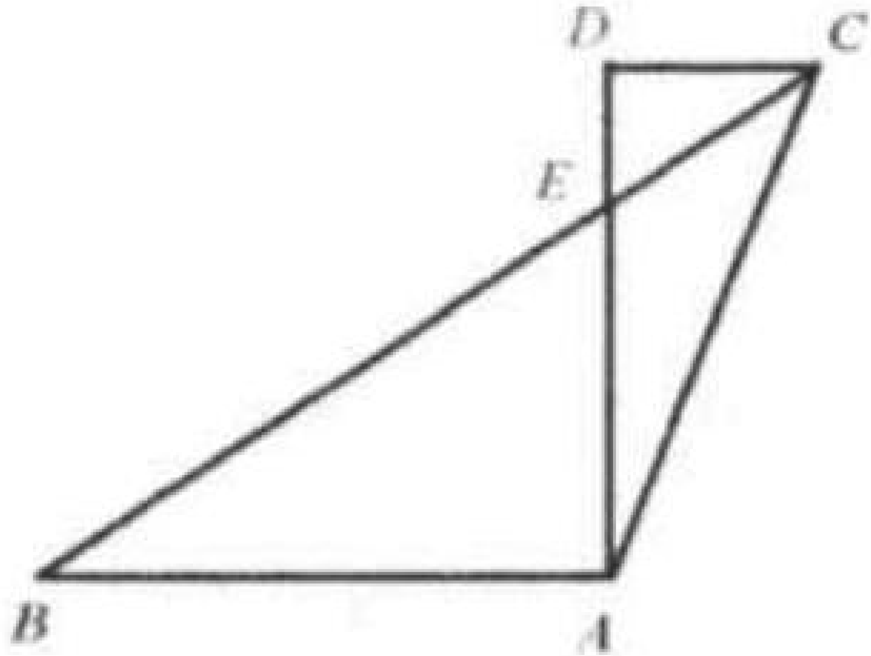
Problem

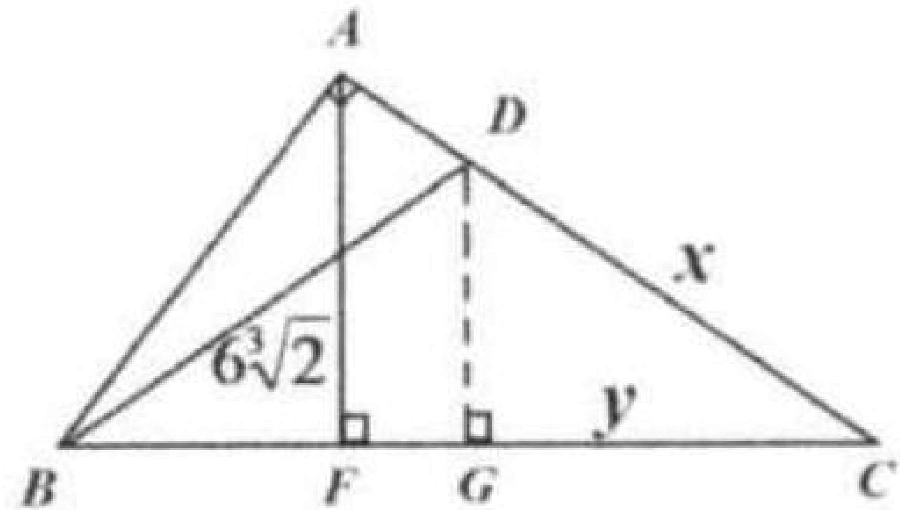
As shown in the figure, $AB//CD.AD \perp AB.AD$ and BC meet at E such that $EB = 2AC$. Show that $\angle ACD = 3\angle BCD$.



Solution

(C).

Draw $DG \perp BC$ where G is on BC. Let $AC = x$ and $GC = y$. We know that BCD is isosceles so $BC = 2y$.



Since $\triangle DCG, \triangle ACF$, and $\triangle BCA$ are similar, we have: $\frac{DC}{CG} = \frac{AC}{CF} = \frac{BC}{AC}$

$$\Rightarrow \quad \frac{6\sqrt[3]{2}}{y} = \frac{x}{6\sqrt[3]{2}} = \frac{2y}{x}.$$

So $y = \frac{36\sqrt[3]{4}}{x}$ and $y = \frac{2x^2}{6\sqrt[3]{2}} \cdot x^3 = \frac{36\sqrt[3]{4} \times 6\sqrt[3]{2}}{2} = 6^3 \Rightarrow \quad x = 6.$