# MGMT90141 Business Analysis and Decision Making
# Review and Revision – Mock Exam Paper 2

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# Question 1

Personal Mini Warehouses is planning to expand its successful Melbourne business into Sydney. The company must determine the best possible combination of large space and small space storage rooms to build to reach the maximum monthly earnings. Each large space storage room costs $2 to advertise and requires 100 square foot. Each small space storage room costs $4 to advertise and requires 50 square foot. The advertising budget available is $400, and the area available is 8,000 square foot. The number of large space storage rooms must not exceed 60. The monthly earning of each large space storage room is $50, and that of each small space storage room is $20.

# Question 1

(i) Formulate the linear programming (LP) model for the Personal Mini Warehouses. (7 marks)

Let $x_1$ be the number of large spaces developed;
$x_2$ be the number of small spaces developed

Maximize monthly earnings or $z = 50x_1 + 20x_2$

Subject to
$$2x_1 + 4x_2 \leq 400$$
$$100x_1 + 50x_2 \leq 8000$$
$$x_1 \leq 60$$
$$x_1 \text{ and } x_2 \geq 0$$

# Question 1

Variable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$10 | Storage rooms Large | 60 | 0 | 50 | 1E+30 | 10 |
| $C$10 | Storage rooms Small | 40 | 0 | 20 | 5 | 20 |

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$16 | Advertising budget available LHS | ? | 0 | 400 | 1E+30 | 120 |
| $B$17 | Square footage required LHS | ? | 0.4 | 8000 | 1500 | 2000 |
| $B$18 | Rental limit expected LHS | 60 | 10 | 60 | 20 | 20 |

(ii) For the optimal solution, how much of the advertising budget is spent? (3 marks)

In the optimal solution, $x_1 = 60$ and $x_2 = 40$. Using these values in the first constraint gives us $2x_1 + 4x_2 = 2(60) + 4(40) = \$280$.

# Question 1

Variable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$10 | Storage rooms Large | 60 | 0 | 50 | 1E+30 | 10 |
| $C$10 | Storage rooms Small | 40 | 0 | 20 | 5 | 20 |

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$16 | Advertising budget available LHS | ? | 0 | 400 | 1E+30 | 120 |
| $B$17 | Square footage required LHS | ? | 0.4 | 8000 | 1500 | 2000 |
| $B$18 | Rental limit expected LHS | 60 | 10 | 60 | 20 | 20 |

(iii) For the optimal solution, how much square footage will be used? (3 marks)

For the second constraint we have $100x_1 + 50x_2 = 100(60) + 50(40)$ = 8,000 square foot.

# Question 1

Variable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$10 | Storage rooms Large | 60 | 0 | 50 | 1E+30 | 10 |
| $C$10 | Storage rooms Small | 40 | 0 | 20 | 5 | 20 |

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$16 | Advertising budget available LHS | ? | 0 | 400 | 1E+30 | 120 |
| $B$17 | Square footage required LHS | ? | 0.4 | 8000 | 1500 | 2000 |
| $B$18 | Rental limit expected LHS | 60 | 10 | 60 | 20 | 20 |

(iv) Would the solution change if the budget were only $300 instead of 400? (4 marks)

No, the solution would not change. The value 300 is between the lower bound of 280 (400 – 120) and the upper bound of infinity.

The shadow price is 0, so the earnings would remain the same and the optimal solution would not change. Indeed, there is slack of 400 – 280 = 120, which is greater than 400-300=100.

# Question 1

Variable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$10 | Storage rooms Large | 60 | 0 | 50 | 1E+30 | 10 |
| $C$10 | Storage rooms Small | 40 | 0 | 20 | 5 | 20 |

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$16 | Advertising budget available LHS | ? | 0 | 400 | 1E+30 | 120 |
| $B$17 | Square footage required LHS | ? | 0.4 | 8000 | 1500 | 2000 |
| $B$18 | Rental limit expected LHS | 60 | 10 | 60 | 20 | 20 |

(v) What would the optimal solution be if the profit on the large spaces were reduced from \$50 to \$45? (4 marks)

As the new coefficient for $x_1$ is between the lower bound of 40 (50 – 10) and the upper bound of infinity, the current solution remains optimal, that is $x_1 = 60$ and $x_2 = 40$. But the monthly earnings change to 45(60) + 20(40) = \$3,500.

# Question 1

Variable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$10 | Storage rooms Large | 60 | 0 | 50 | 1E+30 | 10 |
| $C$10 | Storage rooms Small | 40 | 0 | 20 | 5 | 20 |

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$16 | Advertising budget available LHS | ? | 0 | 400 | 1E+30 | 120 |
| $B$17 | Square footage required LHS | ? | 0.4 | 8000 | 1500 | 2000 |
| $B$18 | Rental limit expected LHS | 60 | 10 | 60 | 20 | 20 |

(vi) How much would earnings increase if the square footage requirement were increased from 8,000 to 9,000? (4 marks)

The shadow price for this constraint is 0.4, and the upper bound of the range of feasibility is 9,500. The increase of 1,000 units will result in an increase in earnings of 1,000*(0.4 per unit) = $400.

