PONTIFICIA
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Contents

1 Introduction

The question of sickness at work presents us with a principal-agent problem: though in cases of true sickness it is best for the employee's personal wellbeing and indeed even firm productivity (FUENTE) that they should stay home, it would also be possible to call in sick without any affliction, skirting work undetected.

The common solution to this problem is third-party certification, a specialized professional to attest to the condition and allow for or even demand of the employer that sick leave be granted. And who else to carry out this evaluation than physicians themselves, knowledgeable on the subject as they are, making diagnosis, treatment referral and certification of sickness a one-stop-shop for the patients.

The problem with this oversized, multi-faceted role physicians play in healthcare systems is that it causes conflict of interests. In cases where sick leave calls (and insurance claims) call for state-subsidied payments to be issued, we can speak of a second principal-agent problem between the physician, exercising the role of "gatekeeper", and the taxpayer, bankrolling public healthcare and insurance.

There is evidence to the fact that even in the best of cases physicians have no incentive to place doubt on their patients claim to unobservable symtoms (FUENTE), and in murkier scenarios they could be motivated to gain the reputation of being "lenient" to enjoy increased demand for their services (MARKUSSEN).

In Chile, the country of our sample, the healthcare system is bi-modal: citizens may opt for one of many private health insurers (ISAPRES) or the sole public option (FONASA), and 82% of them are affiliates of the latter. In their case, the authority behind sick leave granting is COMPIN (fuente?), with the power to oversee, investigate and even sanction physicians that fall under it. A worker granted sickness absence is entitled to a work incapacity benefit (SIL), periodic payments paid in function of sick leave length (fuente?).

Anecdotal evidence speaks to the presence of fraud in the system. A survey by the U. Andrés Bello Public Health Institute (YEAR) has it that 51% of those surveyed know someone who was granted sick leave without any sickness, 62% believe that physicians "frequently" create irregular businesses for the sale of sick leave certificates, and 56% think it would be "easy" to purchase one.

EXPERIMENTO?

2 Literature Review

It is a well established factum that taking sick leave is subject to an economic calculus on part of the workers, rather than being an orthogonal, merely health-concerned matter. Johansson and Palme (2005) begin their article with a quote by Nobel Laureate Ragnar Frisch: "Regarding the high absence rate at the Department: Acquiring minor diseases, such as colds or flu, is an act of choice". Their paper is among many others—Paola et al. (2014), Markussen et al. (2012), Stearns and White (2018), Henrekson and Persson (2004)—which give empirical evidence of such a choice being driven by economic incentives, through an event study on exogenous institutional regime changes in the sampled nation's public insurance system. This line of research, however, is concerned with the actions of workers themselves and their subsequent effect on macroeconomic employment variables, whereas our main focus shall be the role played by physicians.

As for the literature on physicians themselves, it has become commonplace to regard them as agents with two sources of utility: their income and their "altruistic" interest on their patients' well-being. Empirical support for the latter can be found in experimental evidence both on medical students (Brosig-Koch et al. (2017), Hennig-Schmidt and Wiesen (2014)) as well as doctors themselves (Kesternich et al. (2015), Brosig-Koch et al. (2016)). Crea (2019) finds no evidence for this, whereas Godager and Wiesen (2013) do, and explore its heterogeneity across physicians. The fact that physicians are also concerned with revenue, rather than being purely altruistic, is also well evidenced, see Clemens and Gottlieb (2014), Hennig-Schmidt et al. (2011), Autor et al. (2014), and also Robertson et al. (2012) for a review on the matter. Therein lies the dilemma with giving physicians the status of gatekeepers for different services and certifications, like disability insurance (as in autor). As Markussen and Røed (2017, p. 1) put it: "In essence, the GPs [general practitioners] have been assigned the task of protecting the public (or private) insurer's purse against the customers who form the basis for their own livelihood".

Both factors being well established in the literature on physicians, one strand of it would seek to design an optimal contract for medical care in the presence of such an economic calculus, see Choné and Ma (2011) or Gaynor et al. (2023); another, more in line with our approach, would evaluate the effects increased competition among doctors has on their rendered services. In general, Currie et al. (2023) propose that increased competition would lead to physicians offering more services that please the clients yet relatively hurt their own utility (like drug prescriptions), and less services which bring them, physicians, more utility, at the expense of patient utility (like unwanted, expensive surgical procedures). Iversen and Lurås (2000) and Iversen (2004) provide empirical evidence that somewhat supports it: physicians with a shortage of customers will provide more services, thus obtaining more income *per customer*. This line of work is more in line with our own approach, as the altruistic motivations of physicians are set momentarily aside and they're modeled as purely profit-seeking. This reasoning applies to scenarios like ours, where the issuance of sick leaves—particularly for short-term absences, as is the case in most situations[1]—does not have a significant impact on client health.

To our knowledge, the only article dealing specifically with sick leave certicate grant-

[1] For cases where sickness leave is medically required we introduced the upper limit $\bar{\kappa}_{\max}$, such that physicians would never be so strict as to be negligent.

ing as a function of competition among physicians is Markussen and Røed (2017). Carlsen et al. (2020) deal with sick leave as well, but make only the narrower point that in a Bayesian context doctors have almost no incentive to distrust patients' self-reported, unverifiable symptoms. Markussen and Røed's methodology is similar to our own: after performing "raw" regression analysis, they set-up a model of patient choice as a McFadden Logit over observables X_i, including physician leniency (assumed to be observable for prospective patients), and as such can estimate the role leniency plays in demand for their services. They then perform a series of exercises, some of their findings include: physicians with variable wage (i.e. dependent on clientele) certify 7% more absence days per month than fixed wage physicians; half of this difference has a causal interpretation, as observed from within-physician responses to market conditions; in general, more lenient gatekeeping gives the GP (physician) more customers, and more customers make the GP less lenient. Most of these stylized facts are taken up in our equilibrium models, in which patients take into account both sick leave "leniency" ($\bar{\kappa_j}$) as well as physician quality (V_j), such that physicians which offer better medical attention and thus enjoy higher demand can afford to be "stricter", and those who don't will have the incentive to gain clientele through leniency. Not captured is the physician fixed effect, the idiosyncratic motive to leniency: we assume leniency as merely a strategic choice based on expected demand, where prior to choosing leniency physicians only differ in "quality" (V_j) and visit revenue/cost of visit to the patient (r_j/τ_j).

Despite the different subject matter, the main source of inspiration for this paper is Schnell (2017), such that ours can be seen as an attempt to replicate her model and framework, intially devised for opioid markets, to the market for sick leaves. Schnell seeks to model the market scenario for the opioid crisis, with a primary market composed of physician prescriptions and a secondary black market. The presence of the latter, she concludes, makes unilateral interventions ineffective: curbing both excessive physician prescriptions as well as illegal black market sales is required to make more than a dent on the number of opioids consumed in America.

Her paper includes four benchmark for the patient-physician market, building up to her main model including patient search, such that patients with a hire taste for opioids are assortatively matched with physicians more willing to prescribe them, and a secondary market. In our model we keep the former but not the latter, as the "black market" for sick leaves falls *within* the primary market of physicians, composed of those willing to knowingly issue fraudulent sickness certificates. Search was repurposed to fit a two-dimensional frame of physicians, characterized both by their strictness as well as their service quality. What we call the "explicit" search model is in the vein of Schnell's sequential patient search, though more fleshed out in its dynamic programming framework for the reason just mentioned. Then there's the "implicit" model, which defines patient behavior according to a modified McFadden Logit. We show in the APPENDIX that such a framework wouldn't have altered Schnell's main conclusions. Our model differs from hers chiefly in the fact that physicians take into account *other* physicians' behavior when selecting their own strategy, such that market equilibrium requires a Nash equilibrium in physician strategies. We discuss in the APPENDIX that the source of this feature, not presentt in Schnell's model, is the lack of additive separability across patients in the physician's utility function, such that her optimal behavior takes into account *aggregate* patient demand as well as *marginal*.

