Homework 2

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Contents

1 Evaluation of Disturbance Torques

The first step in order to apply an Attitude Control on the Bepi-Colombo spacecraft is to evaluate the disturbance torques it may be subjected to during its trajectory, such as: aero dynamical torques, gravity gradients and solar radiation pressure torques.
In particular, in our example we will consider the aerodynamic torque negligible and therefore we will focus on the last two instead.

We know that in the initial part of the trajectory (which we will call inertial pointing phase) the spacecraft should reach a constant attitude from its initial one.

$$ATT_{desired} = \begin{bmatrix} \theta_G \\ \psi_G \\ \varphi_G \end{bmatrix} = \begin{bmatrix} 40.6° \\ 0° \\ 0° \end{bmatrix} \tag{1.1}$$

$$ATT_{initial} = \begin{bmatrix} \theta_0 \\ \psi_0 \\ \varphi_0 \end{bmatrix} = \begin{bmatrix} 40.60083° \\ 0° \\ 0° \end{bmatrix} \tag{1.2}$$

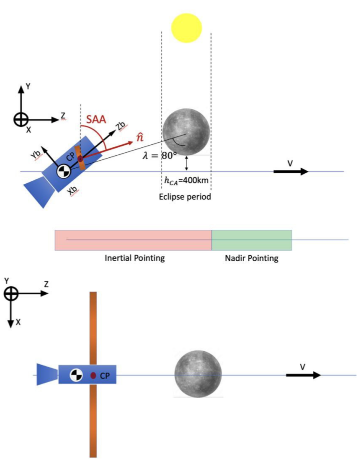


Figure 1: Geometry of the Problem

1.1 Gravity Gradient

The gravity gradient is modelled by the following equation,

$$G_x = \frac{3}{2}\frac{GM_{Mer}}{R_{BM}^3}(I_z - I_y)\sin(2\beta)\cos(\phi) \tag{1.3}$$

where, by applying some simple trigonometry, we find that $\beta = \lambda + \theta$.
Here, the first thing that we notice is that Gx is changing as the spacecraft moves along its

trajectory, so we will have to compute its value at each instant to perform the integration. Although at this stage we can't know the exact values of Gx we can compute its minimum value and its maximum value, which corresponds to calculating Gx at $t = t_0$ and at the time at which the spacecraft is in the middle of the eclipse. Note that the maximum value here discussed may not be the actual maximum one, but by a later verification we can confirm that it is.

$$Gx_{min} = \frac{3}{2}\frac{GM_{Mer}}{R^3_{BM0}}(I_z - I_y)\sin(2(\theta_0 + \lambda_0)) = -2.3620 \cdot 10^{-5}\, Nm \tag{1.4}$$

$$Gx_{max} = \frac{3}{2}\frac{GM_{Mer}}{(R_M + h_{ca})^3}(I_z - I_y)\sin(2\theta) = 5.198 \cdot 10^{-4}\, Nm \tag{1.5}$$

1.2 Solar Radiation Pressure Torque

Similarly to the gravity gradient, the Solar Radiation Pressure Force is also modelled by the following equation,

$$\vec{F_{srp}} = -\frac{\phi}{c}\left(\frac{1\,AU}{R_{Bepi}}\right)^2 A\hat{n}\cdot\hat{s}((1 - C_s)\hat{s} + 2C_s(\hat{n}\cdot\hat{s})\hat{n}) \tag{1.6}$$

As we can see the force applied by the solar pressure has two components: one on the $\hat{n}$ direction and one on the $\hat{s}$ direction. Therefore we can compute the total force applied using pythagora's theorem and we can also define the angle between F_{srp} and $F_{srp,s}$ as δ.

$$F_{srp} = \sqrt{F^2_{srp,s} + F^2_{srp,n}} \tag{1.7}$$

$$\delta = \arccos\left(-\frac{F_{srp,s}}{F_{srp}}\right) \tag{1.8}$$

Finally, we can compute T_{srp} as

$$\vec{T_{srp}} = \vec{F_{srp}} \times \vec{b} \tag{1.9}$$

$$T_{srp} = F_{srp} \cdot b \cdot \cos(\theta + \delta) \tag{1.10}$$

Again, this means that theoretically F_{srp} changes with time but, since in the first part of the problem $\theta \approx const$ and $\delta = const$, we can compute a first approximation of T_{srp} as

$$T_{srp} = F_{srp} \cdot b \cdot \cos(\theta_G + \delta) = 2.8574 \cdot 10^{-4}\, Nm \tag{1.11}$$

2 Inertial Pointing

Our objective in this part of the flight is to keep the the desired attitude while the spacecraft performs the fly-by of Mercury.