# Question 2 (i)

Eddie Kelly is running for reelection as mayor of a small town in Victoria. Jessica Martinez, Kelly's campaign manager during this election, is planning the marketing campaign, and there is some stiff competition. Martinez has selected four ways to advertise: television ads, radio ads, billboards, and newspaper ads. The costs of these, the audience reached by each type of ad, and the maximum number of each is shown in the following table. In addition, Martinez has decided that there should be at least six ads on TV, and at least six ads on radio. The amount spent on billboards and newspaper together must not exceed the amount spent on TV ads. While fundraising is still continuing, the monthly budget for advertising has been set at $15,000.

| Types of ad | Cost per ad | Audience reached per ad | Maximum number |
|---|---|---|---|
| TV | $800 | 30,000 | 10 |
| Radio | $400 | 22,000 | 10 |
| Billboards | $500 | 24,000 | 10 |
| Newspaper | $100 | 8,000 | 10 |

Formulate the LP model to determine the number of ads of each type to be placed such that the total number of people reached is maximized. (10 marks)

# Question 2 (i)

Let $x_1$ be the number of TV ads;
$x_2$ be the number of radio ads;
$x_3$ be the number of billboards ads;
$x_4$ be the number of newspaper ads

Maximize the total number of people reached or $z$
$= 30{,}000x_1 + 22{,}000x_2 + 24{,}000x_3 + 8{,}000x_4$

Subject to

$x_1 \geq 6$ (min TV ads)
$x_2 \geq 6$ (min radio ads)
$x_1 \leq 10$ (max TV ads)
$x_2 \leq 10$ (max radio ads)
$x_3 \leq 10$ (max billboards ads)
$x_4 \leq 10$ (max newspaper ads)
$-800x_1 + 500x_3 + 100x_4 \leq 0$ (spending requirement)
$800x_1 + 400x_2 + 500x_3 + 100x_4 \leq 15{,}000$ (ads budget)
$x_1, x_2, x_3,$ and $x_4 \geq 0$

# Question 2 (ii)

The East Coast company's long-range planning group developed forecasts of the anticipated annual demand at the distribution centers as:

| Distribution center | Annual demand |
|---|---|
| Detroit | 12,000 |
| Toledo | 8,000 |
| Houston | 10,000 |
| Miami | 8,000 |
| St. Louis | 6,000 |

Because of an anticipated increase in demand, the company plans to increase capacity by constructing a new warehouse in one or more of the following cities: Atlanta, Boston, Chicago, or Denver. The estimated annual fixed cost and the annual capacity for the four proposed warehouses are as follows:

| Proposed warehouse | Annual fixed cost | Annual capacity |
|---|---|---|
| Atlanta | $30,000 | 28,000 |
| Boston | $25,000 | 26,000 |
| Chicago | $20,000 | 24,000 |
| Denver | $15,000 | 22,000 |

# Question 2 (ii)

The shipping cost per unit from each warehouse to each distribution center:

| | Distribution center | | | | |
|---|---|---|---|---|---|
| Warehouse | Detroit | Toledo | Houston | Miami | St. Louis |
| Atlanta | 3 | 4 | 4 | 5 | 2 |
| Boston | 6 | 5 | 6 | 15 | 8 |
| Chicago | 7 | 4 | 10 | 6 | 7 |
| Denver | 6 | 6 | 7 | 8 | 9 |

Formulate the LP model to determine which warehouse(s) should the company operate AND how many products should be shipped from warehouse *i* to distribution center *j* to reach the minimum total costs. (15 marks)

# Question 2 (ii)

Let $x_{ij}$ be the units of products to be shipped from warehouse $i$ to center $j$;
$y_i = 1$ if a warehouse is constructed in city $i$, 0 otherwise, where i=1,…,4 and j=1,…,5.

Minimize the total cost or $z$

$$= (3x_{11} + 4x_{12} + 4x_{13} + 5x_{14} + 2x_{15} + 6x_{21} + 5x_{22} + 6x_{23} + 15x_{24} + 8x_{25} + 7x_{31} + 4x_{32} + 10x_{33} + 6x_{34} + 7x_{35} + 6x_{41} + 6x_{42} + 7x_{43} + 8x_{44} + 9x_{45}) + (30{,}000y_1 + 25{,}000y_2 + 20{,}000y_3 + 15{,}000y_4)$$

