An Online Learning Approach to Generative Adversarial Networks

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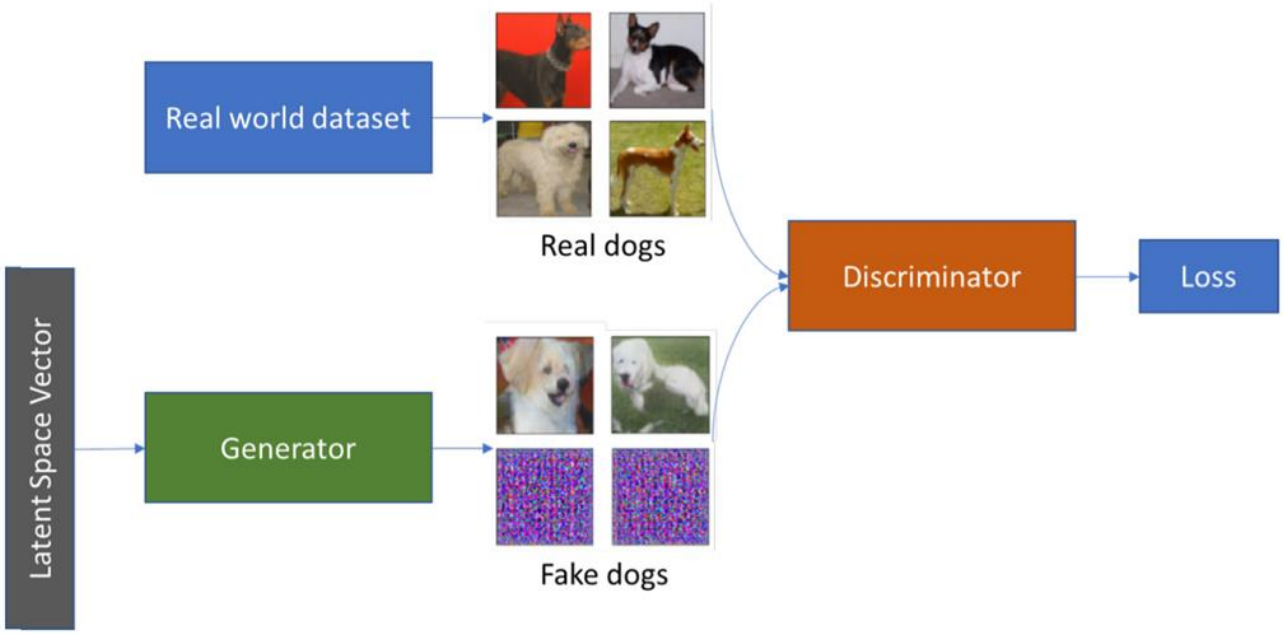
Outline

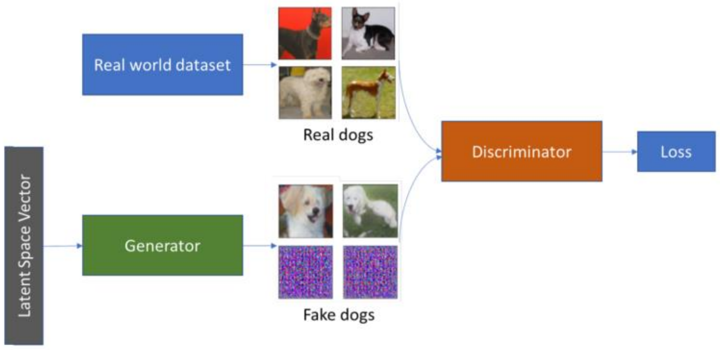
- Background of Generative Adversarial Networks (GAN).
- Mixed Nash Equilibrium (MNE).
- Theoretical algorithm: CHEKHOV GAN.
- Proof of the convergence.
- Practical algorithm and Experiments.

Generative Adversarial Network (GAN)



Generative Adversarial Network (GAN)

$$M(G,D) = \max_{D}\min_{G} L(D,G) = \frac{1}{N}\sum_{i=1}^{N} logD(x_i) + log(1 - D(G(z_i)))$$



Goal of Generator

$$\min_{u\in K_1} \boxed{\max_{v\in K_2} M(u,v)}$$

Goal of Discriminator

$$\max_{v\in K_2} \boxed{\min_{u\in K_1} M(u,v)}$$

	Rock	*Paper*	*Scissors*
Rock	*0*	*1*	*-1*
Paper	*-1*	*0*	*1*
Scissors	*1*	*-1*	*0*

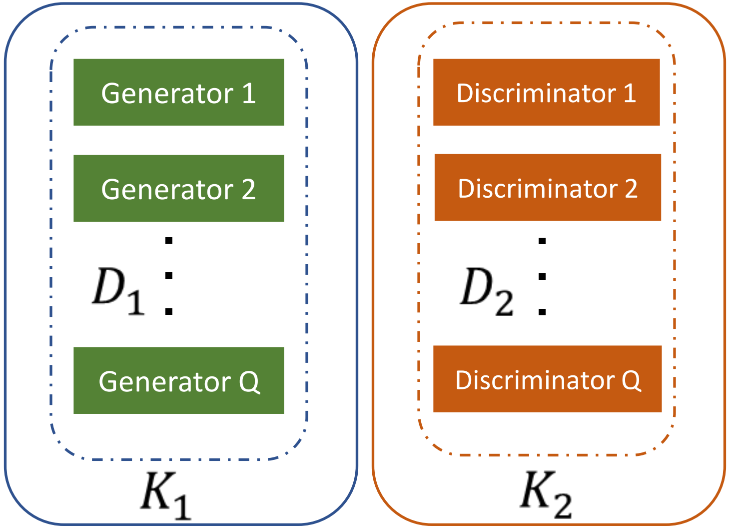
Pure Nash Equilibrium

$$M(u^*,v^*) \le \min_{u\in K_1} M(u,v^*) \qquad M(u^*,v^*) \ge \max_{v\in K_2} M(u^*,v)$$

Does a Pure Nash Equilibrium always exist?