3 Model

MENTION PREVIOUS SECTION FOR CONTINUITY

Note on language: For ease of reference, we refer to any patient –otherwise called client or worker– as "he", and to any doctor –or physician– as "she".

At this point it shall be noted that the first person plural (we) employed over the course of this article is to be read as a royal *we*, being that we claim sole authorship over this paper.

We consider $i = 1, ..., I$ patients and $j = 1, ..., J$ doctors.

Patients are characterized by the tuple $(\kappa_i, \gamma_i) \in (\mathbb{R}_0^+)^2$, their "medical need" and "taste for sick leave" respectively, following the ex-ante cumulative distributions $F(k)$ and $G(\gamma)$.

Doctors are described by their "service quality" $V_j \in \mathbb{R}_0^+$, following the *ex-post*, empirical distribution $H(V)$.

A patient i visits a doctor for treatment and may be granted a sick leave certificate. After being assigned a doctor j, his utility function –implicitly dependent on his characteristic (κ_i, γ_i) tuple– is defined piece-wise as follows:

$$U_i(V_j) = \begin{cases} \gamma_i + V_j\kappa_i - \tau & \text{if he's granted a certificate,} \\ V_j\kappa_i - \tau & \text{if he only visits the doctor,} \\ 0 & \text{if he doesn't see the doctor,} \end{cases}$$

As we see, there's three components to patient utility: an interaction between the patient's medical need κ_i and the physician's service quality V_j which implies their complimentarity, his "taste" for sick leave γ_i in the case he's granted one, and τ, the cost of visit, normalized across doctors.

Whereas patients may visit at most one doctor, a doctor may see several physicians. We define Q_j as the expected number of patients of doctor j, the demand for her services. We say "expected" because, as we'll see later on, patients may opt for a mixed strategy, assigning a certain *probability* to visiting j, and Q_j will be defined over the ex-ante probabilities of all patients and not their ex-post realization.

As doctor j has the option to grant a sick leave certificate to a given patient i which visits her, so we likewise define X_j as the *expected* number of such certificates doctor j will dole out, given her ex-ante client demand and how many of them would be granted one.

We now define the physician's utility function as follows:

$$U_j(Q_j, X_j) = R_j(Q_j) - P_j(X_j)$$

where $R_j(\cdot)$ is an individual, concave *revenue* function defined over expected total clients, and $P_j(\cdot)$ is a convex *punishment* function on X_j, grouping her personal preference as well as institutional incentives. The implication is that after a given number of patients the disutility of an additional certificate issued would outweigh doctor j financial incentives for further clientele. We assume both $R_j(\cdot)$ and $P_j(\cdot)$ to be twicely differentiable.

The j subindex indicates that we will allow $R_j(\cdot)$ and $P_j(\cdot)$ to vary across physicians, meaning that a given doctor j will now be initially characterized by both her parameter V_j as well as the form of her $R_j(\cdot)$ and $P_j(\cdot)$ functions.

We stress again that we define physician utility in terms of the *expected* realization of patient demand and granted sick leaves, as befits the logic of this game, where doctors take action before patients (we will specify the timing of our game below).

Following Schnell (2017), we focus on *threshold equilibria*, wherein each physician's strategy is the choice of a value $\bar{\kappa_j}$, such that of the patients who visit j, those with a κ_i value above or at that threshold will recieve a certificate, and those under it won't.

In both frameworks we will develop, with and without search, the set $\{\bar{\kappa_j}\}_{j=1}^{J}$ composed of all physicians' choice of $\bar{\kappa_j}$ will be public knowledge to patients at the moment they choose their strategy, whereas doctors themselves don't observe it at the moment they make their choice of threshold, because they will select it simultaneously – and *then* patients make their move.

Our models are different iterations of a general game with the following timing:

- First stage: Physicians simultaneously choose $\bar{\kappa_j}$.
- Second stage: Observing $\{\bar{\kappa_j}\}_{j=1}^{J}$, each patient chooses or is assigned some doctor j.
- Third stage: Each patient can choose to see their doctor and incur a visit cost τ, or refrain from doing so, leaving him and his physician with null utility.

Conditional upon his visit, the utility of patient i from seeing doctor j is:

$$y_i(V_j, \bar{\kappa_j}) = \begin{cases} \gamma_i + V_j\kappa_i - \tau & \text{if } \kappa_i \geq \bar{\kappa_j}, \\ V_j\kappa_i - \tau & \text{if } \kappa_i < \bar{\kappa_j}, \end{cases}$$

Patient utility U_i as previously defined, now taking $\bar{\kappa_j}$ explicitly as an argument as well as V_j, can be expressed as a left-censored function over y_i:

$$U_i(V_j, \bar{\kappa_j}) = \max\{y_i(V_j, \bar{\kappa_j}), 0\}$$

3.1 Non-search Equilibrium

We first devote attention to a non-search baseline, where all patients are randomly, symmetrically assigned to a physician, and their only say in the matter is whether they'll then visit doctor j, i.e. the second stage of the game is out of their hands, and they only make choices in the third stage after assingment.

A patient won't visit his assigned physician if his expected utility from such a visit is negative, we call this a *free disposal* requirement. As such, a doctor j's expected client demand, as a function of $\bar{\kappa_j}$ and given the parameter V_j, will be the following:

$$Q_j(\bar{\kappa_j}) \;=\; \frac{I}{J}\left[\int_{\tau/V_j}^{\infty} dF(k) + \int_{\min\{\bar{\kappa_j},\tau/V_j\}}^{\tau/V_j} \int_{\tilde{\gamma}(k)}^{\infty} dG(\gamma)\, dF(k)\right] \tag{N.1}$$

where the left term consists of the mass of patients who just by virtue of doctor j's service quality V_j would be willing to pay a visit (i.e. $\kappa_i \geq \tau/V_j$), and the right term would be patients who only see doctor j out of the expectation of getting sick leave ($\kappa_i \geq \bar{\kappa_j}$ & $\gamma_i \geq \tau - V_j\kappa_i$), but wouldn't visit otherwise. We define $\tilde{\gamma}(k) := \tau - V_j k$ as the lower limit of the inner integral.

Given that each patient with a κ_i higher or equal to $\bar{\kappa_j}$ is granted a sick leave certificate, the expected total number of such certificates granted by j, as a function of $\bar{\kappa_j}$, is:

$$X_j(\bar{\kappa_j}) \;=\; \frac{I}{J}\int_{\bar{\kappa_j}}^{\infty} \int_{\tilde{\gamma}(k)}^{\infty} dG(\gamma)\, dF(k) \tag{N.2}$$

Given (N.1) and (N.2), each physician solves for the following unconstrained optimization:

$$\bar{\kappa_j}^* \equiv \arg\max_{\bar{\kappa_j}} R_j(Q_j) - P_j(X_j) \tag{N.3}$$

We now SEE LATER

Lemma 3.1. *No value in* $(\frac{\tau}{V_j}, \infty)$ *can be the optimal solution of (N.3).*

PROOF: APPENDIX

Lemma 3.2. *When* $\bar{\kappa_j} \in [0, \frac{\tau}{V_j}]$,

$$\frac{\partial Q_j}{\partial \bar{\kappa_j}} = \frac{\partial X_j}{\partial \bar{\kappa_j}}$$

PROOF: APPENDIX

This proposition implies that when $\bar{\kappa_j}$ is low enough that the marginal patient is indifferent, any new clients gained by doctor j are those she entices through the expectation of getting a certificate, meaning that in the vicinity of $\bar{\kappa_j}$ the change in expected patients $\Delta Q_j/\Delta\bar{\kappa_j}$ through a variation in $\bar{\kappa_j}$ is the same as the change in expected certificates granted $\Delta X_j/\Delta\bar{\kappa_j}$.

