

An Online Learning Approach to Generative Adversarial Networks

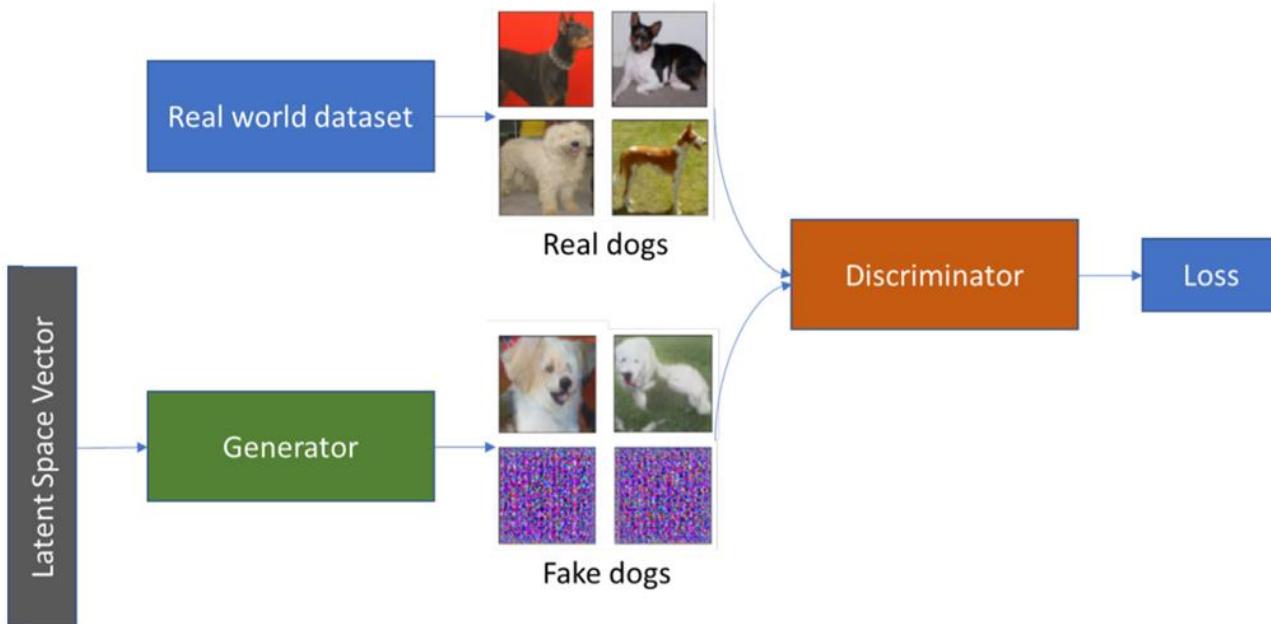
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Dec. 8

Outline

- Background of Generative Adversarial Networks (GAN).
- Mixed Nash equilibrium (MNE).
- Theoretical algorithm: CHEKHOV GAN.
- Proof of the convergence.
- Practical algorithm and Experiments.

Generative Adversarial Network (GAN)



Generative Adversarial Network (GAN)

$$M(G, D) = \max_D \min_G L(D, G) = \frac{1}{N} \sum_{i=1}^N \log D(x_i) + \log(1 - D(G(z_i)))$$

Goal of Generator

$$\min_{u \in K_1} \max_{v \in K_2} M(u, v)$$

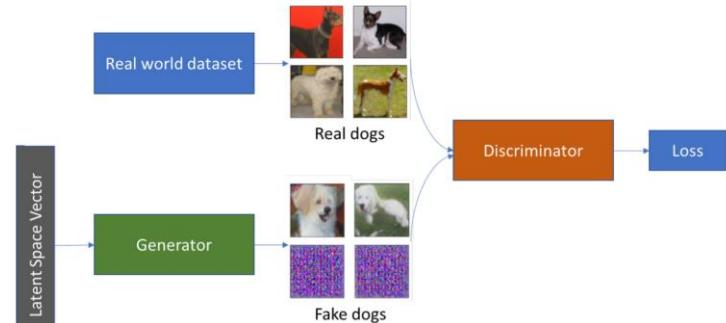
Goal of Discriminator

$$\max_{v \in K_2} \min_{u \in K_1} M(u, v)$$

Pure Nash Equilibrium

$$M(u^*, v^*) \leq \min_{u \in K_1} M(u, v^*) \quad M(u^*, v^*) \geq \max_{v \in K_2} M(u^*, v)$$

Does a Pure Nash Equilibrium always exist?

