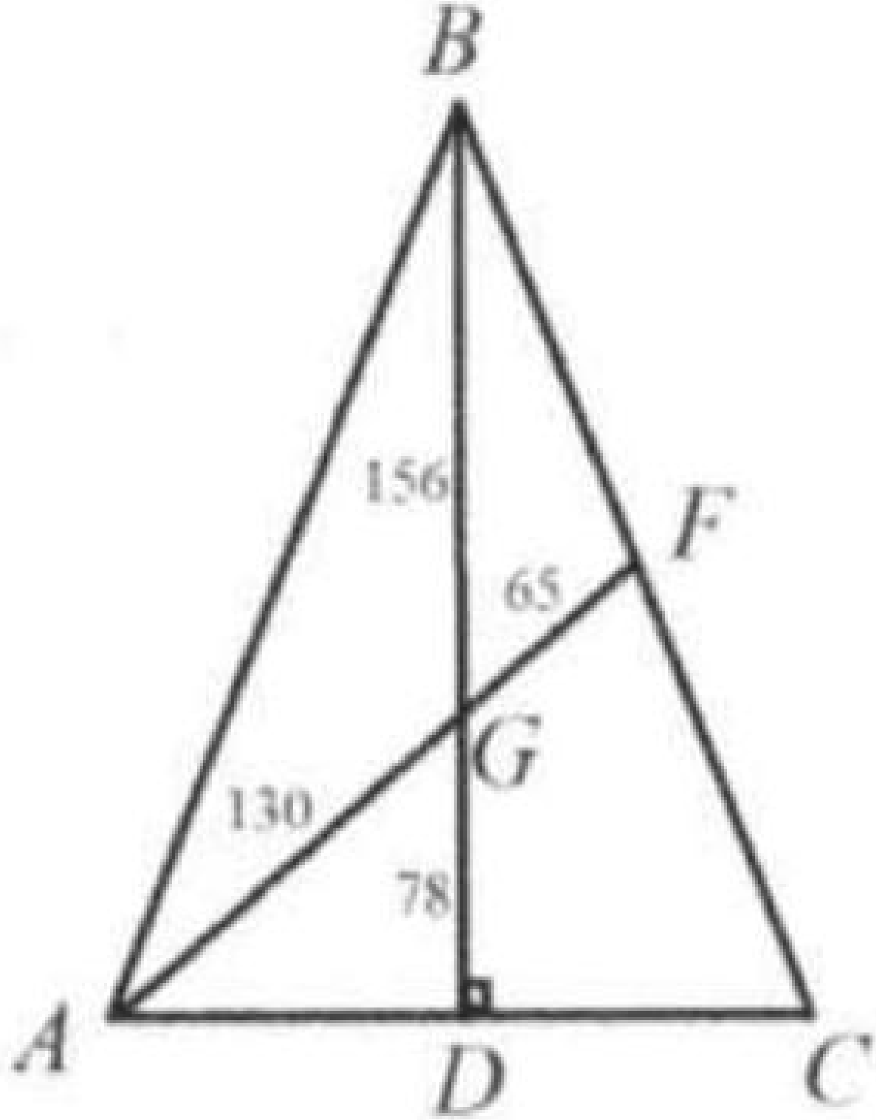
Problem 2

Problem

Triangle ABC is an isosceles triangle. BD is the altitude to base AC. AF is the median to $BC.AF$ meets BD at G. Find the number of square inches in the area of triangle ABG if $BD = 234$ and $AF = 195$.



Solution

8112.
Method 1:
AF, CE, and BD are three medians. They meet at G. Triangle ABC is divided into six smaller equal areas.
$GD = \frac{1}{3}BD = \frac{1}{3} \times 234 = 78. AG = \frac{2}{3}AF = \frac{2}{3} \times 195 = 130.$
Triangle ADG is a 3-4-5 right triangle $(3 \times 26, 4 \times 26, 5 \times 26)$ and $AD = 104$.

$$S_{\triangle ADG} = \frac{78 \times 104}{2} = 4056$$
$$S_{\triangle ABG} = 2S_{\triangle MDG} = 2 \times 4056 = 8112.$$

Method 2:

Note that ABC is an isosceles triangle, so the altitude is also a median.

$GD = \frac{1}{3}BD = \frac{1}{3} \times 234 = 78. AG = \frac{2}{3}AF = \frac{2}{3} \times 195 = 130.$

$AD = \sqrt{AG^2 - DG^2} = \sqrt{130^2 - 78^2} = 104$. The area of triangle ADG is

$$S_{\triangle ADG} = \frac{78 \times 104}{2} = 4056.$$
$$\frac{S_{ABGG}}{S_{\triangle ADG}} = \frac{BG}{DG} = 2$$
$$S_{\triangle ADG} = 2S_{\triangle ADG} = 2 \times 4056 = 8112.$$