**QUESTION 1**

A manufacturer produces three products, X, Y and Z with two machines A and B. The time taken to produce each unit of X is 20 minutes for machine A and 26 minutes for machine B. Each unit produced of Y takes 24 minutes using machine A and 28 minutes using machine B. Lastly, the time taken to produce each unit of Z is 20 minutes for machine A and 16 minutes for machine B. In a particular week, the manufacturer has a limited time of working hours. In particular, 48 hours of work are available on machine A and 52 hours are available on machine B. Moreover, each week starts with a stock of 60 units of X, 50 units of Y and 40 units of Z.

Weekly demand for the products are stochastic but the manufacture estimates an average of 70 units for each product X, Y and Z being demanded on a weekly basis.

(i) Using the information above, formulate the linear programming (LP) model for the manufacturer, if they wish to plan production in order to end the week with the maximum possible stock. (6 marks)

(ii) This question requires you to write out the Excel formula for specific cells from the Excel sheet provided below. Write out the Excel formulas to cells B13, B17 and B19 as if you are typing out the formulae in Excel. Please clearly label your answers and write out your answers in your script booklet and NOT on the exam paper itself. (4 marks)

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **Question 1** | | | |
| 2 | | | | |
| 3 | **Product** | **X** | **Y** | **Z** |
| 4 | Machine A | 20 | 24 | 20 |
| 5 | Machine B | 26 | 28 | 16 |
| 6 | | | | |
| 7 | **Optimal Solutions** | | | |
| 8 | | | | |
| 9 | Decision variables | X | Y | Z |
| 10 | Number of Products | | | |
| 11 | | | | |
| 12 | Objective function | | | |
| 13 | Maximum Possible Stock | | | |
| 14 | | | | |
| 15 | Constraints | LHS | Inequality | RHS |
| 16 | Time on Machine A | | ≤ | 2880 |
| 17 | Time on Machine B | | ≤ | 3120 |
| 18 | Minimum X Products | | ≥ | 10 |
| 19 | Minimum Y Products | | ≥ | 20 |
| 20 | Minimum Z Products | | ≥ | 30 |

The LP has been solved and the sensitivity analysis report generated. For the following questions, please refer to this sensitivity analysis output to answer the questions.

Variable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$10 | Number of Products X | 64 | 0 | 1 | 0.625 | 0 |
| $C$10 | Number of Products Y | 20 | 0 | 1 | 0.2 | 1E+30 |
| $D$10 | Number of Products Z | 56 | 0 | 1 | 0 | 0.384615385 |

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$16 | Time on Machine A LHS | ? | 0.05 | 2880 | 675 | 200 |
| $B$17 | Time on Machine B LHS | 3120 | 0 | 3120 | 260 | 540 |
| $B$18 | Minimum X Products LHS | 64 | 0 | 10 | 54 | 1E+30 |
| $B$19 | Minimum Y Products LHS | 20 | -0.2 | 20 | 61.36363636 | 20 |
| $B$20 | Minimum Z Products LHS | 56 | 0 | 30 | 26 | 1E+30 |

(iii) For the optimal solution, how much time in hours is spent on using Machine A? (2 marks)

(iv) For the optimal solution, how much stock of Product X is left? How much stock of Product Y is left? How much stock of Product Z is left? (3 marks)

(v) Would the solution change if the number of hours available on machine A was 50 hours instead of 48 hours? If yes, then what is the value of the objective function? (4 marks)

(vi) What would the optimal solution be if the machines can now produce an extra 0.5 units of Product Y? To answer this question, think about what it means in terms of the objective coefficient of Product Y in the objective function. (4 marks)

(vii) Define reduced cost. Explain why the reduced costs for products X, Y and Z are equal to 0? (2 marks)

