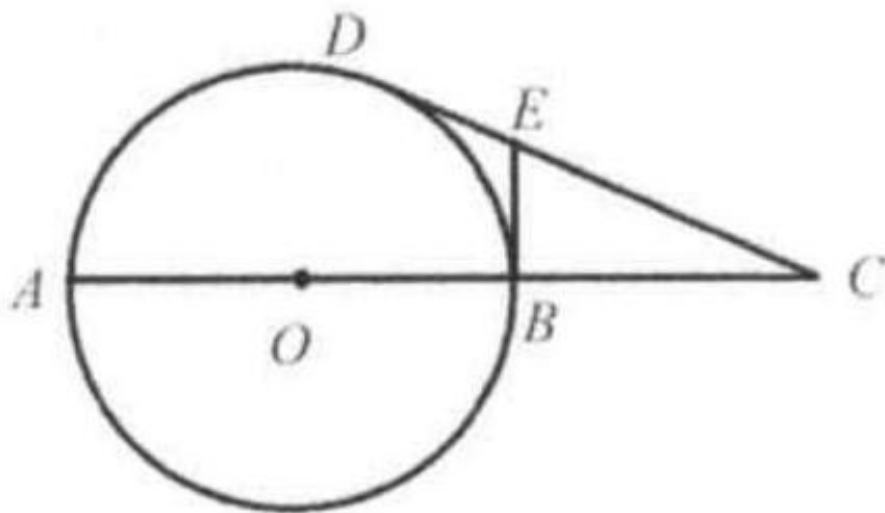


Problem

AB is the diameter of the circle O . Extend AB to C . CD is tangent to the circle at D . BE is tangent to the circle at B and meets CD at E . Show that $CA = \sqrt{3}CD$ if $DE = \frac{1}{2}EC$.



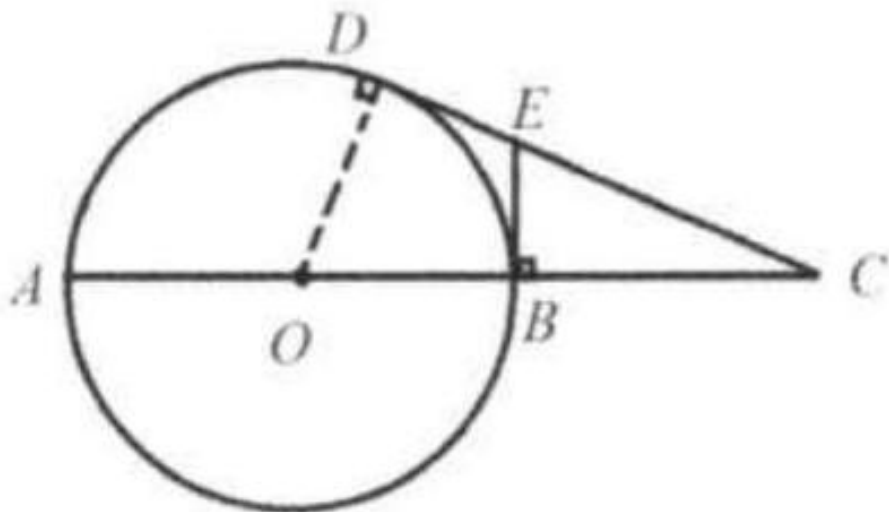
Solution

Connect OB .

Since both CD and BE are tangents of circle, $BE = DE$.

Since $DE = \frac{1}{2}EC$, $BE = \frac{1}{2}EC$. So $\angle C = 30^\circ$. Thus $OC = 2OD$.

Since $OB = OD$, $CB = OC - OB = OC - OD = 2OD -$



$$OD = OD = OB = OA.$$

$$\text{Thus } CA = 3CB.$$

We know that $CD^2 = CA \times CB = \frac{1}{3}CA^2$.

$$\text{So } CA^2 = 3CD^2 \Rightarrow CA = \sqrt{3}CD.$$