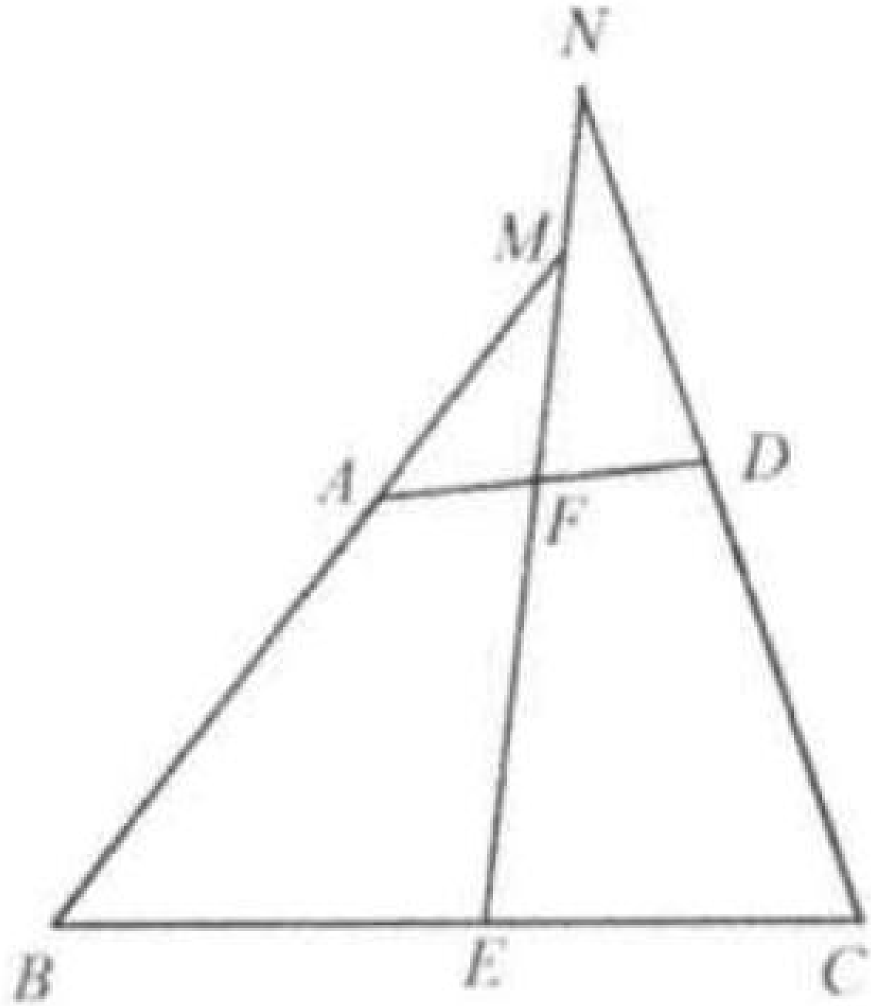
Example 12

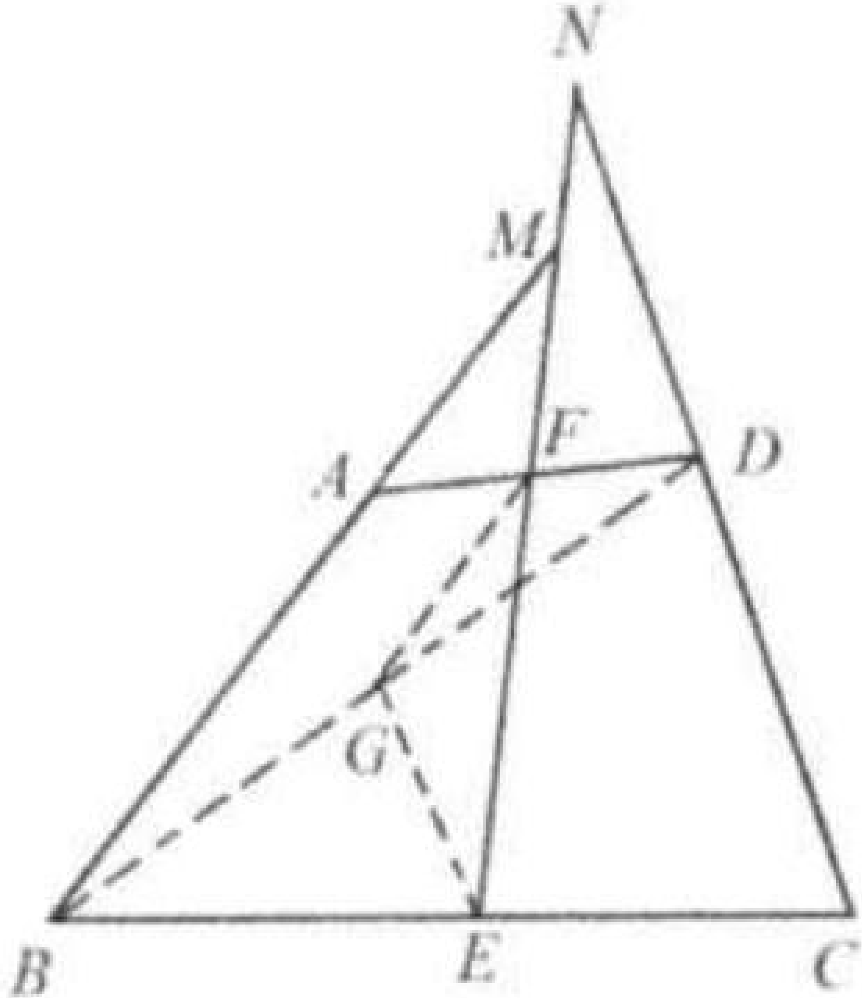
$ABCD$ is a convex quadrilateral. $AB = CD.E, F$ are midpoints of BC, AD, respectively. The extensions of BA and CD meet the extension of EF at M, N, respectively. Show that $\angle BME = \angle CNE$.

