Advanced Spacecraft Dynamics - Homework III

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Contents

1 Introduction

In this report we will analyse and apply an attitude control law to a spacecraft equipped with 4 reaction wheels. We will be provided with an initial attitude of the spacecraft and a commanded attitude computable at each time, meaning that our attitude control law will have to initially align the S/C's body frame with the commanded one and then keep the two aligned.

The reaction wheels are placed in a ortho-skew array, and their principal axes $\hat{a}_i$ are reported here

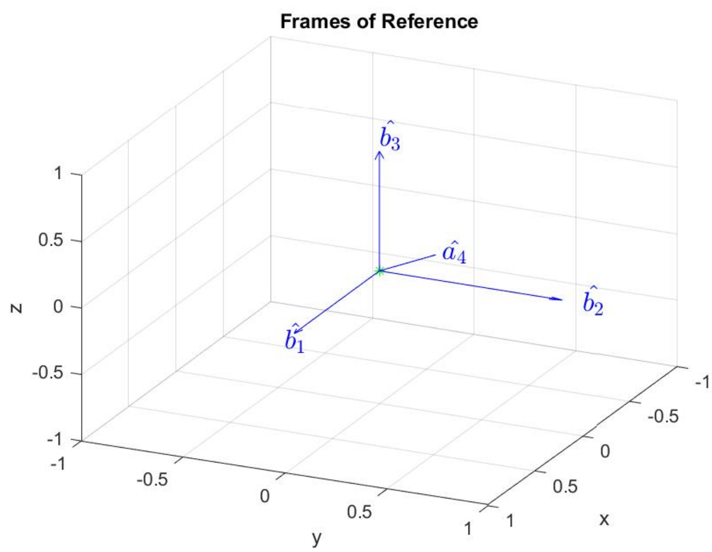

Figure 1: Wheels Principal Axes

Note that $J_c^{(B)}$ is provided in the text and we will assume that the wheels contribution to it is negligible, therefore we'll have $J_c^{(B)} = J = const.$

In the following analysis we will consider multiple reference frames, which we'll describe here below,

$$\underline{\underline{B}} : \text{Body Reference Frame}$$

$$\underline{\underline{C}} : \text{Commanded Reference Frame}$$

$$\underline{\underline{N}} : \text{Inertial Reference Frame}$$

$$\underline{\underline{P}} : \text{Perifocal Reference Frame}$$

One could start solving this problem in multiple ways, which includes defining different states for the propagation, but in our case we will limit to including only the error quaternions inside the state for the sake of execution time.

2 Propagation Procedure

In order to propagate our system forward we first need to define a state containing all the necessary quantities. In our case we define the state X as

$$X = \begin{bmatrix} \underline{\omega} \\ \underline{\omega_s} \\ \underline{r} \\ \underline{v} \\ \underline{x_e} \end{bmatrix} \quad , \quad \underline{x_e} = \begin{bmatrix} q_{e0} \\ \underline{q_e} \end{bmatrix} \tag{2.1}$$

where x_e contains the error quaternions associated to the rotation $\underset{B \leftarrow C}{R}$.
This means that we will not propagate neither one reference frame, but we will just propagate the error attitude between the two using the error quaternion.

2.1 Initial Conditions

The initial conditions for the body angular velocity ω and the angular velocities of the wheels ω_s are both provided and reported here below in *rad/s*.

$$\underline{\omega} = \begin{bmatrix} -0.1 \\ 0.05 \\ 0 \end{bmatrix} \quad , \quad \underline{\omega_s} = \begin{bmatrix} 0.5 \\ 0.5 \\ -0.5 \\ -0.5 \end{bmatrix} \tag{2.2}$$

On the other hand, the initial conditions for the position vector $\underline{r}$ and velocity ulv are not provided directly, but they must be retrieved from the Classical Orbital Elements provided.
In fact, from the informations given in the text, one can compute the COE as

$$COE = \begin{bmatrix} a \\ e \\ i \\ \Omega \\ \omega \\ \theta^* \end{bmatrix} = \begin{bmatrix} 26564\,km \\ 0.73 \\ 63.4^\circ \\ 0^\circ \\ -90^\circ \\ 90^\circ \end{bmatrix} \quad , \quad a = \sqrt[3]{\mu \left(\frac{T_{Mol}}{2\pi} \right)^2} \tag{2.3}$$

then, one can easily compute $\underline{r}$ and $\underline{v}$ from COE and use these two quantities in the propagator.

Finally, in order to find the error quaternions, it is necessary to first compute the commanded attitude quaternions in the correct reference frame.
In fact, since we have that x_b contains the quaternions associated to the rotation matrix from $\underline{\underline{N}}$ to $\underline{\underline{B}}$, we need to find a similar rotation matrix from $\underline{\underline{N}}$ to $\underline{\underline{C}}$.

