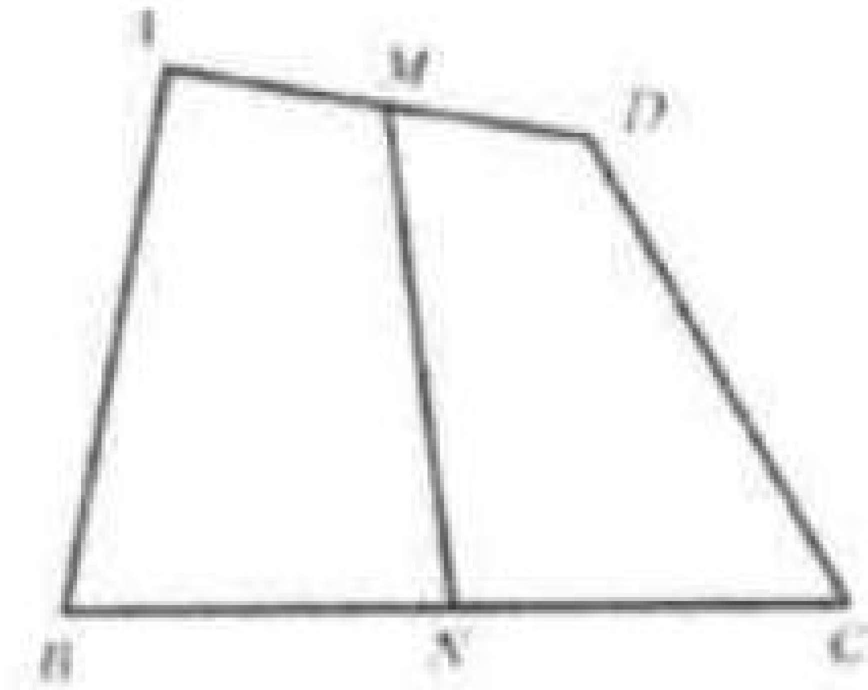
Problem

$ABCD$ is a convex quadrilateral. M and N are midpoints of AD, BC, respectively. Show that $MN \leq \frac{1}{2}(AB + DC)$.



Solution

If $AB//DC, ABCD$ is a trapezoid $ABCD$.

By Theorem 2.3, $MN = \frac{1}{2}(AB + DC)$.

Otherwise, Connect AC. Take P, the midpoint of AC. Connect PA, PN.

Since M and P are midpoints of AD, AC, respectively, by Theorem 2.1,

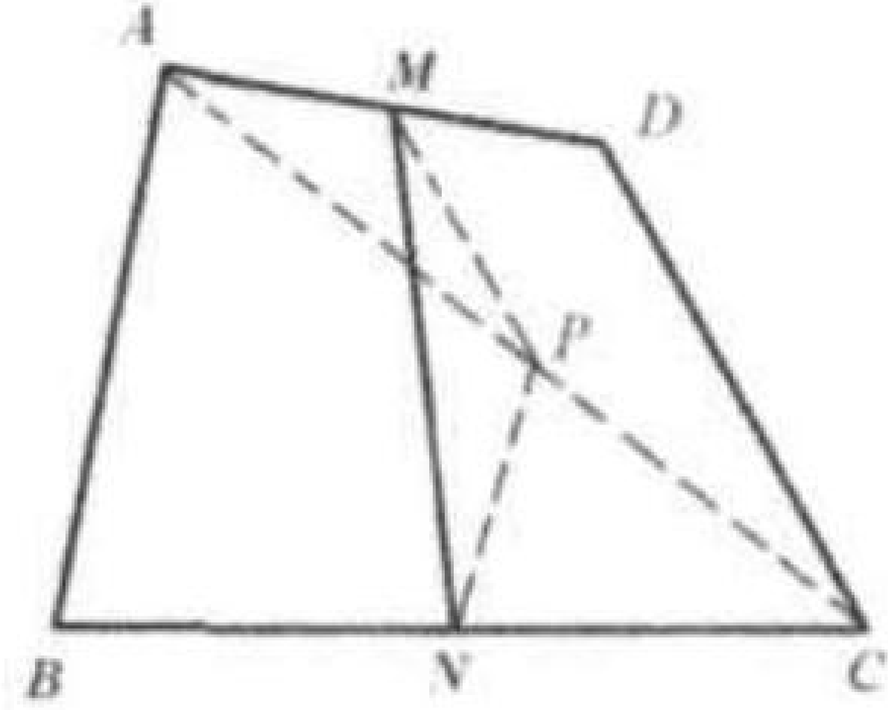
$$MP = \frac{1}{2}DC$$

Since N and P are midpoints of BC, AC, respectively, by Theorem 2.1,

$$NP = \frac{1}{2}AB$$

$$(1) + (2) : MP + NP = \frac{1}{2}(DC + AB)$$

By the triangle inequality theorem, $MP + NP > MN$.



Thus $MN < \frac{1}{2}(AB + DC)$.
Therefore, we have $MN \leq \frac{1}{2}(AB + DC)$.