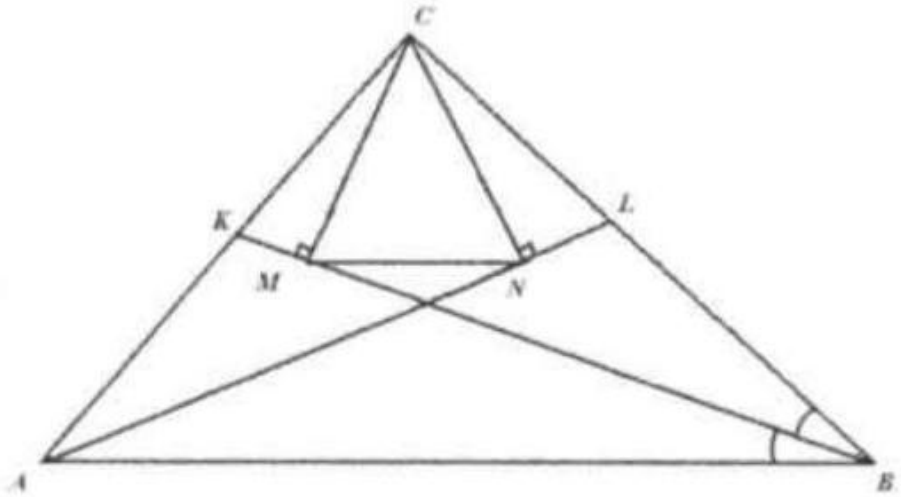
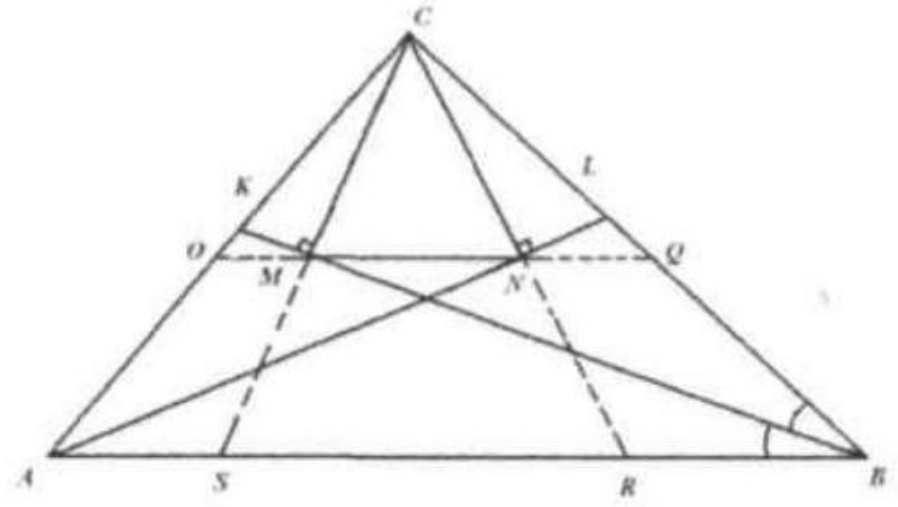
Problem

In triangle ABC, the angle bisector of angle A intersects BC at point L, and the angle bisector of angle B intersects AC at point K. Let M and N be the feet of the perpendiculars from C to BK and AL, respectively. Show that $MN = \frac{1}{2}(BC + AC - AB)$.



Solution

Extend MN such that it intersects lines AC and BC at point O and Q, respectively. Extend CM to meet AB at, say, S. then triangle BCM and triangle BSM are congruent. Hence $BS = BC = 120$. Similarly, extend CN to meet AB at, say, R, and triangle ACN and triangle ARN are congruent. Hence $AR = AC$. So $CM = MS$, and $CN = NR$. So MN is the midline of triangle CSR.



$MN = \frac{SR}{2}$.
$SR = AR - AS = AC - (AB - BS) = AC - (AB - BC) = BC + AC - AB$.
Thus $MN = \frac{1}{2}(BC + AC - AB)$.