Mixed Nash Equilibrium (MNE)

$$\mathbb{E}_{(\mathbf{u},\mathbf{v})\sim\mathcal{D}_1\times\mathcal{D}_2}[M(\mathbf{u},\mathbf{v})] \le \min_{\mathbf{u}\in\mathcal{K}_1} \mathbb{E}_{\mathbf{v}\sim\mathcal{D}_2}[M(\mathbf{u},\mathbf{v})], \ \& \ \mathbb{E}_{(\mathbf{u},\mathbf{v})\sim\mathcal{D}_1\times\mathcal{D}_2}[M(\mathbf{u},\mathbf{v})] \ge \max_{\mathbf{v}\in\mathcal{K}_2} \mathbb{E}_{\mathbf{u}\sim\mathcal{D}_1}[M(\mathbf{u},\mathbf{v})]$$



Definition 1.1. *Let $\varepsilon > 0$, the two distributions D_1, D_2 are called $\varepsilon - MNE$ if*

$$\mathbb{E}_{(u,v)\sim K_1\times K_2}[M(\boldsymbol{u},\boldsymbol{v})] \le \min_{\boldsymbol{u}\in K_1} \mathbb{E}_{v\sim D_2}[M(\boldsymbol{u},\boldsymbol{v})] + \varepsilon$$

$$\mathbb{E}_{(u,v)\sim K_1\times K_2}[M(\boldsymbol{u},\boldsymbol{v})] \ge \min_{\boldsymbol{v}\in K_2} \mathbb{E}_{u\sim D_1}[M(\boldsymbol{u},\boldsymbol{v})] - \varepsilon$$

$$if\ \varepsilon\ is\ sub - linear\ to\ T$$

$$\varepsilon = O(1/\sqrt{T})$$

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AN ONLINE LEARNING APPROACH TO GENERATIVE ADVERSARIAL NETWORKS

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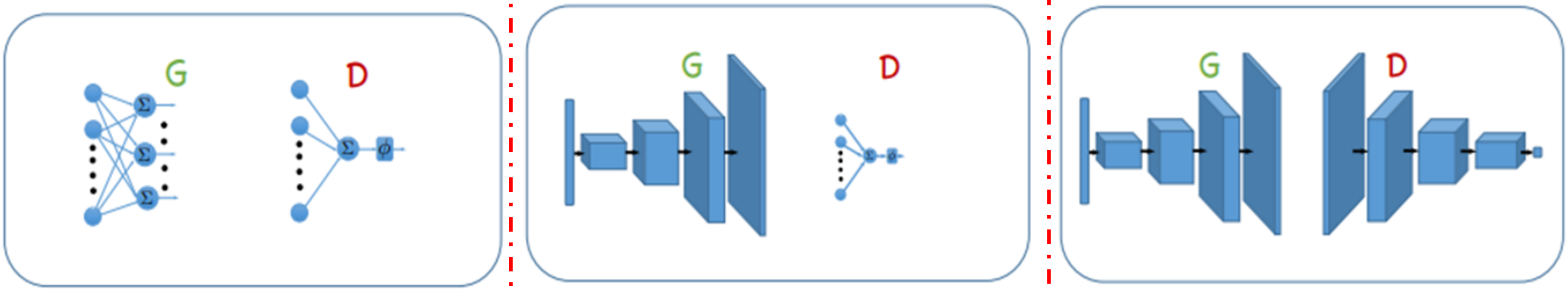
- Proposed an algorithm that guarantees finding a MNE for GAN (Semi-Shallow).
- Designed a new GAN training algorithm that applies in practice (Deep).

G: Convex
D: Arbitrary

Three Different Types of GAN

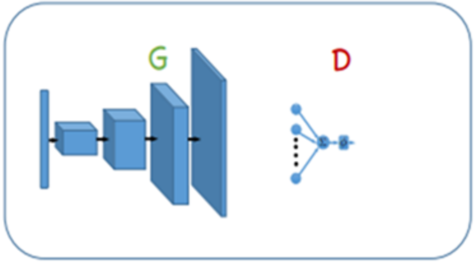


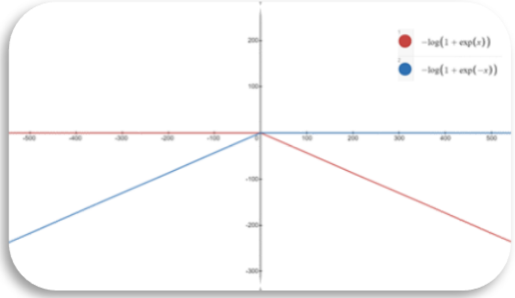
(a) G: Convex D: Concave

(b) G: Arbitrary D: Concave

(c) G: Arbitrary D: Arbitrary

Concavity of the Discriminator





$$F(a) = -log(1 + exp(-a))$$

Proposition 2.1. *Consider the GAN objective in Equation (1) and assume that the discriminator is a single-layer network with a sigmoid activation function, meaning* $D_v(x) = 1/(1 + exp(-v^T x))$, *where* $v \in R^n$. *Then the GAN objective is concave in* v.

