SSY285 - Assignment M3

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Introduction

Consider the system shown in Figure 1 which could be described as a dynamic model of DC motor with flywheel.

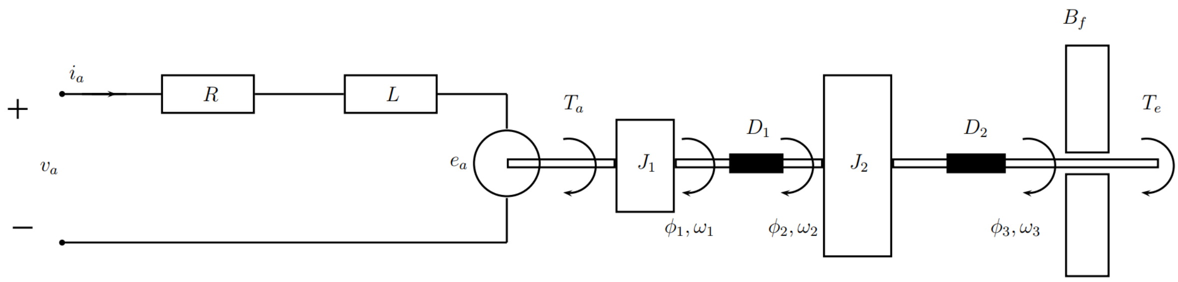

Figure 1: DC motor with flywheel

We get the following state-space matrics:

$$\underbrace{\begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \\ \dot{\omega}_1 \\ \dot{\omega}_2 \end{bmatrix}}_{\dot{x}(t)} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & \frac{D_2}{B} & -\frac{D2}{B} & 0 & 0 \\ -\frac{D_1}{J_1} & \frac{D_1}{J_1} & 0 & -\frac{K_E K_T}{J_1 R} & 0 \\ \frac{D_1}{J_2} & -\frac{D_1+D_2}{J_2} & \frac{D_2}{J_2} & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \omega_1 \\ \omega_2 \end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{B} \\ \frac{K_T}{J_1 R} & 0 \\ 0 & 0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} v_a \\ T_e \end{bmatrix}}_{u(t)} \quad (1)$$

The output equation is as follows:

$$\underbrace{\begin{bmatrix} \phi_2 \\ \omega_2 \end{bmatrix}}_{y(t)} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{C_1} \underbrace{\begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \omega_1 \\ \omega_2 \end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} v_a \\ T_e \end{bmatrix}}_{u(t)} \tag{2}$$

$$A_d = \begin{bmatrix} 0.4010 & 0.5964 & 0.0026 & 0.0004 & 0.0002 \\ 0.2076 & 0.7749 & 0.0175 & 0.0001 & 0.0009 \\ 0.0585 & 0.5686 & 0.3729 & 0.0000 & 0.0004 \\ -782.8328 & 773.7475 & 9.0853 & -0.0489 & 0.5964 \\ 342.5299 & -370.9589 & 28.4290 & 0.1491 & 0.7749 \end{bmatrix} \tag{3}$$

$$B_d = \begin{bmatrix} 0.0032 & 0.0003 \\ 0.0002 & 0.0032 \\ 0.0000 & 0.3168 \\ 4.4993 & 1.3088 \\ 0.5851 & 8.7634 \end{bmatrix} \tag{4}$$

Question a)

Since both noises added in are white noise with zero mean and uncorrelated, we can propose that the covariance matrix Q_w is diagonal corresponding to the variance of each noise input.

As required by the question context, the disturbance bounds correspond to a confidence interval of 99.7%, thus we can calculate variance through the following equation:

$$\frac{\sigma^2}{(x-\mu)^2} = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{+\infty} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} dx = 0.997 \tag{5}$$

where, μ is mean, σ^2 represents variance. As specified in the question, the voltage disturbances are bounded by $\pm 0.3V$ and the torque disturbances are estimated to be less than $\pm 10\%$ of maximum applied external torque.