The latter is slightly trickier to compute because we have to compose two rotations, one from $\underline{\underline{C}}$ to $\underline{\underline{P}}$ defined as R_B and one from $\underline{\underline{P}}$ to $\underline{\underline{N}}$ which is defined from a 3-1-3 Euler Angles rotation.

Then, we can finally compute the initial error quaternions to feed to x_{e0} as,

$$\begin{cases} q_{e0,0} & = q_{c0,0}\, q_{b0,0} + \underline{q_{c,0}}^T \cdot \underline{q_{b,0}} \\ q_{e,0} & = -\underline{q_{c,0}}\, q_{b0,0} + q_{c0,0}\, \underline{q_{b,0}} - \underline{\tilde{q_{c,0}}} \cdot \underline{q_{b,0}} \end{cases} \tag{2.4}$$

2.2 Local Variables

At each time step inside the dynamical model it is fundamental to compute the commanded attitude as

$$\underline{\omega_c} = \begin{bmatrix} 0 \\ -\frac{h}{r^2} \\ 0 \end{bmatrix} \qquad , \qquad \underline{\dot{\omega_c}} = \begin{bmatrix} 0 \\ 2\frac{h}{r^3} v_r \\ 0 \end{bmatrix} \tag{2.5}$$

Then, one must compute the matrix A, which here will be renamed to Ja to avoid confusion with the Gain matrix

$$Ja = \begin{bmatrix} I_{s1}\hat{a_1} & I_{s2}\hat{a_2} & I_{s3}\hat{a_3} & I_{s4}\hat{a_4} \end{bmatrix} \tag{2.6}$$

Another fundamental step is that of defining both the *desired* and *error* angular rates ω_d and ω_e.

$$\underline{\omega_d} = \underline{\omega}^{(B)} - \underline{\omega_c}^{(C)} \tag{2.7}$$

$$\underline{\omega_e} = \underline{\omega}^{(B)} - \underset{B \leftarrow C}{R}\, \underline{\omega_c}^{(C)} \tag{2.8}$$

One could also account for the presence of external disturbance torques here by introducing the M_c parameter, but since in our case there are no external torques we define

$$\underline{M_c} = 0 \tag{2.9}$$

Finally, one can compute the commanded torque as

$$\underline{T_c} = \underline{\tilde{\omega}} J \underline{\omega} - M_c + J\underline{\dot{\omega_c}} - JA^{-1}B\underline{\omega_d} - sign(q_{e0,0})JA^{-1}\underline{q_e} \tag{2.10}$$

2.3 State Update

With the local variables computed previously, and after analytically inverting a few equations, one can update the angular rates state components with the following relations,

$$\dot{\underline{\omega}} = J^{-1}\left[\underline{M_c} - \tilde{\underline{\omega}} J \underline{\omega} - \tilde{\underline{\omega}} A \underline{\omega_s} + \underline{T_c}\right] \tag{2.11}$$

$$\dot{\underline{\omega_s}} = -Ja\,\left(Ja\,Ja^T\right)^{-1}\underline{T_c} \tag{2.12}$$

While the remaining state components are updated according to their standard definitions, reported here below

$$\dot{\underline{r}} = \underline{v} \tag{2.13}$$

$$\dot{\underline{v}} = -\frac{\mu}{r^2}\hat{r} \tag{2.14}$$

$$\dot{q_{e0}} = -\frac{1}{2}\underline{q_e}\,\underline{\omega_d} \tag{2.15}$$

$$\dot{\underline{q_e}} = \frac{1}{2}\left[q_{e0} I_{3\text{x}3} + \tilde{\underline{q_e}}\right]\underline{\omega_e} \tag{2.16}$$

3 Results

3.1 Section A

In this section we are required to evaluate the quantities reported below without ulterior constraints in the time domain $[0, 1200]$ sec.

Indeed, the feedback control law that we have defined is working properly. In fact we can see that the behaviour of each graph is similar to the one we expected, with the S/C that reaches a steady state configuration in which its attitude is very close to the commanded one.