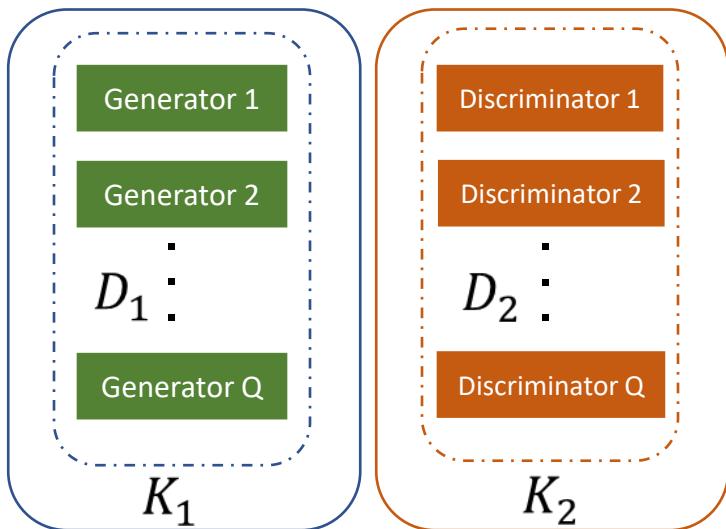


	Rock	Paper	Scissors
Rock	0	1	-1
Paper	-1	0	1
Scissors	1	-1	0



Mixed Nash Equilibrium (MNE)

$$\mathbb{E}_{(\mathbf{u}, \mathbf{v}) \sim \mathcal{D}_1 \times \mathcal{D}_2} [M(\mathbf{u}, \mathbf{v})] \leq \min_{\mathbf{u} \in \mathcal{K}_1} \mathbb{E}_{\mathbf{v} \sim \mathcal{D}_2} [M(\mathbf{u}, \mathbf{v})], \text{ & } \mathbb{E}_{(\mathbf{u}, \mathbf{v}) \sim \mathcal{D}_1 \times \mathcal{D}_2} [M(\mathbf{u}, \mathbf{v})] \geq \max_{\mathbf{v} \in \mathcal{K}_2} \mathbb{E}_{\mathbf{u} \sim \mathcal{D}_1} [M(\mathbf{u}, \mathbf{v})]$$



Definition 1.1. Let $\varepsilon > 0$, the two distributions D_1, D_2 are called ε – MNE if

$$\mathbb{E}_{(u, v) \sim K_1 \times K_2} [M(u, v)] \leq \min_{u \in K_1} \mathbb{E}_{v \sim D_2} [M(u, v)] + \varepsilon$$

$$\mathbb{E}_{(u, v) \sim K_1 \times K_2} [M(u, v)] \geq \min_{v \in K_2} \mathbb{E}_{u \sim D_1} [M(u, v)] - \varepsilon$$

if ε is sub – linear to T

$$\varepsilon = O(1/\sqrt{T})$$

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AN ONLINE LEARNING APPROACH TO GENERATIVE ADVERSARIAL NETWORKS

Paulina Grnarova, Kfir Y. Levy, Aurelien Lucchi, Thomas Hofmann, Andreas Krause

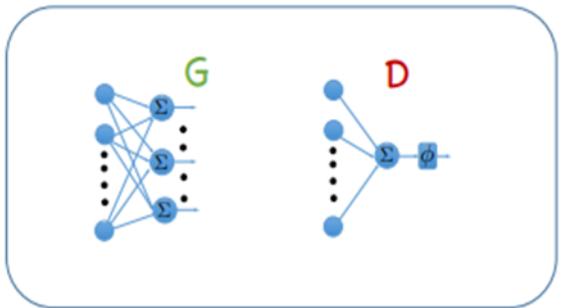
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- Proposed an algorithm that guarantees finding a MNE for GAN (Semi-Shallow).
- Designed a new GAN training algorithm that applies in practice (Deep).

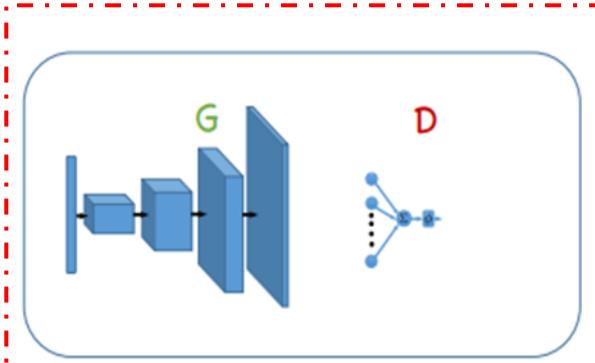
Three Different Types of GAN

G: Convex
D: Arbitrary



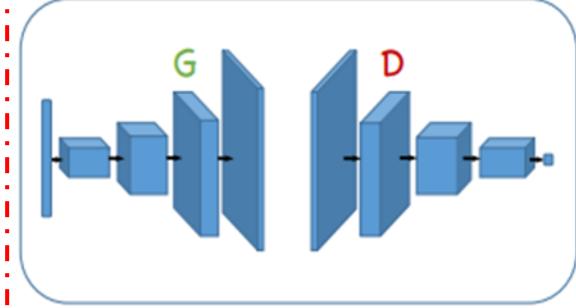
(a)

G: Convex
D: Concave



(b)

