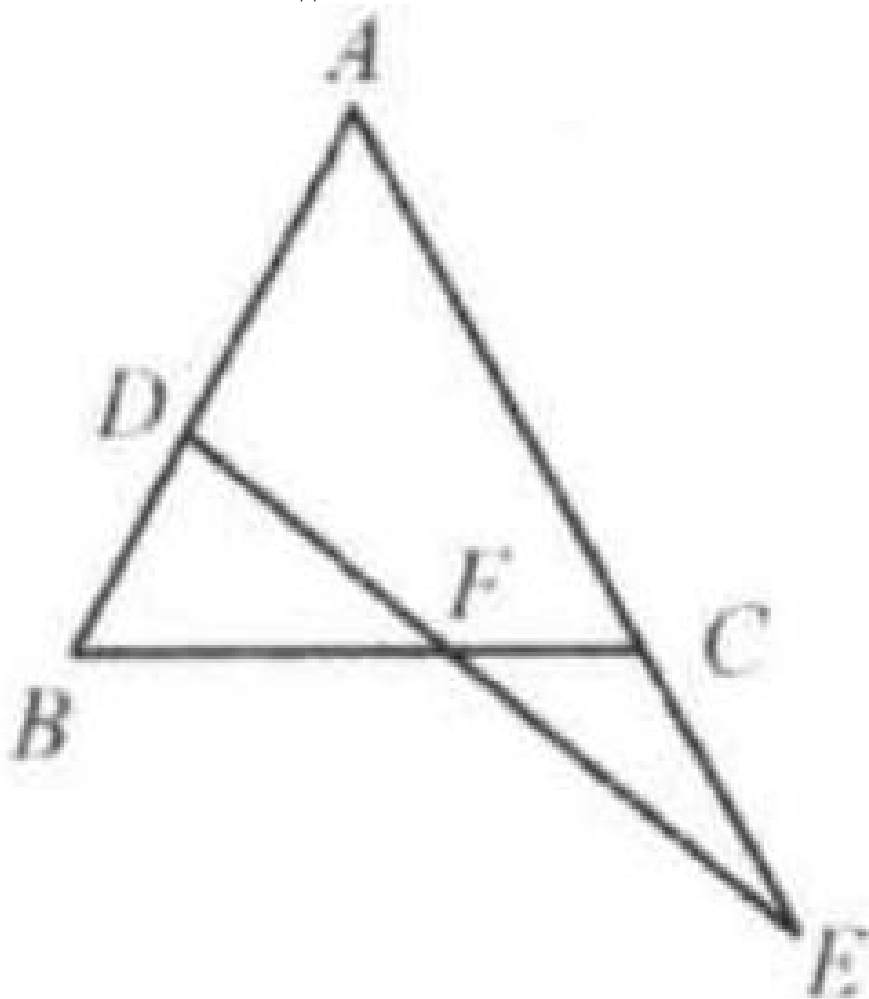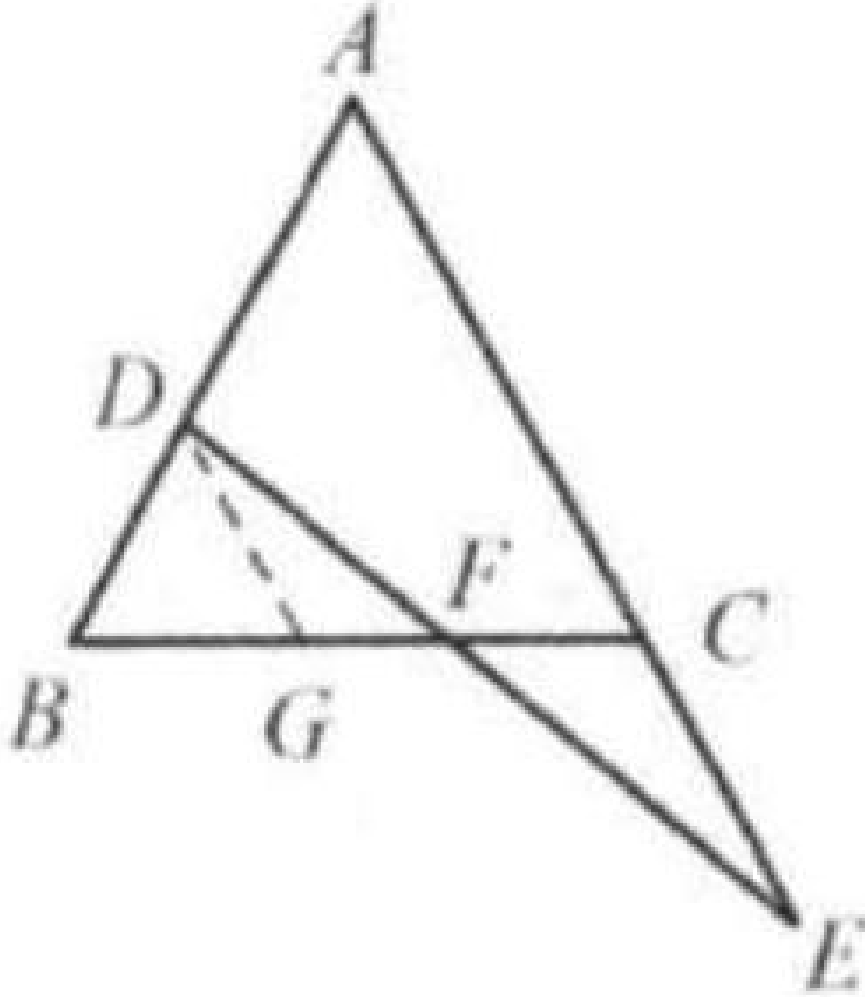# Example 1

