MGMT90141 Business Analysis and Decision Making
Review and Revision – Mock Exam 1

Dr. Lusheng Shao

Senior Lecturer of Operations Management
Department of Management and Marketing
Faculty of Business and Economics
The Spot 10.030
Lusheng.shao@unimelb.edu.au

Examination

- Date: 17th November 2020 (Tuesday)
- Time: 10:00am (Melbourne time)
- Venue: online, open book

Examination

- Reading time: 15 minutes
- Writing time: 3 hours
- Write down your tutorial time on the front page of the answer, e.g., Mon 9am.
- Exam paper will be made available under the Assignment section in Canvas.
- Hand-write your answer on A4 papers, scan/photograph and convert it to a PDF file, and then upload the PDF file to Canvas

Examination

- Section one: Answer 1 compulsory question
- Section two: Answer 3 out of 4 selective questions
- Each question carries 25 marks
- Hurdle requirement on the final exam

Five topics

- LP sensitivity analysis
- LP applications in marketing, operations and finance
- Network models and integer programming models
- Decision analysis
- Probability distributions and regression models

Prepare for the final exam

- Lecture slides
- Lecture examples
- Tutorial questions
- Mock exam papers
- Equivalent book chapters
- Weekly summary notes

Question 1

(i) Formulate the linear programming (LP) model for the DCI. (7 marks)

Let x_{11} be the number of cases of model A manufactured;
x_{12} be the number of cases of model A purchased;
x_{21} be the number of cases of model B manufactured;
x_{22} be the number of cases of model B purchased.

Minimize total cost or $z = 10x_{11} + 14x_{12} + 6x_{21} + 9x_{22}$

Subject to

$x_{11} + x_{12} = 100$	(Demand for model A)
$x_{21} + x_{22} = 150$	(Demand for model B)
$4x_{11} + 3x_{21} \leq 600$	(Injection-molding time)
$6x_{11} + 8x_{21} \leq 1080$	(Assembly time)
x_{11}, x_{12}, x_{21}, and $x_{22} \geq 0$	

Question 1

6	Variable Cells						
7			Final	Reduced	Objective	Allowable	Allowable
8	Cell	Name	Value	Cost	Coefficient	Increase	Decrease
9	B9	Model A Manufactured	100	0	10	1.75	1E+30
10	C9	Model A Purchased	0	1.75	14	1E+30	1.75
11	B10	Model B Manufactured	60	0	6	3	2.333333333
12	C10	Model B Purchased	90	0	9	2.333333333	3
13							
14	Constraints						
15			Final	Shadow	Constraint	Allowable	Allowable
16	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
17	B16	Demand for model A LHS	100	12.25	100	11.42857143	100
18	B17	Demand for model B LHS	150	9	150	1E+30	90
19	B18	Injection molding time LHS	?	0	600	1E+30	20
20	B19	Assembly time LHS	?	-0.375	1080	53.33333333	480

(ii) For the optimal solution, how much injection-molding time is spent? (3 marks)

In the optimal solution, x_{11} = 100 and x_{21} = 60. Using these values in the third constraint gives us $4x_{11} + 3x_{22} = 4(100) + 3(60) = 580$ minutes.

Question 1

	Variable Cells							
7			Final	Reduced	Objective	Allowable	Allowable
8	Cell	Name	Value	Cost	Coefficient	Increase	Decrease
9	B9	Model A Manufactured	100	0	10	1.75	1E+30
10	C9	Model A Purchased	0	1.75	14	1E+30	1.75
11	B10	Model B Manufactured	60	0	6	3	2.333333333
12	C10	Model B Purchased	90	0	9	2.333333333	3
13							
14	Constraints						
15			Final	Shadow	Constraint	Allowable	Allowable
16	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
17	B16	Demand for model A LHS	100	12.25	100	11.42857143	100
18	B17	Demand for model B LHS	150	9	150	1E+30	90
19	B18	Injection molding time LHS	?	0	600	1E+30	20
20	B19	Assembly time LHS	?	-0.375	1080	53.33333333	480

(iii) For the optimal solution, how much assembly time is spent? (3 marks)

For the fourth constraint we have $6x_{11} + 8x_{21} = 6(100) + 8(60) = 1{,}080$ minutes.

