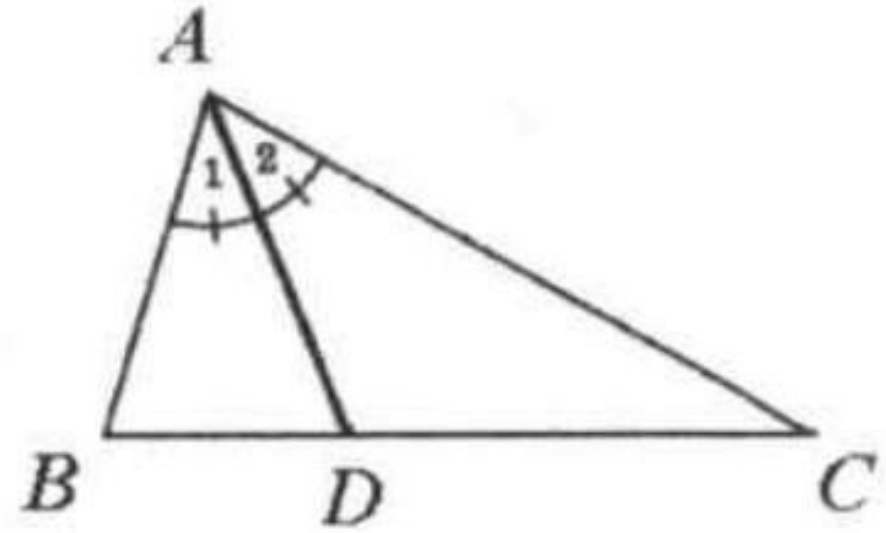
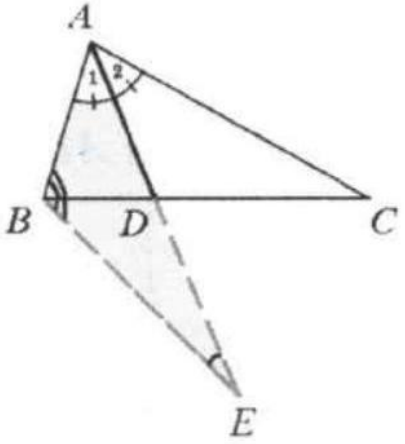
Example 12

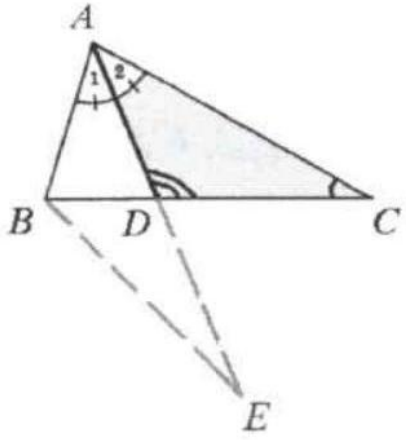
Prove the angle bisector length formula
$AD^2 = AB \times AC - BD \times DC$

Proof: Extend AD to E such that $\angle ABE = \angle ADC$.

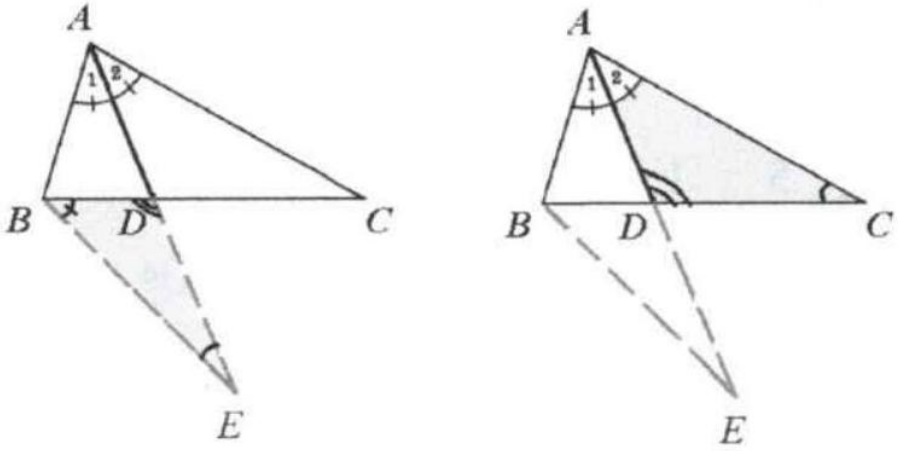


We see that $\triangle ABE \sim \triangle ADC$. Thus $\frac{AB}{AD} = \frac{AE}{AC} \Rightarrow AB \times AC = AD \times AE$





We see that $\triangle BDE \sim \triangle ADC$. Thus $\frac{BD}{AD} = \frac{DE}{DC} \Rightarrow BD \times DC = AD \times DE$



(1) - (2): $AB \times AC - BD \times DC = AD(AE - DE) = AD^2$. QED.