Given Proposition 3.2, the following equation, which is the FOC of (N.3):

$$R_j'(Q_j)\frac{\partial Q_j}{\partial\bar{\kappa_j}} - P_j'(X_j)\frac{\partial X_j}{\partial\bar{\kappa_j}} = 0$$

may be simplified into

$$R'(Q_j) = P'(X_j) \tag{N.4}$$

We can see that (N.3) offers three possibilities by way of solution: a *corner* solution towards ∞, where doctor j is content with her "captured" clientele (those who would visit her even without expecting sick leave), and offers no certificates at all[2]; the corner solution $\bar{\kappa_j} = 0$, maximum leniency; and an *inner* solution in $[0, \frac{\tau}{V_j}]$ fulfilling equation (N.4) and thus the FOC of (N.3). We formalize equilibrium below.

Equilibrium in non-search models. *Given a doctor-patient market specified by* $(\{(\kappa_i, \gamma_i)\}_{i=1}^{I}, \{(V_j, R_j(\cdot), P_j(\cdot))\}_{i=1}^{J})$*, we define an equilibrium as a set of physician thresholds* $\{\bar{\kappa_j}\}_{j=1}^{J}$ *satisfying (N.3) and their corresponding equilibrium (expected) patient demand and granted sick leave certificates* $\{(Q_j, X_j)\}_{j=1}^{J}$ *as described by (N.1), (N.2), respectively.*

3.2 Search Equilibria

a Introducing search

We introduce patient "search" as a general framework in which patients can choose freely among all physicians.

We define for each patient i a vector $S_i \in \Delta(\mathcal{J})$, where $\mathcal{J}$ is the the $1 \times J$ vector composed of all $1, ..., J$ physicians. S_i will be his *strategy* for this game, representing his probabilistic choice of visit for each doctor j, such that each component $s_{i1}, ..., s_{iJ}$ of S_i stands for the probability that he'll visit doctors $1, ..., J$ respectively.

In order to describe a proper probability distribution, the following criteria must be met:

i. $\forall j,\ s_{ij} \geq 0$

ii. $\sum_{i=1}^{J} s_{ij} \leq 1$

We will allow the sum of all components to be less than one, implying the presence of an *outside option* for patients, that is, to not visit any doctors. Such an option is

[2] We may rationalize this by interpreting it as physician j being as strict as is medically responsible. We could have written this as $\bar{\kappa_j}$ having an upper bound $\kappa_{\max}$, where any physician would be obliged to grant sick leave to a patient with $\kappa_i > \kappa_{\max}$.

important, as patient rationality in our models will include "*free disposal*", meaning that a patient will never visit a doctor if his expected utility from such a visit is less than 0, i.e. $s_{ij} = 0$ if $U_i(V_j, \bar{\kappa_j}) = 0$. This makes the third stage trivial, as under free choice, patient i will only be assigned with positive probability to doctors he will be willing to visit.

We can re-interpret the non-search model as each patient being made to play by the strategy $\{S_i : s_{i1} = s_{i2} ... = s_{iJ} = \frac{1}{J}\}$, and it's the lack of a free choice which makes the third stage non-trivial.

Given the collection of all patients' strategies $(\{S_i\}_{i=1}^{I}$, each with a respective s_{ij} for doctor j, we can formulate expected clientele and granted sick leaves for j in a manner very much alike the previous section (indeed, as a generalization):

$$Q_j(\bar{\kappa_j}, \bar{\kappa}_{-j}) \; = \; \int_{\tau/V_j}^{\infty} \boldsymbol{s_{ij}} \, dF(k) + \int_{\min\{\bar{\kappa_j}, \tau/V_j\}}^{\tau/V_j} \int_{\tilde{\gamma}(k)}^{\infty} \boldsymbol{s_{ij}} \, dG(\gamma) \, dF(k) \tag{S.1}$$

$$X_j(\bar{\kappa_j}, \bar{\kappa}_{-j}) \; = \; \int_{\bar{\kappa_j}}^{\infty} \int_{\tilde{\gamma}(k)}^{\infty} \boldsymbol{s_{ij}} \, dG(\gamma) \, dF(k) \tag{S.2}$$

As before, the left term in equation (S.1) includes patients who would visit j even without sick leave, whereas the right term those who only do so because they expect one.

Notice that we now present Q_j and X_j not only in terms of the threshold $\bar{\kappa_j}$ chosen by doctor j, but also in terms of the thresholds of the other $J-1$ doctors, which we abreviate as $\bar{\kappa}_{-j}$. Whereas before doctors were simply alloted a given number of patients, now they will *compete* for them, as patients' S_i strategy will consider the whole of $(\{\bar{\kappa_j}\}_{j=1}^{J}$ when considering which physician(s) they will visit with positive probability.

b Some analytic results

We shall further specify the form S_i will take. Consider now a $1 \times J$ vector U_i, where each component $u_{i1}, ..., u_{iJ}$ indicates the utility patient i expects from a visit to doctors $1, ..., J$ respectively, corresponding to $U_i(V_j, \bar{\kappa_j})$ for each doctor j. As a shorthand, we shall write $u_{i,-j}$ to indicate the $J-1$ components of U_i excluding u_{ij}.

In the models considered, each component s_{ij} of S_i will be defined as:

$$s_{ij} \equiv g_i(u_{ij}, u_{i,-j})$$

where $g_i(\cdot)$ is a continuous function weakly increasing in the first argument u_{ij}, and weakly decreasing in the remaining arguments given by $u_{i,-j}$. Our model specifications will consist in giving this function $g_i(u_{ij}, u_{i,-j})$ a specific form.

As we show in the APPENDIX, such prerrequisites over s_{ij} are enough to prove most of the qualitative relations we expect to see regarding equilibrium aggregates and doctor strategies, but not enough to prove that $\bar{\kappa_j}$ and $\bar{\kappa}_{-j}$ are strategic complements – i.e., that for any $l \neq j$, $\frac{\partial^2 U_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}} \geq 0$

c The "implicit" search model

The "implicit" search model is a McFadden Logit choice model modified to give null probability visits to doctors which afford patient i non-positive utility, that way the free disposal requirement is fulfilled.

We call it the "implicit" search model because strictly speaking patient search is not formally included, yet one arrives at result qualitatively similar to specifications that do (like our own). The choice strategy noisily assigns positive probability to physicians who render i high utility, and the higher this u_{ij} expected utility is, the higher the probability of visit.

Instead of *explicitly* defining search and subsequent choice, probability of visit depends upon expected utility from doctor j, u_{ij}, and a weighing parameter λ. The lower the value of λ, the noisier patient "search" is: they give high assingment probability to sub-optimal –yet feasible– physician choices.

