

### Homework 3

A spacecraft is equipped with an attitude control system composed of 4 reaction wheels, arranged in an **ortho-skew array**. The inertia matrix of the spacecraft is

$$J_c^{(B)} = \begin{bmatrix} 900 & 50 & -100 \\ 50 & 1100 & 150 \\ -100 & 150 & 1250 \end{bmatrix} \text{ Kg m}^2$$

Principal inertia moments of the rotors:

$$I_s^{(j)} = 1 \text{ Kg m}^2 \quad I_T^{(j)} = 0.5 \text{ Kg m}^2 \quad (j=1, \dots, 4)$$

Initial rotor speeds:

$$\omega_{s1}(0) = \omega_{s2}(0) = 0.5 \text{ sec}^{-1} \quad \omega_{s3}(0) = \omega_{s4}(0) = -0.5 \text{ sec}^{-1}$$

Initial attitude and angular velocity:

$$\underline{q}(0) = \begin{bmatrix} 0.1 & 0.3 & -0.5 \end{bmatrix}^T \quad q_0(0) = \sqrt{1 - |\underline{q}(0)|^2}$$

$$\underline{\omega}(0) = \begin{bmatrix} -0.1 & 0.05 & 0 \end{bmatrix}^T \text{ sec}^{-1}$$

The spacecraft travels a Molniya orbit, with period of 0.5 sidereal days,  $e = 0.73$ ,  $i = 63.4 \text{ deg}$ ,  $\Omega = 0$ ,  $\omega = -90 \text{ deg}$

The spacecraft is at the ascending node at the initial time

Objective: track the desired attitude i.e.

$$\hat{b}_{c1} = \hat{\theta} \quad \hat{b}_{c2} = -\hat{h} \quad \hat{b}_{c3} = -\hat{r}$$

For the purpose of obtaining the gain matrices  $A$  and  $B$ , use the following parameters for  $\xi$  and  $\omega_n$ :

$$\omega_n = 0.03 \text{ sec}^{-1} \quad \text{and} \quad \xi = 1$$

Consider the feedback law and the actuator dynamics.

(A) Portray the time evolution of

(i) error quaternion  $\underline{q}_e(t)$  and  $q_{eo}(t)$

(ii) error angular rate  $\underline{w}_e(t)$

(iii) commanded torque components  $\underline{T}_c(t)$

(iv) actual torque components  $\underline{T}_a(t)$

(v) rotor angular rates  $\{w_{s1}(t), \dots, w_{sf}(t)\}$

(vi) rotor angular accelerations  $\{\dot{w}_{s1}(t), \dots, \dot{w}_{sf}(t)\}$

in the time interval  $[0, 1200] \text{ sec}$ , and comment on  $\underline{T}_a$  and  $\underline{T}_c$

(B) Portray the time histories of (i), (ii), (iii) in an orbital period and comment on (possible) remarkable behaviors

(C) Portray the time evolution of (i) through (vi) in the time interval  $[0, 1200] \text{ sec}$ , with the inclusion of saturation of the devices, i.e.

$$|\dot{w}_{si}(t)| \leq 30 \text{ deg/sec}^2$$

and comment on the time histories obtained in (C) and (A)