$$M(G, D) = \max_D \min_G L(D, G) = \frac{1}{N} \sum_{i=1}^{N} logD(x_i) + log(1 - D(G(z_i)))$$

$$logD_v(x_i) = log(1) - log(1 + exp(-v^T x_i)) = -log(1 + exp(-v^T x_i))$$

$$\begin{aligned} log(1 - D_v(G_u(z_i))) &= log(1 - 1/(1 + exp(-v^T(G_u(z_i))))) \\ &= log(\frac{exp(-v^T(G_u(z_i)))}{(1 + exp(-v^T(G_u(z_i))))}) \\ &= log(1) - log(1 + 1/exp(-v^T G_u(z_i))) \\ &= -log(1 + exp(v^T G_u(z_i))) \end{aligned}$$

Algorithm 1 CHEKHOV GAN [4]

Input: Number of steps: T; Game objective: $M(\cdot, \cdot)$

for $t = 1$ `to` T **do**

Calculate:

$\boxed{\mathbf{u}_t \leftarrow \arg\min_{u \in K_1} \sum_{\tau=0}^{t-1} f_\tau(\mathbf{u})}$ $\triangleright A_1$

$\boxed{\mathbf{v}_t \leftarrow \arg\max_{v \in K_2} \sum_{\tau=0}^{t-1} \nabla g_\tau(\mathbf{v}_\tau)^T \mathbf{v} - \frac{\sqrt{T}}{2\eta_0} ||\mathbf{v}||^2}$ $\triangleright A_2$

Update:

$f_t(\cdot) = M(\cdot, \mathbf{v}_t)$; $g_t(\cdot) = M(\mathbf{u}_t, \cdot)$

end for

Output: $D_1 \sim Uni\{\mathbf{u}_1, ..., \mathbf{u}_T\}$; $D_2 \sim Uni\{\mathbf{v}_1, ..., \mathbf{v}_T\}$

Theorem 3.1. *Let K_2 be a convex set, M be a semi-concave zero-sum game, and assume M is L-Lipschitz continuous. Then the mixed strategies (D_1, D_2) produced by Algorithm 1 are $\varepsilon - MNE$, where $\varepsilon = \mathcal{O}(1/\sqrt{T})$.*

Proof Sketch

Theorem 3.1. *Let K_2 be a convex set, M be a semi-concave zero-sum game, and assume M is L-Lipschitz continuous. Then the mixed strategies (D_1, D_2) produced by Algorithm 1 are $\varepsilon - MNE$, where $\varepsilon = \mathcal{O}(1/\sqrt{T})$.*

Theorem 3.2. *The mixed strategies (D_1, D_2) that Algorithm 1 outputs are $\varepsilon - MNE$, where $\varepsilon := (B_T^{A_1} + B_T^{A_2})/T$ and $B_T^{A_1}$, $B_T^{A_2}$ are bounds on the regret of A_1, A_2.*

$B_T^{A_1}$ is $\mathcal{O}(\sqrt{T})$ $B_T^{A_2}$ is $\mathcal{O}(\sqrt{T})$

Proof of Theorem 3.2

Theorem 3.2. *The mixed strategies (D_1, D_2) that Algorithm 1 outputs are $\varepsilon - MNE$, where $\varepsilon := (B_T^{A_1} + B_T^{A_2})/T$ and $B_T^{A_1}$, $B_T^{A_2}$ are bounds on the regret of A_1, A_2.*

Proof. Let $f_t(\mathbf{u}) := \mathbf{M}(\mathbf{u}, \mathbf{v_t})$, $g_t(\mathbf{v}) := \mathbf{M}(\mathbf{u_t}, \mathbf{v})$. Recall that K_1, K_2 are domain of f, g; $B_T^{A_1}$, $B_T^{A_2}$ are bounds on the regret of A_1, A_2. Then

$$\sum_{t=1}^{T} M(\mathbf{u}_t, \mathbf{v}_t) - min_{\mathbf{u}\in K_1} \sum_{t=1}^{T} M(\mathbf{u}, \mathbf{v}_t) \le B_T^{A_1}$$

$$max_{\mathbf{v}\in K_2} \sum_{t=1}^{T} M(\mathbf{u}_t, \mathbf{v}) \ge \mathbb{E}_{v\sim K_2}[\sum_{t=1}^{T} M(\mathbf{u}_t, \mathbf{v})]$$

$$max_{\mathbf{v}\in K_2} \sum_{t=1}^{T} M(\mathbf{u}_t, \mathbf{v}) - \sum_{t=1}^{T} M(\mathbf{u}_t, \mathbf{v}_t) \le B_T^{A_2} \longrightarrow \mathbb{E}_{v\sim K_2}[\frac{1}{T}\sum_{t=1}^{T} M(\mathbf{u}_t, \mathbf{v})] - \frac{1}{T}\sum_{t=1}^{T} M(\mathbf{u}_t, \mathbf{v}_t) \le \frac{1}{T} B_T^{A_2}$$

Proof of Theorem 3.2

Theorem 3.2. *The mixed strategies* (D_1, D_2) *that Algorithm 1 outputs are* $\varepsilon - MNE$*, where* $\varepsilon := (B_T^{A_1} + B_T^{A_2})/T$ *and* $B_T^{A_1}$, $B_T^{A_2}$ *are bounds on the regret of* A_1, A_2.

$$\sum_{t=1}^{T} M(\mathbf{u}_t, \mathbf{v}_t) - min_{\mathbf{u}\in K_1} \sum_{t=1}^{T} M(\mathbf{u}, \mathbf{v}_t) \le B_T^{A_1}$$

$$\mathbb{E}_{v\sim K_2}[\frac{1}{T}\sum_{t=1}^{T} M(\mathbf{u}_t, \mathbf{v})] - \frac{1}{T}\sum_{t=1}^{T} M(\mathbf{u}_t, \mathbf{v}_t) \le \frac{1}{T} B_T^{A_2}$$

$$\mathbb{E}_{v\sim K_2}[\frac{1}{T}\sum_{t=1}^{T} M(\mathbf{u}_t, \mathbf{v})] - min_{\mathbf{u}\in K_1} \frac{1}{T}\sum_{t=1}^{T} M(\mathbf{u}, \mathbf{v}_t) \le \boxed{\frac{1}{T}(B_T^{A_1} + B_T^{A_2})} := \varepsilon$$