(i)
Let $x$ = # product X manufactured
$y$ = # product Y manufactured
$z$ = # product Z manufactured
Max $z = (x + 60 – 70) + (y + 50 – 70) + (z + 40 – 70) = x + y + z – 60$
Such that,
Machine A: $20x + 24y + 20z \leq 2880$
Machine B: $26x + 28y + 16z \leq 3120$
Minimum x: $x \geq 70\text{-}60 = 10$
Minimum y: $y \geq 70\text{-}50 = 20$
Minimum z: $y \geq 70\text{-}40 = 30$

(ii)
B13: =B10+C10+D10-60 (or equivalent)
B17: =SUMPRODUCT(B5:D5,B10:D10) (or
equivalent)
B19: =C10

(iii)
Optimal solutions are, $x = 64$, $y = 20$ and $z = 56$
Using Machine A constraint, $20x + 24y + 20z \leq 2880$
Using Machine A constraint, $20(64) + 24(20) + 20(56) = 2880$ minutes = 48 hours

(iv)
Optimal solutions are, $x = 64$, $y = 20$ and $z = 56$
Product X = 60 + 64 – 70 = 54
Product Y = 50 + 20 – 70 = 0
Product Z = 40 + 56 – 70 = 26

(v)
50 – 48 = 2 hours = 120 minutes
It is within the range of feasibility (within allowable increase)
Therefore the Shadow Price is valid
Objective function value increases by 120 Å~ 0.05 = 6 units
Therefore, the objective function value is 80 + 6 = 86

(vi)
This means the coefficient of y is increase to 1.5
1.5 doesn't falls within the range of optimality
Optimal solution will change
Re-run the model in Excel to find the answer

(vii)
Reduce cost is how much the objective function coefficient of each decision variable would have to improve before that variable could assume a positive value in the optimal solution.
Reduce cost is zero because all the optimal solutions are positive already.

**QUESTION 2**

**(i) 15 marks**

The Grand Strand Oil Company produces regular and premium gasoline for independent service stations. The Grand Strand refinery manufactures the gasoline products by blending 3 petroleum components. The gasolines are sold at different prices, and the petroleum components have different costs. Data available show that regular gasoline can be sold for $2.9 per gallon and premium gasoline for $3 per gallon. For the current production planning period, Grand Strand can obtain the 3 petroleum components at the cost per gallon and in the quantities as follows:

| Petroleum component | Cost/gallon | Maximum available |
|---|---|---|
| 1 | $2.5 | 5,000 gallons |
| 2 | $2.6 | 10,000 gallons |
| 3 | $2.84 | 10,000 gallons |

Product specifications, as shown in the following table, for the regular and premium gasolines restrict the amounts of each component that can be used in each gasoline product. Current commitments to distributors require Grand Strand to product at least 10,000 gallons of regular gasoline.

| Product | Specifications |
|---|---|
| Regular gasoline | At most 30% component 1 |
| | At least 40% component 2 |
| | At most 20% component 3 |
| Premium gasoline | At least 25% component 1 |
| | At most 45% component 2 |
| | At least 30% component 3 |

Formulate the LP model to determine the amounts of gallons of components 1, 2, and 3 should Grand Strand mix or blend into regular and premium gasoline to reach the maximum profits.

(i)

Let $x_{11}$ = gallons of component 1 in regular gasoline
$x_{12}$ = gallons of component 1 in premium gasoline
$x_{21}$ = gallons of component 2 in regular gasoline
$x_{22}$ = gallons of component 2 in premium gasoline
$x_{31}$ = gallons of component 3 in regular gasoline
$x_{32}$ = gallons of component 3 in premium gasoline

Maximize the total profit or $z$

$= 2.9(x_{11}+x_{21}+x_{31}) + 3(x_{12}+x_{22}+x_{32}) - 2.5(x_{11}+x_{12}) - 2.6(x_{21}+x_{22}) - 2.84(x_{31}+x_{32})$

$= 0.4x_{11} + 0.5x_{12} + 0.3x_{21} + 0.4x_{22} + 0.06x_{31} + 0.16x_{32}$

Subject to

$x_{11} + x_{12} \leq 5000$ (Component 1 capacity)

