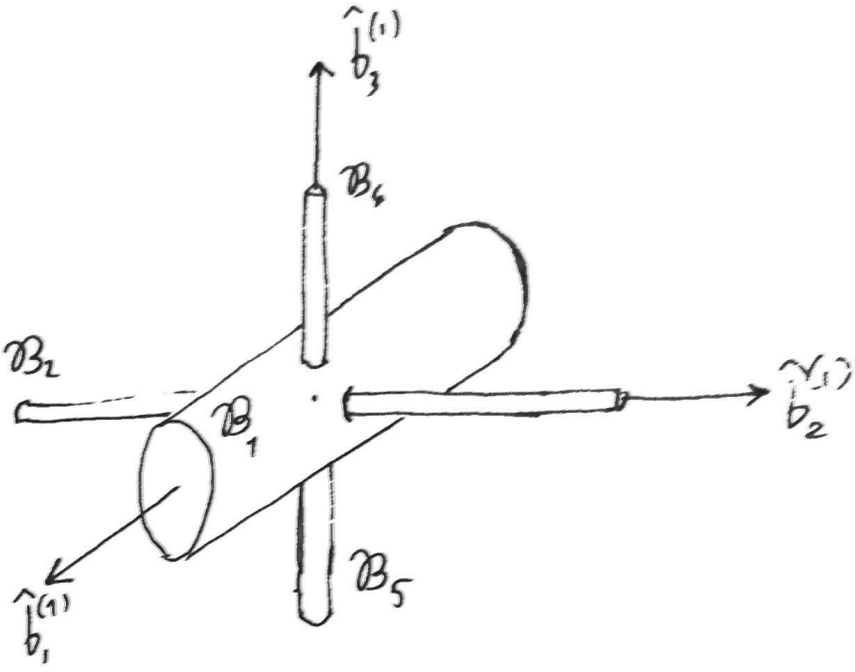
Homework 2

Consider a spacecraft composed of a main cylindrical body $\mathcal{B}_1$ and 4 appendages



Body $\mathcal{B}_1$, with radius R and length L

$m_{B1} = 2000$ Kg

$R = 1\,m$

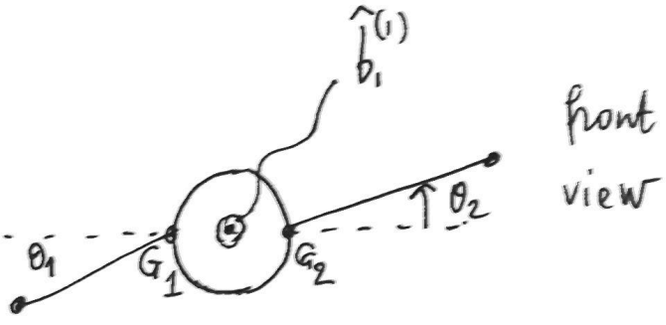
$L = 4\,m$

Bodies $\mathcal{B}_2$ and $\mathcal{B}_3$, with radius r_A and length l_A

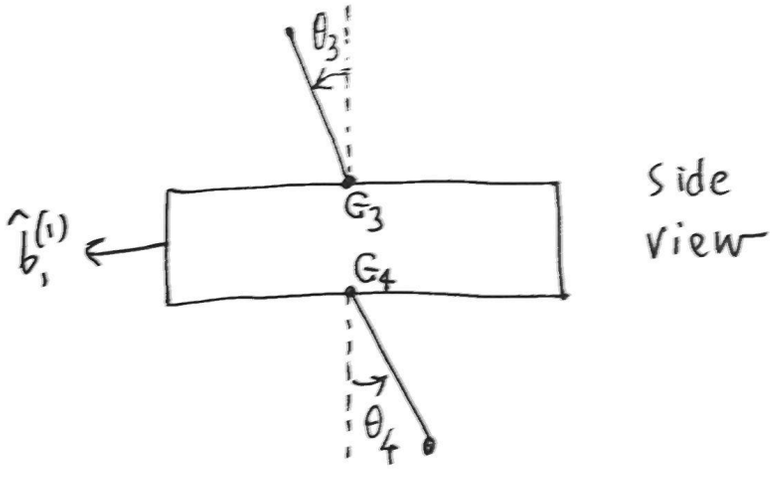
Bodies $\mathcal{B}_4$ and $\mathcal{B}_5$, with radius r_B and length l_B

$m_{B2} = m_{B3} = m_{B4} = m_{B5} = 100$ Kg

$r_A = r_B = 0.05\,m$



front view



side view

$G_1, \dots, G_4$ are revolute joints (1 degree of freedom).

Each of them is associated with an elastic torque and a friction torque about the respective rotation axis, with coefficients

$k_1 = 100$ Nm for the elastic torque (proportional to θ_i)

$k_2 = 50$ Nm sec for the friction torque (proportional to $\dot{\theta}_i$)

The spacecraft travels a circular orbit of radius $R_0 = 7000$ km and the inertial frame coincides with $\underline{\underline{B}}_1$ at t_0 (initial time). Thus the position and velocity of the center of mass of $\mathcal{B}_1$ at t_0 are

$$\underrightarrow{v}_{P1}(t_0) = \begin{bmatrix} 0 & \sqrt{\frac{\mu}{R_0}} & 0 \end{bmatrix} \underline{\underline{N}}^T \qquad \text{where} \quad \underline{\underline{N}} = \underline{\underline{B}}_1(t_0)$$

$$\underrightarrow{r}_{P1}(t_0) = \begin{bmatrix} R_0 & 0 & 0 \end{bmatrix} \underline{\underline{N}}^T$$

The remaining initial conditions are

$$\underrightarrow{\omega}_1(t_0) = \begin{bmatrix} 1 & 0.1 & 0.1 \end{bmatrix} \text{sec}^{-1} \, \underline{\underline{B}}_1^T$$

$$\theta_1(t_0) = -30 \text{ deg} \quad \theta_2(t_0) = 30 \text{ deg} \quad \theta_3(t_0) = -30 \text{ deg} \quad \theta_4(t_0) = 10 \text{ deg}$$

$$\sigma_1(t_0) = -0.2 \text{ sec}^{-1} \quad \sigma_2(t_0) = 0.2 \text{ sec}^{-1} \quad \sigma_3(t_0) = 0.1 \text{ sec}^{-1} \quad \sigma_4(t_0) = -0.3 \text{ sec}^{-1}$$

Consider two cases

(A) $\ell_A = 2m$ $\ell_B = 9m$, propagation time $[0, 1800]$ sec

(B) $\ell_A = 2m$ $\ell_B = 3m$, propagation time $[0, 7200]$ sec

Portray the time histories of

(i) $\underline{\omega}_1(t)$, $\underline{v}_{P1}(t)$

(ii) $\sigma_i(t)$ $(i = 1, .., 4)$

(iii) $\theta_i(t)$ $(i = 1, .., 4)$

Comment on the asymptotic rotational state of the spacecraft