

# Reconhecimento de Padrões

Artigo: Large Margin Gaussian Mixture Classifier With a Gabriel  
Graph Geometric  
Representation of Data Set Structure

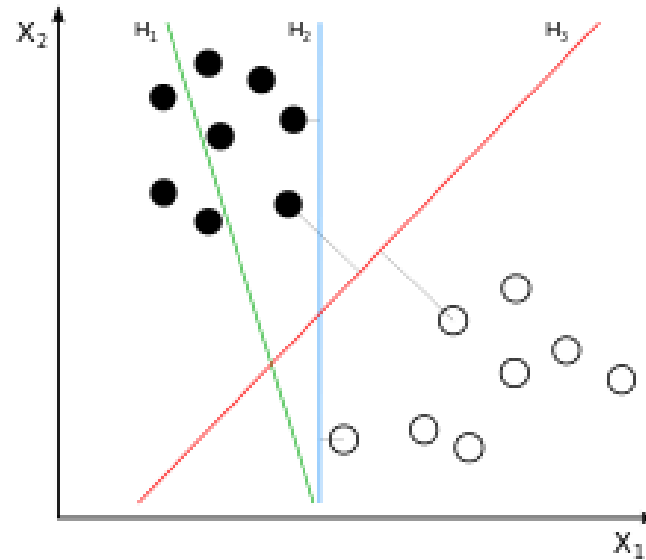
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# Support Vector Machine (SVM)

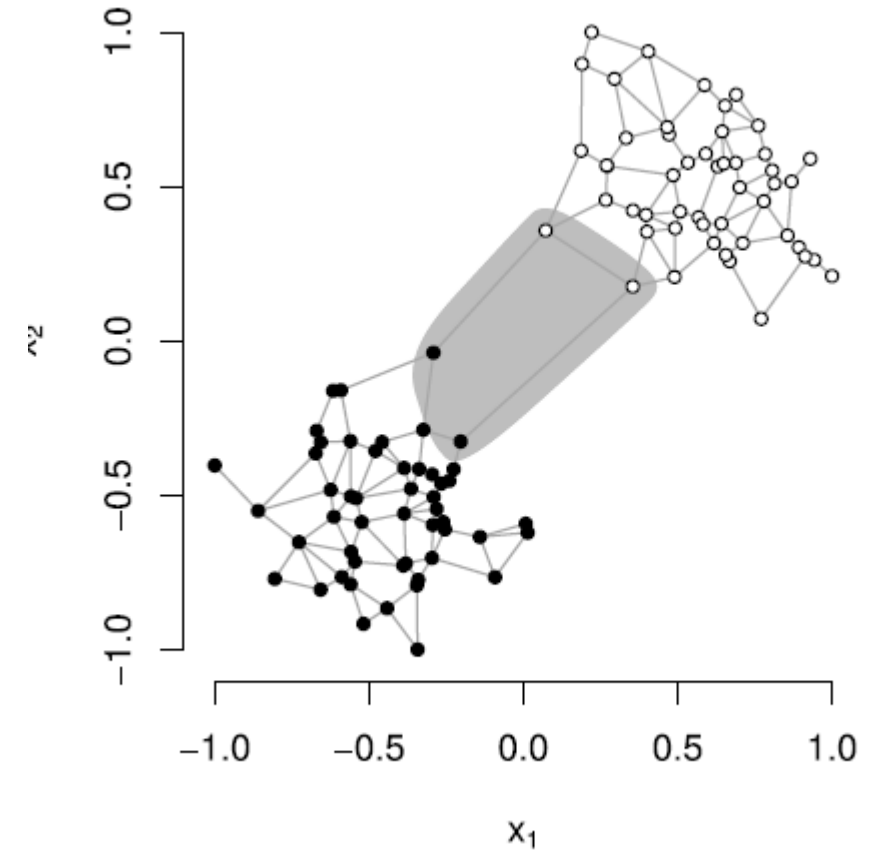
Algoritmo de aprendizado supervisionado, cujo objetivo é classificar determinado conjunto de pontos de dados que são mapeados para um espaço de características multidimensional usando uma função kernel



$H_1$  does not separate the classes.   
 $H_2$  does, but only with a small margin.  
 $H_3$  separates them with the maximal margin.