$x_{21} + x_{22} \leq 10000$ (Component 2 capacity)

$x_{31} + x_{32} \leq 10000$ (Component 3 capacity)

$x_{11} \leq 0.3(x_{11} + x_{21} + x_{31})$

i.e., $0.7x_{11} - 0.3x_{21} - 0.3x_{31} \leq 0$ (Product specification 1)

$-0.4x_{11} + 0.6x_{21} - 0.4x_{31} \geq 0$ (Product specification 2)

$-0.2x_{11} - 0.2x_{21} + 0.8x_{31} \leq 0$ (Product specification 3)

$0.75x_{12} - 0.25x_{22} - 0.25x_{32} \geq 0$ (Product specification 4)

$-0.45x_{12} + 0.55x_{22} - 0.45x_{32} \leq 0$ (Product specification 5)

$-0.3x_{12} - 0.3x_{22} + 0.7x_{32} \geq 0$ (Product specification 6)

$x_{11} + x_{21} + x_{31} \geq 10000$ (Regular gasoline demand)

$x_{11}, x_{12}, x_{21}, x_{22}, x_{31}$ and $x_{32} \geq 0$ (Non-negativity)

**(ii) 10 marks**

Epsilon Airlines services predominately the eastern and south-eastern United States. The vast majority of Epsilon's customers make reservations through Epsilon's website, but a small percentage of customers make reservations via phone. Epsilon employs call-center personnel to handle these reservations along with any problems with the website reservation system and for the rebooking of flights for customers if their plans change or their travel is disrupted. Having too many employees on hand is a waste of money, but having too few results in very poor customer service and the potential loss of customers.

Epsilon analysts have estimated the minimum number of call-center employees needed by day of week for the upcoming vacation season. These estimates are as follows:

| Day | Minimum number of employees needed |
|---|---|
| Monday | 75 |
| Tuesday | 50 |
| Wednesday | 45 |
| Thursday | 60 |
| Friday | 90 |
| Saturday | 75 |
| Sunday | 45 |

The call-center employees work five consecutive days and then have two consecutive days off. An employee may start any day of the week. Each call-center employee receives the same salary. Assume that the schedule cycles and ignore start-up and stopping of the schedule.

Formulate the LP model that will minimize the total number of call-center employees needed to meet the minimum requirements.

(ii)

Let $x_i$ be the number of call-center employees who start working on day i (i= 1 = Monday; i= 2 = Tuesday, etc.)

Minimize the total number of employees or $z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$

Subject to

| | |
|---|---|
| $x_1 + x_4 + x_5 + x_6 + x_7 \geq 75$ | (Minimum employees on Monday) |
| $x_1 + x_2 + x_5 + x_6 + x_7 \geq 50$ | (Minimum employees on Tuesday) |
| $x_1 + x_2 + x_3 + x_6 + x_7 \geq 45$ | (Minimum employees on Wednesday) |
| $x_1 + x_2 + x_3 + x_4 + x_7 \geq 60$ | (Minimum employees on Thursday) |
| $x_1 + x_2 + x_3 + x_4 + x_5 \geq 90$ | (Minimum employees on Friday) |
| $x_2 + x_3 + x_4 + x_5 + x_6 \geq 75$ | (Minimum employees on Saturday) |
| $x_3 + x_4 + x_5 + x_6 + x_7 \geq 45$ | (Minimum employees on Sunday) |
| $x_i \geq 0$ | for all i |

**Question 3**

**(i) 15 marks**

Bennet & Biden Co. has two plants for manufacturing shipping containers. These shipping containers allows customers to transport goods efficiently across different countries. Plant 1 can produce up to 400 units of shipping containers per day, whereas plant 2 can produce as many as 500 units of shipping containers per day. Instead of shipping the shipping containers directly to the customers, each shipping container must be sent to a distribution warehouse first. After packaging the shipping containers at the distribution warehouse, the units are then shipped to the three customers.

