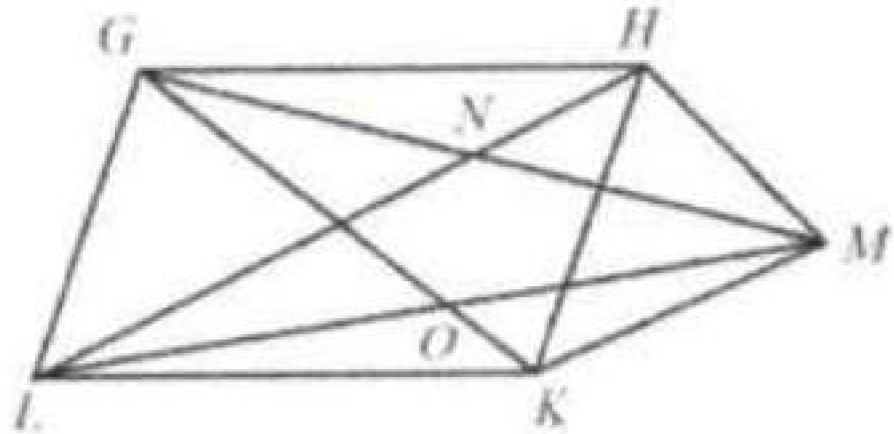
Problem 4

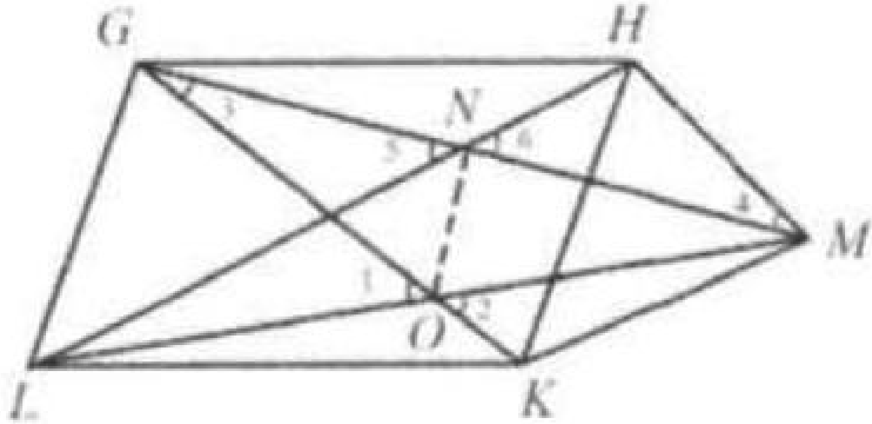
Problem

(1914 Phillips Academy Prize Exam) $GHKL$ is a parallelogram; HM and KM are drawn parallel to the diagonals; GM and LM cut the diagonals at N and O. Prove that NO is equal to one half of GL.



Solution

Method 1:
$\angle 1 = \angle 2$.
$\angle LTO = \angle MKO$ since $LH//MK$, and since $THMK$ is a parallelogram, $HT = MK$. Since $GHKL$ is a parallelogram, the diagonals bisect each other so $HT = LT$. Therefore, $LT = MK \Rightarrow \triangle LTO \cong \triangle MKO$ by AAS, giving $LO = MO$, so O



is the midpoint of LM.

Likewise, $\triangle NTG \cong \triangle NHM$ by AAS, since $\angle 3 = \angle 4, \angle 5 = \angle 6$, and $HM = K = GT$. Thus N is the midpoint of GM.

Since NO is the midline of $\triangle MGL, NO = \frac{1}{2}GL$ and $NO//GL$.

Method 2:

Lable the point of ntersection of LH and GK be T.

Since $TN//KM$, and T is the midpoint of LH, N is the midpoint of GM.

Since $TO//HM$, and T is the midpoint of LH, O is the midpoint of LM.

Thus NO is the midline of $\Delta MGL, NO = \frac{1}{2}GL$ and $NO//GL$.