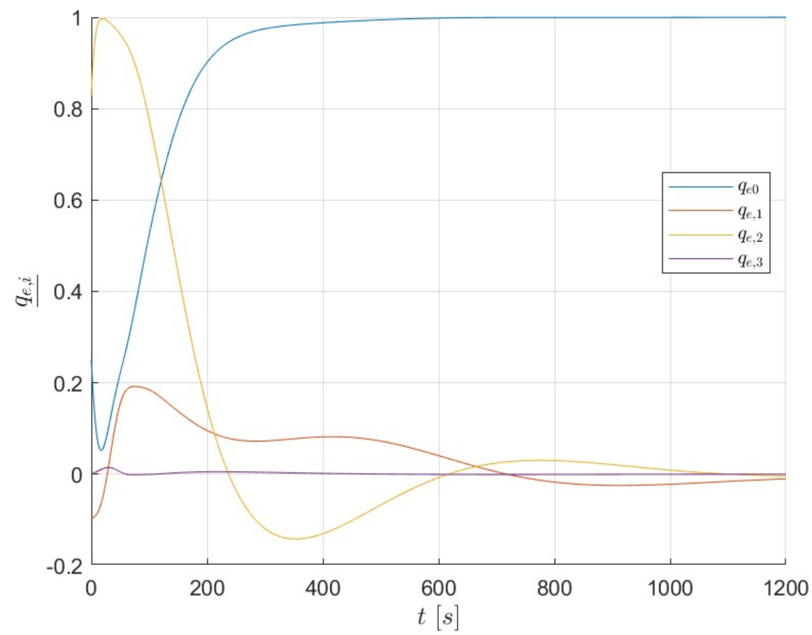

Figure 2: Error Quaternions

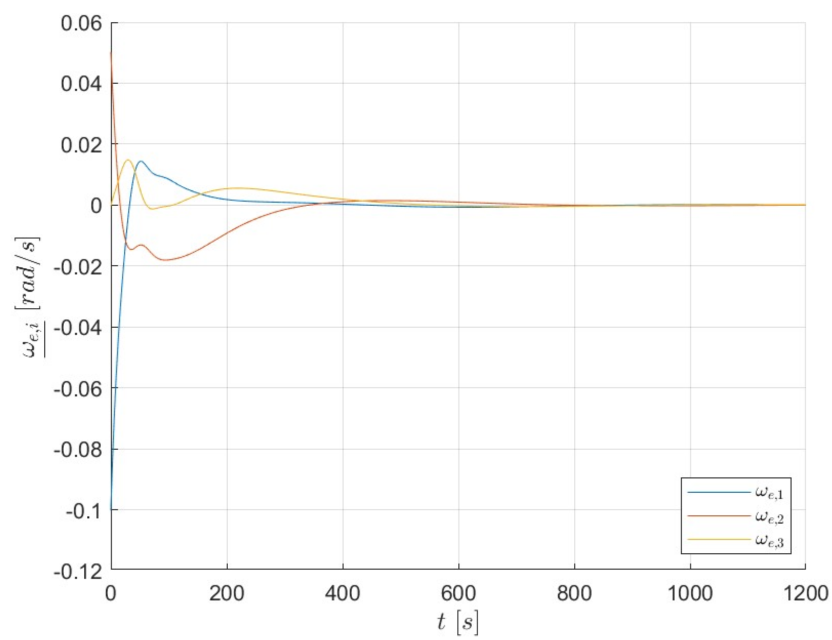

Figure 3: Error Angular Rate

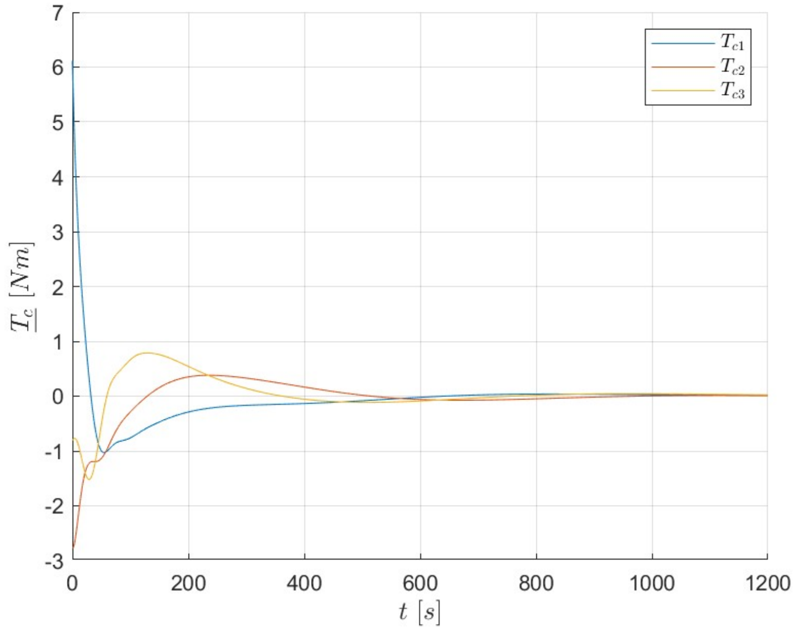

Figure 4: Commanded Torque

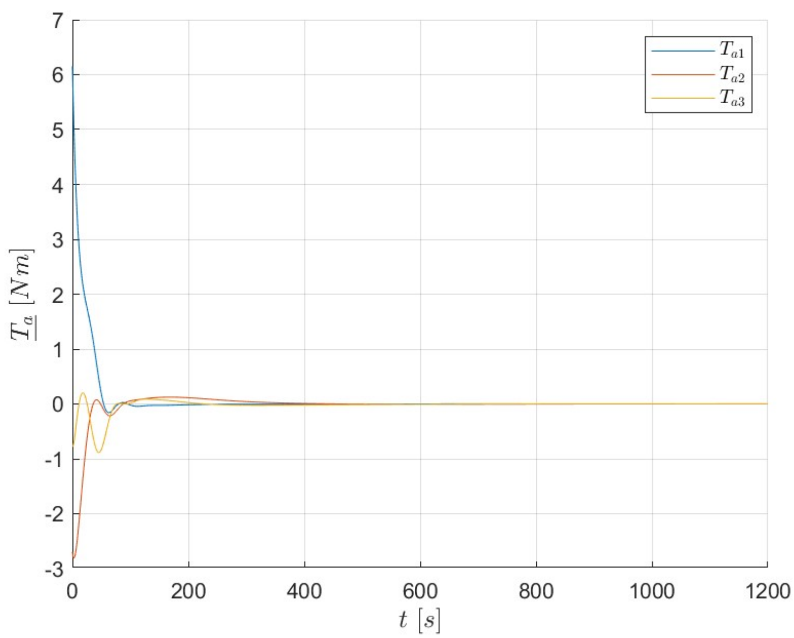

Figure 5: Applied Torque

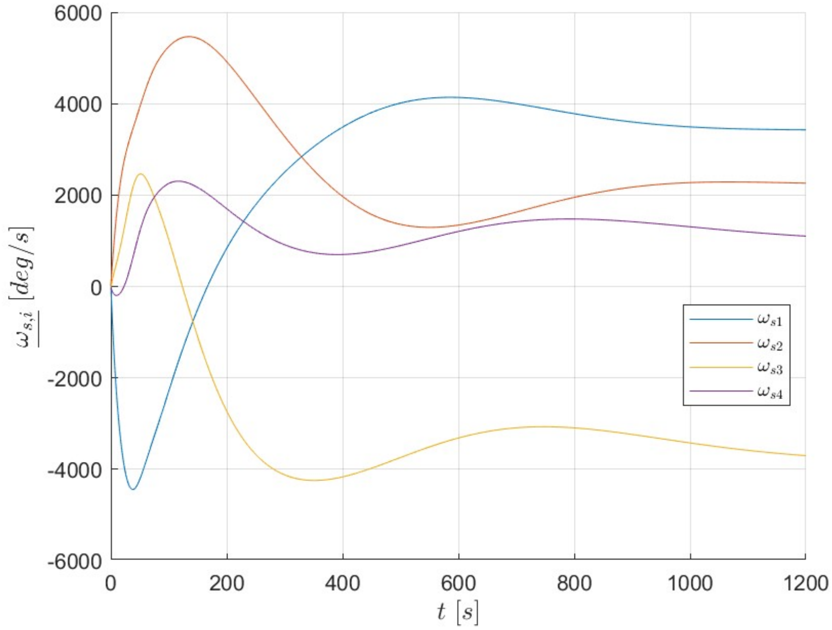

Figure 6: Rotor Angular Rates

