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Contents

1 Introduction

The question of sickness at work presents us with a principal-agent problem: though in cases of true sickness it is best for the employee's personal wellbeing and indeed even firm productivity Chunyu et al. (2024) that they should stay home, it would also be possible to call in sick without any affliction, skipping work undetected.

The common solution to this problem is third-party certification, a specialized professional to attest to the condition and allow for or even demand of the employer that sick leave be granted. And who else to carry out this evaluation than physicians themselves—knowledgeable on the subject as they are—making diagnosis, treatment referral and certification of sickness a one-stop-shop for the patients.

The problem with this oversized, multi-faceted role physicians play in healthcare systems is that it causes conflict of interests. In cases where sick leave calls (and insurance claims) call for state-subsidied payments to be issued, we can speak of a second principal-agent problem between the physician, exercising the role of "gatekeeper", and the taxpayer, bankrolling public healthcare and insurance.

There is evidence to the fact that even in the best of cases physicians have no incentive to place doubt on their patients claim to unobservable symptoms (Carlsen et al.; 2020), and in murkier scenarios they could be motivated to gain the reputation of being "lenient" to enjoy increased demand for their services (Markussen and Røed; 2017).

In Chile, the country of our sample, the healthcare system is bi-modal: citizens may opt for one of many private health insurers (ISAPRES) or the sole public option (FONASA), and 82% of them are affiliates of the latter. In their case, the authority behind sick leave granting is COMPIN, with the power to oversee, investigate and even sanction physicians that fall under it. A worker granted sickness absence is entitled to a work incapacity benefit (SIL), periodic payments paid in function of sick leave length.

Anecdotal evidence speaks to the presence of fraud in the system. A survey by the U. Andrés Bello Public Health Institute (2024) has it that 51% of those surveyed know someone who was granted sick leave without any sickness, 62% believe that physicians "frequently" create irregular businesses for the sale of sick leave certificates, and 56% think it would be "easy" to purchase one.

The presence of fraud is not unknown to the overseeing authority. In September 2021, COMPIN sanctioned 188 physicians who were not able to justify their unusually high issuance rate. Oteíza (2023) finds that this one time intervention had a spillover on non-sanctioned doctors, who on average reduced their issuance rate by at least 5.22%, on conservative estimates. This is further proof that sick leave granting *is* influenced by profit-seeking, and readjusted expectations of the probability of being met with punishment does influence physicians' willingness to provide them to patients.

This paper seeks to develop a framework within which physicians' behavior may be rationalized and modeled. It presents physician in a dual role as both medical caretakers and issuers of sick leave, and patients value both aspects separately according to population distributions. Patient strategies are the probabilities with which they'll visit any given physician, and physician strategies is their choice of "leniency", how willing they are to grant sick leave according to patient characteristics.

A generalized framework of patient search is laid out and contrasted with a non-search baseline, i.e. a particular kind of equilibrium in which patient strategies are set as randomized over all physicians with no distinction, that is to say, in which they're assigned a physician with no say on the matter but whether to visit. Once patient 'search' comes into play, physicians take into consideration the behavior of the rest of their colleagues—and its implication on their own market share—when opting for a given level of strictness in sick leave issuance, meaning there's a *strategic* element now involved, where physicians will be wary to lose out on too many patients to more *lenient* colleagues.

We then present two particular specifications of the 'search' framework: an "implicit" search model, where patient behavior takes on the form of a modified McFadden Logit[1], and an "explicit" search model, akin to sequential job search models, in which patients visit physicians which render them services above their "reserve utility".

2 Literature Review

It is a well established factum that taking sick leave is subject to an economic calculus on part of the workers, rather than being an orthogonal, merely health-concerned matter. Johansson and Palme (2005) begin their article with a quote by Nobel Laureate Ragnar Frisch: "Regarding the high absence rate at the Department: Acquiring minor diseases, such as colds or flu, is an act of choice". Their paper is among many others—Paola et al. (2014), Markussen et al. (2012), Stearns and White (2018), Henrekson and Persson (2004)—which give empirical evidence of such a choice being driven by economic incentives, through an event study on exogenous institutional regime changes in the sampled nation's public insurance system. This line of research, however, is concerned with the actions of workers themselves and their subsequent effect on macroeconomic employment variables, whereas our main focus shall be the role played by physicians.

As for the literature on physicians themselves, it has become commonplace to regard them as agents with two sources of utility: their income and their "altruistic" interest on their patients' well-being. Empirical support for the latter can be found in experimental evidence both on medical students (Brosig-Koch et al. (2017), Hennig-Schmidt and Wiesen (2014)) as well as doctors themselves (Kesternich et al. (2015), Brosig-Koch et al. (2016)). Crea (2019) finds no evidence for this, whereas Godager

[1]See McFadden (1972).

and Wiesen (2013) do, and explore its heterogeneity across physicians. The fact that physicians are also concerned with revenue, rather than being purely altruistic, is also well evidenced, see Clemens and Gottlieb (2014), Hennig-Schmidt et al. (2011), Autor et al. (2014), and also Robertson et al. (2012) for a review on the matter. Therein lies the dilemma with giving physicians the status of gatekeepers for different services and certifications, like disability insurance (as in autor). As Markussen and Røed (2017, p. 1) put it: "In essence, the GPs [general practitioners] have been assigned the task of protecting the public (or private) insurer's purse against the customers who form the basis for their own livelihood".

Both factors being well established in the literature on physicians, one strand of it would seek to design an optimal contract for medical care in the presence of such an economic calculus, see Choné and Ma (2011) or Gaynor et al. (2023); another, more in line with our approach, would evaluate the effects increased competition among doctors has on their rendered services. In general, Currie et al. (2023) propose that increased competition would lead to physicians offering more services that please the clients yet relatively hurt their own utility (like drug prescriptions), and less services which bring them, physicians, more utility, at the expense of patient utility (like unwanted, expensive surgical procedures). Iversen and Lurås (2000) and Iversen (2004) provide empirical evidence that somewhat supports it: physicians with a shortage of customers will provide more services, thus obtaining more income *per customer*. This line of work is more in line with our own approach, as the altruistic motivations of physicians are set momentarily aside and they're modeled as purely profit-seeking. This reasoning applies to scenarios like ours, where the issuance of sick leaves—particularly for short-term absences, as is the case in most situations[2]—does not have a significant impact on client health.

To our knowledge, the only article dealing specifically with sick leave certicate granting as a function of competition among physicians is Markussen and Røed (2017). Carlsen et al. (2020) deal with sick leave as well, but make only the narrower point that in a Bayesian context doctors have almost no incentive to distrust patients' self-reported, unverifiable symptoms. Markussen and Røed's methodology is similar to our own: after performing "raw" regression analysis, they set-up a model of patient choice as a McFadden Logit over observables X_i, including physician leniency (assumed to be observable for prospective patients), and as such can estimate the role leniency plays in demand for their services. They then perform a series of exercises, some of their findings include: physicians with variable wage (i.e. dependent on clientele) certify 7% more absence days per month than fixed wage physicians; half of this difference has a causal interpretation, as observed from within-physician responses to market conditions; in general, more lenient gatekeeping gives the GP (physician) more customers, and more customers make the GP less lenient. Most of these stylized facts are taken up in our equilibrium models, in which patients take into account both sick leave "leniency" ($\bar{\kappa_j}$) as well as physician quality (V_j), such that physicians which

[2] For cases where sickness leave is medically required we introduced the upper limit $\bar{\kappa}_{\max}$, such that physicians would never be so strict as to be negligent.

offer better medical attention and thus enjoy higher demand can afford to be "stricter", and those who don't will have the incentive to gain clientele through leniency. Not captured is the physician fixed effect, the idiosyncratic motive to leniency: we assume leniency as merely a strategic choice based on expected demand, where prior to choosing leniency physicians only differ in "quality" (V_j) and visit revenue/cost of visit to the patient (r_j, τ_j).

Despite the different subject matter, the main source of inspiration for this paper is Schnell (2022), such that ours can be seen as an attempt to replicate her model and framework, intially devised for opioid markets, to the market for sick leaves. Schnell seeks to model the market scenario for the opioid crisis, with a primary market composed of physician prescriptions and a secondary black market. The presence of the latter, she concludes, makes unilateral interventions ineffective: curbing both excessive physician prescriptions as well as black market sales is required to make more than a dent on the number of opioids consumed in America.

Her paper includes four benchmark for the patient-physician market, building up to her main model including patient search, such that patients with a hire taste for opioids are assortatively matched with physicians more willing to prescribe them, and a secondary market. In our model we keep the former but not the latter, as the "black market" for sick leaves falls *within* the primary market of physicians, composed of those willing to knowingly issue fraudulent sickness certificates. Search was repurposed to fit a two-dimensional frame of physicians, characterized both by their strictness as well as their service quality. What we call the "explicit" search model is in the vein of Schnell's sequential patient search, though more fleshed out in its dynamic programming framework for the reason just mentioned. Then there's the "implicit" model, which defines patient behavior according to a modified McFadden Logit. We show in the Appendix A.2 that such a framework wouldn't have altered Schnell's main conclusions.

Our model differs from Schnell's chiefly in the fact that physicians take into account *other* physicians' behavior when selecting their own strategy, such that market equilibrium requires a Nash equilibrium in physician strategies. We discuss in the Appendix A.1 that the source of this feature, not present in Schnell's model, is the lack of additive separability across patients in the physician's utility function, such that her optimal behavior takes into account *aggregate* patient demand as well as *marginal*.

3 Model

3.1 The Physician-Patient Market

We now present the model itself before proceeding with further discussion. We consider $i = 1, ..., I$ patients and $j = 1, ..., J$ physicians.[3]

Patients are characterized by their "medical need" $\kappa_i \in \mathbb{R}_0^+$ and their "taste for sick leave" $\gamma_i \in \mathbb{R}_0^+$, respectively following the ex-ante cumulative distributions $F(\kappa)$ and $G(\gamma)$, which are public knowledge. Physicians are described by their "service quality" $V_j \in \mathbb{R}^+$, also known both to all patients and other physicians.