Solution: Method 1:

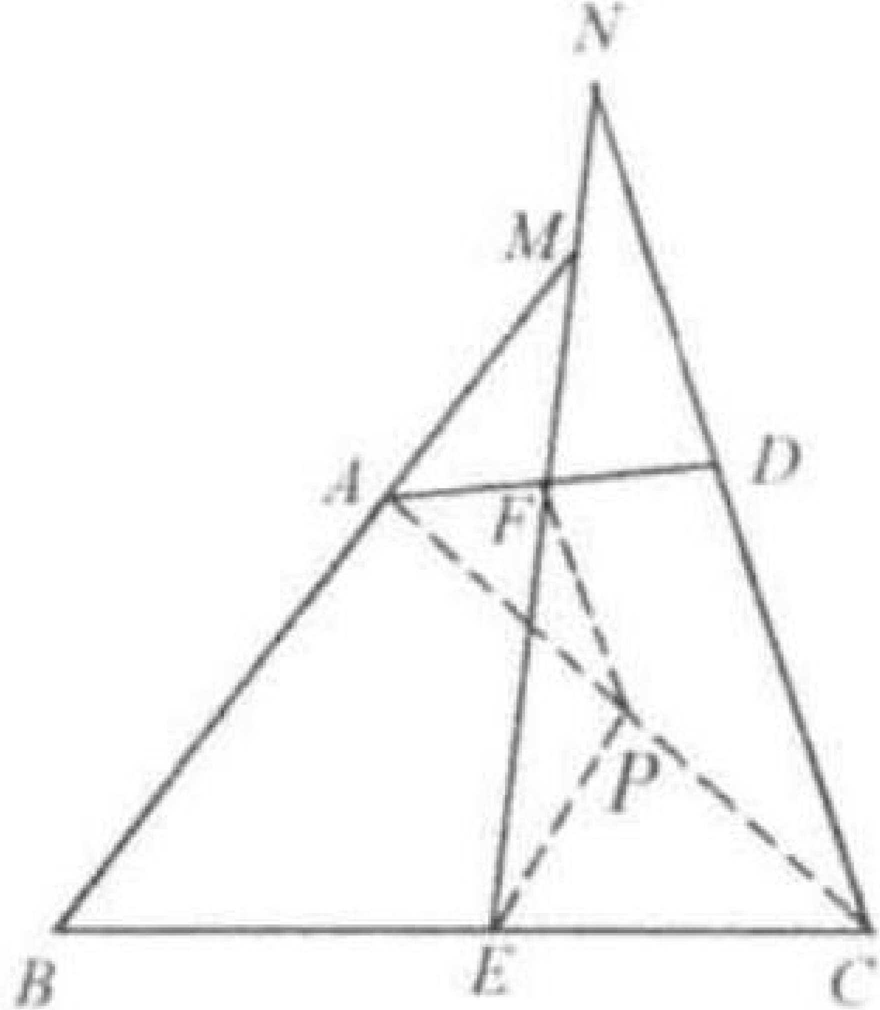
Connect BD. Take G, the midpoint of BD. Connect GF, GE.



Since GF is the midline of $\triangle DAB$.
$GF//AB$ and $GF = \frac{1}{2}AB$
GE is the midline of $\triangle BDC$.
$GE//CD$ and $GE = \frac{1}{2}CD$.
Since $AB = CD, GF = GE$.



Thus $\angle GFE = \angle GEF$.
We know that $GF//AB$, so $\angle BME = \angle GFE$.
We also know that $GE//CD//CN$, so $\angle CNE = \angle GEF$
Therefore, $\angle BME = \angle CNE$.
Method 2:
Connect AC. Take P, the midpoint of AC. Connect EP, EP.
Since EP is the midline of $\triangle CAB, EP//AB$ and $EP = \frac{1}{2}AB$.
FP is the midline of $\triangle ADC$.
$FP//CD$ and $FP = \frac{1}{2}CD$.



Since $AB = CD, EP = FP$. Thus $\angle PFE = \angle PEF$.
We know that $EP//AB$, so $\angle BME = \angle PEF$.
We also know that $FP//CD//CN$, so $\angle CNE = \angle PFE$
Therefore, $\angle BME = \angle CNE$.