Question 1

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B9	Model A Manufactured	100	0	10	1.75	1E+30
C9	Model A Purchased	0	1.75	14	1E+30	1.75
B10	Model B Manufactured	60	0	6	3	2.333333333
C10	Model B Purchased	90	0	9	2.333333333	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B16	Demand for model A LHS	100	12.25	100	11.42857143	100
B17	Demand for model B LHS	150	9	150	1E+30	90
B18	Injection molding time LHS	?	0	600	1E+30	20
B19	Assembly time LHS	?	-0.375	1080	53.33333333	480

(iv) Would the solution change if the injection-molding time available were only 580 minutes instead of 600 minutes? (4 marks)

No, the solution would not change. The shadow price is 0 and the value 580 is between the lower bound of 580 (600 – 20) and the upper bound of infinity. Thus, the change in the objective value is 0*(-20)=0 and hence the solution does not change.

Question 1

	Variable Cells						
7			Final	Reduced	Objective	Allowable	Allowable
8	Cell	Name	Value	Cost	Coefficient	Increase	Decrease
9	\$B\$9	Model A Manufactured	100	0	10	1.75	1E+30
10	\$C\$9	Model A Purchased	0	1.75	14	1E+30	1.75
11	\$B\$10	Model B Manufactured	60	0	6	3	2.333333333
12	\$C\$10	Model B Purchased	90	0	9	2.333333333	3

	Constraints						
15			Final	Shadow	Constraint	Allowable	Allowable
16	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
17	\$B\$16	Demand for model A LHS	100	12.25	100	11.42857143	100
18	\$B\$17	Demand for model B LHS	150	9	150	1E+30	90
19	\$B\$18	Injection molding time LHS	?	0	600	1E+30	20
20	\$B\$19	Assembly time LHS	?	-0.375	1080	53.33333333	480

(v) What would the optimal solution be if the purchase cost for each model B case were increased from \$9 to \$10? (4 marks)

As the new coefficient for x_{22} is between the lower bound of 6 (9 – 3) and the upper bound of 11.33 (9 + 2.33), the current solution remains optimal, that is x_{11} = 100, x_{12} = 0, x_{21} = 60, and x_{22} = 90. Only the total cost change from \$2,170 to \$2,260.

Question 1

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B9	Model A Manufactured	100	0	10	1.75	1E+30
C9	Model A Purchased	0	1.75	14	1E+30	1.75
B10	Model B Manufactured	60	0	6	3	2.333333333
C10	Model B Purchased	90	0	9	2.333333333	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B16	Demand for model A LHS	100	12.25	100	11.42857143	100
B17	Demand for model B LHS	150	9	150	1E+30	90
B18	Injection molding time LHS	?	0	600	1E+30	20
B19	Assembly time LHS	?	-0.375	1080	53.33333333	480

(vi) How much would total cost increase or decrease if the assembly time available were 1100 minutes instead of 1080 minutes? (4 marks)

The shadow price for this constraint is -0.375, and the upper bound is 1,133.33 (1,080 + 53.33). The increase of 20 minutes of the assembly time will result in a decrease in total cost of 20(-0.375) = -$7.5.

Question 2 (i)

Let x_1 be the number of suits produced;
x_2 be the number of sport coats produced;
x_3 be the hours of overtime for the cutting operation;
x_4 be the hours of overtime for the sewing operation.

