1 Appendix

1.1 The cause of the "strategic effect"

The main difference between our equilibrium and that of [**?**] is the presence of a "strategic effect", wherein doctor j takes into account the behavior of other doctors in the selection of her own $\bar{\kappa_j}$. Of the different modifications we made to her framework, we'll argue it's the absence of *additive separability* across patients in physician utility $U_j(\cdot)$ which accounts for this.

In our context of unbounded maximization by doctor j, the absence of additive separability implies she can't consider each patient individually when it comes to whether she's willing to allow (or induce) their visit, as such a decision is no longer independent of other patients' visits; the marginal utility of an additional patient is dependent on the aggregate of clients up to that point, both in terms of the visit itself as well as in the number of sick leave certificates granted up to that point.

Let's illustrate this point. Consider for a moment a finite number of patients $1, ..., k$, where each patient is inputed directly as an argument in our doctor j's $U_j(\cdot)$, like so: $U_j(1, ..., k)$. If $U_j(\cdot)$ has the property of additive separability, this means it may be reformulated like so:

$$U_j(1, ..., k) = v_{j1}(1) + ... + v_{jk}(k) = \sum_{i=1}^{k} v_{ji}(i)$$

Unconstrained optimization in this context implies she's willing to see any patient whose $v_{ji}(i)$ is non-negative, such that her optimal level of utility is:

$$U_j^*(1, ..., k) = \sum_{i:\ v_{ji}(i) \geq 0} v_{ji}(i)$$

In the context of our doctor-patient model, where physician utility is increasing in κ_i, selection is achieved by the doctor by choosing a κ_j^* which excludes all patients i whose $\kappa_i < \kappa_j^*$. Supposing our patients are well-ordered in κ_i, the choice of such a κ_j^* would be one where the marginal consumer i affords a non-negative $v_{ji}(i)$, and the inframarginal consumer $i + 1$ fulfills $v_{j,i+1}(i + 1) < 0$. Ignoring for a moment that patients themselves have a *choice* of visiting – depending upon a second dimension γ –, we would then have:

$$U_j^*(1, ..., k) = \sum_{i:\ \kappa_i \geq \kappa_j^*} v_{ji}(i)$$

If instead of a discrete set we consider a mass of consumers $\mathcal{I}$ characterized by their level of κ_i, and we make the simplifying assumption that v_{ji} takes on the same form v_j for every $i \in \mathcal{I}$, our $U_j(\cdot)$ could be expressed as:

$$U_j^*(\mathcal{I}) = \int_{\kappa_j^*}^{\infty} v_j(k)\, dF(k)$$

$U_j^*(\mathcal{I})$ represents the optimal value of $U_j(\mathcal{I})$, where doctor j only sees patients who provide her with non-negative marginal utility, i.e. such that $\kappa_i \geq \kappa_j^*$. We can find this optimal value κ_j^* by looking at *threshold* equilibria, where doctor utility is also dependent on the threshold $\bar{\kappa_j}$ over which she's willing to see patients:

$$U_j(\mathcal{I}, \bar{\kappa_j}) = \int_{\bar{\kappa_j}}^{\infty} v_j(k)\, dF(k)$$

The optimal value of threshold $\bar{\kappa_j}$ is κ_j^*. Assuming $U_j(\cdot)$ is twice differentiable and concave in $\bar{\kappa_j}$, such a solution may be arrived at through the FCO:

$$\frac{\partial U_j(\mathcal{I})}{\partial \bar{\kappa_j}} \equiv -v_j(\bar{\kappa_j}) f(\bar{\kappa_j}) = 0$$

Solving for this would yield $\bar{\kappa_j} = \kappa_j^*$.

[**?**] is an example of just such a treatment, which specifies the physician's additive utility by patient in the following manner:

$$v_{ji}(\kappa_i) \equiv R_j + \beta_j h(\kappa_i)$$

where R_j is a parameter standing for revenue by visit, and $\beta_j h(\kappa_i)$ represents the physician's "altruistic" utility over the health impact of a prescription drug to a patient with "pain level" κ_i.