# Introdução

- O hiperplano de margem máxima também pode ser obtido a partir da geometria do conjunto de dados;
- Algoritmo proposto não requer parâmetros do usuário e não é baseado num algoritmo de otimização.



Fonte: TORRES, L. C. B. et al

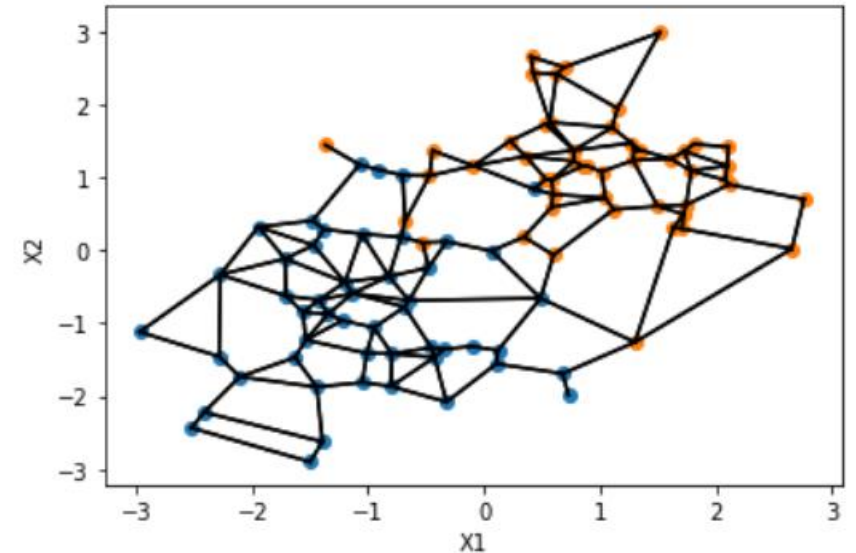
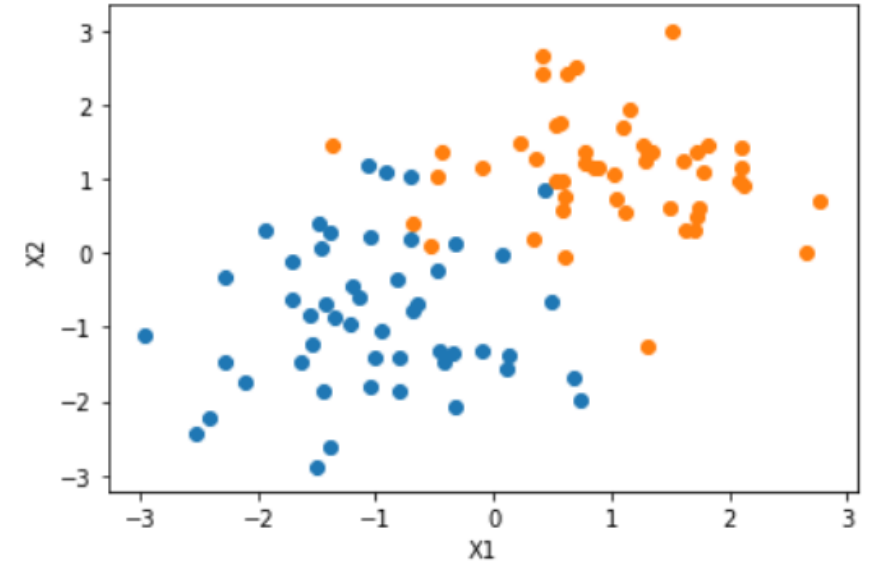
# Grafo de Gabriel

## A. Gabriel Graph Formulation

Considering the data set  $\mathcal{S} = \{\mathbf{x}_i, y_i\}_{i=1}^N$  with  $\mathbf{x}_i \in \mathbb{R}^n$  and  $y_i \in \{C_1, C_2\}$ , the Gabriel graph  $\ddot{G}$  of  $\mathcal{S}$  is defined as the graph with a set of vertices  $\mathcal{V} = \{\mathbf{x}_i\}_{i=1}^N$  and edges  $\mathcal{E}$  that obeys the following definition. An edge connecting the vertices  $\mathbf{x}_i$  and  $\mathbf{x}_j$  from  $\mathcal{V}$  belongs to  $\mathcal{E}$  only, and only if

