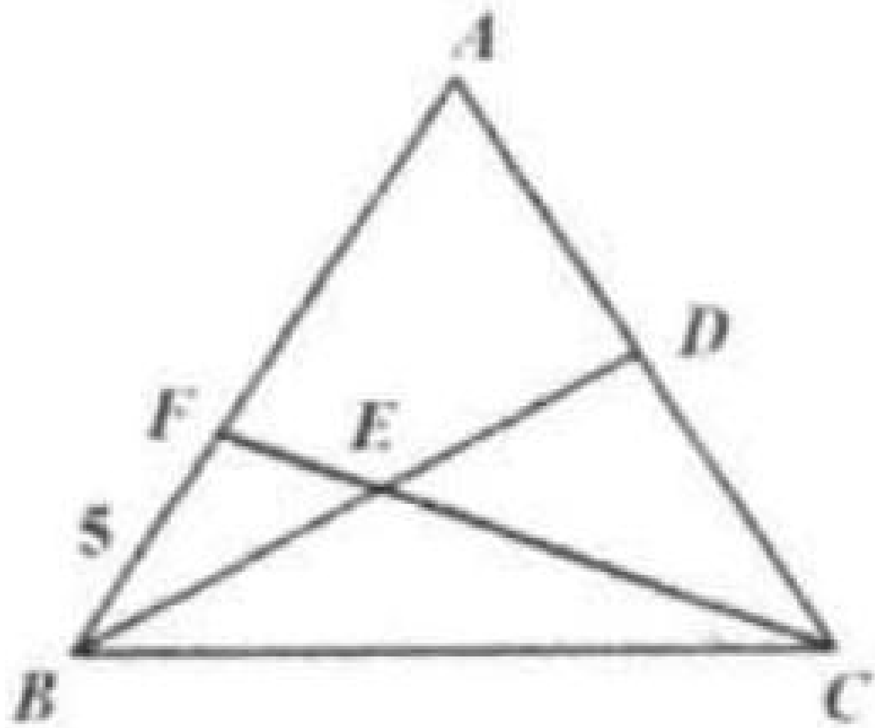
Example 2

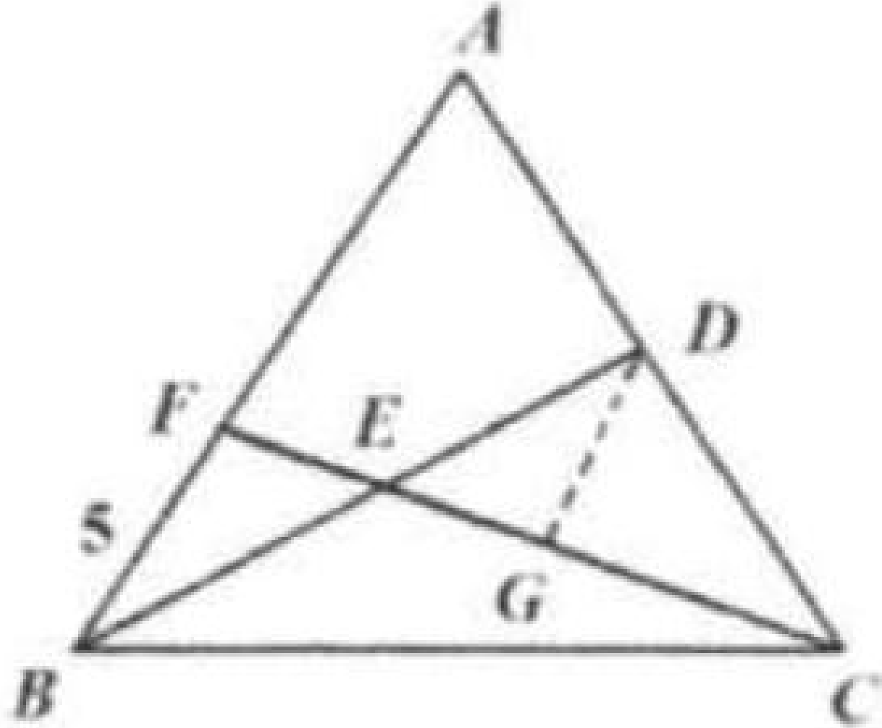
(AMC) In triangle ABC, BD is a median. CF intersects BD at E so BE = ED. Point F is on AB. Then, if $BF = 5, BA$ equals:
(A) 10
(B) 12
(C) 15
(D) 20
(E) none of these

Solution: (C).

