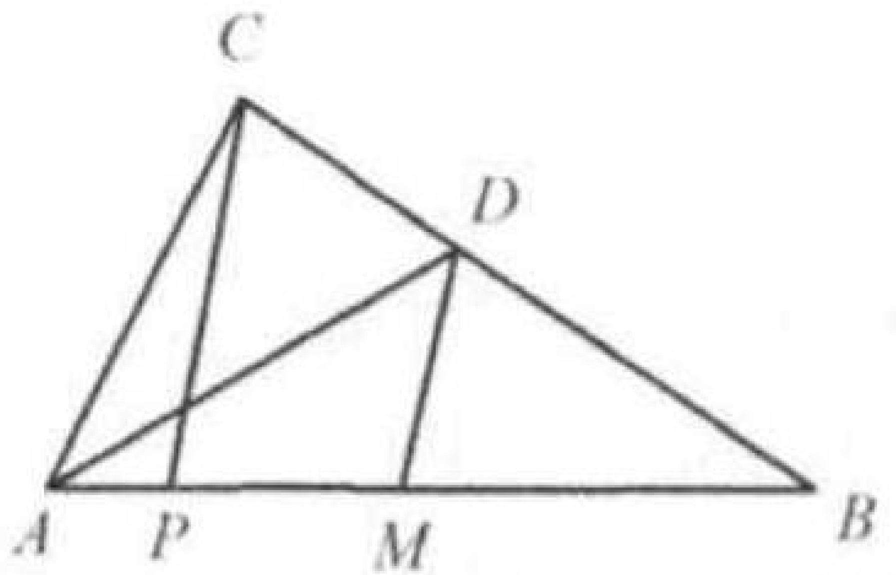
Example 13

Let M be the midpoint of side AB of triangle ABC. Let P be a point on AB between A and M, and let MD be drawn parallel to PC and intersecting BC at D. If the ratio of the area of triangle BPD to that of triangle ABC is denoted by r, then

(A) $\frac{1}{2} < r < 1$ depending upon the position of P

(B) $r = \frac{1}{2}$ independent of the position of P

(C) $\frac{1}{2} \leq r < 1$ depending upon the position of P



(D) $\frac{1}{3} < r < \frac{2}{3}$ depending upon the position of P

(E) $r = \frac{1}{3}$ independent of the position of P

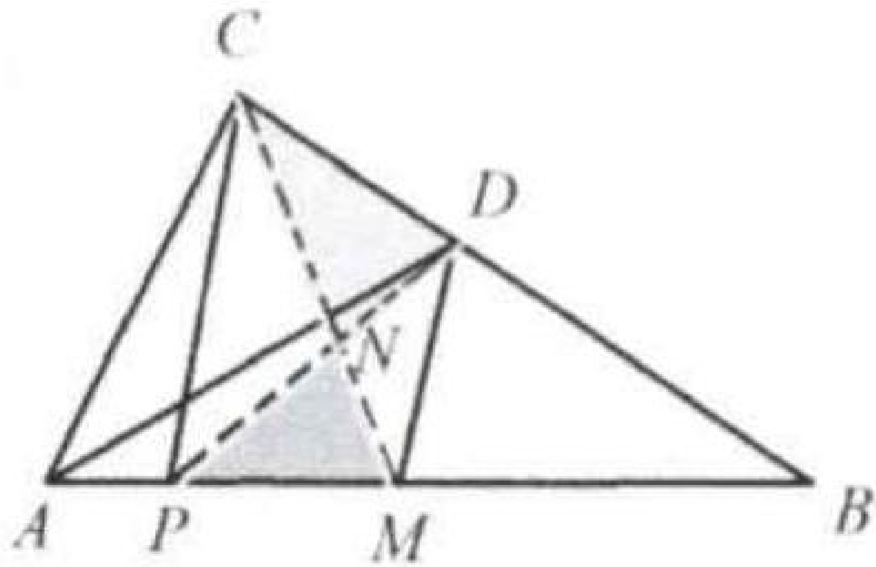
Solution: (B).

Let $r = \frac{S_{\triangle BPD}}{S_{\triangle ABC}}$

Draw the median CM. Connect DP. Let the intersection point be N.

Since $PC//MD, S_{\triangle CDN} = S_{\triangle MPN}$.

Thus $S_{\triangle BPD} = S_{\triangle BCM}$



Since CM is the median, $S_{\triangle BPD} = S_{\triangle BCM} = \frac{1}{2} S_{\triangle ABC}$
Substituting (2) into (1): $r = \frac{S_{\triangle BPD}}{S_{\triangle ABC}} = \frac{1}{2}$.