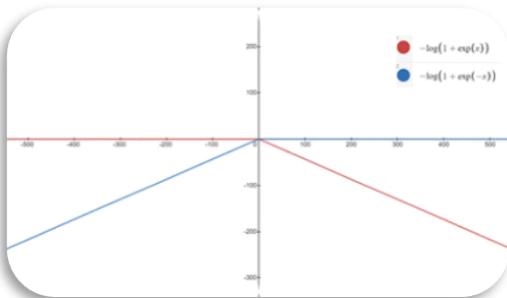
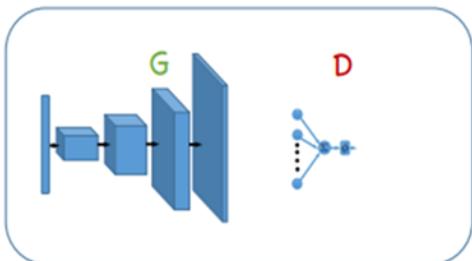
G: Arbitrary
D: Concave



(c)

G: Arbitrary
D: Arbitrary

Concavity of the Discriminator



$$F(a) = -\log(1 + \exp(-a))$$

Boston University

Proposition 2.1. Consider the GAN objective in Equation (1) and assume that the discriminator is a single-layer network with a sigmoid activation function, meaning $D_v(x) = 1/(1 + \exp(-v^T x))$, where $v \in R^n$. Then the GAN objective is concave in v .

$$M(G, D) = \max_D \min_G L(D, G) = \frac{1}{N} \sum_{i=1}^N \log D(x_i) + \log(1 - D(G(z_i)))$$

$$\log D_v(x_i) = \log(1) - \log(1 + \exp(-v^T x_i)) = -\log(1 + \exp(-v^T x_i))$$

$$\begin{aligned} \log(1 - D_v(G_u(z_i))) &= \log(1 - 1/(1 + \exp(-v^T(G_u(z_i))))) \\ &= \log\left(\frac{\exp(-v^T(G_u(z_i)))}{(1 + \exp(-v^T(G_u(z_i)))))}\right) \\ &= \log(1) - \log(1 + 1/\exp(-v^T G_u(z_i))) \\ &= -\log(1 + \exp(v^T G_u(z_i))) \end{aligned}$$



Algorithm 1 CHEKHOV GAN [4]**Input:** Number of steps: T ; Game objective: $M(\cdot, \cdot)$ **for** $t = 1$ **to** T **do**

Calculate:

$$\mathbf{u}_t \leftarrow \arg \min_{\mathbf{u} \in K_1} \sum_{\tau=0}^{t-1} f_\tau(\mathbf{u}) \quad \triangleright A_1$$

$$\mathbf{v}_t \leftarrow \arg \max_{\mathbf{v} \in K_2} \sum_{\tau=0}^{t-1} \nabla g_\tau(\mathbf{v}_\tau)^T \mathbf{v} - \frac{\sqrt{T}}{2\eta_0} \|\mathbf{v}\|^2 \quad \triangleright A_2$$

Update:

$$f_t(\cdot) = M(\cdot, \mathbf{v}_t); g_t(\cdot) = M(\mathbf{u}_t, \cdot)$$

end for**Output:** $D_1 \sim \text{Uni}\{\mathbf{u}_1, \dots, \mathbf{u}_T\}; D_2 \sim \text{Uni}\{\mathbf{v}_1, \dots, \mathbf{v}_T\}$

Theorem 3.1. Let K_2 be a convex set, M be a semi-concave zero-sum game, and assume M is L -Lipschitz continuous. Then the mixed strategies (D_1, D_2) produced by Algorithm 1 are ε – MNE, where $\varepsilon = \mathcal{O}(1/\sqrt{T})$.

Proof Sketch

Theorem 3.1. *Let K_2 be a convex set, M be a semi-concave zero-sum game, and assume M is L -Lipschitz continuous. Then the mixed strategies (D_1, D_2) produced by Algorithm 1 are $\varepsilon - \text{MNE}$, where $\varepsilon = \mathcal{O}(1/\sqrt{T})$.*

Theorem 3.2. *The mixed strategies (D_1, D_2) that Algorithm 1 outputs are $\varepsilon - \text{MNE}$, where $\varepsilon := (B_T^{A_1} + B_T^{A_2})/T$ and $B_T^{A_1}, B_T^{A_2}$ are bounds on the regret of A_1, A_2 .*

$$B_T^{A_1} \text{ is } \mathcal{O}(\sqrt{T}) \quad B_T^{A_2} \text{ is } \mathcal{O}(\sqrt{T})$$

Proof of Theorem 3.2

Theorem 3.2. *The mixed strategies (D_1, D_2) that Algorithm 1 outputs are $\varepsilon - \text{MNE}$, where $\varepsilon := (B_T^{A_1} + B_T^{A_2})/T$ and $B_T^{A_1}, B_T^{A_2}$ are bounds on the regret of A_1, A_2 .*

