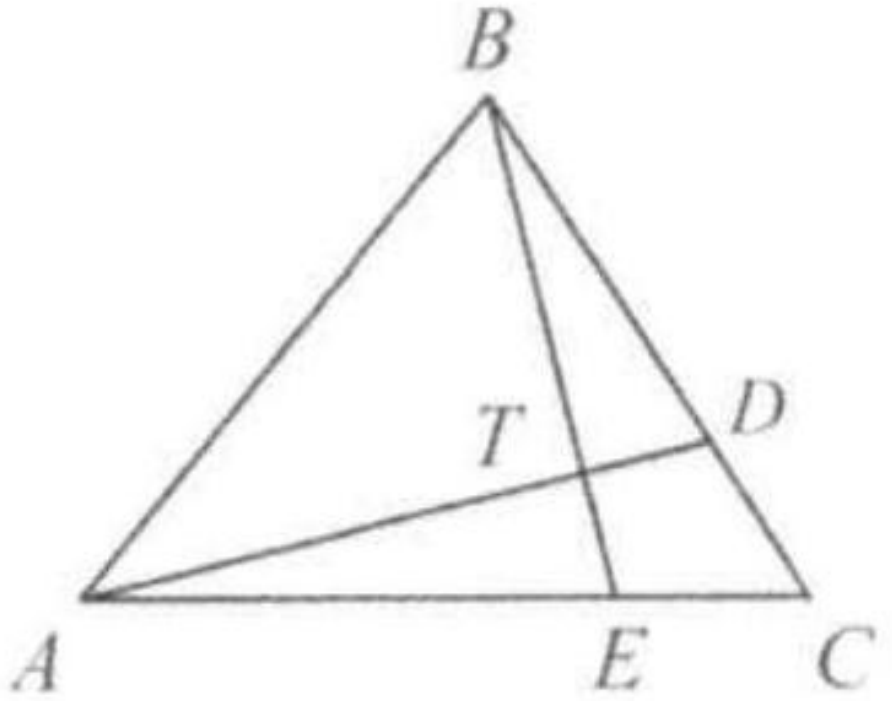
# Example 3

(2004 AMC 10B Problem 20) In $\triangle ABC$ points $D$ and $E$ lie on $BC$ and $AC$, respectively. Suppose that $AD$ and $BE$ intersect at T so that $AT/DT = 3$ and $BT/ET = 4$. What is the value of $CD/BD$ ?

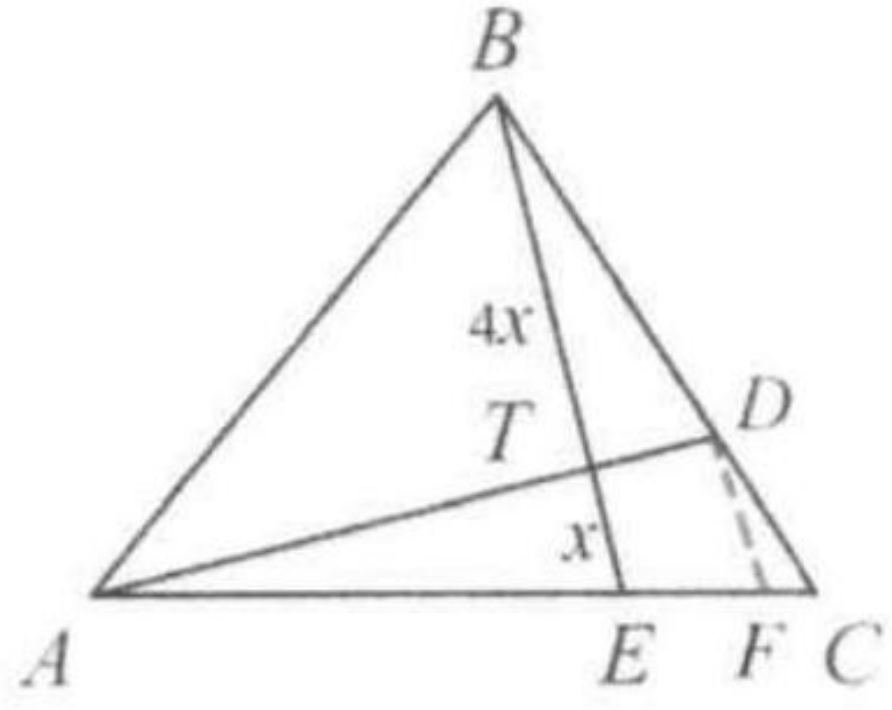
(A) $\frac{1}{8}$
(B) $\frac{2}{9}$
(C) $\frac{3}{10}$
(D) $\frac{4}{11}$
(E) $\frac{5}{12}$

Solution: (D).

