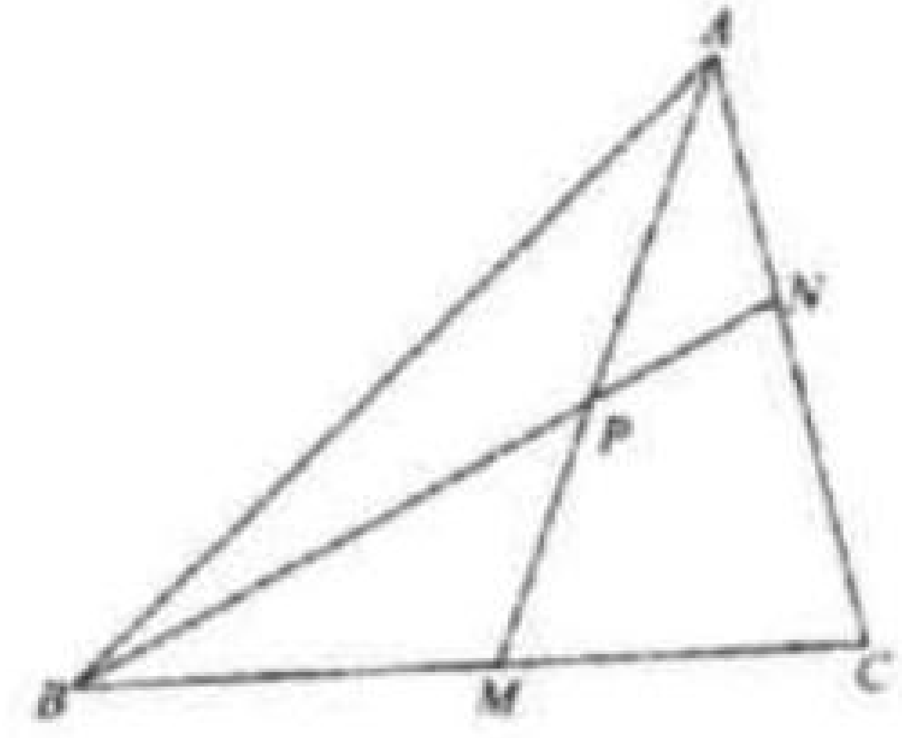
Problem

As shown in the figure, in triangle ABC, M is the midpoint of BC. $AN = \frac{1}{3}AC$. Connect BN and denote the point where BN meets AM to be P. Show that $BP = 3PN$.



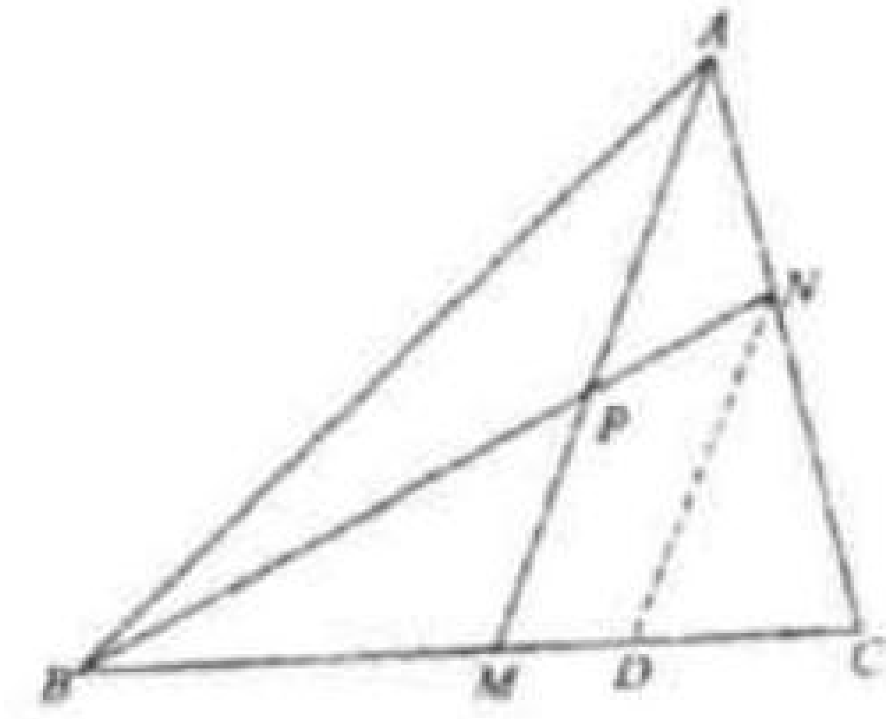
Solution

This problem is the same as Example 4. Here, we show two new, different ways to solve it.

Method 1:

Draw a line through N parallel to AM to meet CM at D.

Since $\frac{AN}{AC} = \frac{1}{3}$, $\frac{MD}{MC} = \frac{1}{3}$.



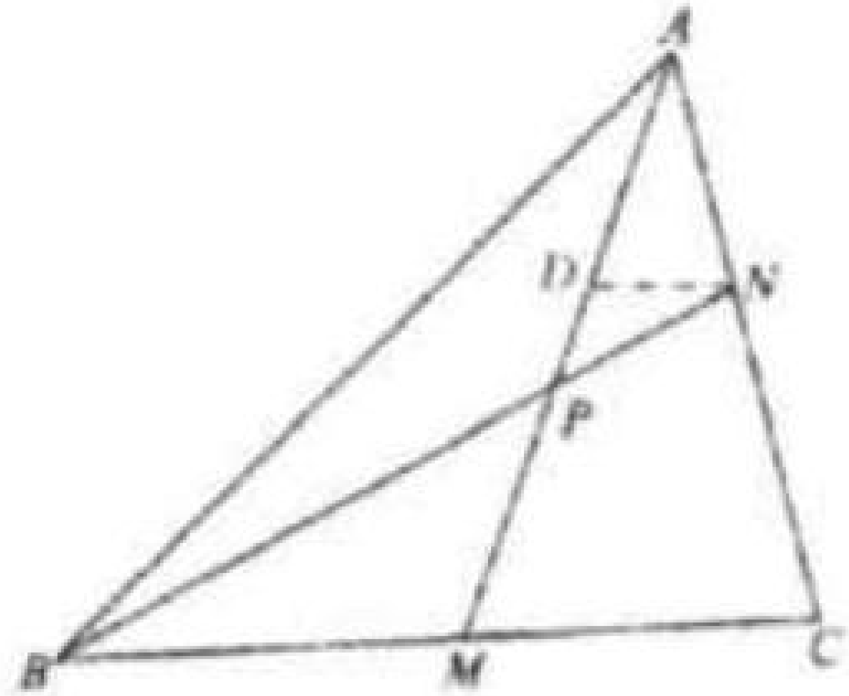
We know that $BM = MC$, so $\frac{MD}{BM} = \frac{1}{3}$.

We also know that $MP//DN$, so $\frac{PN}{BP} = \frac{1}{3}$

$\Rightarrow \quad BP = 3PN$.

Method 2:

Draw a line through N parallel to BC to meet AM at D.



Since $\triangle ADN \sim \triangle AMC$, $\frac{AN}{AC} = \frac{1}{3}$.
Since $MB = MC, 3DN = MC = MB$.
Since $\triangle PDN \sim \triangle PMA$, $\frac{DN}{MC} = \frac{PN}{PB} = \frac{1}{3}$
$\Rightarrow \quad BP = 3PN$.