Proof. Let $f_t(\mathbf{u}) := \mathbf{M}(\mathbf{u}, \mathbf{v}_t)$, $g_t(\mathbf{v}) := \mathbf{M}(\mathbf{u}_t, \mathbf{v})$. Recall that K_1, K_2 are domain of f, g ; $B_T^{A_1}, B_T^{A_2}$ are bounds on the regret of A_1, A_2 . Then

$$\sum_{t=1}^T M(\mathbf{u}_t, \mathbf{v}_t) - \min_{\mathbf{u} \in K_1} \sum_{t=1}^T M(\mathbf{u}, \mathbf{v}_t) \leq B_T^{A_1} \quad \max_{\mathbf{v} \in K_2} \sum_{t=1}^T M(\mathbf{u}_t, \mathbf{v}) \geq \mathbb{E}_{v \sim K_2} [\sum_{t=1}^T M(\mathbf{u}_t, \mathbf{v})]$$

$$\max_{\mathbf{v} \in K_2} \sum_{t=1}^T M(\mathbf{u}_t, \mathbf{v}) - \sum_{t=1}^T M(\mathbf{u}_t, \mathbf{v}_t) \leq B_T^{A_2} \quad \longrightarrow \quad \mathbb{E}_{v \sim K_2} [\frac{1}{T} \sum_{t=1}^T M(\mathbf{u}_t, \mathbf{v})] - \frac{1}{T} \sum_{t=1}^T M(\mathbf{u}_t, \mathbf{v}_t) \leq \frac{1}{T} B_T^{A_2}$$

Proof of Theorem 3.2

Theorem 3.2. *The mixed strategies (D_1, D_2) that Algorithm 1 outputs are $\varepsilon - \text{MNE}$, where $\varepsilon := (B_T^{A_1} + B_T^{A_2})/T$ and $B_T^{A_1}, B_T^{A_2}$ are bounds on the regret of A_1, A_2 .*

$$\sum_{t=1}^T M(\mathbf{u}_t, \mathbf{v}_t) - \min_{\mathbf{u} \in K_1} \sum_{t=1}^T M(\mathbf{u}, \mathbf{v}_t) \leq B_T^{A_1}$$

$$\mathbb{E}_{v \sim K_2} \left[\frac{1}{T} \sum_{t=1}^T M(\mathbf{u}_t, \mathbf{v}) \right] - \frac{1}{T} \sum_{t=1}^T M(\mathbf{u}_t, \mathbf{v}_t) \leq \frac{1}{T} B_T^{A_2}$$

$$\mathbb{E}_{v \sim K_2} \left[\frac{1}{T} \sum_{t=1}^T M(\mathbf{u}_t, \mathbf{v}) \right] - \min_{\mathbf{u} \in K_1} \frac{1}{T} \sum_{t=1}^T M(\mathbf{u}, \mathbf{v}_t) \leq \boxed{\frac{1}{T} (B_T^{A_1} + B_T^{A_2})} := \varepsilon$$

Definition 1.1. Let $\varepsilon > 0$, the two distributions D_1, D_2 are called $\varepsilon - \text{MNE}$ if

$$\mathbb{E}_{(u,v) \sim K_1 \times K_2} [M(\mathbf{u}, \mathbf{v})] \leq \min_{\mathbf{u} \in K_1} \mathbb{E}_{v \sim D_2} [M(\mathbf{u}, \mathbf{v})] + \varepsilon$$

$$\mathbb{E}_{(u,v) \sim K_1 \times K_2} [M(\mathbf{u}, \mathbf{v})] \leq \min_{\mathbf{u} \in K_1} \mathbb{E}_{v \sim D_2} [M(\mathbf{u}, \mathbf{v})] + \varepsilon$$

$$\mathbb{E}_{(u,v) \sim K_1 \times K_2} [M(\mathbf{u}, \mathbf{v})] \geq \min_{\mathbf{v} \in K_2} \mathbb{E}_{u \sim D_1} [M(\mathbf{u}, \mathbf{v})] - \varepsilon$$

Proof of Regret Bounds

$B_T^{A_1}$ is $\mathcal{O}(\sqrt{T})$ Let d_2 is the diameter of K_2 , M is L – Lipschitz.

$$\begin{aligned}
 \text{Regret}_T^{A_1} &\leq \sum_{t=1}^T (f_t(u_t) - f_t(u_{t+1})) \\
 &= \sum_{t=1}^{T-1} (f_t(u_t) - f_t(u_{t+1}) + f_{t+1}(u_{t+1}) - f_{t+1}(u_{t+1})) + (f_T(u_T) - f_T(u_{T+1})) \\
 &= \sum_{t=1}^{T-1} (f_{t+1}(u_{t+1}) - f_t(u_{t+1})) + \sum_{t=1}^{T-1} (f_t(u_t) - f_{t+1}(u_{t+1})) + (f_T(u_T) - f_T(u_{T+1})) \\
 &= \sum_{t=1}^{T-1} (M(u_{t+1}, v_{t+1}) - M(u_{t+1}, v_t)) + (f_1(u_1) - f_T(u_{T+1})) \\
 &\leq Ld_2\sqrt{T}/\sqrt{2} + (f_1(u_1) - f_T(u_{T+1})) = Ld_2\sqrt{T}/\sqrt{2} + 2C
 \end{aligned}$$

