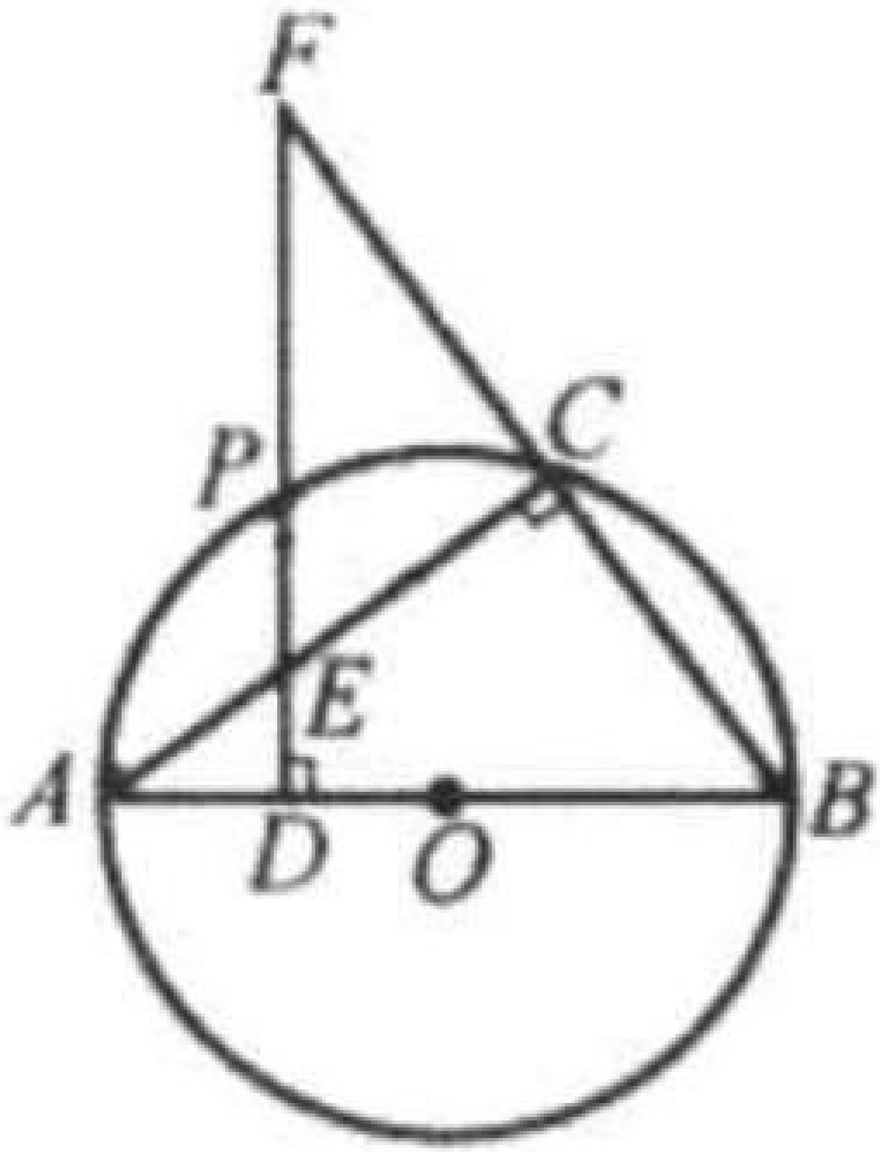
Problem

AB is the diameter of circle O. C is a point on the circumference. P is a point on the circumference PF is perpendicular to AB. PF meets AC at E, AB at D and the extension of BC at F. Show that $DP^2 = DE \cdot DF$.



Solution

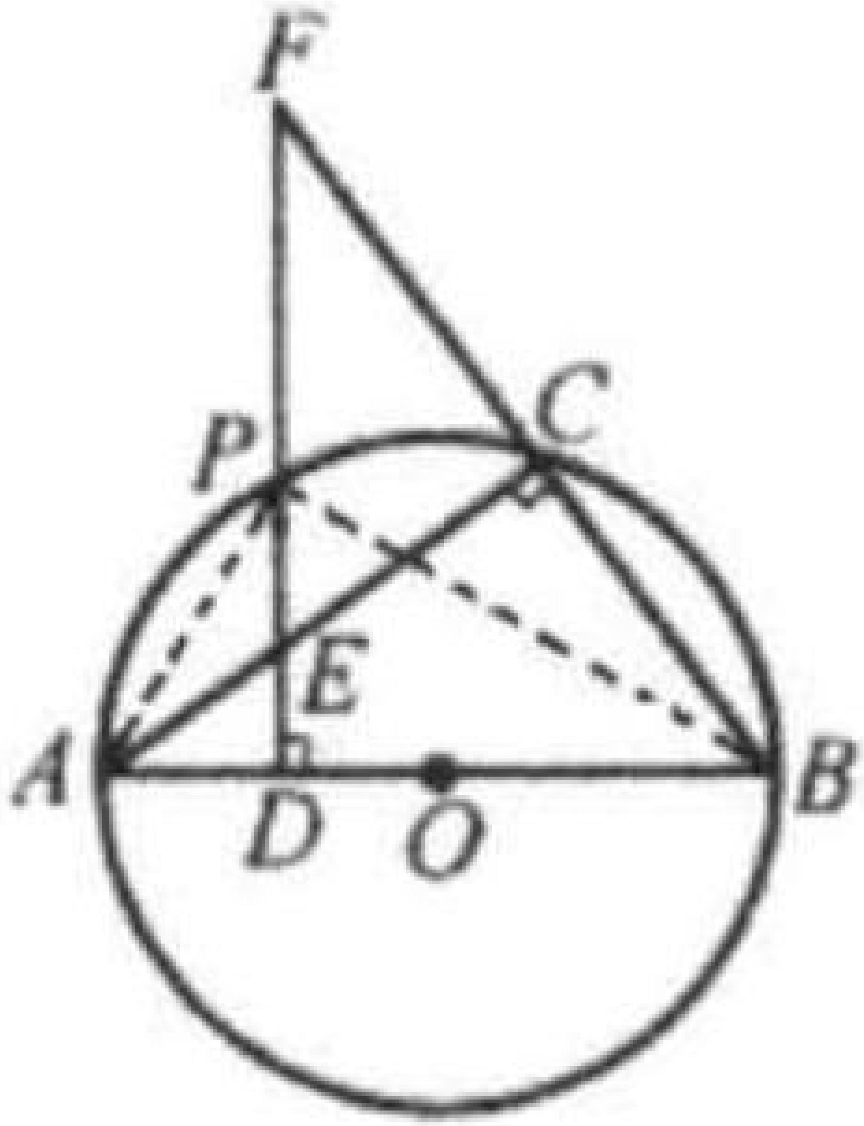
Connect PA and $PB.AB$ is the hypotenuse of right triangles APB and ACB, so $\angle APB = 90°$ and $\angle ACB = 90°$.

Since triangle APB is a right triangle, $DP^2 = AD \cdot DB$.

Instead of showing that $DP^2 = DE \cdot DF$, we can now prove that $AD \cdot DB = DE \cdot DF$.

Note that $\triangle ADE \sim \triangle FDB$.

We know that $\angle F = \angle EAD$, and $\angle ADE = \angle FDB = 90°$.



Therefore $\triangle ADE \sim \triangle FDB \Rightarrow \frac{AD}{DE} = \frac{DF}{DB} \Rightarrow AD \cdot DB = DE \cdot DF$.