We define the components s_{ij} of the patient's strategy vector S_i as follows:

$$s_{ij} = \frac{\alpha_{ij}}{\sum_{k=1}^{J} \alpha_{ik}}, \quad \text{where} \quad \alpha_{ij} = \begin{cases} e^{\lambda u_{ij}}, & \text{if} \quad u_{ij} > 0 \\ 0, & \text{if} \quad u_{ij} = 0 \end{cases} \tag{S}$$

Unlike the "explicit" search model, the probability that patient i visits doctor j is *strictly* growing in κ_i, rather than being a piece-wise constant function with different levels. There is a discrete jump in probability at $\bar{\kappa_j}$, but elsewhere above 0 the function is smoothly increasing, rewarding physicians with high V_j.

The same lemmas as in the non-search model still hold.

Lemma 3.3. *No value in* $(\frac{\tau}{V_j}, \infty)$ *can be the optimal solution of (N.3).*

PROOF: APPENDIX

Lemma 3.4. *When* $\bar{\kappa_j} \in [0, \frac{\tau}{V_j}]$,

$$\frac{\partial Q_j}{\partial \bar{\kappa_j}} = \frac{\partial X_j}{\partial \bar{\kappa_j}}$$

PROOF: APPENDIX

Thus, the possible solutions for $(N.3)$ remain threefold: the corner solutions 0 and ∞, and an inner solution where the simplified FOC $R'(Q_j) = P'(X_j)$ holds.

Equilibrium is achieved in a similar fashion, though now taking into account the patients' S_i strategies.

Equilibrium in "implicit" search. *Given a doctor-patient market specified by* $(\{(\kappa_i, \gamma_i)\}_{i=1}^{I}, \{(V_j, R_j(\cdot), P_j(\cdot))\}_{i=1}^{J})$*, we define an equilibrium as a set of physician thresholds* $\{\bar{\kappa_j}\}_{j=1}^{J}$ *satisfying (N.3) and patient strategies* $\{S_i\}_{i=1}^{I}$ *as defined by (S), with their corresponding equilibrium (expected) patient demand and granted sick leave certificates* $\{(Q_j, X_j)\}_{j=1}^{J}$ *as described by (S.1), (S.2), respectively.*

d The "explicit" search model

What we call the "explicit" model is in which patient strategy S_i is explicitly the result of a sequential search algorithm on part of the patients. This is in line with Schnell (2017), though made trickier by the fact that our model allows for physician services apart from contingent "prescriptions" (in our case, sick leave certificates). In our model some patients *are* willing to visit physicians who don't intend to grant them sick leave, given a high enough benefit from the medical service proper, $V_j\kappa_i$.

FURTHER EXPLANATION REQUIRED, TO BE CONTINUED

4 Computation

Given a set of physicians $\{(V_j, \tau_j, R_j(\cdot), P_j(\cdot))\}_{j=1}^{J}$, the distribution functions for patient parameters $F(\kappa)$ and $G(\gamma)$, and one search model parameter z, which is λ in the Logit model, β in the sequential model, we develop an algorithm to compute market equilibrium.

Equilibrium aggregates are computed on the basis of Monte-Carlo matrix calculus. Set J as the number of physicians[3], I as the size of the sample drawn randomly from $F(\kappa)$ and $G(\gamma)$. For both models we define a class which can output a matrix S where each column is a patient's strategy vector S_i following that model, when given as input the arrayed set of physicians' quality and visit cost $\{(V_j, \tau_j)\}_{i=1}^{J}$, an arrayed set of patients $(\{(\kappa_i, \gamma_i)\}_{i=1}^{I}$, the model parameter z and a given vector of physician strategies $\{\bar{\kappa_j}\}_{j=1}^{J}$.

We input as "patients" our I samples from $F(\kappa)$ and $G(\gamma)$, then a $J \times I$ matrix U is computed, where each component u_{ji}[4] corresponds to the utility the sampled patient i would get from a visit to physician j. This step is the same for both classes.

What differs between both models is the computation of the matrix S of patient strate-

[3]Or, as we'll later interpret it, as the number of bins, where each bin j is a unique combination of $(V_j, \tau_j, R_j(\cdot), P_j(\cdot))$.

[4]As we have defined our matrices $J \times I$ for ease of visualization, we will refer to matrix components as x_{ji} in this section rather than x_{ij} as we do elsewhere in the paper.

gis out of the utility matrix U:

- Implicit search model (Logit): First, an 'α-matrix' is calculated over matrix U, where each α_{ji} is $e^{\lambda u_{ji}}$ if $u_{ji} > 0$ and 0 if not. Then, for each patient i, that is, for each column, each component s_{ji} of the S matrix is evaluated as $\alpha_{ji}/\sum_{k=1}^{J}\alpha_{ki}$.

- Explicit search model (Sequential): Recall equation (NUMBER) characterizing patient thresholds. We first compute the I-dimensional vector of the sampled patients' respective $\bar{U}_i$. Define U_i as the set $\{u_{ji}\}_{j=1}^{J}$ of utility patient i recieves from a visit to each physician. In matrix terms U_i would be the ith column of the $J \times I$ matrix U. The operation to compute each $\bar{U}_i$ is the following:

$$\bar{U}_i \equiv \underset{x \in U_i}{\arg\min} \left\| x - \frac{\beta}{1-\beta} \sum_{j=1}^{J} \left\{ \frac{\mathbb{1}[u_{ij} \geq x] \cdot (u_{ij} - x)}{\mathbb{1}[u_{ij} \geq x]} \right\} \right\| \tag{1}$$

  where the norm $\|\cdot\|$ is defined in $\mathbb{R}$ as simply the absolute value $|\cdot|$. This is to say, for each sampled patient i we evaluate x for each u_{ji} in U_i. In plain words, if patient i where to say: 'I will only visit physicians which grant me at least as much utility as physician j', the optimal choice of j and its respective u_{ji} would be the minimal in U_i for the evaluation of the absolute value in the left-hand side of (1). This is computationally less intensive than seeking to compute the exact root of (NUMBER), which would be a redundant exercise, because there's a discrete number of physicians above that mark, and selecting instead to use 'the lowest u_{ji} above the root of (NUMBER)' as threshold instead of the root proper would result in the same vector of strategies S_i for each patient[5]

[5]Granted, this is not strictly true, as in our formulation a 'u_{ji}' may be selected as threshold which is actually below the root proper in $\mathbb{R}$, but closer to it than the first one above it. The effect of this 'estimation noise' on our overall results is negligible.