Maximize the total profit or $z = 190x_1 + 150x_2 - 15x_3 - 10x_4$

Subject to

$1.2x_1 + 0.8x_2 - x_3 \leq 200$	(Cutting time available)
$0.7x_1 + 0.6x_2 - x_4 \leq 180$	(Sewing time available)
$6x_1 + 4x_2 \leq 1200$	(Fabric available)
$x_3 + x_4 \leq 100$	(Maximum overtime)
$x_1 \geq 100$	(Minimum number of suits)
$x_2 \geq 75$	(Minimum number of sport coats)
$x_1, x_2, x_3,$ and $x_4 \geq 0$	(Non-negativity)

Question 2 (ii)

Let x_{11} = gallons of component 1 in regular gasoline
x_{12} = gallons of component 1 in premium gasoline
x_{21} = gallons of component 2 in regular gasoline
x_{22} = gallons of component 2 in premium gasoline
x_{31} = gallons of component 3 in regular gasoline
x_{32} = gallons of component 3 in premium gasoline

Maximize the total profit or z

$= 2.9(x_{11}+x_{21}+x_{31}) + 3(x_{12}+x_{22}+x_{32}) - 2.5(x_{11}+x_{12}) - 2.6(x_{21}+x_{22}) - 2.84(x_{31}+x_{32})$

$= 0.4x_{11} + 0.5x_{12} + 0.3x_{21} + 0.4x_{22} + 0.06x_{31} + 0.16x_{32}$

Question 2 (ii)

Subject to	$x_{11} + x_{12} \leq 5000$	(Component 1 capacity)
	$x_{21} + x_{22} \leq 10000$	(Component 2 capacity)
	$x_{31} + x_{32} \leq 10000$	(Component 3 capacity)
	$x_{11} \leq 0.3(x_{11} + x_{21} + x_{31})$	
i.e.,	$0.7x_{11} – 0.3x_{21} – 0.3x_{31} \leq 0$	(Product specification 1)
	$-0.4x_{11} + 0.6x_{21} – 0.4x_{31} \geq 0$	(Product specification 2)
	$-0.2x_{11} – 0.2x_{21} + 0.8x_{31} \leq 0$	(Product specification 3)
	$0.75x_{12} – 0.25x_{22} – 0.25x_{32} \geq 0$	(Product specification 4)
	$-0.45x_{12} + 0.55x_{22} – 0.45x_{32} \leq 0$	(Product specification 5)
	$-0.3x_{12} – 0.3x_{22} + 0.7x_{32} \geq 0$	(Product specification 6)
	$x_{11} + x_{21} + x_{31} \geq 10000$	(Regular gasoline demand)
	$x_{11}, x_{12}, x_{21}, x_{22}, x_{31}$ and $x_{32} \geq 0$	(Non-negativity)

Question 3 (i)

Let x_i be the number of call-center employees who start working on day i (i = 1 = Monday; i = 2 = Tuesday, etc.)

Minimize the total number of employees or $z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$

Subject to

$x_1 + x_4 + x_5 + x_6 + x_7 \geq 75$ (Minimum employees on Monday)
$x_1 + x_2 + x_5 + x_6 + x_7 \geq 50$ (Minimum employees on Tuesday)
$x_1 + x_2 + x_3 + x_6 + x_7 \geq 45$ (Minimum employees on Wednesday)
$x_1 + x_2 + x_3 + x_4 + x_7 \geq 60$ (Minimum employees on Thursday)
$x_1 + x_2 + x_3 + x_4 + x_5 \geq 90$ (Minimum employees on Friday)
$x_2 + x_3 + x_4 + x_5 + x_6 \geq 75$ (Minimum employees on Saturday)
$x_3 + x_4 + x_5 + x_6 + x_7 \geq 45$ (Minimum employees on Sunday)
$x_i \geq 0$ for all i

Question 3 (ii)

	Destination j					
Origin i	W4	W5	C6	C7	C8	C9
P1	4 x_{14}	7 x_{15}				
P2	8 x_{24}	5 x_{25}				
P3	5 x_{34}	6 x_{35}				7 x_{39}
W4		2 x_{45}	6 x_{46}	4 x_{47}	8 x_{48}	4 x_{49}
W5	2 x_{54}		3 x_{56}	6 x_{57}	7 x_{58}	7 x_{59}

Question 3 (ii)

Minimize shipping cost or $z = 4\,x_{14} + 7\,x_{15} + 8\,x_{24} + 5\,x_{25} + 5\,x_{34} + 6\,x_{35} + 7\,x_{39} + 2\,x_{45} + 6\,x_{46} + 4\,x_{47} + 8\,x_{48} + 4\,x_{49} + 2\,x_{54} + 3\,x_{56} + 6\,x_{57} + 7\,x_{58} + 7\,x_{59}$

