Example 14

(AMC) Vertex E of equilateral triangle ABE is in the interior of square $ABCD$, and F is the point of intersection of diagonal BD and line segment AE. If length AB is $\sqrt{1+\sqrt{3}}$ then the area of $\triangle ABF$ is

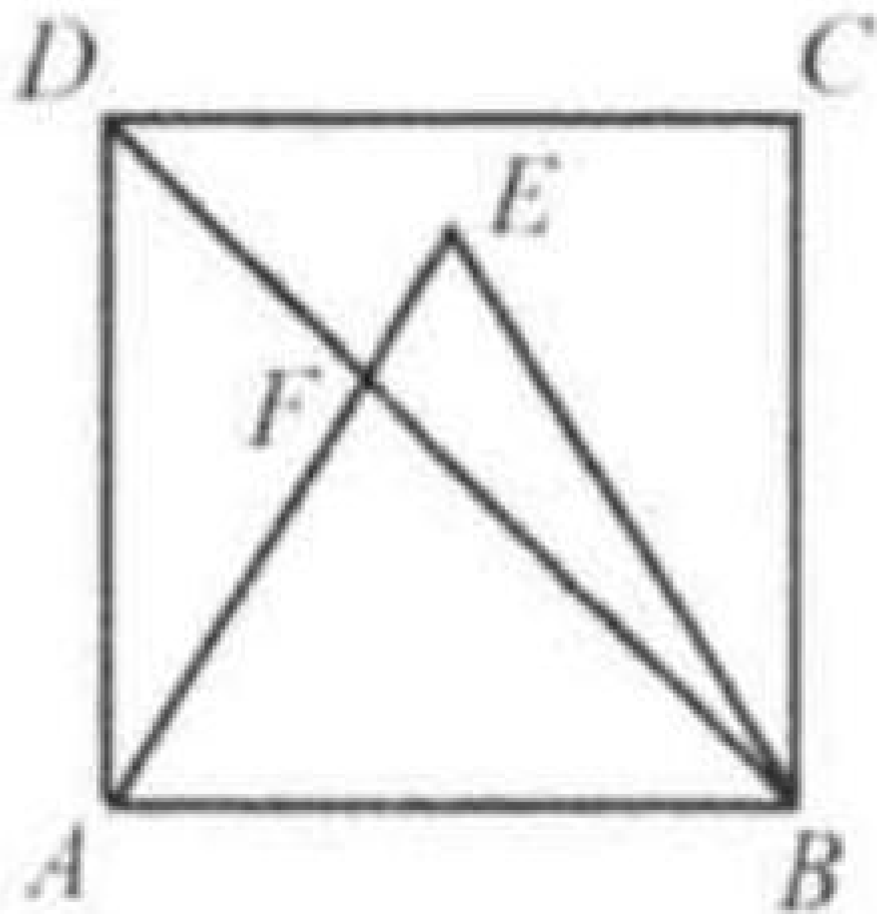
(A) 1

(B) $\frac{\sqrt{2}}{2}$

(C) $\frac{\sqrt{3}}{2}$

(D) $4-2\sqrt{3}$

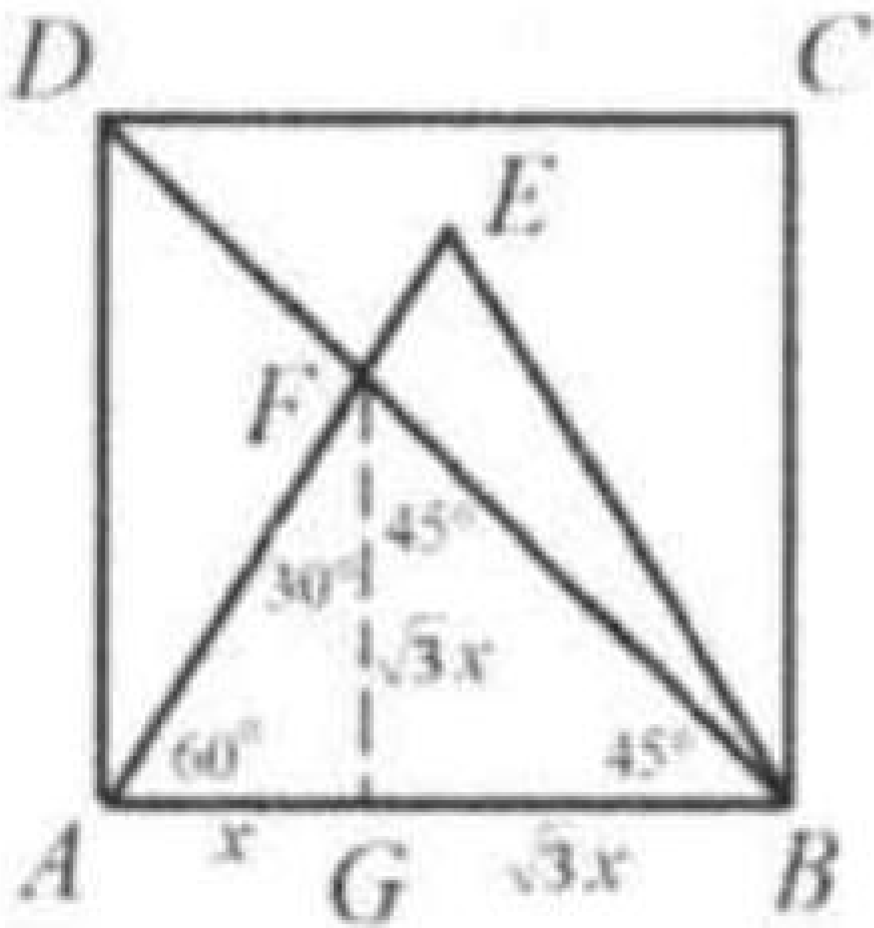
(E) $\frac{1}{2}+\frac{\sqrt{3}}{4}$



Solution: (C).

Let FG be an altitude of triangle ABE, and let x denote the length of AG.
From the adjoining figure it may be seen that

$$\sqrt{1+\sqrt{3}} = AB = x(1+\sqrt{3}) \quad \Rightarrow \quad 1+\sqrt{3} = x^2(1+\sqrt{3})^2 \quad \Rightarrow$$
$$1 = x^2(1+\sqrt{3})$$



The area of triangle ABE is $\frac{1}{2}(AB)(FG) = \frac{1}{2}x^2(1+\sqrt{3})\sqrt{3} = \frac{\sqrt{3}}{2}$.