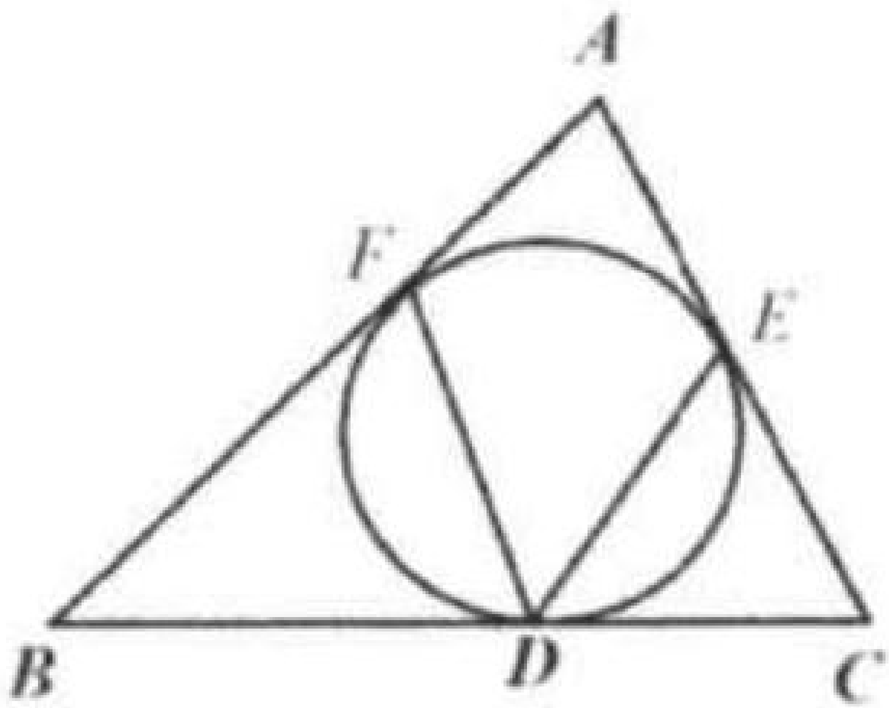
Problem 1

Problem

A circle is inscribed in triangle ABC. The tangent points are D, E, F as shown. Show that $\angle FDE = 90^\circ - \frac{1}{2}\angle A$.



Solution

Connect EF.

Since both AF and AE are tangent to circle $O, AF = AE$ and $\angle AFE = \angle AEF = \alpha$.

Thus $\angle FDE = \alpha(\angle FDE, \angle AFE,$ and $\angle AEF$ face the same arc FE).

In $\triangle AEF, \angle A = 180^\circ - 2\alpha \quad \Rightarrow \quad 2\alpha = 180^\circ - \angle A$

That is $\alpha = \angle FDE = 90^\circ - \frac{1}{2}\angle A$.