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Weighted k-shell decomposition for complex networks based on potential edge weights

Bo Wei [a], Jie Liu [a], Daijun Wei [b], Cai Gao [a], Yong Deng [a,c,*]

[a] School of Computer and Information Science, Southwest University, Chongqing 400715, China

[b] School of Science, Hubei University for Nationalities, Enshi 445000, China

[c] School of Engineering, Vanderbilt University, TN 37235, USA

HIGHLIGHTS

- Existing methods consider the edges equally when designing the identifying method for the unweighted networks.
- We propose an edge weighting method based on adding the degree of its two end nodes and a weighted k-shell decomposition method.
- The monotonicity of the k-shell index is improved by the weighted k-shell decomposition.
- The proposed method is effective in detecting the node influence.

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ABSTRACT

Identifying influential nodes in complex networks has attracted much attention because of its great theoretical significance and wide application. Existing methods consider the edges equally when designing identifying methods for the unweighted networks. In this paper, we propose an edge weighting method based on adding the degree of its two end nodes and for the constructed weighted networks, we propose a weighted k-shell decomposition method (*Wks*). Further investigations on the epidemic spreading process of the Susceptible–Infected–Recovered (SIR) model and Susceptible–Infected (SI) model in real complex networks verify that our method is effective for detecting the node influence.

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1. Introduction

The evaluation of node importance has been a fundamental issue in the research of complex networks. Many mechanisms are highly affected by a tiny fraction of influential nodes such as spreading, cascading, synchronizing and controllability [1–5]. It is well-known that degree, closeness and betweenness centralities [6–8] are three firstly developed measures to distinguish which nodes are more important than others. With the rapid development of investigating the node importance, many centrality measures are proposed in recent years, such as PageRank [9], LeaderRank [10,11], semi-local centrality [12–14] and so forth [15–25]. However, designing an effective method to identify the node importance is still an open issue. In Ref. [26], Pei et al. searched for the influential spreaders by following the real spreading dynamics in many online social networks and found that the widely-used degree and PageRank fail in ranking users' influence. Borge-Holthoefer et al. presented the influential spreaders in rumor dynamics may constrain the spreading of the rumor [27]. What is more, it is a very

* Corresponding author at: School of Computer and Information Science, Southwest University, Chongqing 400715, China. Tel.: +86 23 68254555; fax: +86 23 68254555.

E-mail addresses: ydeng@swu.edu.cn, prof.deng@hotmail.com (Y. Deng).

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difficult task to find multiple influential spreaders in complex networks [28,29]. For more recent overview of identifying the node importance and some related application, we refer to Ref. [28,30–36].

Recently, Kitsak et al. [37] proposed an interesting measure, k-shell decomposition (*Ks*), to categorize the nodes into core nodes and fringe nodes. They show that the most influential nodes are those located in the core of the network. After this original work, many work contribute to improving the k-shell decomposition. In order to overcome the original k-shell decomposition which assigns many nodes in the same k-shell, Zeng et al. [38] proposed the mixed degree decomposition (*MDD*) method by considering both the residual degree and the exhausted degree; Basaras et al. [39] posed the μ-power community index ($\mu - PCI$), which balanced the principles of the coreness and betweenness centralities; more recently, Bae and Kim [40] proposed a novel measure, coreness centrality (*Cnc*), to estimate the spreading influence of a node in a network by using the k-shell indices of its neighbors. In Ref. [41,42], the k-shell decomposition was extended to weighted complex networks.

In this paper, we continue to improve the k-shell decomposition method in complex networks. In most of unweighted networks, the edges are treated equally, however, each edge may have underlying different significance in network structures, functions [43–45]. Therefore, it is very important to take the edges' potential importance into consideration when we design the centrality measure for unweighted networks. Here, we first propose an edge weighting method based on adding the degree of its two end nodes. The intuition for this weighting method is based on the observation that an edge is more important when its two end nodes have larger degree. Then we use the weighted k-shell decomposition method (*Wks*) in the new weighted complex networks. The rank results show that our method gives wider rank list and overcomes the original k-shell decomposition assigning many nodes in an identical k-shell. To evaluate the effectiveness of the proposed method, we apply the SIR model for investigating the epidemic spreading process. By measuring the rank correlation between the ranked list generated by centrality measures and the ones generated by simulation results via SIR, it shows that our method can rank the spreading ability of nodes more accurately than centrality measures such as degree, local centrality, k-shell and coreness centrality. Moreover, we use the SI model to simulate the epidemic spreading process of the top-20 nodes and show that our proposed method is effective under the SI model.

