



Student ID _____

Semester / Year:

Faculty / Dept:

Subject Code:

Subject Name:

Writing Time: hrs

Reading Time: minutes

Open Book Status:

Number of Pages (including this page): 11

Authorised Materials:

Eg. Non-programmable calculators without added text ability

Instructions to Students:

This examination contributes 50% to the final subject mark.

This examination paper includes 2 sections and has a total of 100 marks.

Section 1: Contains 1 compulsory question. You are required to answer this question. This section accounts for 25 marks.

Section 2: Contains 4 selective questions. You are required to answer only 3 questions. This section accounts for 75 marks.

SECTION 1 (compulsory)

Question 1

A manufacturer produces two products, X and Y with two machines A and B. The time taken to produce each unit of X is 29 minutes for machine A and 24 minutes for machine B. Each unit produced of Y takes 22 minutes using machine A and 30 minutes using machine B. In a particular week, the manufacturer has a limited time of working hours. In particular, 50 hours of work are available on machine A and 37 hours are available on machine B. Moreover, each week starts with a stock of 70 units of X and 80 units of Y.

Weekly demand for the products are stochastic but the manufacture estimates an average of 90 units of X and 90 units of Y demanded on a weekly basis.

- (i) Using the information above, formulate the linear programming (LP) model for the manufacturer, if they wish to plan production in order to end the week with the maximum possible stock.

(6 marks)

- (ii) This question requires you to write out the Excel formula for specific cells from the Excel sheet provided below. Write out the Excel formulas to cells B13, B16 and B17 as if you are typing out your formulae in Excel. Please clearly label your answers and write out your answers in your answer booklet and NOT on the exam paper itself.

(4 marks)

	A	B	C	D
1	Question 1			
2				
3	Product	X	Y	
4	Machine A	29	22	
5	Machine B	24	30	
6				
7	Optimal Solutions			
8				
9	<u>Decision variables</u>	X	Y	
10	Number of Products			
11				
12	<u>Objective function</u>			
13	Maximum Possible Stock			
14				
15	<u>Constraints</u>	LHS	Inequality	RHS
16	Time on Machine A		\leq	3000
17	Time on Machine B		\leq	2220
18	Minimum X Products		\geq	20
19	Minimum Y Products		\geq	10

-Please Turn Over-

The LP has been solved and the sensitivity analysis report generated. For the following questions, please refer to this sensitivity analysis output to answer the questions.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$10	Number of Products X	80	0	1	1E+30	0.2
\$C\$10	Number of Products Y	10	0	1	0.25	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$B\$16	Time on Machine A LHS	?	0	3000	1E+30	460
\$B\$17	Time on Machine B LHS	2220	0.041666667	2220	380.6896552	1440
\$B\$18	Minimum X Products LHS	80	0	20	60	1E+30
\$B\$19	Minimum Y Products LHS	10	-0.25	10	48	10

- (iii) For the optimal solution, how much time in hours is spent on using Machine A?
(2 marks)
- (iv) For the optimal solution, how much stock of Product X is left? How much stock of Product Y is left?
(3 marks)
- (v) Would the solution change if the number of hours available on machine B was 40 hours instead of 37 hours? If yes, then what is the value of the change in the objective function?
(4 marks)
- (vi) What would the optimal solution be if the machines can now produce an extra 0.5 units of Product X? To answer this question, think about what it means in terms of the objective coefficient of Product X in the objective function.
(4 marks)
- (vii) How much would the stock increase by if the amount of time available on Machine A was 60 hours instead of 50 hours?
(2 marks)

SECTION 2 (answer three out of four questions)

Question 2

- (i) The program manager for Australian Broadcasting Network (ABN), Venus Hingis, would like to determine the best way to allocate the time for the 6pm to 6:30pm evening news broadcast. Specifically, she would like to determine the number of minutes of broadcast time to devote to local news, national news, weather, and sports. Over the 30-minute broadcast, 10 minutes are set aside for advertising due to ABN requirements.

The station's broadcast policy states that at least 16% of the time available should be devoted to local news coverage; the time devoted to local news or national news must be at least 49% of the total evening news broadcast time; the time devoted to the weather segment must be equal to or more than the time devoted to the sports segment; the time devoted to the sports segment should be no longer than the 65% of the total time spent on the local and national news broadcast; and the time devoted to the weather segment should not be greater than the 40% of the broadcast time spent on local news and sports coverage. The production costs per minute are \$375 for local news, \$250 for national news, \$155 for weather, and \$105 for sports.

Formulate a Linear Programming (LP) model to help Venus Hingis to determine the number of minutes of broadcast time to devote to local news, national news, weather, and sports that will minimise the total production cost. You do not need to solve the model.

(9 marks)

- (ii) Airtaker has been awarded a contract for a large number of air-conditioners. To meet this demand, it will use its existing plants in Geelong and Wollongong, and considering new plants in Ballarat, Cairns and Newcastle. Finished air-conditioners are to be shipped to Brisbane, Melbourne and Sydney. Pertinent information is given in the table below.