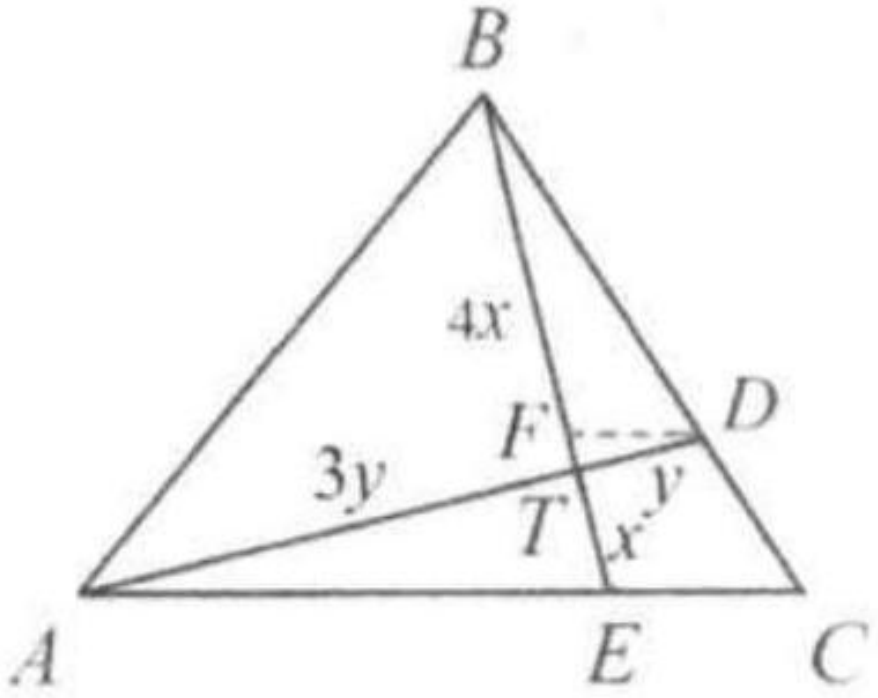



Method 1:
Let $F$ be a point on $AC$ such that $DF$ is parallel to $BE$. Let $BT = 4x$ and $ET = x$.
Because $\triangle ATE$ and $\triangle ADF$ are similar, we have

$\frac{DF}{x} = \frac{AD}{AT} = \frac{4}{3}$ and $DF = \frac{4x}{3}$.
Also, $\triangle BEC$ and $\triangle DFC$ are similar, so
$\frac{CD}{BC} = \frac{DF}{BE} = \frac{4x/3}{5x} = \frac{4}{15}$.




Thus $\frac{CD}{BC} = \frac{CD/BC}{1-CD/BC} = \frac{4/15}{1-4/15} = \frac{4}{11}$.
Method 2:
Let $F$ be a point on $BE$ such that $DF$ is parallel to $AC$. Let $BT = 4x$ and $ET = x$.
Let $AT = 3y$ and $DT = y$.
Because $\triangle ATE$ and $\triangle DTF$ are similar, we have
$\frac{AT}{DT} = \frac{ET}{FT} = \frac{3}{1} \Rightarrow \frac{x}{FT} = \frac{3}{1} \Rightarrow FT = \frac{x}{3}$.
So $EF = x + \frac{x}{3} = \frac{4}{3}x$ and $BF = 4x - \frac{x}{3} = \frac{11}{3}x$



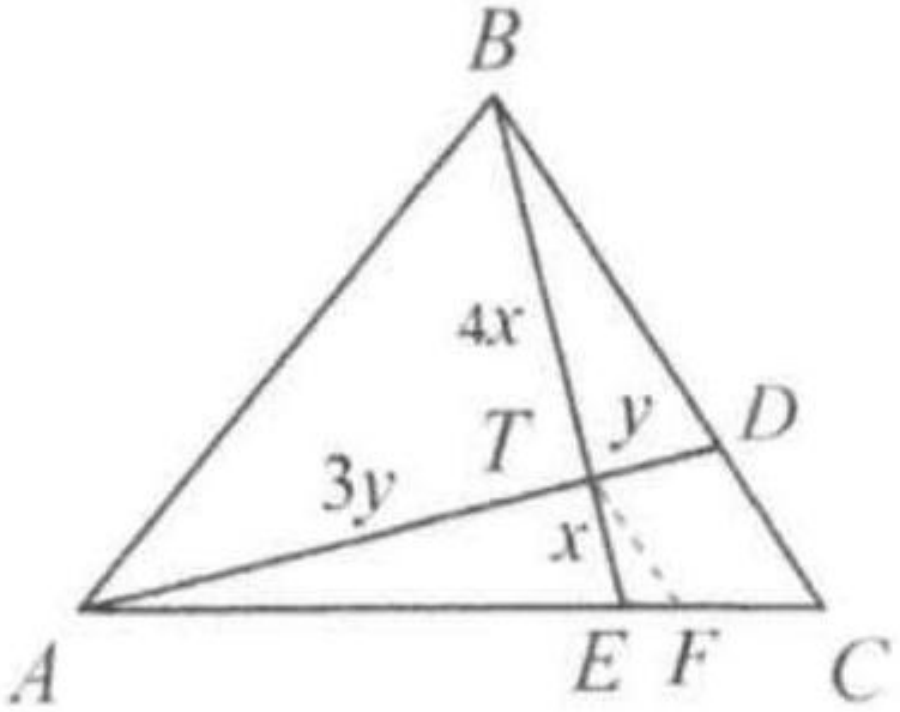

Also, $\triangle BEC$ and $\triangle BFD$ are similar, so
$\frac{BF}{EF} = \frac{BD}{CD} \quad \Rightarrow \quad \frac{\frac{11}{3}x}{\frac{4x}{3}} = \frac{BD}{CD} \quad \Rightarrow \frac{CD}{BC} = \frac{4}{11}$.
$(1) \div (2) : \frac{CD}{BD} = \frac{4}{11}$.

