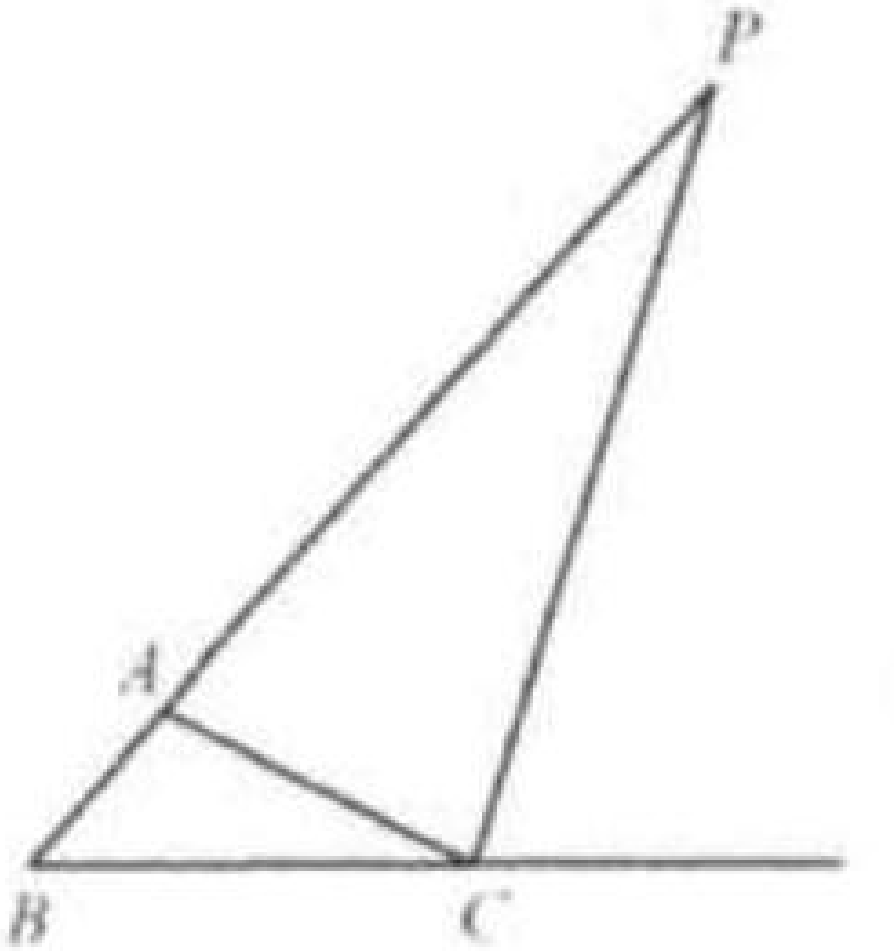
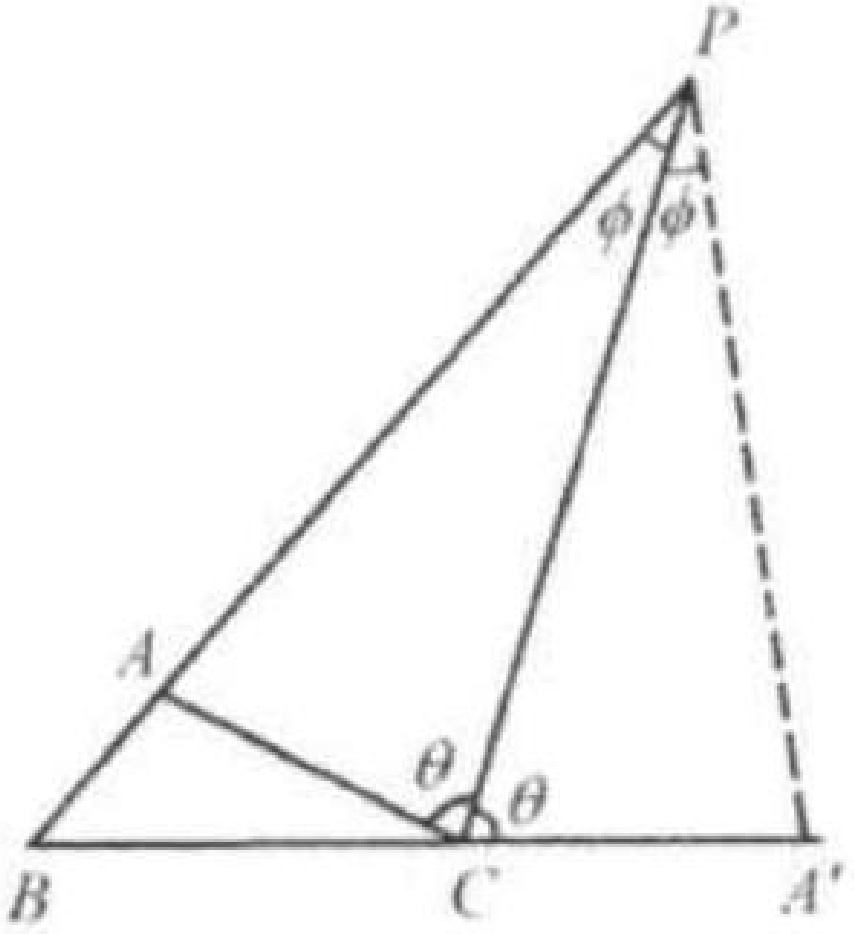
Example 3

(AMC) In triangle ABC the ratio $AC : CB$ is 3:4. The bisector of the exterior angle at C intersects BA extended at P(A is between P and B). The ratio $PA : AB$ is:
(A) $1 : 3$
(B) $3 : 4$
(C) $4 : 3$
(D) $3 : 1$
(E) $7 : 1$

Solution: (D).



Draw PA' such that $\angle BPC = \angle A'PC$, then $\triangle ACP \cong \triangle A'\ CP$ (ASA) and $AC = A'C, PA = PA'$. Since PC bisects $\angle BPA'$ in $\triangle BPA'$,
$\frac{BC}{CA'} = \frac{PB}{PA'}$ or $\frac{BC}{CA} = \frac{PB}{PA} = \frac{4}{3}$.
$AB = PB - PA$ since A is between P and B.



$\frac{AB}{PA} = \frac{PB}{PA} - \frac{PA}{PA} = \frac{4}{3} - 1 = \frac{1}{3}$, so $\frac{PA}{AB} = \frac{3}{1}$.