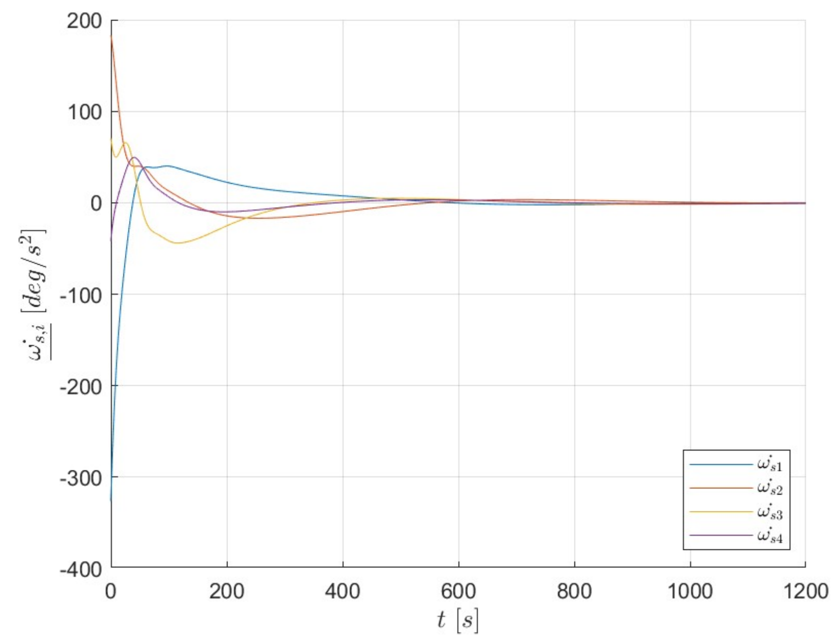

Figure 7: Rotor Angular Accelerations

3.2 Section B

The procedure followed in this section is identical to the one followed in the previous one, with the exception of the time domain analysed. In fact in this case the final time of propagation will be after a full period of the orbit, meaning that the time domain will be $[0, T_{Mol}]$.

