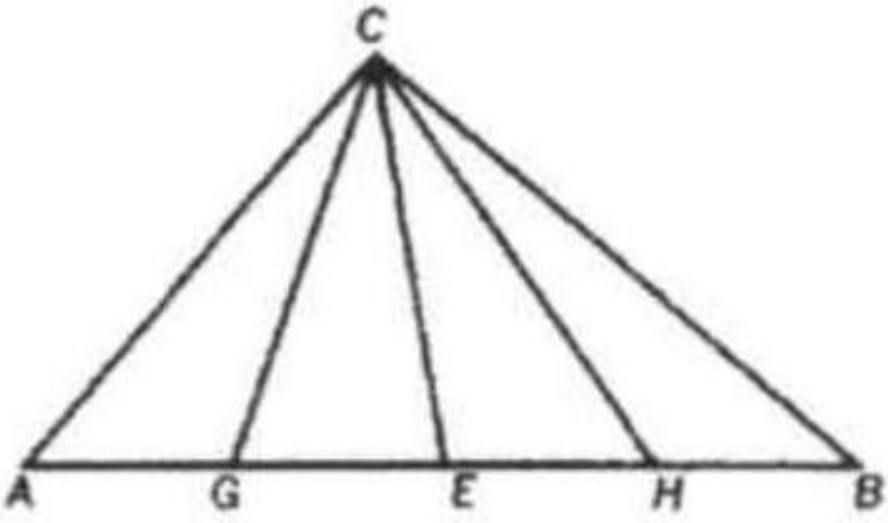
Example 7

Hypotenuse AB of right $\triangle ABC$ is divided into four congruent segments by points G, E, and H, in the order A, G, E, H, and B. If $AB = 20$, find the sum of the squares of the measures of the line segments from C to G, E, and H.



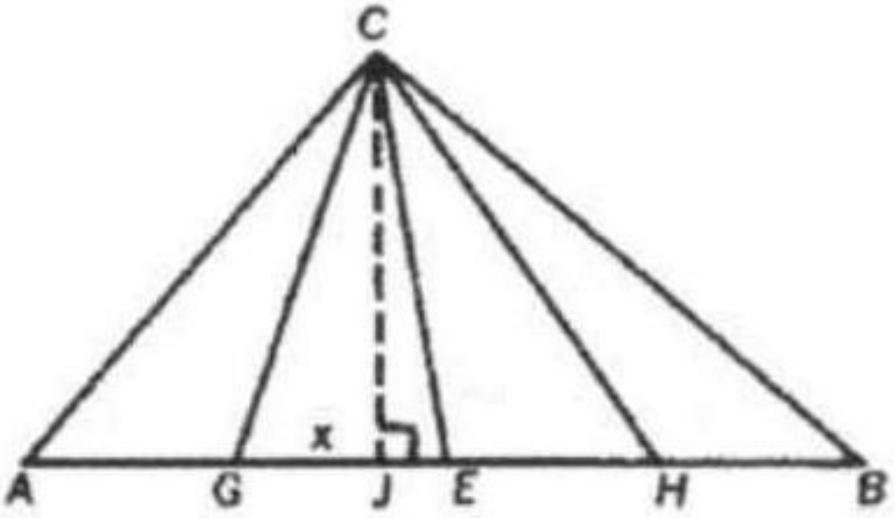
Solution: 350 .

Draw CJ perpendicular to AB at J. Since $AB = 20, CE = 10$.

Let $GJ = x$, and $JE = 5 - x$.

By the Pythagorean Theorem, in right triangles $\triangle CJG$ and $\triangle CJE$,

$(CG)^2 - x^2 = 10^2 - (5 - x)^2$



or $(CG)^2 = 75 + 10x$.
Similarly, in $\triangle CJH$ and $\triangle CJE, (CH)^2 = (10 - x)^2 - (5 - x)^2$, or
$(CH)^2 = 175 - 10x$.
By the addition of (1) and (2):
$(CG)^2 + (CH)^2 + (CE)^2 = 75 + 10x + 175 - 10x + 100 = 350$.