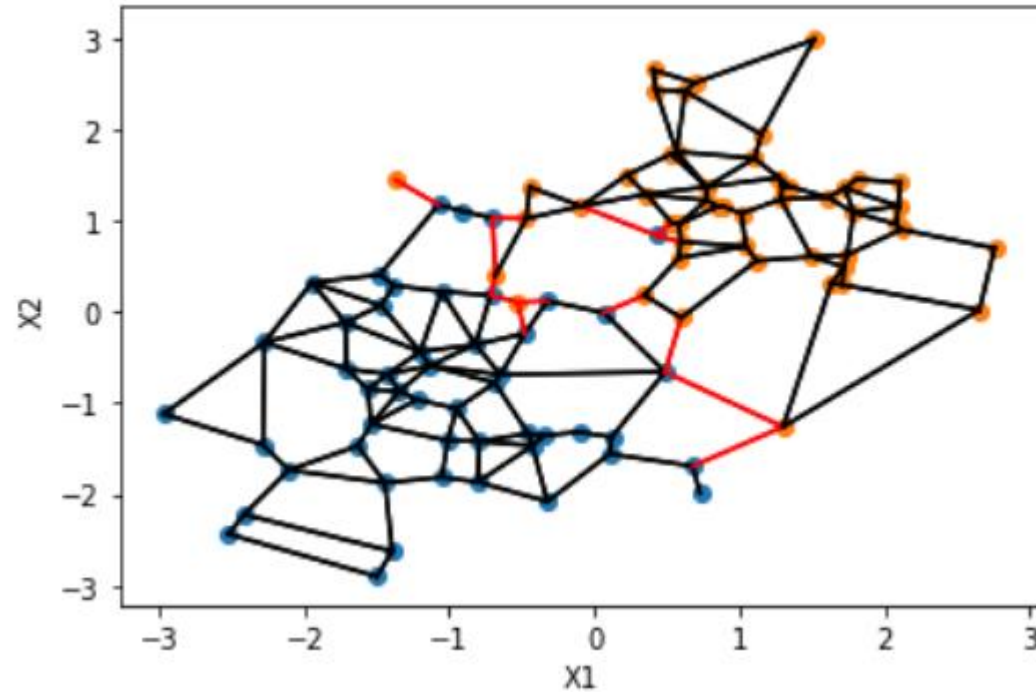
$$\|\mathbf{x}_i - \mathbf{x}_j\|^2 \leq (\|\mathbf{x}_i - \mathbf{x}_k\|^2 + \|\mathbf{x}_j - \mathbf{x}_k\|^2) \quad (1)$$

$\forall \mathbf{x}_k \in V$  and  $i \neq j \neq k$ , where  $\|\cdot\|$  is the Euclidean distance between vertices. Fig. 2(a) shows an example of graph resulting from the previous definition.



# Support Edges (SEs)

São as arestas localizadas na região de separação



# Class Overlapping

$$q(x_i) = \frac{|\hat{\mathcal{D}}(x_i)|}{|\mathcal{D}(x_i)|} \quad (2)$$

- 1) For all  $x_i \in \ddot{G}$ , compute  $q(x_i)$  according to (2).
- 2) Group  $q(x_i)$  per class such that  $\mathcal{Q}^+$  and  $\mathcal{Q}^-$  holds the membership measures for the patterns with labels  $+1$  and  $-1$ , respectively. In other words,  $\mathcal{Q}^+$  is the set of all  $q(x_i)$  belonging to class  $+1$  and  $\mathcal{Q}^-$  for class  $-1$ .
- 3) Compute the class thresholds  $t^+$  and  $t^-$  as the mean of the membership measures belonging to  $\mathcal{Q}^+$  and  $\mathcal{Q}^-$

$$t^+ = \frac{\sum_{q(x_i) \in \mathcal{Q}^+} q(x_i)}{|\mathcal{Q}^+|}, \quad t^- = \frac{\sum_{q(x_i) \in \mathcal{Q}^-} q(x_i)}{|\mathcal{Q}^-|}. \quad (3)$$

- 4) Remove from  $\ddot{G}$  all vertices whose  $q(x_i)$  are less than  $t^+$  and  $t^-$ .

