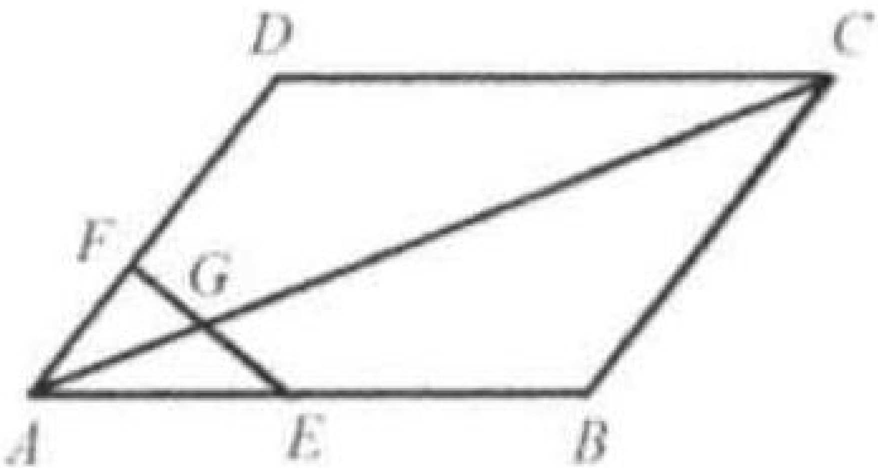
Example 20

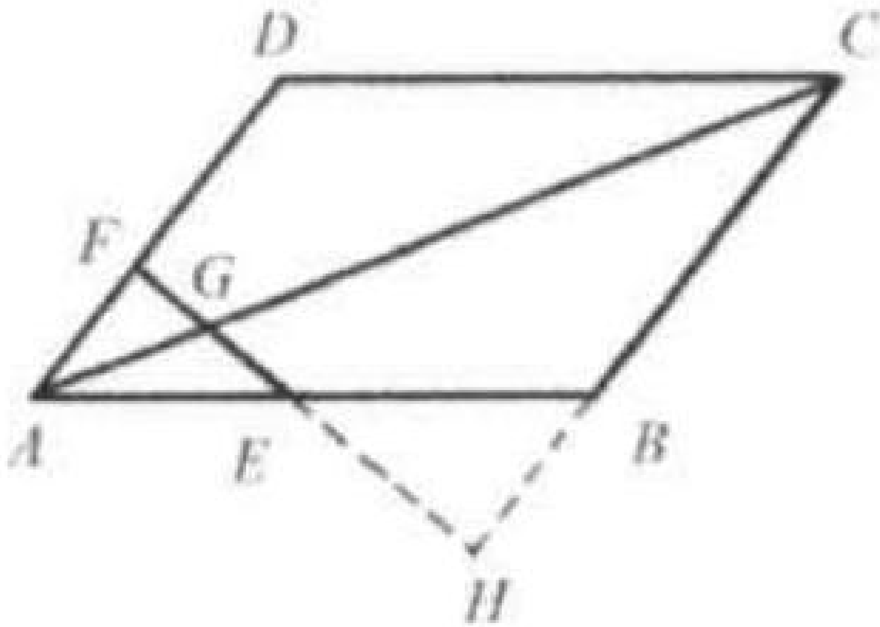
In parallelogram $ABCD$, point E is the midpoint of AB, and point F is on AD so that $\frac{AF}{AD} = \frac{1}{3}$. Let G be the point of intersection of AC and FE. Find $\frac{AC}{AG}$.



Solution: 5. Extend FE through E to H and to meet the extension of CB at H.

We know that $AD//CH$. So $\triangle AEF \sim \triangle BEH$ (Figure 1).

$\frac{AF}{BH} = \frac{AE}{EB} \quad \Rightarrow BH = AF$. So we know that $CH = CB$ $+BH = AD + AF = 4AF$.



