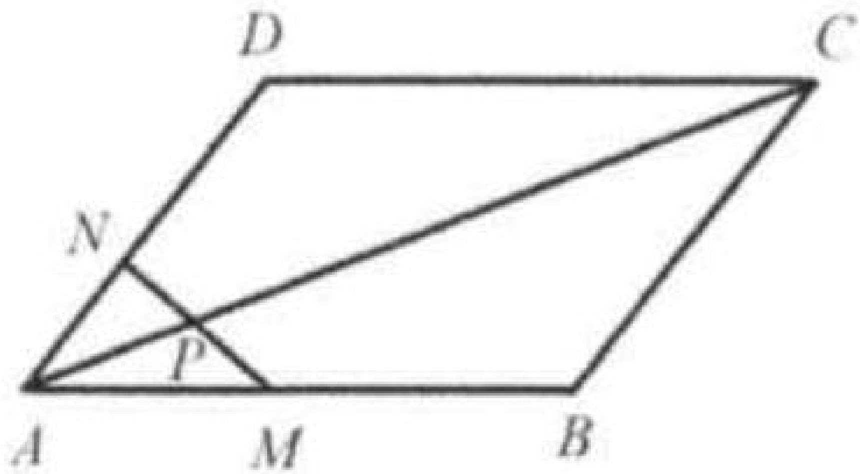
Problem

In parallelogram $ABCD$, point M is on AB so that $\frac{AM}{MB} = \frac{17}{1000}$, and point N is on AD so that $\frac{AN}{ND} = \frac{17}{2009}$. Let P be the point of intersection of AC and MN. Find $\frac{PC}{PA}$.



Solution

178.

Extend NM through M to E and to meet the extension of CB at E. We label the line segments as shown in the figures.

We know that $AD//CE$. So $\triangle AMN \sim \triangle BME$ (Figure 1). $\frac{AN}{BE} = \frac{AM}{MB} \quad \Rightarrow$

$$\frac{17y}{BE} = \frac{17x}{1000x} \Rightarrow \quad BE = 1000y.$$

We know that $AN//CE$. So $\triangle APN \sim \triangle CPE$ (Figure 2).

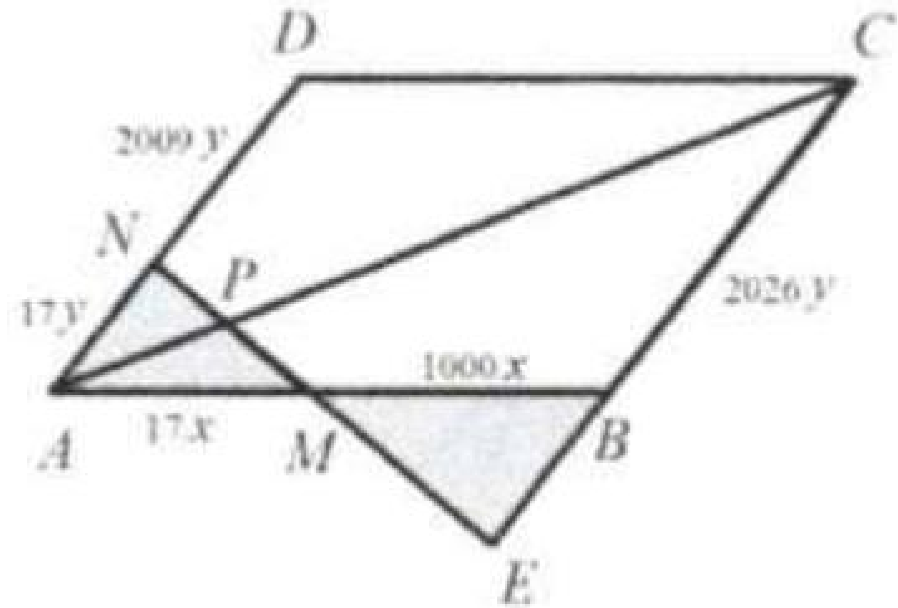


Figure 1

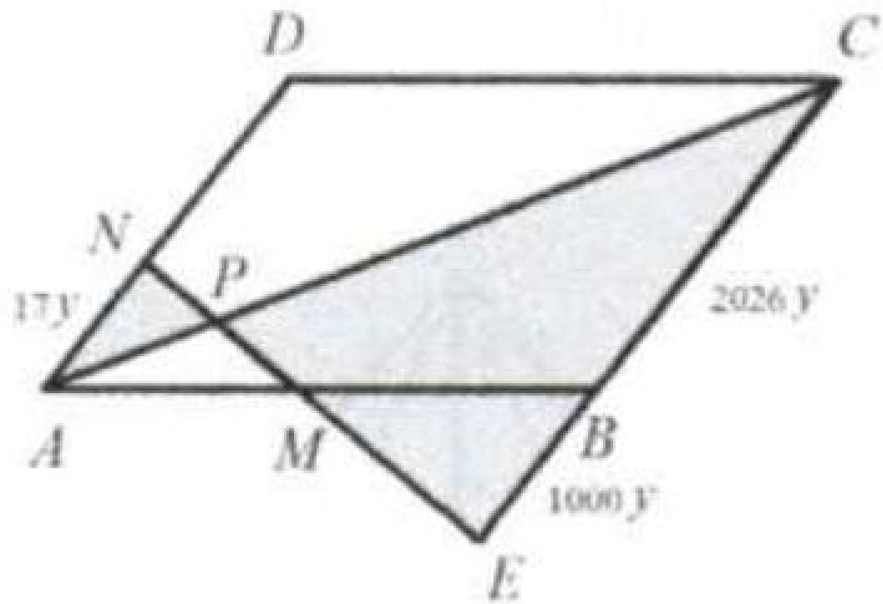


Figure 2

$\frac{PC}{PA} = \frac{CE}{AN} = \frac{2026y+1000y}{17y} = 178.$