Bennet & Biden Co. would also like to keep 75 units of shipping containers as backstock at each distribution warehouse, just in case for unexpected fluctuations of demand in the near future. The current demand amounts of customer 1, customer 2, and customer 3 are 400, 300, and 200 respectively.

The transportation cost per unit of shipping container is given in the table below. P denotes plants, W denotes the distribution warehouses and C denotes the customers. Please read the tables from left to right as this is the direction of the flow of the shipping containers. Note that not all paths are possible.

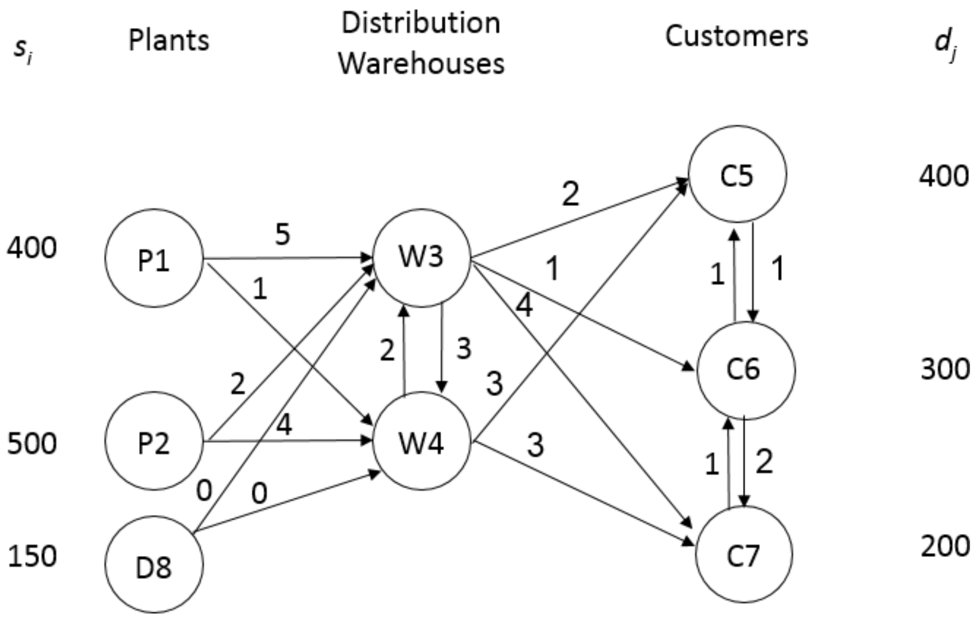
| | W3 | W4 |
|---|---|---|
| P1 | 5 | 1 |
| P2 | 2 | 4 |

| | C5 | C6 | C7 |
|---|---|---|---|
| W3 | 2 | 1 | 4 |
| W4 | 3 | - | 3 |

| | W3 | W4 |
|---|---|---|
| W3 | - | 3 |
| W4 | 2 | - |

| | C5 | C6 | C7 |
|---|---|---|---|
| C5 | - | 1 | - |
| C6 | 1 | - | 2 |
| C7 | - | 1 | - |

Draw a network diagram for this transshipment problem based on the information above. Then, formulate an integer linear programming (IP) model that will determine how to fulfill each customer's order while not exceeding the capacity of any plant at the minimum cost. You do not need to solve the model.