Subject to

$$x_{14} + x_{15} \le 400$$
$$x_{24} + x_{25} \le 600$$
$$x_{34} + x_{35} + x_{39} \le 300$$

$$(x_{44} + x_{45} + x_{46} + x_{47} + x_{48} + x_{49}) - (x_{14} + x_{24} + x_{34} + x_{54}) = 0$$
$$(x_{54} + x_{55} + x_{56} + x_{57} + x_{58} + x_{59}) - (x_{15} + x_{25} + x_{35} + x_{45}) = 0$$

$$x_{46} + x_{56} = 300$$
$$x_{47} + x_{57} = 300$$
$$x_{48} + x_{58} = 300$$
$$x_{39} + x_{49} + x_{59} = 400$$

$x_{ij} \ge 0$ for all i and j

Question 4

(i) Determine the optimal decision for Rona and Jerry if they wish to minimize the travel time. (2 marks)

Let QC = Queen City; EW = Expressway

EV(QC) = 0.85(30) + 0.15(30) = 30 minutes
EV(EW) = 0.85(25) + 0.15(45) = 28 minutes
Decision = Expressway

(ii) What is the expected value of perfect information? (2 marks)

EVwPI = 0.85(25) + 0.15(30) = 25.75 minutes
EVwoPI = min(30, 28) = 28 minutes
EVPI = |25.75 – 28| = 2.25 minutes

Question 4 (iii)

Clear

State of nature s_j	Prior probabilities $P(s_j)$	Conditional probabilities $P(C \mid s_j)$	Joint probabilities $P(C \cap s_j)$	Posterior probabilities $P(s_j \mid C)$
Open	0.85	0.8	0.68	0.98
Jammed	0.15	0.1	0.015	0.02
			$P(C) = 0.695$	1.00

