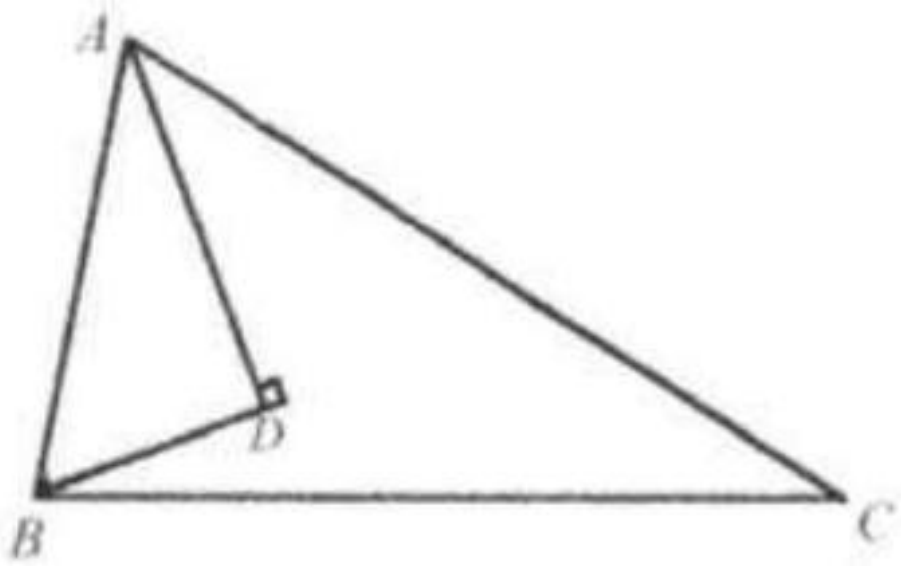
Problem 6

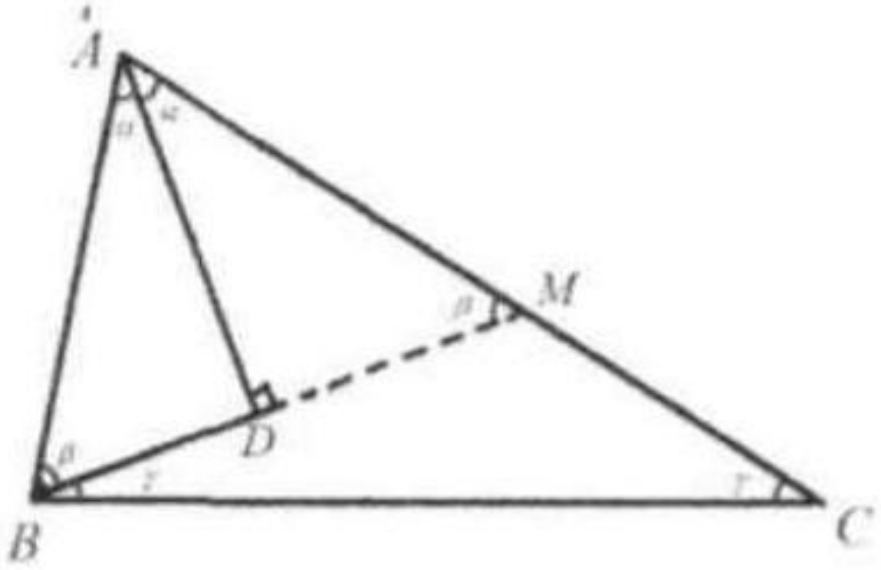
Problem

As shown in the figure below, in $\triangle ABC, \angle ABD = 2\angle C. AD$ is the angle bisector of $\angle A. BD \perp AD$ at D. Show that $BD = \frac{1}{2}(AC - AB)$.



Solution

Extend BD to meet AC at M.
Since AD is the angle bisector of $\angle A.$ and $BD \perp AD$,
$\angle BAD = \angle MAD = \alpha, AB = AM, \angle ABD = \angle AMD = \beta.$
Since $\angle ABD = \angle AMD = 2\angle C, \angle MCB = \angle MBC = \gamma$
$AM = 2BD = MC = AC - AM \Rightarrow BD = \frac{1}{2}(AC - AB).$


A
M
D
B
C