Let $x_{ij}$ = # shipping containers to be shipped from i to j

Minimize $z = 5x_{13} + 1x_{14} + \ldots + 1x_{76} + 0x_{83} + 0x_{84}$

Subject to

| | |
|---|---|
| $x_{13} + x_{14} + x_{15} + x_{16} + x_{17}$ | $\leq 400$ |
| $x_{23} + x_{24} + x_{25} + x_{26} + x_{27}$ | $\leq 500$ |
| $x_{83} + x_{84} + x_{85} + x_{86} + x_{87}$ | $\leq 150$ |
| $x_{33} + x_{34} + x_{35} + x_{36} + x_{37}$ | $\leq 1050$ |
| $x_{43} + x_{44} + x_{45} + x_{46} + x_{47}$ | $\leq 1050$ |
| $x_{13} + x_{23} + x_{43} + x_{83}$ | $= 1050 + 75$ |
| $x_{14} + x_{24} + x_{34} + x_{84}$ | $= 1050 + 75$ |
| $x_{15} + x_{25} + x_{85} + x_{35} + x_{45} + x_{65} - x_{56}$ | $= 400$ |
| $x_{16} + x_{26} + x_{85} + x_{36} + x_{56} + x_{76} - x_{65} - x_{67}$ | $= 300$ |
| $x_{17} + x_{27} + x_{87} + x_{37} + x_{47} + x_{67} - x_{76}$ | $= 200$ |

For all $x_{ij}$ along non-existing shipping routes = 0

$x_{ij} \geq 0$ for all i and j and integer

**(ii) 10 marks**

The McConnell Nuts Company sells three different half-kilogram bags of peanuts mixes: Party Nuts, Mixed Nuts and Premium Mixed Nuts. These generate per bag revenue of $1.05, $2.12, and $2.87, respectively. The table below shows the makeup of each mix, the available ingredients for the next week of production, and the cost of each ingredient per kilogram.

| **Ingredients** | | | | |
|---|---|---|---|---|
| | **Peanuts** | **Cashews** | **Almond** | **Brazil Nuts** |
| **Party Nuts** | 100% | - | - | - |
| **Mixed Nuts** | 50% | 10% | 30% | 10% |
| **Premium Mixed Nuts** | - | 40% | 30% | 30% |

| | **Kilogram Available** | **Cost per kilogram** |
|---|---|---|
| **Peanuts** | 150 | $1.87 |
| **Cashews** | 100 | $2.99 |
| **Almond** | 200 | $3.47 |
| **Brazil Nuts** | 50 | $5.49 |

Formulate a linear programming (LP) model to determine the number of kilogram of peanuts mixes to produce to reach the maximum total profits. You do not need to solve the model.

Hint: There should only be three decision variables.

Let

$X_1$ = # kg of Party Nuts

$X_2$ = # kg of Mixed Nuts

$X_3$ = # kg of Premium Mixed Nuts

$$
\begin{aligned}
\text{Max total profit or } z = {} & 2*(1.05X_1 + 2.12X_2 + 2.87X_3) \\
& - 1.87*(X_1 + 0.5X_2) \\
& - 2.99*(0.1X_2 + 0.4X_3) \\
& - 3.47*(0.3X_2 + 0.3X_3) \\
& - 5.49*(0.1X_1 + 0.3X_3)
\end{aligned}
$$

s.t

$X_1 + 0.5X_2 \leq 150$

$0.1X_2 + 0.4X_3 \leq 100$

$0.3X_2 + 0.3X_3 \leq 200$

$0.1X_2 + 0.3X_3 \leq 50$

$X_1, X_2, X_3 \geq 0$

**Question 4**

Joshua Tang, a store manager of a local general store in Ararat, is considering signing a new contract with Daylesford Fruits or with a local Ararat farm for daily supplies of fresh cherries. Daylesford Fruits may reject the contract, but Joshua Tang may also negotiate to sign the contract for 2 or 5 years. Joshua Tang must decide to sign the contact with Daylesford Fruits or signing a contract with the local farm in Ararat, but he knows that if he signs with the local Ararat farm, his profits will not be as high in comparison to Daylesford Fruits. If any contract is rejected, it is assumed that there will be a quantifiable cost to the store. Joshua Tang would like to maximise the store profits and the profits for the different contract lengths are summarised in the following payoff table (in thousands of Australian dollars).

