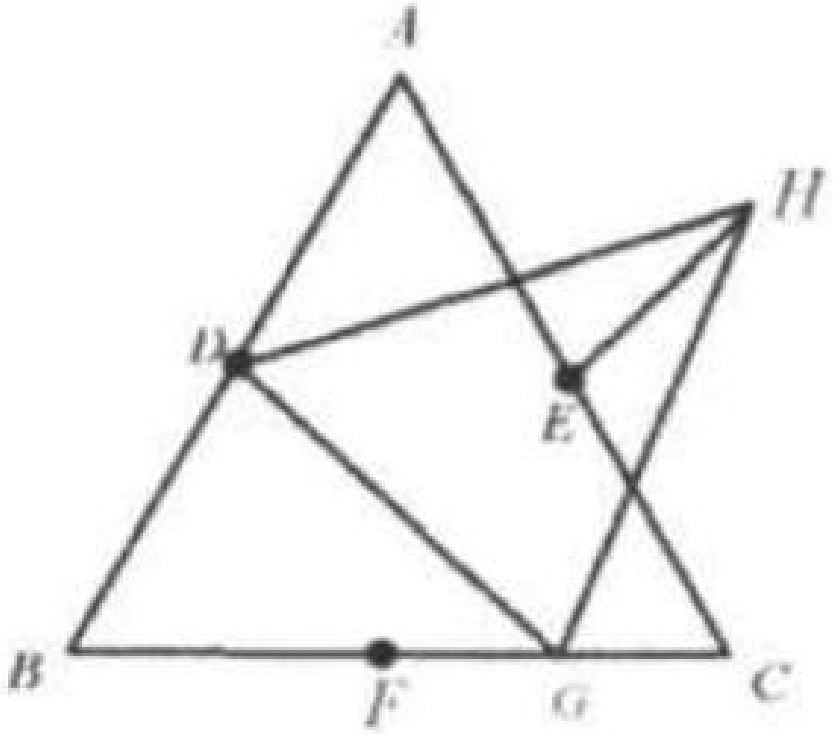
Problem 8

Problem

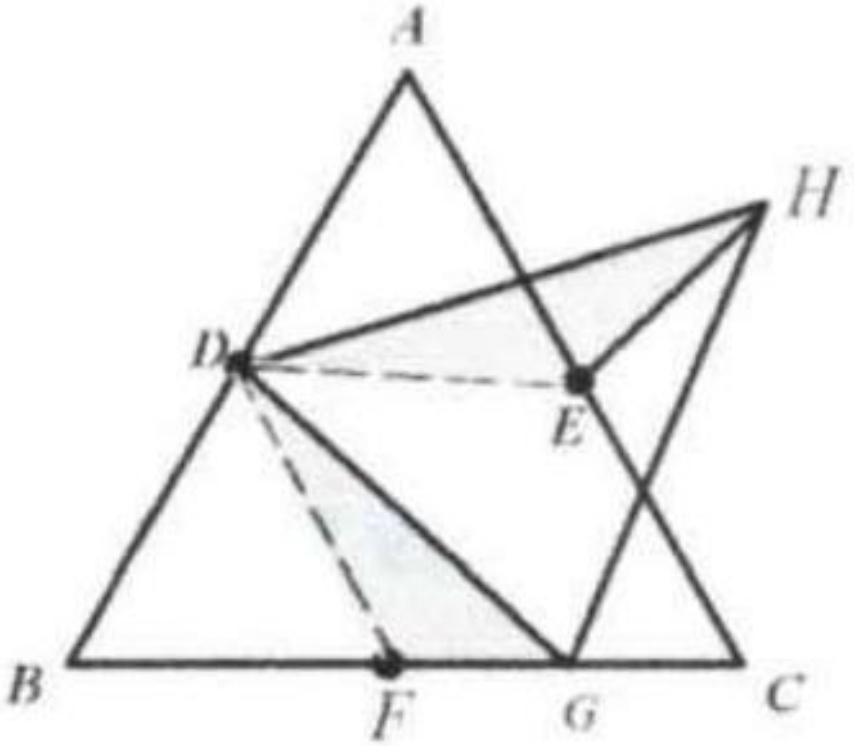
In equilateral $\triangle ABC$, points D, E, F are the midpoints of AB, AC, BC, respectively. G is a point of FC. Show that $FG = EH$ if $\triangle DGH$ is an equilateral triangle as well.



Solution

Connect DE, DF.
Since $\triangle ABC$ is an equilateral triangle, $DE = \frac{1}{2}BC = \frac{1}{2}AC = DF$.
$DE//BC, DF//AC$.

$\angle EDF = \angle ACB = 60°$.



Since $\triangle DGH$ is an equilateral triangle, $DH = DG, \angle HDG = 60°$.
Thus $\angle FDG = 60° - \angle GDE = \angle EDH$.
So $\triangle FDG \cong \triangle EDH$. $FG = EH$.