Definition 1.1. *Let* $\varepsilon > 0$*, the two distributions* D_1, D_2 *are called* $\varepsilon - MNE$ *if*

$$\mathbb{E}_{(u,v)\sim K_1\times K_2}[M(\boldsymbol{u}, \boldsymbol{v})] \le \min_{\boldsymbol{u}\in K_1} \mathbb{E}_{v\sim D_2}[M(\boldsymbol{u}, \boldsymbol{v})] + \varepsilon$$

$$\mathbb{E}_{(u,v)\sim K_1\times K_2}[M(\boldsymbol{u}, \boldsymbol{v})] \ge \min_{\boldsymbol{v}\in K_2} \mathbb{E}_{u\sim D_1}[M(\boldsymbol{u}, \boldsymbol{v})] - \varepsilon$$

$$\mathbb{E}_{(u,v)\sim K_1\times K_2}[M(\mathbf{u}, \mathbf{v})] \le \min_{\mathbf{u}\in K_1} \mathbb{E}_{v\sim D_2}[M(\mathbf{u}, \mathbf{v})] + \varepsilon$$

Proof of Regret Bounds

$B_T^{A_1}$ is $\mathcal{O}(\sqrt{T})$ *Let* d_2 *is the diameter of* K_2, M *is* $L-$ *Lipschitz.*

$$
\begin{aligned}
Regret_T^{A_1} &\leq \sum_{t=1}^{T}(f_t(u_t) - f_t(u_{t+1})) \\
&= \sum_{t=1}^{T-1}(f_t(u_t) - f_t(u_{t+1}) + f_{t+1}(u_{t+1}) - f_{t+1}(u_{t+1})) + (f_T(u_T) - f_T(u_{T+1})) \\
&= \sum_{t=1}^{T-1}(f_{t+1}(u_{t+1}) - f_t(u_{t+1})) + \sum_{t=1}^{T-1}(f_t(u_t) - f_{t+1}(u_{t+1})) + (f_T(u_T) - f_T(u_{T+1})) \\
&= \sum_{t=1}^{T-1}(M(u_{t+1}, v_{t+1}) - M(u_{t+1}, v_t)) + (f_1(u_1) - f_T(u_{T+1})) \\
&\leq Ld_2\sqrt{T}/\sqrt{2} + (f_1(u_1) - f_T(u_{T+1})) = Ld_2\sqrt{T}/\sqrt{2} + 2C
\end{aligned}
$$

Proof of Regret Bounds

$B_T^{A_2}$ is $\mathcal{O}(\sqrt{T})$ *Let* d_2 *is the diameter of* K_2, M *is* $L-$ *Lipschitz.*

$$Regret_T \leq 2\eta \sum_{t=1}^{T} || \nabla_{f_t} ||^2 + \frac{R(u) - R(x_1)}{\eta}$$

$$= 2\eta L^2 T + \frac{d^2}{\eta} \qquad \eta = \frac{d}{\sqrt{2T}L}$$

$$Regret_T^{A_2}(g_1, \ldots, g_T) \leq L d_2 \sqrt{2T}$$

Algorithm 1 CHEKHOV GAN [4]

Input: Number of steps: T; Game objective: $M(\cdot,\cdot)$
  for $t = 1$ `to` T **do**
    Calculate:
    $\mathbf{u}_t \leftarrow \arg\min_{u \in K_1} \sum_{\tau=0}^{t-1} f_\tau(\mathbf{u})$ $\triangleright A_1$
    $\mathbf{v}_t \leftarrow \arg\max_{v \in K_2} \sum_{\tau=0}^{t-1} \nabla g_\tau(\mathbf{v}_\tau)^T \mathbf{v} - \frac{\sqrt{T}}{2\eta_0} ||\mathbf{v}||^2$ $\triangleright A_2$
    Update:
    $f_t(\cdot) = M(\cdot, \mathbf{v}_t)$; $g_t(\cdot) = M(\mathbf{u}_t, \cdot)$
  end for
Output: $D_1 \sim Uni\{\mathbf{u}_1, ..., \mathbf{u}_T\}$; $D_2 \sim Uni\{\mathbf{v}_1, ..., \mathbf{v}_T\}$

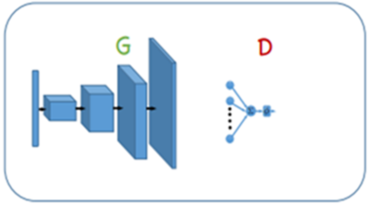
GAN is a minimax zero-sum game.

Standard method aim to find a Pure Nash Equilibrium.

Pure Nash Equilibrium not always exist

We aim to find a Mixed Nash Equilibrium

Finding a Mixed Nash Equilibrium guarantees convergence on Semi-Shallow GAN.



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Update:

$f_t(\cdot) = M(\cdot, \mathbf{v}_t)$; $g_t(\cdot) = M(\mathbf{u}_t, \cdot)$

end for

Output: $D_1 \sim Uni\{\mathbf{u}_1, ..., \mathbf{u}_T\}$; $D_2 \sim Uni\{\mathbf{v}_1, ..., \mathbf{v}_T\}$

Theoretical CHEKHOV GAN does not work in practice!