Proof of Regret Bounds

$B_T^{A_2}$ is $\mathcal{O}(\sqrt{T})$ Let d_2 is the diameter of K_2 , M is L – Lipschitz.

$$\begin{aligned} \text{Regret}_T &\leq 2\eta \sum_{t=1}^T \| \nabla_{f_t} \|^2 + \frac{R(u) - R(x_1)}{\eta} \\ &= 2\eta L^2 T + \frac{d^2}{\eta} \quad \eta = \frac{d}{\sqrt{2TL}} \end{aligned}$$

$$\text{Regret}_T^{A_2}(g_1, \dots, g_T) \leq L d_2 \sqrt{2T}$$

GAN is a minimax zero-sum game.

Standard method aim to find a Pure Nash equilibrium.

Pure Nash Equilibrium not always exist

Algorithm 1 CHEKHOV GAN [4]

Input: Number of steps: T ; Game objective: $M(\cdot, \cdot)$

for $t = 1$ **to** T **do**

Calculate:

$$\mathbf{u}_t \leftarrow \arg \min_{\mathbf{u} \in K_1} \sum_{\tau=0}^{t-1} f_\tau(\mathbf{u})$$

$$\mathbf{v}_t \leftarrow \arg \max_{\mathbf{v} \in K_2} \sum_{\tau=0}^{t-1} \nabla g_\tau(\mathbf{v}_\tau)^T \mathbf{v} - \frac{\sqrt{T}}{2\eta_0} \|\mathbf{v}\|^2$$

Update:

$$f_t(\cdot) = M(\cdot, \mathbf{v}_t); g_t(\cdot) = M(\mathbf{u}_t, \cdot)$$

end for

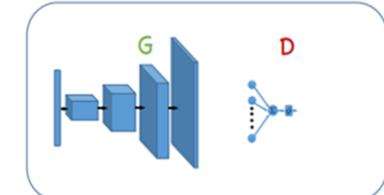
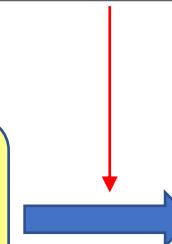
Output: $D_1 \sim \text{Uni}\{\mathbf{u}_1, \dots, \mathbf{u}_T\}; D_2 \sim \text{Uni}\{\mathbf{v}_1, \dots, \mathbf{v}_T\}$

$\triangleright A_1$

$\triangleright A_2$

We aim to find a Mixed Nash Equilibrium

Finding a Mixed Nash Equilibrium guarantees convergence on Semi-Shallow GAN.



Algorithm 1 CHEKHOV GAN [4]

Input: Number of steps: T ; Game objective: $M(\cdot, \cdot)$ **for** $t = 1$ **to** T **do**

Calculate:

$$\mathbf{u}_t \leftarrow \arg \min_{\mathbf{u} \in K_1} \sum_{\tau=0}^{t-1} f_\tau(\mathbf{u}) \quad \triangleright A_1$$

$$\mathbf{v}_t \leftarrow \arg \max_{\mathbf{v} \in K_2} \sum_{\tau=0}^{t-1} \nabla g_\tau(\mathbf{v}_\tau)^T \mathbf{v} - \frac{\sqrt{T}}{2\eta_0} \|\mathbf{v}\|^2 \quad \triangleright A_2$$

Update:

$$f_t(\cdot) = M(\cdot, \mathbf{v}_t); g_t(\cdot) = M(\mathbf{u}_t, \cdot)$$

end for

Output: $D_1 \sim Uni\{\mathbf{u}_1, \dots, \mathbf{u}_T\}; D_2 \sim Uni\{\mathbf{v}_1, \dots, \mathbf{v}_T\}$

Theoretical CHEKHOV GAN does not work in practice!

Practical CHEKHOV GAN

Algorithm 2 Practical CHEKHOV GAN [4], modified

Input: Number of steps: T ; Game objective: $M(\cdot, \cdot)$; Number of past states K ; Spacing m .

Initialize: Set loss/reward $f_0(\cdot) = 0, g_0(\cdot) = 0$, and $Q_1.insert(f_0), Q_2.insert(g_0)$

```

for  $t = 1$  to  $T$  do
     $\mathbf{u}_{t+1} \leftarrow u_t - \eta_t \cdot \nabla_{u_t} \left( \frac{1}{|Q_1|} \sum_{f \in Q_1} f(u) + \frac{C}{\sqrt{t}} \|u\|^2 \right)$ 
     $\mathbf{v}_{t+1} \leftarrow v_t - \eta_t \cdot \nabla_{v_t} \left( \frac{1}{|Q_2|} \sum_{g \in Q_2} g(v) - \frac{C}{\sqrt{t}} \|v\|^2 \right)$ 
    Calculate:  $f_t(\cdot) = M(\cdot, v_t); g_t(\cdot) = M(u_t, \cdot)$ 
    Update  $Q_1$  and  $Q_2$ :
        if  $t \bmod m == 0$  and  $|Q| == K$  then  $\triangleright |Q| = |Q_1| = |Q_2|$ 
             $Q_1.remove\_last(); Q_2.remove\_last();$ 
             $Q_1.insert(f_t); Q_2.insert(g_t).$ 
        else if  $|Q| < K$  then
             $Q_1.insert(f_t); Q_2.insert(g_t).$ 
        else
             $Q_1.replace\_first(f_t); Q_2.replace\_first(g_t).$ 
        end if
    end for

