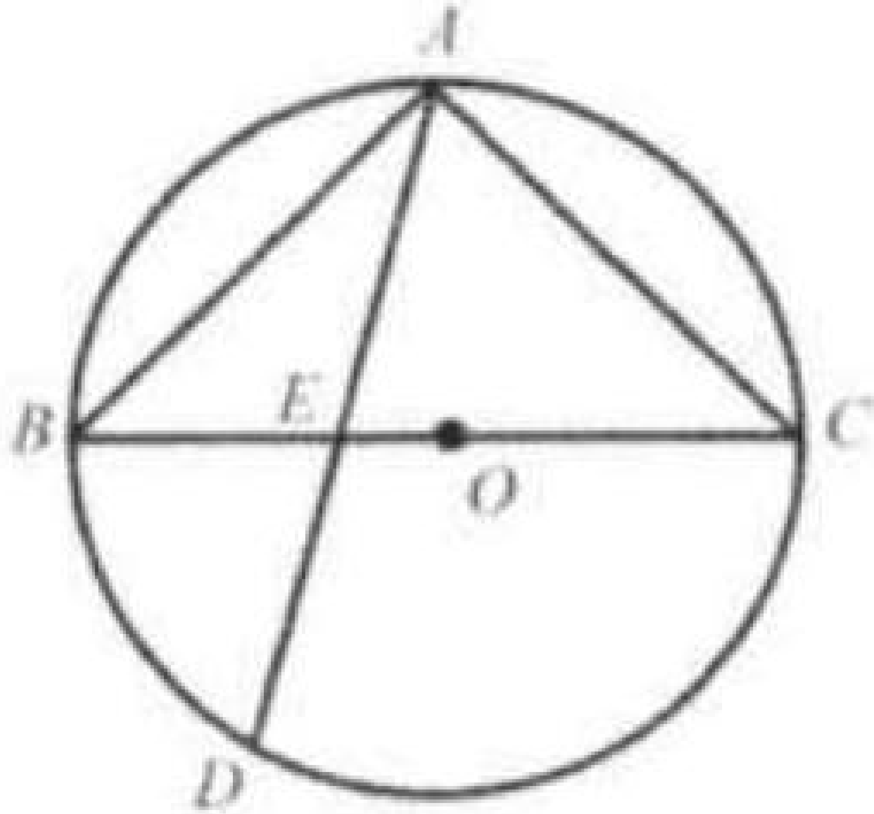
Problem 9

Problem

In a circle with center O chord $AB =$ chord AC. Chord AD cuts BC in E. If $AC = 12$ and $AE = 8$, then AD equals:

(A) 27
(B) 24
(C) 21
(D) 20
(E) 18



Solution

(E).

Since $AB = AC, \angle ACB = \angle ABC = \alpha$.

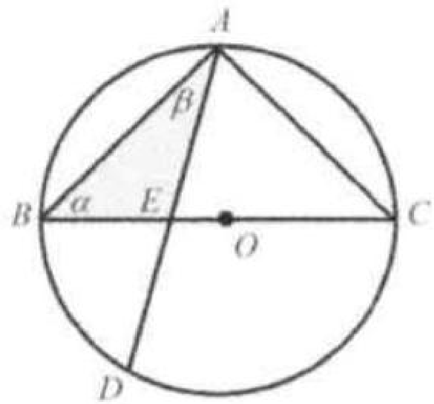
Connect BD.

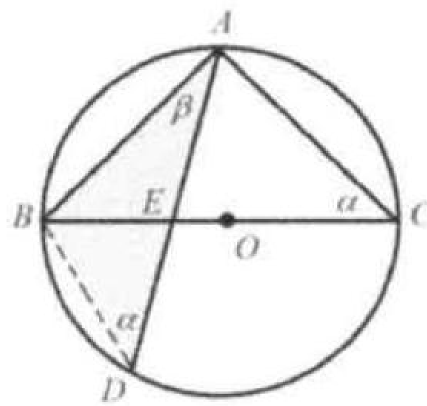
$\angle ADB = \angle ACB = \alpha$(they face the same arc AB).

Then triangles ABE and ADB are similar to each other, so the following equality holds true: $\frac{AB}{AD} = \frac{AE}{AB}$

$\Rightarrow \quad \frac{12}{AD} = \frac{8}{12}$

$\Rightarrow \quad AD = 18$





Method 2:

Connect CD.

