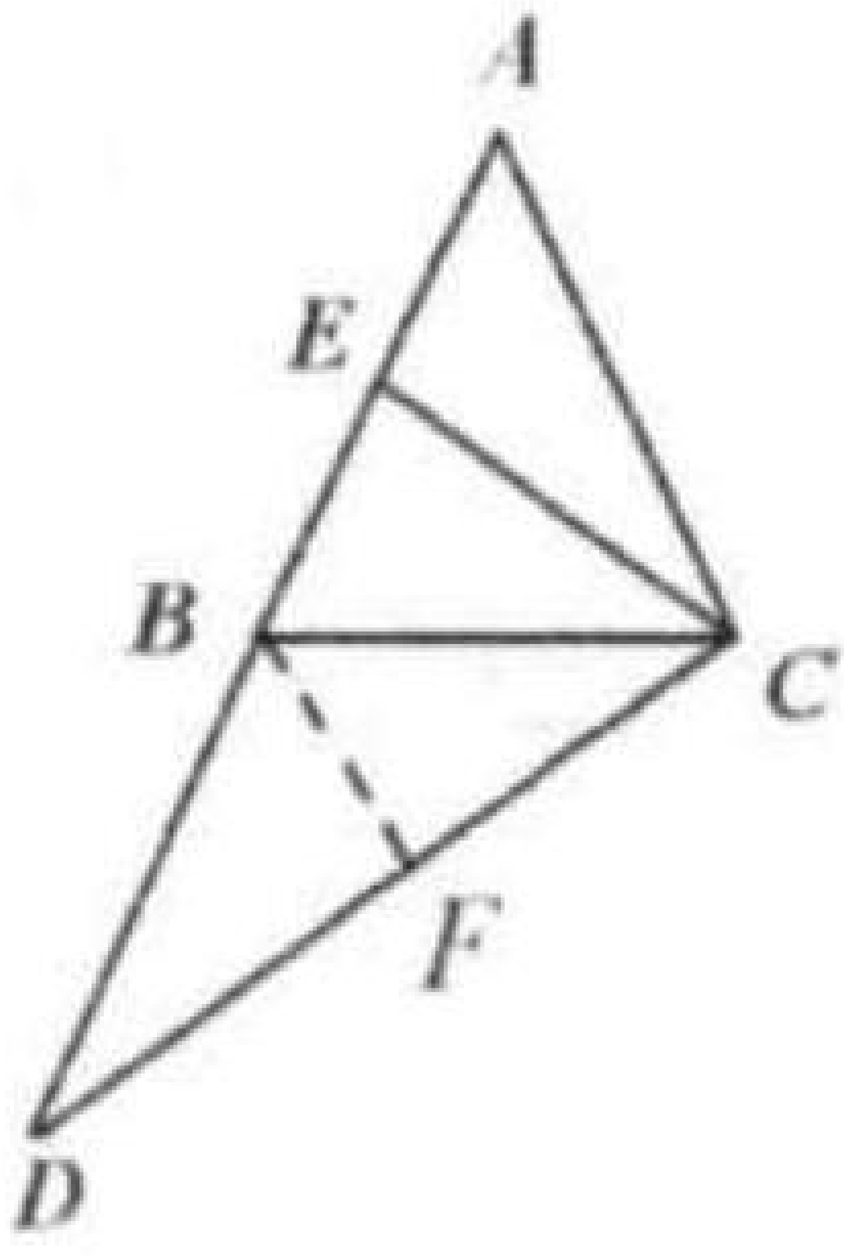
Example 8

Given $\triangle ABC, AB = AC, E$ is the midpoint of AB. Extend AB to D such that $BD = BA$. Prove: $CD = 2CE$.

Solution: Method 1:

Take F, the midpoint of CD. Connect B.

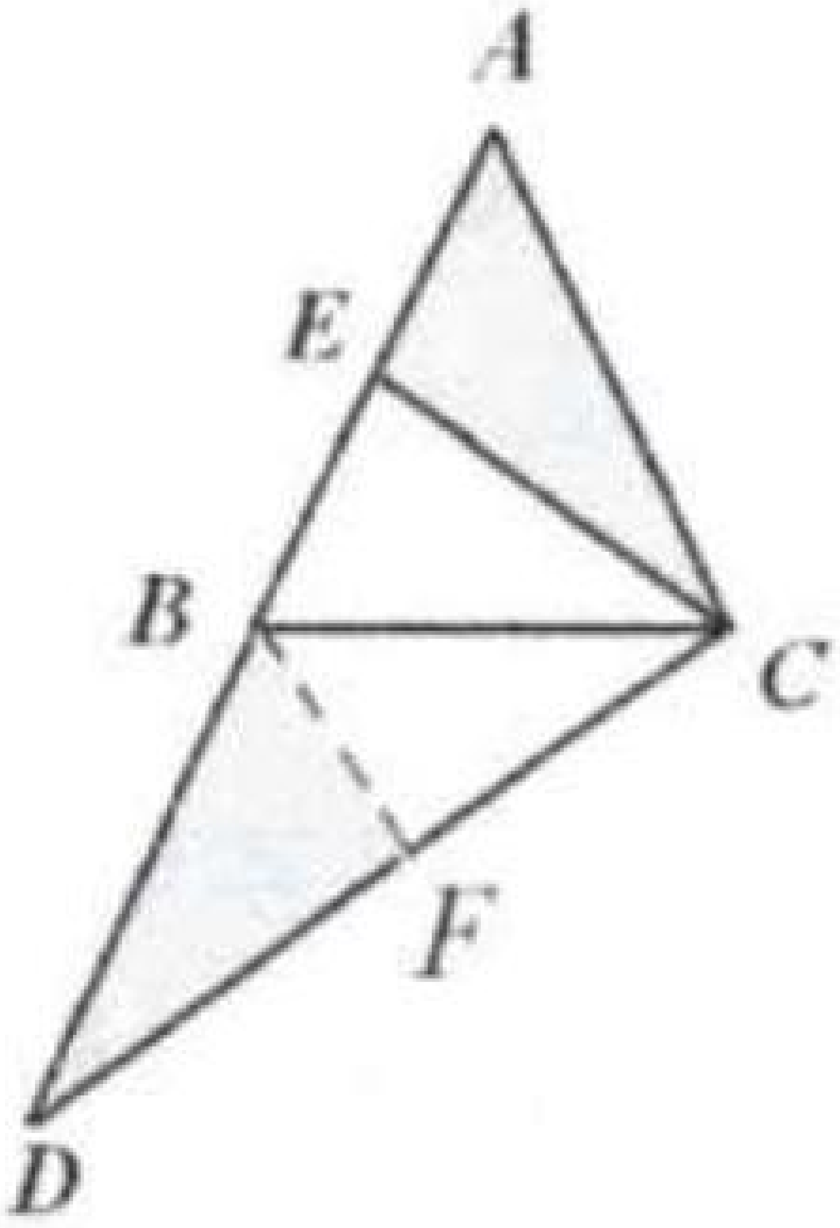
Since points B, F are the midpoints of AD, CD, respectively, $BF//AC$ and $BF = \frac{1}{2}AC$.



