Example 8

(1995 AMC) In the figure, AB and CD are diameters of the circle with center $O, AB \perp CD$, and chord DF intersects AB at E. If $DE = 6$ and $EF = 2$, then the area of the circle is

(A) 23π
(B) $\frac{47}{2}\pi$
(C) 24π
(D) $\frac{49}{2}\pi$
(E) 25π

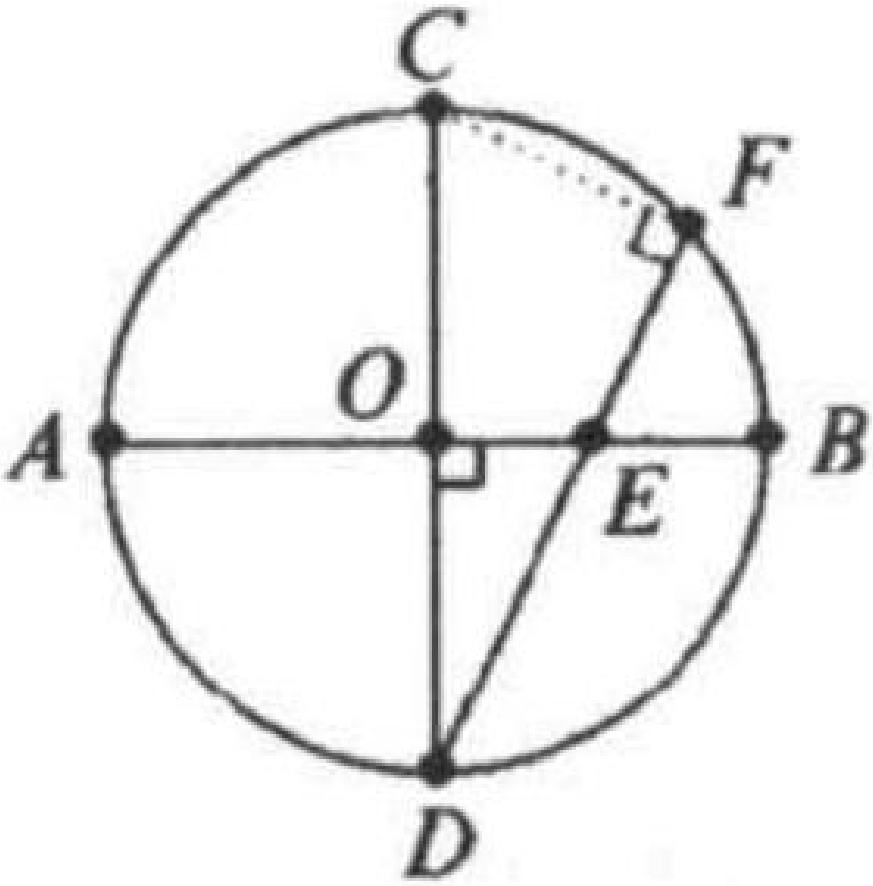
Solution: (C).

Draw segment FC. Angle CFD is a right angle since arc CFD is a semicircle. Then right triangles DOE and DFC are similar to each other, so the following equality holds true:

$$\frac{DO}{DF} = \frac{DE}{DC}.$$

Let $DO = r$ and $DC = 2r$. Substituting this into the equality above, we have

$$\frac{r}{8} = \frac{6}{2r} \quad \Rightarrow \quad 2r^2 = 48 \quad \Rightarrow$$



$r^2 = 24.$

The area of the circle is $\pi r^2 = 24\pi$.