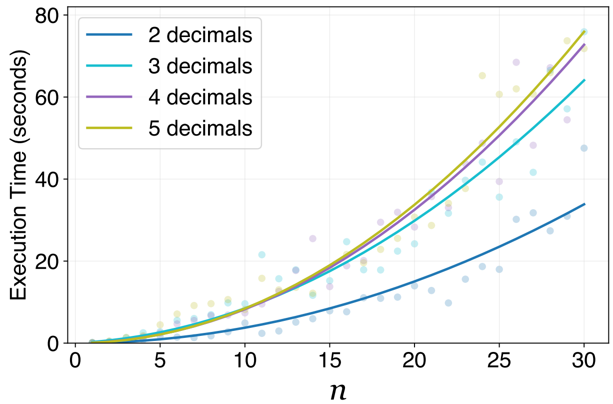
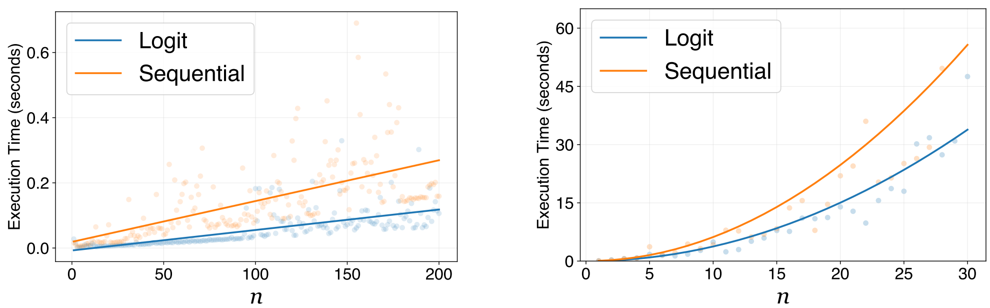


Figure 1: Coefficient β_t of *log_trade* over *log_distance* over the years, including FEs



(a) Coefficient β_t of *log_trade* over *for first case*

(b) Coefficient β_t of *log_trade* over *for second case*

Figure 2: Comparison of Coefficient β_t of *log_trade* over *log_distance* over the years, including FEs

5 Data

PENDING.

6 Calibration

PENDING.

7 Counterfactuals & Comments

PENDING.

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8 Appendix

8.1 The cause of the "strategic effect"

The main difference between our equilibrium and that of Schnell (2017) is the presence of a "strategic effect", wherein doctor j takes into account the behavior of other doctors in the selection of her own $\bar{\kappa_j}$. Of the different modifications we made to her framework, we'll argue it's the absence of *additive separability* across patients in physician utility $U_j(\cdot)$ which accounts for this.

In our context of unbounded maximization by doctor j, the absence of additive separability implies she can't consider each patient individually when it comes to whether she's willing to allow (or induce) their visit, as such a decision is no longer independent of other patients' visits; the marginal utility of an additional patient is dependent on the aggregate of clients up to that point, both in terms of the visit itself as well as in the number of sick leave certificates granted up to that point.

Let's illustrate this point. Consider for a moment a finite number of patients $1, ..., k$, where each patient is inputed directly as an argument in our doctor j's $U_j(\cdot)$, like so: $U_j(1, ..., k)$. If $U_j(\cdot)$ has the property of additive separability, this means it may be reformulated like so:

$$U_j(1, ..., k) = v_{j1}(1) + ... + v_{jk}(k) = \sum_{i=1}^{k} v_{ji}(i)$$

Unconstrained optimization in this context implies she's willing to see any patient whose $v_{ji}(i)$ is non-negative, such that her optimal level of utility is:

$$U_j^*(1, ..., k) = \sum_{i:\ v_{ji}(i) \geq 0} v_{ji}(i)$$

In the context of our doctor-patient model, where physician utility is increasing in κ_i, selection is achieved by the doctor by choosing a κ_j^* which excludes all patients i whose $\kappa_i < \kappa_j^*$. Supposing our patients are well-ordered in κ_i, the choice of such a κ_j^* would be one where the marginal consumer i affords a non-negative $v_{ji}(i)$, and the inframarginal consumer $i+1$ fulfills $v_{j,i+1}(i+1) < 0$. Ignoring for a moment that patients themselves have a *choice* of visiting – depending upon a second dimension γ –, we would then have:

$$U_j^*(1, ..., k) = \sum_{i:\ \kappa_i \geq \kappa_j^*} v_{ji}(i)$$

If instead of a discrete set we consider a mass of consumers $\mathcal{I}$ characterized by their level of κ_i, and we make the simplifying assumption that v_{ji} takes on the same form v_j for every $i \in \mathcal{I}$, our $U_j(\cdot)$ could be expressed as:

$$U_j^*(\mathcal{I}) = \int_{\kappa_j^*}^{\infty} v_j(k)\, dF(k)$$

$U_j^*(\mathcal{I})$ represents the optimal value of $U_j(\mathcal{I})$, where doctor j only sees patients who provide her with non-negative marginal utility, i.e. such that $\kappa_i \geq \kappa_j^*$. We can find this optimal value κ_j^* by looking at *threshold* equilibria, where doctor utility is also dependent on the threshold $\bar{\kappa_j}$ over which she's willing to see patients:

$$U_j(\mathcal{I}, \bar{\kappa_j}) = \int_{\bar{\kappa_j}}^{\infty} v_j(k)\, dF(k)$$

The optimal value of threshold $\bar{\kappa_j}$ is κ_j^*. Assuming $U_j(\cdot)$ is twice differentiable and concave in $\bar{\kappa_j}$, such a solution may be arrived at through the FCO:

$$\frac{\partial U_j(\mathcal{I})}{\partial \bar{\kappa_j}} \equiv -v_j(\bar{\kappa_j}) f(\bar{\kappa_j}) = 0$$

Solving for this would yield $\bar{\kappa_j} = \kappa_j^*$.

Schnell (2017) is an example of just such a treatment, which specifies the physician's additive utility by patient in the following manner:

$$v_{ji}(\kappa_i) \equiv R_j + \beta_j h(\kappa_i)$$

where R_j is a parameter standing for revenue by visit, and $\beta_j h(\kappa_i)$ represents the physician's "altruistic" utility over the health impact of a prescription drug to a patient with "pain level" κ_i.

The optimal threshold κ_j^* is then obtained out of the maximization[6]:

$$\max_{\bar{\kappa_j}} \int_{\bar{\kappa_j}}^{\infty} R_j + \beta_j h(k)\, dF(k)$$

Which gives out the following FOC:

$$R_j = -\beta_j h'(\bar{\kappa_j})$$

which, as is immediately apparent, doesn't depend upon the behavior of other doctors —more specifically, *their* choice of $\bar{\kappa_j}$. It merely establishes that, at the threshold, marginal benefit by patient –revenue R_j– must equal marginal "cost" –altruistic "cost" $\beta_j h(\bar{\kappa_j})$–. A way to interpret this is that doctors will be willing to see any and all patients which render them positive marginal utility, unconcerned with their total market share, and thus, what other physicians may do to take away clientele.

Such a treatment is rendered inviable by our choice of utility function. Schnell's parameter of revenue would in our model imply the linearity of our revenue *function* $R_j(\cdot)$ and our $P(\cdot)$ function over *aggregate* licenses granted. Strict convexity of $P(\cdot)$ would forestall its formulation as Schnell-like $\beta_j h'(\bar{\kappa_j})$ terms for each patient, because

[6] Once again, ignoring patient choice and γ,. We're presenting a *simplified* version for illustrative purposes.

the impact on doctor j's utility in granting patient i a license would no longer independent from the granting of licenses of other patients. Likewise, strict concavity of $R_j(\cdot)$ belies a simple "r" parameter such that $R_j'(Q_j) = rQ_j$.

More formally, when $U_j(1, ..., k)$ *isn't* additively separable across patients, the value κ_j^* such that if $\kappa_i \geq \kappa_j^*$ patient i provides positive marginal utility, isn't independent of current clientele, because what before was a properly defined object, marginal utility by patient i, $v_j(\kappa_i)$, can no longer be so identified. The marginal utility i provides to j as the kth client (assuming some order over clients) is not necessarily the same he'd provide as the $k+1$th client, and so, as the kth client he could provide 0 utility, impliying $\kappa_i = \kappa_j^*$, whereas as the $k+1$th he could be inframarginal, such that $\kappa_i < \kappa_j^*$.