Practical CHEKHOV GAN

Algorithm 2 Practical CHEKHOV GAN [4], modified

Input: Number of steps: T; Game objective: $M(\cdot,\cdot)$; Number of past states K; Spacing m.
Initialize: Set loss/reward $f_0(\cdot)=0, g_0(\cdot)=0$, and $Q_1.insert(f_0), Q_2.insert(g_0)$

for $t = 1$ `to` T **do**
  $\mathbf{u}_{t+1} \leftarrow u_t - \eta_t \cdot \nabla_{u_t}(\frac{1}{|Q_1|}\sum_{f\in Q_1} f(u) + \frac{C}{\sqrt{t}}||u||^2)$
  $\mathbf{v}_{t+1} \leftarrow v_t - \eta_t \cdot \nabla_{v_t}(\frac{1}{|Q_2|}\sum_{g\in Q_2} g(v) - \frac{C}{\sqrt{t}}||v||^2)$
  Calculate: $f_t(\cdot) = M(\cdot, v_t)$; $g_t(\cdot) = M(u_t, \cdot)$
  Update Q_1 and Q_2:
  if t mod $m == 0$ and $|Q| == K$ **then** $\triangleright\ |Q| = |Q_1| = |Q_2|$
    Q_1.remove_last(); Q_2.remove_last().
    Q_1.insert(f_t); Q_2.insert(g_t).
  else if $|Q| < K$ **then**
    Q_1.insert(f_t); Q_2.insert(g_t).
  else
    Q_1.replace_first(f_t); Q_2.replace_first(g_t).
  end if
end for

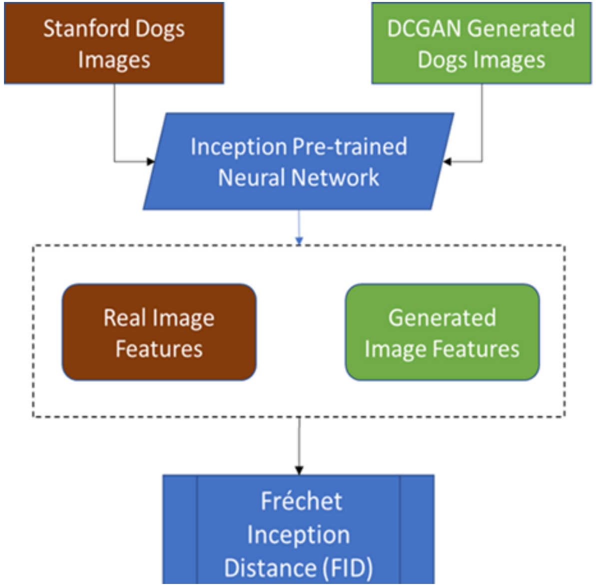
Output: $D_1 \sim Uni\{\mathbf{u}_1, ..., \mathbf{u}_K \in Q_1\}$; $D_2 \sim Uni\{\mathbf{v}_1, ..., \mathbf{v}_K \in Q_2\}$

Experiments

Data set:
Cifar-10 (paper)
Stanford Dogs Images (mine)

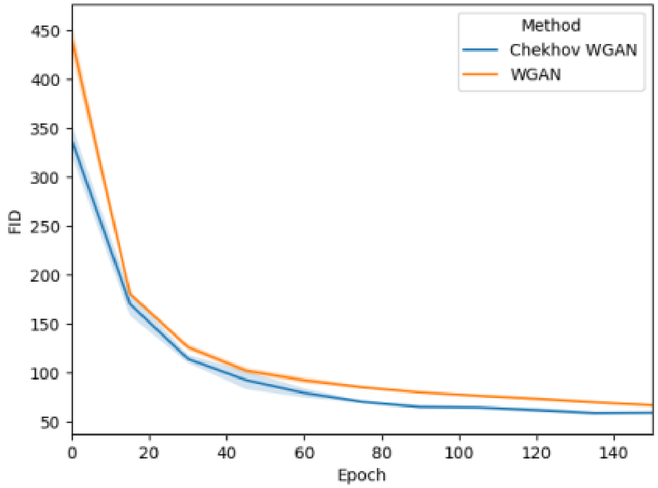
Evaluation metrics:
Fréchet Inception Distance

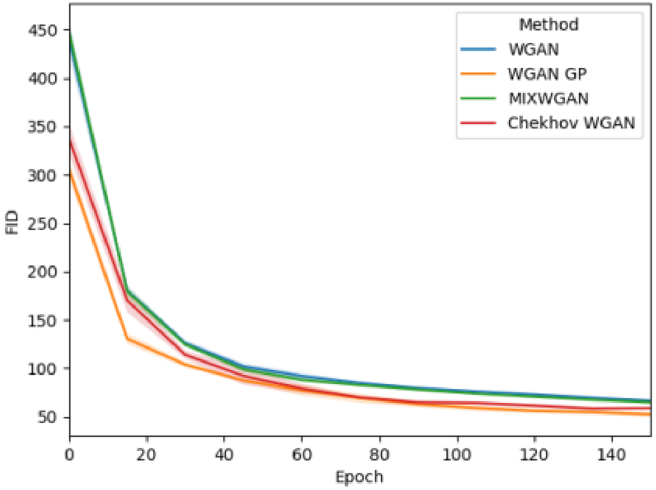
$$FID = \|\mu_r - \mu_s\|^2 + Tr\left(\Sigma_r + \Sigma_g - 2\sqrt{\Sigma_r\Sigma_g}\right)$$