Subject to

$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} - 28{,}000y_1 \le 0$ (capacity – Atlanta)
$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} - 26{,}000y_2 \le 0$ (capacity – Boston)
$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} - 24{,}000y_3 \le 0$ (capacity – Chicago)
$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} - 22{,}000y_4 \le 0$ (capacity – Denver)
$x_{11} + x_{21} + x_{31} + x_{41} = 12{,}000$ (demand – Detroit)
$x_{12} + x_{22} + x_{32} + x_{42} = 8{,}000$ (demand – Toledo)
$x_{13} + x_{23} + x_{33} + x_{43} = 10{,}000$ (demand – Houston)
$x_{14} + x_{24} + x_{34} + x_{44} = 8{,}000$ (demand – Miami)
$x_{15} + x_{25} + x_{35} + x_{45} = 6{,}000$ (demand – St Louis)
$x_{ij} \ge 0$ and integer; $y_i = 0$ or 1 for all $i$ and $j$

# Question 3 (i)

Prentice Hall, Inc., a publisher headquartered in London, wants to assign three recently hired college graduates, Adam, Brian, and Carlos to regional sales districts in Birmingham, Manchester, and Liverpool. But the firm also has an opening in Newcastle and would send one of the three there if it were more economical than a move to Birmingham, Manchester, or Liverpool. It will cost $1,000 to relocate Adam to Newcastle, $800 to relocate Brian there, and $1,500 to move Carlos. Formulate the LP model to determine the optimal assignment of personnel to offices. (8 marks)

| | Office | | |
|---|---|---|---|
| Personnel | Birmingham | Manchester | Liverpool |
| Adam | $800 | $1,100 | $1,200 |
| Brian | $500 | $1,600 | $1,300 |
| Carlos | $500 | $1,000 | $2,300 |

# Question 3 (i)

Let $x_{ij}$ = 1 if personnel $i$ is relocated to city $j$, 0 otherwise.
Note that $x_{11}$ = 1 if Adam is relocated to Birmingham,
$x_{44}$ = 1 if the dummy person is relocated to Newcastle.

Minimize the total cost or $z$

$$= 800x_{11} + 1100x_{12} + 1200x_{13} + 1000x_{14}$$
$$+ 500x_{21} + 1600x_{22} + 1300x_{23} + 800x_{24}$$
$$+ 500x_{31} + 1000x_{32} + 2300x_{33} + 1500x_{34}$$
$$+ 0x_{41} + 0x_{42} + 0x_{43} + 0x_{44}$$

Subject to

$$x_{11} + x_{12} + x_{13} + x_{14} = 1 \text{ (Adam)}$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1 \text{ (Brian)}$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1 \text{ (Carlos)}$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1 \text{ (Dummy)}$$
$$x_{11} + x_{21} + x_{31} + x_{41} = 1 \text{ (Birmingham)}$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 1 \text{ (Manchester)}$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1 \text{ (Liverpool)}$$
$$x_{14} + x_{24} + x_{34} + x_{44} = 1 \text{ (Newcastle)}$$
$$x_{ij} \geq 0 \quad \text{for all } i \text{ and } j$$

# Question 3 (ii)

The distribution system for the Herman Company consists of three plants, two warehouses, and four customers. Plant capacities and shipping costs per unit (in dollars) from each plant to each warehouse are as follows:

| | Warehouse | | |
|---|---|---|---|
| Plant | 1 | 2 | Capacity |
| 1 | 4 | 7 | 400 |
| 2 | 8 | 5 | 600 |
| 3 | 5 | 6 | 300 |

Customer demand and shipping costs per unit (in dollars) from each warehouse to each customer are:

| | Customer | | | |
|---|---|---|---|---|
| Warehouse | 1 | 2 | 3 | 4 |
| 1 | 6 | 4 | 8 | 4 |
| 2 | 3 | 6 | 7 | 7 |
| Demand | 300 | 300 | 300 | 400 |

Formulate the LP model for the problem. (Hints: use P1, P2, and P3 to denote the plants, use W4 and W5 to denote the warehouses, and use C6, C7, C8, and C9 to denote the customers.) (17 marks)

# Question 3 (ii)

Define $x_{ij}$ as the number of units shipped from plant i to warehouse j;
Define $x_{jk}$ as the number of units shipped from warehouse j to customer k.

Minimize shipping cost or $z = 4\,x_{14} + 7\,x_{15} + 8\,x_{24} + 5\,x_{25} + 5\,x_{34} + 6\,x_{35} + 6\,x_{46} + 4\,x_{47} + 8\,x_{48} + 4\,x_{49} + 3\,x_{56} + 6\,x_{57} + 7\,x_{58} + 7\,x_{59}$