```

Output: $D_1 \sim Uni\{\mathbf{u}_1, \dots, \mathbf{u}_K \in Q_1\}; D_2 \sim Uni\{\mathbf{v}_1, \dots, \mathbf{v}_K \in Q_2\}$

Experiments

Data set:

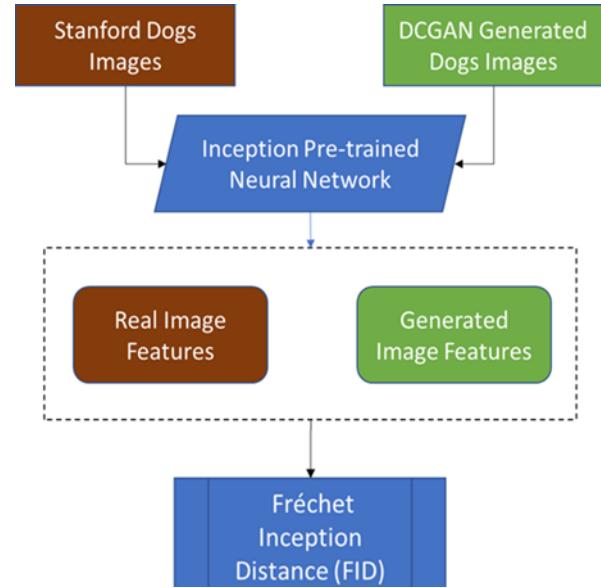
Cifar-10 (paper)

Stanford Dogs Images (mine)

Evaluation metrics:

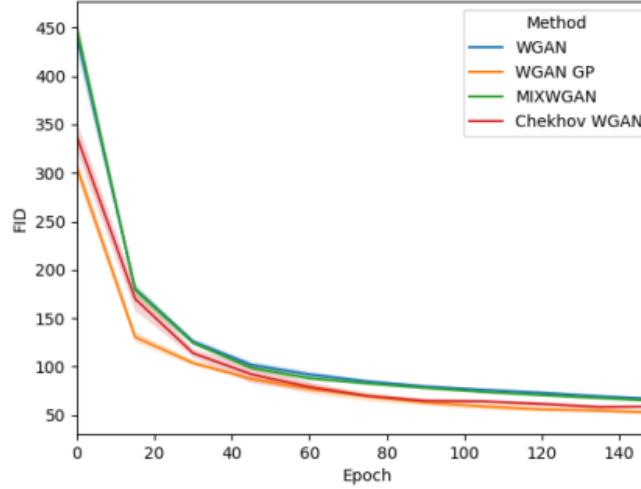
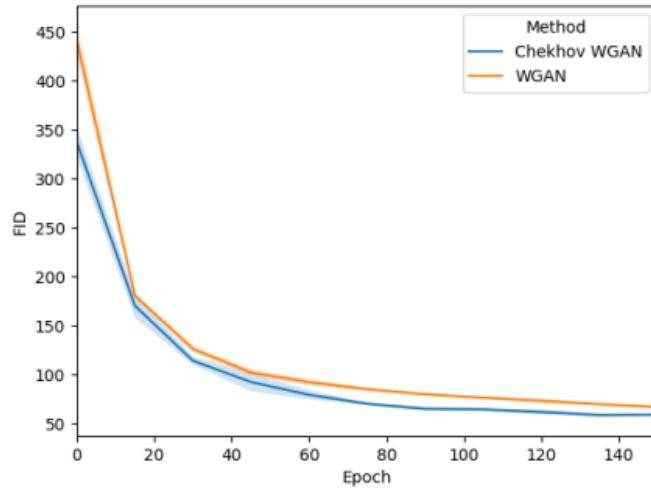
Fréchet Inception Distance

$$FID = \|\mu_r - \mu_s\|^2 + Tr \left(\Sigma_r + \Sigma_g - 2\sqrt{\Sigma_r \Sigma_g} \right)$$