Triangle ABC is an isosceles triangle. D is a point the on AB. Extend AC
to E and connect DE so that $BD = CE$. Prove: $DF = FE$.

Proof: Method 1:
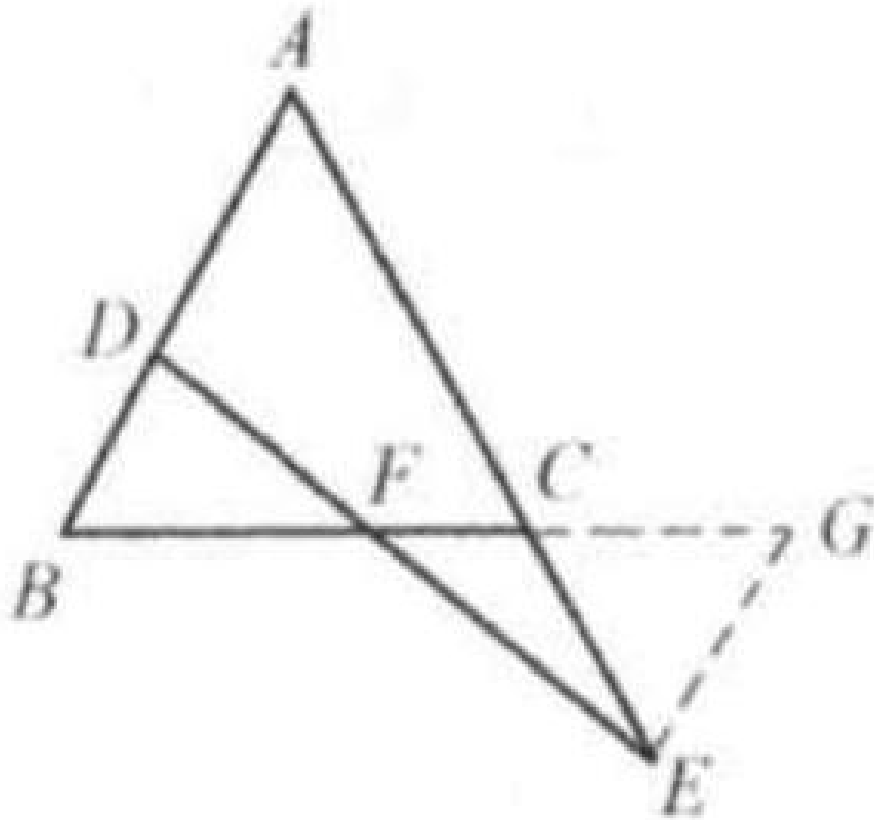
Draw $DG//CE$. We have ,

$\angle B = \angle DGB = \angle C$. So $BD = DG = CE$.
We also know that $\angle GDF = \angle CEF, \angle DGF = \angle ECF$.
Thus $\triangle DGF \cong \triangle ECF (ASA)$.
So $DF = FE$.