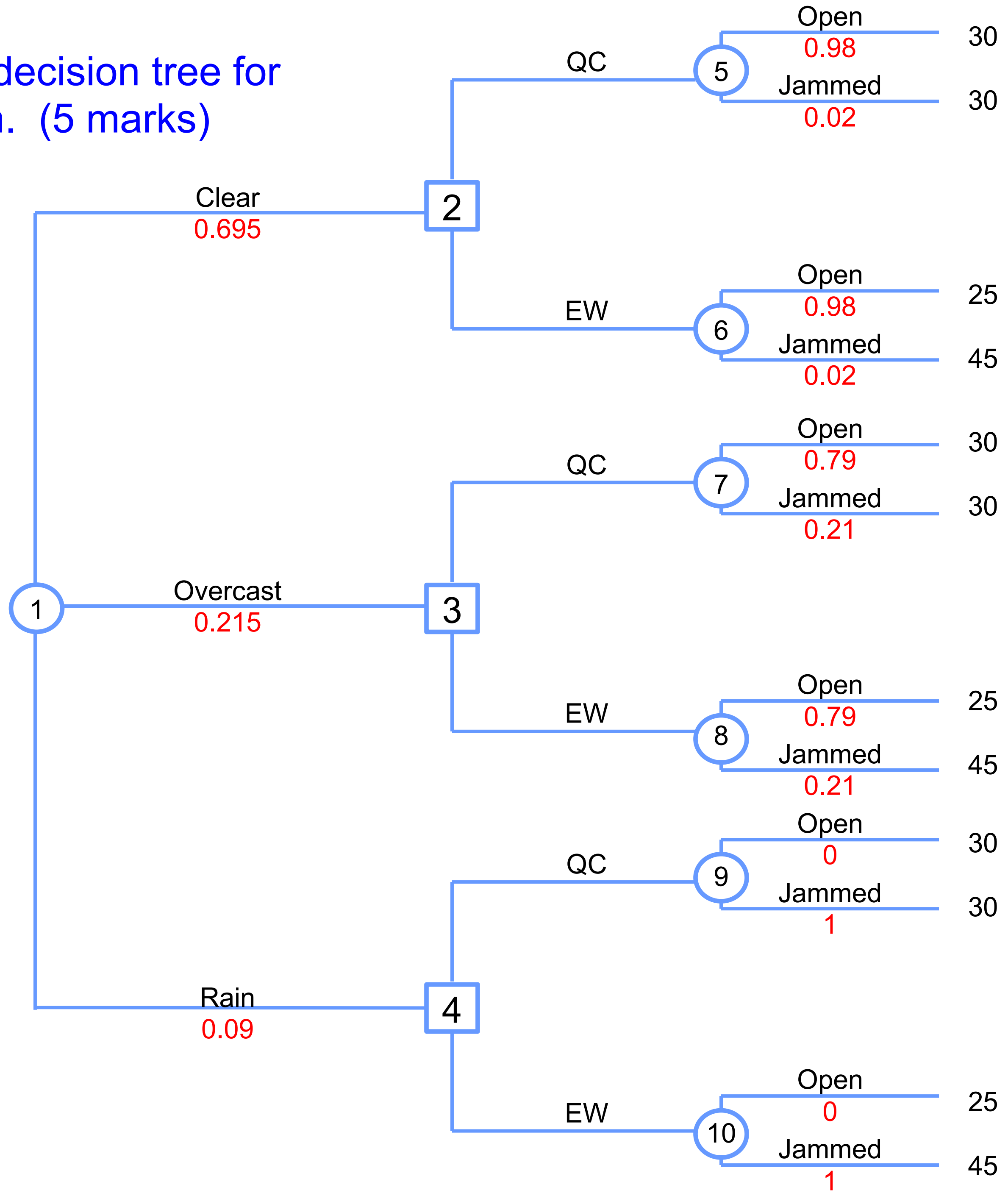
Overcast

State of nature s_j	Prior probabilities $P(s_j)$	Conditional probabilities $P(O \mid s_j)$	Joint probabilities $P(O \cap s_j)$	Posterior probabilities $P(s_j \mid O)$
Open	0.85	0.2	0.17	0.79
Jammed	0.15	0.3	0.045	0.21
			$P(O) = 0.215$	1.00

