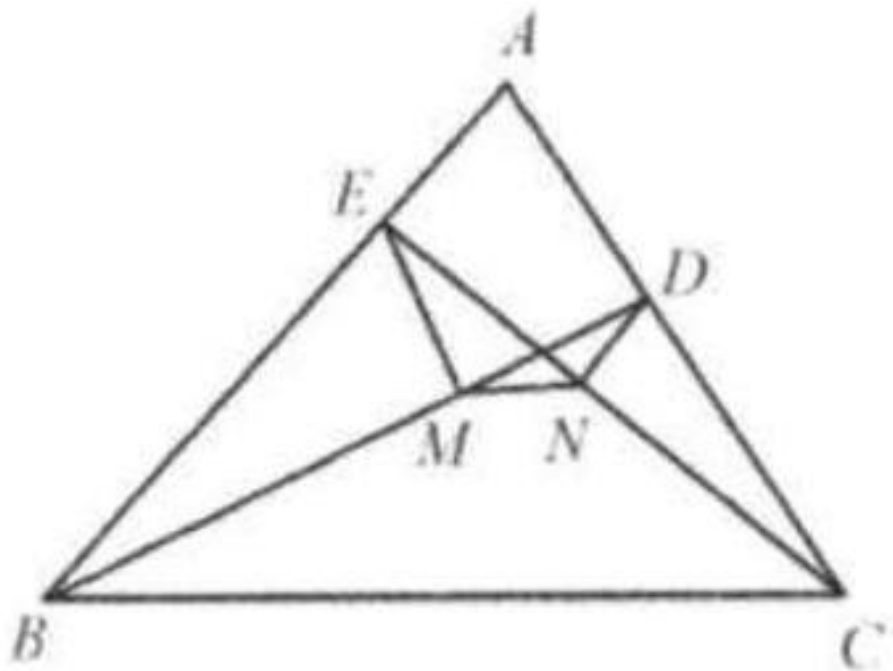


## Problem

As shown in the figure,  $BD, CE$  are the altitudes on  $AC, AB$  of  $\triangle ABC$ , respectively.  $EM \perp BD$  at  $M, DN \perp CE$  at  $N$ . Show that  $MN \parallel BC$ .



## Solution

$$\angle BEC = \angle BDC = 90^\circ.$$

Thus points  $B, C, D$ , and  $E$  are concyclic.

Draw the circle as shown.

So  $\angle CED = \angle CBD = \alpha$  (they face the same arc  $CD$ ).

$$\angle EMD = \angle END = 90^\circ.$$

Thus points  $E, M, N$ , and  $D$  are concyclic.

So  $\angle NED = \angle NMD = \alpha$  (they face the same chord  $DN$ ).

Since  $\angle CBD = \angle NMD = \alpha, MN \parallel BC$ .