In order to do so, we can use only the reaction wheels by applying a control law on the dynamics of the system.
The control law that we will adopt is the PD Control Law (Proportional - Derivative), which implies the choice of two constants K_p, K_d which can be related by the equation for damping. At this point, we can define what our control torque is going to be and we can add it to the dynamical model of rotations about x-axis.
Note that in our example $\dot{\theta} > 0$ represents a negative rotation of the spacecraft around the x-axis. Hence, to correct this, we will consider T_d to be oriented in the positive x direction of rotation, while we will consider a positive value of T_c to be oriented in the negative one.

$$T_c = K_p(\theta_G - \theta) + K_d(\dot{\theta_G} - \dot{\theta}) \tag{2.1}$$

$$I_x\ddot{\theta} = \frac{-T_d + T_c}{Ix} \tag{2.2}$$

$$K_d = 2\,\xi\sqrt{K_p Ix} \tag{2.3}$$

Conveniently, we will impose the critical damping condition for a minimum response rate and therefore choose $\xi = 1$.
Now, we can define a state $s = (\theta, \dot{\theta}, \omega)$ and update it using matlab's ode45 function according to the following relation.

$$s = \begin{bmatrix} \theta \\ \dot{\theta} \\ \omega \end{bmatrix} \qquad \dot{s} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{\omega} \end{bmatrix} \tag{2.4}$$

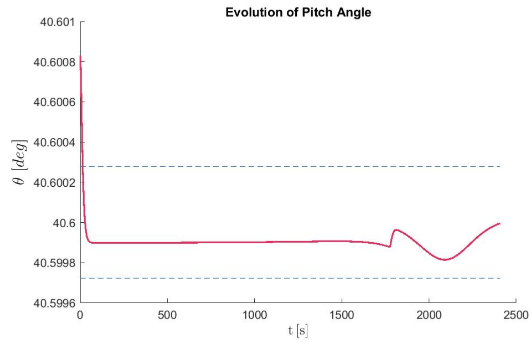
$$\begin{cases} \ddot{\theta} = \frac{-T_d + T_c}{I_x} \\ \dot{\omega} = \frac{Tc}{I_w} \end{cases} \tag{2.5}$$

Finally, we can choose a value of K_p which yields acceptable results for this first part.

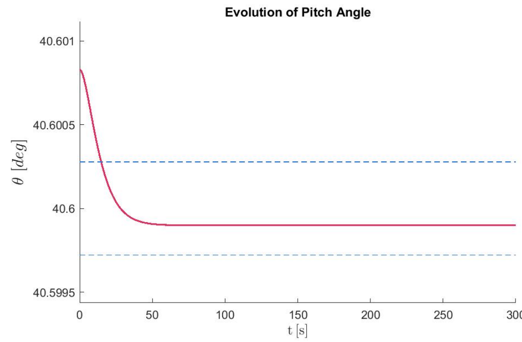
$$K_p = 160 \tag{2.6}$$

$$K_d = 2375 \tag{2.7}$$

By applying this procedure from our starting point we obtain an evolution of the attitude as follows.