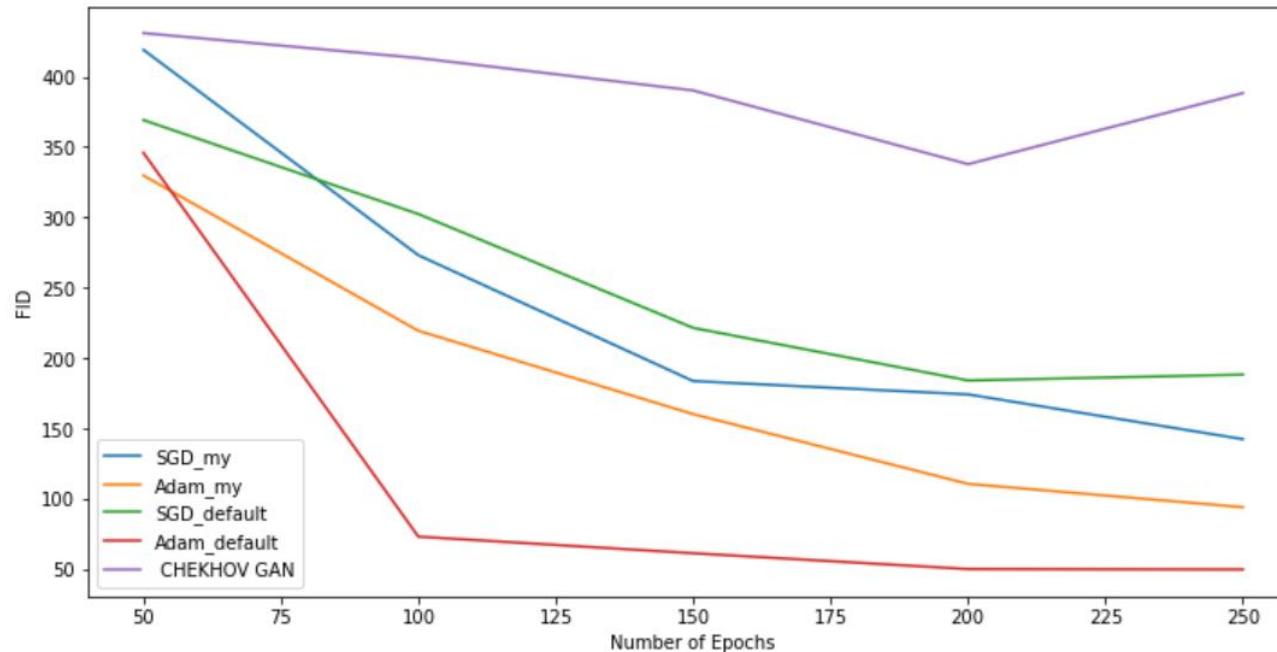
Experiments

Cifar-10 (paper)



Experiments

Stanford Dogs Images (mine)



Discussion

- Converge but the speed?
- Experiments on Semi-Shallow GAN that show standard GAN training method does not converge but CHEKHOV GAN converges.
- The practical Chekhov GAN is using gradients and is very different from the theoretical Chekhov GAN, so does the algorithm still guarantee convergence?
- The proposed algorithm requires significantly large GPU memory and is also computational.
- The experiments in the original paper do not show many advantages of Checkhov GAN, though Chekhov WGAN consistently outperforms WGAN by achieving better FID scores.
- Proof convergence of Deep GAN architecture.

References

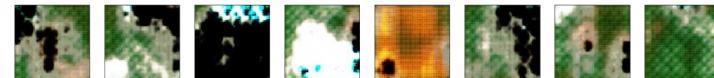
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Thanks!

Questions?



Fig.1 Image samples from Stanford Dog Dataset.



24 Fig.3 Generated using my implementation of SGD.

Fig.2 Generated using my implementation of Adam.

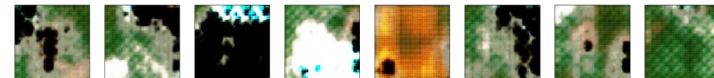


Fig.4 Generated using my implementation of CHECKOV GAN.