The rest of this paper is organized as follows. In Section 2, we begin with a brief introduction of the k-shell decomposition method and propose our method. We apply the SIR and SI models to evaluate the effectiveness of the proposed method in six real complex networks in Section 3. Finally, some conclusions are summarized in the last section.

2. Materials and methods

For a given unweighted complex network $G = (N, E)$, N is the number of nodes, and E is the number of edges. e_{ij} represents the connection between node i and node j. The value of e_{ij} is defined as 1 if node i is connected to node j, and 0 otherwise. The degree of node i, denoted by k_i, is defined as follows:

Definition 2.1 (*Dc*)**.** The degree of node *i*, is defined as

$$k_i = \sum_{j}^{N} e_{ij}, \tag{1}$$

where *i* is the focal node, *j* represents the neighbors of the node *i*.

The node degree is a fundamental indicator in the study of complex networks and can be used to measure the importance of nodes [12,13,17].

The k-shell decomposition method partitions a network into sub-structures that are directly linked to centrality [37]. The decomposition algorithm can be stated as follows: First, all the nodes with degree $k = 1$ are removed. After removing, there may appear new nodes with $k = 1$. We keep on removing those nodes until all nodes with $k = 1$ are removed. Then, the removing nodes belong to the $k - s = 1$ shell index. Next, we repeat the removing process by a similar manner for the nodes of degree $k = 2$ to get the $k - s = 2$ shell. This procedure is repeated until all nodes of the networks are removed and assigned to one of the k-shells index. In Fig. 1(a), we illustrate schematically the layered structure obtained by applying the k-shell decomposition method.

In unweighted networks, the edges are treated equally in many works. In fact, the edges are different if its connection nodes influence is different. Thus, we can define this potential edge weights in the following way:

Definition 2.2 (*Edge Weighted*)**.** The weight of edge *eij*, denoted by w_{ij}, is defined as

$$w_{ij} = k_i + k_j. \tag{2}$$

The edges' weights by (2) are shown in Fig. 1(b). Next, we encounter the decomposition method by considering the degree and the edges weights. We assign each node for a weighted degree by the following measure:

$$k_i^w = \alpha k_i + (1 - \alpha)\Sigma_{j \in \Gamma_i} w_{ij}, \tag{3}$$

where Γ_i are a set of neighboring nodes of *i*, and the α is a positive tuning parameter from 0 to 1 that can be set according to the research setting and data. If this parameter is close to 0, then the high edge weight is taken as favorable. Since the

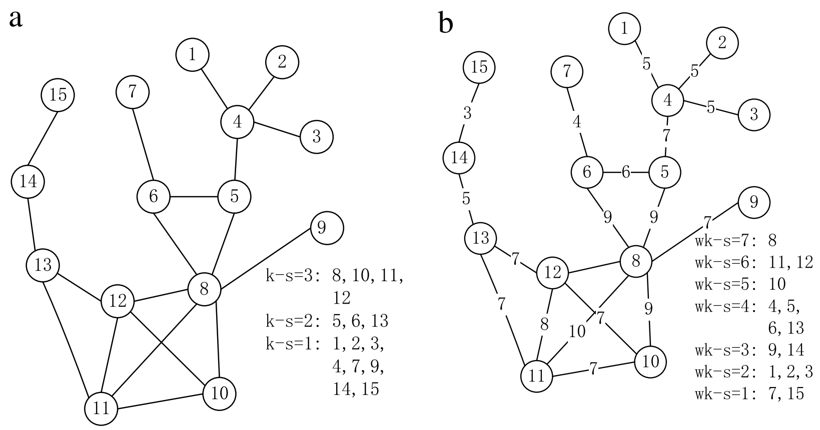

Fig. 1. The decomposition analysis is applied to this network by the k-shell decomposition method (a) and the wk-shell decomposition method (b). The $k - s$ index and $wk - s$ index are represented on the right of the network respectively. There are 4 nodes with identical $k - s = 3$, however, those 4 nodes can be identified by the wk-shell decomposition. Although k-shell decomposition efficiently detects a group of influential nodes, the nodes in the same shell are not distinguishable by the k-shell decomposition.

weighted degree may be no longer integers, the weighted degree is round down to the nearest integer. After these preparing, we generalize the k-shell decomposition method for the structured weighted networks, which is called weighted k-shell decomposition method (*Wks*). The pruning routine is the same as the k-shell decomposition method but is based on both the degree of a node and the edges' weights according to (3). If we set $\alpha = 1$ in (3), the weighted k-shell decomposition method is the same as the k-shell decomposition method. In this paper, we only discuss the case when $\alpha = 0.5$, which treats the weights and degree equally. In Fig. 1(b), we show the results obtained by the weighted k-shell decomposition method.