Subject to

$$
\begin{aligned}
& x_{14} + x_{15} && \le 400 \text{ (P1)} \\
& x_{24} + x_{25} && \le 600 \text{ (P2)} \\
& x_{34} + x_{35} && \le 300 \text{ (P3)} \\
& (x_{14} + x_{24} + x_{34}) - (x_{46} + x_{47} + x_{48} + x_{49}) && = 0 \text{ (W1)} \\
& (x_{15} + x_{25} + x_{35}) - (x_{56} + x_{57} + x_{58} + x_{59}) && = 0 \text{ (W2)} \\
& x_{46} + x_{56} && = 300 \text{ (C1)} \\
& x_{47} + x_{57} && = 300 \text{ (C2)} \\
& x_{48} + x_{58} && = 300 \text{ (C3)} \\
& x_{49} + x_{59} && = 400 \text{ (C4)} \\
& x_{ij}\,,\ x_{jk} \ge 0 && \text{for all } i \text{ and } j
\end{aligned}
$$

# Question 4

Blake Electronics Corporation is considering the introduction of a new product. The company can use one of the two designs for this product. Design A will be more profitable if consumer preferences remain basically unchanged. Design B will be more profitable if consumer preferences change. Initiate estimates indicate there is a 60 percent chance consumer preferences will remain unchanged. The payoff table for this decision is shown below. All figures are in dollars of net profit.

| | Consumer preferences | |
|---|---|---|
| Decision alternative | Unchanged, P(U) = 0.6 | Changed, P(C) = 0.4 |
| Design A | $42,000 | $10,000 |
| Design B | $33,000 | $20,000 |

# Question 4

(i) Determine the optimal decision for the company if management wishes to maximize expected profit. (2 marks)

EV(A) = 0.6(42,000) + 0.4(10,000) = $29,200
EV(B) = 0.6(33,000) + 0.4(20,000) = $27,800
Decision = Design A

(ii) What is the expected value of perfect information? (2 marks)

EVwPI = 0.6(42,000) + 0.4 (20,000) = $33,200
EVwoPI = max(29200, 27800) = $29,200
EVPI = $33,200 - $29,200 = $4,000

# Question 4

A market research firm has offered, for a fee, to conduct a survey on consumer preferences for the company. Past experience shows that, when consumer preferences are unchanged, the survey will reveal that fact 85 percent of the time (use *RU* denotes reveal unchanged, $P(RU|U) = 0.85$). When consumer preferences have changed, the survey reveals that fact 75 percent of the time (use *RC* denotes reveal changed, $P(RC|C) = 0.75$).

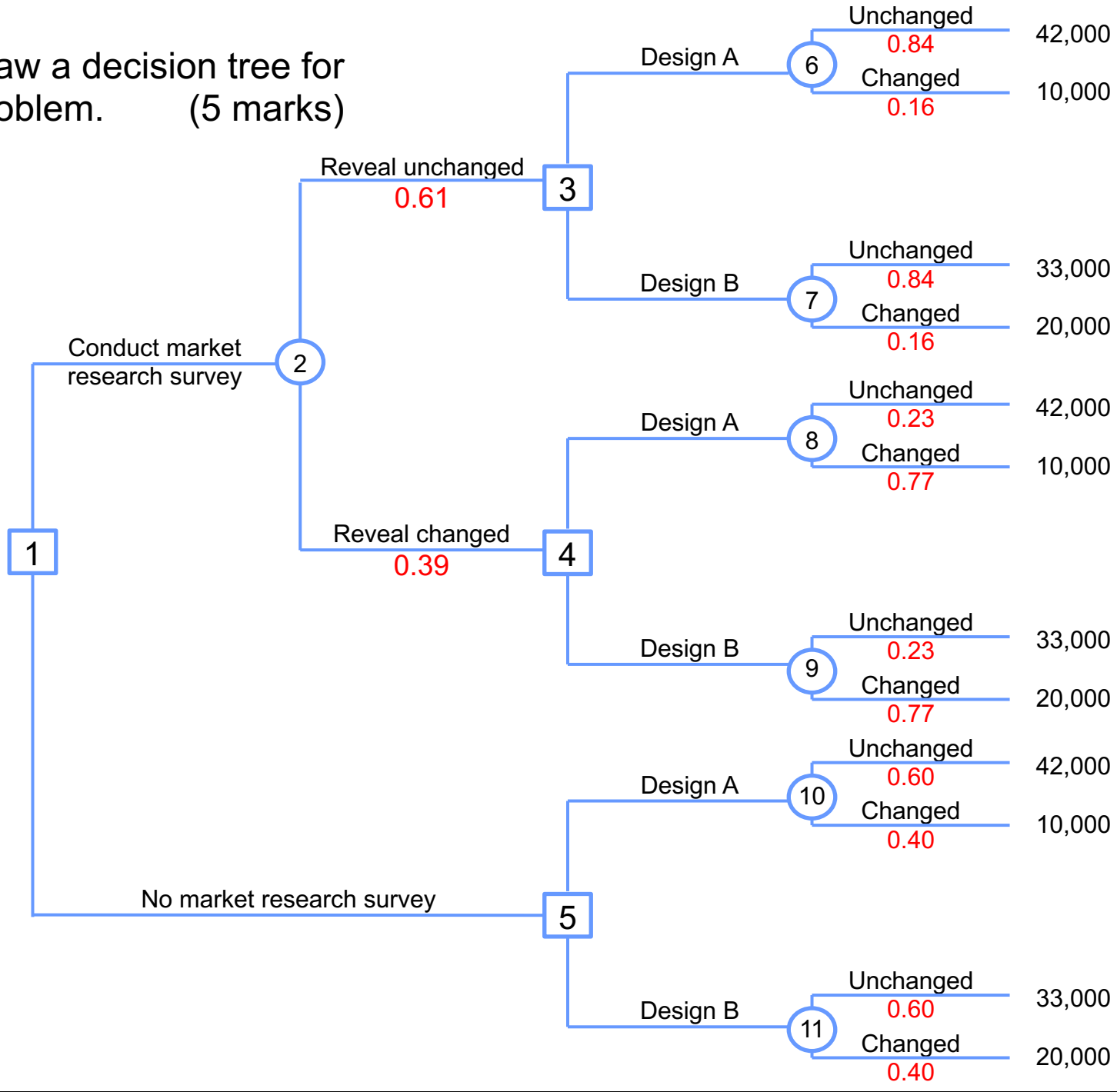
(iii) Apply the Bayes' Theorem to compute joint probabilities and posterior probabilities. (6 marks)