A patient i can visit a physician for treatment and may be also granted sick leave. After the patient is assigned a physician j, his utility function—implicitly dependent on his characteristic (κ_i, γ_i) tuple—is defined piecewise as follows:

$$U_i(V_j) = \begin{cases} \gamma_i + V_j\kappa_i - \tau_j & \text{if he's granted sick leave,} \\ V_j\kappa_i - \tau_j & \text{if he only visits the physician,} \\ 0 & \text{if no visit takes place} \end{cases}$$

As we see, there are three components to patient utility: an interaction between the patient's medical need κ_i and the physician's service quality V_j which implies their complimentarity, his "taste" for sick leave γ_i in the case he's granted one, and τ_j, which we define as the cost of visit for physician j. As well as being complementary, κ_j as multiplied by V_j would give it the interpretation of being the marginal willigness to pay for physician quality.[4]

Whereas patients may visit at most one physician, a physician may see several patients. We define Q_j as the expected number of patients for physician j, the demand for her services. We say 'expected' because, as we'll see later on, patients may opt for a mixed strategy, assigning a certain *probability* to visiting j, and we define Q_j as the sum of the ex-ante probability of visit of all patients, not their ex-post realization.

As physician j has the option to issue sick leave to a given patient i which visits her, we likewise define X_j as the *expected* number of sick leaves granted by physician j, given her ex-ante patient demand and how many of them would be granted one.

We define the physician's utility function as follows:

$$U_j(Q_j, X_j) = R_j(Q_j) - P(X_j)$$

where $R_j(\cdot)$ is an individual, concave *revenue* function defined over expected total clients, and $P(\cdot)$ is a convex *punishment* function on X_j, composed of the probability

[3]Note on language: For ease of reference, we refer to any patient as "he", and to any physician as "she" (s we have done already). At this point it shall be noted that the first person plural (we) employed over the course of this article is to be read as including the reader or perhaps as a royal *we*.

[4]Conceptually, willingness to pay would also depend on patient i's wealth level w_i. To steer away from discussions on wealth disparity, we interpret κ_i as the *average* willingness to pay for V_J.

of being fined for a certain number of sick leaves issued and the magnitude of the fine. The implication is that after a given number of patients, the disutility of an additional (expected) sick leave issued would outweigh physician j's financial incentive for further clientele. The j subindex indicates that we will allow $R_j(\cdot)$ (revenue by visit) to vary across physicians. We assume both $R_j(\cdot)$ and $P(\cdot)$ to be twicely differentiable.

We stress again that we define physician utility in terms of the *expected* realization of patient demand and granted sick leaves, as befits the logic of this game, where doctors take action before patients (we will specify the timing of our game below).

Following Schnell (2022), we focus on *threshold equilibria*, wherein each physician's strategy is the choice of a value $\bar{\kappa_j} \in \mathbb{R}_0^+$, such that of the patients who visit j, those with a κ_i value above or at that threshold will be granted sick leave, and those strictly under it won't. As a result of this, each physician will be identified by their given 'service quality', revenue function, cost of visit to the patient and choice of threshold: $\{V_j, R_j(\cdot), \tau_j, \bar{\kappa_j}\}$. The set $\{\bar{\kappa_j}\}_{j=1}^{J}$ will be known to patients when deciding on their behavior strategy.

Our models are different iterations of a general game with the following timing:

- First stage: Physicians simultaneously choose $\bar{\kappa_j}$.
- Second stage: Observing $\{V_j, R_j(\cdot), \tau_j, \bar{\kappa_j}\}$. The set $\{\bar{\kappa_j}\}_{j=1}^{J}$, each patient chooses (or is assigned to) some doctor j.
- Third stage: Each patient can choose to see their physician and incur a visit cost τ_j, or refrain from doing so.

Conditional upon his visit (as not to visit renders null utility), the utility of patient i from seeing physician j is:

$$U_i(V_j, \bar{\kappa_j}) = \begin{cases} \gamma_i + V_j\kappa_i - \tau_j & \text{if } \kappa_i \geq \bar{\kappa_j}, \\ V_j\kappa_i - \tau_j & \text{if } \kappa_i < \bar{\kappa_j} \end{cases}$$

We will usually use u_{ij} as a shorthand.

3.2 Non-Search Equilibrium

For illustration purposes, we first devote attention to a non-search baseline, where each patient is randomly assigned to a physician, with an equal probability of being matched to any of the J physicians. Their only say in the matter is whether they'll then visit physician j, which is to say, the second stage of the game is out of their hands, and they only make choices in the third stage after assignment.

A patient won't visit his assigned physician if his expected utility from such a visit is negative, we call this the *free disposal* requirement. As such, a physician j's expected patient demand, as a function of her threshold $\bar{\kappa_j}$ (and given the parameter V_j) will be the following:

$$Q_j(\bar{\kappa_j}) \;=\; \frac{I}{J}\left[\int_{\tau_j/V_j}^{\infty} dF(\kappa) + \int_{\min\{\bar{\kappa_j},\tau_j/V_j\}}^{\tau_j/V_j} \int_{\tilde{\gamma}(\kappa)}^{\infty} dG(\gamma)\, dF(\kappa)\right] \tag{1}$$

where the left term consists of the mass of patients who just by virtue of physician j's service quality V_j would be willing to pay a visit (i.e. $\kappa_i \geq \tau_j/V_j$), and the right term would be patients who only see physician j solely out of the expectation of getting sick leave ($\kappa_i \geq \bar{\kappa_j}$ & $\gamma_i \geq \tau_j - V_j\kappa_i$), and wouldn't visit otherwise. We define $\tilde{\gamma}(\kappa) := \tau_j - V_j\kappa$ as the lower limit of the inner integral over γ.

Given that each patient with a κ_i higher or equal to $\bar{\kappa_j}$ is granted sick leave, the expected total number of such certificates granted by j, as a function of $\bar{\kappa_j}$, is:

$$X_j(\bar{\kappa_j}) \;=\; \frac{I}{J}\int_{\bar{\kappa_j}}^{\infty}\int_{\tilde{\gamma}(k)}^{\infty} dG(\gamma)\, dF(k) \tag{2}$$

which includes in theory patients from both the left and right term of equation (1).

For values of Q_j and X_j which follow (1) and (2) respectively, each physician solves for the following constrained optimization:

$$\bar{\kappa_j}^* \;\equiv\; \arg\max_{\bar{\kappa_j}} R_j(Q_j) - P_j(X_j) \qquad \text{s.a.} \quad 0 \leq \bar{\kappa_j} \leq \bar{\kappa}_{\max} \tag{3}$$

where $\bar{\kappa}_{\max} < \infty$ is the maximum value the choice of threshold of any physician may take, as we mentioned earlier.[2]

An inner solution to problem (3) takes on the following form because of the first order condition:

$$R_j'(Q_j)\frac{\partial Q_j}{\partial \bar{\kappa_j}} = P_j'(X_j)\frac{\partial X_j}{\partial \bar{\kappa_j}} \tag{4}$$

Proposition 3.1. *In the non-search model as described thus far, when the solution $\bar{\kappa_j}^*$ of problem (3) $\in (0, \frac{\tau_j}{V_j}]$, this $\bar{\kappa_j}^*$ is described by:*

$$R_j'(Q_j(\bar{\kappa_j})) = P_j'(X_j(\bar{\kappa_j}))$$

Proof. For $Q_j(\bar{\kappa_j})$ and $X_j(\bar{\kappa_j})$ as defined by (1) and (2), when $\bar{\kappa_j} \in (0, \frac{\tau_j}{V_j}]$:

$$\frac{\partial Q_j}{\partial \bar{\kappa_j}} = -\frac{I}{J}\int_{\tilde{\gamma}(\bar{\kappa_j})}^{\infty} dG(\gamma)\, f(\bar{\kappa_j}) = \frac{\partial X_j}{\partial \bar{\kappa_j}}$$

such that both derivatives are cancelled in equation (4). As a result, if the optimal $\bar{\kappa_j}^* \in (0, \frac{\tau_j}{V_j}]$, the proposition follows. □

This proposition implies that when for low enough values of κ_i, any new patients gained by physician j are those she entices through the expectation of getting sick leave, meaning that in the vicinity of $\bar{\kappa_j} \in (0, \frac{\tau_j}{V_j}]$ the change in expected patients $\Delta Q_j / \Delta \bar{\kappa_j}$ through a variation in $\bar{\kappa_j}$ is the same as the change in expected sick leaves issued $\Delta X_j / \Delta \bar{\kappa_j}$.

We can see that (3) offers three possibilities by way of solution: two corner solutions at 0 and $\bar{\kappa}_{\max}$, and an inner solution fulfilling equation (4). The proposition below proves such an inner solution can't hold for $\bar{\kappa_j} \in (\frac{\tau_j}{V_j}, \bar{\kappa}_{\max})$.

Proposition 3.2. *In the non-search model as described thus far, no $\bar{\kappa_j}$ in $(\frac{\tau_j}{V_j}, \bar{\kappa}_{\max})$ can be the solution of problem (3).*

Proof. For $\bar{\kappa_j} \in (\frac{\tau_j}{V_j}, \bar{\kappa}_{\max})$,

$$Q_j(\bar{\kappa_j}) \;=\; \frac{I}{J}\left[\int_{\tau_j/V_j}^{\infty} dF(\kappa)\right] \text{ and thus } \frac{\partial Q_j}{\partial \bar{\kappa_j}} \;=\; 0$$

whereas

$$\frac{\partial X_j}{\partial \bar{\kappa_j}} = -\frac{I}{J}\int_{\tilde{\gamma}(\bar{\kappa_j})}^{\infty} dG(\gamma)\, f(\bar{\kappa_j}) \;<\; 0$$

so equation (4) can never hold, which would be required of $\bar{\kappa_j}$ as an inner solution to problem (3). □

The intuition for this proposition is that $\kappa_i = \tau_j / V_j$ defines the threshold above which patients are willing to visit their assigned physician j regardless of whether they get sick leave, so choosing such a $\bar{\kappa_j}$ is always either too strict or inefficiently lenient. We can speak of a "captured clientele". A similar behavior will be observed in the models with patient search, only that, since patients freely choose physicians, a "captured" patient won't be one which always visits j, rather one that always asigns *positive probability* to visiting physician j, no matter her choice of threshold $\bar{\kappa_j}$.

Equilibrium in the non-search model. *Given a physician-patient market specified by $\{\{(\kappa_i, \gamma_i)\}_{i=1}^{I}, \{(V_j, R_j(\cdot), \tau_j)\}_{j=1}^{J}, P(\cdot)\}$, we define equilibrium as a set of physician thresholds $\{\bar{\kappa_j}\}_{j=1}^{J}$ satisfying (3) and their corresponding equilibrium (expected) patient demand and issued sick leaves $\{(Q_j, X_j)\}_{j=1}^{J}$ as described by (1) and (2), respectively.*

This non-search baseline model, based off Schnell (2022), already presents some elements which will be present in our more complex models later, but completely neglects patient behavior, who are reduced to being assigned a physician and choosing to visit her or not. Patient search will allow us to formalize their free choice of physician as a function of the model parameters and physician strategies.