As illustrated in Fig. 1, the network is divided into seven different layers by the proposed method, while the nodes can only be assigned to three layers by the original k-shell decomposition method. The proposed method takes the advantage of the nodes' location and the different weighted edges to identify the influences of nodes. Based on our method, node 8 is identified as the most influential node, but it could not be distinguished from nodes 10, 11, and 12 by the original k-shell decomposition.

3. Results

3.1. Experimental setup

In order to verify the effectiveness of the proposed method, we carry out it in the following six real-world networks: (i) Netscience—the network of co-authorships between scientists who are themselves publishing on the topic of networks. There are in total 1589 scientists in this collaboration network [46]. We here consider the largest component with 379 scientists. (ii) *C. elegans*—the neural network of the *Caenorhabditis elegans* worm [47]. This network contains 453 nodes that represent neurons. (iii) Email—the Email network of University Rovira i Virgili (URV) of Spain contains faculty, researchers, technicians, managers, administrators, and graduate students [48]. (iv) Blog—the network constructed by uniting the blogs from two political camps in US [49]. Links in this network exist between blog sites which are based on a crawl of each 120 blog front page. (v) CNCG—the collaboration network in computational geometry [50]. (vi) Router—the router-level topology of the Internet, collected by the Rocket fuel Project [51]. It is an extremely sparse network with an average degree only being 2.49.

In Table 1, we list some statistical properties of the six networks. We provide the network hierarchies obtained by applying the proposed method, the original k-shell decomposition method. As shown in Table 1, by applying the proposed method, we obtain more detailed information about the networks internal structure.

The SIR model is a widely used tool to examine the spreading influence of top ranked nodes [5,16,53,54] and evaluate the effectiveness of immunization strategies [55,56]. In this model, individuals belong to one of the following states: (i) Susceptible S stands for the number of individuals susceptible to (not yet infected) the disease; (ii) Infected I denotes the number of individuals that have been infected and are able to spread the disease to susceptible individuals; (iii) Recovered R represents individuals that have been recovered and will never be infected again. At each time step, for each infected node, one selects its susceptible neighbors to spread the disease with probability β and then enters the recovered state with probability $\lambda = 1$. Under this assumption, the infecting process stops when there is no new infected node. Notice that in order to mimic the limited spreading capability of individuals, we set the infecting β relatively small values, i.e., $\beta \in (0, 0.1]$. In the case of large β values, where spreading can reach a large fraction of the nodes, the role of individual node is no longer important and spreading would cover almost all the network, independently of where it originates from [37].

Table 1
Some statistical properties and rank results for the six complex networks. For each network we list node number (N), edge number (E), average degree ($\langle k \rangle$), epidemic threshold (β^c_{rand}) [52]. Here N_{Wks} and N_{Ks} are the total number of the wk-shells and k-shells respectively. The M_{Wks} and M_{Ks} denote the total number of nodes in the cores obtained by the *Wks* and *Ks* methods respectively.

Network	N	E	$\langle k \rangle$	β^c_{rand}	N_{Ks}	N_{Wks}	M_{Ks}	M_{Wks}
Netscience	379	914	4.82	0.12	8	94	52	1
C. elegans	453	2025	8.97	0.02	10	259	25	1
Email	1133	5451	9.62	0.05	12	401	11	1
Blogs	1222	16714	27.3552	0.01	36	940	56	1
CNCG	3621	9461	5.23	0.05	21	395	22	1
Router	5022	6258	2.49	0.07	26	255	53	1

Table 2
The relations between *Wks* and *Dc*, *Lc*, *Ks* and *Cnc* centrality measures in six real networks.

Network	$\tau(Wks, Dc)$	$\tau(Wks, Lc)$	$\tau(Wks, Ks)$	$\tau(Wks, Cnc)$
Netscience	0.75	0.66	0.67	0.83
C. elegans	0.60	0.91	0.61	0.70
Email	0.91	0.86	0.87	0.94
Blogs	0.88	0.97	0.86	0.92
CNCG	0.53	0.69	0.63	0.86
Router	0.51	0.66	0.37	0.59

