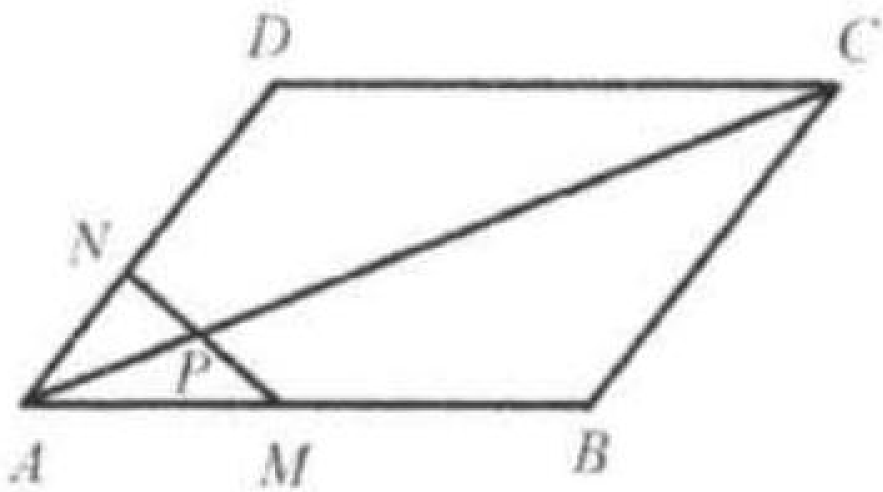
Problem 20

Problem

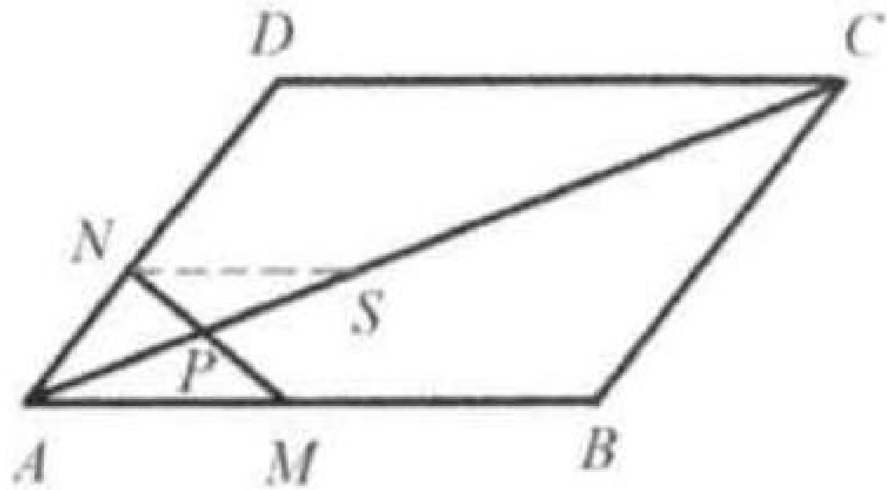
(2009 AIME) In parallelogram $ABCD$, point M is on AB so that $\frac{AM}{AB} = \frac{17}{1000}$, and point N is on AD so that $\frac{AN}{AD} = \frac{17}{2009}$. Let P be the point of intersection of AC and MN. Find $\frac{AC}{AP}$.



Solution

177. Method 1:

Let point S be on AC such that NS is parallel to AB.
Because $\triangle ASN$ is similar to $\triangle ACD, AS/AC = (AP + PS)/AC$
$= AN/AD = 17/2009.$
Because $\triangle PSN$ is similar to $\triangle PAM, PS/AP = SN/AM$



$= \frac{\frac{17}{2009}CD}{\frac{17}{1000}AB} = \frac{1000}{2009}$, and so $\frac{PS}{AP} + 1 = \frac{3009}{2009}$.

Hence $\frac{\frac{17}{2009}AC}{AP} = \frac{3009}{2009}$, and $\frac{AC}{AP} = 177$.

Method 2:

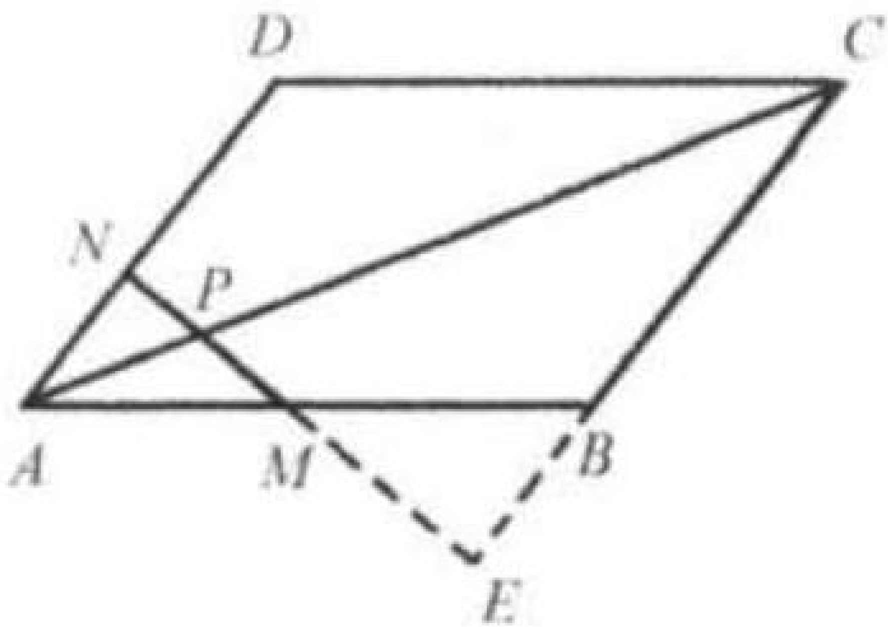
Extend NM through M to E and to meet the extension of CB at E.