Method 2:
Draw $EG//BG$ to meet the extension of BC at G.
We see that $\angle B = \angle G = \angle ACB = \angle ECG$, so $CE = GE = BD$.
In $\triangle DFB$ and $\triangle EFG$, $\angle BDF = \angle GEF, GE = BD, \angle B = \angle G$.
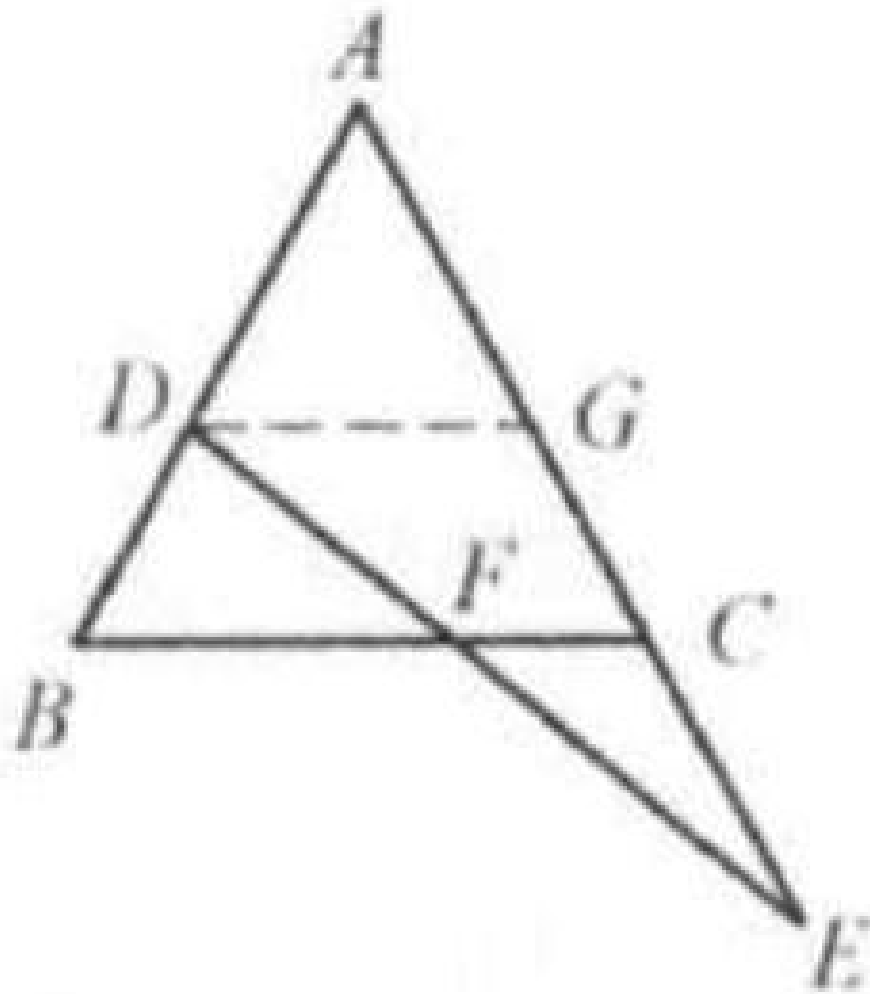Thus $\triangle DFB \cong \triangle EFG (ASA)$.

So $DF = FE$.
Method 3:
Draw $DG//BC$.
Since triangle ABC is an isosceles triangle, we know that $BD = CG = CE$.
Thus C, F are midpoints of EG, ED, respectively.

So CF bisects and $DF = EF$.