We know that $AF//CH$. So $\triangle AGF \sim \triangle CGH$ (Figure 2). $\frac{AF}{CH} = \frac{AG}{GC} \Rightarrow$

$$\frac{AF}{4AF} = \frac{AG}{GC} = \frac{1}{4} \quad \Rightarrow \quad \frac{GC}{AG} = 4 \quad \Rightarrow \quad \frac{AC - AG}{AG} = 4 \Rightarrow$$
$$\frac{AC}{AG} - 1 = 4 \quad \Rightarrow \quad \frac{AC}{AG} = 5.$$

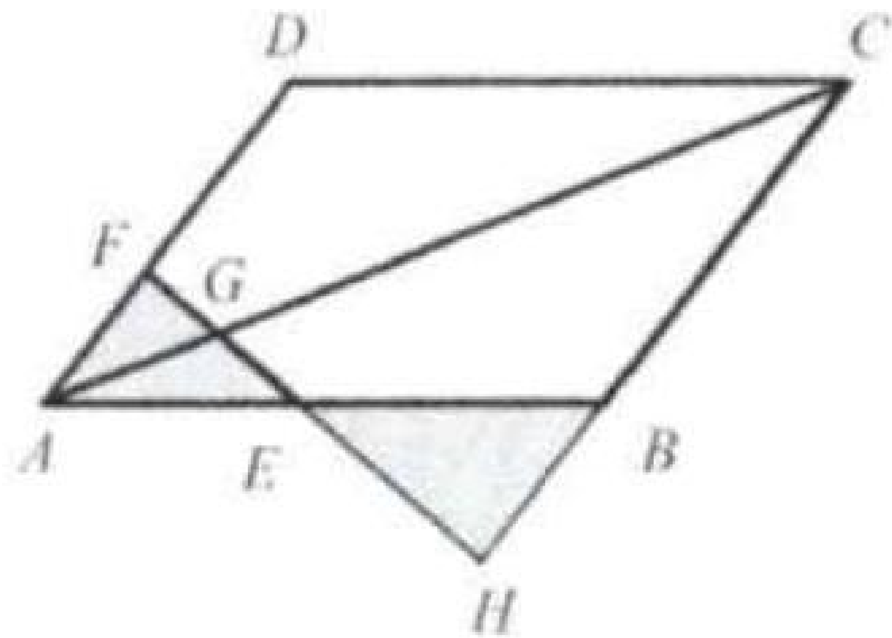


Figure 1

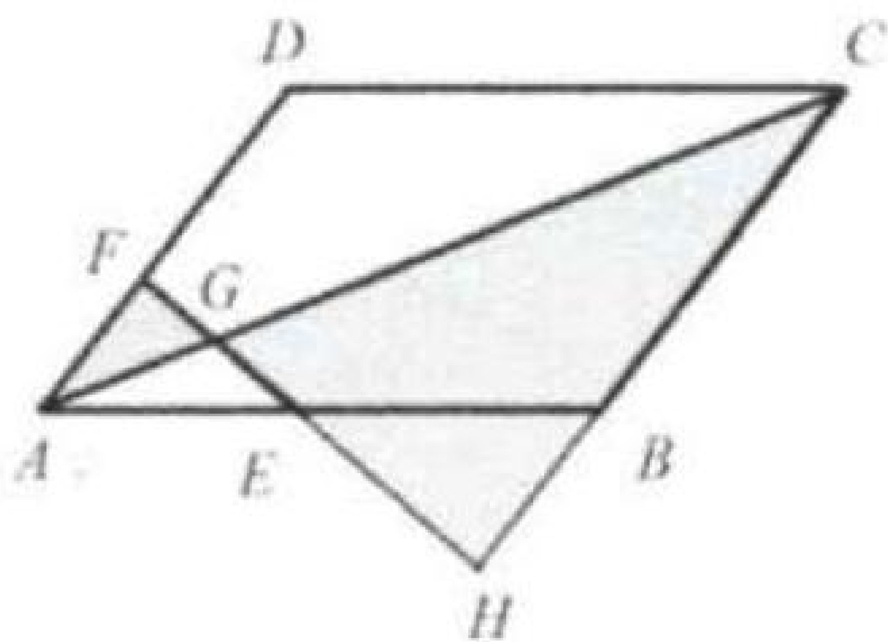


Figure 2