Experiments

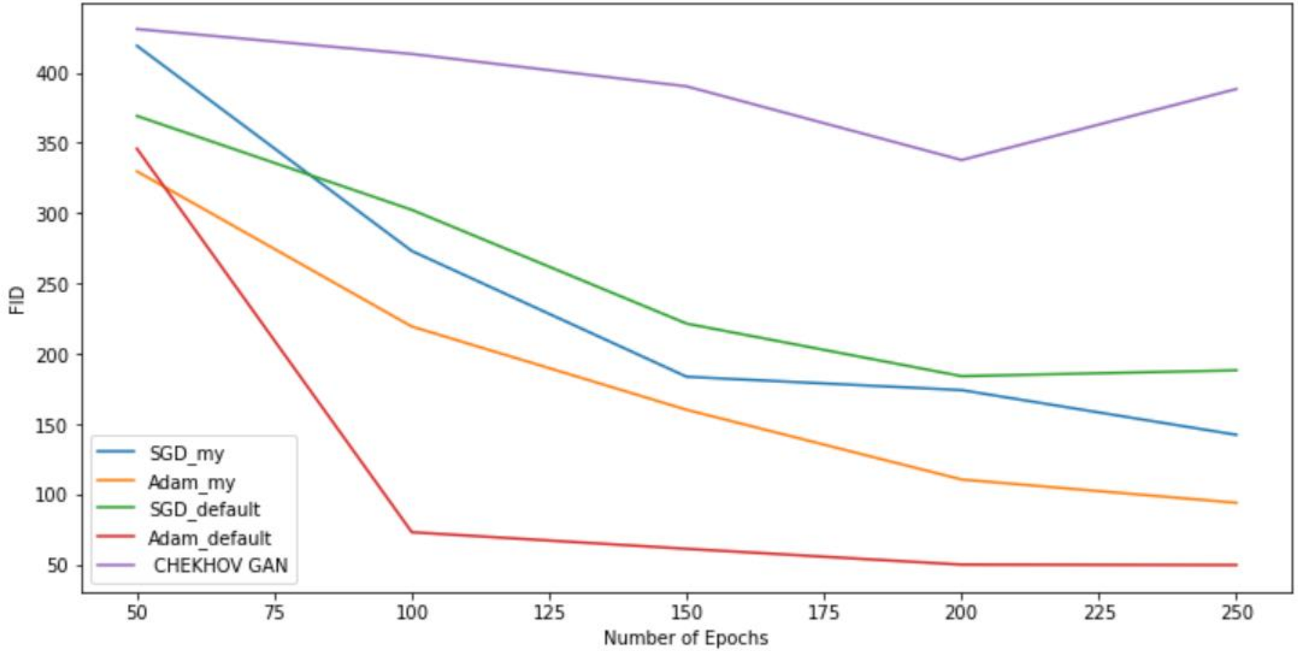
Cifar-10 (paper)





Experiments

Stanford Dogs Images (mine)



Discussion

- Converge but the speed?
- Experiments on Semi-Shallow GAN that show standard GAN training method does not converge but CHEKHOV GAN converges.
- The practical Chekhov GAN is using gradients and is very different from the theoretical Chekhov GAN, so does the algorithm still guarantee convergence?
- The proposed algorithm requires significantly large GPU memory and is also computational.
- The experiments in the original paper do not show many advantages of Checkhov GAN, though Chekhov WGAN consistently outperforms WGAN by achieving better FID scores.
- Proof convergence of Deep GAN architecture.

References

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Thanks!

Questions?

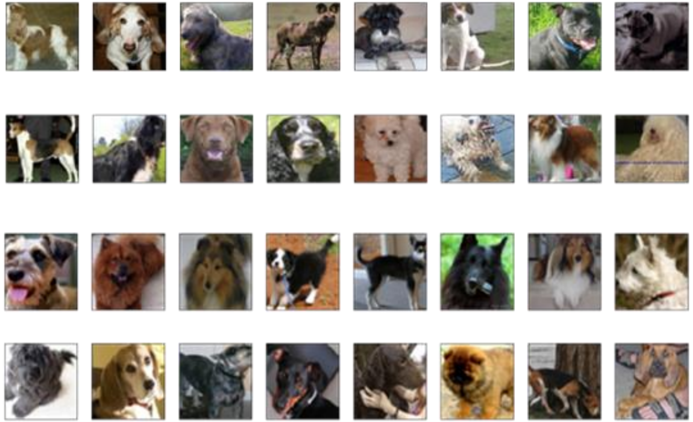


Fig.1 Image samples from Stanford Dog Dataset.

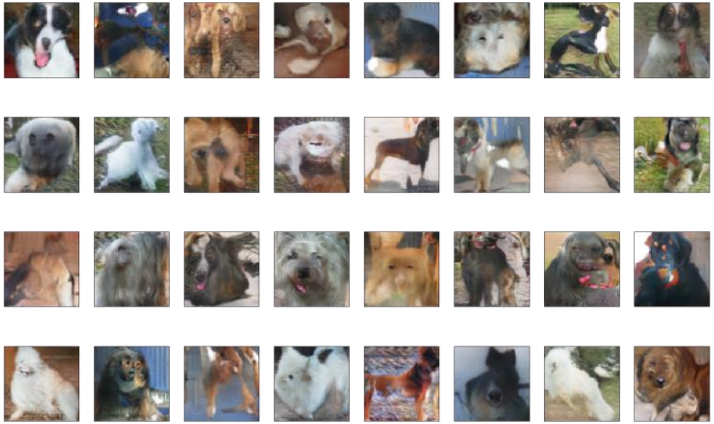


Fig.2 Generated using my implementation of Adam.