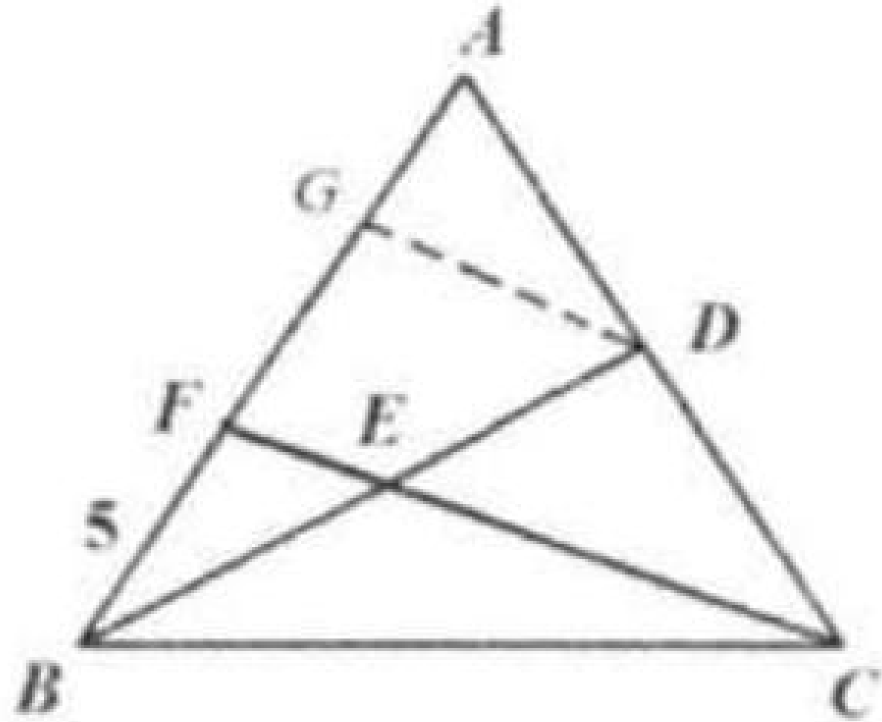


Method 1:
Draw $DG//AB$ to meet CF at G. Since D is the midpoint of AC,
$AF = 2DG$.
Since $BE = ED, \angle EBF = \angle EDG$ (alternate interior angles) and

$\angle BEF = \angle DEG$ (vertical angles),
$\triangle EFB \cong \triangle EGD$ and $DG = BF = 5$.



$AF = 2DG = 10. AB = 5 + 10 = 15.$
Method 2:
Draw $DG//CF$ to meet AB at G. Since D is the midpoint of AC, $AG = GF$.
Since $DG//EF$, and E is the midpoint of BD,



$GF = BF$.

So $AG = GF = BF = 5$.

$AB = BF + FG + AG = 5 + 5 + 5 = 15$.

Method 3:

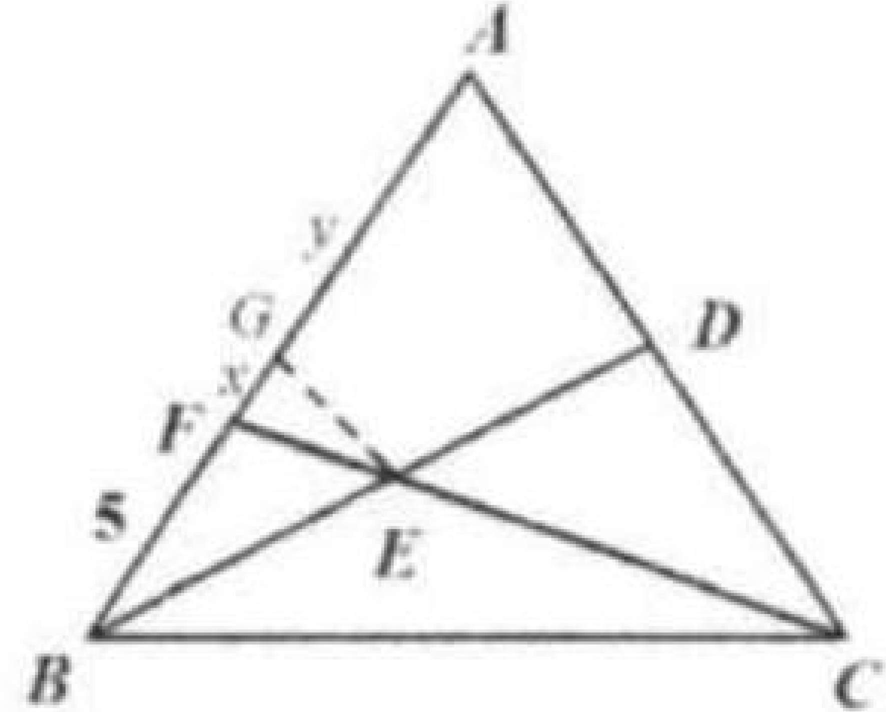
Draw $EG//AC$ to meet AB at G. Since E is the midpoint of BD,

