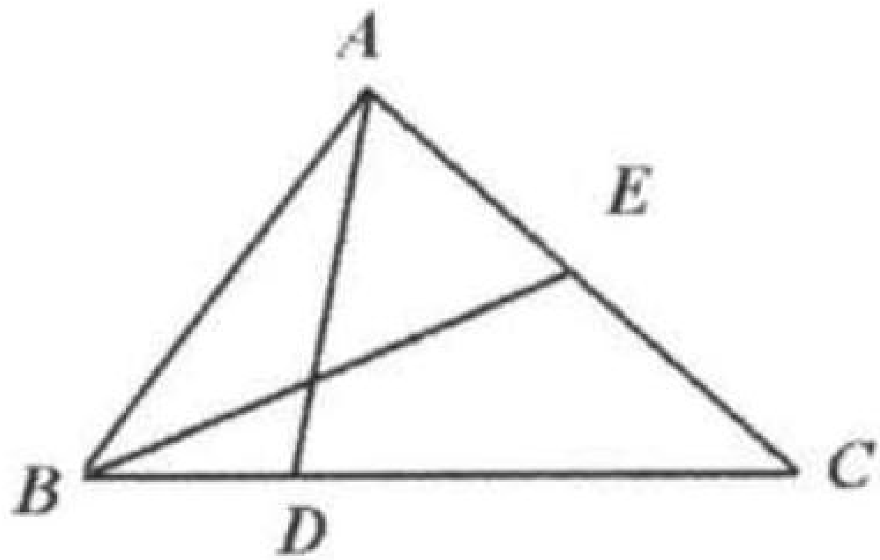
Example 3

In $\triangle ABC$, point E is the midpoint of AC. D is on BC and $BD = 1/3\ BC$. Show that AD bisects BE.

Solution: Method 1:

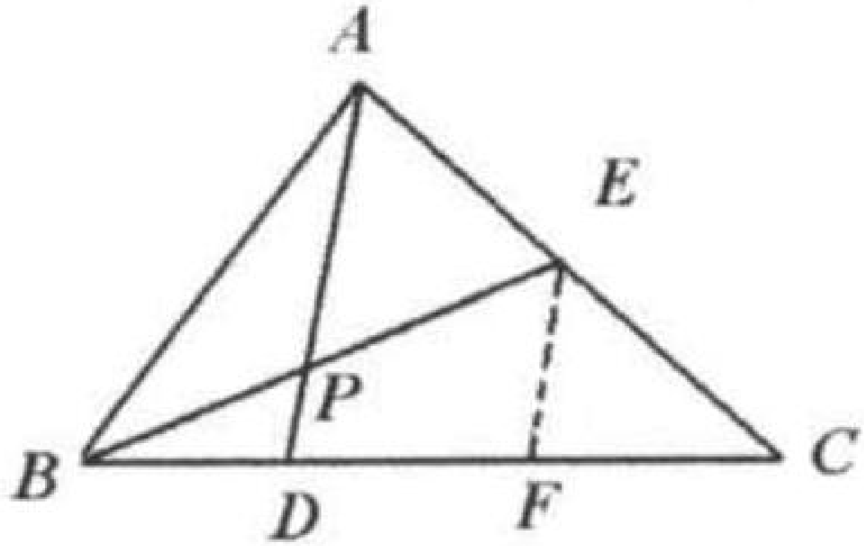
Let the point of intersection of BE and AD be P.



Connect EF where F is the midpoint of DC.

Since E is the midpoint of AC and F is the midpoint of $DC, AD//EF$.

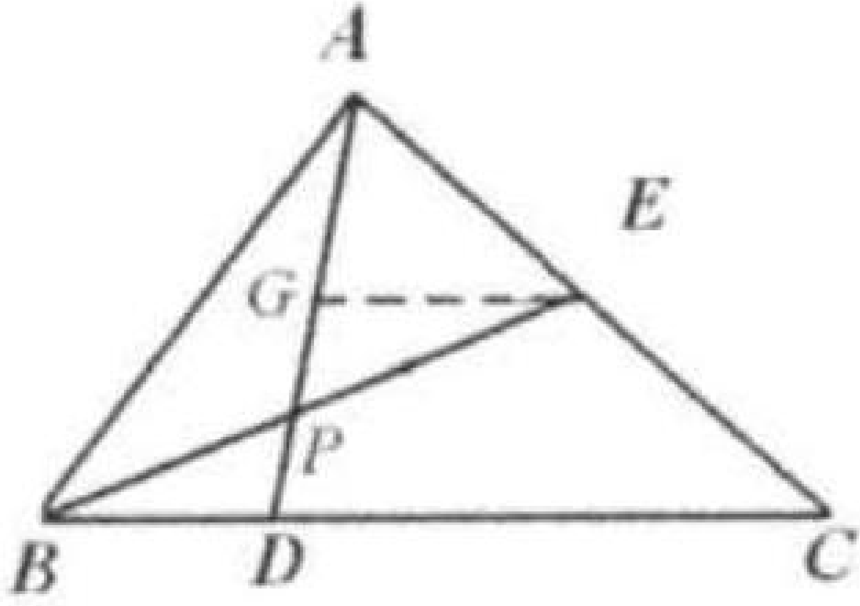
Therefore $EF = \frac{1}{2}AD$.



In $\triangle BEF, PD//EF, BD = DF$. Therefore PD bisects BE , or in other words, AD bisects BE.

Method 2:

Draw $EG//DC$ to meet AD at G.



Since point E is the midpoint of AC, by Theorem 2.2, G is the midpoint of AD and $EG = \frac{1}{2}DC$.

We know that $BD = 1/3BC$. So $DC = \frac{2}{3}BC$ and $EG = \frac{1}{2}DC = \frac{1}{2} \times \frac{2}{3}BC = \frac{1}{3}BC = BD$.

Thus $\triangle EGP \cong \triangle BDP(EG = BD, \angle GEP = \angle DBP$ and $\angle GEP = \angle DBP)$.

So $BP = PE$. In other words, AD bisects BE.