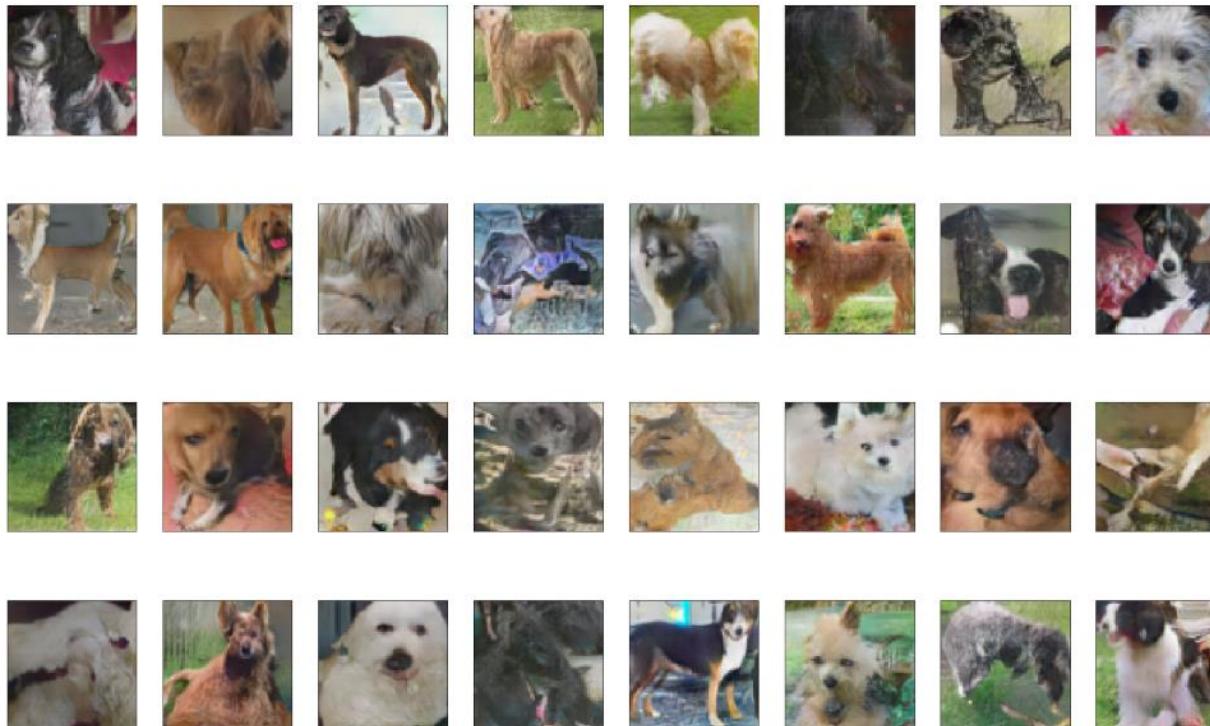


Figure 5: Result of DCGAN training using default Adam optimizer.

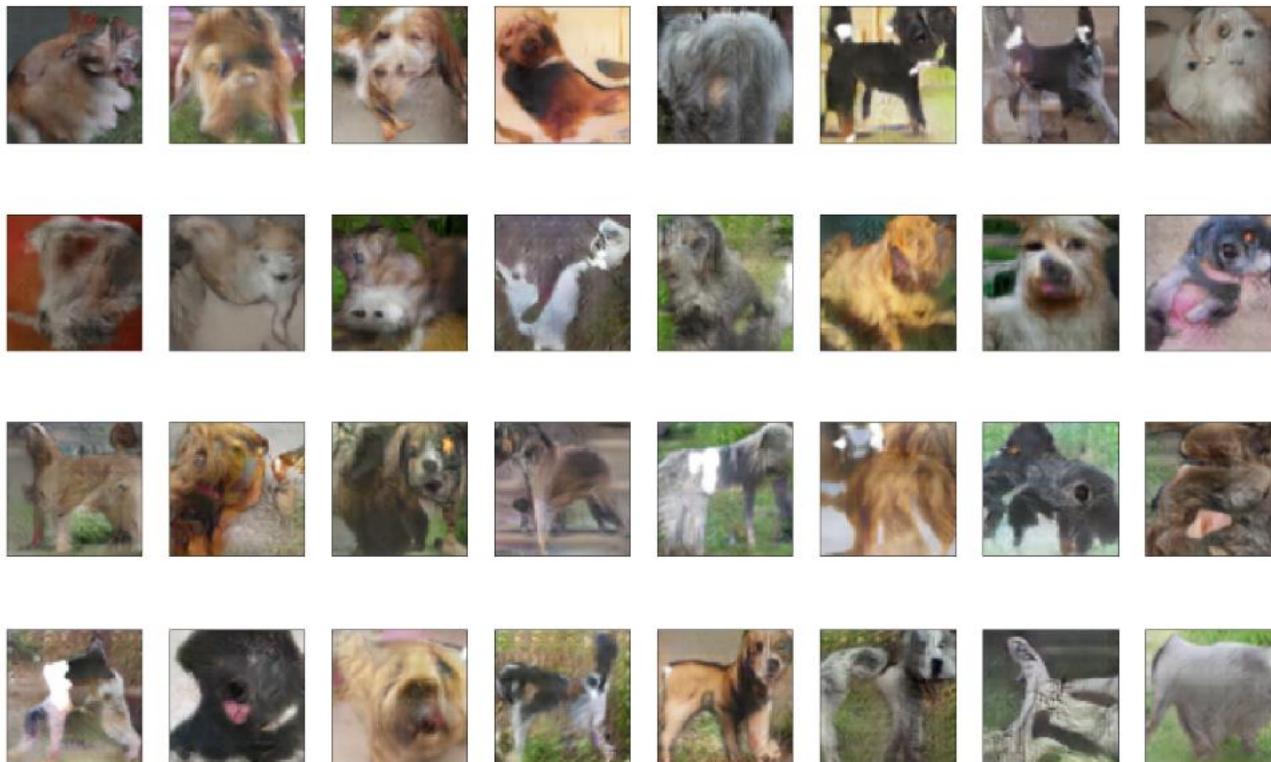


Figure 7: Result of DCGAN training using default SGD optimizer.

Thanks!

Questions?