Actually, in MATLAB, we can define function *calcVariance* to formulate variance:

```
% function
function sigma2_val = calcVariance(ub, realization_ub)
    syms sigma2 x
    eq = realization_ub == int(1/sqrt(2*pi*sigma2)*exp(-(x^2)/(2*sigma2)),x,-ub,ub);
    sigma2_val = double(solve(eq,sigma2));
    mu = 0;
```

```
    pd = makedist('Normal',mu,abs( sqrt(sigma2_val)));
    v_aw = linspace(-ub*2,ub*2,100);
    y = pdf(pd,v_aw);
end
```

We can then use the command $calcVariance$ to calculate variances:

```
sigma2_va    = calcVariance(0.3, 0.997);
sigma2_Te    = calcVariance(0.1, 0.997);
```

Therefore, Q_w:

$$Q_w = \begin{bmatrix} \sigma_{va}^2 & 0 \\ 0 & \sigma_{Te}^2 \end{bmatrix} = \begin{bmatrix} 0.0102 & 0 \\ 0 & 0.0011 \end{bmatrix} \tag{6}$$

The N matrix will be exactly the same as B_d since the system is linear and disturbances are directly added to the inputs.

$$N = \begin{bmatrix} 0.0032 & 0.0003 \\ 0.0002 & 0.0032 \\ 0.0000 & 0.3168 \\ 4.4993 & 1.3088 \\ 0.5851 & 8.7634 \end{bmatrix} \tag{7}$$

Question b)

Suppose the same assumptions as question a, with the same procedures, we can easily calculate the covariance matrix Q_v for the measurement disturbance vector v:

At first, calculate the variances for ϕ_2 and ω_2

```
sigma2_phi2 = calcVariance(0.02, 0.997);
sigma2_w2    = calcVariance(0.01, 0.997);
```

Then, we get the Q_v:

$$Q_v = \begin{bmatrix} \sigma_{\phi_2}^2 & 0 \\ 0 & \sigma_{\omega_2}^2 \end{bmatrix} = \begin{bmatrix} 4.5416 \times 10^{-5} & 0 \\ 0 & 1.1354 \times 10^{-5} \end{bmatrix} \tag{8}$$

Question c)

The system description equations are defined by:

$$x(k+1) = Ax(k) + Bu(k) + Nu(k) \tag{9}$$
$$y(k) = Cx(k) + v(k) \tag{10}$$

In the context of this report, the matrices are designated as follows: A corresponds to A_d as introduced earlier, B aligns with B_d as outlined in the introductory section, and C is synonymous with C_1 as previously defined. Additionally, the matrix N is explicitly defined in question a.

In MATLAB, we can then compute the discrete-time Kalman filter by:

```
% Calculate Kalman gain
sysmodel = ss(Ad, [doubleBd N], C1, 0, Ts)
Qw = diag([sigma2_va,sigma2_Te]); % input noise
Qv = diag([sigma2_phi2 sigma2_w2]); % output noise
% kest, kalman gain and covariance value
[kest,L_kalman,P] = kalman(sysmodel, Qw , Qv)
```

Then, we can get the observer gain L and covariance matrix P as follows:

```
L_kalman =
    0.0005    0.0009
    0.0005    0.0018
    0.0003    0.0113
   -0.0001    0.6970
   -0.0292    1.6755
P =
    0.0000    0.0000   -0.0000    0.0001    0.0001
    0.0000    0.0000    0.0000    0.0000    0.0000
   -0.0000    0.0000    0.0001    0.0011    0.0032
    0.0001    0.0000    0.0011    0.3504    0.0253
    0.0001    0.0000    0.0032    0.0253    0.1120
```

Furthermore, we can calculate the observer's eigenvalues as follows:

```
% Check observer stability (observer eigenvalues)
eig(Ad-L_kalman*C1)
```

Consequently, the eigenvalues are:

```
ans =
  -0.6883 + 0.0000i
   0.9995 + 0.0000i
   0.1438 + 0.5209i
   0.1438 - 0.5209i
  -0.0001 + 0.0000i
```

As we can see, all the observer's eigenvalues are located inside the unit circle, which means the observer is stable. Since the observer is stable, all the states in the system are detectable.