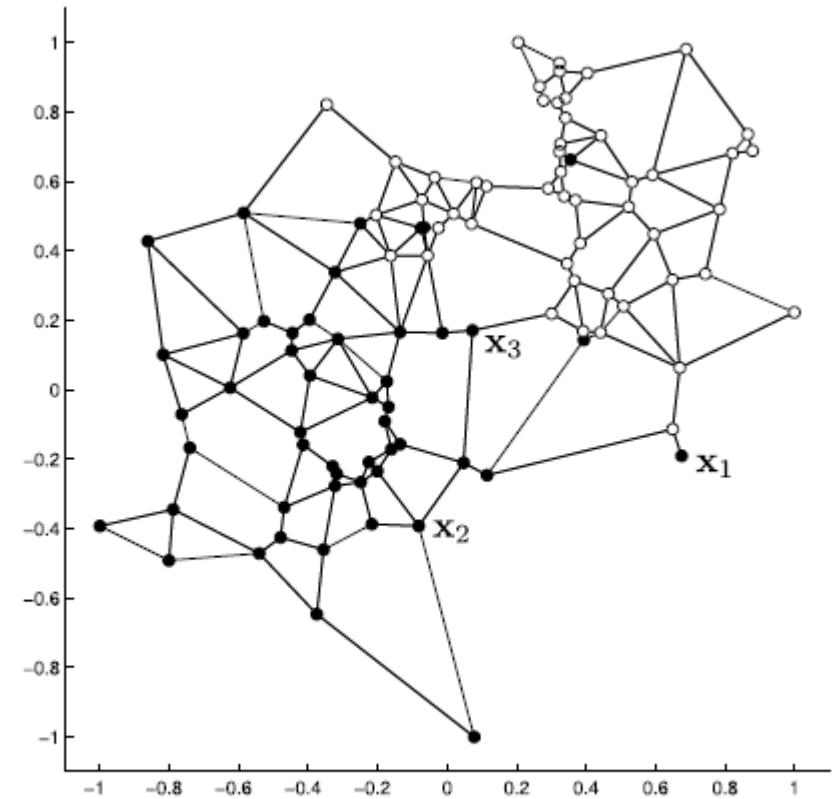
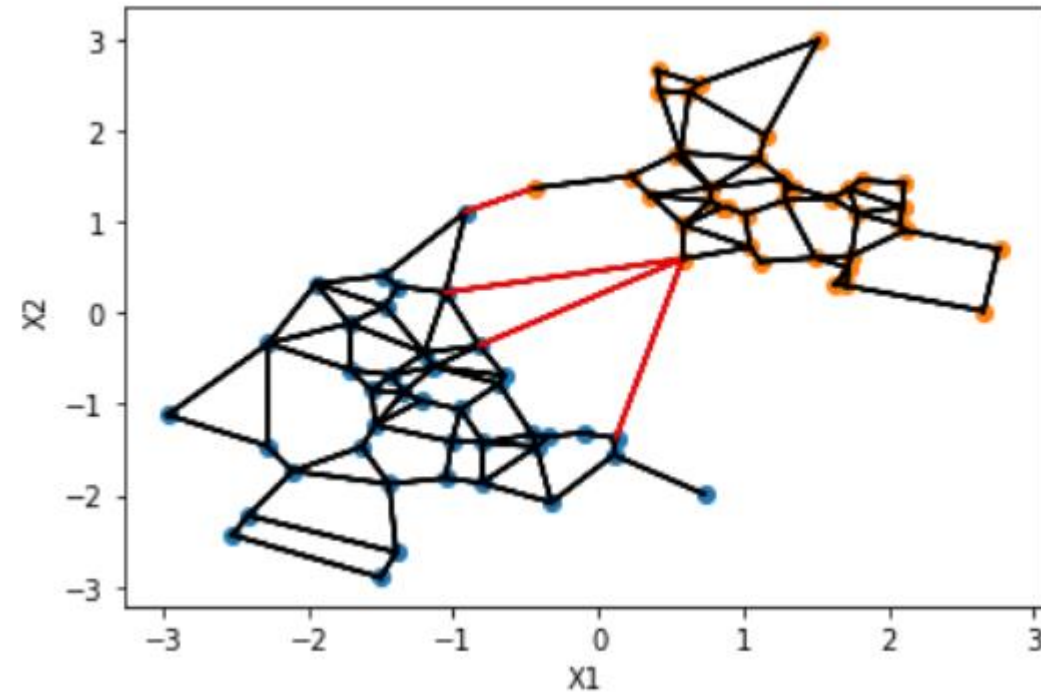