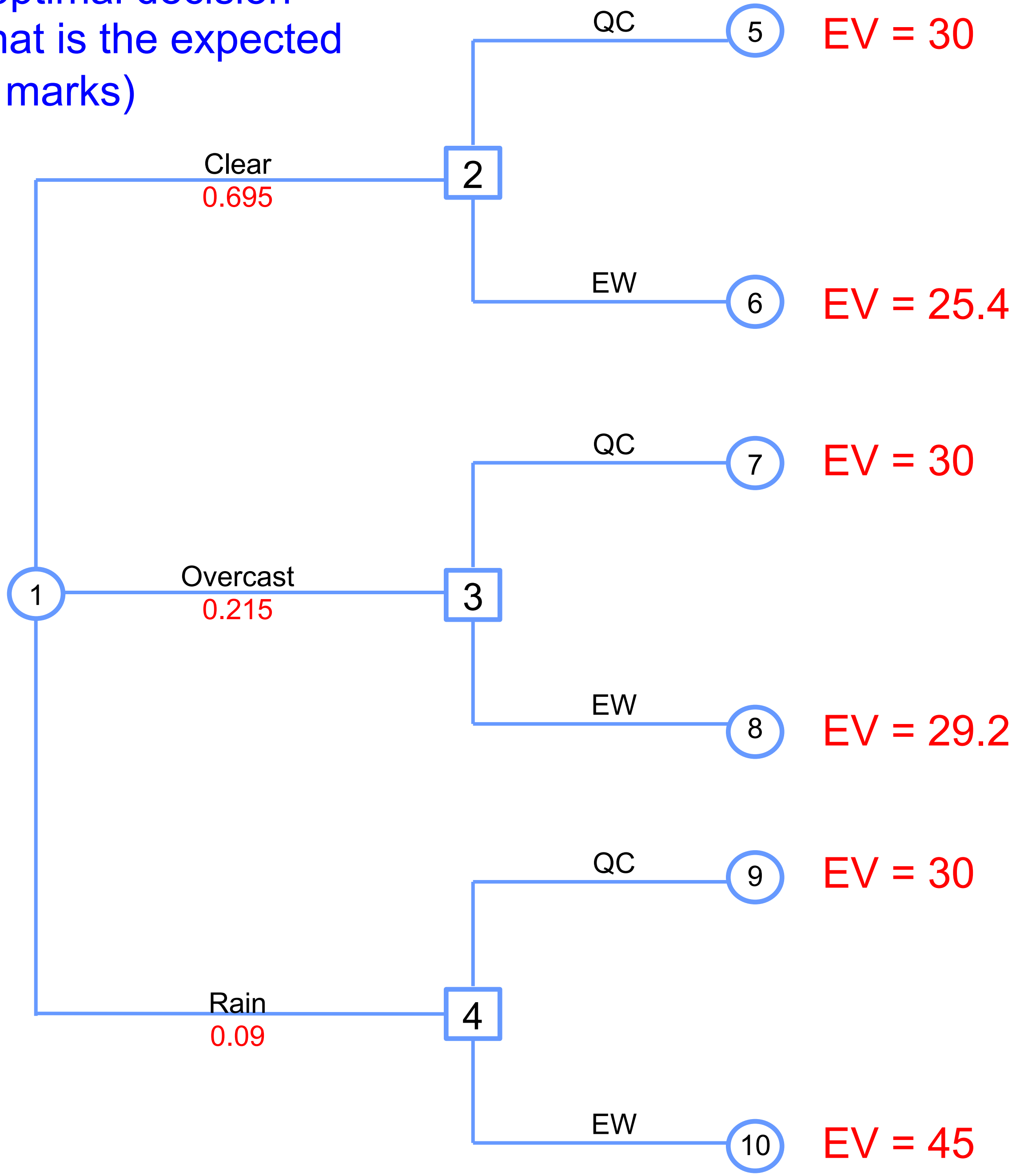
Rain

State of nature s_j	Prior probabilities $P(s_j)$	Conditional probabilities $P(R \mid s_j)$	Joint probabilities $P(R \cap s_j)$	Posterior probabilities $P(s_j \mid R)$
Open	0.85	0	0	0
Jammed	0.15	0.6	0.09	1
			$P(R) = 0.09$	1.00

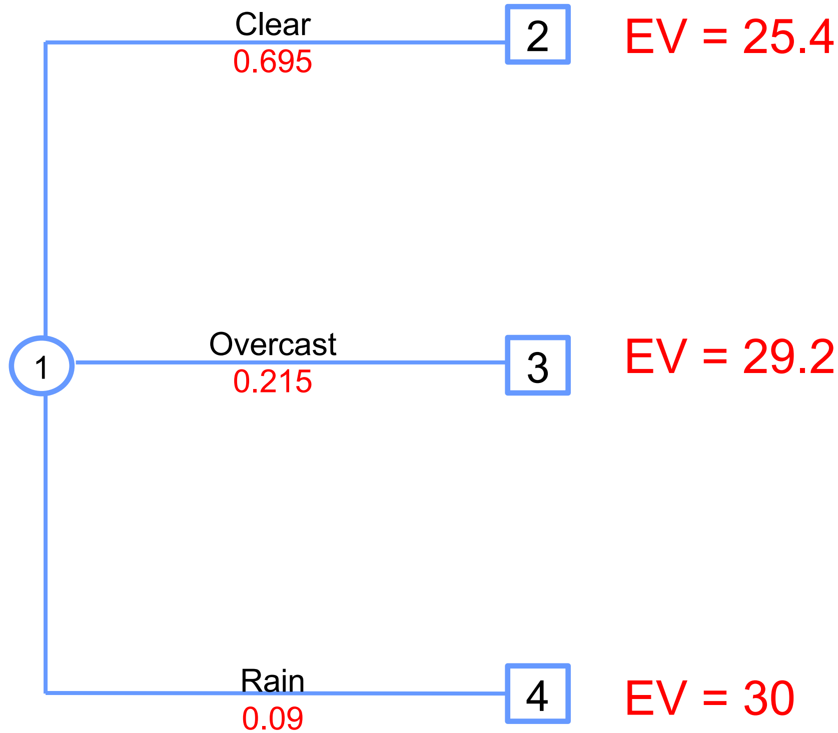
(iv) Draw a decision tree for this problem. (5 marks)