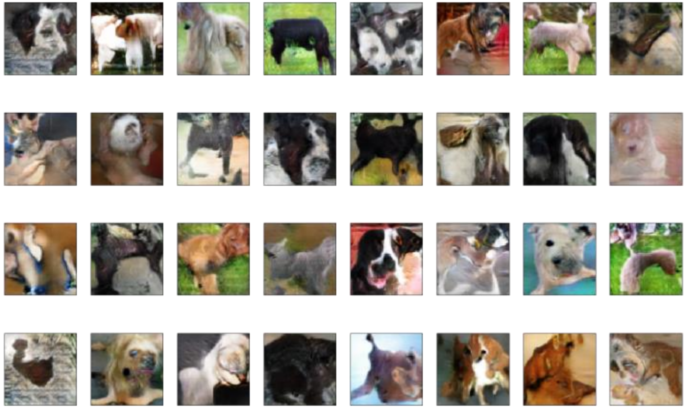


Fig.3 Generated using my implementation of SGD.

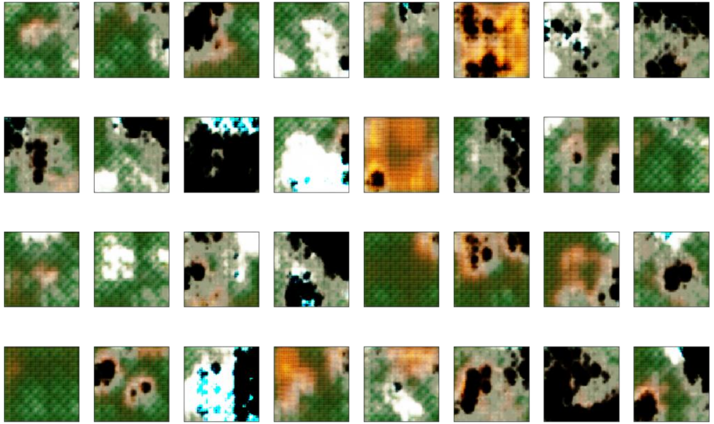


Fig.4 Generated using my implementation of CHECKOV GAN.

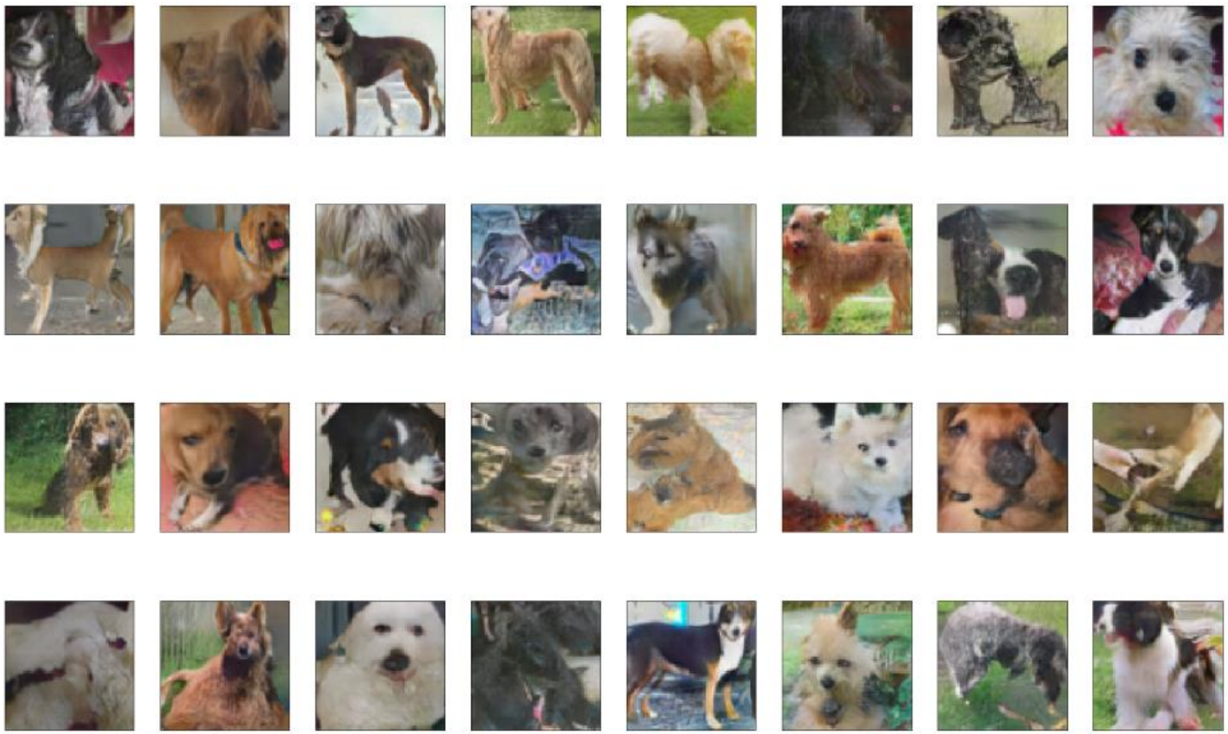


Figure 5: Result of DCGAN training using default Adam optimizer.

