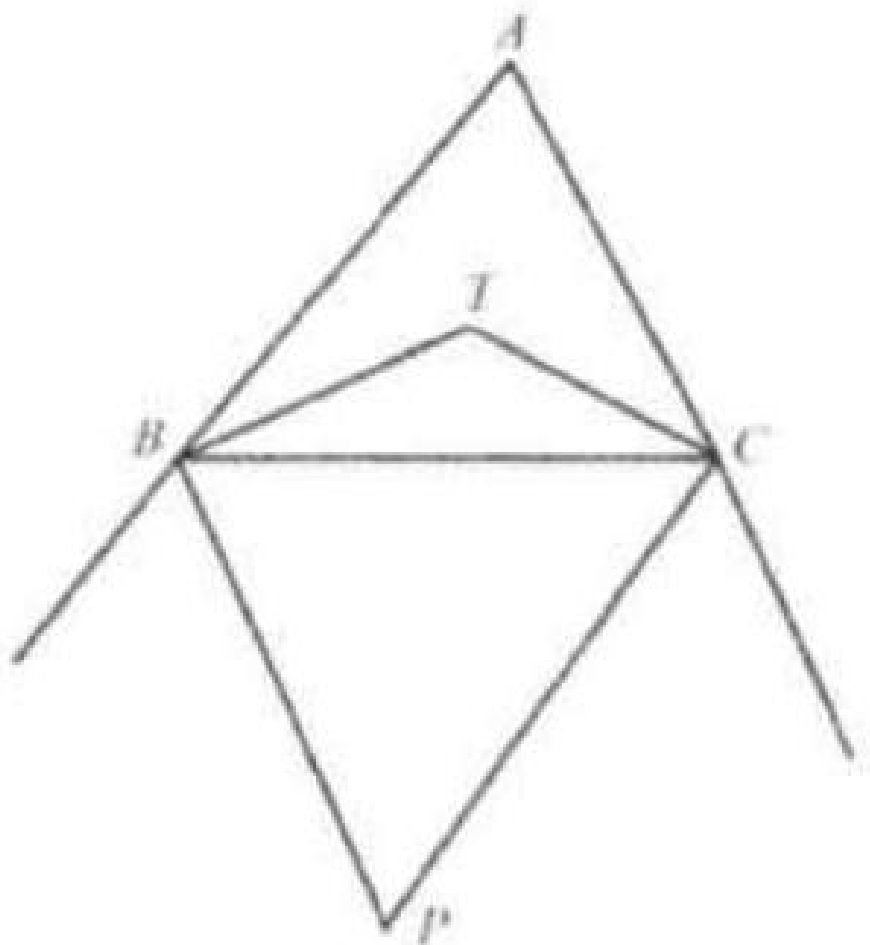


Problem 11

Problem

In $\triangle ABC$, the angle bisectors of $\angle B, \angle C$ meet at T , and the exterior angle bisectors of $\angle B, \angle C$ meet at P . Find $\angle BPC$ if $\angle A = 72^\circ$.



$\angle EFI = \angle EAI = \alpha$ (they face the same arc EI).

$\angle FEI = \angle FAI = \alpha$ (they face the same arc FI).

In $\triangle FEI$, $\angle FIE = 180^\circ - 2\alpha = 180^\circ - \angle A$

But $\angle FDE = \frac{1}{2}\angle FIE$. So $\angle FDE = 90^\circ - \frac{1}{2}\angle A$.

Solution

54°. Label $\angle TBC = \angle TBA = \alpha$, $\angle TCB = \angle TCA = \beta$, $\angle CBP = \gamma$.

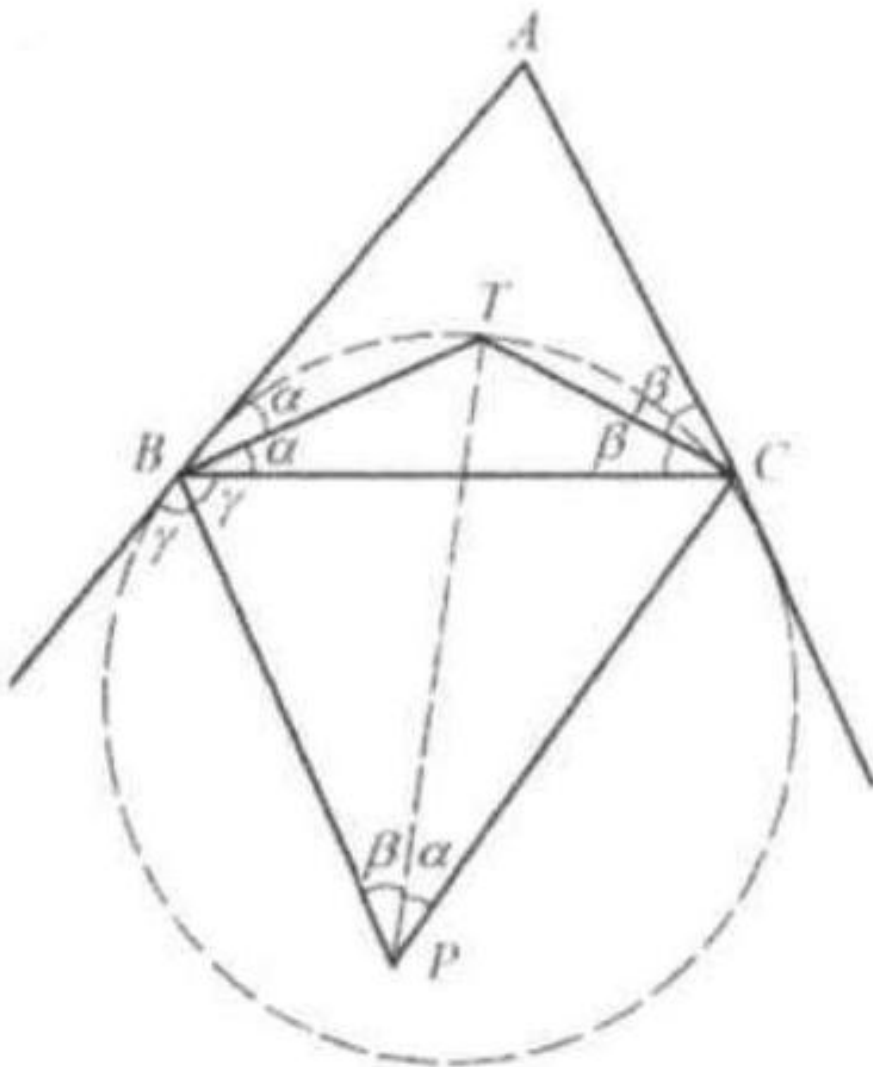
Since $2\alpha + 2\gamma = 180^\circ$, $\alpha + \gamma = 90^\circ$.

Thus $\angle TBP = 90^\circ$.

Similarly, $\angle TCP = 90^\circ$.

So points B, P, C , and T are concyclic and TP is the diameter of the circle.

Therefore, $\angle BPT = \angle TCB = \beta$, $\angle CPT = \angle TBC = \alpha$.



$$\angle BPC = \alpha + \beta = \frac{(180^\circ - \angle A)}{2} = \frac{(180^\circ - 72^\circ)}{2} = 54^\circ.$$

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