Question d)

We first establish the closed-loop controller in the following way:

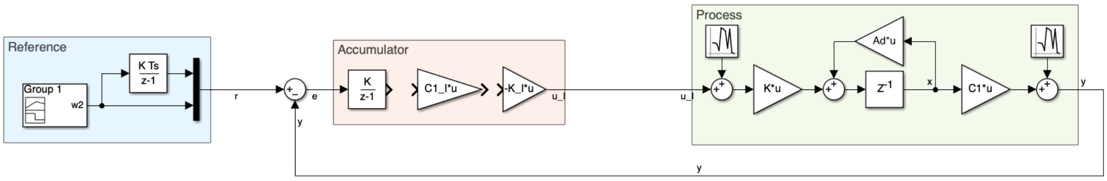

Figure 2: Simulink closed-loop control block

The step response is shown as follows:

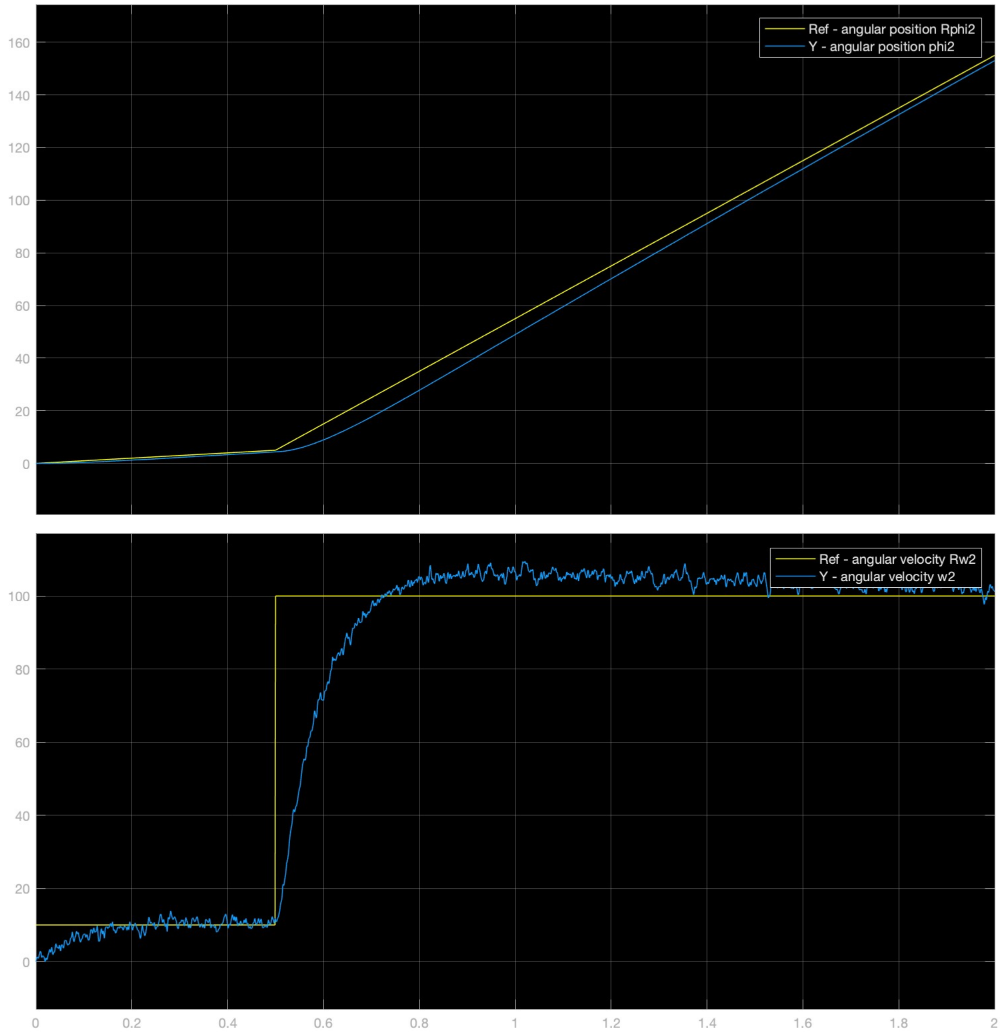

Figure 3: Step response

The inputs are shown as follows:

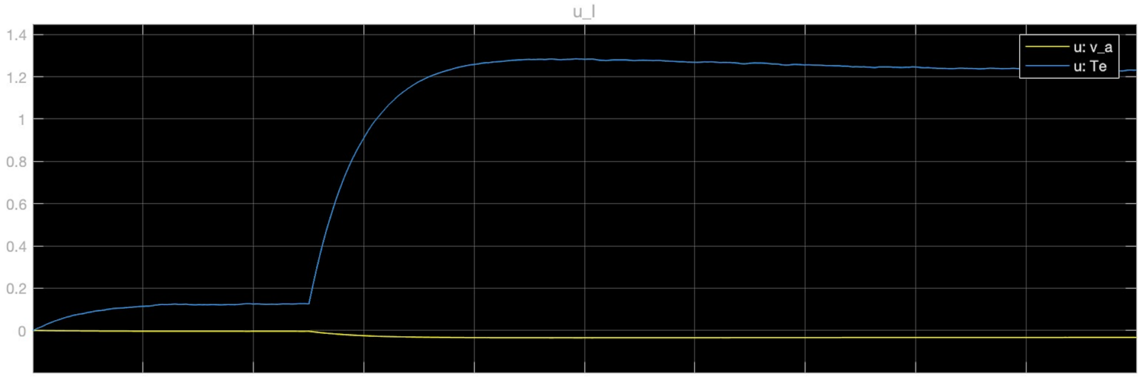

Figure 4: Inputs

We then establish the control with integral action: We first extend the state-space model with integral actions in continuous time as follows to allow us to integrate ω_2's error:

$$A_{ce} = \begin{bmatrix} A_c & 0 \\ -C_{1fd} & 0 \end{bmatrix} \text{and } B_{ce} = \begin{bmatrix} B_c \\ 0 \end{bmatrix} \tag{11}$$

where C_{1fb} means selecting ω_2 from x_e vector (fb means feedback):

$$C_{1fb} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

$$x_e = \begin{bmatrix} \phi_1 & \phi_2 & \phi_3 & \omega_1 & \omega_2 & x_I \end{bmatrix}^T \tag{13}$$

And C_{1e} as:

$$C_{1e} = \begin{bmatrix} C_{1fb} & 0 \end{bmatrix} \tag{14}$$

Moreover, weighting matrix Q_x and Q_u for LQ controller are choosing as:

```
Qx = diag([0 0 0 10 1000 10]); % set suitable parameters
Qu = diag([1000 0.1]);
```

In general, we need to discretize the augmented state-space model in this step, but we can directly use the following MATLAB command to establish our discrete-time LQR controller:

```
% A, B matrixs are CT matrixs, Ts is sampling time.
lqrd(Ac, Bc, Qx, Qu, Ts);
```

Then, the Simulink model with integral action and Kalman filter is established as follows:

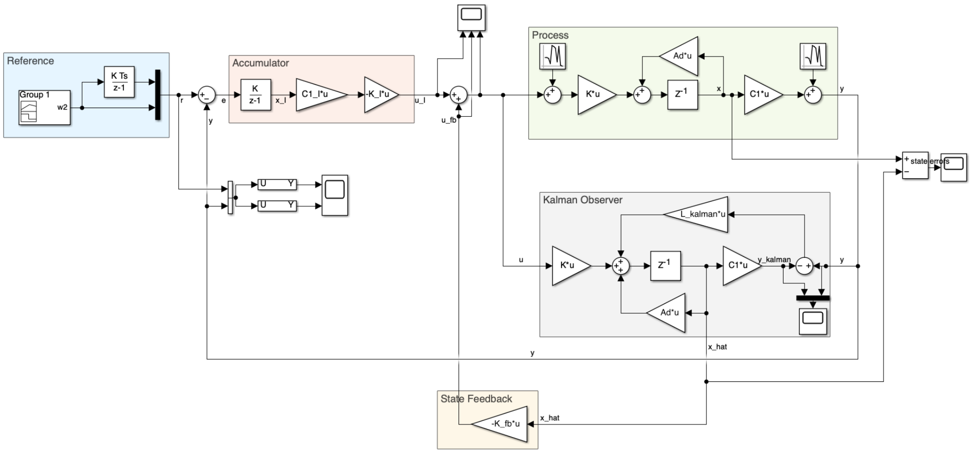

Figure 5: Simulink closed-loop control block with integral action and Kalman filter