| State of nature $s_j$ | Prior probabilities $P(s_j)$ | Conditional probabilities $P(RU \mid s_j)$ | Joint probabilities $P(RU \cap s_j)$ | Posterior probabilities $P(s_j \mid RU)$ |
|---|---|---|---|---|
| *U* | 0.6 | 0.85 | 0.51 | 0.84 |
| *C* | 0.4 | 0.25 | 0.1 | 0.16 |
| | | | $P(RU) = 0.61$ | 1.00 |

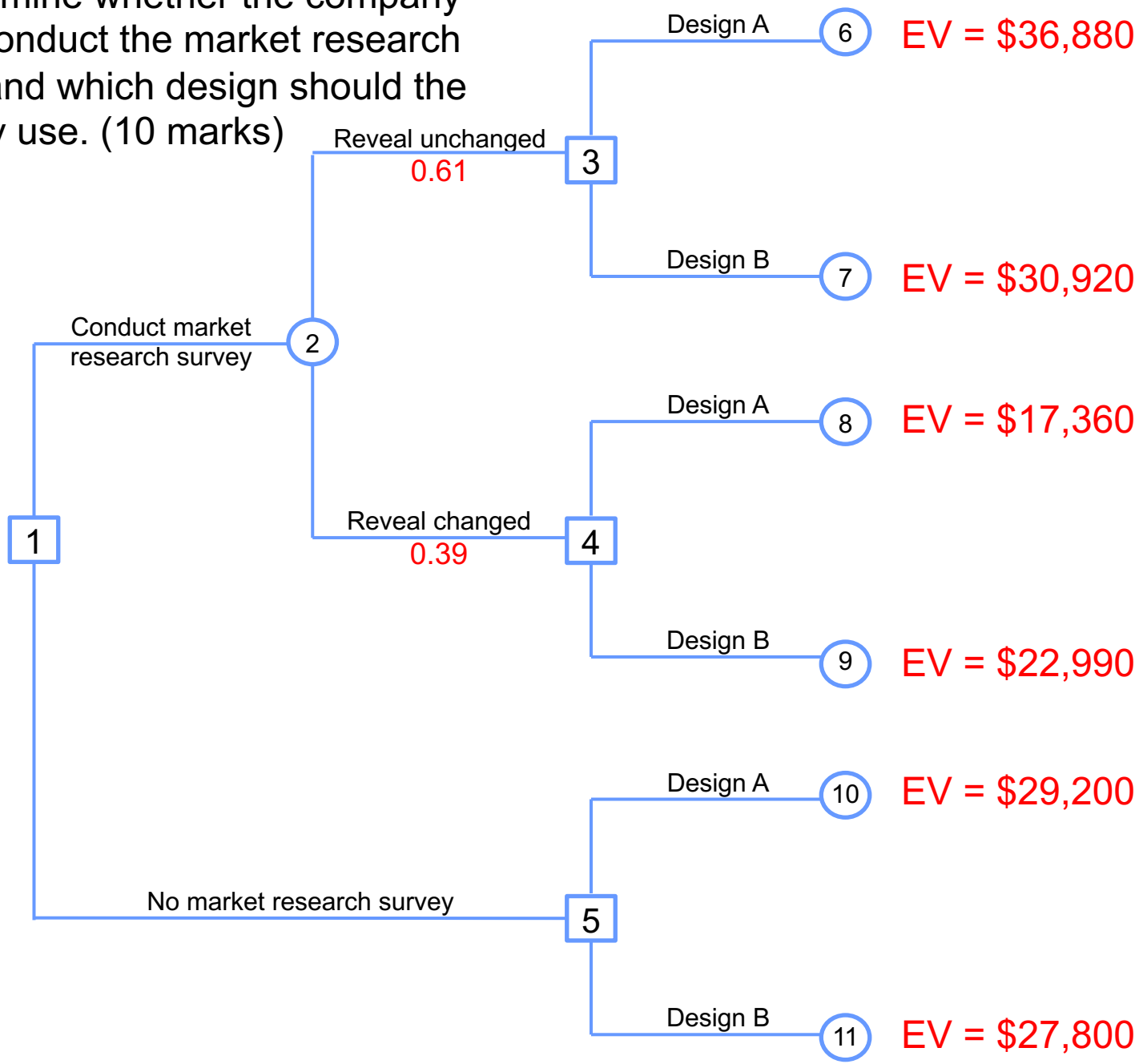
