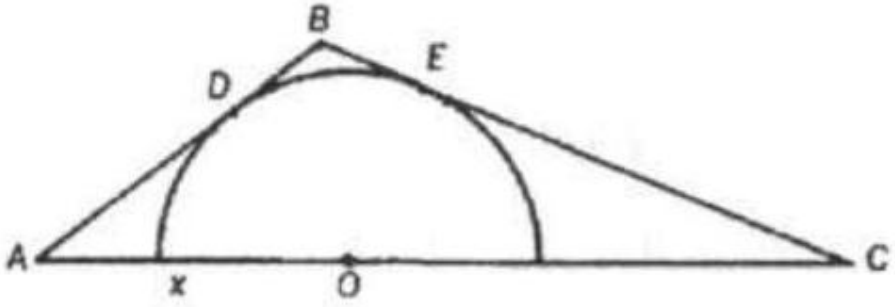
Problem 10

Problem

In $\triangle ABC$, in which $AB = 12, BC = 18$, and $AC = 25$, a semicircle is drawn so that its diameter lies on AC, and so that it is tangent to AB and BC. If O is the center of the circle, find the measure of AO.



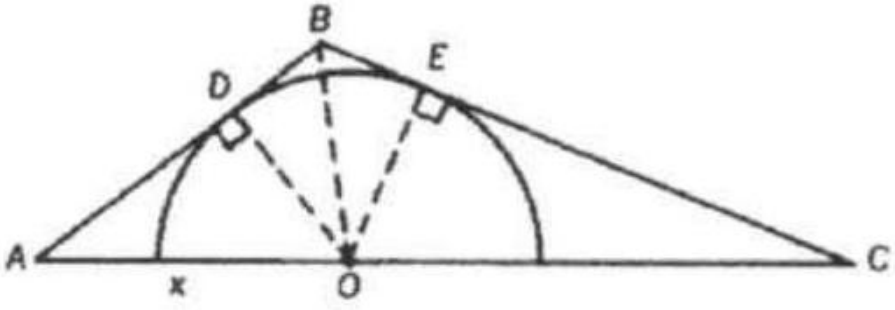
Solution

10.

Draw radii OD and OE to the points of contact of tangents AB and BC, respectively. $OD = OE$ (radii), and $\angle BDO = \angle BEO = 90°$.

Since $DB = BE$, right $\triangle BDO \cong$ right $\triangle BEO$, and $\angle DBO = \angle EBO$.

In $\triangle ABC, BO$ bisects $\angle B$ so that $\frac{AB}{AO} = \frac{BC}{OC}$.



Let $AO = x$; then $\frac{12}{x} = \frac{18}{25-x}$ and $x = 10 = AO$.