| | STATE OF NATURE | | |
|---|---|---|---|
| DECISION ALTERNATIVE | Reject, s1 | 2 Years, s2 | 5 Years, s3 |
| DAYLESFORD FRUITS, d1 | -2 | 3 | 9 |
| LOCAL ARARAT FARM, d2 | -1 | 1 | 5 |

(i) Apply the Optimistic Approach to recommend a decision to Joshua Tang. (1 mark)

Joshua Tang made an initial assessment on what he thinks Daylesford Fruits and the local Ararat farm may do and came up with the following probabilities:

Daylesford Fruits: P(Reject) = 0.4, P(2 Years) = 0.5, P(5 Years) = 0.1

The local Ararat farm: P(Reject) = 0.2, P(2 Years) = 0.3, P(5 Years) = 0.5

(ii) Draw a decision tree and recommend a decision strategy to Joshua Tang on whom he should sign the contract with. Clearly show your calculations and provide a justification for your decision strategy. (5 marks)

For a consulting fee of $1,500, David Ernesto Consultancy (DEC) will review the plans for the Daylesford Fruits contract and indicate the overall chances of a favourable outcome to Joshua Tang. Denote favourable review by F, and unfavourable review by U. Joshua Tang believes that the following conditional probabilities are realistic appraisals of DEC's evaluation accuracy. No review is conducted on the local Ararat farm because there was insufficient information for DEC to do so.

P(F|Rejection) = 0.1, P(F|2 years) = 0.5, P(F|5 years) = 0.7

P(U|Rejection) = 0.9, P(U|2 years) = 0.5, P(U|5 years) = 0.3

(iii) Apply Bayes' Theorem to compute the posterior probabilities for Daylesford Fruits for both the Favourable and Unfavourable review cases. (6 marks)

(iv) Draw the decision tree, determine the recommended decision strategy(ies) and the expected value for the case where Joshua Tang has already decided to conduct the review on Daylesford Fruits. Make the assumption that after receiving the review from DEC, Joshua Tang only has two options of whom he can sign the contract with; namely, Daylesford Fruits or the local Ararat farm.

Draw the decision tree with clear labels, show all your calculations and provide justification(s) for your decision strategy(ies) to Joshua Tang. (10 marks)

(v) Construct a risk profile for the optimal decision strategy from part (iv). (2 marks)

(vi) Why should we review the risk profile associated with an optimal decision alternative? (1 mark)

(i)

Recommend $d_1$ (show relevant working)

(ii)

(Draw decision tree with correct nodes and labels)

EV(Daylesford Fruits) = 0.4*(-2) + 0.5*(3) + 0.1*(9) = 1.6

EV(Ararat Farm) = 0.2*(-1) + 0.3*(1) + 0.5*(5) = 2.6

Since EV(Ararat Farm) > EV(Daylesford Fruits), Joshua Tang should sign with Ararat farm.

(iii)

If the review is **favourable (F)**, then the probability table is given as follows:

| States of Nature | Prior $P(s_j)$ | Conditional $P(F \mid s_j)$ | Joint $P(F \cap s_j)$ | Posterior $P(s_j \mid F)$ |
|---|---|---|---|---|
| Reject | 0.40 | 0.10 | 0.04 | 0.1111 |
| 2 years | 0.50 | 0.50 | 0.25 | 0.6944 |
| 5 years | 0.10 | 0.70 | 0.07 | 0.1944 |
| | | P(F) = | 0.36 | 1 |

If the review is **unfavorable (U)**, then the probability table is given as follows:

| States of Nature | Prior $P(s_j)$ | Conditional $P(U \mid s_j)$ | Joint $P(U \cap s_j)$ | Posterior $P(s_j \mid U)$ |
|---|---|---|---|---|
| Reject | 0.40 | 0.90 | 0.36 | 0.5625 |
| 2 years | 0.50 | 0.50 | 0.25 | 0.3906 |
| 5 years | 0.10 | 0.30 | 0.03 | 0.0469 |
| | | P(U) = | 0.64 | 1 |

(iv)

(Draw decision tree with correct nodes and label)

EV(4) = 3.6108

EV(5) = 2.6

EV(2) = max{3.6108,2.6} = 3.6 108

EV(6) = 0.4689

EV(7) = 2.6

EV(3) = max{0.4689,2.6} = 2.6

EV(1) = 0.36*(3.6108) + 0.64*(2.6) = 2.9638 (EV for this case)

If favourable, then sign with Daylesford Fruits since EV(Daylesford) > EV(Ararat).

