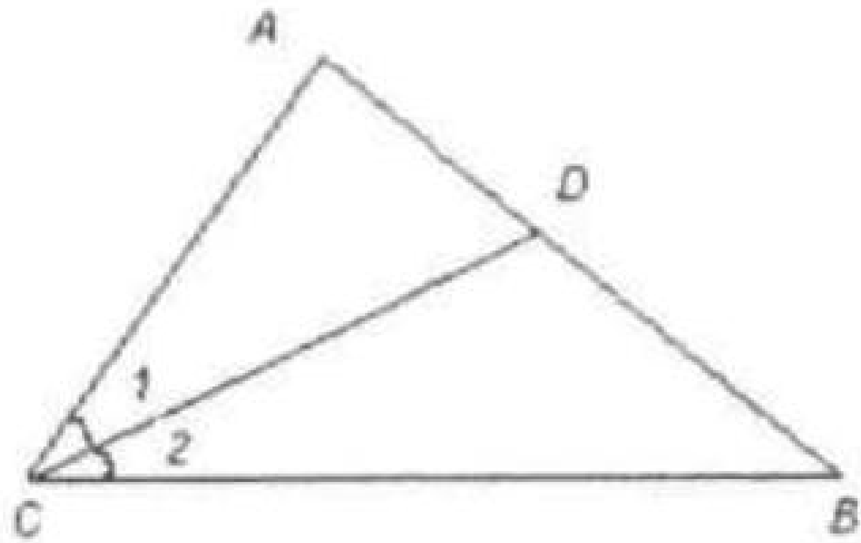
Example 4

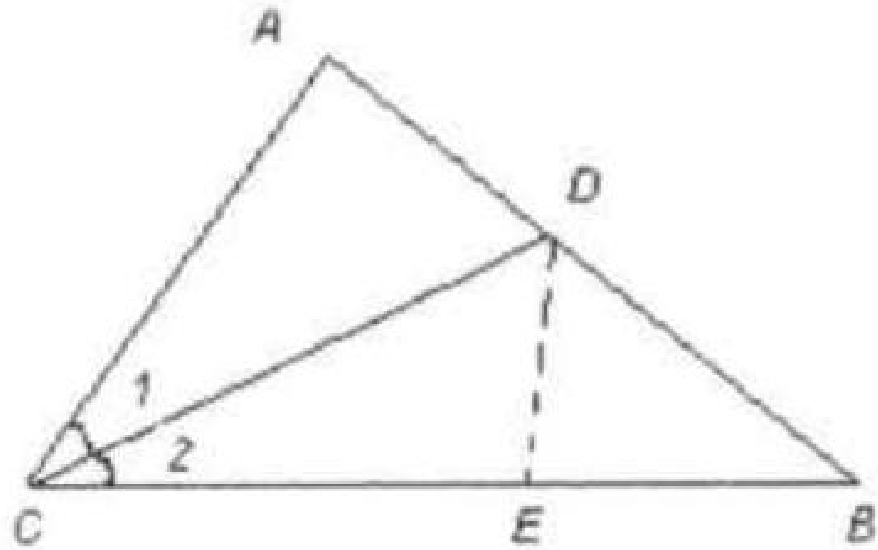
As shown in the figure below, in $\triangle ABC, \angle CAB = 2\angle ABC$. CD is the angle bisector of $\angle ACB$. Show that $BC = AC + AD$.

Solution: Method 1:



Draw DE so that $CE = AC$. Since $\angle 1 = \angle 2$, and $CD = CD$, we have $\triangle ACD \cong \triangle ECD$. Therefore $\angle CED = \angle CAB$, and $AD = DE$.

Since $\angle CAB = 2\angle ABC, \angle CED = 2\angle ABC$, and $\angle CED =$



$\angle DBE + \angle EDB$.

Hence $\angle EDB = \angle EBD \Rightarrow DE = EB$.