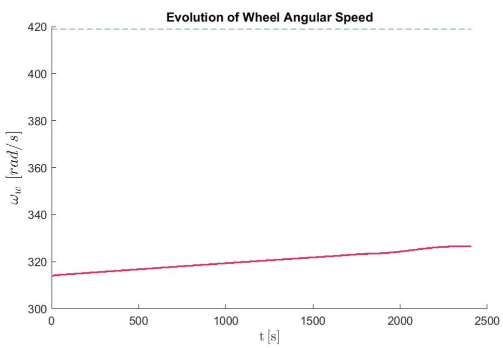


(a) Pitch angle over time

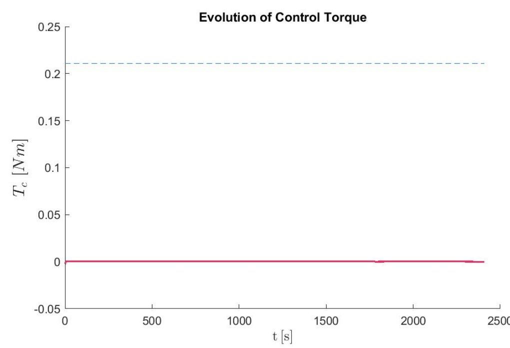


(b) Pitch angle over an initial portion of time

Figure 2: Pitch Angle

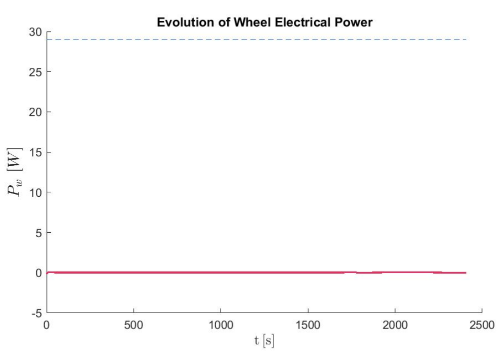


(a) Wheel Angular Velocity over time

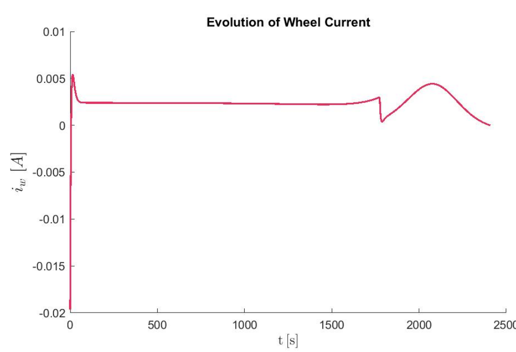


(b) Control Torque over time

Figure 3: Wheel Angular Velocity and Control Torque



(a) Wheel Electrical Power over time



(b) Wheel Current over time

Figure 4: Wheel Current and Electrical Power

3 Nadir Pointing

In the following part we are asked to change the attitude of the spacecraft in order to keep the Y_b axis aligned with the radial direction between Mercury and the spacecraft itself. Contrary to the previous part, here our desired state is not constant but it is changing in time. By considering $z(t)$ as the trajectory of the spacecraft we can write the following relations.

$$\begin{cases} z(t) = z0 + v_0 \cdot t & z_0 = \\ \theta_G = \arctan\left(\frac{R_M + h_{ca}}{z}\right) \\ \dot{\theta_G} = -\frac{v_0(R_M + h_{ca})}{(R_M + h_{ca})^2 + z^2} \end{cases} \tag{3.1}$$

Now, we could follow the same procedure as before but, if we do, we notice that the wheels are not sufficient to keep the nadir pointing inside the fixed range.

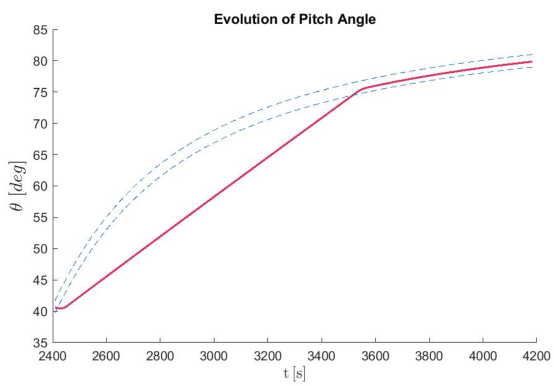
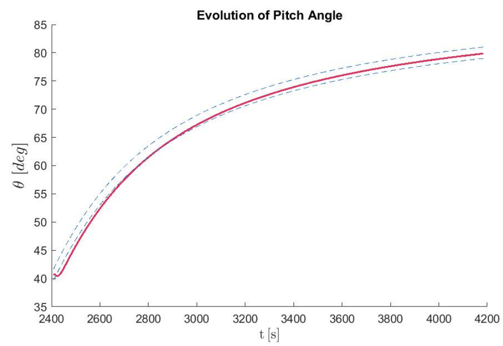