If unfavourable, then sign with Local Ararat farm since EV(Ararat) > EV(Daylesford).

(v)

| Payoff (in thousands of Australian dollars) | Probability |
|---|---|
| -2 | 0.36*0.1111 = 0.04 |
| -1 | 0.64*0.2 = 0.128 |
| 1 | 0.64*0.3 = 0.192 |
| 3 | 0.36*0.6944 = 0.25 |
| 5 | 0.64*0.5 = 0.32 |
| 9 | 0.36*0.1944 = 0.07 |
| Total | 1 |

(vi)

This can help you identify and understand the risks that you could face. Thus, it can potentially help you manage these risks, and minimise their impact on your plans (e.g. cause the decision maker to choose another decision alternative even though the expected value of the other decision alternative is not as good).

**Question 5**

**Normal Probability Distribution**

As per the Australian Bureau of Statistics, the average 2018 mortgage sizes in Victoria, Australia is approximately $\$409{,}000$. Assume mortgage sizes vary with standard deviations of $\$19{,}500$ and that it is normally distributed.

(i) What is the probability that the mortgage size falls below $\$380{,}000$? (2 marks)

P(X < 380,000)
= P(Z < 380,000-409,000/19,500 )
= P(Z < -1.49)
= 0.0681

(ii) What is the probability that the mortgage size fall between $\$370{,}000$ and $\$450{,}000$? (3 marks)

P(370,000 < X < 450,000)
= P(X < 450,000) – P(X < 370,000)
= P(Z < (450,000-409,000)/19,500) – P(Z < (370,000-409,000)/19,500)
= P(Z < 2.10) – P(Z < -2)
= 0.9821 – 0.0228
= 0.9593

(iii) The State Government of Victoria wants to know the range of mortgage sizes fall in between the lower and upper $10\%$ of the mortgage sizes. Calculate the lower and upper limits of the range. (4 marks)

P(X < a) = 0.10
P(Z < (a-409,000)/19,500) = 0.10
From table, (a-409,000)/19,500 = -1.28
a = -1.28 × 19,500 + 409,000
a = $384,040

P(X > a) = 0.10
P(X < a) = 0.90
P(Z < (a-409,000)/19,500 = 0.10
From table, (a-409,000)/19,500 = 1.28
a = 1.28 × 19,500 + 409,000
a = $433,960

(iv) Find the value of $d$ such that the probability between $\$385{,}000$ and $d$ will have a probability of $0.4503$. In other words, find the value of $d$ such that $P(385{,}000 < X < d) = 0.4503$, where $X$ represents the 2018 mortgage sizes in Victoria, Australia. (4 marks)

P(385,000 < X < d) = 0.4503
P(X < d) – P(X < 385,000) = 0.4495
P(Z < (d-409,000)/19,500) – P(Z < (385,000-409,000)/19,500) = 0.4503
P(Z < (d-409,000)/ 19,500) – P(Z < -1.23) = 0.4503
P(Z < (d-409,000)/ 19,500) – 0.1093 = 0.4503

P(Z < (d-409,000)/19,500) = 0.5596
(d-409,000)/ 19,500) = 0.15
d = 0.15 × 19,500 + 409,000 = 411,925 (

