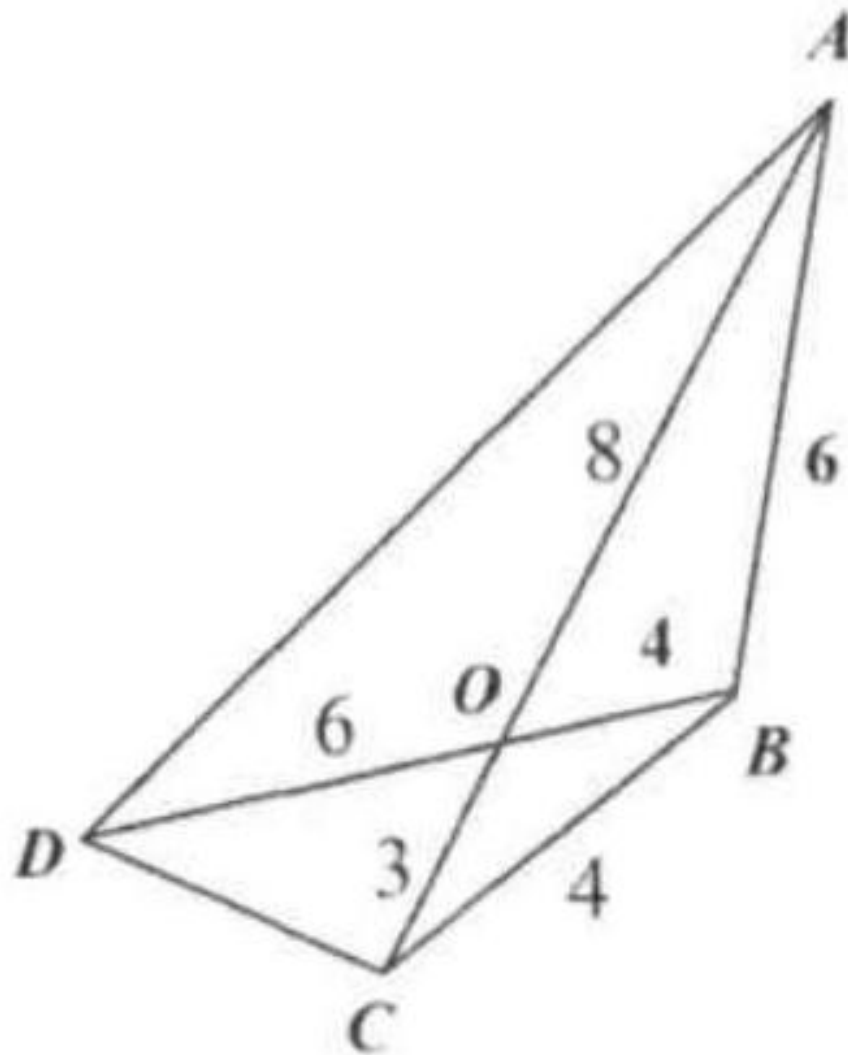# Example 12

As shown in the figure, two diagonals AC and BD of quadrilateral $ABCD$ intersect at O. Find the length of AD if $BO = 4$, $OD = 6$, $AO = 8$, $OC = 3$, and $AB = 6$.
(A) $9\sqrt{7}$
(B) $8\sqrt{7}$
(C) $6\sqrt{7}$
(D) $8\sqrt{2}$ $\sqrt{166}$
(E)

Solution: (E).

Denote F as the foot of the perpendicular from A to an extended diagonal DB, and denote BF and FA by x and y respectively (see figure). By the Pythagorean Theorem, $x^2 + y^2 = 6^2$ and $(x+4)^2 + y^2 = 8^2$.
Subtracting the first of these equations from the second yields
$$8x + 16 = 28, x = 3/2.$$
Substitute the value of x into the first equation and solve for y^2 :
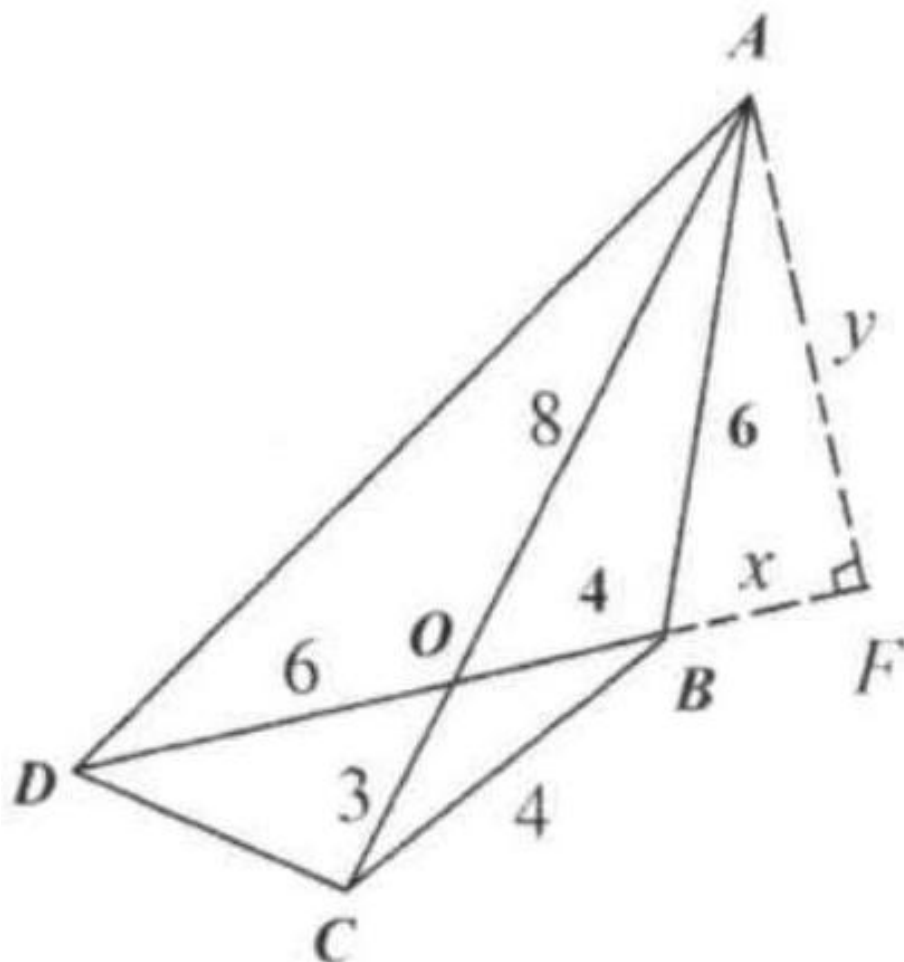
$$\left(\tfrac{3}{2}\right)^2 + y^2 = 6^2 \Rightarrow \quad y^2 = \tfrac{135}{4}.$$

Therefore $AD^2 = (10 + x)^2 + y^2 = \left(\tfrac{23}{2}\right)^2 + \tfrac{135}{4} = \tfrac{664}{4} = 166.$

$$AD = \sqrt{166}.$$