In this section we notice the presence of a sort of instability at a certain point of the orbit. This is evident in each graph but in particular in the evolution of the error quaternions we notice a big spike towards the end of the orbital period. Moreover, by testing the propagation for more than one orbital period it is possible to show that this behaviour is recurrent and it is not a one-off event.
Then, since we have that generally $\underline{q_e}$ has a null steady state value, we can set a tolerance on them such that when this tolerance value is surpassed we will have an indication of where this instability happens in the orbit.
By following this procedure one obtains Figure 11, where we can see that the S/C observes the error attitude spike when it is moving towards the perigee.
Note that in this representation the initial point for the integration is shown as $\underline{x_0}$.

From this brief analysis one may conclude that this behaviour is caused by the combined effects of the rapidly decreasing norm of $\underline{r}$ and possibly the change in radial velocity v_r.

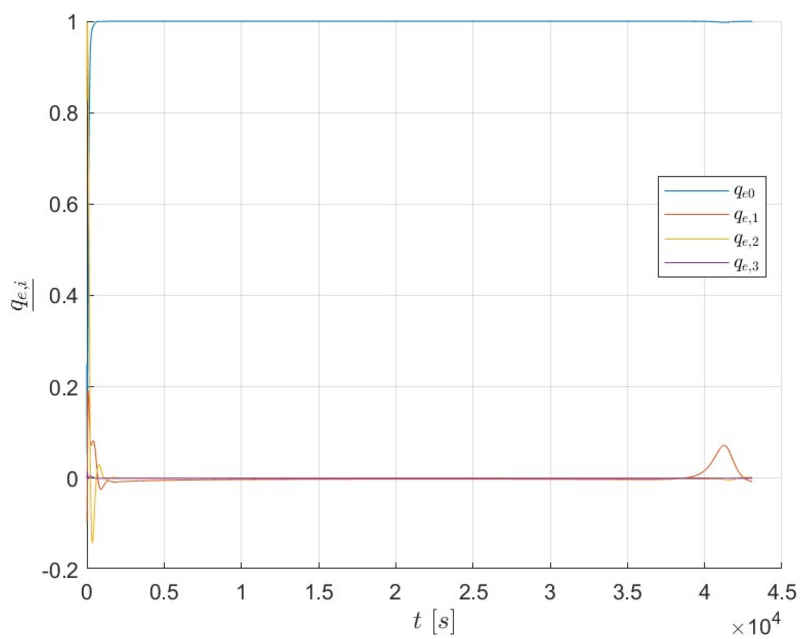

Figure 8: Error Quaternions

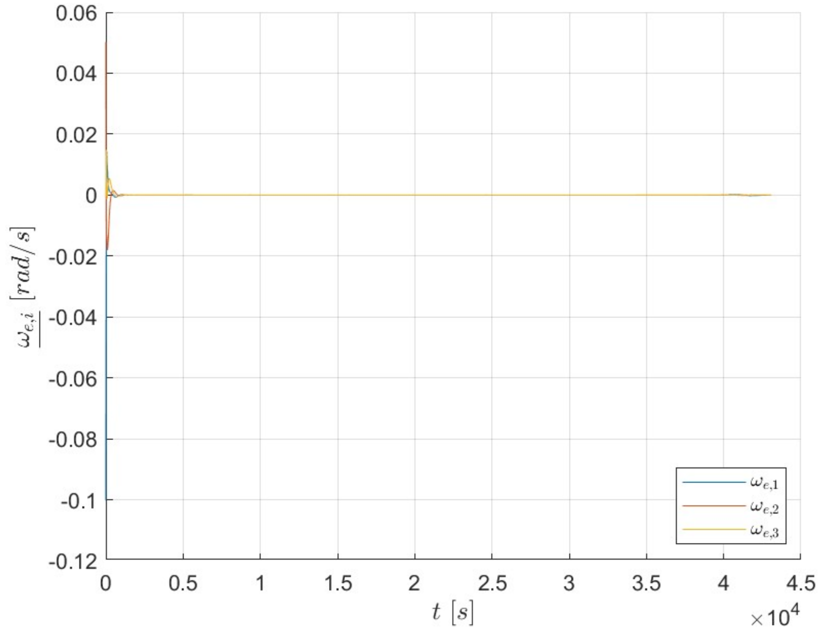

Figure 9: Error Angular Rate

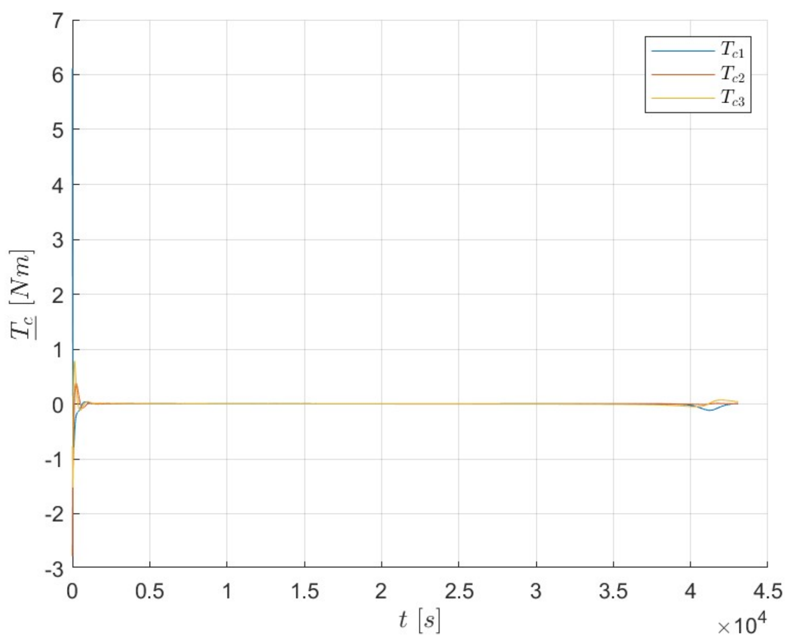