(v) What is the optimal decision strategy, and what is the expected travel time? (10 marks)



Question 4 (v)



Decision 1a: If it is clear, Rona and Jerry select Expressway

Decision 1b: If it is overcast, Rona and Jerry select Expressway

Decision 1c: If it is rain, Rona and Jerry select Queen City

The expected travel time = 0.695(25.4) + 0.215(29.2) + 0.09(30) = 26.6 minutes

Question 5

(i) What is the probability that a household views television between 5 and 10 hours a day? (6 marks)

If $x = 5$, $z = \frac{5-8.35}{2.5} = -1.34$

If $x = 10$, $z = \frac{10-8.35}{2.5} = 0.66$

$P(-1.34 \leq z \leq 0.66) = P(z \leq 0.66) - P(z \leq -1.34)$

According to Table 1, $P(z \leq 0.66) = 0.7454$

According to Table 2, $P(z \leq -1.34) = 0.0901$

$P(-1.34 \leq z \leq 0.66) = 0.7454 - 0.0901 = 0.6553$

Question 5

(ii) How many hours of television viewing must a household have in order to be in the top 3% of all television viewing households? (4 marks)

If the probability or area in the upper tail of the curve is 3% or 0.03, then the area under the curve to the left of unknown *z* value must equal 0.97. According to Table 1, *z* = 1.88. $z = \frac{x - 8.35}{2.5} = 1.88.$
x = 1.88(2.5) + 8.35 = 13.05 hours.

(iii) What is the probability that a household views television more than 3 hours a day? (3 marks)

If *x* = 3, $z = \frac{3 - 8.35}{2.5} = -2.14$
According to Table 2, $P(z \geq -2.14) = 1 - P(z \leq -2.14) = 1 - 0.0162 = 0.9838$

Question 5

(iv) Develop an estimated regression equation that can be used to predict the rating for an elliptical trainer. (7 marks)

i	x_i	y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})(y_i - \bar{y})$	$(x_i - \bar{x})^2$
1	3700	87	1825	13	23725	3330625
2	2500	84	625	10	6250	390625
3	2800	82	925	8	7400	855625
4	1900	74	25	0	0	625
5	1000	73	-875	-1	875	765625
6	800	69	-1075	-5	5375	1155625
7	1700	68	-175	-6	1050	30625
8	600	55	-1275	-19	24225	1625625
Total	15000 $\bar{x} = 1875$	592 $\bar{y} = 74$			68900	8155000

Question 5

(iv) Develop an estimated regression equation that can be used to predict the rating for an elliptical trainer. (7 marks)

$$b_1 = \frac{\sum_i^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_i^n (x_i - \bar{x})^2} = \frac{68900}{8155000} = 0.01$$

$$b_0 = \bar{y} - b_1\bar{x} = 74 - 0.01(1875) = 55.25$$

$$\hat{y} = 55.25 + 0.01x$$

(v) Use the estimated regression equation to predict the rating for an elliptical trainer with a price of $1500. (1 mark)

$$\hat{y} = 55.25 + 0.01(1500) = 70.25$$

Question 5

(vi) Given that SSE = 173.88. Compute the coefficient of determination r^2. (4 marks)

i	y_i	$y_i - \bar{y}$	$(y_i - \bar{y})^2$
1	87	13	169
2	84	10	100
3	82	8	64
4	74	0	0
5	73	-1	1
6	69	-5	25
7	68	-6	36
8	55	-19	361
Total	592 $\bar{y} = 74$		756

SSE = 173.88; SST = 756; SSR = 756 – 173.88 = 582.12

$$r^2 = \frac{\text{SSR}}{\text{SST}} = \frac{582.12}{756} = 0.77$$

$r_{xy} = +\sqrt{0.77} = 0.88$ (It reflects a strong relationship between price and rating)

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