κ_j^* is not longer independent of clientele mass $\mathcal{I}$ as before, but a function of it, $\kappa_j^*(\mathcal{I})$. More specifically, in the models we consider it will depend on the *cardinality* of current clientele, $|\mathcal{I}|$, such that our physician's choice of marginal consumer will depend on her aggregate level of patient demand, where before it didn't. This effect is introduced through the *strict non-linearity* of our physician utility function $U_j(\cdot)$, either through the strict concavity of $R_j(\cdot)$ over expected patient demand Q_j, or the strict convexity of $P_j(\cdot)$ over total expected sick leaves granted.

When either of those is the case, $aggregate$ levels enter into the equation and form part of the optimality condition. Our general FOC reflects this:

$$R_j'(Q_j)\frac{\partial Q_j}{\partial \bar{\kappa_j}} = P_j'(X_j)\frac{\partial X_j}{\partial \bar{\kappa_j}} \tag{2}$$

where either or both of $R_j'(\cdot)$ and $P_j'(\cdot)$ are a non-constant function over aggregates.

When such aggregates, either Q_j or X_j, come into play, physicians come to consider their *market share*, which doesn't depend exclusively on their own choice of $\bar{\kappa_j}$. Each patient's strategy S_i is constructed taking into account the whole of $\{(V_j, \bar{\kappa_j})\}_{i=1}^{J}$.

Imagine equation (2) holds for some doctor j, and some doctor $l \neq j$ decides to lower her $\bar{\kappa_l}$, enough that it changes the value of s_{ij} for some mass of clients. The value of Q_j would then change, and therefore that of $R_j'(Q_j)$. If $P_j'(X_j)$ didn't vary by the same amount, (2) would no longer be an equality, leading j to modify her choice of $\bar{\kappa_j}$ to make equality hold.

This intelligible line of reasoning links the presence of a "strategic effect" to the non-additive separability of $U_j(\cdot)$: doctor j takes into account other physicians' strategy in her own choice of $\bar{\kappa_j}$ because of the present of *aggregate* amounts of clientele in her optimality conditions, which is so because utility isn't additively separable across clients.

8.2 Schnell (2017) with Logit choice

To prove our point that it is additive separability which accounts for a possible strategic effect, we reformulate Schnell (2017) in the manner of a McFadden Logit – with some quirks.

The way in which Schnell devises her physician's utility formula is implicitly Bernoulli-like:

$$\int_{\kappa}\int_{\gamma} u(\kappa,\gamma)\cdot p(\kappa,\gamma)\, dG(\gamma)dF(\kappa)$$

where $u(\kappa,\gamma)$ is the utility function of the physician from patients characterised by a given (κ,γ) tuple, and $p(\kappa,\gamma)$ stands for the proportion of clients atomized in such a tuple the physician expects to have visit her.

$u(\kappa,\gamma)$ we'll leave as Schnell defined it: $\beta_j h(\kappa)+R_j$. $p(\kappa,\gamma)$ lends itself nicely as the s_{ij} we have been using thus far, which implicitly depends on the (κ_i,γ_i) tuple which characterizes patient i:

$$\int_{\kappa}\int_{\gamma} [\beta_j h(\kappa)+R_j]\cdot s_{ij}(\kappa,\gamma)\, dG(\gamma)dF(\kappa)$$

In our modeling section, our "implicit" search model was a left-censored Logit choice model, which gave null probability of assignment to physicians from which patient i expected non-positive utility. It was defined as:

$$s_{ij}=\frac{\alpha_{ij}}{\sum_{k=1}^{J}\alpha_{ik}},\quad \text{where}\quad \alpha_{ij}=\begin{cases} e^{\lambda u_{ij}}, & \text{if} \quad u_{ij}>0 \\ 0, & \text{if} \quad u_{ij}=0 \end{cases}$$

Schnell's opioid-focused physician model provides an additional simplification: patients which aren't given a drug prescription, i.e. those such that $\kappa_i<\bar{\kappa_j}$, don't garner positive utility from a visit. As such, in the original model as in this reformulation, patients "below" $\bar{\kappa_j}$ have a null probability of visit, such that the inferior limit of integration for κ is $\bar{\kappa_j}$.

As for γ, the limit of the inner integral remains the same as in Schnell in the absence of a secondary market, a value of γ_i such that $u_{ij}=0$, i.e. $h(\kappa_i)+\gamma_i-\tau^d-\tau^o=0$.[7]

The double-integral maximized as physician utility is then as follows:

$$\max_{\bar{\kappa_j}}\int_{\bar{\kappa_j}}^{\infty}\int_{\tau^d-\tau^o-h(\kappa)}^{\infty}[\beta_j h(\kappa)+R_j]\cdot\frac{e^{\lambda u_{ij}}}{\sum\limits_{k:\, u_{ik}>0} e^{\lambda u_{ik}}}\, dG(\gamma)dF(\kappa)$$

The FOC of such an equation is:

$$[\beta_j h(\bar{\kappa_j})+R_j]\int_{\tau^d-\tau^o-h(\bar{\kappa_j})}^{\infty}\frac{e^{\lambda u_{ij}}\mid\bar{\kappa_j}}{\sum\limits_{k:\, u_{ik}>0} e^{\lambda u_{ik}}\mid\bar{\kappa_j}}\, dG(\gamma)f(\bar{\kappa_j})=0 \tag{3}$$

[7] Schnell splits the costs a patient will face into costs of visit (τ^d), cost of purchase (τ^o) and search cost (τ^s)

Where we use "$| \ \bar{\kappa_j}$" to clumsily indicate that we integrate γ over patients where $\kappa_i = \bar{\kappa_j}$. As an integral over a strictly positive value and the atom of a density function, respectively, the factors B and C in the equation are non-negative. For this reason, the only way for (3) to be fulfilled is the following condition:

$$R_j = -\beta_j h(\bar{\kappa_j}) \tag{4}$$

This is the same result as in the original model in ?? in the absence of a secondary market, a condition which stipulates that marginal utility in $\bar{\kappa_j}$ must be 0, i.e. that revenue (R_j) must equal "altruistic" loss ($\beta_j h(\bar{\kappa_j})$), which pre-supposes that the marginal patient granted prescription opioids suffers a net loss in utility (the negative externalities outweigh its medical benefit as a palliative).

Condition (4) means a value κ_j is chosen irrespective of the strategies employed by other doctors, it quite literally doesn't enter into the equation. As we have argued, additive separability over clients means the *total* demand for physician j's services doesn't influence marginal utility by a given patient i, and so has no sway in the optimality condition. Doctor j states simply that she won't grant a prescription (or sick leave, as in our case) below $\bar{\kappa_j}$, *come what may*. In order for her to care about *aggregate* values, like her total expected demand, making her care about her market share and thus about the strategies of other doctors, *strict non-linearity* must be introduced into her utility function.