Figure 10: Commanded Torque

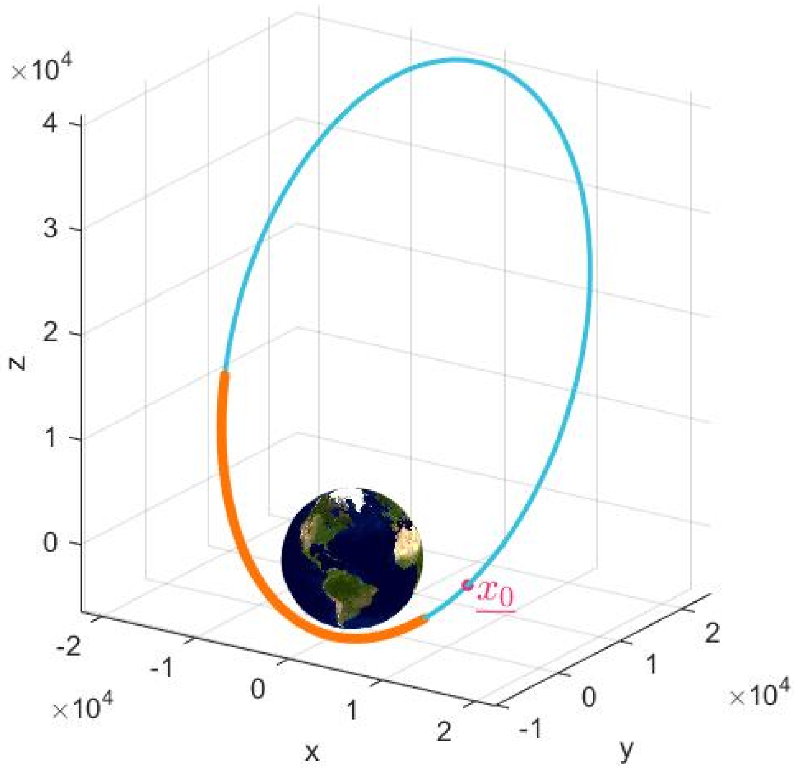

Figure 11: Orbit Representation

3.3 Section C

In this section we come back to the same time domain used in section A but this time we must consider the saturation of the reaction wheels.
We recall that

$$\underline{\dot{\omega_{s,th}}} = -Ja\,\left(Ja\,Ja^T\right)^{-1}\underline{T_c} \tag{3.1}$$

In order to account for this inside the code we implemented the following algorithm for each i-th wheel,

$$\begin{cases} \dot{\omega_{s,i}} = \dot{\omega_{s,th,i}} & , \quad |\dot{\omega_{s,th,i}}| < \dot{\omega_{s,max}} \\ \dot{\omega_{s,i}} = \dot{\omega_{s,max}} \cdot sign(\dot{\omega_{s,th,i}}) & , \quad |\dot{\omega_{s,th,i}}| \geq \dot{\omega_{s,max}} \end{cases} \tag{3.2}$$

If we compare the results obtained in this section with the ones in section A, we notice that the key difference is the response time of the system to the control applied.

In fact, if we consider the saturation of the wheels, we won't be able to apply instantaneously any torque needed but we are constrained inside an interval of values and this definitely slows the stabilization process down.
In other words, the S/C cannot reach its commanded state as quickly because it is only capable of changing its state at a maximum pace.