Fig. 3. Data set with overlapping.

Fonte: TORRES, L. C. B. et al.

# Class Overlapping



# Mistura de Gaussianas

- Cada vértice das arestas de suporte (SE) se torna o centro de uma gaussiana.
- Desvio padrão de  $3\sigma$  representa 99,73% das amostras.

$$R = 3\sigma, \quad \sigma = \frac{R}{3} \quad (8)$$

$$R = \frac{1}{2}\|c - m\| = \frac{1}{2}\|d - m\| \quad (9)$$

$$m = \frac{1}{2}(c + d), \quad (c, d) \in SE \quad (10)$$

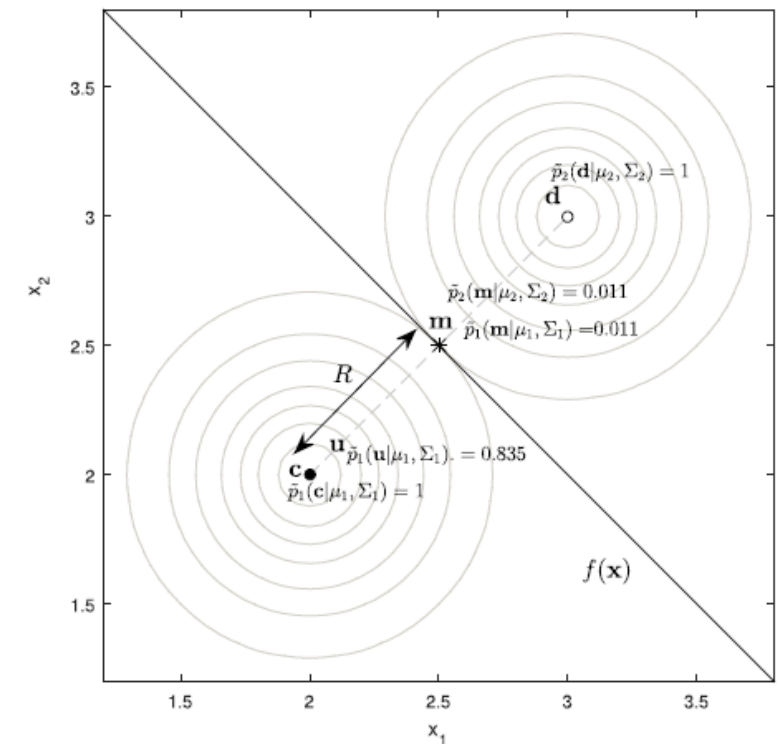


Fig. 5. Two multivariate normal distributions and a midpoint separator in the lower density region.

Fonte: TORRES, L. C. B. et al.