8.3 On the strategic complementarity of $\bar{\kappa_j}$'s

We had defined the components s_{ij} of S_i as:

$$s_{ij} \equiv g_i(u_{ij}, u_{i,-j})$$

where $g_i(\cdot)$ is a continuous function weakly increasing in the first argument u_{ij}. Our defining s_{ij} this way has two interlinked corollaries:

Corollary 8.1.

$$s_{ij} \mid \kappa_i < \bar{\kappa_j} \quad \leq \quad s_{ij} \mid \kappa_i \geq \bar{\kappa_j} \quad \textit{(with strict inequality if } \gamma_i > 0\textit{)}.$$

Proof. For a fixed value of V_j and κ_i, for all i, the value of $U_i(V_j, \bar{\kappa_j})$ is $V_j\kappa_i - \tau_j$ if $\kappa_i < \bar{\kappa_j}$, and $\gamma_i + V_j\kappa_i - \tau_j$ if $\kappa_i \geq \bar{\kappa_j}$, where $\gamma_i \geq 0$. Given that s_{ij} is weakly increasing in u_{ij}, and we see that u_{ij} is weakly higher if $\kappa_i \geq \bar{\kappa_j}$, the corollary follows. □

Corollary 8.2.

$$\frac{\Delta s_{ij}}{\Delta \bar{\kappa_j}} \leq 0 \qquad\qquad \frac{\Delta s_{ij}}{\Delta \bar{\kappa_l}} \geq 0, \forall l \neq j$$

Proof. From the argumentation in corollary 1 follows that u_{ij} is weakly decreasing in $\bar{\kappa_j}$, $\forall j$. Take some $l \neq j$, then s_{ij} is defined in turn as weakly decreasing in u_{il}, which implies it is increasing in $\bar{\kappa_l}$. □

Both corollaries hinge upon our definition of $U_i(\cdot)$ as a step function over κ_i, such that it is discontinuous at $\kappa_i = \bar{\kappa_j}$, where there's a discrete jump of magnitude $\gamma_i \geq 0$.

Corollary 8.3.

$$\frac{\partial Q_j}{\partial \bar{\kappa_j}}, \frac{\partial X_j}{\partial \bar{\kappa_j}} \leq 0 \qquad\qquad \frac{\partial Q_j}{\partial \bar{\kappa_l}}, \frac{\partial X_j}{\partial \bar{\kappa_l}} \geq 0, \forall l \neq j$$

Proof. We recall our function of patient demand for physician j is:

$$Q_j(\bar{\kappa_j}, \bar{\kappa}_{-j}) \;=\; \int_0^\infty \int_0^\infty s_{ij}(k,\gamma)\, dG(\gamma)\, dF(k)$$

Were physician j to increase her threshold from $\bar{\kappa_j}$ to $\bar{\kappa_j}' = \bar{\kappa_j} + \epsilon, \epsilon > 0$, for the mass of patients whose $\kappa_i \in [\bar{\kappa_j}, \bar{\kappa_j} + \epsilon)$, utility would weakly decrease and thus so would their s_{ij}, falling to $s_{ij}' \leq s_{ij}$ (see first corollary, think of s_{ij}' as $s_{ij} \mid \kappa_i < \bar{\kappa_j}$ as opposed to $s_{ij} \mid \kappa_i \geq \bar{\kappa_j}$).

The difference that would make for patient demand *caeteris paribus* given the $\bar{\kappa}_{-j}$ strategies of the other physicians (ommitted in our notation) is:[8]

$$\Delta Q_j = Q_j(\bar{\kappa_j} + \epsilon) - Q_j(\bar{\kappa_j}) = \int_{\bar{\kappa_j}}^{\bar{\kappa_j}+\epsilon} \int_0^\infty \{s_{ij}' - s_{ij}\}\, dG(\gamma)\, dF(k)$$

By the definition of a partial derivative:[9]

$$\begin{aligned}\frac{\partial Q_j}{\partial \bar{\kappa_j}} &= \lim_{\epsilon \to 0} \frac{\int_{\bar{\kappa_j}}^{\bar{\kappa_j}+\epsilon} \int_0^\infty \{s_{ij}' - s_{ij}\}\, dG(\gamma)\, dF(k)}{\epsilon} \\ &= \int_0^\infty \{s_{ij}(\bar{\kappa_j}, \gamma)' - s_{ij}(\bar{\kappa_j}, \gamma)\}\, dG(\gamma)\, f(\bar{\kappa_j})\end{aligned}$$

where the derivative is negative if s_{ij}' is strictly lower than s_{ij}, meaning a shift in leniency would affect the utility and optimal strategy of some positive mass of patients around $\bar{\kappa_j}$; or null, if $s_{ij}' = s_{ij}$.

In a similar vein, consider some physician $l \neq j$ shifting her threshold from $\bar{\kappa_l}$ to $\bar{\kappa_l}' = \bar{\kappa_l} + \epsilon, \epsilon > 0$. The fall in patient strategies in $[\bar{\kappa_j}, \bar{\kappa_j} + \epsilon)$ from s_{il} to $s_{il}^\star \leq s_{il}$ could provide a windfall for j (and all other physicians), raising s_{ij} to $s_{ij}^\star \geq s_{ij}$ (see previous corollary). In that case:

$$\Delta Q_j = \int_{\bar{\kappa_l}}^{\bar{\kappa_l}+\epsilon} \int_0^\infty \{s_{ij}^\star - s_{ij}\}\, dG(\gamma)\, dF(k)$$

[8] Besides simplifying $Q_j(\bar{\kappa_j}, \bar{\kappa}_{-j})$ into $Q_j(\bar{\kappa_j})$, we also forego the arguments of $s_{ij}(k,\gamma)$, as we've done many times already, to avoid notation clutter.

[9] The notation for $s_{ij}(\bar{\kappa_j}, \gamma)'$ is to be taken to mean the strategy $s_{ij}' \leq s_{ij}$ of the mass of patients at different levels of γ atomized at $\kappa_i = \bar{\kappa_j}$.

thus rendering:

$$\frac{\partial Q_j}{\partial \bar{\kappa_l}} = \lim_{\epsilon \to 0} \frac{\int_{\bar{\kappa_l}}^{\bar{\kappa_l}+\epsilon} \int_0^\infty \{s_{ij}{}' - s_{ij}\} \, dG(\gamma) \, dF(k)}{\epsilon}$$
$$= \int_0^\infty \{s_{ij}(\bar{\kappa_l}, \gamma)^\star - s_{ij}(\bar{\kappa_l}, \gamma)\} \, dG(\gamma) \, f(\bar{\kappa_l})$$

As for X_j, the steps taken are almost the same, only that after raising the threshold by ϵ the amount of sick leaves granted goes around $\bar{\kappa_j}$ goes to 0, thus giving out:

$$\frac{\partial X_j}{\partial \bar{\kappa_j}} = -\int_0^\infty s_{ij}(\bar{\kappa_j}, \gamma) \, dG(\gamma) \, f(\bar{\kappa_j})$$

such that it is simply required that j has positive demand for the mass of patients whose $\kappa_i = \bar{\kappa_j}$ for this derivative to be strictly negative, else it is null.

Likewise:

$$\frac{\partial X_j}{\partial \bar{\kappa_l}} = -\int_0^\infty s_{ij}(\bar{\kappa_l}, \gamma) \, dG(\gamma) \, f(\bar{\kappa_l})$$

□

Before dealing with PROPOSITION we need one final corollary.

