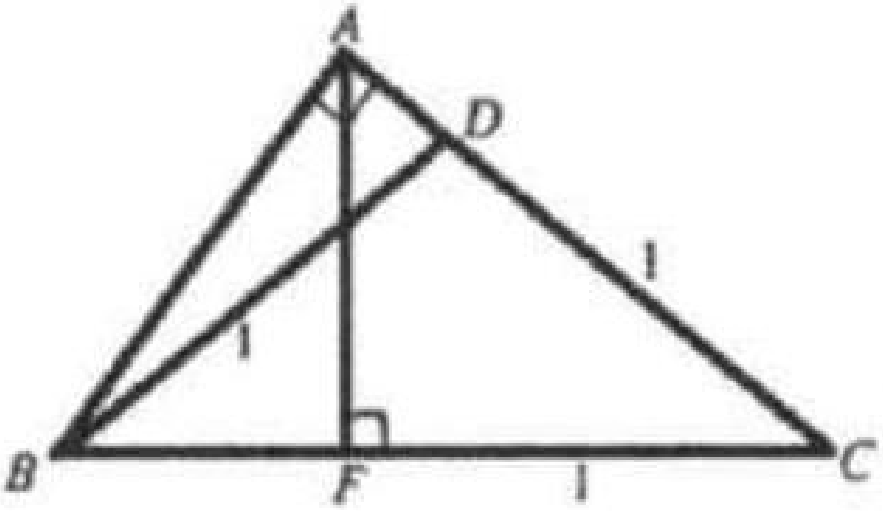
Example 8

(AMC) In $\triangle ABC, D$ is on AC and F is on BC. Also $AB \perp AC, AF \perp BC$, and $BD = DC = FC = 1$. Find AC.

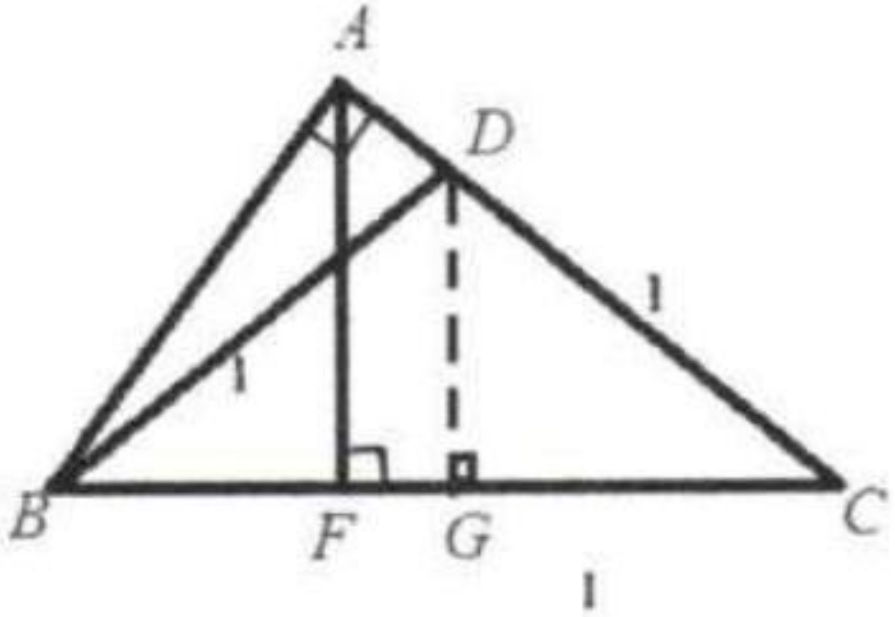
Solution: $\sqrt[3]{2}$

Draw DG so that $DG \perp BC$ and G lies on BC. Let $AC = x$



and $GC = y$. Note that $BC = 2y$, since $\triangle BCD$ is isosceles.

Since $\triangle DCG \sim \triangle ACF \sim \triangle BCA$, we obtain the equal ratios: $\frac{DC}{GC} = \frac{AC}{CF} = \frac{BC}{AC} \quad \Rightarrow \quad \frac{1}{y} = \frac{x}{1} = \frac{2y}{x}$. Thus $y = \frac{1}{x}$ and $y = \frac{x^2}{2}$, implying that



$x^3 = 2$, or $x = \sqrt[3]{2}$.