The ranked lists are obtained by applying all centrality measures in each network. In principle, the ranked list generated by an effective ranking method should be as consistent as possible with the ranked list generated by the real spreading process. In order to quantify the correctness of the ranking methods, we use Kendalls tau as the rank correlation coefficient. The Kendalls tau coefficient considers a set of joint observations from two random variables X and Y (in this paper, X is the values of a certain centrality measure and Y is the simulation results for all nodes in the SIR model). Any pair of observations (x_i, y_i) and (x_j, y_j) are said to be concordant if the ranks for both elements agree: that is, if both $x_i > x_j$ and $y_i > y_j$ or if both $x_i < x_j$ and $y_i < y_j$. They are said to be discordant if $x_i > x_j$ and $y_i < y_j$ or if $x_i < x_j$ and $y_i > y_j$. If $x_i = x_j$ or $y_i = y_j$, the pair is neither concordant nor discordant. The Kendalls coefficient is defined as

$$\tau = \frac{n_c - n_d}{0.5n(n-1)} \tag{4}$$

where n_c and n_c denote the number of concordant and discordant pairs respectively. The higher τ value is, the more accurate the ranked list of a centrality measure could generate is. The most ideal case is $\tau = 1$, where the ranked list generated by the centrality measure coincides exactly with the ranked list generated by the real spreading process.

3.2. Ranking effectiveness

To verify the effectiveness of the proposed method, we compare its τ values with the τ values for *Dc*, *Lc*, *Ks*, and *Cnc* under different spreading probabilities β and show the results in Fig. 2. As shown in Fig. 2, our proposed method can achieve the best performance on a wide range of probabilities β in Netscience and Router networks. For the *C. elegans* and CNCG, our method performs best when the spreading probabilities are not larger than 0.05. But for the Blogs and Email, our method cannot give a good performance when the probabilities β increase to certain values. In Ref. [37], it showed that the most efficient spreaders are those locating in the core of the network. However, the k-shell decomposition method usually assigns many nodes in the same k-shell. As the Kendall's tau measures the ranking of all the nodes, the *Ks* method cannot provide a high correlation of the ranked list generated by the SIR model.

Table 2 shows the relations between the proposed method and *Dc*, *Lc*, *Ks* and *Cnc* centralities in six networks, where Kendall's Tau is used to measure the correlation between proposed method and other four measures. From Table 2, we can see the *Wks* centrality has the strongest correlation with local centrality for the *C. elegans*, Blogs and Router networks. Moreover, the proposed method has rather high correlation with coreness centrality and weak correlation with k-shell centrality. Both local centrality and coreness centrality have been shown to identify the node importance validly [12,13,40]. Thus, to some extent, our method is effective as well, which will be further shown in the next subsection.

3.3. Performance on the SI model

As the local centrality outperforms the degree centrality and the coreness centrality is more effective than the k-shell method, in the following, we only compare performances of the proposed method, local and coreness centralities. In the SI model, nodes can only have two states: susceptible and infected. At each time step, the infected nodes infect their susceptible

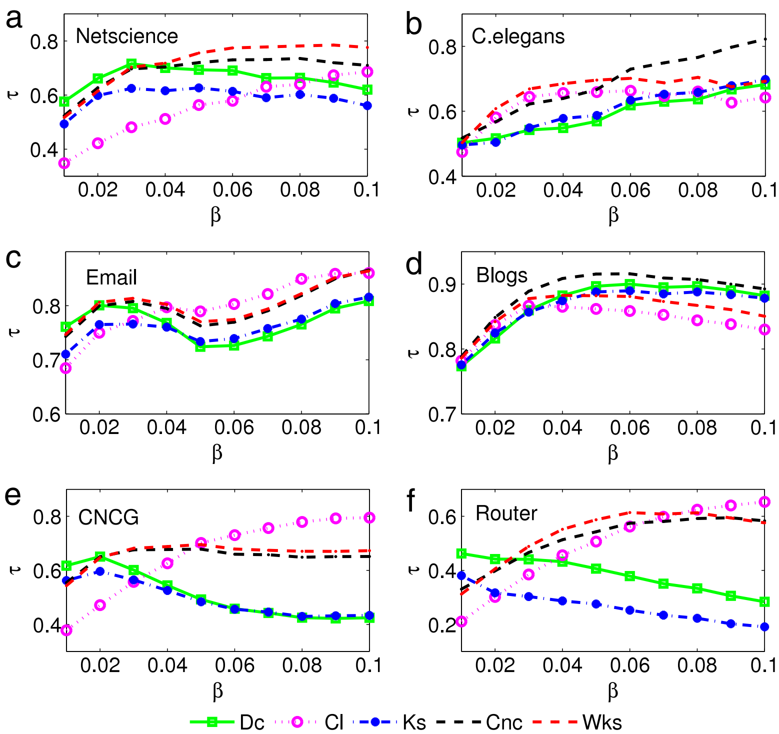

Fig. 2. The Kendall's tau values obtained by comparing the ranked lists generated by all five centrality measures and the ranked lists generated by the SIR spreading process in six real networks. The results are obtained by averaging over 500 independent realizations.