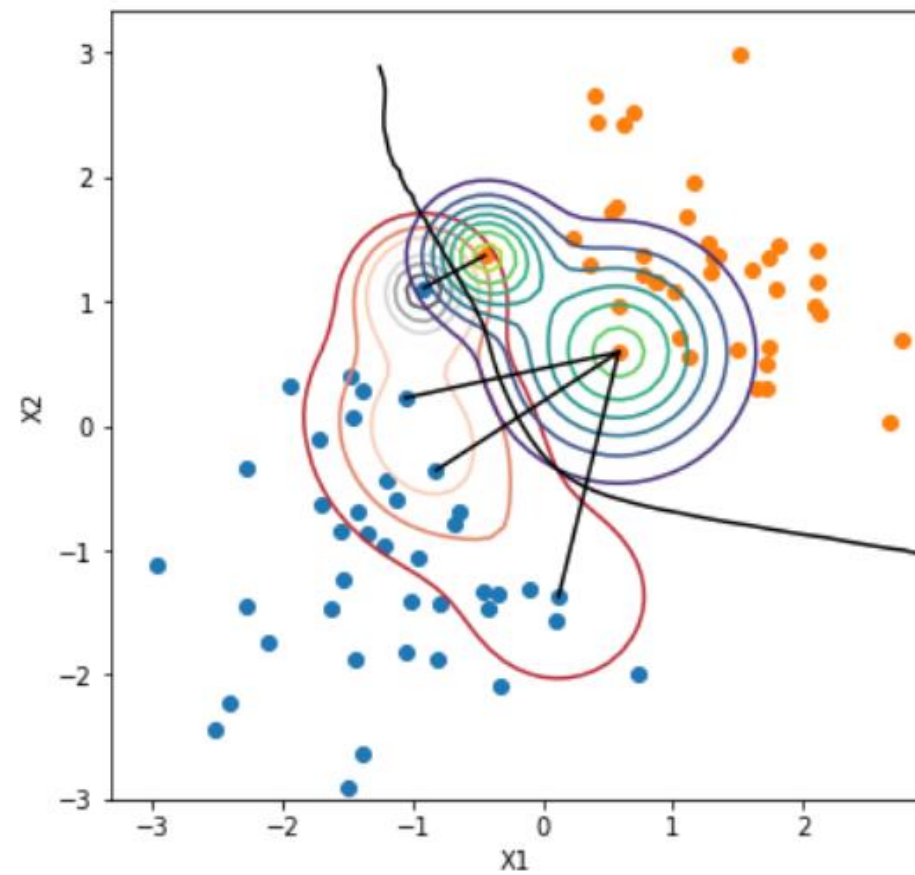
# Mistura de Gaussianas

$$P(\mathbf{x}|S_1, \dots, S_p) = \sum_{k=1}^p \pi_k \frac{1}{\sqrt{(2\pi)^n |\Sigma_k|}} \exp \left( -\frac{1}{2} (\mathbf{x}_k - \boldsymbol{\mu}_k)^T \Sigma_k^{-1} (\mathbf{x}_k - \boldsymbol{\mu}_k) \right)$$

$$f(\mathbf{x}_i) = \begin{cases} +1, & \text{if } \tilde{p}(\mathbf{x}_i, \theta_1|C_1)P(C_1) \geq \tilde{p}(\mathbf{x}_i, \theta_2|C_2)P(C_2) \\ -1, & \text{if } \tilde{p}(\mathbf{x}_i, \theta_1|C_1)P(C_1) < \tilde{p}(\mathbf{x}_i, \theta_2|C_2)P(C_2) \end{cases}$$

where  $\tilde{p}(\mathbf{x}_i, \theta_1|C_1)$  and  $\tilde{p}(\mathbf{x}_i, \theta_2|C_2)$  are the likelihoods of the positive and negative classes, respectively, estimated with  $\tilde{S}\tilde{V}$  only,  $\theta_1$  and  $\theta_2$  their vectors of parameters.

