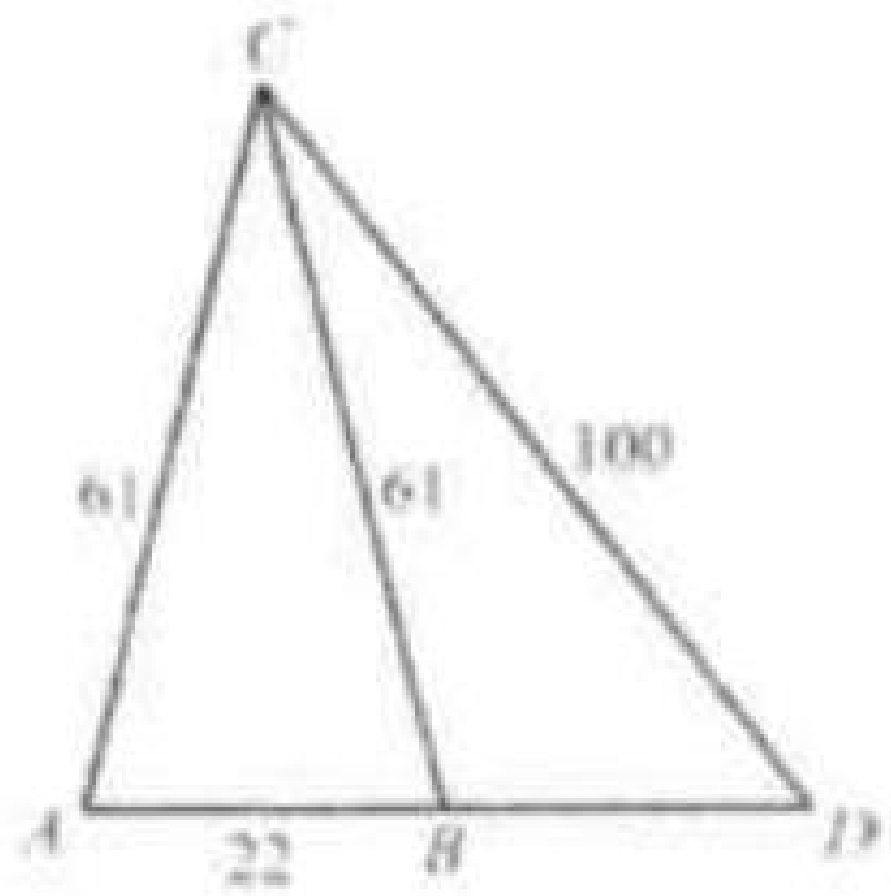
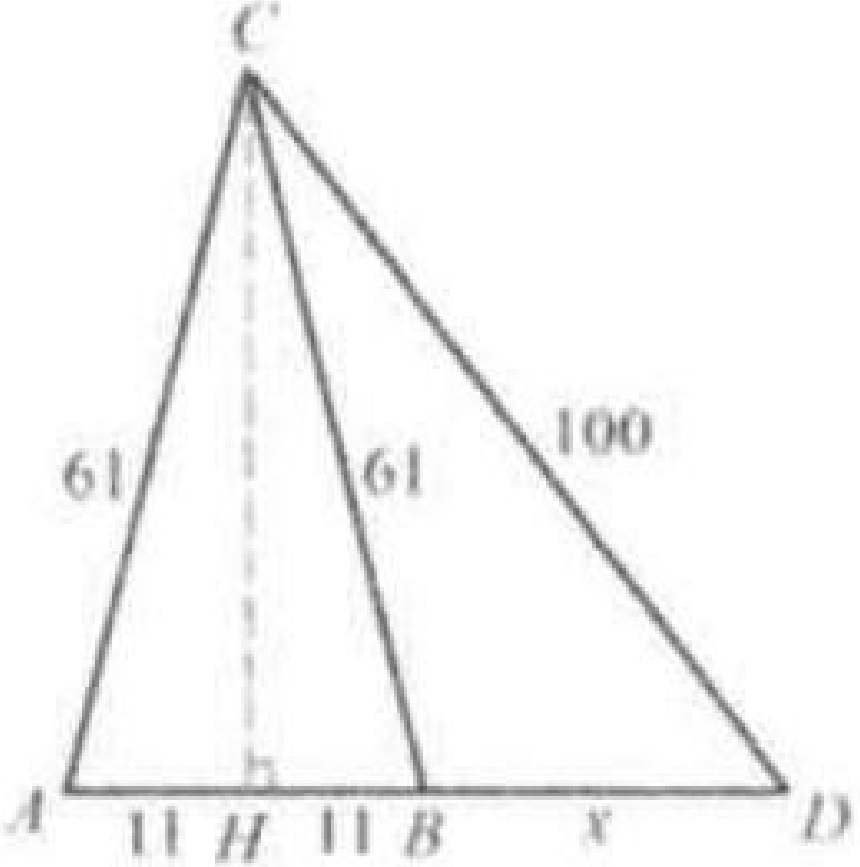
Example 10

In $\triangle ABC$, we have $AC = BC = 61$ and $AB = 22$. Suppose that D is a point on line AB such that B lies between A and D and $CD = 100$. What is BD ?

(A) 22
(B) 42
(C) 52
(D) 69
(E) 64



Solution: (D).
Let CH be an altitude of $\triangle ABC$. Applying the Pythagorean Theorem to $\triangle CHB$ and to $\triangle CHD$ produces
$100^2 - (x+11)^2 = CH^2 = 61^2 - 11^2 = 60^2$, so $(x+11)^2 = 100^2 - 60^2 = 6400 \quad \Rightarrow \quad x + 11 = 80$
Thus $BD = x = 80 - 11 = 69$.
Note that 11-60-61 and 60-80-100 are all Pythagorean Triples.


C
61
61
100
A
H
B
X
D