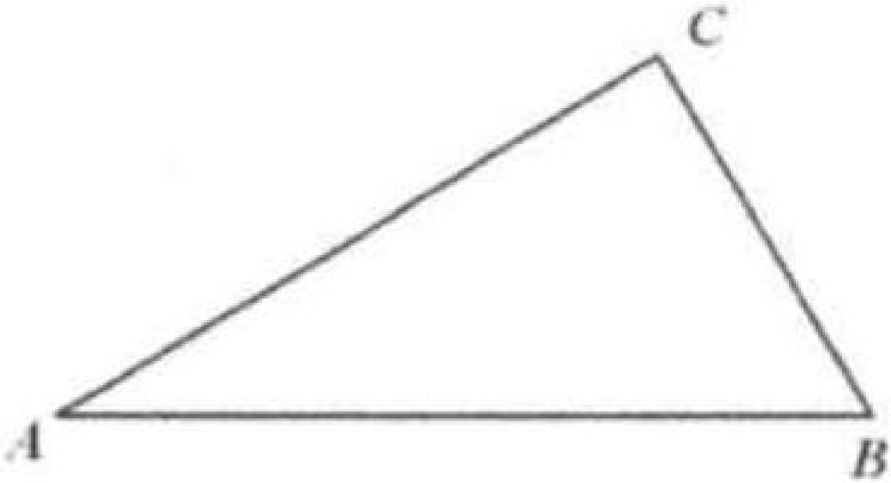
Problem 6

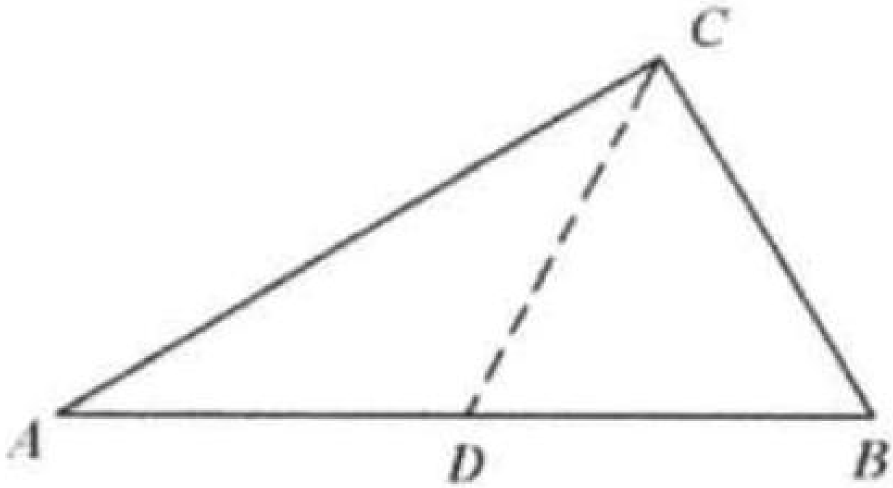
Problem

In $\triangle ABC, \angle B = 2\angle A$ and $AB = 2BC$. Show that $AB^2 = AC^2 + BC^2$.



Solution

Since $AB > BC, \angle C > \angle A$.
Draw CD to meet AB at D such that $\angle ACD = \angle A$.
$\triangle ADC$ is an isosceles triangle and $AD = DC$.
$\angle CDB = \angle ACD + \angle A = 2\angle A = \angle B$



Therefore, $\triangle BCD$ is also an isosceles triangle, and so $DC = BC$.

$$AB = AD + DB = 2BC$$
$$\Rightarrow \quad AD + DB = 2DC = 2AD$$
$$\Rightarrow \quad AD = DB = DC.$$

This tells us that DC is the median of right triangle ABC with $\angle C = 90°$.

By the Pythagorean Theorem, we have $AB^2 = AC^2 + BC^2$.