# Question 4

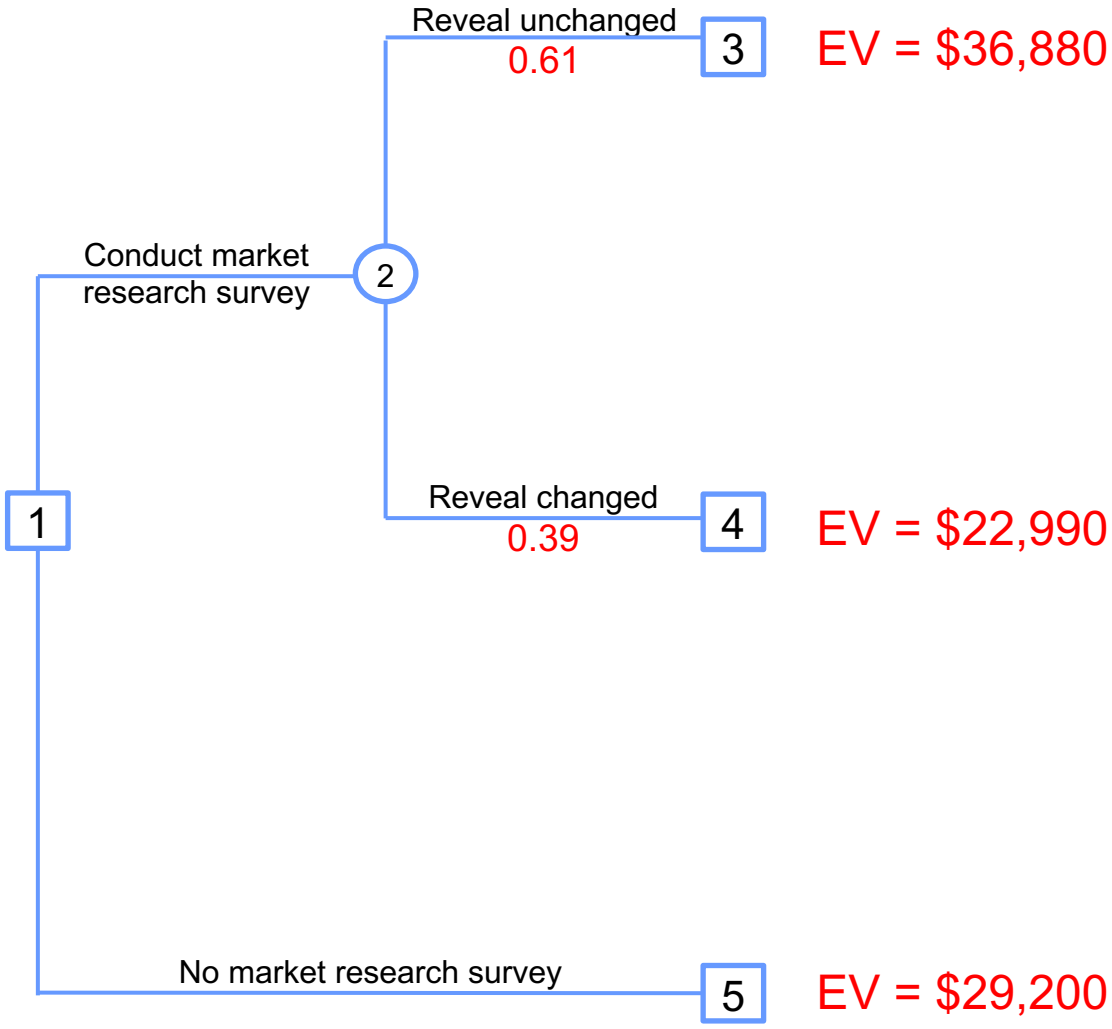
| State of nature $s_j$ | Prior probabilities $P(s_j)$ | Conditional probabilities $P(RC \mid s_j)$ | Joint probabilities $P(RC \cap s_j)$ | Posterior probabilities $P(s_j \mid RC)$ |
|---|---|---|---|---|
| *U* | 0.6 | 0.15 | 0.09 | 0.23 |
| *C* | 0.4 | 0.75 | 0.3 | 0.77 |
| | | | $P(RC) = 0.39$ | 1.00 |

