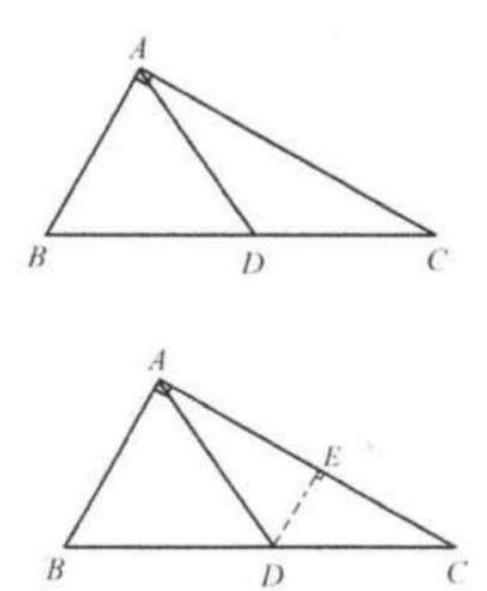
Example 15

The measures of the sides of a right triangle are 60, 80, and 100. Find the measure of a line segment, drawn from the vertex of the right angle to the hypotenuse, that divides the triangle into two triangles of equal perimeters.

Solution: Let AB=60, AC=80, and BC=100. If $\triangle ABD$ is to have the same perimeter as $\triangle ACD$, then AB+BD must equal AC+DC, since both triangles share AD; that is, 60+BD=80+100-BD. Therefore, BD=60 and DC=40.

Draw DE perpendicular to AC. Right $\triangle EDC \sim \text{right } \triangle ABC$; therefore, $\frac{ED}{AB} = \frac{DC}{BC}$.



By substituting the appropriate values, we have $\frac{ED}{60} = \frac{40}{100}$, and ED = 24. By the Pythagorean Theorem, for $\triangle EDC$, we find EC = 32; then, by subtraction, AE = 48. Again, using the Pythagorean Theorem, in $\triangle AED, AD = 24\sqrt{5}$.