$AB = 2AG = 2BG = 2y = 5 + x + y \Rightarrow y = 5 + x$

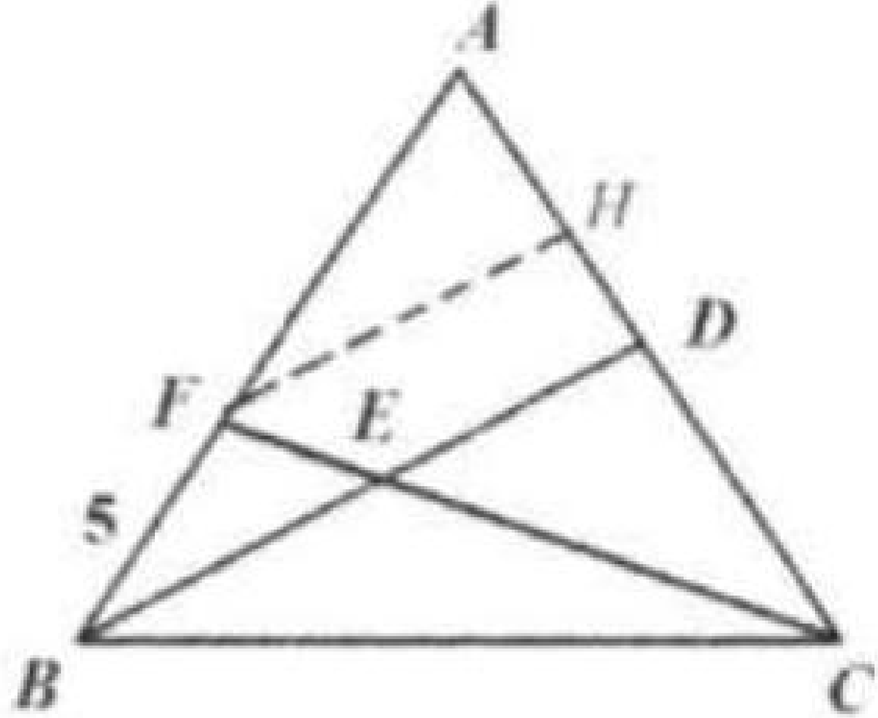
Since $EG//AC, \triangle FAC \sim \triangle FGE$ and

$(x + y)/x = 4 \Rightarrow y = 3x$

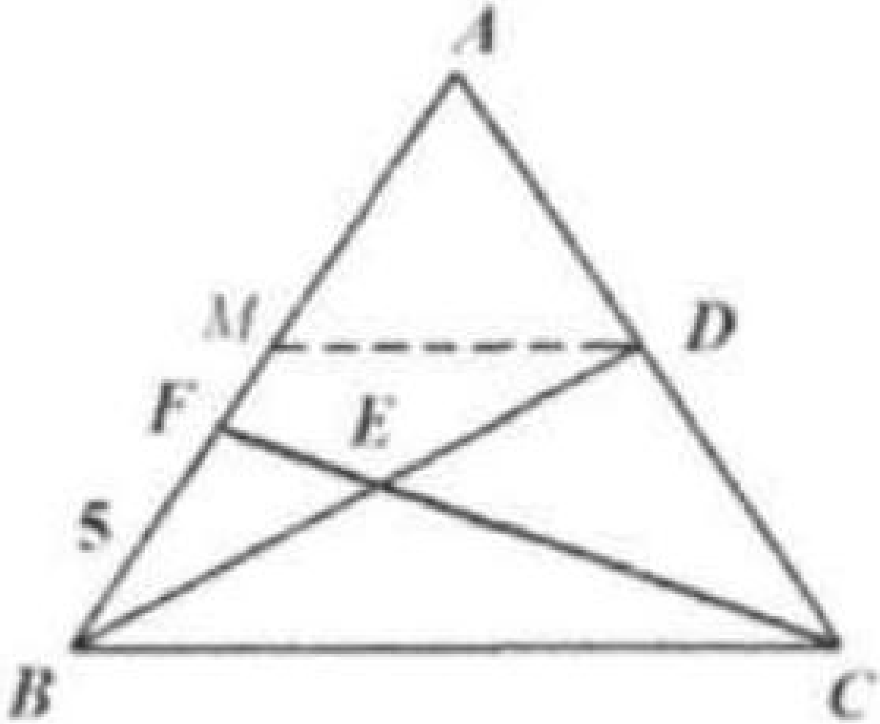
Substituting (2) into (1): $3x = 5 + x \Rightarrow 2x = 5$