3.3 General Search Framework

We introduce patient 'search' as a general framework in which patients can choose freely among all physicians.

We define for each patient i a vector $S_i \in \Delta(\mathcal{J})$, where $\mathcal{J}$ is the the J-dimensional vector composed of all $1, ..., J$ physicians. S_i represents the *strategy* of patient i in this game, specifying the probability of visiting each physician j. Each component $s_{i1}, \ldots, s_{iJ}$ within S_i denotes the probability that patient i will visit physicians $1, \ldots, J$, respectively.

In order to play the role of a proper subprobability measure, for each patient i the components $s_{ij} \in S_i$ must fulfill the following criteria:

i. $\forall j,\ s_{ij} \geq 0$

ii. $\sum_{i=1}^{J} s_{ij} \leq 1$

We will allow the sum of all components for each patient i to be less than one (hence *sub*-probability), implying the presence of an *outside option* for patients, that is, to not visit any physicians. To have this option is important, as patient rationality in our models entails a "*free disposal* " property, meaning that a patient will never visit a physician if his expected utility from such a visit is less than 0, that is, $s_{ij} = 0$ if $U_i(V_j, \bar{\kappa_j}) = 0$. This makes the third stage of the game as described in Section 3.1 trivial, as under free choice patient i will only be "assigned" with positive probability to physicians he will be willing to visit.

We can re-interpret the non-search model as each patient being made to play by the strategy $\{S_i : s_{i1} = s_{i2} ... = s_{iJ} = \frac{1}{J}\}$, and it is the lack of a free choice which makes the third stage non-trivial.

We shall further specify the form S_i will take. Consider now the J-dimensional vector U_i, where each component $u_{i1}, ..., u_{iJ}$ indicates the utility patient i expects from a visit to physicians $1, ..., J$ respectively. As a shorthand, we shall write $u_{i,-j}$ to indicate the $J - 1$ components of U_i excluding u_{ij}.

In the models considered, each component s_{ij} of S_i will be defined as:

$$s_{ij} \equiv g_i(u_{ij}, u_{i,-j})$$

where $g_i(\cdot)$ is a continuous function weakly increasing in the first argument u_{ij}, and weakly decreasing in the remaining arguments given by $u_{i,-j}$. Our model specifications will consist in giving this function $g_i(u_{ij}, u_{i,-j})$ a specific form.

Defining s_{ij} over u_{ij} makes it dependent upon patient i's characteristic tuple (κ_i, γ_i). We will use the $s_{ij}(\kappa_i, \gamma_i)$ formulation to define physician expected demand and sick leaves issued as functions over patient strategies:

$$Q_j(\bar{\kappa_j}, \bar{\kappa}_{-j}) = \int_0^\infty \int_0^\infty s_{ij}(\kappa, \gamma)\, dG(\gamma)\, dF(\kappa) \tag{5}$$

$$X_j(\bar{\kappa_j}, \bar{\kappa}_{-j}) = \int_{\bar{\kappa_j}}^\infty \int_0^\infty s_{ij}(\kappa, \gamma)\, dG(\gamma)\, dF(\kappa) \tag{6}$$

At face value, these definitions appear less informative than their non-search equivalents in (1) and (2).

Given the collection of all patients' strategies $(\{S_i\}_{i=1}^{I}$, each with a respective s_{ij} for doctor j, we can formulate expected clientele and granted sick leaves for j in a manner very much alike the previous section (indeed, as a generalization):

As before, the left term in equation (5) includes patients who would visit j even without sick leave, whereas the right term those who only do so because they expect one.

Notice that we now present Q_j and X_j not only in terms of the threshold $\bar{\kappa_j}$ chosen by doctor j, but also in terms of the thresholds of the other $J-1$ doctors, which we abreviate as $\bar{\kappa}_{-j}$. Whereas before doctors were simply alloted a given number of patients, now they will *compete* for them, as patients' S_i strategy will consider the whole of $(\{\bar{\kappa_j}\}_{j=1}^{J}$ when considering which physician(s) they will visit with positive probability.

This specification of s_{ij} is sufficient to make the following proposition:

Proposition 3.3. *In the non-search model as described thus far, no $\bar{\kappa_j}$ in $(\frac{\tau_j}{V_j}, \bar{\kappa}_{\max})$ can be the solution of problem (3).*

Consider some two physicians j, l, such that $\bar{\kappa_j} \neq \bar{\kappa_l}$.

3.4 The "Implicit" Search Model

The "implicit" search model is a McFadden Logit choice model modified to give null probability visits to doctors which afford patient i non-positive utility, that way the free disposal requirement is fulfilled.

We call it the "implicit" search model because strictly speaking patient search is not formally included, yet one arrives at result qualitatively similar to specifications that do (like our own). The choice strategy noisily assigns positive probability to physicians who render i high utility, and the higher this u_{ij} expected utility is, the higher the probability of visit.

Instead of *explicitly* defining search and subsequent choice, probability of visit depends upon expected utility from doctor j, u_{ij}, and a weighing parameter λ. The lower the value of λ, the noisier patient "search" is: they give high assingment probability to sub-optimal –yet feasible– physician choices.

We define the components s_{ij} of the patient's strategy vector S_i as follows:

$$s_{ij} = \frac{\alpha_{ij}}{\sum_{k=1}^{J} \alpha_{ik}}, \quad \text{where} \quad \alpha_{ij} = \begin{cases} e^{\lambda u_{ij}}, & \text{if} \quad u_{ij} > 0 \\ 0, & \text{if} \quad u_{ij} = 0 \end{cases} \tag{S}$$

Unlike the "explicit" search model, the probability that patient i visits doctor j is *strictly* growing in κ_i, rather than being a piece-wise constant function with different levels. There is a discrete jump in probability at $\bar{\kappa_j}$, but elsewhere above 0 the function is smoothly increasing, rewarding physicians with high V_j.

The same lemmas as in the non-search model still hold.

Lemma 3.4. *No value in* $(\frac{\tau}{V_j}, \infty)$ *can be the optimal solution of (3).*

PROOF: APPENDIX

Lemma 3.5. *When* $\bar{\kappa_j} \in [0, \frac{\tau}{V_j}]$,

$$\frac{\partial Q_j}{\partial \bar{\kappa_j}} = \frac{\partial X_j}{\partial \bar{\kappa_j}}$$

PROOF: APPENDIX

Thus, the possible solutions for (3) remain threefold: the corner solutions 0 and ∞, and an inner solution where the simplified FOC $R'(Q_j) = P'(X_j)$ holds.

Equilibrium is achieved in a similar fashion, though now taking into account the patients' S_i strategies.

Equilibrium in "implicit" search. *Given a doctor-patient market specified by* $(\{(\kappa_i, \gamma_i)\}_{i=1}^{I}, \{(V_j, R_j(\cdot), P_j(\cdot))\}_{i=1}^{J})$*, we define an equilibrium as a set of physician thresholds* $\{\bar{\kappa_j}\}_{j=1}^{J}$ *satisfying (3) and patient strategies* $\{S_i\}_{i=1}^{I}$ *as defined by (S), with their corresponding equilibrium (expected) patient demand and granted sick leave certificates* $\{(Q_j, X_j)\}_{j=1}^{J}$ *as described by (5), (6), respectively.*

3.5 The "Explicit" Search Model

What we call the "explicit" model is in which patient strategy S_i is explicitly the result of a sequential search algorithm on part of the patients. This is in line with Schnell (2022), though made trickier by the fact that our model allows for physician services apart from contingent "prescriptions" (in our case, sick leave certificates). In our model some patients *are* willing to visit physicians who don't intend to grant them sick leave, given a high enough benefit from the medical service proper, $V_j\kappa_i$.

FURTHER EXPLANATION REQUIRED, TO BE CONTINUED

4 Computation

4.1 Algorithms for Model Computation

Given a set of physicians $\{(V_j, R_j(\cdot), \tau_j)\}_{j=1}^{J}$, punishment function $P(\cdot)$, distribution functions for patient parameters $F(\kappa)$ and $G(\gamma)$, and one search model parameter z, which is λ in the Logit model, β in the sequential model, we develop an algorithm to compute market equilibrium.

Equilibrium aggregates are computed on the basis of Monte-Carlo matrix calculus. Set J as the number of physicians,[5] I as the size of the sample drawn randomly from $F(\kappa)$ and $G(\gamma)$. For both models we define a class which can output a matrix S where each column is a patient's strategy vector S_i following that model, when given as input the arrayed set of physicians' quality and visit cost $\{(V_j, \tau_j)\}_{i=1}^{J}$, an arrayed set of patients $(\{(\kappa_i, \gamma_i)\}_{i=1}^{I}$, the model parameter z and a given vector of physician strategies $\{\bar{\kappa_j}\}_{j=1}^{J}$.

We input as "patients" our I samples from $F(\kappa)$ and $G(\gamma)$, then a $J \times I$ matrix U is computed, where each component u_{ji} corresponds to the utility the sampled patient i would get from a visit to physician j.[6] This step is the same for both classes.