The step response is shown as follows:

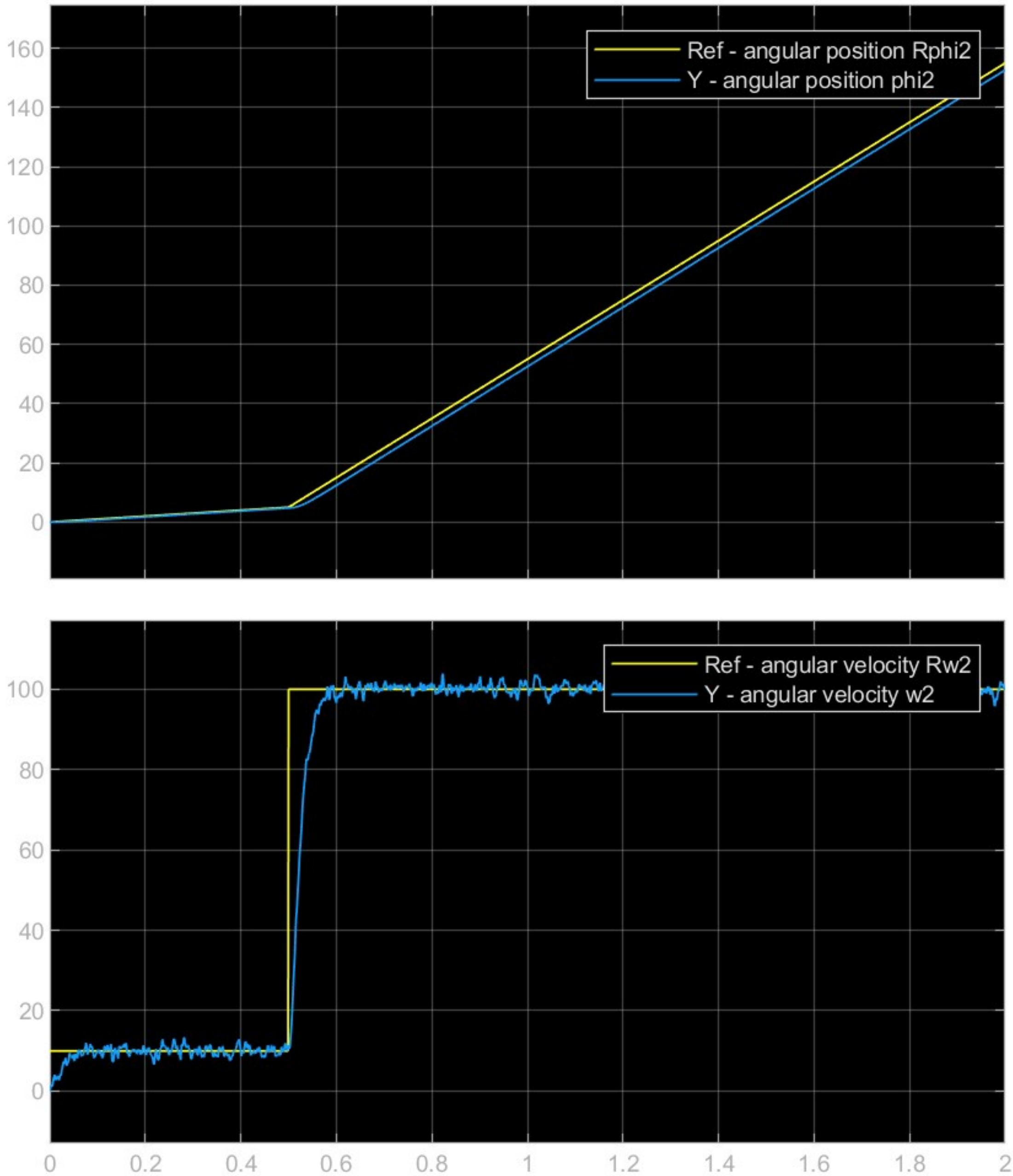

Figure 6: Step response

The inputs are shown as follows:

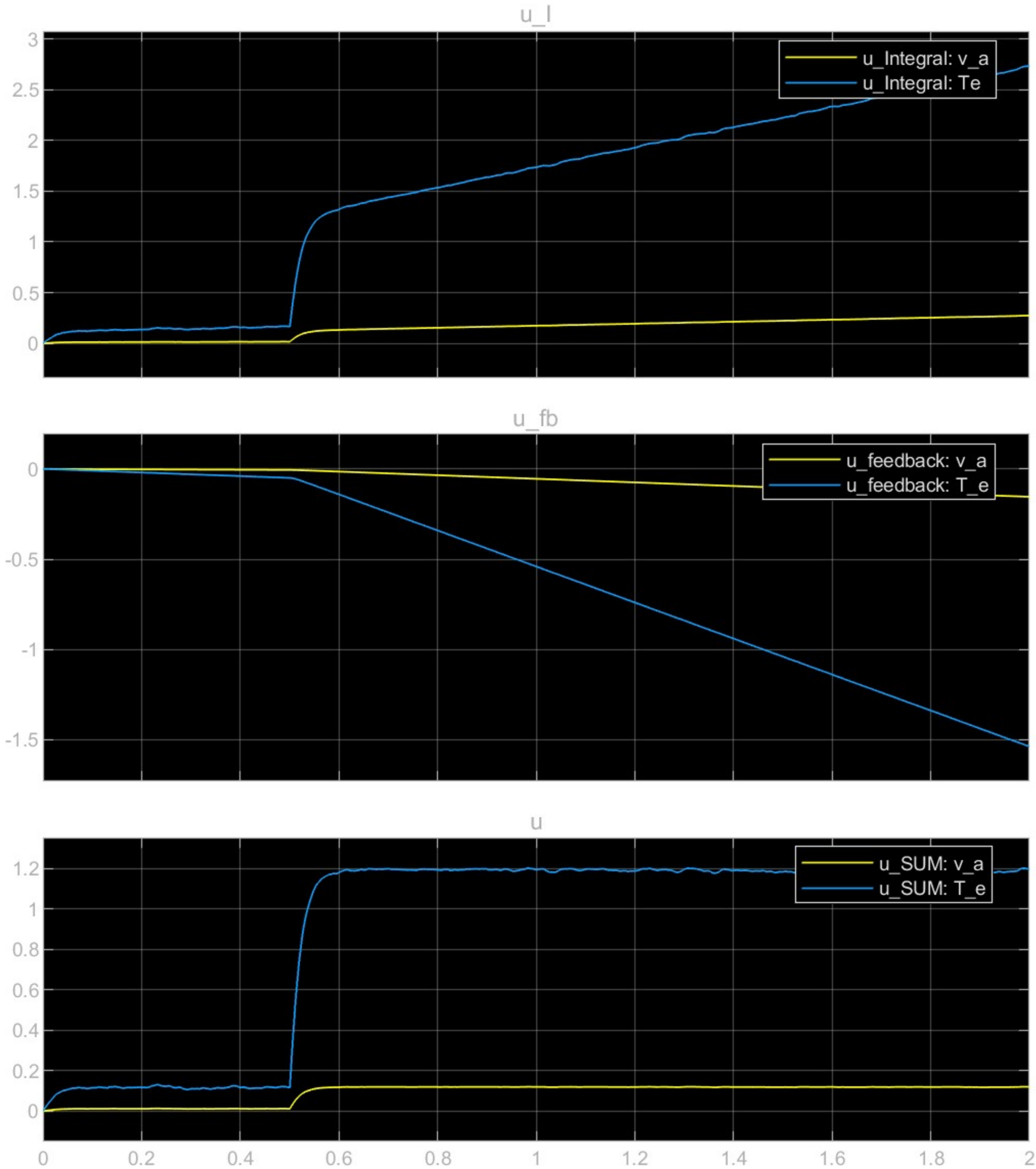

Figure 7: Inputs

Comparing the two different controllers above, we have the following conclusions:

- The integral action will lead to less error than only feedback.(Not only the offset error but also the transfer error)