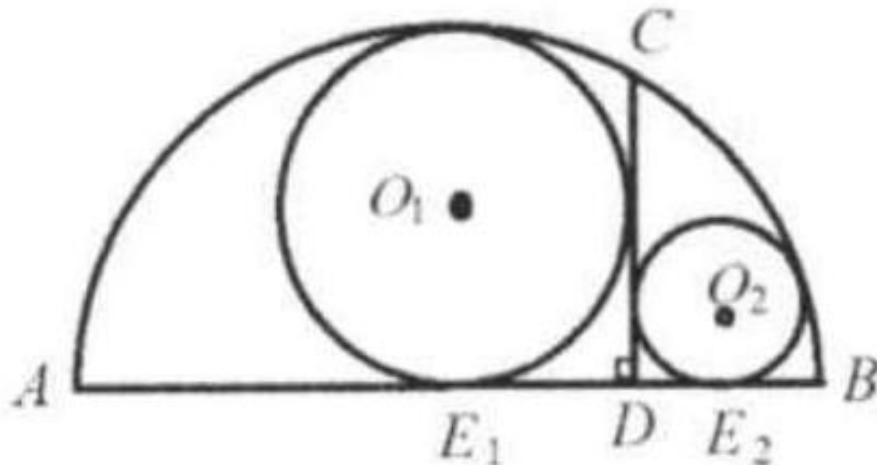# Problem

AB is the diameter of the semicircle as shown in the figure. $CD \perp AB$, with the foot at D. Two circles O_1 ($r_1 = 12$) and O_2 ($r_2 = 8$) inscribed in the semicircle and are tangent to CD. The tangent points to AB are E_1 and E_2, respectively. Find the length of AB.
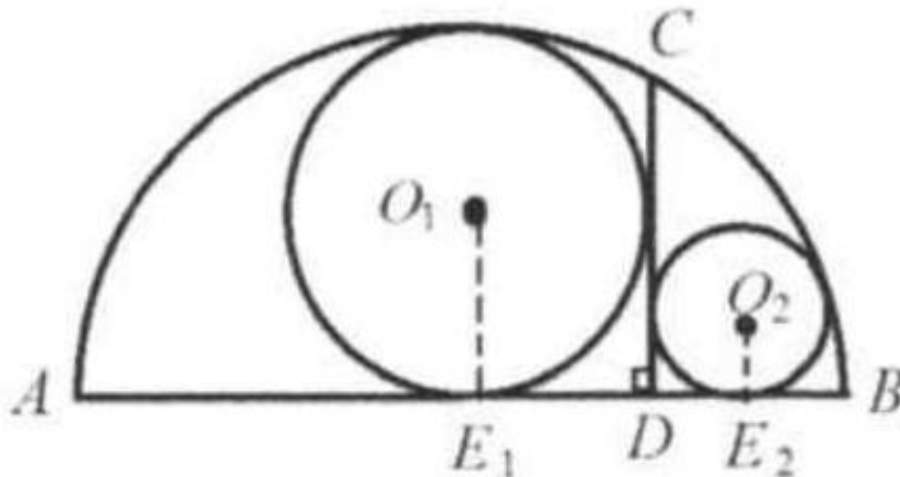

Solution

Connect $O_1 E_1$ and $O_2 E_2$.
$E_1 D = O_1 E_1 = 12. DE_2 = O_2 E_2 = 8$.
Let $AD = x, DB = y$.
Then $AE_1 = x - 12, E_1 B = y + 12, AE_2 = x + 8, E_2 B = y - 8$.

Then we have $\frac{1}{AE_1} + \frac{1}{E_1 B} = \frac{1}{O_1 E_1}$

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$$\frac{1}{AE_2} + \frac{1}{E_2 B} = \frac{1}{O_2 E_2}$$

Or $\quad \frac{1}{x-12} + \frac{1}{y+12} = \frac{1}{12}$

$$\frac{1}{x+8} + \frac{1}{y-8} = \frac{1}{8}$$

Solving we get $x = 32, y = 18$.
$$AB = x + y = 50.$$