Method 5:

Let $F$ be a point on $AC$ such that $TF$ is parallel to $BC$. Let $BT = 4x$ and $ET = x$. Let $AT = 3y$ and $DT = y$.
Because $\triangle ATF$ and $\triangle ADC$ are similar, we have

$$\frac{AT}{AD} = \frac{FT}{CD} \Rightarrow \frac{3y}{4y} = \frac{FT}{CD} \Rightarrow \frac{FT}{CD} = \frac{3}{4}$$

Also, $\triangle BEC$ and $\triangle TEF$ are similar, so
$\frac{FT}{BC} = \frac{ET}{BE} \quad \Rightarrow \quad \frac{FT}{BD+CD} = \frac{x}{5x}$




$$\Rightarrow \quad \frac{FT}{BD+CD} = \frac{1}{5}$$

(1) $\div$ (2): $\frac{BD+CD}{CD} = \frac{15}{4} \quad \Rightarrow \frac{BD}{CD} + 1 = \frac{15}{4}$

$$\Rightarrow \frac{BD}{CD} = \frac{11}{4} \Rightarrow \frac{CD}{BD} = \frac{4}{11}.$$

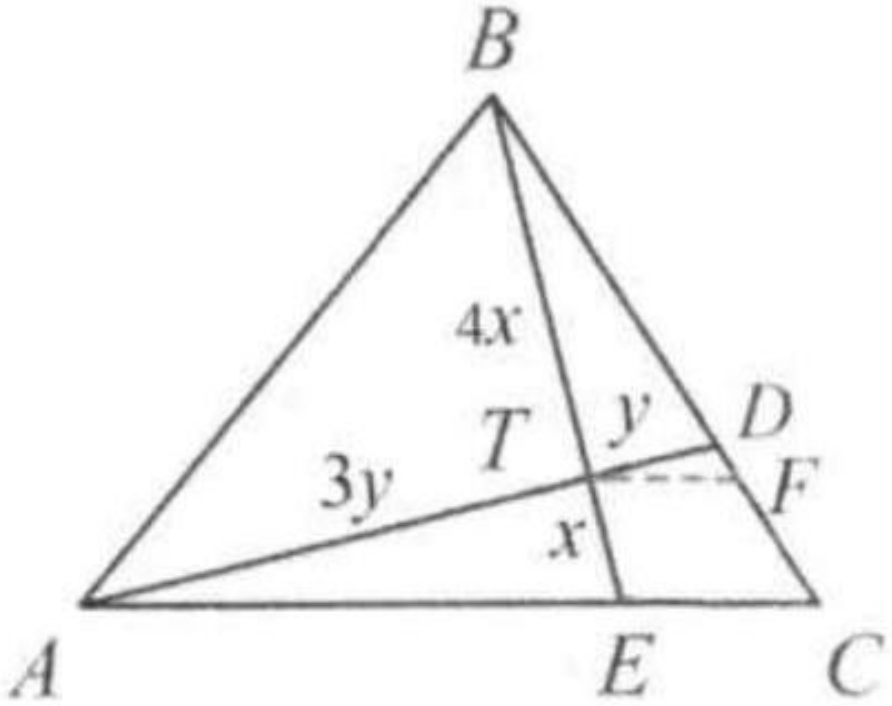
Method 6:

Let $F$ be a point on $BC$ such that $TF$ is parallel to $BC$. Let $BT = 4x$ and $ET = x$. Let $AT = 3y$ and $DT = y$.

Because $\triangle ADC$ and $\triangle TDF$ are similar, we have

$\frac{AT}{CF} = \frac{DT}{DF} \Rightarrow \frac{3y}{CF} = \frac{y}{DF} \Rightarrow \frac{CF}{DF} = 3 \Rightarrow$

$\frac{CD-DF}{DF} = 3 \Rightarrow \frac{CD}{DF} = 4$




Because $\triangle BTF$ and $\triangle BEC$ are similar, so

$$\frac{BT}{TE} = \frac{BF}{CF} \quad \Rightarrow \quad \frac{4x}{x} = \frac{BF}{CF} = 4$$

$$\frac{BF}{CF} = \frac{BD+DF}{CF} = 4 \Rightarrow \quad \frac{BD+DF}{3DF} = 4 \quad \Rightarrow \quad \frac{BD}{DF} + 1 = 12$$

$$\Rightarrow \quad \frac{BD}{DF} = 11$$

$$(1) \div (2) : \frac{CD}{BD} = \frac{4}{11}.$$