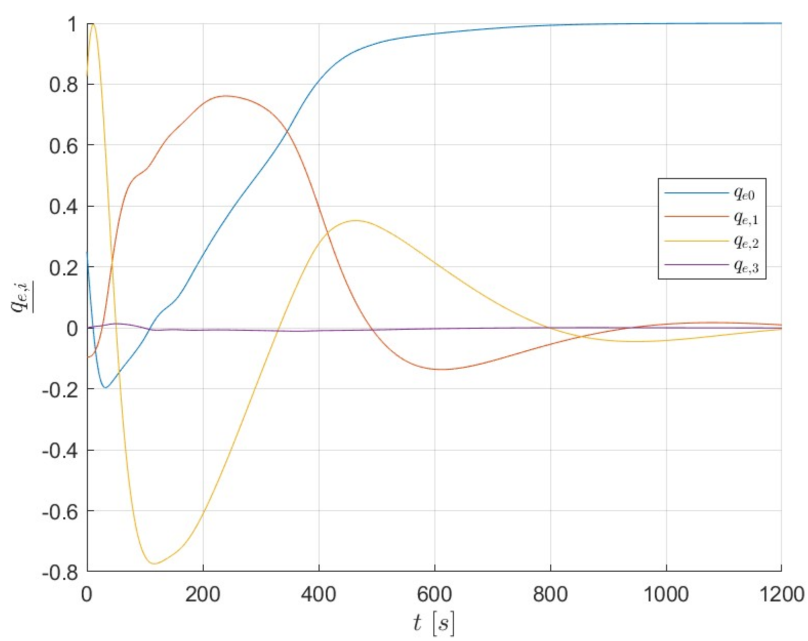

Figure 12: Error Quaternions

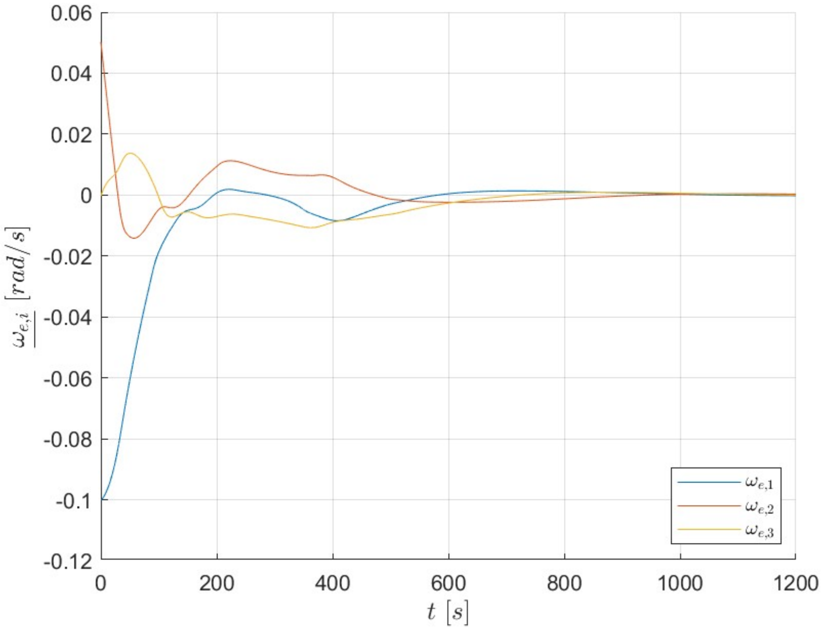

Figure 13: Error Angular Rate

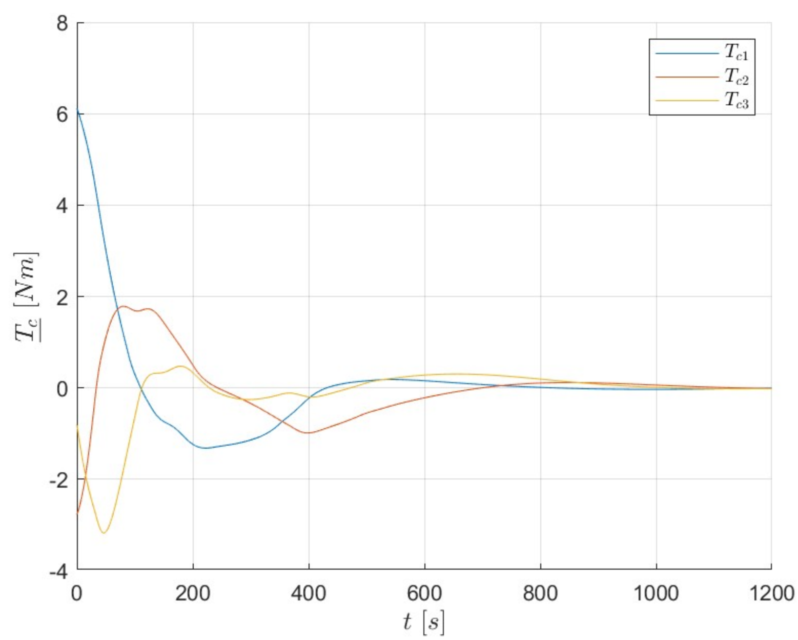

Figure 14: Commanded Torque

