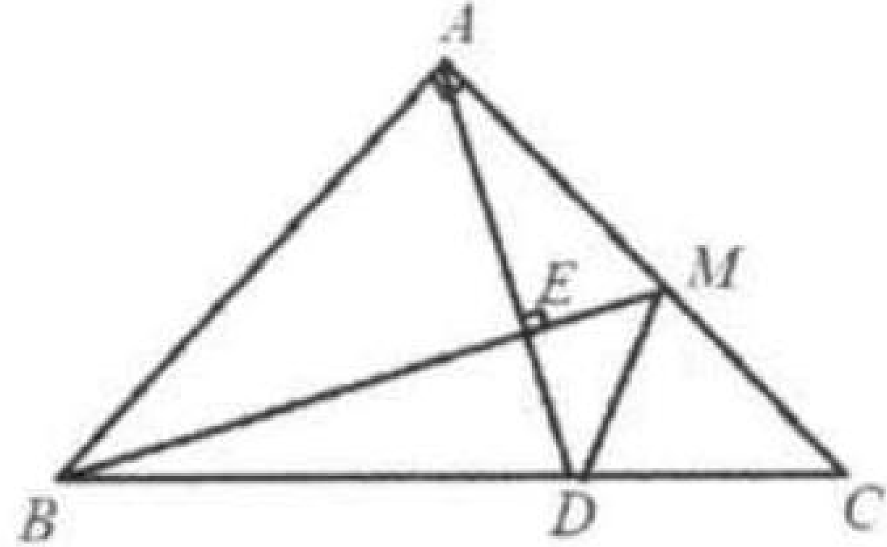
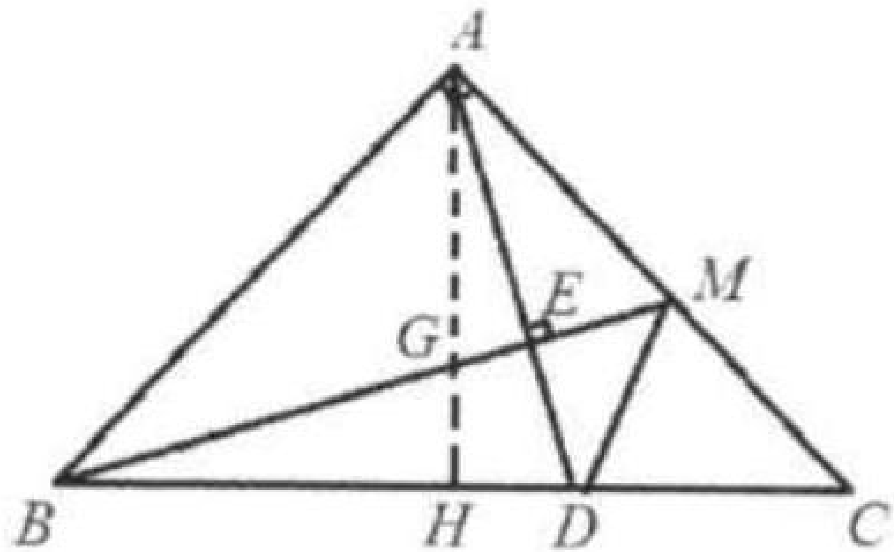
Problem

Given $\triangle ABC, \angle A = 90^\circ. AB = AC. M$ is the midpoint of $AC. AE \perp BM$ with the feet at E. Extend AE to meet BC at D. Prove that $\angle AMB = \angle CMD$.



Solution

Draw the angle bisector of $\angle A$ to meet BM at G and BC at H.
Since $AB = AC, \angle A = 90^\circ, \angle BAG = 45^\circ$.
$\angle EBA = 90^\circ - \angle EAB = 90^\circ - (\angle BAG + \angle EAG) = 90^\circ - (45^\circ + \angle EAG) = 45^\circ - \angle EAG = \angle CAD$.
In $\triangle ABG$ and $\triangle ADC$, since $\angle EBA = \angle CAD, \angle BAG = \angle C$,



$AB = AC, \triangle ABG \cong \triangle ADC$. Thus $AG = CD$.
In $\triangle AMG$ and $\triangle CMD$, since
$\angle GAM = \angle DCM = 45°, AM = CM, AG = CD$, $\triangle AMG \cong \triangle CMD$. Thus
$\angle AMB = \angle CMD$.