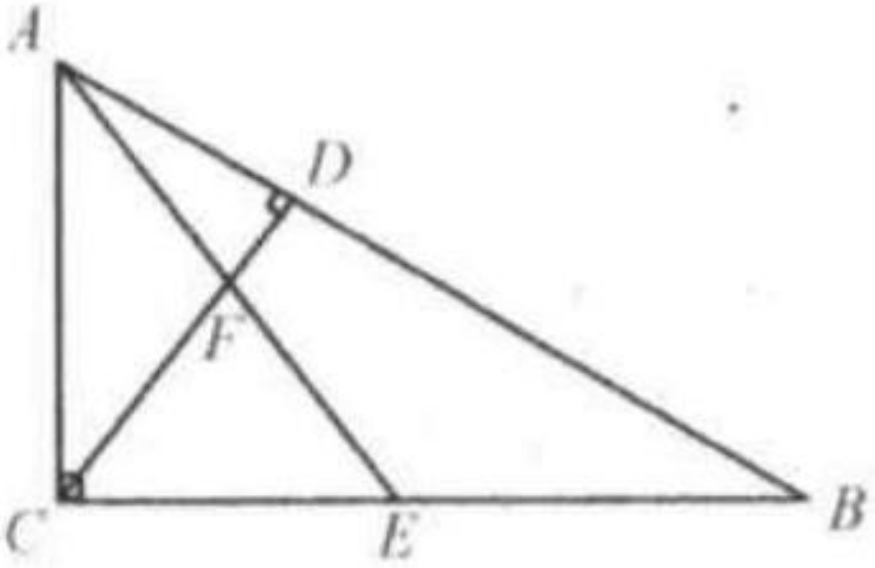
Problem 7

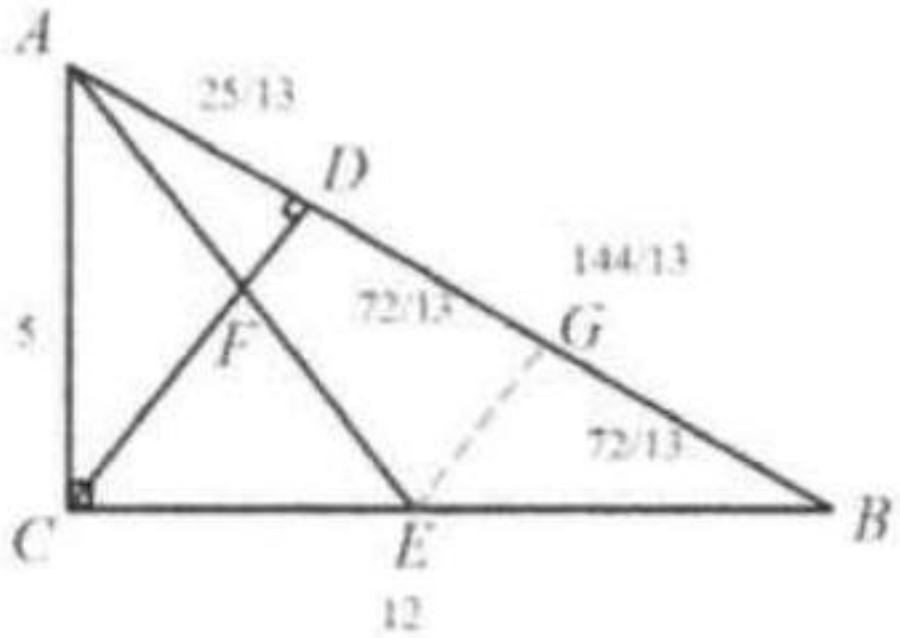
Problem

(2013 Mathcounts National Sprint 28) In right triangle ABC, shown here, $AC = 5$ units and $BC = 12$ units. Points D and E lie on AB and BC, respectively, so that CD is perpendicular to AB and E is the midpoint of BC. Segments AE and CD intersect at point F. What is the ratio of AF to FE ? Express your answer as a common fraction.



Solution

25/72. Triangle ABC is a 5-12-13 right triangle, so $AB = 13$. We can determine from similar triangles $AD = \frac{25}{13}$ and $DB = \frac{144}{13}$.



Draw $EG//FD$.
Since $CE = EB, DG = BG = \frac{1}{2} \times \frac{144}{13} = \frac{72}{13}$.
Since triangle AFD is similar to triangle AEC,
$\frac{AF}{FE} = \frac{AD}{DG} = \frac{\frac{25}{13}}{\frac{72}{13}} = \frac{25}{72}$.