What differs between both models is the computation of the matrix S of patient strategis out of the utility matrix U:

- **Implicit search model (Logit):** First, an 'α-matrix' is calculated over matrix U, where each α_{ji} is $e^{\lambda u_{ji}}$ if $u_{ji} > 0$ and 0 if not. Then, for each patient i, that is, for each column, each component s_{ji} of the S matrix takes on the values $s_{ji} = \alpha_{ji} / \sum_{k=1}^{J} \alpha_{ki}$.
- **Explicit search model (Sequential):** Recall equation (NUMBER) characterizing patient thresholds. We first compute the I-dimensional vector of the sampled patients' respective $\bar{U}_i$. Define U_i as the set $\{u_{ji}\}_{j=1}^{J}$ of utility patient i recieves from a visit to each physician. In matrix terms U_i would be the ith column of the $J \times I$ matrix U. The operation to compute each $\bar{U}_i$ is the following:

  $$\bar{U}_i \equiv \underset{x \in U_i}{\arg\min} \left\| x - \frac{\beta}{1-\beta} \sum_{j=1}^{J} \left\{ \frac{\mathbb{1}[u_{ji} \geq x] \cdot (u_{ji} - x)}{\mathbb{1}[u_{ji} \geq x]} \right\} \right\|$$

  where the norm $\|\cdot\|$ is defined in $\mathbb{R}$ as simply the absolute value $|\cdot|$. This is to say, for each sampled patient i we evaluate x for each u_{ji} in U_i. In plain words, if patient i where to say: "I will only visit physicians which grant me at least as much utility as physician j", the optimal choice of j and its respective u_{ji} would be the minimal in U_i for the evaluation of the absolute value in the left-hand side of (4.1). This is computationally less intensive than seeking to compute the exact root of (NUMBER), which would be a redundant exercise, because there's a discrete number of physicians above that mark, and selecting instead to use "the lowest u_{ji} above the root of (NUMBER)" as threshold instead of the root proper would result in the same vector of strategies S_i for each patient.[7]

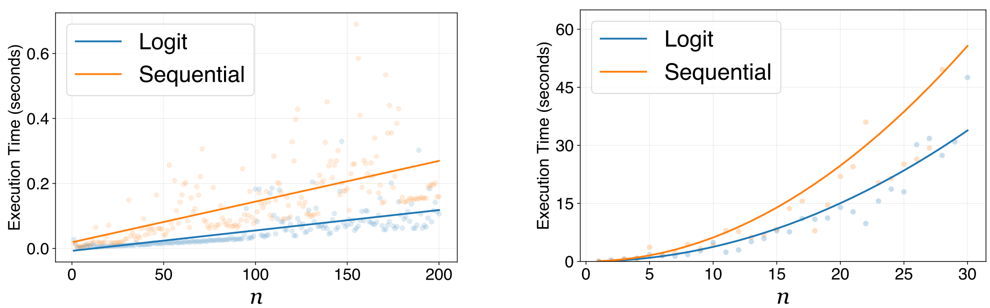
[5] Or, as we'll later interpret it, as the number of bins, where each bin j is a unique combination of $(V_j, \tau_j, R_j(\cdot), P_j(\cdot))$.

[6] As we have defined our matrices $J \times I$ for ease of visualization, we will refer to matrix components as x_{ji} in this section rather than x_{ij} as we do elsewhere in the paper.

[7] Granted, this is not strictly true, as in our formulation a 'u_{ji}' may be selected as threshold which

Once having computed our I-dimensional vector of patients' $\bar{U}_i$, we evaluate column-wise the binary operator $\mathbb{1}\{u_{ji} \geq \bar{U}_i\}$ over matrix U, and the $J \times I$ matrix S of patient strategies takes on the values $s_{ji} = \mathbb{1}\{u_{ji} \geq \bar{U}_i\} / \sum_{k=1}^{J} \mathbb{1}\{u_{ki} \geq \bar{U}_i\}$.

The S matrix in both cases is computed in linear time, $\mathcal{O}(n)$ in big-O notation, where input size n is the number of physicians J. Both consist of a series of ufunc or vectorized operations, where the inclusion of one more physician introduces a new row with I elements on which operations are performed at each stage. The additional step of calculating the $\bar{U}_i$ vector makes the execution of the 'explicit' search model relatively slower (see Figure 1a).



(a) Linear polynomial fit of execution time for S matrix in both models

(b) Quadratic polynomial fit of execution time for $\bar{\kappa}^*$ vector in both models

Figure 1: Execution time comparison between the *implicit* and *explicit* search models

The steps following the computation of the S matrix are the same for both models. The J-dimensional vector Q of each physician's expected demand Q_j is achieved through the row-wise summation of all patient's demand for j, i.e. performing $\sum_{i=1}^{I} s_{ji}$ for each j. The vector X of each physician's expected sick leaves issued X_j performs that same summation, conditional on the patients' κ_i being above or at physician j's chosen threshold $\bar{\kappa}_j$, that is, $\sum_{i=1}^{I} \mathbb{1}[\kappa_i \geq \bar{\kappa}_j]\, s_{ji}$. Both sums are normalized to fit the actual number of patients in the physician-patient market.

For a given vector $\bar{\kappa}$, computing patient strategies, physician aggregates and then physicians' utility—defined as $\{R_j(Q_j) - P(X_j)\}$—entails a sequence of calculations done in linear time. However, the calculation of the *equilibrium* vector $\bar{\kappa}^*$ is performed in quadratic time $\mathcal{O}(n^2)$, as it requires each of the aforementioned steps to be done some x number of times *per* physician, such that the inclusion of an additional physician increases the number of operations required *per* physician as well as the amount of physicians whose $\bar{\kappa_j}$ needs to be computed. See Figure 1b.

The algorithm to find the equilibrium J-vector $\bar{\kappa}^*$ of physician strategies is as fol-

is actually below the root proper in $\mathbb{R}$, but closer to it than the nearest one above it. The effect of this "estimation noise" on our overall results is negligible.

lows:

i Input an initial guess $\bar{\kappa}^0$ for physician thresholds.

ii For each physician, we fix the the value of $\bar{\kappa}_{-j}$, the threshold values of the other $J-1$ physicians, and compute equilibrium aggregates and physician j's utility for different choices of $\bar{\kappa}_j$ across a grid. In particular, $U_j(\bar{\kappa}_j, \bar{\kappa}_{-j})$ is computed for each decimal step between 0 and $\bar{\kappa}_{\max}$, the maximum threshold physicians may choose.

iii The choice of threshold k_j which rendered j the most utility in the previous step is selected, and a grid is set-up spanning the 19 centesimal values in $[k_j - 0.05, k_j + 0.05]$,[8] each value therein input as physician j's threshold $\bar{\kappa}_j$ to compute U_j again, and the value within the grid which maximizes utility is chosen as $\bar{\kappa}_j^1$.

iv Having performed the previous step for all J physicians, we input as a new guess the vector $\bar{\kappa}^1 = \{\bar{\kappa}_1^1, ..., \bar{\kappa}_J^1\}$ to run steps ii and iii again. This defines an equilibrium-searching loop $\bar{\kappa}^n = \Phi(\bar{\kappa}^{n-1})$, and the loop is concluded when a fixed point is found, that is, the vector $\bar{\kappa}^*$ such that $\bar{\kappa}^* = \Phi(\bar{\kappa}^*)$.

v Optionally, having found a two decimals fixed point $\bar{\kappa}^*$, there's an algorithm in place to find an x-decimal fixed point $\bar{\kappa}^{*x}$. It starts by running a modified version of step iii on the two-decimal solution, setting up a grid of the 19 *millesimal* values in $[\bar{\kappa}_j^* - 0.005, \bar{\kappa}_j^* + 0.005]$ for each physician j,[9] selecting that which renders highest utility for each, and inputting this new vector as guess to run this step again, and so on until the fixed point $\bar{\kappa}^{*3} = \Phi(\bar{\kappa}^{*3})$ is found.

  This goes on like this, calculating the t decimals solution out of the $t-1$ decimals solution by setting up $10^{-(t-1)}$-sized grids for each physician in each iteration, until the specified amount of decimals wanted from the solution vector $\bar{\kappa}^{*x}$ is reached.

On the one hand, having to perform several operations *per* physician is quite computationally intensive and runs the cost of quadratic execution time. On the other hand, altough each iteration takes some time, for the right parameters the algorithm is quite efficient in the number of iterations needed for convergence, usually taking between 2 and 5. The option to find fixed points to $n+1$ decimal places incurs progressively smaller execution time costs compared to n decimals, as the initial guess for $n+1$ decimals becomes increasingly accurate. See Figure 2.

[8] If k_j is 0 (or $\bar{\kappa}_{\max}$), only the 10 centesimal values above (below) and at k_j are computed.

[9] For $\bar{\kappa}_j^* = 0$ or $\bar{\kappa}_{\max}$ a similar logic to the footnote above follows.

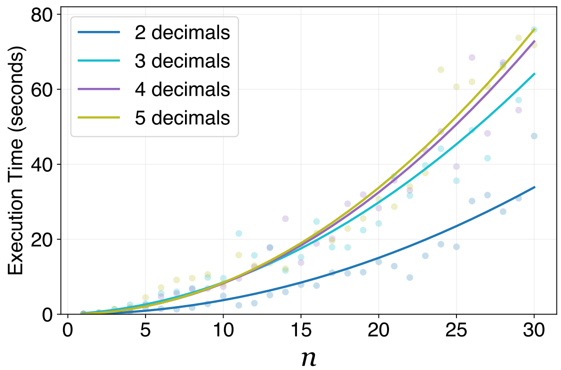
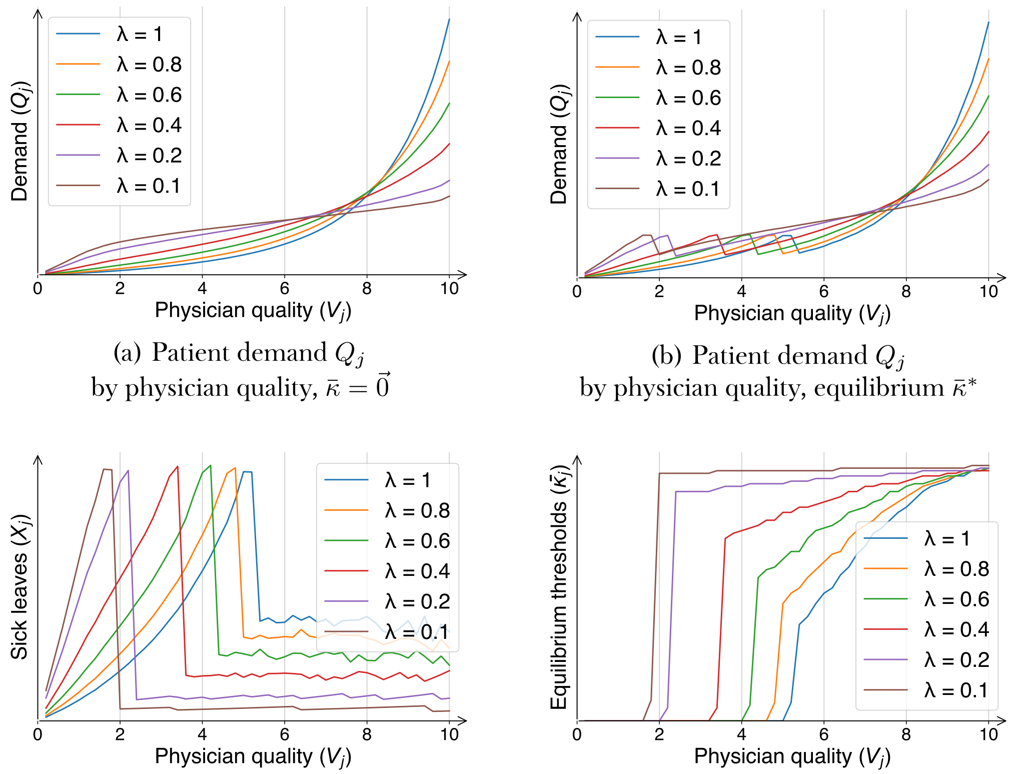


Figure 2: Quadratic polynomial fit of execution time of $\bar{\kappa_j}^*$ for different decimal approximations in Logit model

Once the equilibrium vector $\bar{\kappa_j}^*$ is calculated, it may be used to compute the equilibrium aggregates Q_j and X_j for all physicians, as well as physician utility.

