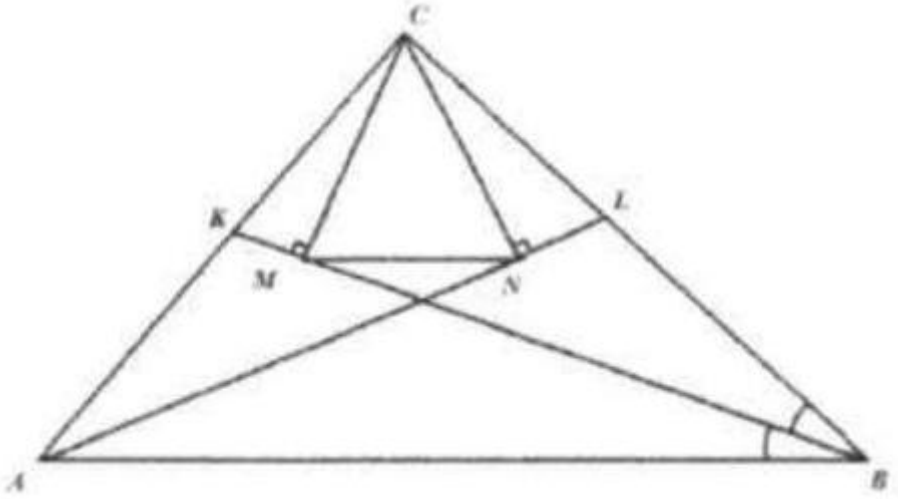
# Example 8

(AIME I) In triangle $ABC, AB = 125, AC = 117$ and $BC = 120$. The angle bisector of angle $A$ intersects $BC$ at point $L$, and the angle bisector of angle $B$ intersects $AC$ at point $K$. Let $M$ and $N$ be the feet of the perpendiculars from $C$ to $BK$ and $AL$, respectively. Find $MN$.

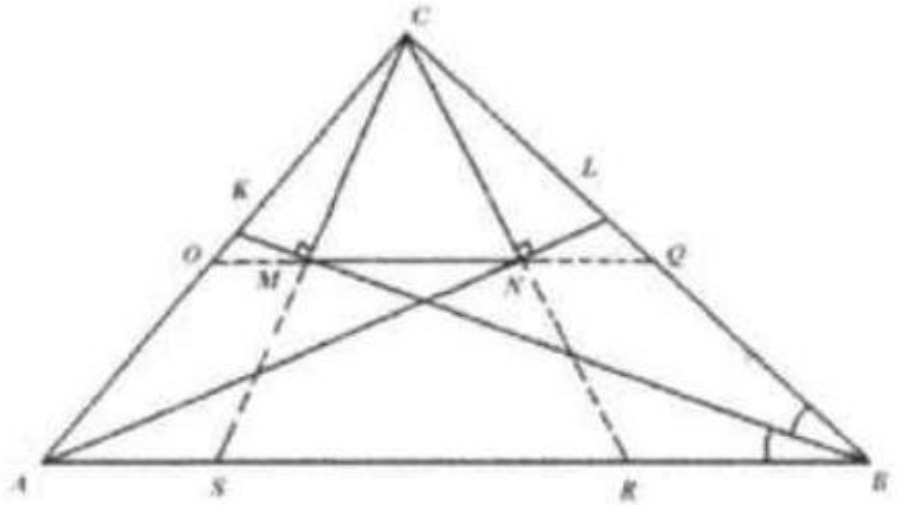



Solution: 56.

Extend $MN$ such that it intersects lines $AC$ and $BC$ at point $O$ and $Q$, respectively. Extend $CM$ to meet $AB$ at, say, $S$. then triangle $BCM$ and triangle $BSM$ are congruent. Hence $BS = BC = 120$.

Similarly, extend $CN$ to meet $AB$ at, say, $R$, and triangle $ACN$ and triangle $ARN$ are congruent. Hence $AR = AC = 117$. So $CM = MS$, and $CN = NR$.

So $MN$ is the midline of triangle $CSR$ (and $OQ$ is the midline of $AB$ ).




$$MN = \frac{AB}{2} - \frac{AS}{2} - \frac{BR}{2} = \frac{AB}{2} - \frac{AB-BS}{2} - \frac{AB-AR}{2}$$
$$= \frac{BS}{2} + \frac{AR}{2} - \frac{AB}{2} = \frac{BC+AC-AB}{2} = \frac{120+117-125}{2} = 56$$