The optimal threshold κ_j^* is then obtained out of the maximization[1]:

$$\max_{\bar{\kappa_j}} \int_{\bar{\kappa_j}}^{\infty} R_j + \beta_j h(k)\, dF(k)$$

Which gives out the following FOC:

$$R_j = -\beta_j h'(\bar{\kappa_j})$$

which, as is immediately apparent, doesn't depend upon the behavior of other doctors —more specifically, *their* choice of $\bar{\kappa_j}$. It merely establishes that, at the threshold, marginal benefit by patient –revenue R_j– must equal marginal "cost" –altruistic "cost" $\beta_j h(\bar{\kappa_j})$–. A way to interpret this is that doctors will be willing to see any and all patients which render them positive marginal utility, unconcerned with their total market share, and thus, what other physicians may do to take away clientele.

Such a treatment is rendered inviable by our choice of utility function. Schnell's parameter of revenue would in our model imply the linearity of our revenue *function* $R_j(\cdot)$ and our $P(\cdot)$ function over *aggregate* licenses granted. Strict convexity of $P(\cdot)$ would forestall its formulation as Schnell-like $\beta_j h'(\bar{\kappa_j})$ terms for each patient, because the impact on doctor j's utility in granting patient i a license would no longer independent from the granting of licenses of

[1] Once again, ignoring patient choice and γ,. We're presenting a *simplified* version for illustrative purposes.

other patients. Likewise, strict concavity of $R_j(\cdot)$ belies a simple "r" parameter such that $R_j'(Q_j) = rQ_j$.

More formally, when $U_j(1,...,k)$ *isn't* additively separable across patients, the value κ_j^* such that if $\kappa_i \geq \kappa_j^*$ patient i provides positive marginal utility, isn't independent of current clientele, because what before was a properly defined object, marginal utility by patient i, $v_j(\kappa_i)$, can no longer be so identified. The marginal utility i provides to j as the kth client (assuming some order over clients) is not necessarily the same he'd provide as the $k+1$th client, and so, as the kth client he could provide 0 utility, impliying $\kappa_i = \kappa_j^*$, whereas as the $k+1$th he could be inframarginal, such that $\kappa_i < \kappa_j^*$.

κ_j^* is not longer independent of clientele mass $\mathcal{I}$ as before, but a function of it, $\kappa_j^*(\mathcal{I})$. More specifically, in the models we consider it will depend on the *cardinality* of current clientele, $|\mathcal{I}|$, such that our physician's choice of marginal consumer will depend on her aggregate level of patient demand, where before it didn't. This effect is introduced through the *strict non-linearity* of our physician utility function $U_j(\cdot)$, either through the strict concavity of $R_j(\cdot)$ over expected patient demand Q_j, or the strict convexity of $P_j(\cdot)$ over total expected sick leaves granted.

When either of those is the case, *aggregate* levels enter into the equation and form part of the optimality condition. Our general FOC reflects this:

$$R_j'(Q_j)\frac{\partial Q_j}{\partial \bar{\kappa_j}} = P_j'(X_j)\frac{\partial X_j}{\partial \bar{\kappa_j}} \tag{1}$$

where either or both of $R_j'(\cdot)$ and $P_j'(\cdot)$ are a non-constant function over aggregates.

When such aggregates, either Q_j or X_j, come into play, physicians come to consider their *market share*, which doesn't depend exclusively on their own choice of $\bar{\kappa_j}$. Each patient's strategy S_i is constructed taking into account the whole of $\{(V_j, \bar{\kappa_j})\}_{i=1}^{J}$.

