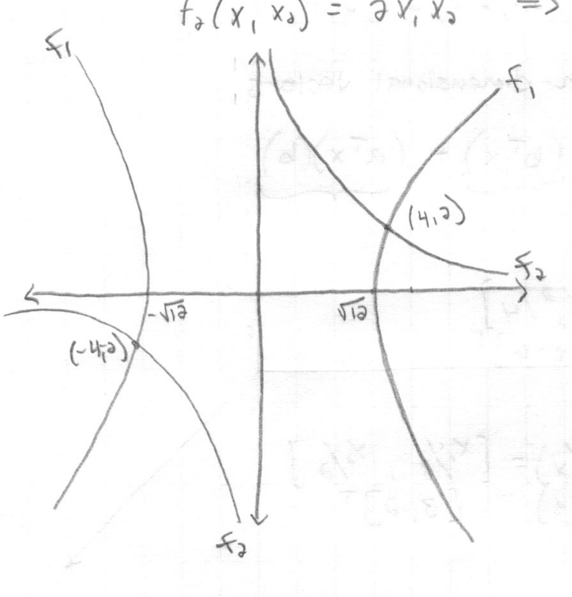
5.9 $f_1(x_1\ x_2) = x_1^2 - x_2^2 \Rightarrow f_1(x_1\ x_2) = x_1^2 - x_2^2 = 12$

$f_2(x_1\ x_2) = 2x_1 x_2 \Rightarrow f_2(x_1\ x_2) = 2x_1 x_2 = 16,\ x_2 = 8/x_1$



Intersections: $x_1^2 - (8/x_1)^2 = 12$

$x_1^4 - 12x_1^2 - 64 = 0$

$x_1^2 = 16, -4 \Rightarrow x_1 = \pm 4$

$x_2 = \pm 2$

$[4,2]^T \quad [4,-2]^T$

Page 22: 2.1, 2.2, 2.3, 2.4

2.1 $A \in \mathbb{R}^{m \times n}$, $\operatorname{rank} A = m$. Show that $m < n$

Suppose $n < m$. Column Rank(A) = n. Rank(A) = maximum # of linearly independent columns of A, then rank(A) cannot be greater than n. This contradicts the assumption that Rank(A) = m

2.2 $Ax = b$, $A \in \mathbb{R}^{m \times n}$, unique solution if/only if $\operatorname{rank} A = \operatorname{rank}[A, b] = n$

The rank is the maximum number of linearly independent vectors in a matrix. If the matrix $[A,b]$ has the same rank as A, then both matrices possess n linearly independent column vectors with one distinct / unique solution

2.3 $\bar{a}_i = [1, a_i^T]^T \in \mathbb{R}^{n+1}$, $i = 1, \dots, k$. Since $k \geq n+2$ then the vectors $\bar{a}_1, \dots, \bar{a}_k$ must be linearly independent in $\mathbb{R}^{n+1}$. $\exists\ a_1 \dots a_k$, s.t. → not all zero

$$\sum_{i=0}^{k} \alpha_i a_i = 0$$

$\Rightarrow$ the first component is $\sum_{i=0}^{k} \alpha_i = 0$, while the last n components have the form $\sum_{i=0}^{k} \alpha_i a_i = 0$

2.4 $M = \begin{bmatrix} M_{m-k,k} & I_{m-k} \\ M_{k,k} & O_{k,m-k} \end{bmatrix}$ $M_{k,k}$ is $k \times k$, $M_{m-k,k}$ is $(m-k) \times k$, $I_{(m-k)}$ identity, $O_{k,m-k}$ $k \times (m-k)$ zero matrix

a. $|\det M| = |\det M_{k,k}|$

$$\begin{bmatrix} M_{m-k,k} & I_{m-k} \\ M_{k,k} & O_{k,m-k} \end{bmatrix} \begin{bmatrix} I_k & O \\ -M_{m-k,k} & I_{m-k} \end{bmatrix} = \begin{bmatrix} O & I_{m-k} \\ M_{k,k} & O \end{bmatrix}$$

$$\begin{bmatrix} O & I_{m-k} \\ M_{k,k} & O \end{bmatrix} \begin{bmatrix} O & I_k \\ I_{m-k} & O \end{bmatrix} = \begin{bmatrix} I_k & O \\ O & M_{k,k} \end{bmatrix}$$

$$\det M = \det\left(\begin{bmatrix} I_k & O \\ O & M_{k,k} \end{bmatrix}\right) \det\left(\begin{bmatrix} O & I_k \\ I_{m-k} & O \end{bmatrix}\right)$$

$$\det\left(\begin{bmatrix} O & I_k \\ I_{m-k} & O \end{bmatrix}\right) = \pm 1$$