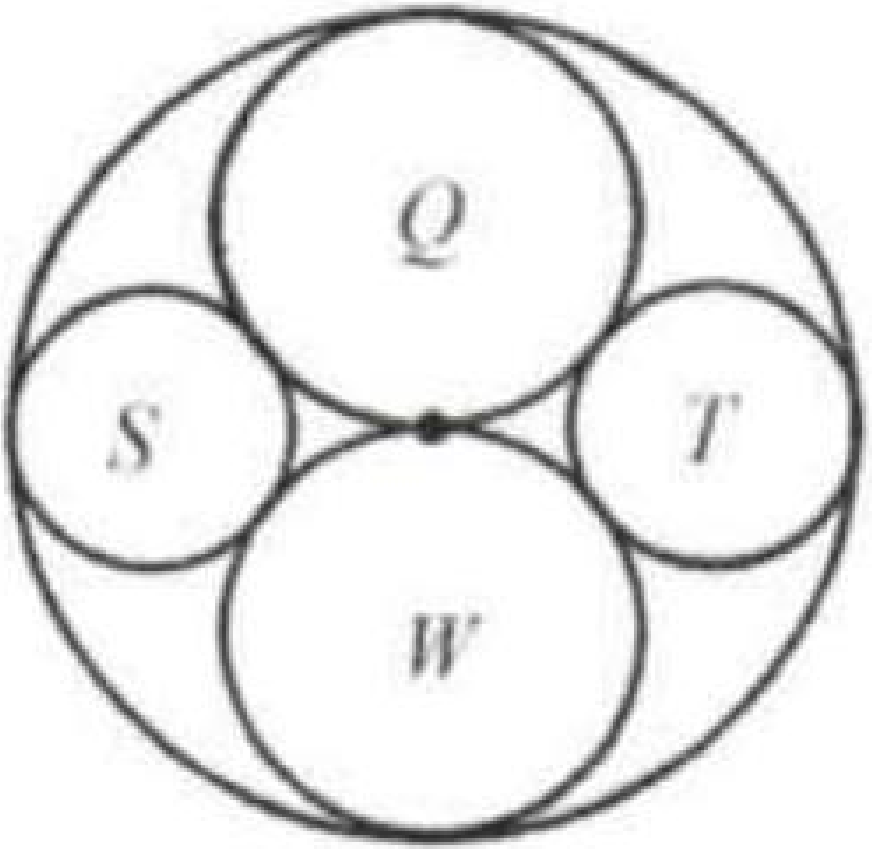
Problem 9

Problem

(1996 Mathcounts National Sprint Problem 25) In the diagram, circle Q is congruent to circle W , and both are tangent to circle O and to each other. Circle S and circle T are congruent and



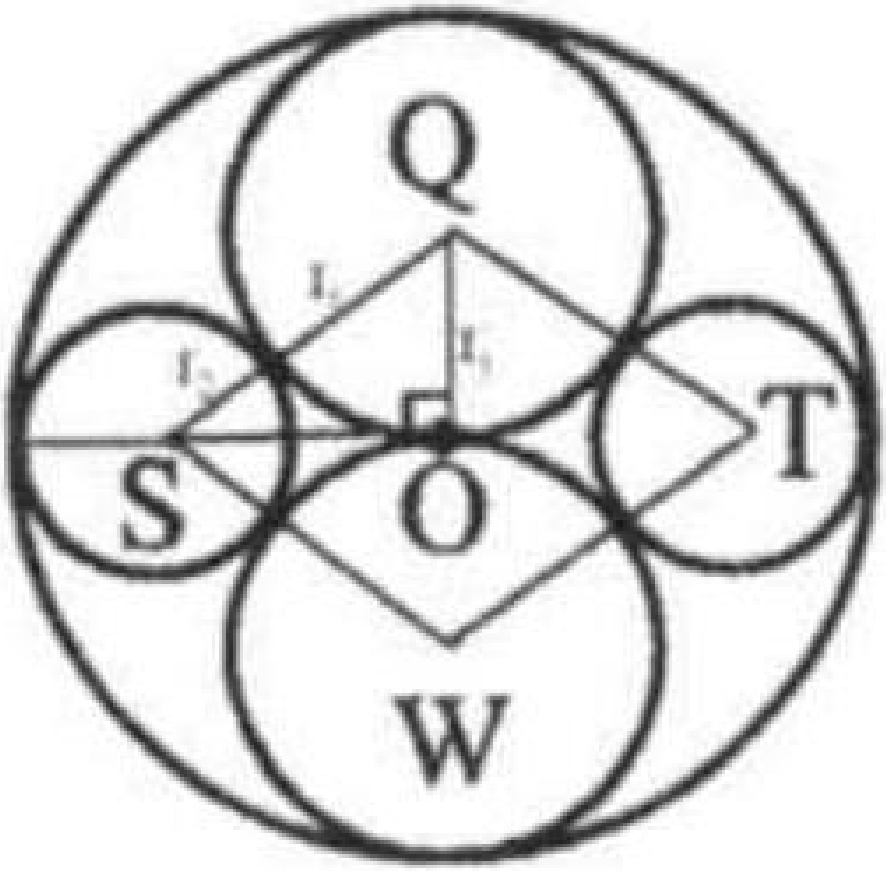
are tangent to circle O, to circle Q and to circle W. Find the ratio of the area of the smallest circle to the largest circle.

Solution

1: 9. Let the radius of the circle O be R, the radius of the circle Q be r_1, and the radius of the circle S be r_2. We know that $r_1 = \frac{R}{2}$.

By the Pythagorean Theorem,

$(R - r_2)^2 = r_1^2 + (r_1 + r_2)^2 \Rightarrow R^2 - 2Rr_2 = 2r_1r_2 \Rightarrow$



$R^2 - 2Rr_2 = 2r_1r_2 \Rightarrow r_2 = \frac{R}{3}$

The ratio of the areas of the smallest circle and largest circle is

$\frac{\pi {r_2}^2}{\pi R^2} = \frac{{r_2}^2}{R^2} = \frac{\left(\frac{R}{3}\right)^2}{R^2} = \frac{1}{9}$.