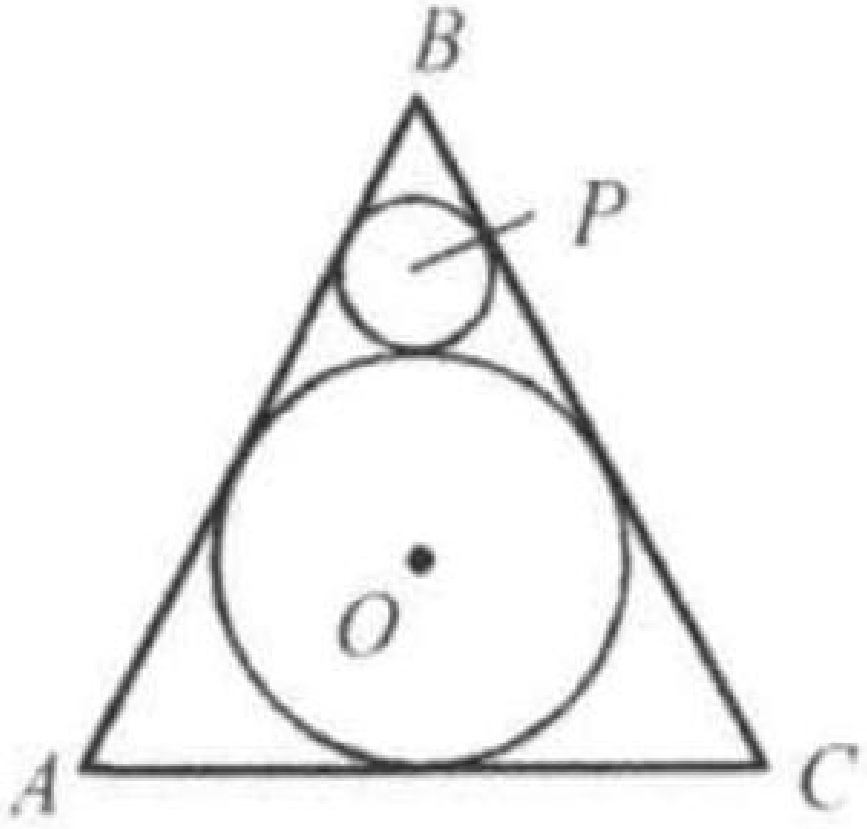
Example 1

Circle O is inscribed in equilateral triangle ABC. Circle P of radius 1 is tangent to circle O and segments AB and BC. Find the area of triangle ABC.
(A) 27
(B) $9\sqrt{3}$
(C) $36\sqrt{3}$
(D) $27\sqrt{3}$
(E) 47

Solution: (D).
Draw $PF \perp AB$ and $OE \perp AB$.



Connect BD.

Triangle BEO is a $30^\circ - 60^\circ - 90^\circ$ triangle and $BP = 2PF = 2$.

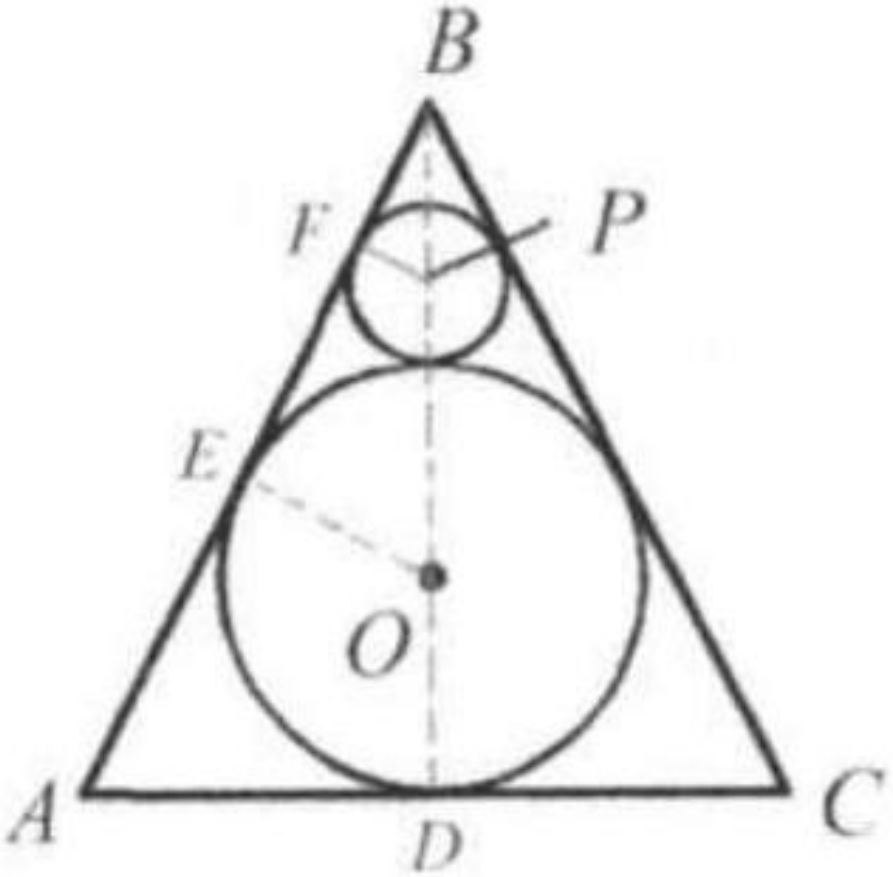
Let $OE = r$.

We know that $BO = 2OE \quad \Rightarrow \quad 2r = r + 3 \quad \Rightarrow r = 3$

We know that $r = \frac{1}{6}a\sqrt{3}$, where a is the length of the side of triangle ABC. So

$3 = \frac{1}{6}a\sqrt{3} \quad \Rightarrow \quad a = 6\sqrt{3}$

We know that $S_{\triangle ABC} = \frac{1}{4}a^2\sqrt{3}$. So



$S_{\triangle ABC} = \frac{1}{4} \times (6\sqrt{3})^2\sqrt{3} = 27\sqrt{3}$