Figure 5: Failure of Reaction Wheels in Nadir Pointing

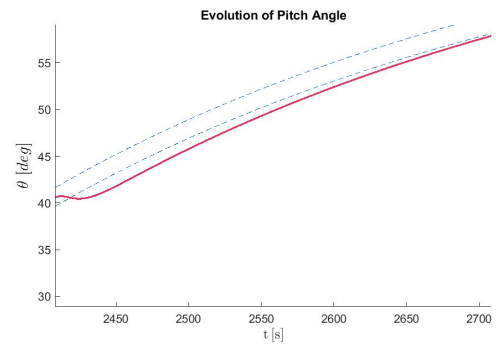
Therefore, to obtain a successful nadir pointing, we must use the thrusters. After numerous attempts at obtaining a stable result, one could notice that the most critical part of the attitude control is the initial phase, where a large increase of θ is required in a brief time. Hence, a good idea would be firing the thrusters to achieve this initial rotation and then, continue to control the attitude by using reaction wheels. In fact, this is exactly what we are going to do, in particular we choose to give an initial thrust of the duration of $t_{off} = 1\,s$ and these are the results. So, for $t < t_{off}$ we can write,

$$\begin{cases} \ddot{\theta} = \frac{T_{th}}{I_x} \\ \dot{\omega} = \frac{Tc}{I_w} \end{cases} \tag{3.2}$$

where T_{th} is the torque provided by the thrusters, which in our case is $20\,Nm$.
After t_{off} we can then proceed by integrating the previous model to obtain the following results.

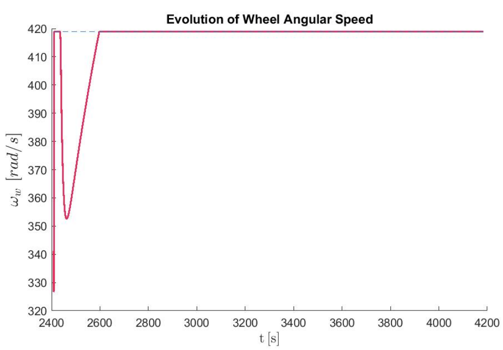


(a) Pitch angle over time

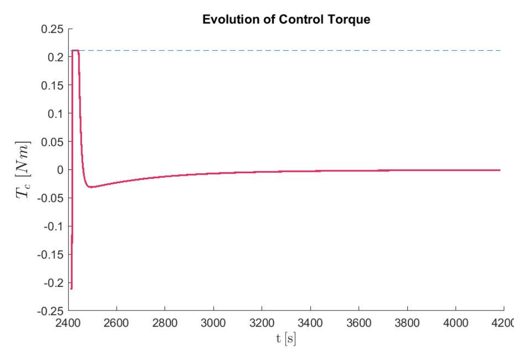


(b) Pitch angle over an initial portion of time

Figure 6: Pitch Angle

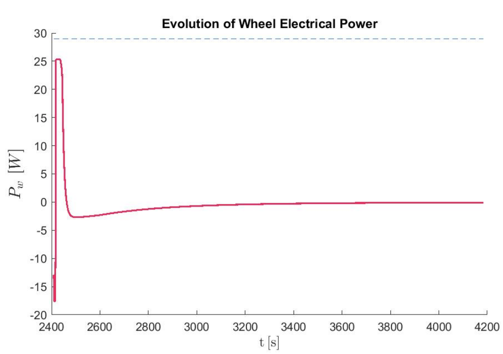


(a) Wheel Angular Velocity over time

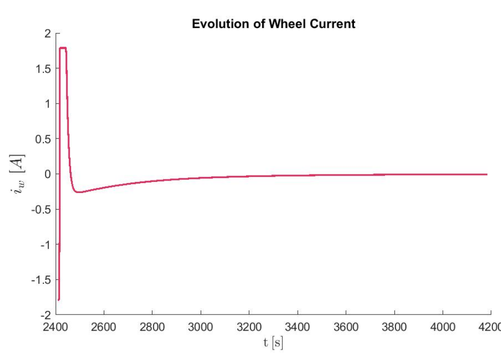


(b) Control Torque over time

Figure 7: Wheel Angular Velocity and Control Torque



(a) Wheel Electrical Power over time



(b) Wheel Current over time

Figure 8: Wheel Current and Electrical Power

Now, these results are by no means perfect, in fact trying to obtain decent results from the integration of our model was not trivial.

In the angular velocity of the wheels for example, there is the presence of a big spike right after the firing of thrusters which shouldn't be there, but as we can see from the graph the wheel would later reach saturation after about $2597.3\,s$ from t_0 or about $189\,s$ after the eclipse.