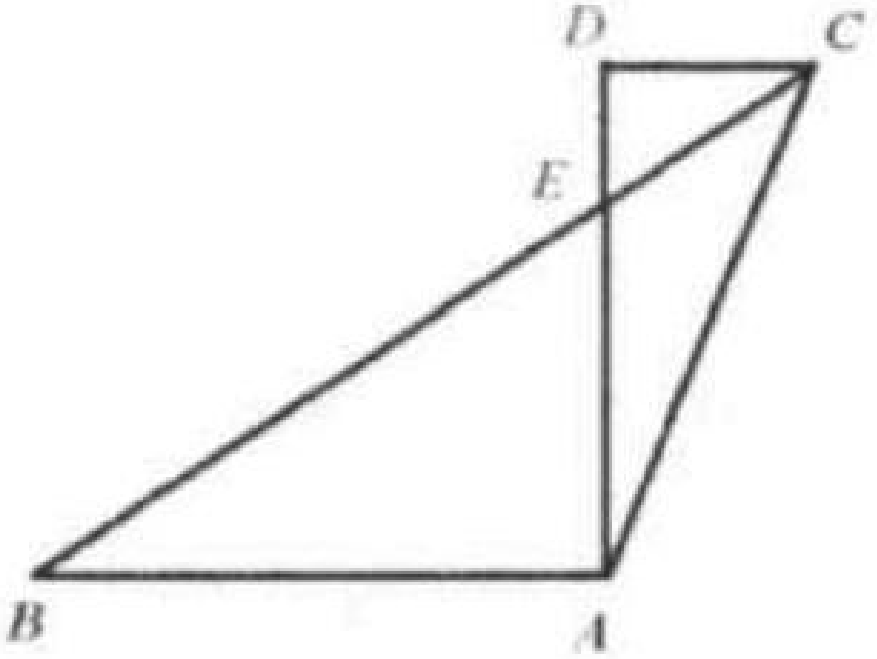
Problem 8

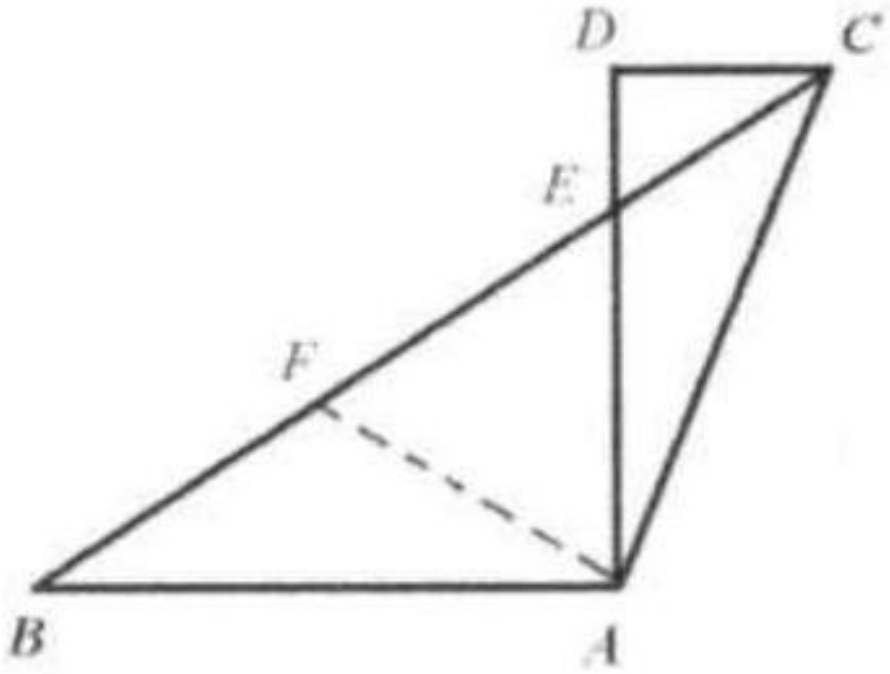
Problem

As shown in the figure, $AB//CD.AD \perp AB.AD$ and BC meet at E such that $EB = 2AC$. Show that $\angle ACD = 3\angle BCD$.

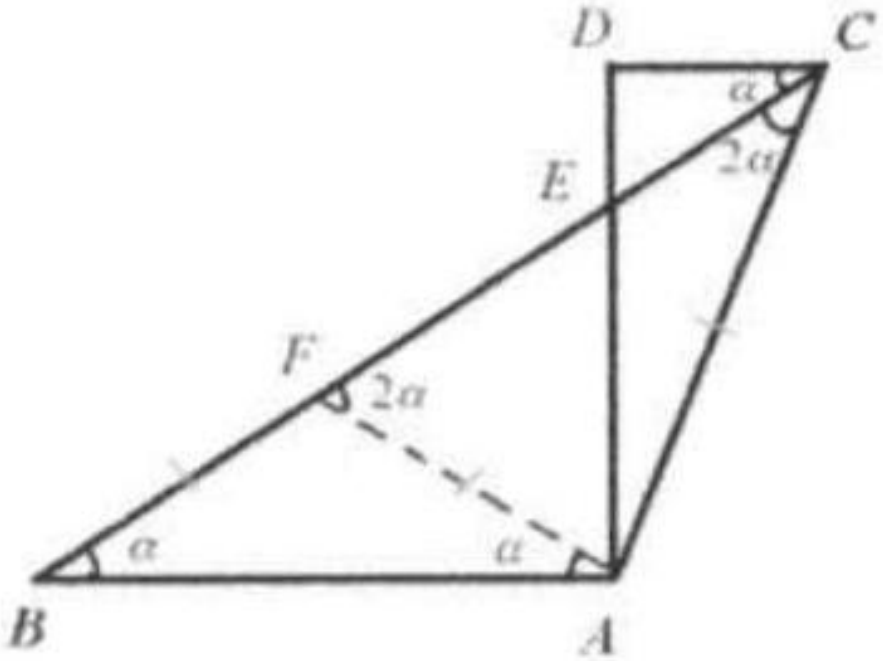


Solution

Draw AF, the median of right triangle BAE. Since AF is the median, by Theorem 1.3, $AF = BF = NC$.
Thus $AF = AC$.
Thus both triangles AFB and CAF are isosceles triangles.



Let $\angle B = \angle FAB = \alpha$.
$\angle CFA$ is the exterior angle of triangle ABF. So $\angle CFA = \angle ACF = 2\alpha$.
Since $AB//CD, \angle B = \angle DCE = \alpha$.
$\angle ACD = \angle ACF + \angle DCE = 3\alpha = 3\angle BCD$.


D
C
α
2α
E
F
2α
α
α
B
A