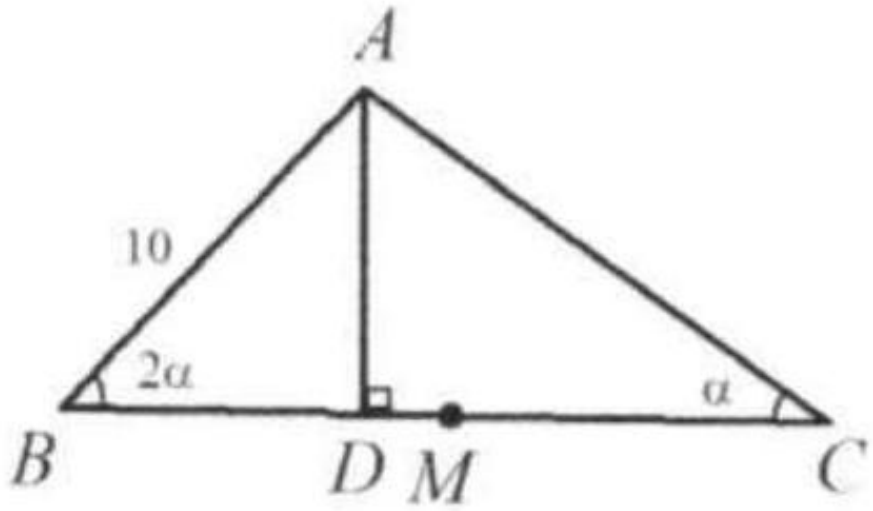
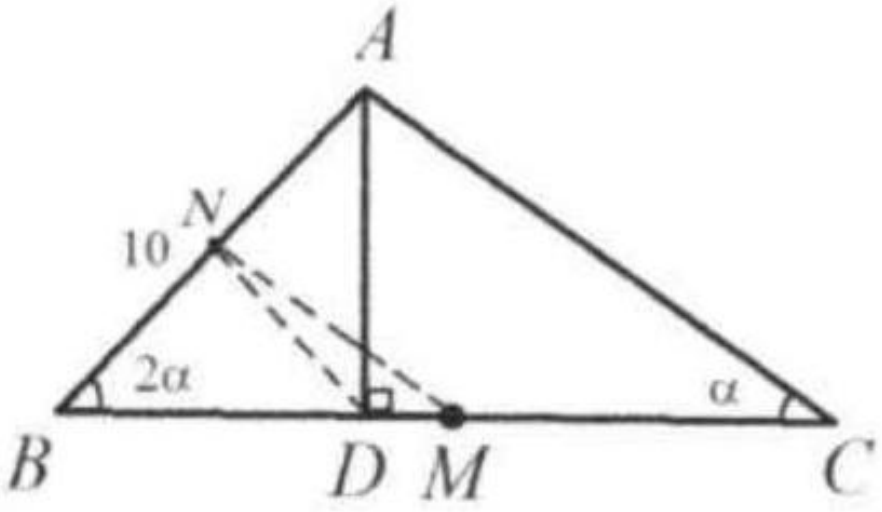
Problem

In $\triangle ABC, \angle B = 2\angle C. AD \perp BC. M$ is the midpoint of $BC. AB$ is 10 cm . Find the length of MD.



Solution

5 cm . Take N, the midpoint of AB. Connect DN, MN. DN is the median of right triangle ABD. So $DN = \frac{1}{2}AB = 10/2 = 5, \angle NDB = \angle B = 2\alpha$.



MN is the midline of $\triangle BAC$. So $MN//AC$. Thus $\angle NMB = \angle NMD = \angle C = \alpha$.

We know that $\angle NDB$ is the exterior angle of $\triangle NDM$. So
$\angle NDB = \angle MND + \angle NMD = \angle MND + \angle\alpha$, or
$\angle MND + \alpha = 2\alpha \quad \Rightarrow \quad \angle MND = \alpha.$
$\triangle NMD$ is an isosceles triangle with $ND = MD = 5$.