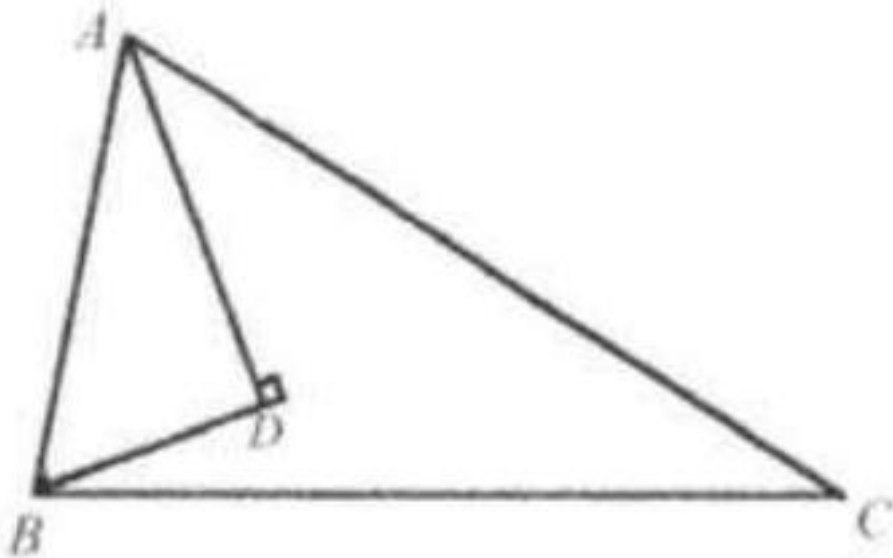


Problem

As shown in the figure below, in $\triangle ABC$, $\angle ABD = 2\angle C$. AD is the angle bisector of $\angle A$. $BD \perp AD$ at D . Show that $BD = \frac{1}{2}(AC - AB)$.



Solution

Extend BD to meet AC at M .

Since AD is the angle bisector of $\angle A$. and $BD \perp AD$,
 $\angle BAD = \angle MAD = \alpha$, $AB = AM$, $\angle ABD = \angle AMD = \beta$.

Since $\angle ABD = \angle AMD = 2\angle C$, $\angle MCB = \angle MBC = \gamma$
 $AM = 2BD = MC = AC - AM \Rightarrow BD = \frac{1}{2}(AC - AB)$.