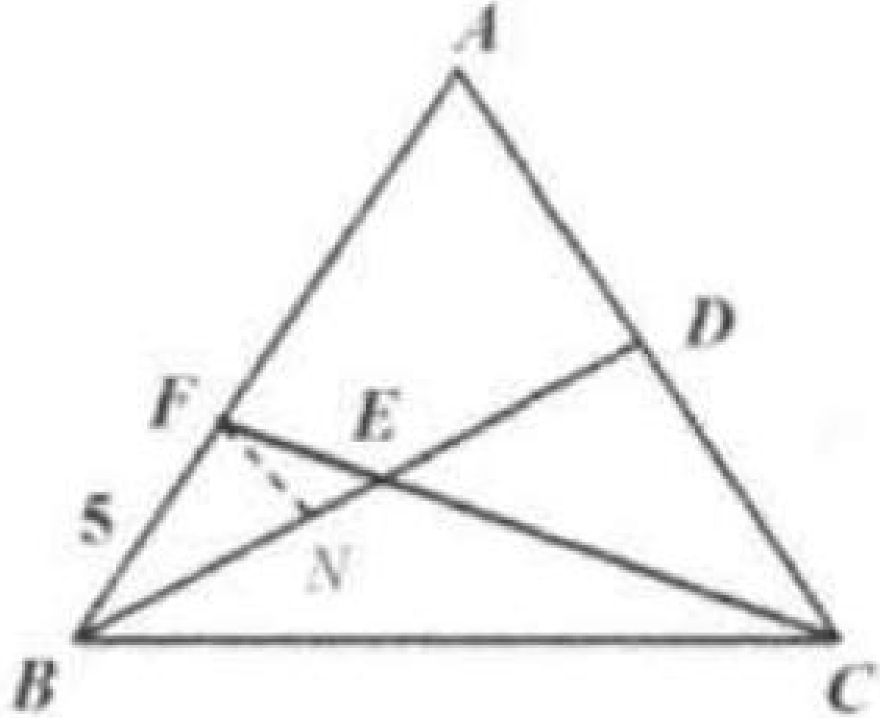
So $AB = 2y = 10 + 2x = 10 + 5 = 15$.
Note: the following ways to draw the auxiliary line will not work.



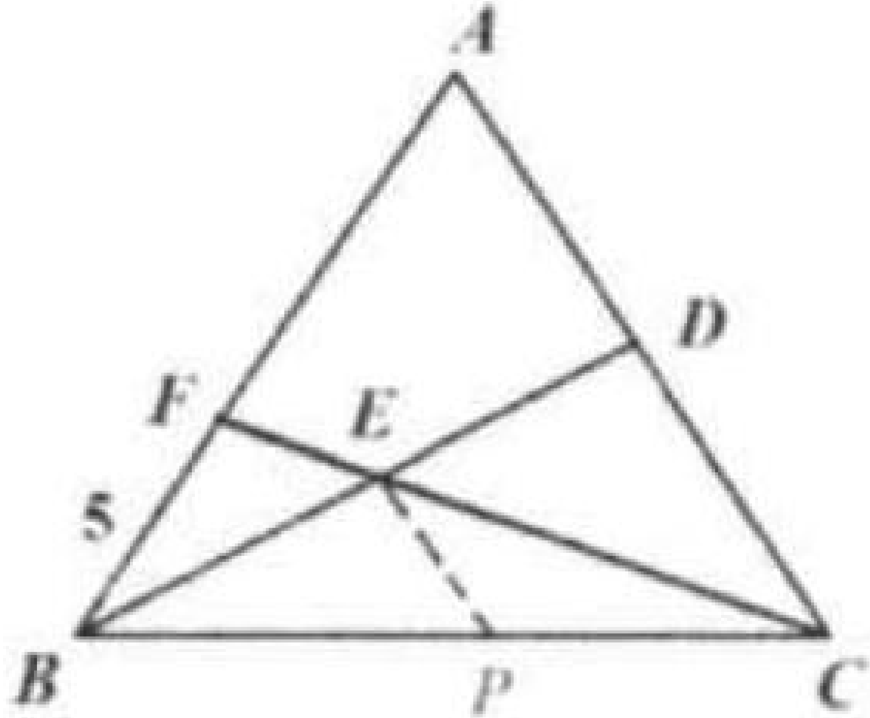
$FH//BD$



$DM//BC$



$FN//AC$



EP // AC