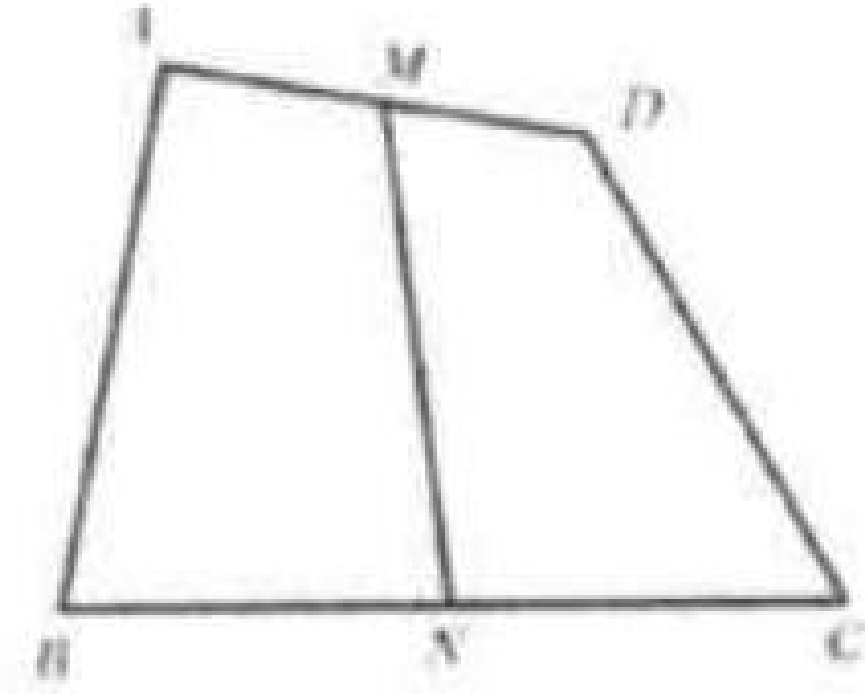


Problem 5

Problem

$ABCD$ is a convex quadrilateral. M and N are midpoints of AD , BC , respectively. Show that $MN \leq \frac{1}{2}(AB + DC)$.



Solution

If $AB \parallel DC$, $ABCD$ is a trapezoid $ABCD$.

By Theorem 2.3, $MN = \frac{1}{2}(AB + DC)$.

Otherwise, Connect AC . Take P , the midpoint of AC . Connect PA , PN .

Since M and P are midpoints of AD , AC , respectively, by Theorem 2.1,

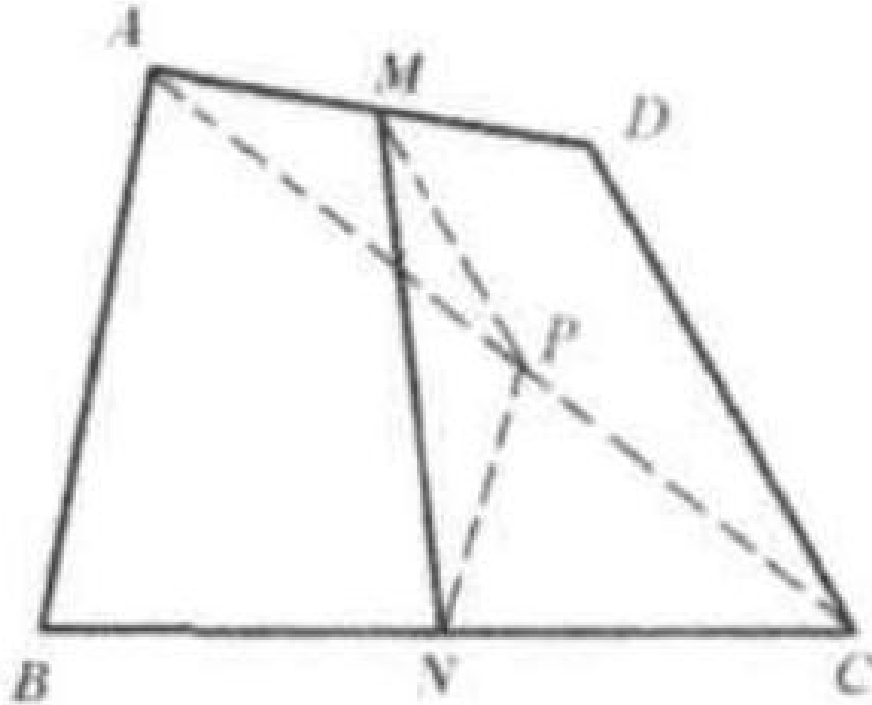
$$MP = \frac{1}{2}DC$$

Since N and P are midpoints of BC, AC , respectively, by Theorem 2.1,

$$NP = \frac{1}{2}AB$$

$$(1) + (2) : MP + NP = \frac{1}{2}(DC + AB)$$

By the triangle inequality theorem, $MP + NP > MN$.



$$\text{Thus } MN < \frac{1}{2}(AB + DC).$$

Therefore, we have $MN \leq \frac{1}{2}(AB + DC)$.