABE$ are similar.



$$\frac{AB}{AE} = \frac{AD}{AC} \quad \Rightarrow \quad AB \times AC = AD \times AE.$$

Method 2:
Connect EC. Triangle AEC is a right triangle.
Since $\angle ABC$ and $\angle AEC$ face the same arc, $\angle ABC = \angle AEC$.
We also know that $\angle ADB = \angle ACE = 90°$. Therefore $\triangle AEC$ and $\triangle ABD$ are similar.

$$\frac{AB}{AD} = \frac{AE}{AC} \quad \Rightarrow \quad AB \times AC = AD \times AE.$$


A
O
B
D
C
E