4.2 Illustrative Examples



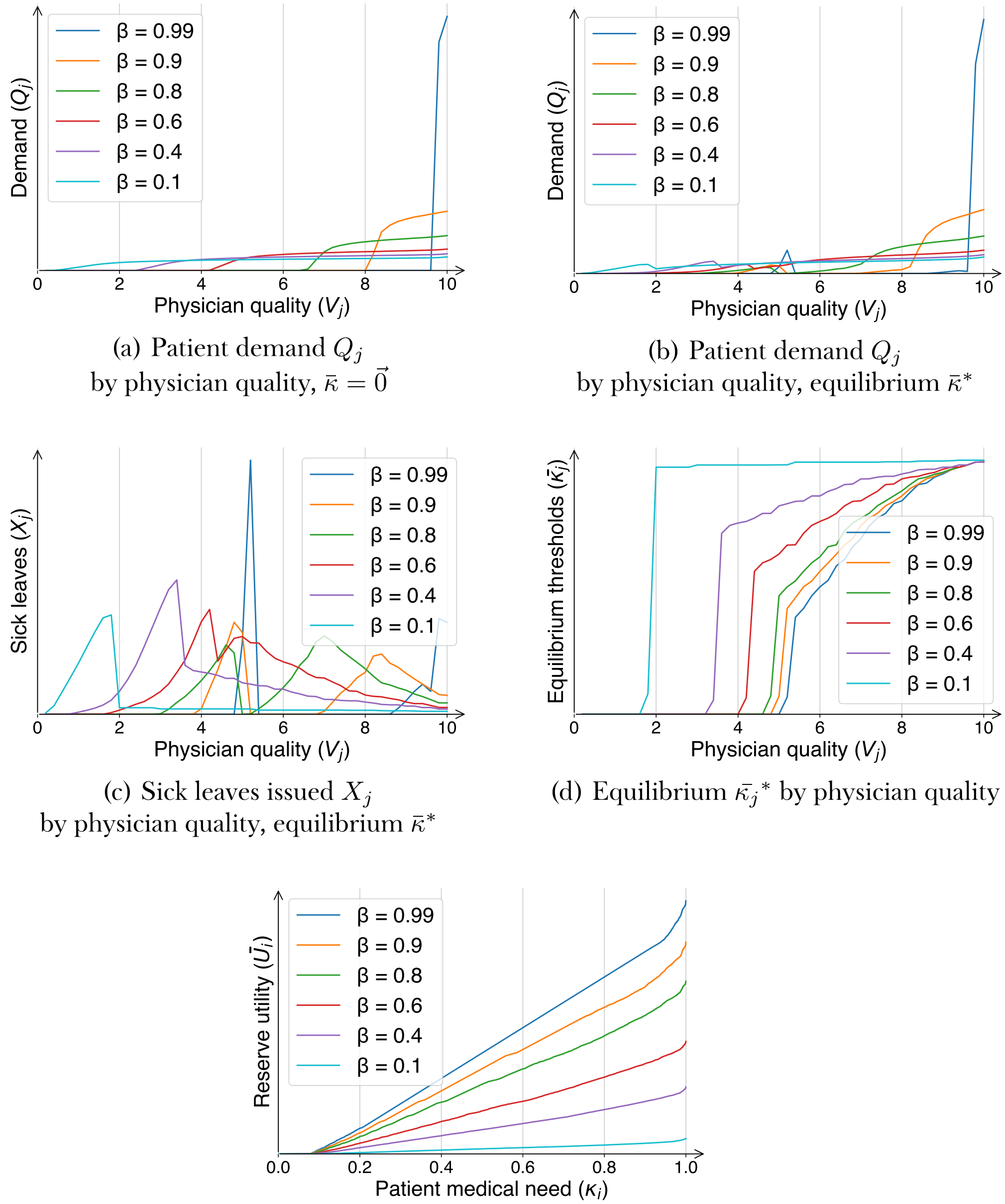
(a) Patient demand Q_j by physician quality, $\bar{\kappa} = \vec{0}$

(b) Patient demand Q_j by physician quality, equilibrium $\bar{\kappa}^*$

(c) Sick leaves issued X_j by physician quality, equilibrium $\bar{\kappa}^*$

(d) Equilibrium $\bar{\kappa_j}^*$ by physician quality

Figure 3: Physician aggregates and strategies for different values of λ in the *implicit* search model



(a) Patient demand Q_j
by physician quality, $\bar{\kappa} = \vec{0}$

(b) Patient demand Q_j
by physician quality, equilibrium $\bar{\kappa}^*$

(c) Sick leaves issued X_j
by physician quality, equilibrium $\bar{\kappa}^*$

(d) Equilibrium $\bar{\kappa_j}^*$ by physician quality

(e) Patient reserve utility $\bar{U}_i$
by medical necessity (κ_i), equilibrium $\bar{\kappa}^*$

Figure 4: Physician aggregates and strategies for different values of β
in the *explicit* search model

5 Data

We use administrative data provided by FONASA from all sick leaves issued from January 2018 to October 2022 in the public health system.

Overall totals by patients and physicians in the sampled period have the following descriptive statistics.

Statistic	Physicians	Patients
Mean	88	3
Std	199	3
Min	1	1
Max	8,743	73
Percentiles		
10th	2	1
50th	24	2
90th	216	7
99th	1007	14
Count	48,611	1,445,696

Table 1: Summary Statistics for Physicians and Patients

6 Calibration

The first step required in calibration is to divy up the physicians into bins. The way this is achieved by by setting up a three part 'quality' score, composed of the physician's percentile in three categories: average days of sickness leave granted, percentage of sick leaves issued over 30 days, amount of sick leaves issued over 30 days.

The logic behind these categories can be surmissed in the models section. As physician quality (V_j) increases, so too their choice of threshold ($\bar{\kappa_j}$) in order to avoid excessive patient demand. As a result, the composition of their patient-base tends to a higher average κ_i, including a higher percentage of $\kappa_i \geq \bar{\kappa}_{\max}$. Sickness leave of ≥ 30 days is subject to higher scrutiny and requires special approval, as well as being indicative of serious illness/injury. As such, issuances of ≥ 30 days, composing close to 21% of our sample, are taken to be such given to patients whose $\kappa_i \geq \bar{\kappa}_{\max}$, and a higher percentage of those in Q_j taken to be indicative of higher V_j.

Taking the average percentile in the three categories mentioned above, we set up four bins of physician quality, with the following frequency in the sample

	%
V_{high}	39.65
$V_{mid-high}$	21.69
$V_{mid-low}$	20.00
V_{low}	18.66

Table 2: Frequency of physician quality V_j bins

We perform calibration of the Logit model. We perform GMM on 9 sample moments as can be seen below. We use the subsample of day sick leaves because it allows for linear independence from the parameters determining $G(\gamma)$, as a patient with $\kappa_i \geq \bar{\kappa}_{\max}$ will be granted sick leave by any physician, such that:

$$s_{ij} = \frac{\alpha_{ij}}{\sum_{k=1}^{J} \alpha_{ik}} = \frac{e^{\lambda u_{ij}}}{\sum_{k=1}^{J} e^{\lambda u_{ik}}} = \frac{e^{\lambda \{V_j \kappa_i + \cancel{\gamma_i} - \tau_j\}}}{\sum_{k=1}^{J} e^{\lambda \{V_k \kappa_i + \cancel{\gamma_i} - \tau_k\}}} = \frac{e^{\lambda \{V_j \kappa_i + - \tau_j\}}}{\sum_{k=1}^{J} e^{\lambda \{V_k \kappa_i + - \tau_k\}}}$$

We assume $F(\kappa)$ to be an exponential function where we normalize the shape parameter λ_F (subcscript F, subscript S for the model one) to 1. $G(\gamma)$ is a normal distribution whose parameters we'll seek to shape. We also look to calibrate the value of each of the four bins, and finally the value of $\kappa_{\max}$. We perform a separate GMM prior to the main one to get the parameters determining $P(\cdot) = \text{fine} * \Pr[\text{fine}|X_j] = t * (\theta X_j)^2$.

Parameter	Value	Source
Quality bins		
V_{high}	-	2nd GMM
$V_{mid-high}$	-	2nd GMM
$V_{mid-low}$	-	2nd GMM
V_{low}	-	2nd GMM
Punishment function		
t	-	1st GMM
θ	-	1st GMM
Medical need distribution $F(\kappa)$		
λ_F	-	Normalization
Taste distribution $G(\gamma)$		
μ	-	2nd GMM
σ	-	2nd GMM
Maximum threshold level		
$\bar{\kappa}_{\max}$	-	2nd GMM
Logit model parameter		
λ_s	-	2nd GMM
Revenue/cost of visit		
r_j	-	Data
τ_j	-	Data

Table 3: Model parameters

Data calibration relies on nine moments: sick leaves issued by bin, sick leaves over 30 days by bin, proportion of ≥ 30 in total.

Moment	Data	Simulated
Total sick leaves issued		
V_{high}	967,956	-
$V_{mid-high}$	1,055,544	-
$V_{mid-low}$	1,068,364	-
V_{low}	1,481,964	-
Sick leaves issued of ≥ 30 days		
V_{high}	656,920	-
$V_{mid-high}$	239,168	-
$V_{mid-low}$	53,270	-
V_{low}	4,832	-
Proportion of ≥ 30 days sick leaves over total	2,086	-

Table 4: Model parameters

7 Counterfactuals & Comments

PENDING.

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Appendix

A.1 The cause of the "strategic effect"

The main difference between our equilibrium and that of Schnell (2022) is the presence of a "strategic effect", wherein doctor j takes into account the behavior of other doctors in the selection of her own $\bar{\kappa_j}$. Of the different modifications we made to her framework, we'll argue it's the absence of *additive separability* across patients in physician utility $U_j(\cdot)$ which accounts for this.

In our context of unbounded maximization by doctor j, the absence of additive separability implies she can't consider each patient individually when it comes to whether she's willing to allow (or induce) their visit, as such a decision is no longer independent of other patients' visits; the marginal utility of an additional patient is dependent on the aggregate of clients up to that point, both in terms of the visit itself as well as in the number of sick leave certificates granted up to that point.