We label the line segments as shown in the figure 1. We know that $AD//CE$.

So $\triangle AMN \sim \triangle BME$ (Figure 1).

$\frac{AN}{BE} = \frac{AM}{MB} \Rightarrow \quad \frac{17y}{BE} = \frac{17x}{983x} \Rightarrow BE = 983y$.

We know that $AN//CE$. So $\triangle APN \sim \triangle CPE$ (Figure 2).



$$\frac{AN}{CE} = \frac{AP}{PC} \quad \Rightarrow \frac{17y}{(2009+983)y} = \frac{AP}{AC-AP} \quad \Rightarrow$$

$$\frac{AC}{AP} - 1 = \frac{2992}{17} \quad \Rightarrow \quad \frac{AC}{AP} = \frac{2992}{17} + 1 = 177.$$

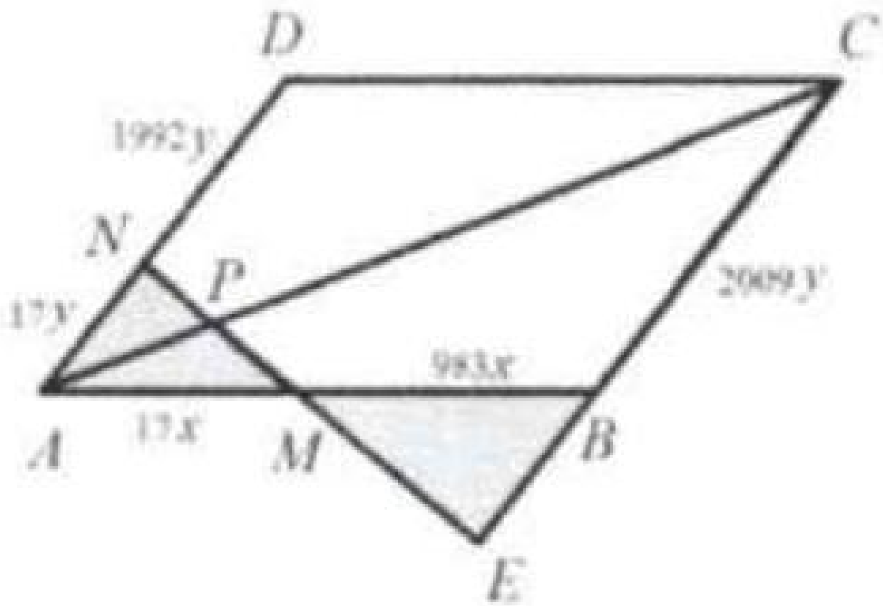


Figure 1

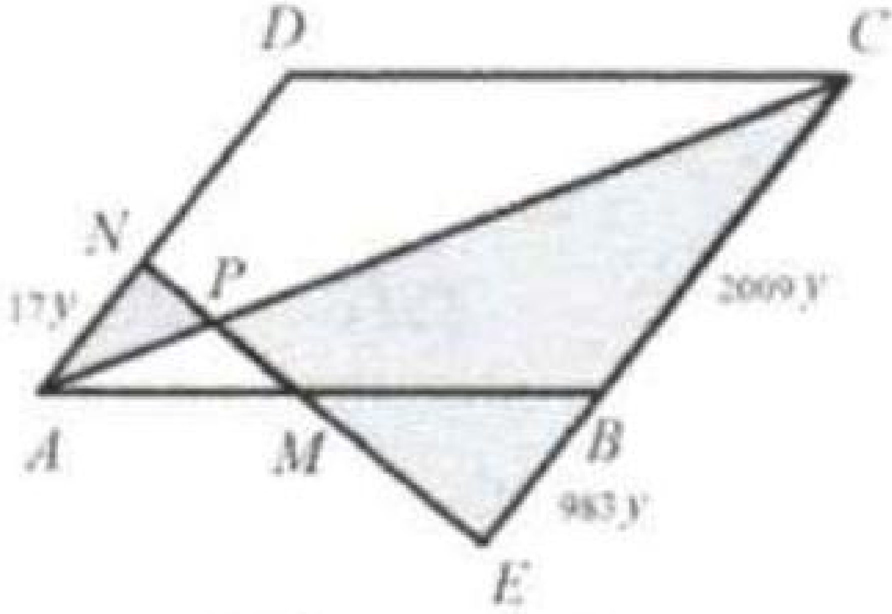


Figure 2