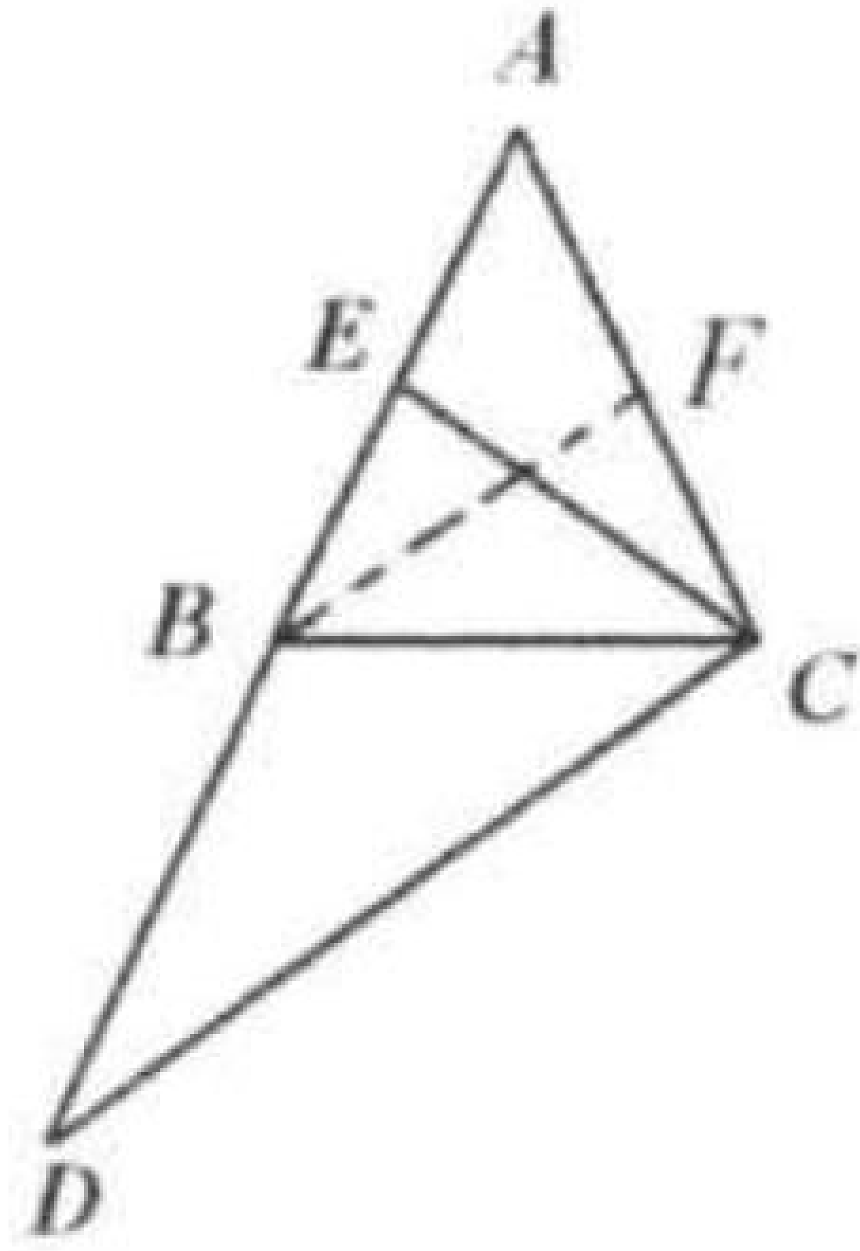
Since point E is the midpoint of $AB, BE = AE$

$\frac{1}{2}AB == \frac{1}{2}AC = BF.$

Since $BF//AC, \angle A = \angle DBF.BD = AB = AC, AE = BF.$ $\triangle AEC \cong \triangle DFB.$



Thus $DF = CE$, or $\frac{1}{2}CD = CE \quad \Rightarrow \quad CD = 2CE$.
Method 2:
Take F, the midpoint of AC.
Connect BF.
Since point B is the midpoint of AD, F is the midpoint of AC,



$BF = \frac{1}{2}DC$

Since $\triangle ABC$ is an isosceles triangle, $BF = CE$

Substituting (2) into (1): $CE = \frac{1}{2}DC \Rightarrow \quad CD = 2CE$.