Let's illustrate this point. Consider for a moment a finite number of patients $1, ..., k$, where each patient is inputed directly as an argument in our doctor j's $U_j(\cdot)$, like so: $U_j(1, ..., k)$. If $U_j(\cdot)$ has the property of additive separability, this means it may be reformulated like so:

$$U_j(1, ..., k) = v_{j1}(1) + ... + v_{jk}(k) = \sum_{i=1}^{k} v_{ji}(i)$$

Unconstrained optimization in this context implies she's willing to see any patient whose $v_{ji}(i)$ is non-negative, such that her optimal level of utility is:

$$U_j^*(1, ..., k) = \sum_{i:\, v_{ji}(i) \geq 0} v_{ji}(i)$$

In the context of our doctor-patient model, where physician utility is increasing in κ_i, selection is achieved by the doctor by choosing a κ_j^* which excludes all patients i whose $\kappa_i < \kappa_j^*$. Supposing our patients are well-ordered in κ_i, the choice of such a κ_j^* would be one where the marginal consumer i affords a non-negative $v_{ji}(i)$, and the inframarginal consumer $i+1$ fulfills $v_{j,i+1}(i+1) < 0$. Ignoring for a moment that patients themselves have a *choice* of visiting – depending upon a second dimension γ –, we would then have:

$$U_j^*(1, ..., k) = \sum_{i:\, \kappa_i \geq \kappa_j^*} v_{ji}(i)$$

If instead of a discrete set we consider a mass of consumers $\mathcal{I}$ characterized by their level of κ_i, and we make the simplifying assumption that v_{ji} takes on the same form v_j for every $i \in \mathcal{I}$, our $U_j(\cdot)$ could be expressed as:

$$U_j^*(\mathcal{I}) = \int_{\kappa_j^*}^{\infty} v_j(k)\, dF(k)$$

$U_j^*(\mathcal{I})$ represents the optimal value of $U_j(\mathcal{I})$, where doctor j only sees patients who provide her with non-negative marginal utility, i.e. such that $\kappa_i \geq \kappa_j^*$. We can find this optimal value κ_j^* by looking at *threshold* equilibria, where doctor utility is also dependent on the threshold $\bar{\kappa_j}$ over which she's willing to see patients:

$$U_j(\mathcal{I}, \bar{\kappa_j}) = \int_{\bar{\kappa_j}}^{\infty} v_j(k)\, dF(k)$$

The optimal value of threshold $\bar{\kappa_j}$ is κ_j^*. Assuming $U_j(\cdot)$ is twice differentiable and concave in $\bar{\kappa_j}$, such a solution may be arrived at through the FCO:

$$\frac{\partial U_j(\mathcal{I})}{\partial \bar{\kappa_j}} \equiv -v_j(\bar{\kappa_j}) f(\bar{\kappa_j}) = 0$$

Solving for this would yield $\bar{\kappa_j} = \kappa_j^*$.

Schnell (2022) is an example of just such a treatment, which specifies the physician's additive utility by patient in the following manner:

$$v_{ji}(\kappa_i) \equiv R_j + \beta_j h(\kappa_i)$$

where R_j is a parameter standing for revenue by visit, and $\beta_j h(\kappa_i)$ represents the physician's "altruistic" utility over the health impact of a prescription drug to a patient with "pain level" κ_i.

The optimal threshold κ_j^* is then obtained out of the maximization[10]:

$$\max_{\bar{\kappa_j}} \int_{\bar{\kappa_j}}^{\infty} R_j + \beta_j h(k)\, dF(k)$$

Which gives out the following FOC:

$$R_j = -\beta_j h'(\bar{\kappa_j})$$

which, as is immediately apparent, doesn't depend upon the behavior of other doctors —more specifically, *their* choice of $\bar{\kappa_j}$. It merely establishes that, at the threshold, marginal benefit by patient –revenue R_j– must equal marginal "cost" –altruistic "cost" $\beta_j h(\bar{\kappa_j})$–. A way to interpret this is that doctors will be willing to see any and all patients which render them positive marginal utility, unconcerned with their total market share, and thus, what other physicians may do to take away clientele.

Such a treatment is rendered inviable by our choice of utility function. Schnell's parameter of revenue would in our model imply the linearity of our revenue *function* $R_j(\cdot)$ and our $P(\cdot)$ function over *aggregate* licenses granted. Strict convexity of $P(\cdot)$ would forestall its formulation as Schnell-like $\beta_j h'(\bar{\kappa_j})$ terms for each patient, because

[10] Once again, ignoring patient choice and γ,. We're presenting a *simplified* version for illustrative purposes.

the impact on doctor j's utility in granting patient i a license would no longer independent from the granting of licenses of other patients. Likewise, strict concavity of $R_j(\cdot)$ belies a simple "r" parameter such that $R'_j(Q_j) = rQ_j$.

More formally, when $U_j(1, ..., k)$ *isn't* additively separable across patients, the value κ_j^* such that if $\kappa_i \geq \kappa_j^*$ patient i provides positive marginal utility, isn't independent of current clientele, because what before was a properly defined object, marginal utility by patient i, $v_j(\kappa_i)$, can no longer be so identified. The marginal utility i provides to j as the kth client (assuming some order over clients) is not necessarily the same he'd provide as the $k+1$th client, and so, as the kth client he could provide 0 utility, impliying $\kappa_i = \kappa_j^*$, whereas as the $k+1$th he could be inframarginal, such that $\kappa_i < \kappa_j^*$.

κ_j^* is not longer independent of clientele mass $\mathcal{I}$ as before, but a function of it, $\kappa_j^*(\mathcal{I})$. More specifically, in the models we consider it will depend on the *cardinality* of current clientele, $|\mathcal{I}|$, such that our physician's choice of marginal consumer will depend on her aggregate level of patient demand, where before it didn't. This effect is introduced through the *strict non-linearity* of our physician utility function $U_j(\cdot)$, either through the strict concavity of $R_j(\cdot)$ over expected patient demand Q_j, or the strict convexity of $P_j(\cdot)$ over total expected sick leaves granted.

When either of those is the case, *aggregate* levels enter into the equation and form part of the optimality condition. Our general FOC reflects this:

$$R'_j(Q_j)\frac{\partial Q_j}{\partial \bar{\kappa_j}} = P'_j(X_j)\frac{\partial X_j}{\partial \bar{\kappa_j}} \tag{7}$$

where either or both of $R'_j(\cdot)$ and $P'_j(\cdot)$ are a non-constant function over aggregates.

When such aggregates, either Q_j or X_j, come into play, physicians come to consider their *market share*, which doesn't depend exclusively on their own choice of $\bar{\kappa_j}$. Each patient's strategy S_i is constructed taking into account the whole of $\{(V_j, \bar{\kappa_j})\}_{i=1}^J$.

Imagine equation (7) holds for some doctor j, and some doctor $l \neq j$ decides to lower her $\bar{\kappa_l}$, enough that it changes the value of s_{ij} for some mass of clients. The value of Q_j would then change, and therefore that of $R'_j(Q_j)$. If $P'_j(X_j)$ didn't vary by the same amount, (7) would no longer be an equality, leading j to modify her choice of $\bar{\kappa_j}$ to make equality hold.

This intelligible line of reasoning links the presence of a "strategic effect" to the non-additive separability of $U_j(\cdot)$: doctor j takes into account other physicians' strategy in her own choice of $\bar{\kappa_j}$ because of the present of *aggregate* amounts of clientele in her optimality conditions, which is so because utility isn't additively separable across clients.

A.2 Schnell (2022) with Logit choice

To prove our point that it is additive separability which accounts for a possible strategic effect, we reformulate Schnell (2022) in the manner of a McFadden Logit – with some quirks.

The way in which Schnell devises her physician's utility formula is implicitly Bernoulli-like:

$$\int_\kappa \int_\gamma u(\kappa,\gamma)\cdot p(\kappa,\gamma)\, dG(\gamma)dF(\kappa)$$

where $u(\kappa,\gamma)$ is the utility function of the physician from patients characterised by a given (κ,γ) tuple, and $p(\kappa,\gamma)$ stands for the proportion of clients atomized in such a tuple the physician expects to have visit her.

$u(\kappa,\gamma)$ we'll leave as Schnell defined it: $\beta_j h(\kappa)+R_j$. $p(\kappa,\gamma)$ lends itself nicely as the s_{ij} we have been using thus far, which implicitly depends on the (κ_i,γ_i) tuple which characterizes patient i:

$$\int_\kappa \int_\gamma [\beta_j h(\kappa)+R_j]\cdot s_{ij}(\kappa,\gamma)\, dG(\gamma)dF(\kappa)$$

In our modeling section, our "implicit" search model was a left-censored Logit choice model, which gave null probability of assignment to physicians from which patient i expected non-positive utility. It was defined as:

$$s_{ij} = \frac{\alpha_{ij}}{\sum_{k=1}^J \alpha_{ik}}, \quad \text{where} \quad \alpha_{ij} = \begin{cases} e^{\lambda u_{ij}}, & \text{if} \quad u_{ij} > 0 \\ 0, & \text{if} \quad u_{ij} = 0 \end{cases}$$

Schnell's opioid-focused physician model provides an additional simplification: patients which aren't given a drug prescription, i.e. those such that $\kappa_i < \bar{\kappa_j}$, don't garner positive utility from a visit. As such, in the original model as in this reformulation, patients "below" $\bar{\kappa_j}$ have a null probability of visit, such that the inferior limit of integration for κ is $\bar{\kappa_j}$.