(iv) Draw a decision tree for this problem. (5 marks)

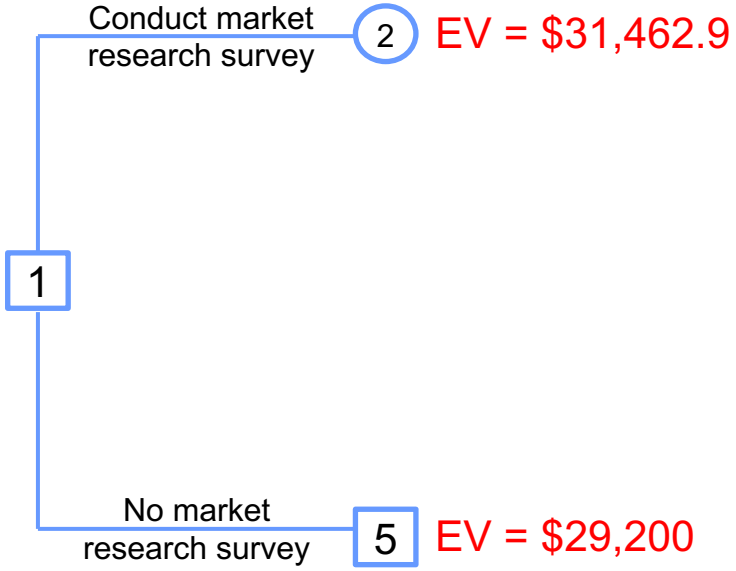


(v) Determine whether the company should conduct the market research survey, and which design should the company use. (10 marks)




Reveal unchanged
0.61
3
EV = $36,880
Conduct market
research survey
2
1
Reveal changed
0.39
4
EV = $22,990
No market research survey
5
EV = $29,200

# Question 4




Decision 1: Blake Electronics Corporation should conduct the market research survey

Decision 2: If the market research reveals unchanged, the company should use design A.

If the market research reveals changed, the company should use design B.

# Question 5

Trading volume on the New York Stock Exchange is heaviest during the first half hour (early morning) and last half hour (late afternoon) of the trading day. The early morning trading volumes (millions of shares) for 13 days in January and February are shown here. The probability distribution of trading volumes is approximately normal.

| | | | | | | |
|---|---|---|---|---|---|---|
| 214 | 202 | 174 | 163 | 198 | 171 | 265 |
| 212 | 211 | 194 | 201 | 211 | 180 | |

(i) Compute the mean and standard deviation to use as estimates of the population mean and standard deviation (round up to integers).

(5 marks)

# Question 5

| $x_i$ | $\bar{x}$ | $(x_i - \bar{x})$ | $(x_i - \bar{x})^2$ |
|---|---|---|---|
| 214 | 200 | 14 | 196 |
| 202 | 200 | 2 | 4 |
| 174 | 200 | -26 | 676 |
| 163 | 200 | -37 | 1369 |
| 198 | 200 | -2 | 4 |
| 171 | 200 | -29 | 841 |
| 265 | 200 | 65 | 4225 |
| 212 | 200 | 12 | 144 |
| 211 | 200 | 11 | 121 |
| 194 | 200 | -6 | 36 |
| 201 | 200 | 1 | 1 |
| 211 | 200 | 11 | 121 |
| 180 | 200 | -20 | 400 |
| | | Total | 8138 |

Sample mean = 200

Sample variance = $s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1} = \frac{8138}{12} = 678.17$

Sample standard deviation = $s = \sqrt{678.17} = 26$

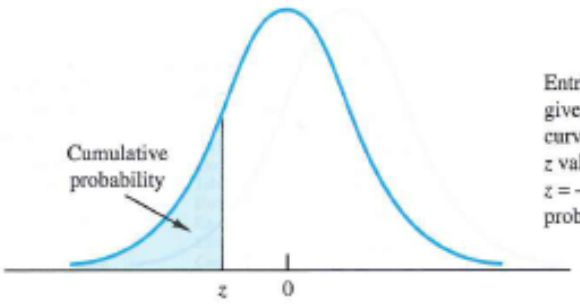
# Question 5

(ii) What is the probability that, on a randomly selected day, the early morning trading volume will be less than 180 million shares? (3 marks)

If $x = 180$, $z = \frac{180-200}{26} = -0.77$

According to Appendix 1, $P(z \leq -0.77) = 0.2206$

## CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION




Entries in this table give the area under the curve to the left of the $z$ value. For example, for $z = -.85$, the cumulative probability is .1977.