Imagine equation (1) holds for some doctor j, and some doctor $l \neq j$ decides to lower her $\bar{\kappa_l}$, enough that it changes the value of s_{ij} for some mass of clients. The value of Q_j would then change, and therefore that of $R_j'(Q_j)$. If $P_j'(X_j)$ didn't vary by the same amount, (1) would no longer be an equality, leading j to modify her choice of $\bar{\kappa_j}$ to make equality hold.

This intelligible line of reasoning links the presence of a "strategic effect" to the non-additive separability of $U_j(\cdot)$: doctor j takes into account other physicians' strategy in her own choice of $\bar{\kappa_j}$ because of the present of *aggregate* amounts of clientele in her optimality conditions, which is so because utility isn't additively separable across clients.

1.2 [?] with Logit choice

To prove our point that it is additive separability which accounts for a possible strategic effect, we reformulate [?] in the manner of a McFadden Logit – with some quirks.

The way in which Schnell devises her physician's utility formula is implicitly Bernoulli-like:

$$\int_{\kappa}\int_{\gamma} u(\kappa,\gamma)\cdot p(\kappa,\gamma)\, dG(\gamma)dF(\kappa)$$

where $u(\kappa,\gamma)$ is the utility function of the physician from patients characterised by a given (κ,γ) tuple, and $p(\kappa,\gamma)$ stands for the proportion of clients atomized in such a tuple the physician expects to have visit her.

$u(\kappa,\gamma)$ we'll leave as Schnell defined it: $\beta_j h(\kappa)+R_j$. $p(\kappa,\gamma)$ lends itself nicely as the s_{ij} we have been using thus far, which implicitly depends on the (κ_i,γ_i) tuple which characterizes patient i:

$$\int_{\kappa}\int_{\gamma} [\beta_j h(\kappa)+R_j]\cdot s_{ij}(\kappa,\gamma)\, dG(\gamma)dF(\kappa)$$

In our modeling section, our "implicit" search model was a left-censored Logit choice model, which gave null probability of assignment to physicians from which patient i expected non-positive utility. It was defined as:

$$s_{ij} = \frac{\alpha_{ij}}{\sum_{k=1}^{J}\alpha_{ik}}, \quad \text{where} \quad \alpha_{ij} = \begin{cases} e^{\lambda u_{ij}}, & \text{if} \quad u_{ij}>0 \\ 0, & \text{if} \quad u_{ij}=0 \end{cases}$$

Schnell's opioid-focused physician model provides an additional simplification: patients which aren't given a drug prescription, i.e. those such that $\kappa_i < \bar{\kappa_j}$, don't garner positive utility from a visit. As such, in the original model as in this reformulation, patients "below" $\bar{\kappa_j}$ have a null probability of visit, such that the inferior limit of integration for κ is $\bar{\kappa_j}$.

As for γ, the limit of the inner integral remains the same as in Schnell in the absence of a secondary market, a value of γ_i such that $u_{ij}=0$, i.e. $h(\kappa_i)+\gamma_i-\tau^d-\tau^o=0$.[2]

The double-integral maximized as physician utility is then as follows:

$$\max_{\bar{\kappa_j}} \int_{\bar{\kappa_j}}^{\infty}\int_{\tau^d-\tau^o-h(\kappa)}^{\infty} [\beta_j h(\kappa)+R_j]\cdot \frac{e^{\lambda u_{ij}}}{\sum\limits_{k:\, u_{ik}>0} e^{\lambda u_{ik}}}\, dG(\gamma)dF(\kappa)$$

The FOC of such an equation is:

$$[\beta_j h(\bar{\kappa_j})+R_j]\int_{\tau^d-\tau^o-h(\bar{\kappa_j})}^{\infty} \frac{e^{\lambda u_{ij}} \mid \bar{\kappa_j}}{\sum\limits_{k:\, u_{ik}>0} e^{\lambda u_{ik}} \mid \bar{\kappa_j}}\, dG(\gamma)f(\bar{\kappa_j}) = 0 \tag{2}$$