As for γ, the limit of the inner integral remains the same as in Schnell in the absence of a secondary market, a value of γ_i such that $u_{ij}=0$, i.e. $h(\kappa_i)+\gamma_i-\tau^d-\tau^o=0$.[11]

The double-integral maximized as physician utility is then as follows:

$$\max_{\bar{\kappa_j}} \int_{\bar{\kappa_j}}^{\infty} \int_{\tau^d-\tau^o-h(\kappa)}^{\infty} [\beta_j h(\kappa)+R_j]\cdot \frac{e^{\lambda u_{ij}}}{\sum\limits_{k:\, u_{ik}>0} e^{\lambda u_{ik}}}\, dG(\gamma)dF(\kappa)$$

The FOC of such an equation is:

$$[\beta_j h(\bar{\kappa_j})+R_j] \int_{\tau^d-\tau^o-h(\bar{\kappa_j})}^{\infty} \frac{e^{\lambda u_{ij}} \mid \bar{\kappa_j}}{\sum\limits_{k:\, u_{ik}>0} e^{\lambda u_{ik}} \mid \bar{\kappa_j}}\, dG(\gamma) f(\bar{\kappa_j}) = 0 \tag{8}$$

Where we use "$\mid \bar{\kappa_j}$" to clumsily indicate that we integrate γ over patients where $\kappa_i=\bar{\kappa_j}$. As an integral over a strictly positive value and the atom of a density function,

[11] Schnell splits the costs a patient will face into costs of visit (τ^d), cost of purchase (τ^o) and search cost (τ^s)

respectively, the factors B and C in the equation are non-negative. For this reason, the only way for (8) to be fulfilled is the following condition:

$$R_j = -\beta_j h(\bar{\kappa_j}) \tag{9}$$

This is the same result as in the original model in ?? in the absence of a secondary market, a condition which stipulates that marginal utility in $\bar{\kappa_j}$ must be 0, i.e. that revenue (R_j) must equal "altruistic" loss ($\beta_j h(\bar{\kappa_j})$), which pre-supposes that the marginal patient granted prescription opioids suffers a net loss in utility (the negative externalities outweigh its medical benefit as a palliative).

Condition (9) means a value κ_j is chosen irrespective of the strategies employed by other doctors, it quite literally doesn't enter into the equation. As we have argued, additive separability over clients means the *total* demand for physician j's services doesn't influence marginal utility by a given patient i, and so has no sway in the optimality condition. Doctor j states simply that she won't grant a prescription (or sick leave, as in our case) below $\bar{\kappa_j}$, *come what may*. In order for her to care about *aggregate* values, like her total expected demand, making her care about her market share and thus about the strategies of other doctors, *strict non-linearity* must be introduced into her utility function.

A.3 On the strategic complementarity of $\bar{\kappa_j}$'s

We had defined the components s_{ij} of S_i as:

$$s_{ij} \equiv g_i(u_{ij}, u_{i,-j})$$

where $g_i(\cdot)$ is a continuous function weakly increasing in the first argument u_{ij}. Our defining s_{ij} this way has two interlinked corollaries:

Corollary A.1.

$$s_{ij} \mid \kappa_i < \bar{\kappa_j} \quad \leq \quad s_{ij} \mid \kappa_i \geq \bar{\kappa_j} \quad \textit{(with strict inequality if } \gamma_i > 0\textit{).}$$

Proof. For a fixed value of V_j and κ_i, for all i, the value of $U_i(V_j, \bar{\kappa_j})$ is $V_j\kappa_i - \tau_j$ if $\kappa_i < \bar{\kappa_j}$, and $\gamma_i + V_j\kappa_i - \tau_j$ if $\kappa_i \geq \bar{\kappa_j}$, where $\gamma_i \geq 0$. Given that s_{ij} is weakly increasing in u_{ij}, and we see that u_{ij} is weakly higher if $\kappa_i \geq \bar{\kappa_j}$, the corollary follows. □

Corollary A.2.

$$\frac{\Delta s_{ij}}{\Delta \bar{\kappa_j}} \leq 0 \qquad\qquad \frac{\Delta s_{ij}}{\Delta \bar{\kappa_l}} \geq 0, \forall l \neq j$$

Proof. From the argumentation in corollary 1 follows that u_{ij} is weakly decreasing in $\bar{\kappa_j}$, $\forall j$. Take some $l \neq j$, then s_{ij} is defined in turn as weakly decreasing in u_{il}, which implies it is increasing in $\bar{\kappa_l}$. □

Both corollaries hinge upon our definition of $U_i(\cdot)$ as a step function over κ_i, such that it is discontinuous at $\kappa_i = \bar{\kappa_j}$, where there's a discrete jump of magnitude $\gamma_i \geq 0$.

Corollary A.3.

$$\frac{\partial Q_j}{\partial \bar{\kappa_j}}, \frac{\partial X_j}{\partial \bar{\kappa_j}} \leq 0 \qquad\qquad \frac{\partial Q_j}{\partial \bar{\kappa_l}}, \frac{\partial X_j}{\partial \bar{\kappa_l}} \geq 0, \forall l \neq j$$

Proof. We recall our function of patient demand for physician j is:

$$Q_j(\bar{\kappa_j}, \bar{\kappa}_{-j}) \;=\; \int_0^\infty \int_0^\infty s_{ij}(k,\gamma)\, dG(\gamma)\, dF(k)$$

Were physician j to increase her threshold from $\bar{\kappa_j}$ to $\bar{\kappa_j}' = \bar{\kappa_j} + \epsilon, \epsilon > 0$, for the mass of patients whose $\kappa_i \in [\bar{\kappa_j}, \bar{\kappa_j} + \epsilon)$, utility would weakly decrease and thus so would their s_{ij}, falling to $s_{ij}' \leq s_{ij}$ (see first corollary, think of s_{ij}' as $s_{ij} \mid \kappa_i < \bar{\kappa_j}$ as opposed to $s_{ij} \mid \kappa_i \geq \bar{\kappa_j}$).

The difference that would make for patient demand *caeteris paribus* given the $\bar{\kappa}_{-j}$ strategies of the other physicians (ommitted in our notation) is:[12]

$$\Delta Q_j = Q_j(\bar{\kappa_j} + \epsilon) - Q_j(\bar{\kappa_j}) = \int_{\bar{\kappa_j}}^{\bar{\kappa_j}+\epsilon} \int_0^\infty \{s_{ij}' - s_{ij}\}\, dG(\gamma)\, dF(k)$$

By the definition of a partial derivative:[13]

$$\begin{aligned}\frac{\partial Q_j}{\partial \bar{\kappa_j}} &= \lim_{\epsilon \to 0} \frac{\int_{\bar{\kappa_j}}^{\bar{\kappa_j}+\epsilon} \int_0^\infty \{s_{ij}' - s_{ij}\}\, dG(\gamma)\, dF(k)}{\epsilon} \\ &= \int_0^\infty \{s_{ij}(\bar{\kappa_j},\gamma)' - s_{ij}(\bar{\kappa_j},\gamma)\}\, dG(\gamma)\, f(\bar{\kappa_j})\end{aligned}$$

where the derivative is negative if s_{ij}' is strictly lower than s_{ij}, meaning a shift in leniency would affect the utility and optimal strategy of some positive mass of patients around $\bar{\kappa_j}$; or null, if $s_{ij}' = s_{ij}$.

In a similar vein, consider some physician $l \neq j$ shifting her threshold from $\bar{\kappa_l}$ to $\bar{\kappa_l}' = \bar{\kappa_l} + \epsilon, \epsilon > 0$. The fall in patient strategies in $[\bar{\kappa_j}, \bar{\kappa_j} + \epsilon)$ from s_{il} to $s_{il}^\star \leq s_{il}$ could provide a windfall for j (and all other physicians), raising s_{ij} to $s_{ij}^\star \geq s_{ij}$ (see previous corollary). In that case:

$$\Delta Q_j = \int_{\bar{\kappa_l}}^{\bar{\kappa_l}+\epsilon} \int_0^\infty \{s_{ij}^\star - s_{ij}\}\, dG(\gamma)\, dF(k)$$

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[12] Besides simplifying $Q_j(\bar{\kappa_j}, \bar{\kappa}_{-j})$ into $Q_j(\bar{\kappa_j})$, we also forego the arguments of $s_{ij}(k,\gamma)$, as we've done many times already, to avoid notation clutter.

[13] The notation for $s_{ij}(\bar{\kappa_j},\gamma)'$ is to be taken to mean the strategy $s_{ij}' \leq s_{ij}$ of the mass of patients at different levels of γ atomized at $\kappa_i = \bar{\kappa_j}$.

thus rendering:

$$\frac{\partial Q_j}{\partial \bar{\kappa_l}} = \lim_{\epsilon \to 0} \frac{\int_{\bar{\kappa_l}}^{\bar{\kappa_l}+\epsilon} \int_0^\infty \{s_{ij}' - s_{ij}\}\, dG(\gamma)\, dF(k)}{\epsilon}$$
$$= \int_0^\infty \{s_{ij}(\bar{\kappa_l},\gamma)^\star - s_{ij}(\bar{\kappa_l},\gamma)\}\, dG(\gamma)\, f(\bar{\kappa_l})$$

As for X_j, the steps taken are almost the same, only that after raising the threshold by ϵ the amount of sick leaves granted goes around $\bar{\kappa_j}$ goes to 0, thus giving out:

$$\frac{\partial X_j}{\partial \bar{\kappa_j}} = -\int_0^\infty s_{ij}(\bar{\kappa_j},\gamma)\, dG(\gamma)\, f(\bar{\kappa_j})$$

such that it is simply required that j has positive demand for the mass of patients whose $\kappa_i = \bar{\kappa_j}$ for this derivative to be strictly negative, else it is null.

Likewise:

$$\frac{\partial X_j}{\partial \bar{\kappa_l}} = -\int_0^\infty s_{ij}(\bar{\kappa_l},\gamma)\, dG(\gamma)\, f(\bar{\kappa_l})$$

□

Before dealing with PROPOSITION we need one final corollary.

Corollary A.4.

$$\frac{\partial^2 Q_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}} = 0 \quad \textit{and} \quad \frac{\partial^2 X_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}} = 0\ , \quad \forall j \textit{ and } l,\ \bar{\kappa_j} \neq \bar{\kappa_l}$$

Proof. Our proof will be heuristic. Consider

$$\frac{\partial Q_j}{\partial \bar{\kappa_j}} = \int_0^\infty \{s_{ij}(\bar{\kappa_j},\gamma)' - s_{ij}(\bar{\kappa_j},\gamma)\}\, dG(\gamma)\, f(\bar{\kappa_j})$$

Patient strategies s_{ij} are taken at the atom $\kappa_i = \bar{\kappa_j}$. For any other physician l such that $\bar{\kappa_l} \neq \bar{\kappa_j}$, an infinitesimally small change to $\bar{\kappa_l}$ doesn't affect the utility of patients at $\bar{\kappa_j}$, who will remain above or below l's threshold same as before. Thus their strategy s_{ij} also remains unaffected. If $\bar{\kappa_l}$ *does* equal $\bar{\kappa_j}$, then the following limit is not properly defined:

$$\lim_{\epsilon \to 0} \frac{\frac{\partial Q_j(\kappa_l+\epsilon)}{\partial \bar{\kappa_j}} - \frac{\partial Q_j(\kappa_l)}{\partial \bar{\kappa_j}}}{\epsilon}$$

because as ϵ approaches 0 from the right the limit is weakly positive (≥ 0), whereas approaching from the left it's strictly 0. Meaning the derivative $\frac{\partial^2 Q_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}}$ doesn't exist for j and $l \neq j$ when $\bar{\kappa_j} = \bar{\kappa_l}$.