neighbors with probability β and remain infected. Via SI model, we can observe the spreading process until all the nodes in the network are infected even in the case of small infection probability. We use the SI model to compare the top-20 nodes' influence in six real networks. Here, we set $\beta = \beta^c_{rand}$ in each network and we only compare the spreading ability of the nodes that neither appear in the top-20 lists by *Wks* centrality nor *Lc* and *Cnc* centralities. Note that, without considering the effect of common nodes in three ranking lists, the different performances of these three methods can be well distinguished. We show the simulation results in six real networks in Fig. 3. To illustrate more clearly, we limit the spreading time step t to 50. From Fig. 3, we can see that the curves which represent the *Wks* centrality in six real networks are in the middle of the curves representing the *Lc* and *Cnc* centralities. The performances of *Lc* are the best in Netscience, but the worst in Router. The similar phenomenon appears in the *Cnc* centrality. Obviously, in *C. elegans* and Blogs, the three methods perform almost parallelly. The *Lc* centrality slightly outperforms in Email and CNCG. All in all, the results confirm the effectiveness of our proposed method.

4. Conclusions

In this paper, we proposed an edge weighting method by adding the degree of its two end nodes to construct a weighted network. We modified the original k-shell decomposition method to identify the node importance. Our approach is to consider both the degree and the links' weights of a node simultaneously. The decomposition results suggest that the *Wks* centrality produces more monotonic ranking than the k-shell method. To evaluate the effectiveness of our method, we compare the ranking of *Wks* centrality with the size of the infected population in the SIR model. Besides the SIR model, we estimate the spreading influence of the top-20 nodes by the proposed method in the SI model. The experimental results on six real networks show that our method is comparable with the local centrality and coreness centrality in identifying the influential nodes. Although the *Wks* method cannot outperform the coreness centrality for certain spreading probabilities, but it can outperform the *ks* centrality and degree centrality. Since the coreness centrality is based on the k-shell indices of its neighbors, we conjecture that if it is based on the wk-shell indices, the effectiveness of coreness centrality can be further improved.

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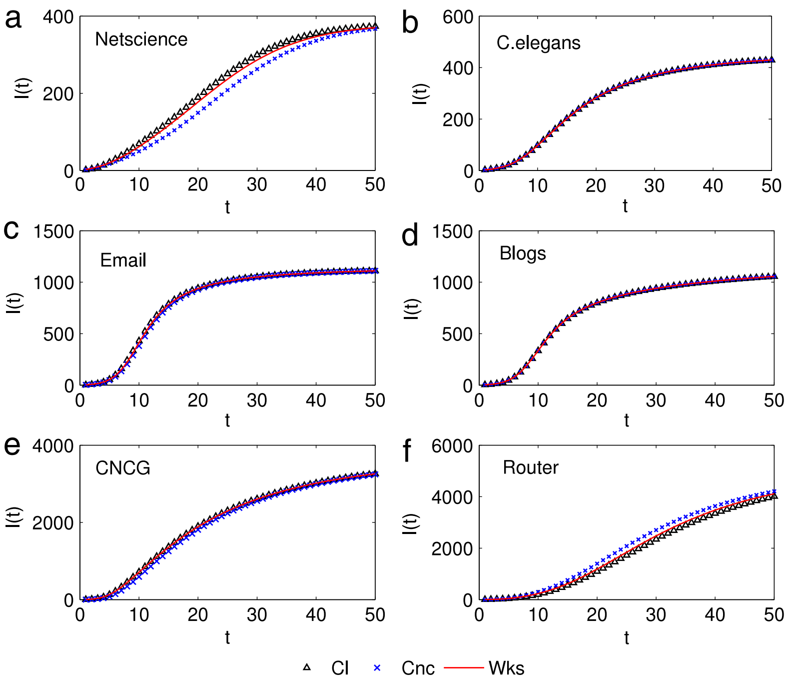

Fig. 3. The number of infected nodes $I(t)$ as a function of time t under SI model, with the initially infected nodes being those either appearing in the top-20 list by *Lc*, *Cnc* and *Wks*, but not appearing in three lists. The results are obtained by averaging over 500 independent realizations where the spreading probability $\beta = \beta^c_{rand}$ for each network.

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