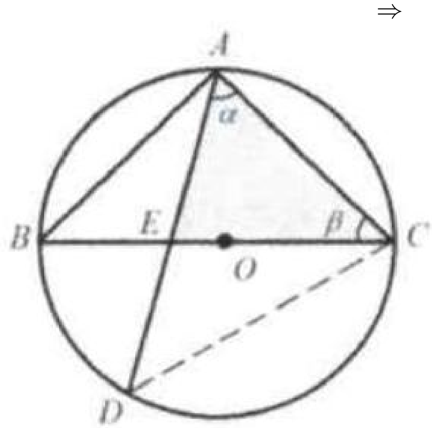
Let $\angle EAC = \angle BAC = \alpha$.

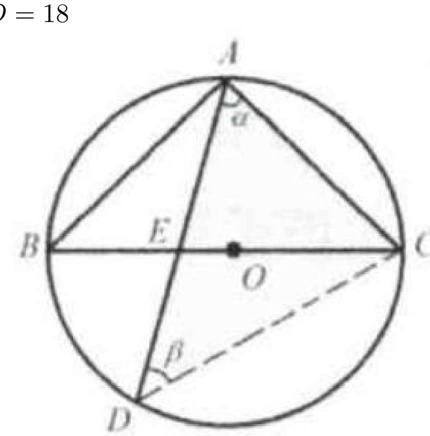
$\angle ACB = \angle ADC = \beta$ (they face the arcs of the same length: arcs AC, AB).

Then triangles AEC and ACD are similar to each other, so the following equality holds true: $\frac{AC}{AE} = \frac{AD}{AC}$

$\Rightarrow \quad \frac{12}{8} = \frac{AD}{12}$

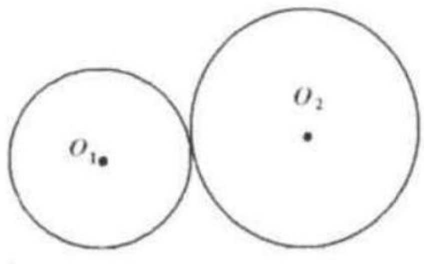
$\Rightarrow \quad AD = 18$

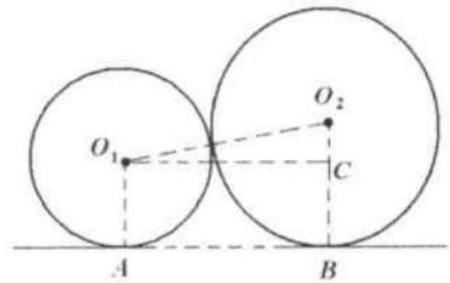




3. When two circles are tangent or intersecting, draw the common tangent

line, the common chords, or connect the centers. 3.1. Circle O_1 and O_2 are tangent. Draw the common tangent line AB. Connect O_1A and O_2B. Connect O_1O_2. Draw $O_1C//AB$.





We have

$AB = O_1C$.

$AB \perp O_1C$.

$AB \perp O_2B$.

$O_1O_2 = r_1 + r_2$.

$O_2C = r_2 - r_1$.

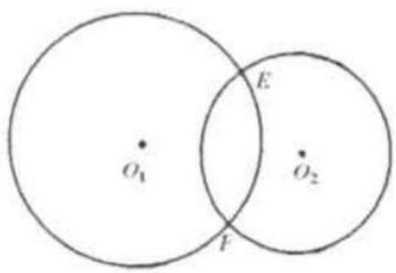
$\triangle O_1\mathrm{CO}_2$ is a right triangle.

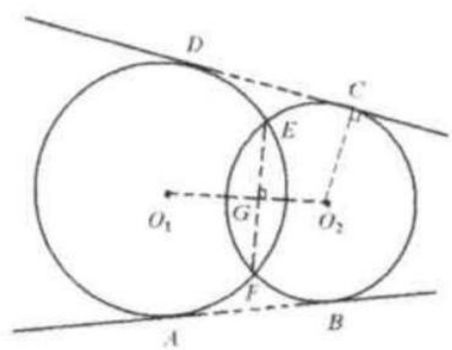
$$O_1C = \sqrt{(r_2 + r_1)^2 - (r_2 - r_1)^2}$$

r_1 and r_2 are the radius of circle O_1 and O_2, respectively.

3.2. Circle O_1 and O_2 are intersecting at E and F. Draw the common tangent lines AB, CD, respectively. Connect O_2C, EF, and O_1O_2.

O_1O_2 is the perpendicular bisector of $EF.O_2C \perp DC$.





Theorem 6.9. Any point on the perpendicular bisector of a line segment is equidistant from the endpoints of the line segment. Two points equidistant from the endpoints of a line segment determine the perpendicular bisector of the line segment.