Where we use "$\mid \bar{\kappa_j}$" to clumsily indicate that we integrate γ over patients where $\kappa_i=\bar{\kappa_j}$. As an integral over a strictly positive value and the atom of a density function, respectively, the factors B and C in the equation are non-negative. For this reason, the only way for (2) to be fulfilled is the following condition:

$$R_j = -\beta_j h(\bar{\kappa_j}) \tag{3}$$

[2] Schnell splits the costs a patient will face into costs of visit (τ^d), cost of purchase (τ^o) and search cost (τ^s)

This is the same result as in the original model in **??** in the absence of a secondary market, a condition which stipulates that marginal utility in $\bar{\kappa_j}$ must be 0, i.e. that revenue (R_j) must equal "altruistic" loss ($\beta_j h(\bar{\kappa_j})$), which presupposes that the marginal patient granted prescription opioids suffers a net loss in utility (the negative externalities outweigh its medical benefit as a palliative).

Condition (3) means a value κ_j is chosen irrespective of the strategies employed by other doctors, it quite literally doesn't enter into the equation. As we have argued, additive separability over clients means the *total* demand for physician j's services doesn't influence marginal utility by a given patient i, and so has no sway in the optimality condition. Doctor j states simply that she won't grant a prescription (or sick leave, as in our case) below $\bar{\kappa_j}$, *come what may.* In order for her to care about *aggregate* values, like her total expected demand, making her care about her market share and thus about the strategies of other doctors, *strict non-linearity* must be introduced into her utility function.

1.3 On the strategic complementarity of $\bar{\kappa_j}$'s

We had defined the components s_{ij} of S_i as:

$$s_{ij} \equiv g_i(u_{ij}, u_{i,-j})$$

where $g_i(\cdot)$ is a continuous function weakly increasing in the first argument u_{ij}. Our defining s_{ij} this way has two interlinked corollaries:

Corollary 1.1.

$$s_{ij} \mid \kappa_i < \bar{\kappa_j} \quad \leq \quad s_{ij} \mid \kappa_i \geq \bar{\kappa_j} \quad \textit{(with strict inequality if } \gamma_i > 0\textit{).}$$

Proof. For a fixed value of V_j and κ_i, the value of $U_i(V_j, \bar{\kappa_j})$ is $V_j\kappa_i - \tau$ if $\kappa_i < \bar{\kappa_j}$, and $\gamma_i + V_j\kappa_i - \tau$ if $\kappa_i \geq \bar{\kappa_j}$, where $\gamma_i \geq 0$. Given that s_{ij} is weakly increasing in u_{ij}, the corollary follows. □

Corollary 1.2.

$$\frac{\partial s_{ij}}{\partial \bar{\kappa_j}} \leq 0 \qquad\qquad \frac{\partial s_{ij}}{\partial \bar{\kappa_l}} \geq 0, \forall l \neq j$$

Proof. From the argumentation in corollary 1 follows that u_{ij} is weakly decreasing in $\bar{\kappa_j}$, $\forall j$. Take some $l \neq j$, then s_{ij} is defined in turn as weakly decreasing in u_{il}, which implies it is increasing in $\bar{\kappa_l}$. □

Both corollaries hinge upon our definition of $U_i(\cdot)$ as a step function over κ_i, such that it is discontinuous at $\kappa_i = \bar{\kappa_j}$, where there's a discrete jump of magnitude γ_i.