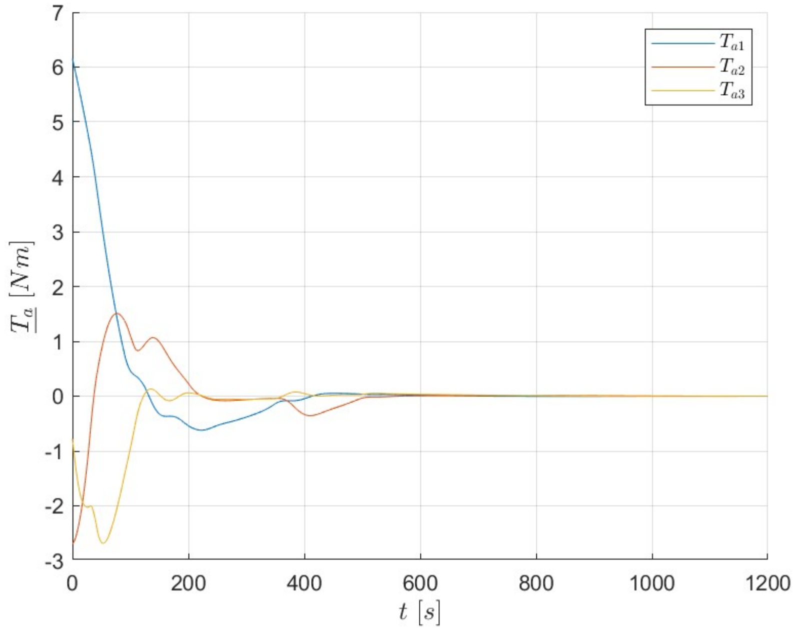

Figure 15: Applied Torque

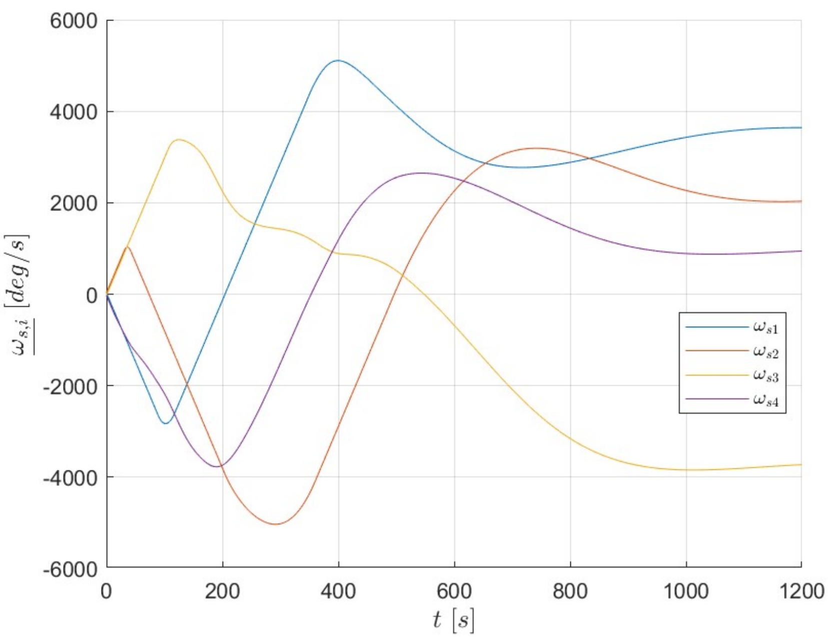

Figure 16: Rotor Angular Rates

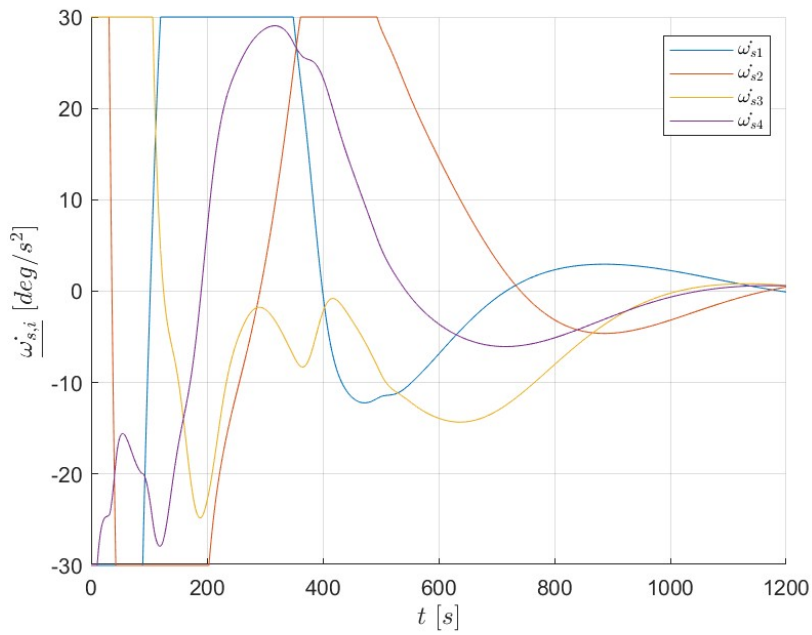

Figure 17: Rotor Angular Accelerations