Therefore, P(385,000 < X < 411,925) = 0.3952

d = 411,925

**Simple Regression**

Eastern Grey Kangaroo is a marsupial mammal that belongs to a small group called macropods in Australia. For a quick research study, a scientist wanted to examine whether you could use nasal length (mm) to predict nasal width (mm) for male Eastern Grey Kangaroos. The table below shows a random group of 12 male Easter Grey Kangaroo's nasal length and width.

| Eastern Grey Kangaroos | Nasal length (mm) | Nasal width (mm) |
|---|---|---|
| **1** | 606 | 227 |
| **2** | 660 | 240 |
| **3** | 630 | 215 |
| **4** | 672 | 231 |
| **5** | 778 | 263 |
| **6** | 616 | 220 |
| **7** | 727 | 271 |
| **8** | 810 | 284 |
| **9** | 778 | 279 |
| **10** | 820 | 272 |
| **11** | 755 | 268 |
| **12** | 710 | 278 |

(v) Develop an estimated regression equation that can be used to predict nasal width (mm) for male Eastern Grey Kangaroos given the nasal length (mm). (6 marks)

Let $x_i$ = nasal length and let $y_i$ = nasal width

| $i$ | $x_i$ | $y_i$ | $x_i - \bar{x}$ | $y_i - \bar{y}$ | $(x_i - \bar{x})(y_i - \bar{y})$ | $(x_i - \bar{x})^2$ |
|---|---|---|---|---|---|---|
| 1 | 606 | 227 | -107.5 | -27 | 2902.5 | 11556.25 |
| 2 | 660 | 240 | -53.5 | -14 | 749 | 2862.25 |
| 3 | 630 | 215 | -83.5 | -39 | 3256.5 | 6972.25 |
| 4 | 672 | 231 | -41.5 | -23 | 954.5 | 1722.25 |
| 5 | 778 | 263 | 64.5 | 9 | 580.5 | 4160.25 |
| 6 | 616 | 220 | -97.5 | -34 | 3315 | 9506.25 |
| 7 | 727 | 271 | 13.5 | 17 | 229.5 | 182.25 |
| 8 | 810 | 284 | 96.5 | 30 | 2895 | 9312.25 |
| 9 | 778 | 279 | 64.5 | 25 | 1612.5 | 4160.25 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 10 | 820 | 272 | 106.5 | 18 | 1917 | 11342.25 |
| 11 | 755 | 268 | 41.5 | 14 | 581 | 1722.25 |
| 12 | 710 | 278 | -3.5 | 24 | -84 | 12.25 |
| **Average** | 713.5 | 254 | | **Total** | 18909 | 63511 |

$$b_1 = \frac{\sum_i^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_i^n (x_i - \bar{x})^2} = \frac{18909}{63511} = 0.2977$$

$$b_0 = \bar{y} - b_1\bar{x} = 254 - 0.2977 * (713.5) = 41.5911$$

$$\hat{y} = 41.5911 + 0.2977x$$

(vi) Using the regression equation found in part (v), interpret the slope coefficient. (2 marks)

For every 1 mm increase in nasal length of a male Eastern Grey Kangaroo, it is related to a 0.2977 mm increase in nasal width.

(vii) Using the regression equation found in part (v) to predict nasal width (mm) for male Eastern Grey Kangaroos given nasal length is 75 $cm$. (2 marks)

When $x = 750$

$\hat{y} = 41.5911 + 0.2977(750)$

$\hat{y} = 264.87\ mm$

(viii) Given that $SSR = 5{,}629.74$ and $SSE = 1{,}492.26$ compute the coefficient of determination $r^2$ and the sample correlation coefficient $r_{xy}$. (2 marks)

$$SST = SSR + SSE = 5629.74 + 1492.26 = 7122$$

The Coefficient of Determination is $r^2 = \frac{\text{SSR}}{\text{SST}} = \frac{5629.74}{7122} = 0.7905$

Sample Correlation Coefficient

$$r_{xy} = (sign\ of\ b_1)\sqrt{r^2} = +\sqrt{0.7905} = 0.8891$$