Corollary 8.4.

$$\frac{\partial^2 Q_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}} = 0 \quad \textit{and} \quad \frac{\partial^2 X_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}} = 0 \ , \quad \forall j \textit{ and } l, \ \bar{\kappa_j} \neq \bar{\kappa_l}$$

Proof. Our proof will be heuristic. Consider

$$\frac{\partial Q_j}{\partial \bar{\kappa_j}} = \int_0^\infty \{s_{ij}(\bar{\kappa_j}, \gamma)' - s_{ij}(\bar{\kappa_j}, \gamma)\} \, dG(\gamma) \, f(\bar{\kappa_j})$$

Patient strategies s_{ij} are taken at the atom $\kappa_i = \bar{\kappa_j}$. For any other physician l such that $\bar{\kappa_l} \neq \bar{\kappa_j}$, an infinitesimally small change to $\bar{\kappa_l}$ doesn't affect the utility of patients at $\bar{\kappa_j}$, who will remain above or below l's threshold same as before. Thus their strategy s_{ij} also remains unaffected. If $\bar{\kappa_l}$ *does* equal $\bar{\kappa_j}$, then the following limit is not properly defined:

$$\lim_{\epsilon \to 0} \frac{\frac{\partial Q_j(\kappa_l + \epsilon)}{\partial \bar{\kappa_j}} - \frac{\partial Q_j(\kappa_l)}{\partial \bar{\kappa_j}}}{\epsilon}$$

because as ϵ approaches 0 from the right the limit is weakly positive (≥ 0), whereas approaching from the left it's strictly 0. Meaning the derivative $\frac{\partial^2 Q_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}}$ doesn't exist for j and $l \neq j$ when $\bar{\kappa_j} = \bar{\kappa_l}$.

□

PROOF OF PROPOSITION

Consider some two physicians j, l, such that $\bar{\kappa_j} \neq \bar{\kappa_l}$. If we breakdown the mixed partial derivative of U_j over $\bar{\kappa_j}$ and $\bar{\kappa_l}$, we get:

$$\begin{aligned}
\frac{\partial^2 U_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}} =& \underbrace{R''(Q_j(\bar{\kappa_j}, \bar{\kappa}_{-j}))}_{\leq 0} \cdot \underbrace{\frac{\partial Q_j}{\partial \bar{\kappa_l}}}_{\geq 0} \cdot \underbrace{\frac{\partial Q_j}{\partial \bar{\kappa_j}}}_{\leq 0} + \underbrace{R'(Q_j(\bar{\kappa_j}, \bar{\kappa}_{-j}))}_{\geq 0} \underbrace{\frac{\partial^2 Q_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}}}_{=0} \\
&- \underbrace{P''(X_j(\bar{\kappa_j}, \bar{\kappa}_{-j}))}_{\geq 0} \cdot \underbrace{\frac{\partial X_j}{\partial \bar{\kappa_l}}}_{\geq 0} \cdot \underbrace{\frac{\partial X_j}{\partial \bar{\kappa_j}}}_{\leq 0} - \underbrace{P'(X_j(\bar{\kappa_j}, \bar{\kappa}_{-j}))}_{\geq 0} \underbrace{\frac{\partial^2 X_j}{\partial \bar{\kappa_j} \partial \bar{\kappa_l}}}_{=0} \\
=& \underbrace{R''(Q_j(\bar{\kappa_j}, \bar{\kappa}_{-j})) \cdot \frac{\partial Q_j}{\partial \bar{\kappa_l}} \cdot \frac{\partial Q_j}{\partial \bar{\kappa_j}}}_{\geq 0} - \underbrace{P''(X_j(\bar{\kappa_j}, \bar{\kappa}_{-j})) \cdot \frac{\partial X_j}{\partial \bar{\kappa_l}} \cdot \frac{\partial X_j}{\partial \bar{\kappa_j}}}_{\leq 0} \quad \geq 0
\end{aligned}$$

POOSITIVE MIXED DERIVATIVE IS STRATEGIC COMPLEMENTARITY

8.4 Proof Sequential

We follow the argument of (QUOTE). Let for the moment $\tilde{F}_i(U) = \Pr\{u_{ij} \leq U\}$ be the distribution of utility patient i recieves from doctor visits, with $\tilde{F}_i(0) = 0$, $\tilde{F}_i(B) = 1$, for $B < \infty$. Each period, the patient draws an 'offer' U from $\tilde{F}_i$, which is to say, his search lead him to a physician j such that $u_{ij} = U$. He may either accept and have physician j assigned to him, recieving utility U per period henceforth, or look for a new physician next period.

Let u_t be the patient's utility for period t, where it's 0 if he doesn't visit a physician and U if he has accepted an assigned physician which renders him utility $u_{ij} = U$. This patient will seek to maximize $\mathbb{E}[\sum_{t=0}^{\infty} \beta^t u_t]$, where $0 < \beta < 1$ is his discount factor.

Let $V(U)$ be the expected value of $\sum_{t=0}^{\infty} \beta^t u_t$ of a patient with an offer U in hand. Assuming no recall, under optimal behavior the value function $V(U)$ satisfies the Bellman equation:

$$V(U) = \max \left\{ \frac{U}{1-\beta}, \;\; \beta \int V(U')\, d\tilde{F}_i(U') \right\} \tag{5}$$

The patient chooses an optimal value $\bar{U}_i$, such that offers $U \geq \bar{U}_i$ are accepted (the visit takes place). As such, the solution takes on the following form:

$$V(U) = \begin{cases} \frac{\bar{U}_i}{1-\beta} + \beta \int V(U')\, d\tilde{F}_i(U') & \text{if} \quad U \leq \bar{U}_i, \\ \frac{U}{1-\beta} & \text{if} \quad U \geq \bar{U}_i. \end{cases} \tag{6}$$

Using equation (6), we can convert the functional equation (5) into an ordinary equation in the reservation utility $\bar{U}_i$. Evaluating $V(\bar{U}_i)$ and using equation (6), we have:

$$\frac{\bar{U}_i}{1-\beta} = \beta \int_0^{\bar{U}_i} \frac{\bar{U}_i}{1-\beta}\, d\tilde{F}_i(U') + \beta \int_{\bar{U}_i}^{B} \frac{U'}{1-\beta}\, d\tilde{F}_i(U')$$

or

$$\begin{aligned}\frac{\bar{U}_i}{1-\beta} \int_0^{\bar{U}_i} d\tilde{F}_i(U') + \frac{\bar{U}_i}{1-\beta} \int_{\bar{U}_i}^{B} d\tilde{F}_i(U') \\ = \beta \int_0^{\bar{U}_i} \frac{\bar{U}_i}{1-\beta}\, d\tilde{F}_i(U') + \beta \int_{\bar{U}_i}^{B} \frac{U'}{1-\beta}\, d\tilde{F}_i(U')\end{aligned}$$

or

$$\bar{U}_i \int_0^{\bar{U}_i} d\tilde{F}_i(U') = \frac{1}{1-\beta} \int_{\bar{U}_i}^{B} \left(\beta U' - \bar{U}_i\right) d\tilde{F}_i(U').$$

Adding $\bar{U}_i \int_{\bar{U}_i}^{B} d\tilde{F}_i(U')$ to both sides gives

$$\bar{U}_i = \frac{\beta}{1-\beta} \int_{\bar{U}_i}^{B} \left(U' - \bar{U}_i\right) d\tilde{F}_i(U') \tag{7}$$

Finally, for our purposes, we discretize $\int_{\bar{U}_i}^{B} \left(U' - \bar{U}_i\right) d\tilde{F}_i(U')$ as a summation over the countable set of J doctors. The discrete equivalent of equation (7) is:

$$\bar{U}_i = \frac{\beta}{1-\beta} \sum_{j=1}^{J} \left\{ \frac{\mathbb{1}[u_{ij} \geq \bar{U}_i] \cdot (u_{ij} - \bar{U}_i)}{\mathbb{1}[u_{ij} \geq \bar{U}_i]} \right\} \tag{8}$$

Which is the function we say characterizes patient thresholds in the explicit search model.