SOURCES	CONSTRUCTION COST	SHIPPING COST TO DESTINATION PER UNIT			
		BRISBANE	MELBOURNE	SYDNEY	CAPACITY
GEELONG	N.A	13	7	10	12,500
WOLLONGONG	N.A	8	9	4	14,500
BALLARAT	400,000	11	3	6	30,000
CAIRNS	350,000	5	11	8	15,000
NEWCASTLE	450,000	6	9	2	28,000
DEMAND		25,000	25,000	40,000	

Formulate an Integer Linear Programming (IP) model that will help Airtaker determine which plants to build and the optimal shipping schedule in order to minimise the total cost. You do not need to solve the model.

(13 marks)

- (iii) In Question 2 (ii), we have been fortunate to assume that there is a route from every source to every destination for Airtaker to ship their air-conditioners. However, establishing a route from every source to every destination may not be possible in real life situations. For example, shipping air-conditioners from Geelong to Sydney may no longer be acceptable. Name three different ways that can handle this situation in your Integer Linear Programming (IP) formulation from Question 2 (ii).

(3 marks)

Question 3

- (i) Rain Gardeners Pty Ltd has a contract to produce 17,000 garden hoses for a large discount chain store in Australia. Rain Gardeners Pty Ltd has four different machines in their warehouse that can produce this kind of hose. Because these machines are from different manufacturers and use slightly differing technologies, their specifications are not the same.

Machine	Fixed Cost to Set Up Production Run (AUD)	Variable Cost Per Hose (AUD)	Capacity
1	650	1.15	6,500
2	750	1.48	8,000
3	1,250	0.98	3,500
4	300	2.01	9,000

Formulate an Integer Linear Programming (IP) model that will minimize total cost. Also include a constraint to ensure that if Machine 1 is used, Machine 2 cannot be, vice versa. You do not need to solve the model.

(10 marks)

- (ii) Jason Yau has up to \$100,000 AUD that he can invest and he wishes to invest in the possible 12 mutual fund alternatives with the following restrictions:

For diversification, no more than \$40,000 AUD can be invested in any one fund. If a fund is chosen for investment, then at least \$20,000 AUD will be invested in it. No more than one of the funds can be in pure Income funds, and at least one pure Stock fund must be selected. The total amount invested in pure Growth funds must be at least as much as the amount invested in pure Corporate Bonds and pure Government Bonds. And lastly, if Jason invests in any of the pure Growth funds, then he must also invest in at least one other combined type of fund alternative.

Using the following expected annual returns, formulate a Mix-Integer Linear Programming (MILP) model that will determine the investment strategy that will maximise expected annual return for Jason. You do not need to solve the model.

FUND ALTERNATIVES	TYPE	EXPECTED ANNUAL RETURN (%)
1	Growth	6.25
2	Growth	6.35
3	Growth & Corporate Bonds	7.55
4	Corporate Bonds	7.05
5	Corporate Bonds	8.55
6	Income	4.75
7	Income	5.55
8	Income & Stock	6.75
9	Stock	6.40
10	Stock	6.65
11	Stock & Government Bonds	6.45
12	Government Bonds	5.75

(15 marks)

Question 4

Engus Productions is considering producing a pilot for a reality television series for Australian Broadcasting Network (ABN). ABN may reject the pilot for the series, but it may also purchase the program for 1 or 2 years. Engus Productions may decide to produce the pilot or transfer the rights of pilot production to a competitor at \$80,000. Engus Production would like to maximise their profits and their profits are summarised in the following payoff table (in thousands of Australian dollars).

DECISION ALTERNATIVE	STATE OF NATURE		
	Reject, s_1	1 Year, s_2	2 Years, s_3
PRODUCE PILOT, d_1	-80	40	120
SELL TO COMPETITOR, d_2	80	80	80

- (i) Apply the optimistic approach to recommend a decision to Engus Productions. (1 mark)

Engus Productions made an initial assessment on what they think ABN may do and came up with the following probabilities:

$$P(\text{Reject}) = 0.3, P(1 \text{ Year}) = 0.3, P(2 \text{ Years}) = 0.4$$

- (ii) Draw a decision tree and recommend a decision strategy to Engus Productions. Clearly show your calculations and provide justification for your decision strategy. (5 marks)
- (iii) Use EVPI to determine the maximum that Engus Productions is willing to pay for inside information on what ABN will do. You do not need to draw a decision tree to answer this question. (3 marks)

For a consulting fee of \$3,000, Research Rapport Agency (RRA) will review the plans for the reality television series and indicate the overall chances of a favourable network reaction to the series. Denote favourable review by F, and unfavourable review by U. Engus Productions believes that the following conditional probabilities are realistic appraisals of RRA's evaluation accuracy.

$$P(F | \text{Rejection}) = 0.2, P(F | 1 \text{ year}) = 0.5, P(U | 2 \text{ years}) = 0.2$$