# Resultados



# Resultados

TABLE I  
AVERAGE VALUES OF AUC, TRAINING TIME, AND CHARACTERISTICS OF THE DATA SETS

Data Set	New Method			SVM-RBF			SVM-Poly			SVM-Linear			$N_d$	$N$	$N^+$	$N^-$
	AUC	Ngv	T(s)	AUC	Nsv	T(s)	AUC	Nsv	T(s)	AUC	Nsv	T(s)				
Appendicitis	<b>0.792±0.165</b>	8	0.002	0.712±0.226	50.7	56.04	0.766±0.193	32.5	168.5	0.652±0.203	31.2	45.66	7	106	21	85
Stalog Australian Credit	0.836±0.040	251.9	0.118	0.864±0.040	312	105.2	<b>0.872±0.048</b>	298.2	571.66	0.857±0.038	198.4	63.95	14	690	307	383
Banknote Authentication	0.997±0.005	177.8	0.177	<b>1.000±0.000</b>	193.3	111.8	0.999±0.003	195.9	986.0	0.991±0.011	69.1	58.29	4	1372	610	762
The Wisconsin Breast Cancer	0.959±0.019	51.7	0.047	<b>0.968±0.020</b>	262.3	96.99	0.967±0.021	83.7	361.7	0.960±0.028	46.2	51.40	9	683	444	239
Breast Cancer Hess Probes	<b>0.814±0.115</b>	45.7	0.047	0.736±0.176	75.2	62.02	0.670±0.165	60.8	211.08	0.555±0.110	47.4	47.56	30	133	99	34
Climate Model Simulation Crashes	0.704±0.173	235.2	0.195	0.510±0.032	112.3	113.0	<b>0.759±0.172</b>	85.9	364.78	0.751±0.100	56.3	53.27	18	540	494	46
Pima Indian Diabetes	<b>0.727±0.056</b>	213.5	0.067	0.717±0.065	424.3	116.6	0.706±0.052	393.2	606.25	0.717±0.050	361.5	59.72	8	768	500	268
EEG Eye State	<b>0.802±0.014</b>	4805.5	44.26	0.797±0.036	6629.2	401.9	0.643±0.062	8732.2	2494.02	0.581±0.015	11637.5	307.05	14	14980	6723	8257
Fertility	<b>0.643±0.282</b>	34.9	0.004	0.500±0	39.1	56.39	0.500±0	34.2	1.94	0.5±0	35.3	46.06	9	100	12	88
Stalog German Credit	<b>0.676±0.049</b>	459.4	0.966	0.649±0.046	564.2	202.03	0.662±0.046	516.4	1266.05	0.668±0.054	477.7	98.34	24	1000	700	300
Glass Identification	<b>0.924±0.106</b>	26.8	0.007	0.880±0.103	72.5	60.34	0.896±0.097	30.1	193.11	0.874±0.175	19.3	46.68	9	214	29	185
Haberman's Survival	0.550±0.118	56.7	0.010	0.534±0.052	165.1	65.14	0.497±0.007	147.4	249.38	0.494±0.010	149.7	48.80	3	306	225	81
Stalog Heart	0.804±0.103	95	0.032	0.828±0.075	133.9	66.15	<b>0.831±0.087</b>	140	250.50	0.824±0.097	88.9	49.37	13	270	150	120
Indian Liver Patient	<b>0.622±0.083</b>	146.2	0.035	0.498±0.011	356.6	97.71	0.497±0.008	315.9	542.03	0.499±0.004	323.3	58.26	10	579	414	165
Ionosphere	0.893±0.049	105.3	0.045	<b>0.938±0.039</b>	153.6	80.94	0.886±0.049	90.4	351.64	0.831±0.066	77.4	53.45	33	351	225	126
Parkinsons	0.792±0.125	31.5	0.008	0.790±0.151	89	63.99	<b>0.867±0.114</b>	56.9	223.69	0.753±0.063	58	48.16	22	195	147	48
Breast Cancer WP	0.566±0.162	76.9	0.036	0.493±0.015	115.5	67.38	<b>0.594±0.127</b>	92.1	257.28	0.591±0.112	78.1	54.10	33	194	46	148
Letter Recognition A Vs All	0.956±0.22	1985	123.055	0.956±0.030	391.5	2600	<b>0.990±0.009</b>	226.8	485.91	0.925±0.023	469.0	879.03	16	20000	789	19211
Mnist 0 Vs All	0.982±0.01	847.3	10516.18	<b>0.992±0.002</b>	1574.9	1160.9	<b>0.992±0.001</b>	976.2	206.86	0.967±0.007	1802.0	1679.01	40	70000	6903	63097
Stalog Shuttlest	0.962±0.02	355	2497.32	<b>0.998±0.001</b>	653.6	202.23	0.974±0.012	3165.1	148.13	0.952±0.001	4903.3	270.0124	9	58000	45586	12414
Av. Rank	1.9750			2.175			2.575			3.275						

Fontes: M. Lichman(2013),  
J. Alcalá-Fdez, et al.,  
K. R. Hess *et al.*,

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