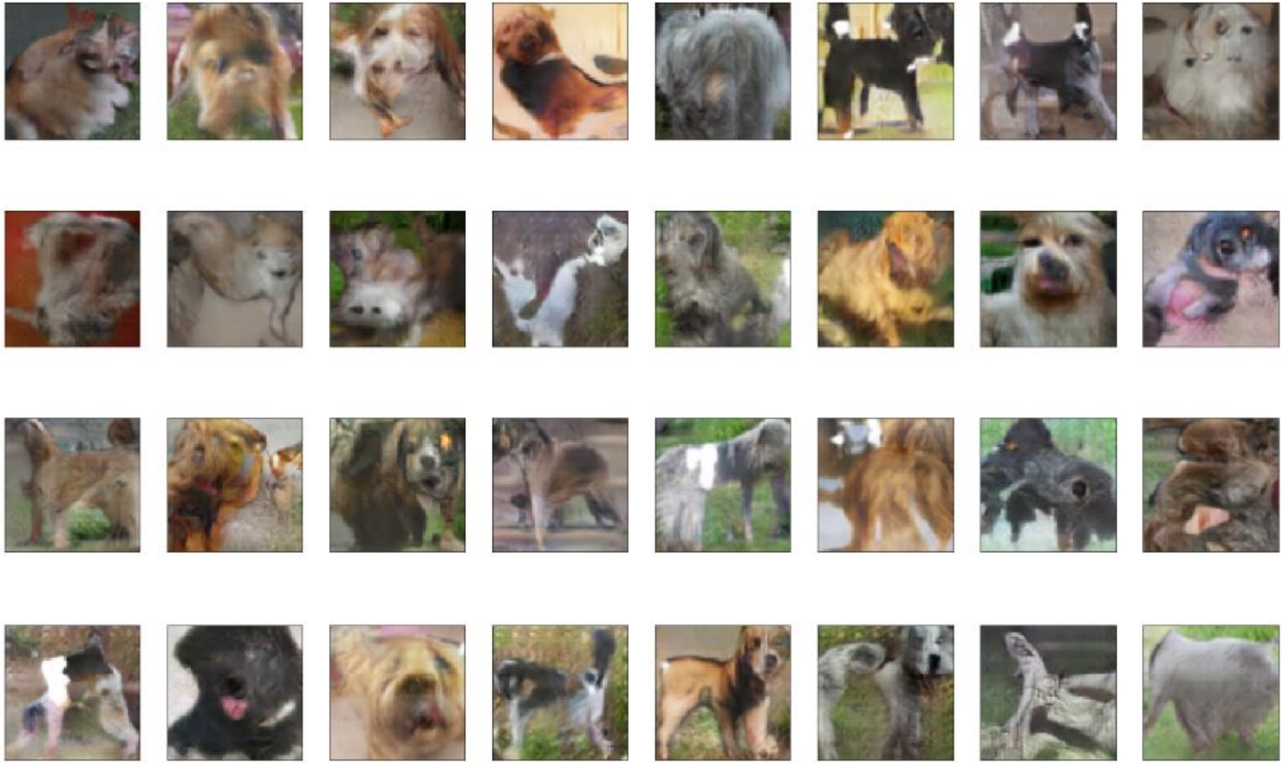
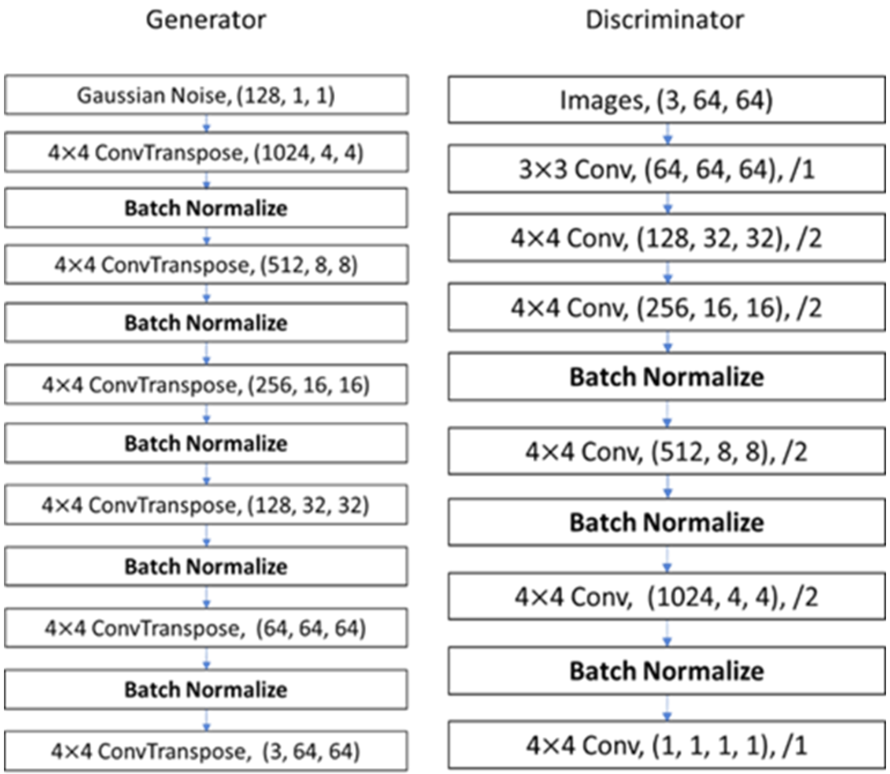


Figure 7: Result of DCGAN training using default SGD optimizer.

Thanks!

Questions?


Generator
Gaussian Noise, (128, 1, 1)
4×4 ConvTranspose, (1024, 4, 4)
Batch Normalize
4×4 ConvTranspose, (512, 8, 8)
Batch Normalize
4×4 ConvTranspose, (256, 16, 16)
Batch Normalize
4×4 ConvTranspose, (128, 32, 32)
Batch Normalize
4×4 ConvTranspose, (64, 64, 64)
Batch Normalize
4×4 ConvTranspose, (3, 64, 64)
Discriminator
Images, (3, 64, 64)
3×3 Conv, (64, 64, 64), /1
4×4 Conv, (128, 32, 32), /2
4×4 Conv, (256, 16, 16), /2
Batch Normalize
4×4 Conv, (512, 8, 8), /2
Batch Normalize
4×4 Conv, (1024, 4, 4), /2
Batch Normalize
4×4 Conv, (1, 1, 1, 1), /1