- (iv) Apply the Bayes' Theorem to compute the posterior probabilities for both Favourable and Unfavourable review cases. (6 marks)
- (v) Draw the decision tree, determine the recommended decision strategy and the expected value for Engus Productions, assuming that the RRA will conduct the review. Clearly draw out the decision tree with clear labels, show all your calculations and provide justifications for your decision strategy to Engus Productions. (10 marks)

Question 5

At Sean's Saxicolous Supermarket, the length of completed customer transactions at the check-out is normally distributed with a mean of 5.22 minutes and a standard deviation of 1.31 minutes. Sean, the owner of this particular supermarket, wants to answer the following questions:

- (i) What is the probability that the length of a completed customer transaction will be 5.22 minutes?
(1 mark)
- (ii) What is the probability that the length of a completed customer transaction will be between 3.5 and 6.5 minutes?
(3 marks)
- (iii) What is the length of time a completed customer transaction needs to be for it to be among the fastest 9% of completed customer transactions?
(3 marks)
- (iv) Find the value of d such that the probability between 4 and d minutes will have a probability of 0.3952. In other words, find the value of d such that $P(4 < X < d) = 0.3952$.
(5 marks)

A research institute conducted a series of experiments in a poorly performing and newly launched supermarket. Customers are asked to give a performance score out of 100 to the cashier after completing their transaction. The institute wants to find out how the supermarket cashier's performance score is associated with the factors such as time to complete the transaction, transaction amount and gender. For the qualitative variable; gender, it is coded as *Male* = 0 and *Female* = 1. Following tables indicate the results of the multiple regression analysis.

SUMMARY OUTPUT					
<i>Regression Statistics</i>					
Multiple R	0.609				
R Square	0.371				
Adjusted R Square	0.293				
Standard Error	6.457				
Observations	28				
ANOVA					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	3	591.182	197.061	4.727	0.010
Residual	24	1000.532	41.689		
Total	27	1591.714			
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>
Intercept	55.222	3.787	14.582	0.000	47.406
Time to complete the transaction (minutes)	-1.479	0.699	-2.115	0.045	-2.922
Transaction amount (\$)	0.045	0.019	2.405	0.024	0.006
Female	6.473	2.542	2.547	0.018	1.228

- (v) Develop an estimated regression equation that relates the cashier's performance score to the factors such as time to complete the transaction, transaction amount and whether the cashier is a female. (2 Marks)
- (vi) Is cashier being a female a significant factor for the performance of this supermarket? Use $\alpha = 0.05$. (2 Marks)
- (vii) Explain how the transaction time is associated with the cashier's performance. (2 Marks)
- (viii) What are the coefficient of determination and adjusted coefficient of determination for this model? How are they different from each other? (3 Marks)
- (ix) What is your opinion about the overall model significance? Use $\alpha = 0.05$. (1 Marks)
- (x) The following table indicate the correlation statistics of the model variables. Briefly define what multicollinearity is, whether it exists in this model and why. (3 Marks)

	<i>Time to complete the transaction</i>	<i>Transaction amount (\$)</i>	<i>Female</i>
Time to complete the transaction	1		
Transaction amount (\$)	0.589024808	1	
Female	-0.239745954	-0.17055409	1

Appendix 1

Cumulative Standard Normal Probabilities

Areas from $-\infty$ to:

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Appendix 2

Cumulative Standard Normal Probabilities

Areas from $-\infty$ to:

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Appendix 3

1. Expected value,

$$EV(d_i) = \sum_{j=1}^N P(s_j) V_{ij}$$

2. Expected value with perfect information (for maximisation problems),

$$EV_{wPI} = \sum_{j=1}^N P(s_j) \max_i V_{ij}$$

3. Expected value without perfect information (for maximisation problems),

$$EV_{woPI} = \max_i [EV(d_i)] = \max_i \left[\sum_{j=1}^N P(s_j) V_{ij} \right]$$

4. Expected value of perfect information, $EVPI = |EV_{wPI} - EV_{woPI}|$

5. Sample variance, $s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$

6. Sample standard deviation, $s = \sqrt{s^2}$

7. Binomial probability function, $f(x) = \binom{n}{x} p^x (1-p)^{(n-x)}$ where, $\binom{n}{x} = \frac{n!}{x!(n-x)!}$

8. Conversion from normal distribution to standard normal distribution, $z = \frac{x-\mu}{\sigma}$

9. Estimated linear regression equation, $\hat{y} = b_0 + b_1 x$

10. Slope,

$$b_1 = \frac{\sum_i^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_i^n (x_i - \bar{x})^2}$$

11. Y-intercept of the regression line, $b_0 = \bar{y} - b_1 \bar{x}$

12. Sum of squares due to error, $SSE = \sum (y_i - \hat{y}_i)^2$

13. Total sum of squares, $SST = \sum (y_i - \bar{y})^2$

14. Sum of squares due to regression, $SSR = \sum (\hat{y}_i - \bar{y})^2$ or $SSR = SST - SSE$

15. Coefficient of determination, $r^2 = \frac{SSR}{SST}$

16. Sample correlation coefficient, $r_{xy} = (\text{sign of } b_1) \sqrt{r^2}$

END OF EXAMINATION PAPER