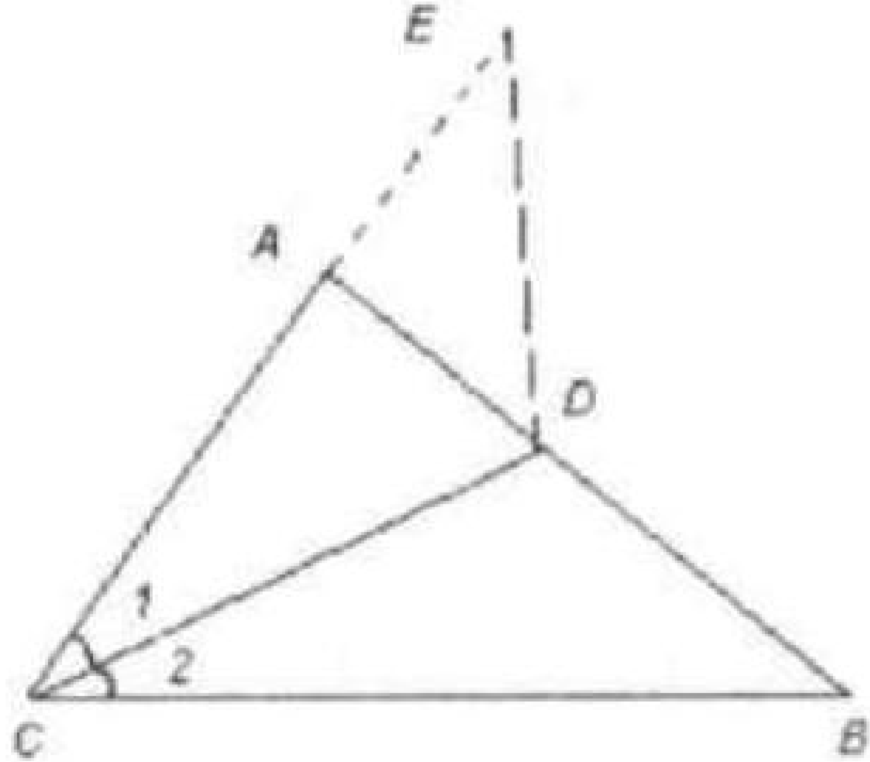
Therefore $BC = CE + EB = AC + DE = AC + AD$.

Method 2:

Extend CA to E such that $AE = AD$. Therefore $\angle E = \angle ADE$, and $\angle CAB = 2\angle E$.

Since $\angle CAB = 2\angle ABC, \angle E = \angle ABC$.

Since $\angle 1 = \angle 2$ and $CD = CD, \triangle CED \cong \triangle CBD$.

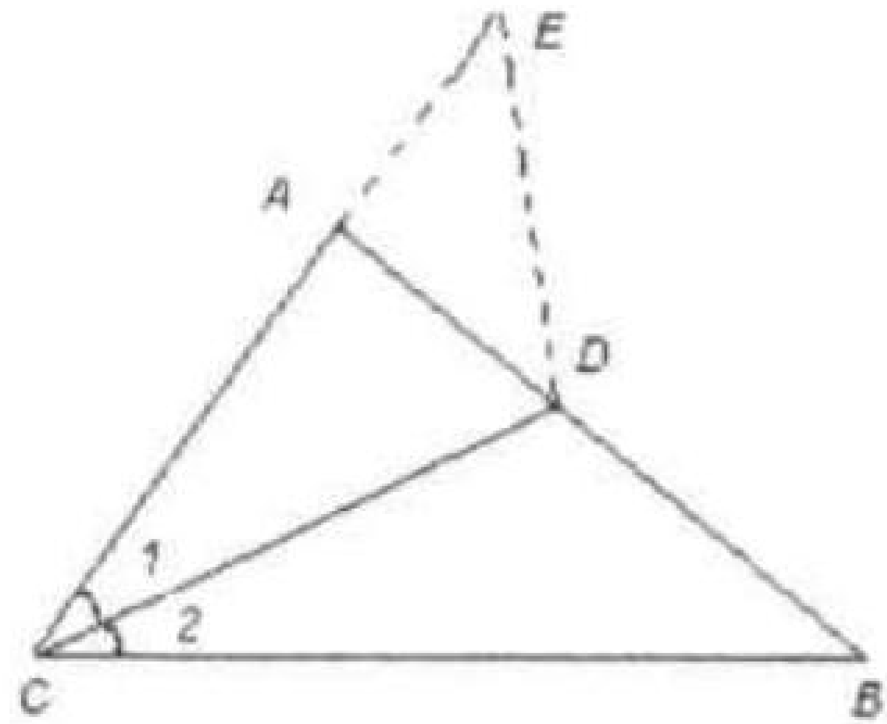


Therefore $CE = CB.BC = CE = CA + AE = CA + AD$.

Method 3:

Extend CA to E so that $CE = CB$. Then $\triangle CDE \cong \triangle CDB, \angle E = \angle B$.

Since $\angle A = 2\angle B$, and $\angle A = \angle E + \angle ADE$, so $2\angle B = \angle E$



$+\angle ADE$ or $2\angle E = \angle E + \angle ADE$, or $\angle E = \angle ADE$, i.e, triangle ADE is an isosceles triangle with $AD = AE$. Therefore $BC = CA + AE = AC + AD$.