Corollary 1.3.

$$\frac{\partial Q_j}{\partial \bar{\kappa_j}}, \frac{\partial X_j}{\partial \bar{\kappa_j}} \leq 0 \qquad\qquad \frac{\partial Q_j}{\partial \bar{\kappa_l}}, \frac{\partial X_j}{\partial \bar{\kappa_l}} \geq 0, \forall l \neq j$$

Proof.

$$\frac{\partial Q_j}{\partial \bar{\kappa_l}} = \frac{\partial Q_j}{\partial s_{ij}} \cdot \frac{\partial s_{ij}}{\partial \bar{\kappa_l}} \text{ where } \frac{\partial Q_j}{\partial s_{ij}} \geq 0 \text{ as the integral of a non-negative term,}$$

$$\text{and } \frac{\partial s_{ij}}{\partial \bar{\kappa_l}} \geq 0 \text{ if } l = j, \leq 0 \text{ if } l \neq j. \text{The same logic follows for } X_j.$$

□

These simple corollaries build-up to the following result: the theoretical ambiguity of strategic complementarity between physicians' $\bar{\kappa_j}$.

Consider some $l \neq j$. If we breakdown the mixed partial derivative of U_j over $\bar{\kappa_j}$ and $\bar{\kappa_l}$, we get:

$$\begin{aligned}\frac{\partial^2 U_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}} &= \underbrace{R''(Q_j(\bar{\kappa_j}, \bar{\kappa}_{-j}))}_{\leq 0} \cdot \underbrace{\frac{\partial Q_j}{\partial \bar{\kappa_l}}}_{\geq 0} \cdot \underbrace{\frac{\partial Q_j}{\partial \bar{\kappa_j}}}_{\leq 0} + \underbrace{R'(Q_j(\bar{\kappa_j}, \bar{\kappa}_{-j}))}_{\geq 0} \underbrace{\frac{\partial^2 Q_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}}}_{?} \\ &- \underbrace{P''(X_j(\bar{\kappa_j}, \bar{\kappa}_{-j}))}_{\geq 0} \cdot \underbrace{\frac{\partial X_j}{\partial \bar{\kappa_l}}}_{\geq 0} \cdot \underbrace{\frac{\partial X_j}{\partial \bar{\kappa_j}}}_{\leq 0} - \underbrace{P'(X_j(\bar{\kappa_j}, \bar{\kappa}_{-j}))}_{\geq 0} \underbrace{\frac{\partial^2 X_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}}}_{?} \end{aligned} \tag{A}$$

Reordering terms we get the following:

$$\begin{aligned}\frac{\partial^2 U_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}} &= \underbrace{[R''(Q_j(\bar{\kappa_j}, \bar{\kappa}_{-j})) \cdot \frac{\partial Q_j}{\partial \bar{\kappa_l}} \cdot \frac{\partial Q_j}{\partial \bar{\kappa_j}} - P''(X_j(\bar{\kappa_j}, \bar{\kappa}_{-j})) \cdot \frac{\partial X_j}{\partial \bar{\kappa_l}} \cdot \frac{\partial X_j}{\partial \bar{\kappa_j}}]}_{\geq 0} \\ &+ \underbrace{[R'(Q_j(\bar{\kappa_j}, \bar{\kappa}_{-j})) \frac{\partial^2 Q_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}} - P'(X_j(\bar{\kappa_j}, \bar{\kappa}_{-j})) \frac{\partial^2 X_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}}]}_{?} \quad \overset{?}{\geq} \quad 0\end{aligned}$$

We see that ambiguity with respect to the sign of the mixed derivative is two-fold:

- Our modest prerrequisites for s_{ij} aren't enough to analytically establish the sign of the mixed derivatives $\frac{\partial^2 Q_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}}$ and $\frac{\partial^2 X_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}}$, though it stands to reason they'd share the same sign, and furthermore that it would be negative: raising $\bar{\kappa_j}$ would depress the positive effect of a raise in $\bar{\kappa_l}$, whereas raising $\bar{\kappa_l}$ would exacerbate the negative effect of a raise in $\bar{\kappa_j}$, for both Q_j and X_j.
- As $R'(\cdot)$ and $P'(\cdot)$ share the same sign, the question of whether the expression in brackets on the second row is positive falls to the *quantitative* level of both derivatives, which will depend on the specific form and parameters chosen for these functions and can't be anticipated by this analysis.