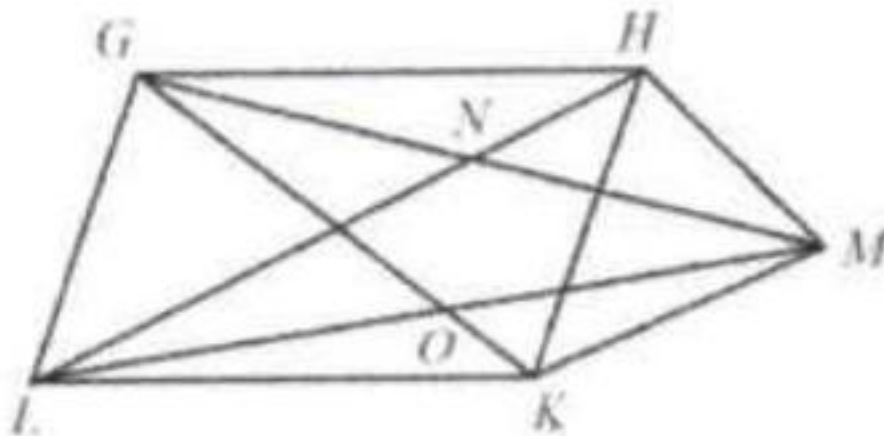


## Problem

(1914 Phillips Academy Prize Exam)  $GHKL$  is a parallelogram;  $HM$  and  $KM$  are drawn parallel to the diagonals;  $GM$  and  $LM$  cut the diagonals at  $N$  and  $O$ . Prove that  $NO$  is equal to one half of  $GL$ .

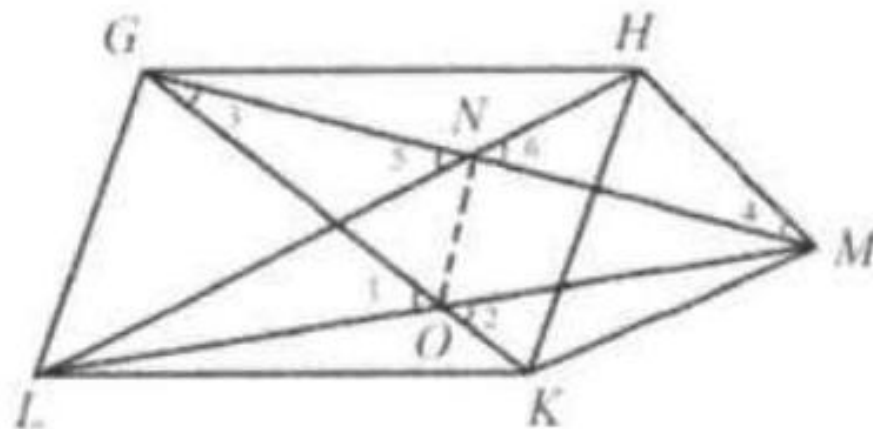


## Solution

Method 1:

$$\angle 1 = \angle 2.$$

$\angle LTO = \angle MKO$  since  $LH \parallel MK$ , and since  $THMK$  is a parallelogram,  $HT = MK$ . Since  $GHKL$  is a parallelogram, the diagonals bisect each other so  $HT = LT$ . Therefore,  $LT = MK \Rightarrow \triangle LTO \cong \triangle MKO$  by *AAS*, giving  $LO = MO$ , so  $O$



is the midpoint of  $LM$ .

Likewise,  $\triangle NTG \cong \triangle NHM$  by AAS, since  $\angle 3 = \angle 4$ ,  $\angle 5 = \angle 6$ , and  $HM = K = GT$ . Thus  $N$  is the midpoint of  $GM$ .

Since  $NO$  is the midline of  $\triangle MGL$ ,  $NO = \frac{1}{2}GL$  and  $NO \parallel GL$ .

Method 2:

Label the point of intersection of  $LH$  and  $GK$  be  $T$ .

Since  $TN \parallel KM$ , and  $T$  is the midpoint of  $LH$ ,  $N$  is the midpoint of  $GM$ .

Since  $TO \parallel HM$ , and  $T$  is the midpoint of  $LH$ ,  $O$  is the midpoint of  $LM$ .

Thus  $NO$  is the midline of  $\triangle MGL$ ,  $NO = \frac{1}{2}GL$  and  $NO \parallel GL$ .