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| −2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| −2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| −2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| −2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| −2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| −2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| −2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| −2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| −2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| −2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| −1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| −1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| −1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| −1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| −1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| −1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| −1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| −1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| −1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| −1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| −.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| −.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| −.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| −.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| −.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| −.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| −.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| −.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| −.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| −.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

# Question 5

(iii) What is the probability that, on a randomly selected day, the early morning trading volume will exceed 230 million shares? (4 marks)

If $x = 230$, $z = \frac{230-200}{26} = 1.15$

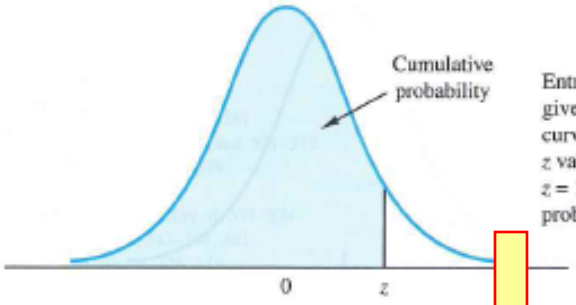
According to Appendix 2, $P(z \geq 1.15)$

$= 1 - P(z \leq 1.15)$

$= 1 - 0.8749$

$= 0.1251$

## CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION




Entries in the table give the area under the curve to the left of the $z$ value. For example, for $z = 1.25$, the cumulative probability is .8944.

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| .0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| .1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| .2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| .3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| .4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| .5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| .6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| .7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| .8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| .9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9913 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9986 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |

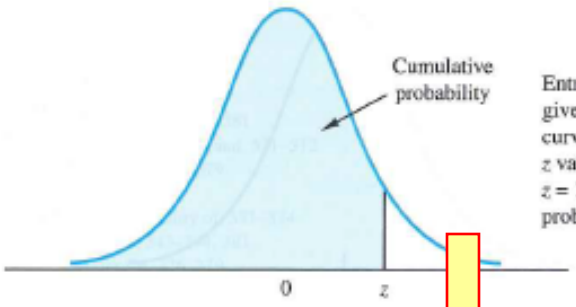
# Question 5

(iv) How many shares would have to be traded for the early morning trading volume on a particular day to be among the busiest 5% of days? (5 marks)

If the probability or area in the upper tail of the curve is 5% or 0.05, then the area under the curve to the left of unknown *z* value must equal 0.95. According to Appendix 2, *z* = 1.645. $z = \frac{x-200}{26} = 1.645$.

*x* = 1.645(26) + 200 = 242.77 million share.

## CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION




Entries in the table give the area under the curve to the left of the $z$ value. For example, for $z = 1.25$, the cumulative probability is .8944.

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| .0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| .1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| .2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| .3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| .4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| .5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| .6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| .7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| .8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| .9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9913 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9986 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |

# Question 5

A sales manager collected the following data on annual sales for new customer accounts and the number of years of experience for a sample of 10 salespersons.

| Salesperson | Years of experience | Annual sales ($1000s) |
|---|---|---|
| 1 | 1 | 80 |
| 2 | 3 | 97 |
| 3 | 4 | 92 |
| 4 | 4 | 102 |
| 5 | 6 | 103 |
| 6 | 8 | 111 |
| 7 | 10 | 119 |
| 8 | 10 | 123 |
| 9 | 11 | 117 |
| 10 | 13 | 136 |

# Question 5

(v) Develop an estimated regression equation that can be used to predict annual sales given the years of experience. (7 marks)

| i | $x_i$ | $y_i$ | $x_i - \bar{x}$ | $y_i - \bar{y}$ | $(x_i - \bar{x})(y_i - \bar{y})$ | $(x_i - \bar{x})^2$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 80 | -6 | -28 | 168 | 36 |
| 2 | 3 | 97 | -4 | -11 | 44 | 16 |
| 3 | 4 | 92 | -3 | -16 | 48 | 9 |
| 4 | 4 | 102 | -3 | -6 | 18 | 9 |
| 5 | 6 | 103 | -1 | -5 | 5 | 1 |
| 6 | 8 | 111 | 1 | 3 | 3 | 1 |
| 7 | 10 | 119 | 3 | 11 | 33 | 9 |
| 8 | 10 | 123 | 3 | 15 | 45 | 9 |
| 9 | 11 | 117 | 4 | 9 | 36 | 16 |
| 10 | 13 | 136 | 6 | 28 | 168 | 36 |
| Total | 70 | 1080 | | | 568 | 142 |
| | $\bar{x} = 7$ | $\bar{y} = 108$ | | | | |

# Question 5

(v) Develop an estimated regression equation that can be used to predict annual sales given the years of experience. (7 marks)

$$b_1 = \frac{\sum_i^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_i^n (x_i - \bar{x})^2} = \frac{568}{142} = 4$$

$$b_0 = \bar{y} - b_1\bar{x} = 108 - 4(7) = 80$$

$$\hat{y} = 80 + 4x$$

(vi) Use the regression equation to predict annual sales for a salesperson with 9 years of experience. (1 mark)

$$\hat{y} = 80 + 4(9) = \$116{,}000$$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