□

PROOF OF PROPOSITION

Consider some two physicians j, l, such that $\bar{\kappa_j} \neq \bar{\kappa_l}$. If we breakdown the mixed partial derivative of U_j over $\bar{\kappa_j}$ and $\bar{\kappa_l}$, we get:

$$\begin{aligned}
\frac{\partial^2 U_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}} &= \underbrace{R''(Q_j(\bar{\kappa_j}, \bar{\kappa}_{-j}))}_{\leq 0} \cdot \underbrace{\frac{\partial Q_j}{\partial \bar{\kappa_l}}}_{\geq 0} \cdot \underbrace{\frac{\partial Q_j}{\partial \bar{\kappa_j}}}_{\leq 0} + \underbrace{R'(Q_j(\bar{\kappa_j}, \bar{\kappa}_{-j}))}_{\geq 0} \underbrace{\frac{\partial^2 Q_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}}}_{=0} \\
&\quad - \underbrace{P''(X_j(\bar{\kappa_j}, \bar{\kappa}_{-j}))}_{\geq 0} \cdot \underbrace{\frac{\partial X_j}{\partial \bar{\kappa_l}}}_{\geq 0} \cdot \underbrace{\frac{\partial X_j}{\partial \bar{\kappa_j}}}_{\leq 0} - \underbrace{P'(X_j(\bar{\kappa_j}, \bar{\kappa}_{-j}))}_{\geq 0} \underbrace{\frac{\partial^2 X_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}}}_{=0} \\
&= \underbrace{R''(Q_j(\bar{\kappa_j}, \bar{\kappa}_{-j})) \cdot \frac{\partial Q_j}{\partial \bar{\kappa_l}} \cdot \frac{\partial Q_j}{\partial \bar{\kappa_j}}}_{\geq 0} - \underbrace{P''(X_j(\bar{\kappa_j}, \bar{\kappa}_{-j})) \cdot \frac{\partial X_j}{\partial \bar{\kappa_l}} \cdot \frac{\partial X_j}{\partial \bar{\kappa_j}}}_{\leq 0} \quad \geq 0
\end{aligned}$$

POOSITIVE MIXED DERIVATIVE IS STRATEGIC COMPLEMENTARITY

A.4 Proof Sequential

We follow the argument of Ljungqvist and Sargent (2012). Let for the moment $\tilde{F}_i(U) = \Pr\{u_{ij} \leq U\}$ be the distribution of utility patient i recieves from doctor visits, with $\tilde{F}_i(0) = 0$, $\tilde{F}_i(B) = 1$, for $B < \infty$. Each period, the patient draws an 'offer' U from $\tilde{F}_i$, which is to say, his search lead him to a physician j such that $u_{ij} = U$. He may either accept and have physician j assigned to him, recieving utility U per period henceforth, or look for a new physician next period.

Let u_t be the patient's utility for period t, where it's 0 if he doesn't visit a physician and U if he has accepted an assigned physician which renders him utility $u_{ij} = U$. This patient will seek to maximize $\mathbb{E}[\sum_{t=0}^{\infty} \beta^t u_t]$, where $0 < \beta < 1$ is his discount factor.

Let $V(U)$ be the expected value of $\sum_{t=0}^{\infty} \beta^t u_t$ of a patient with an offer U in hand. Assuming no recall, under optimal behavior the value function $V(U)$ satisfies the Bellman equation:

$$V(U) = \max\left\{ \frac{U}{1-\beta}, \ \ \beta \int V(U')\, d\tilde{F}_i(U') \right\} \tag{10}$$

The patient chooses an optimal value $\bar{U}_i$, such that offers $U \geq \bar{U}_i$ are accepted (the visit takes place). As such, the solution takes on the following form:

$$V(U) = \begin{cases} \frac{\bar{U}_i}{1-\beta} + \beta \int V(U')\, d\tilde{F}_i(U') & \text{if} \quad U \leq \bar{U}_i, \\ \frac{U}{1-\beta} & \text{if} \quad U \geq \bar{U}_i. \end{cases} \tag{11}$$

Using equation (11), we can convert the functional equation (10) into an ordinary equation in the reservation utility $\bar{U_i}$. Evaluating $V(\bar{U_i})$ and using equation (11), we have:

$$\frac{\bar{U_i}}{1-\beta} = \beta \int_0^{\bar{U_i}} \frac{\bar{U_i}}{1-\beta}\, d\tilde{F}_i(U') + \beta \int_{\bar{U_i}}^{B} \frac{U'}{1-\beta}\, d\tilde{F}_i(U')$$

or

$$\begin{aligned} &\frac{\bar{U_i}}{1-\beta} \int_0^{\bar{U_i}} d\tilde{F}_i(U') + \frac{\bar{U_i}}{1-\beta} \int_{\bar{U_i}}^{B} d\tilde{F}_i(U') \\ &\quad = \beta \int_0^{\bar{U_i}} \frac{\bar{U_i}}{1-\beta}\, d\tilde{F}_i(U') + \beta \int_{\bar{U_i}}^{B} \frac{U'}{1-\beta}\, d\tilde{F}_i(U') \end{aligned}$$

or

$$\bar{U_i} \int_0^{\bar{U_i}} d\tilde{F}_i(U') = \frac{1}{1-\beta} \int_{\bar{U_i}}^{B} \left(\beta U' - \bar{U_i}\right) d\tilde{F}_i(U').$$

Adding $\bar{U_i} \int_{\bar{U_i}}^{B} d\tilde{F}_i(U')$ to both sides gives

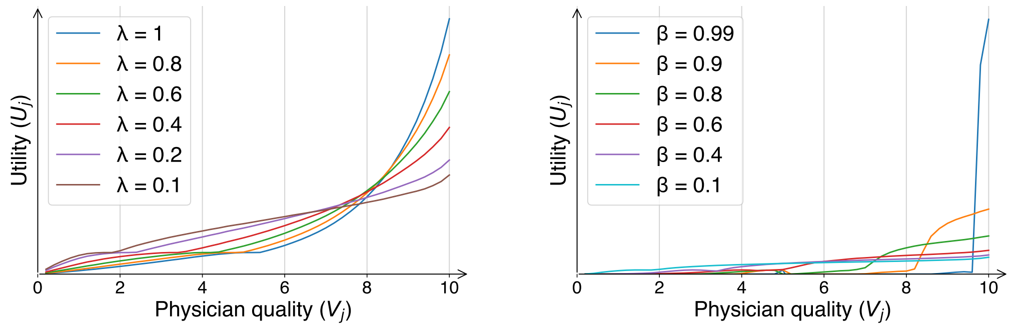
$$\bar{U_i} = \frac{\beta}{1-\beta} \int_{\bar{U_i}}^{B} \left(U' - \bar{U_i}\right) d\tilde{F}_i(U') \tag{12}$$

Finally, for our purposes, we discretize $\int_{\bar{U_i}}^{B} \left(U' - \bar{U_i}\right) d\tilde{F}_i(U')$ as a summation over the countable set of J doctors. The discrete equivalent of equation (12) is:

$$\bar{U_i} = \frac{\beta}{1-\beta} \sum_{j=1}^{J} \left\{ \frac{\mathbb{1}[u_{ij} \geq \bar{U_i}] \cdot (u_{ij} - \bar{U_i})}{\mathbb{1}[u_{ij} \geq \bar{U_i}]} \right\} \tag{13}$$

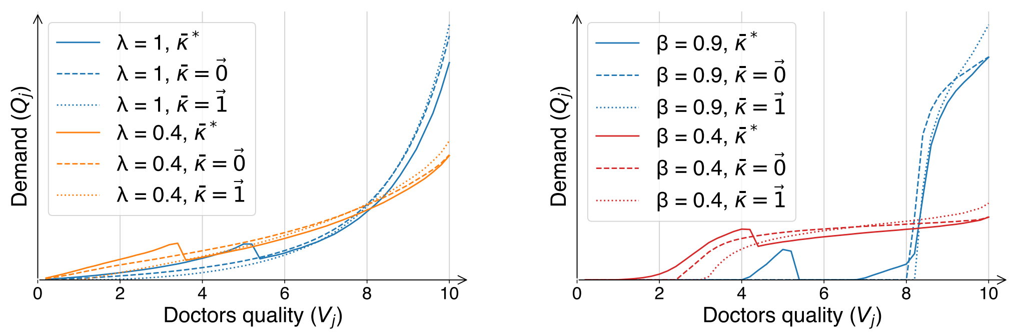
Which is the function we say characterizes patient thresholds in the explicit search model.

A.5 Graphs



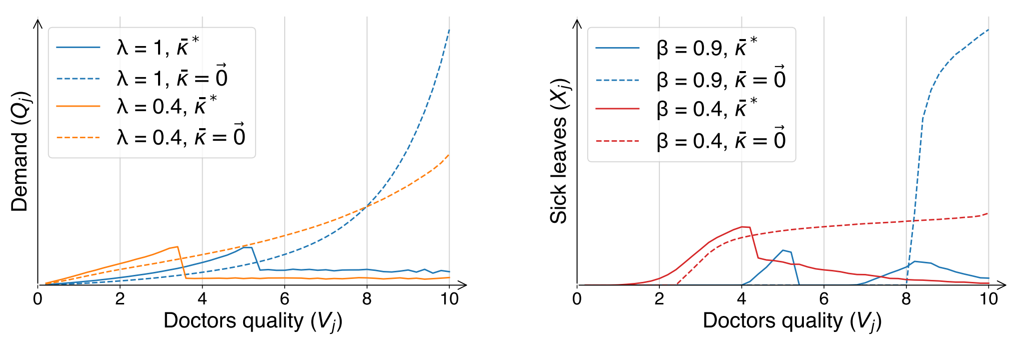
(a) Logit model (b) Sequential model

Figure 5: Equilibrium physician utility (U_j) by quality (V_j)
for different model parameters, equilibrium $\bar{\kappa}^*$



(a) Logit model (b) Sequential model

Figure 6: Comparison of patient demand (Q_j) by physician quality (V_j)
for different model parameters and threshold vectors $\bar{\kappa}$



(a) Logit model (b) Sequential model

Figure 7: Comparison of sick leaves issued (X_j) by physician quality (V_j)
for different model parameters and threshold vectors $\bar{\kappa}$