Space Missions and Systems 2022: Home Test #1

A Venus orbiter, in the aerobraking phase, is tracked by a ground station located on Earth collecting range and range-rate observables. The spacecraft orbit is in the plane containing the Earth and Venus (2D problem). The reference frame is centered in the Venus's center of mass, and the Earth is placed along the x-axis with the coordinates $[X_{\oplus} = -38.2 \times 10^6\ km, Y_{\oplus} = 0\ km]$. The Earth and Venus are considered fixed in space and non-rotating. The spacecraft is subject to the monopole gravity of Venus only and to the drag acceleration due to the planet's thick atmosphere. Consider the acceleration due to atmospheric drag described by:

$$\vec{a}_{drag} = -\frac{1}{2}\rho(z) C_D \frac{A}{m} V^2\, \hat{V}$$

where $\rho(z)$ is the atmospheric density, z is the spacecraft altitude with respect to Venus's surface, $A = 40\ m^2$ is the spacecraft exposed area, $m = 2000\ kg$ is the spacecraft mass, $\vec{V}$ is the velocity of the spacecraft and C_D is the drag coefficient. Assume that the normal direction to the exposed area remains parallel to $\vec{V}$. The atmospheric density can be modeled through an exponential law:

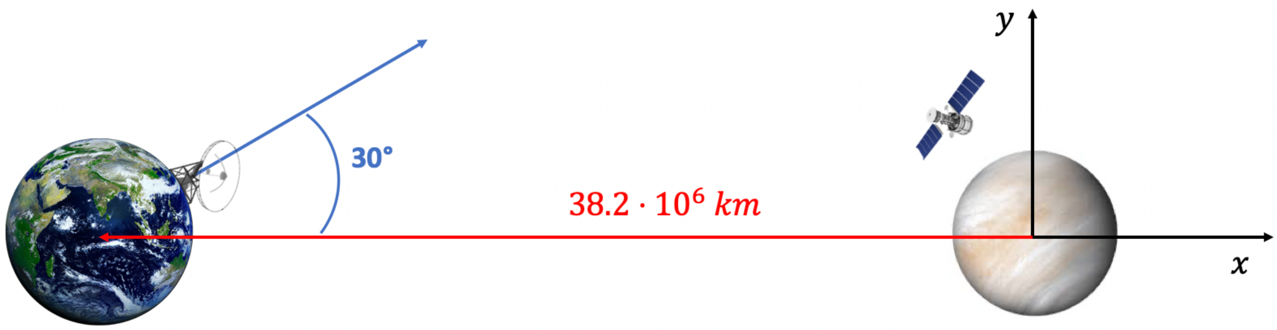
$$\rho(z) = \rho(z_0) e^{-\frac{z - z_0}{H}}$$

where $z_0 = 200\ km$, $\rho(z_0) = 1.64 \times 10^{-10} \frac{kg}{m^3}$ and $H = 14.7\ km$.

The a priori values of the state vector at the initial reference time $t_0 = 0$ is:

	State vector components	Uncertainty
x	-0.8 km	500 m
y	6419.4 km	500 m
v_x	-7.11389 km/s	7 cm/s
v_y	-0.24912 km/s	7 cm/s
GM_{Venus}	324860.3 km³/sec²	0.5 km³/s²
C_D	2.2	0.15

The a priori covariance matrix is assumed to be diagonal. The angle between the station position vector and the positive x-axis of the reference frame is +30.0 degrees (positive y-component). The figure below illustrates the details of the geometry. The ground station equipment provides instantaneous range and range-rate measurements with a putative accuracy of 1 m and 50 $\frac{\mu m}{s}$, respectively.



(1) Use the range and range-rate observables provided by the ground station to obtain an estimate of the six state vector's components and the associated uncertainties at the reference epoch.

(2) Which of the two types of observables is most valuable for the determination of the probe state? Justify your answer.

(3) Plot the evolution of the semimajor axis and eccentricity from t_0 to t_0 + 24 h.

(4) Plot the evolution of the altitude and its uncertainty (*) from t_0 to t_0 + 24 h.

(5) The spacecraft must perform a correction maneuver when the apocenter reaches an altitude below 400 km. At what time it is necessary to perform the first maneuver?

(*) For the uncertainty of derived quantities, see https://en.wikipedia.org/wiki/Propagation_of_uncertainty (section "non-linear combinations")

Additional information:

- The observables are provided in the "observables.txt": the first column contains the observation time (in seconds past t_0), the second column is the corresponding observed range value (in km), and the third column is the corresponding observed range-rate value (in km/s).
- Consider the Earth and Venus fixed in space and non-rotating
- Venus radius: 6052 km
- Earth radius: 6378 km

Upload on Google Classroom a working computer code printing and plotting the relevant results (MATLAB is recommended) and a **comprehensive and short** note (a pdf file) with the mathematical procedure, results, comments, and figures by **Monday 4 April 23:59**.

In the first page of your pdf indicate your **first name**, **last name**, and **student id** (a.k.a. "numero di matricola").

Tips on Matlab:

- Rather than ode45, use ode113, which excels in orbital dynamics problems (just replace ode45 with ode113 in your setup);
- For integration purposes, use at least RelTol=1.0E-13, AbsTol=1.0E-13;
- ALWAYS put labels and units on the plot axes!